UNIT-III

TORSION AND SPRINGS

PART-A [2-MARKS]

1) Write down the expression for power transmitted by a shaft. (Apr/May 2019)

$$P = 2\pi NT/60$$

2) Define helical springs. (Apr/May 2019)

A helical spring is a length of wire or bar wound into a helix. There are mainly two types of helical springs (i) close-coiled (ii) open-coiled

3) Give any two functions of spring. (Nov/Dec 2018)

- To absorb shock or impact loading as in carriage springs
- To store energy as in clock springs
- To apply forces to and to control motions as in brakes and clutches
- To measure forces as in spring balance
- To change the variations characteristics of a member as in flexible mounting of motors

4) Write the expression for polar modulus for a solid shaft and for a hollow shaft (Nov/Dec 2017) (Apr/May 2018)

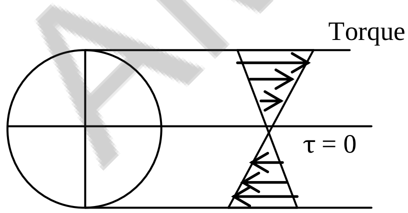
$$J = \frac{\pi}{32} d^4$$

$$J = \frac{\pi}{32} (D^4 - d^4)$$

5) Draw the shear stress distribution of a circular section due to torque. (May/June 2017)

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\tau = \frac{TR}{J}$$



6) What is meant by spring constant? (May/June 2017)

Spring constant (spring index) is the ratio of mean diameter of the spring to the diameter of the wire.

7) Define torsional rigidity. Nov/Dec-2016 , Nov/Dec-2014

From the torsional equation, we know that

$$\frac{T}{J} = \frac{C\theta}{\ell} \Rightarrow \theta = \frac{T\ell}{CJ}$$

Since, c,ℓ, and J are constant for A given shaft, θ(angle of twist) is directly proportional to T(Torque). The term CJ is known as torsional rigidity at it is represented by K.

8) What is a spring? Name the two important types of springs. (Nov/Dec 2016) (Nov/Dec 2017)

Spring is a device which is used to absorb energy by taking very large change in its form without permanent deformation and then release the same when it is required.

The important types of springs are

1) Torsion spring
2) Closed coiled helical spring
3) Open coil helical spring
4) Leaf spring

9) List out the stresses induced in the helical and carriage springs. (May/June 2016)

i) Direct shear stress
ii) Torsional shear stress
iii) Bending stress

10) Draw and discuss the shafts in series and parallel (May/June 2016)

In order to form a composite shaft sometimes two shaft are connected in series. In such case each shaft transmit, the same torque. The angle of twist is the sum of the angle of twist of two shaft connected in series.

When shaft are said to be parallel when the driving torque is applied at the junction of the shaft and the resisting torque is at the other end of the shaft. The angle of twist is same for each shaft.

11) The shearing stress in a solid shaft is not to exceed 40N/mm^2. When the torque transmitted is 2000N.m. Determine the minimum diameter of the shaft (Nov/Dec-2015)

Data

Shear stress(τ) = 40N/mm^2
Torque (T) = 2000N.m = 2000x10^3 N.mm
To find
(i) Minimum diameter of shaft

Solution:

$$T = \frac{\pi}{16} \times \tau \times d^3$$

$$\frac{2000 \times 10^3 \times 16}{\pi \times 40} = D^3$$

$$\boxed{D = 64\text{mm}}$$

12) What are the various types of springs? (Nov/Dec 2015)

1) Helical springs
(i) Closed coil helical springs
(ii) Open coil helical springs
2) Leaf springs
(i) Full-elliptic (ii) Semi-elliptic (iii) cantilever
3) Torsion springs
4) Circular springs
5) flat springs

13) What is meant by torsional stiffness? (Apr/May 2015)

It is the radio of Torque (T) to the angle of twist (θ)

Torsional stiffness(q) = $\frac{T}{\theta}$

14) What are the uses of helical springs? (Apr/May 2015)

1) Railway wagons
2) Cycle seating
3) Pistols
4) brakes

15) Differentiate open coiled and closed coiled helical springs. (Nov/Dec 2014) (Apr/May 2018)

	Open coil helical spring	**Closed coil helical spring**
1	Large gap between adjacent coils	Adjacent coils are very close to each other
2	Tensile and compressive loads can carry	Only tensile load can carry
3	Helix angle is considerable	Helix angle is negligible

16) Write down the expression for torque transmitted by hollow shaft.

$$T = \frac{\pi}{16} \times \tau \left[\frac{D^4 - d^4}{D} \right]$$

Where, T – Torque in N mm
τ - shear stress in N/mm²
D – outer diameter in mm
d – inner diameter in mm

17) Calculate the maximum torque that a shaft of 125 mm diameter can transmit, if the maximum angle of twist is 1° in a length of 1.5m. Take C = 70x10^3 N/mm^2.

Given data:

Diameter, D = 125mm

Angle of twist, Q = $1° \times \frac{\pi}{180} = 0.017$

Length, ℓ = 1.5m = 1500mm

Modulus of rigidity, C = $70x10^3$ N/mm^2

To find

Maximum Torque, T_{max}

Solution: Torsional equation

$$\frac{T}{J} = \frac{C\theta}{\ell}$$

$$T = \frac{JC\theta}{\ell}$$

$$T = \frac{\frac{\pi}{32}\left[D^4\right]}{\ell} \times C\theta$$

$$= \frac{\frac{\pi}{32}\left[125^4\right]}{1500} \times 70 \times 10^3 \times 0.017$$

T = T_{max} = 19.01 x 10^6 N/mm

18) The stiffness of spring is 10N/mm. What is the axial deformation in the spring when a load of 50N is acting?

Given:

K = $10N/mm^2$

W = 50N

$$k = \frac{w}{\delta} \Rightarrow \delta = \frac{w}{k} = \frac{50}{10} = 5mm$$

19) A helical spring is made of 4mm steel wire with a mean radius of 25mm and number of turns of coil 15. What will be deflection of the spring under a load of 6N. Take C = 80 x 10^3 N/mm^2

Given:

d = 4mm
R = 25mm
n = 15
w = 6N
c = 80 x 10^3 N/mm^2

Solution:

Axial deformation, $\delta = \frac{64wR^3n}{cd^4}$

$$\delta = \frac{64 \times 6 \times 25^3 \times 15}{80 \times 10^3 \times 4^4}$$

$$\delta = 4.39mm$$

20) Define Torsion / Twisting moment (Nov/Dec 2018)

When a pair of forces of equal magnitude but opposite directions acting on body, it tends to twist the body. It is known as twisting moment or torsion moment or simply as torque. Torque is equal to the product of the force applied and the distance between the point of application of the force and the axis of the shaft.

21) What are the assumptions made in Tortion equation

- The material of the shaft is homogeneous, perfectly elastic and obeys Hooke's law.
- Twist is uniform along the length of the shaft
- The stress does not exceed the limit of proportionality
- The shaft circular in section remains circular after loading
- Strain and deformations are small.

22) Define polar modulus

It is the ratio between polar moment of inertia and radius of the shaft = J/R polar moment of inertia = J Radius = R

23) Why hollow circular shafts are preferred when compared to solid circular shafts?

The torque transmitted by the hollow shaft is greater than the solid shaft.

For same material, length and given torque, the weight of the hollow shaft will be less compared to solid shaft.

24) Write torsion equation

$T/J = C\theta/L = q/R$

T – Torque
θ – angle of twist in radians
J- Modulus of rigidity
L- Length
Q – shear stress
R- Radius

25) What is spring index(C)?

The ratio of mean or pitch diameter to the diameter of wire for the spring is called the spring index.

26) What is solid length?

The length of a spring under the maximum compression is called its solid length. It is the product of total number of coils and the diameter of wire.

$L_s = n_t \times d$ where, n_t = total number of coils.

27) Define spring rate(stiffness).

The spring stiffness or spring constant is defined as the required per unit deflection of the spring, $K = W/\delta$

Where W = load and δ = Deflection

Part-B

1) Two shafts of the same material and same length are subjected to the same torque. If the first shaft is of a solid circular section and the second shaft is of a hollow circular section whose internal diameter is 2/3 of the outside diameter and the maximum shear stress developed in each shaft is the same, compare the weights of the two shafts. (Apr/May 2019)

Solution. Let
D_S = Diameter of the solid shaft,
D_H = External diameter of the hollow shaft,
d_H = Internal diameter of the hollow shaft, and
τ = Maximum shear stress developed.

$$d_H = \frac{2}{3} D_H \qquad (Given)$$

The torque transmitted by the solid shaft,

$$T_s = \tau \cdot \frac{\pi}{16} D_S^3$$

and, the torque transmitted by the hollow shaft,

$$T_H = \tau \cdot \frac{\pi}{16}\left[\frac{D_H^4 - d_H^4}{D_H}\right] = \tau \cdot \frac{\pi}{16}\left[\frac{D_H^4 - (2/3\,D_H)^4}{D_H}\right]$$

$$= \tau \cdot \frac{\pi}{16}\left[\frac{D_H^4 - \left(\frac{16}{81} D_H^4\right)}{D_H}\right] = \tau \cdot \frac{\pi}{16} \times \frac{65}{81} D_H^3$$

Since both the torques are equal, therefore equating (*i*) and (*ii*), we get

$$T_S = T_H$$

$$\tau \cdot \frac{\pi}{16} D_S^3 = \tau \cdot \frac{\pi}{16} \cdot \frac{65}{81} D_H^3$$

$\therefore$

$$D_H^3 = 1{\cdot}246\, D_S^3, \text{or,}\ \ D_H = 1{\cdot}08\, D_S$$

We know that,

$$\frac{\text{Weight of solid shaft}}{\text{Weight of hollow shaft}} = \frac{W_S}{W_H}$$

$$= \frac{A_S \times l_S \times w_S}{A_H \times l_H \times w_H} = \frac{A_S}{A_H}$$

$$\left[\begin{array}{l} \therefore\ l_S = l_H \\ w_S = w_H \text{ where, } w \text{ stands for weight density} \end{array}\right]$$

$$= \frac{\frac{\pi}{4} D_S^2}{\frac{\pi}{4}(D_H^2 - d_H^2)} = \frac{D_S^2}{[D_H^2 - \left(\frac{2}{3} D_H\right)^2]} = \frac{D_S^2}{D_H^2\left(1 - \frac{4}{9}\right)}$$

$$= \frac{D_S^2}{\frac{5}{9} \times (1.08\, D_S)^2} = \frac{\mathbf{1.543}}{\mathbf{1}} \quad \textbf{(Ans.)}$$

2) A closely coiled helical spring of mean diameter 20cm is made of 3cm diameter rod and has 16 turns. A weight of 3kN is dropped on this spring. Find the height by which the should be dropped before striking the spring so that the spring may be compressed by 18cm. Take C=8 x 10^4 N/mm^2 (Apr/May 2019) (Nov/Dec 2015)

D = 20cm = 200mm

R = $\frac{D}{2}$ = 100mm

d = 3cm = 30mm

n=16

W = 3kN = 3000N

δ = 18cm = 180mm

C = 8x10^4 N/mm^2

h = Height through which the weight W is dropped

W = Gradually applied load which produces the compression of spring equal to 180mm

$$\delta = \frac{64WR^3n}{Cd^4} = \frac{64xWx100^3x16}{8x10^4x30^4}$$

W = 11390N

Work done by falling weight on spring = W(h+δ)=3000(h+180) $\rightarrow 1$

Energy stored in the spring = $\frac{1}{2}$W x δ = $\frac{1}{2}$x11390x180=1025100Nmm $\rightarrow 2$

Equate 1 & 2, we get

3000(h+180) = 1025100

h= 161.7mm

3) A hollow shaft is to transmit 300kW power at 80rpm. If the shear stress is not to exceed 60 N/mm² and the internal diameter is 0.6 of the external diameter, find the external and internal diameters assuming that the maximum torque is 1.4 times the mean. (Nov/Dec 2017) (Nov/Dec 2018)

P = 300kW = 300000W

N = 80rpm

τ = 60N/mm^2

D = 0.6d

$T_{max} = 1.4T_{mean}$

$$P = \frac{2\pi N T_{mean}}{60} \Rightarrow T_{mean} = 35809.8Nm$$

$$T_{max} = 1.4T_{mean} \Rightarrow T_{max} = 50133.7Nm = 50133700Nmm$$

$$T_{max} = \frac{\pi}{16} x \tau x \left[\frac{D^4 - d^4}{D}\right]$$

$$50133700 = \frac{\pi}{16} x 60 x \left[\frac{D^4 - (0.6D)^4}{D}\right]$$

D = 169.2mm $\approx$ 170mm

d = 0.6 x 170 = 102mm

4) A solid cylindrical shaft is to transmit 300kW power at 100rpm. (a) If the shear stress is not to exceed , find its diameter. (b) What percent saving in weight would be obtained if this shaft is replaced by a hollow one whose internal diameter equals to 0.6 of the external diameter, the length, the material and maximum shear stress being the same. (Apr/May 2018)

a)

$$P = \frac{2\pi NT}{60}$$

$$300000 = \frac{2\pi x 100 x T}{60} \Rightarrow T = 28647.8\text{Nm} = 28647800\text{Nmm}$$

$$T = \frac{\pi}{16} \text{ x } \tau \text{ x } D^3$$

$$28647800 = \frac{\pi}{16} \text{ x } 80 \text{ x } D^3 \Rightarrow D = 121.8 \approx 122\text{mm}$$

b)

$D_i = 0.6\ D_o$

Torque transmitted by solid shaft is equal to torque transmitted by hollow shaft

$$T = \frac{\pi}{16} \text{ x } \tau \text{ x } \frac{D_o^4 - D_i^4}{D_o}$$

$$T = \frac{\pi}{16} \text{ x } \tau \text{ x } \frac{D_o^4 - (0.6D_o)^4}{D_o}$$

$$28647800 = \frac{\pi}{16} \text{ x } 80 \text{ x } \frac{D_o^4 - (0.6D_o)^4}{D_o}$$

$D_o = 127.6\text{mm} \approx 128\text{mm}$

$D_i = 0.6\ D_o = 0.6 \text{ x } 128 = 76.8\text{mm}$

W_s = Weight of solid shaft = Weight density x Area of solid shaft x Length

W_h = Weight of hollow shaft = Weight density x Area of hollow shaft x Length

$$\text{Percentage saving in weight} = \frac{W_s - W_h}{W_s} \text{ x } 100$$

(Both of same material and same length weight density and length get cancelled)

$$\text{Percentage saving in weight} = \frac{D^2 - (D_o^2 - D_i^2)}{D^2} \text{ x } 100$$

$$= \frac{122^2 - (128^2 - 76.8^2)}{122^2} \text{ x } 100 = 29.55\%$$

5) A closely coiled helical spring made of 10mm diameter steel wire has 15 coils of 100mm mean diameter. The spring is subjected to an axial load of 100N. Calculate (i) The maximum shear stress induced (ii)The deflection and (iii) Stiffness of the spring Take C=8.16 x 10^4 N/mm^2 (Nov/Dec 2017)

d = 10mm

n = 15

D = 100mm

$R = \frac{D}{2} = 50mm$

W = 100N

$C = 8.16 \times 10^4\ N/mm^2$

i) Maximum shear stress induced

$$\tau = \frac{16WR}{\pi d^3} = \frac{16x100x50}{\pi x10^3} = 24.46N/mm^2$$

ii) Deflection (δ)

$$\delta = \frac{64WR^3n}{Cd^4} = \frac{64x100x50^3x15}{8.16\ x\ 10^4x\ 10^4} = 14.7mm$$

iii) Stiffness of the spring (k):

$$k = \frac{W}{\delta} = \frac{100}{14.7} = 6.802\ N/mm$$

6) A hollow shaft, having an inside diameter 60% of its outer diameter, is to replace a solid shaft transmitting in the same power at the same speed. Calculate percentage saving in material, if the material to be is also the same. (May 2017)

Given:

Let D_0 = outer diameter of the hollow shaft

D_i = Inside diameter of the hollow shaft

= 60% of D_0 = $\frac{60}{100} \times D_0 = 0.6D_0$

D = Diameter of the solid shaft

P = power transmitted hollow (or) solid shaft

N = speed of each shaft

τ = maximum shear stress induced in each shaft since material of both is same

$$p = \frac{2\pi NT}{60}$$

$$T = \frac{p \times 60}{2\pi N} = \text{constant}$$

Torque transmitted by solid shaft is the same as the torque transmitted by hollow shaft

$$T = \frac{\pi}{16}\tau D^3 \quad \text{(Solid shaft)} \quad \rightarrow (1)$$

Torque transmitted by hollow shaft

$$T = \frac{\pi}{16}\tau\left[\frac{D_0{}^4 - D_i{}^4}{D_0}\right] = \frac{\pi}{16}\tau\left[\frac{D_0{}^4 - (0.6D_0)^4}{D_0}\right]$$

$$= \frac{\pi}{16}\tau\left[\frac{D_0{}^4 - 0.1296D_0{}^4}{D_0}\right] = \frac{\pi}{16}\tau \times 0.8704 D_0{}^3 \rightarrow (2)$$

Torque transmitted is same, hence equating equations (1) and (2)

$$\frac{\pi}{16}\tau D^3 = \frac{\pi}{16}\tau \times 0.8704 D_0{}^3$$

$$D = (0.8704)^{1/3} D_0 = 0.9548 D_0$$

Area of solid shaft = $\frac{\pi}{4}D^2 = \frac{\pi}{4}(0.9548D_0)^2 = 0.716D_0{}^2$

Area of hollow shaft = $\frac{\pi}{4}\left[D_0^2 - D_i^2\right]$

$$= \frac{\pi}{4}\left[D_0{}^2 - (0.6D_0)^2\right]$$
$$= \frac{\pi}{4}\left[D_0{}^2 - 0.36D_0{}^2\right]$$
$$= \frac{\pi}{4} \times 0.64D_0{}^2 = 0.502D_0{}^2$$

For the shaft of same material, the weight of the shaft is proportional to the areas.

$\therefore$ saving in material = saving in area $= \frac{\text{Area of solid shaft} - \text{Area of hollow shaft}}{\text{Area of solid shaft}}$

$$= \frac{0.716D_0{}^2 - 0.502D_0{}^2}{0.716D_0{}^2}$$
$$= 0.2988$$

$\therefore$ percentage saving in material = 0.2988 x 100 = 29.88

7) Derive a relation for deflection of a closely coiled helical spring subjected to an axial compressive load 'w'.
(May 2017)

Expression for deflection of spring

Length of one coil = πD (or) $2\pi R$

$\therefore$ Total length of the wire = length of one coil x No of coils

$\ell = 2\pi R \times n$

Strain energy stored by the spring

$$U = \frac{\tau^2}{4C}.\text{volume} = \frac{\tau^2}{4C}\text{volume}$$

$$= \left[\frac{16W.R}{\pi d^3}\right]^2 \times \frac{1}{4C} \times \left[\frac{\pi}{4}d^2 \times 2\pi R.n\right]$$

$$\left[\tau = \frac{16WR}{\pi d^3} \text{ and volume} = \frac{\pi}{4}d^2 \times \text{Total length of wire}\right]$$

$$= \frac{32W^2R^2}{Cd^4}.R.n = \frac{32W^2R^3n}{cd^4}$$

Work done on the spring = Average load x Deflection

$$= \frac{1}{2}W \times \delta$$

Equating the work done on spring to energy stored

$$\frac{1}{2}W.\delta = \frac{32W^2R^3.n}{cd^4}$$

$$\delta = \frac{64WR^3.n}{cd^4}$$

8) A solid shaft has to transmit the power 105kw at 2000 r.p.m. The maximum torque transmitted in each revolution exceeds the mean by 36% . Find the suitable diameter of the shaft, if the shear stress is not to exceed 75N/mm^2 and maximum angle of twist is 1.5 in a length of 3.30m and G = 0.80 x 10^5 N/mm^2.

(AU Nov 2016 – 8Marks)

Given data

Power = p =105 kw

Speed = N = 2000 rpm

$T_{max} = 1.36\ T_{mean}$

Shear stress (τ) = 75 N/mm^2

Angle of twist (Q) = 1.5° $= 1.5 \times \frac{\pi}{180} = 0.026$ radians

Length = L = 3.30m = 3300mm

$G = 0.80 \times 10^5$ N/mm^2

To find

(i) diameter of the shaft

Solution:

We know that

$$\text{Power (p)} = \frac{2\pi NT}{60}$$

$$\text{Torque (T)} = \frac{P \times 60}{2 \times 3.14 \times 2000}$$

T = 0.5015 kN-m

$$\boxed{T = 501.59\,\text{Nm}}$$

$T_{max} = 1.36 \times 501.59$

$T_{max} = 682.16$ Nm

Considering shear stress (z)

$$\text{Torque, } T = \frac{\pi}{16} \times \tau \times D^3$$

$$D^3 = \frac{T \times 16}{\pi \times \tau} = \frac{682.16 \times 10^3 \times 16}{3.14 \times 75}$$

$$D = 35.92\,\text{mm}$$

Considering angle of twist (θ)

$$\frac{T_{max}}{J} = \frac{c\theta}{\ell} \quad \text{where } J = \frac{\pi}{32} \times D^4$$

$$\frac{682.16 \times 10^3}{\frac{\pi}{32} \times D^4} = \frac{0.8 \times 10^5 \times 0.026}{3300}$$

$$\frac{682.16 \times 10^3 \times 3300 \times 32}{3.14 \times 0.8 \times 10^5 \times 0.026} = D^4$$

D = 57.62

D = 58mm

From above two cases we find that suitable diameter for the shaft is 58mm(ie, greater of the two values)

9) A laminated spring carries a central load of 5200 N and it is made of 'n' number of plates, 80mm wide, 7 mm thickness and length 500 mm . Find the number of plates is the maximum deflection is 10mm. Let $E = 2.0 \times 10^5$ N/mm^2 (AU Nov/Dec -2016 -8 marks)

Given data

Load = w = 5200 N

Width of plate = b =80 mm

Thickness of plate = t = 7 mm

Length of plate = ℓ = 500 mm

Maximum deflection = δ = 10 mm

`young modulus = E = 2.0 x 10^5 N/mm^2

To find:

(i) number of plates = (n)

We know that deflection equation for semi-elliptical spring is

$$\delta = \frac{3}{8}\frac{w\ell^3}{nEbt^3}$$

$$n = \frac{3}{8}\frac{w\ell^3}{Ebt^3\delta}$$

$$n = \frac{3}{8}\frac{5000\times500^3}{2.0\times10^5\times80\times7^3\times10}$$

n = 4.27

n = 5 number of plates = 5

10) A closed coiled helical spring is to be made out of 5mm diameter wire, 2m long, so that it deflects by 20mm under an axial load of 50 N. Determine mean diameter of the coil Take c = 8.1 x 10^4 N/mm^2 (Nov/Dec 2016) 16 Marks

Given

Diameter of wire = d = 5mm

Length (ℓ) = 2m = 2000 mm

Deflect (δ) = 20 mm

Axial load (w) = 50 N

C = 8.1 x 10^4 N/mm^2

To find:

(i) mean diameter of the coil (D)

Solution:

Deflection (δ) = $\frac{64wR^3n}{cd^4}(\text{or})\frac{8wD^3n}{cd^4}$

Length of spring (ℓ) πD_n

$$n = \frac{\ell}{\pi D} = \frac{2000}{3.14\times D}$$

$$\delta = \frac{8wD^3n}{cd^4}$$

$$\frac{8cd^4}{8w_n} = D^3$$

$$\frac{20\times 8.1\times 10^4\times(5)^4\times 3.14\,D}{8\times 50\times 2000} = D^3$$

$$\boxed{D = 63.04\text{mm}}$$

11) A solid circular shaft 200 mm in diameter is to be replaced by a hollow shaft the ratio of external diameter to internal diameter being 5:3 . Determine the size of the hollow shaft if max shear stress is to be the same as that of a solid shaft. Also find the percentage saving in mass (March/June 2016) 16 Marks

Solid shaft dia (D) = 200 mm

Hollow shaft internal dia(d) = ?

Hollow shaft external dia(D_1) = ?

$$D_1/d = 5/3$$

$$d = 3/5\,D_1 = 0.6D_1$$

Torque transmitted by solid shaft (T) = $\pi/16\times\tau\times D^3$

$$\boxed{T = 1570796.32\,\tau}$$

Torque transmitted by solid hollow shaft (T) = $\pi/16\times\tau\times\left(\frac{D_1^n - d^n}{D_1}\right)$

D = $0.6D_1$

$$T = \frac{\pi}{16} \times \tau \times \left[\frac{D_1^4 - (0.6D_1)^4}{D_1} \right]$$

$$= \frac{\pi}{16} \times \tau \times \frac{D_1^4}{D_1} \left[1 - (0.6)^4 \right]$$

$$= \frac{\pi}{16} \times \tau \times D_1^3 \times 0.8704$$

$$\boxed{T = 0.1709\,\tau D_1^3} \qquad \rightarrow (2)$$

Equate (1) & (2) , we get

$1570796.32 = 0.1709\ D_1^3$

`$D_1^3 = 9191176.43$

$$\boxed{D_1 = 209.47\,\text{mm} \approx 210\text{mm}}$$

% Saving in weight = $\dfrac{\text{weight of solid shaft} - \text{weight of hollow shaft}}{\text{weight of solid shaft}}$

Weight of solid shaft = A x ρ x ℓ

$$= \frac{\pi}{4}\left(D_1^2 - d^2\right)\rho^{\ell}$$

$$= \frac{\pi}{4}\left(210^2 - 126^2\right)\rho^{\ell}$$

$$= 31415.9\rho^{\ell}$$

% savings in weight = $\dfrac{31415.93\rho^{\ell} - 22167.08\rho^{\ell}}{31415.93\rho^{\ell}} \times 100$

$$\boxed{\%\,\text{Savings in weight} = 29.44\%}$$

12) A closely coiled helical spring made from round steel rod is required to carry a load of 1000 N for a stress of 400MN/m^2 , the spring stiffness being 20N/mm^2 the dia of the helix is 100mm and G for the material is 80GN/m^2 . Calculate (1) the diameter of the wire and and (2) the number of turns required for the spring (8) (May/June 2016)

W = 1000 N

K = 20N/mm

D = 100 mm

τ = 400 MN/m^2

= 400 N/mm^2

d = ? C = 80 x 10^9 x 10^{-6} N/mm^2

n =? = 80 x 10^3 N/mm^2

$$\tau = \frac{16wR}{\pi d^3}$$

$$d^3 = \frac{16wR}{\pi\tau} = \frac{16 \times 1000 \times \left(100/2\right)}{\pi \times 400}$$

$$d^3 = 636.62$$

$$\boxed{d = 8.60\,\text{mm}}$$

$$k = \frac{cd^4}{64R^3n}$$

$$n = \frac{cd^4}{64R^3k} = \frac{80 \times 10^3 \times 8.6^4}{64 \times \left(\frac{100^3}{2}\right) \times 20}$$

$$\boxed{n = 2.73 \approx 3}$$

13) A spiral spring is made of 10mm diameters wire has to close coils, each 100mm mean diameter. Find the axial load the spring will carry if he stress is not exceed 200N/mm^2 Also determine the extension of the spring. Take G = 0.8 x 10^5 N/mm^2 (May/June-2016) 8 Marks

d = 10 mm c (or) G = 0.8 x 10^5 N/mm^2

n = 20

D = 100 mm $\Longrightarrow$ R = D/2 = 50 mm

τ = 200 N/mm^2

W = ?

δ = ?

$$\tau = \frac{16WR}{\pi d^3}$$

$$W = \frac{\tau \times \pi \times d^3}{16R} = \frac{200 \times \pi \times 10^3}{16 \times 50} = 785.40\text{N}$$

$$\delta = \frac{64WR^3n}{cd^4} = \frac{64 \times 785.4 \times 50^3 \times 20}{0.8 \times 10^5 \times 10^4}$$

$$\boxed{\delta = 157.08\,\text{mm}}$$

14) A hollow shaft of external dia 120 mm transmit 300 kw power at 200 rpm. Determine the maximum internal dia. If the max stress in the shaft is not to exceed 60 N/mm^2 (Nov/Dec -2015) 16 Marks

D = 120 mm P = 300 kw N = 200 rpm d = ?

τ = 60 N/mm^2

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{P \times 60}{2\pi N} = \frac{300 \times 10^3 \times 60}{2\pi \times 200} = 14323.914\,\text{Nm}$$

$$T = \frac{\pi}{16} = \left(\frac{D^4 - d^4}{D}\right)$$

$$14323.94 \times 10^3 = \frac{\pi}{16} \times 60 \times \left[\frac{120^4 - d^4}{120}\right]$$

$$\frac{14323.94 \times 10^3 \times 16 \times 120}{\pi \times 60} = 120^4 - d^4$$

$$145902454.8 = 120^4 - d^4$$

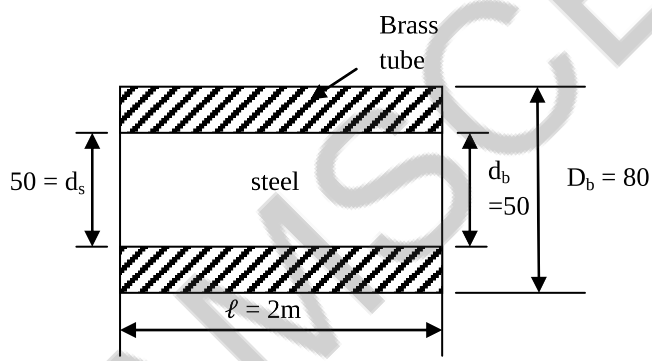
$$d^4 = 61457545.24$$

$$\boxed{d = 88.54\text{mm} \approx 89\text{mm}}$$

15) A brass tube of external dia. 80mm and internal dia 50mm is closely fixed to a steel rod of 50mm dia to form a composite shaft. If a torque of 10 KNm is to be resisted by this shaft, find the max stresses developed in each material and the angle of twist in 2m length. Take modulus of rigidity of brass and steel as 40×10^3 N/mm^2 respectively. (Apr/May 2015) 16 Marks

D_b = 80 mm d_b = 50 mm d_s = 50mm T = 10kNm θ =

$C_b = 40 \times 10^3$ N/mm^2 $c_s = 80 \times 10^3$ N/mm^2 ℓ = 2 m



$$\frac{T_S}{J_S} = \frac{\tau_S}{r_s}$$

$$\tau_S = \frac{T_S}{J_S} \times r_S$$

$$= \frac{10 \times 10^6}{\frac{\pi}{32} \times 50^4} \times \left(\frac{52}{2}\right)$$

$$\tau_S = \frac{10 \times 10^6 \times 25}{613592.31} = 407.44\,\text{N/mm}^2$$

$$\boxed{\tau_S = 407.44\,\text{N/mm}^2}$$

$$\frac{T_b}{J_b} = \frac{\tau_b}{R_b}$$

$$\tau_b = \frac{T_b}{J_b} \times R_b$$

$$= \frac{10 \times 10^6}{\frac{\pi}{32} \times (80^4 - 50^4)} \times \left(\frac{80}{2}\right)$$

$$\tau_S = \frac{10 \times 10^6 \times 40}{3407646.28} = 117.38\,\text{N/mm}^2$$

$$\boxed{\tau_S = 117.38\,\text{N/mm}^2}$$

$$\theta = \frac{T_S \ell_S}{C_S J_S}$$

$$= \frac{10 \times 10^6 \times 2000}{80 \times 10^3 \times \pi/32 \times 50^4} = \frac{10 \times 10^6 \times 2000}{4.9087 \times 10^{10}}$$

$$= 0.407\,\text{rod}$$

$$\theta = 0.407 \times \left(\frac{180}{\pi}\right)$$

$$\boxed{\theta = 23.34}$$

$$\boxed{\theta = \theta_s = \theta_b = 23.34^\circ}$$

16) A close-coiled helical spring is to have a stiffness of 900 N/m in compression, with a max. Load of 45N and a max. Shearing stress of 120N/mm^2. The solid length of the spring (i.e coils touching) is 45 mm. Find

(i) the wire dia
(ii) the mean coil radius
(iii) the number of coils. Take c = 0.4 x 10^5 N/mm^2 (Apr/May 2015) 16 Marks
k = 900 N/m w = 45 N τ = 120N/mm^2

(i) d

$$\delta = \frac{64WR^3n}{cd^4}$$

$$k = W/s = \frac{cd^4}{64R^3n}$$

$$0.9 = \frac{0.4\times10^5\times d^4}{64R^3n}$$

$$d^4 = \left(\frac{0.9\times64}{0.4\times10^5}\right)R^3n \qquad \rightarrow(1)$$

$$\tau = \frac{16WR}{\pi d^3}$$

$$120 = \frac{16\times45\times R}{\pi d^3}$$

$$R = \frac{120\pi d^3}{16\times45}$$

$$\boxed{R = 0.52d^3} \qquad \rightarrow(2)$$

$$\left.\begin{array}{l}\text{solid length of}\\ \text{spring when coils}\\ \text{are touching}\end{array}\right\} \quad nd = 45$$

$$\boxed{n = 45/d} \qquad \rightarrow(3)$$

Substituting equation (2) & (3) values in equation (1)

$$d^4 = \left(\frac{0.9\times64}{0.4\times10^5}\right)\left(0.52d^3\right)^3\times\frac{45}{d}$$

$$= \left(\frac{0.9\times64}{0.4\times10^5}\right)(0.52)^3\times45d^8$$

$$d^4 = \frac{0.4\times10^5}{0.9\times64\times(0.52)^3\times45} = 109.75$$

$$d = (109.75)^{1/4} = 3.24\text{mm}$$

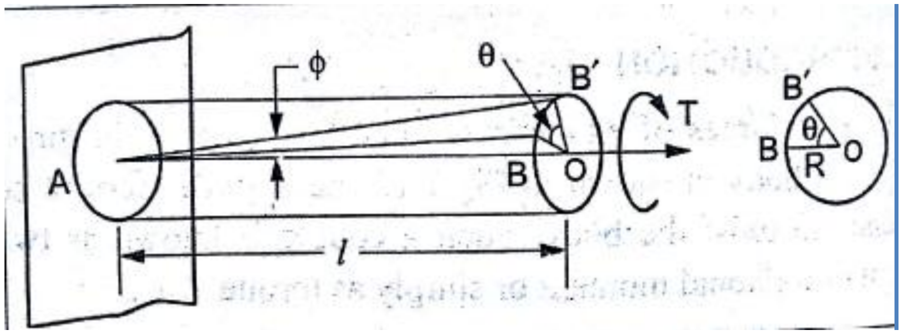
(ii) $R = 0.52d^3$

$R = 0.52 \text{ x } (3.24)^3 = 17.68$ mm

(iii) $n = 45/d$

$n = 45/3.24 = 13.88 \simeq 14$

17) Derive torsion equation (AU Nov/Dec 2014 – 16 Marks)



Consider a shaft of length L, radius R, fixed at one end and subjected to a torque 'T' at the other end is shown in figure.

Let '0' be the centre of circular section 'B' a point on surface and AB be the line on the shaft parallel to the axis of the shaft.

When the shaft is subjected to torque (T), B is moved to B' if 'ϕ' is shear strain and 'θ' is the angle of twist in length 'ℓ'.

Then $R\theta = BB' = \ell\phi \quad \rightarrow (1)$

If 'τ' is the shear stress and 'c' is the modulus of rigidity then

$$\phi = \frac{\tau}{c}$$

Substitute ϕ value in equation (1)

$$R\theta = \ell\phi$$

$$R\theta = \ell \times \frac{\tau}{c}$$

$$\boxed{\frac{c\theta}{\ell} = \frac{\tau}{R}} \quad \rightarrow (2)$$

Polar moment of inertia (J)

From equation (2) we know that

$$\frac{c\theta}{\ell} = \frac{\tau}{R}$$

Where, c – modulus of rigidity – N/mm^2

θ – angle of twist - radian

ℓ - length - mm

τ - shear stress - N/mm^2

R –Radius - mm

We know that

Torque, $T = \frac{\pi}{16} \times \tau \times D^3$

$$\tau = \frac{16 \times T}{\pi D^3}$$

Substitute τ value in equation (2)

$$\frac{c\theta}{\ell} = \frac{\frac{16 \times T}{\pi D^3}}{R} = \frac{\frac{16 \times T}{\pi D^3}}{\frac{D}{2}}$$

$$= \frac{32\,T}{\pi D^3 \times D} = \frac{32\,T}{\pi D^4}$$

$$\frac{c\theta}{\ell} = \frac{T}{\frac{\pi}{32} \times D^4} = \frac{T}{J}$$

Where J (polar moment of inertia) $= \frac{\pi}{32} D^4$

$$\frac{T}{J} = \frac{c\theta}{\ell}$$

Torsional equation = $\boxed{\frac{T}{J} = \frac{c\theta}{\ell} = \frac{\tau}{R}}$

For hollow shaft,

Polar moment of inertia , $J = \frac{\pi}{32}\left[D^4 - d^4\right]$

Where d – inner diameter

D – outer diameter

18) The stiffness of a close coiled helical spring is 1.5 N/mm of compression under a maximum load of 60 N. The maximum shearing stress produced in the wire is 125 N/mm^2 . The solid length of the spring (when the coils are touching) is given as 50 mm. Find:

(i) The diameter of wire
(ii) The mean diameter of the coils
(iii) Number of coils required.
Take C = 4.5 x 10^4 N/mm^2 (Nov/Dec 2018)(Apr/May 2018)(Nov/Dec – 2014)

Given data:

Stiffness, k = 1.5 N/mm

Load, w = 60 N

Stress, τ = 125 N/mm^2

Solid length = n x d = 50 mm

Modulus of rigidity = c =4.5 x 10^4 N/mm^2

To find

(i) diameter of the wire, d

` (ii) diameter of the coil, D

(iii) number of coils , n

Solution:

We know that,

$$\text{Stiffness, } k = \frac{cd^4}{64R^3n}$$

$$1.5 = \frac{4.5\times10^4 \times d^4}{64R^3n}$$

$$\boxed{2.133\times10^{-3} = \frac{d^4}{R^3n}} \qquad \rightarrow (1)$$

$$\text{Shear stress, } \tau = \frac{8WD}{\pi d^3} = \frac{16WR}{\pi d^3}$$

$$125 = \frac{16\times60\times R}{\pi d^3}$$

$$\boxed{0.4090 = \frac{R}{d^3}} \qquad \rightarrow (2)$$

nd = 50

$$\boxed{d = \frac{50}{n}}$$

Substitute 'd' value in equation (1) & (2)

$$\text{eqn}(1) \Rightarrow \frac{\left(\frac{50}{n}\right)^4}{R^3 n} = 2.133 \times 10^{-3}$$

$$\frac{(50)^4}{R^3 n^5} = 2.133 \times 10^{-3}$$

$$\boxed{R^3 n^5 = 2930 \times 10^9} \quad \rightarrow (3)$$

$$\text{Eqn}(2) \quad 0.4090 = \frac{R}{\frac{(50)^3}{n^3}}$$

$$51.125 \times 10^3 = Rn^3$$

$$\boxed{R = \frac{51.125 \times 10^3}{n^3}} \quad \rightarrow (4)$$

Substitute R value in equation (3)

$$\left[\frac{51.125 \times 10^3}{n^3}\right]^3 n^5 = 2.930 \times 10^9$$

$$\frac{1.336 \times 10^4}{n^4} = 2.930 \times 10^9$$

$$n = 14.62 = 15$$

Number of turns, $\boxed{n = 15}$

Substitute n value in equation (4)

$$R = \frac{51.125 \times 10^3}{(14.62)^3}$$

R = 16.36 mm

Diameter of coil $\boxed{D = 32.72\text{mm}}$

We know that nd = 50

14.62 x d = 50

D = 3.42 mm

Diameter of wire, d = 3.42 mm

Results: 1) d = 3.42 mm

2) D = 32.72 mm

3) n = 15

19) Determine the bending stress, shear stress and total work done on an open coiled helical spring subjected to axial force having mean radius of each coil as 'r' and 'n' number of turns. (May/June 2014) 16 Marks

Let,

W = axial load

P = pitch of the spring

d = wire diameter

R = mean radius of spring

Axial load

Torque, T = WR cos d

Bending moment, M = WR sin d

Shear stress, $\tau = \frac{16T}{\pi d^3} = \frac{16WR\cos\alpha}{\pi d^3}$

Bending stress, $\sigma_b = \frac{32M}{\pi d^3} = \frac{32WR\sin\alpha}{\pi d^3}$

Work done

Deflection, $\delta = \frac{64WR^3n}{cd^4}$

The average external work done on the spring under load,

$$w = \frac{1}{2}W\delta$$
$$= \frac{1}{2}W \times \frac{64WR^3n}{cd^4}$$

Work done = $w = \frac{32WR^3n}{cd^4}$