UNIT – I (SNME)

PART – A

1) Write an expression of volumetric strain for a rectangular bar subjected to an axial load P. (Nov/Dec 2018)

$$e_v = \frac{\delta l}{l}(1-2\mu)$$

2) What does the radius of mohr's circle refer to? (May/June 2017)

The radius of mohr's circle refers to the maximum shear stress.

3) Define principle plane (May /June 2016)

The plane which have no shear stress are known as principle planes.

4) Obtain the relation between E and K (May/June 2016) (Apr/May 2018)

$$E = 3K\left(1-\frac{2}{m}\right) = 3K(1-2\mu)$$

$$E \rightarrow \text{Young's modulus } \left(N/mm^2\right)$$

$$K \rightarrow \text{Bulk modulus } \left(N/mm^2\right)$$

$$\frac{1}{m} = \mu \rightarrow \text{poisson's ratio}$$

5) Differentiate elasticity and elastic limit (Nov/Dec 2015)

Elasticity

The body which regains its original position on the removal of the force that property is known as Elasticity

Elastic limit

There is always a limiting values of load upto which the strain totally disappears on the removal of load the stress corresponding to this load is known as Elastic limit

6) What is principle of super position? (Nov/Dec 2015)

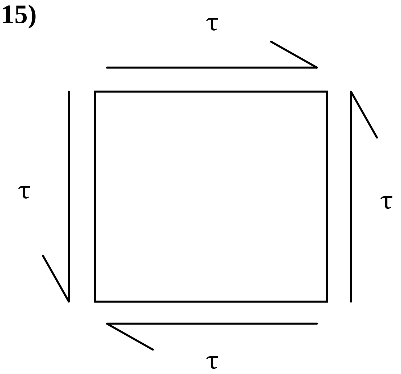
In some cases, interior cross section of a body subjected to external axial forces. In such cases, the forces are split up and their effects are considered on individual section. The total deformation is equal to the algebraic sum of the deformation individual section. This principle of finding the resultant deformation is known as principle of super position.

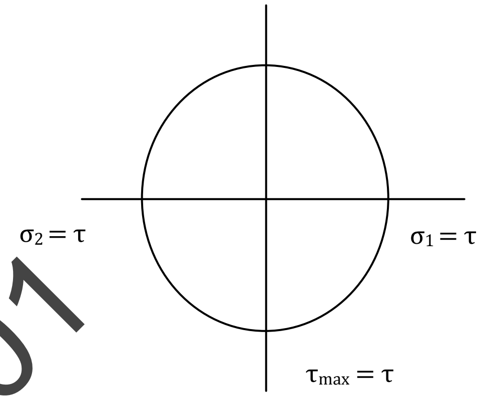
$$\delta l = \frac{P_1 l_1 + P_2 l_2 + P_3 l_3 + \ldots..}{AE}$$

7) What do you mean by thermal stresses?(Apr/ May 2015) (Apr/ May 2019)

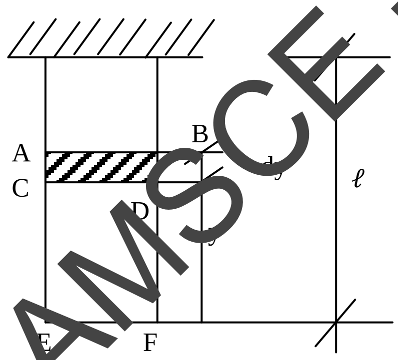
When the temperature varies, the bar will tends to expands or contracts, but the same is prevented by external forces or by fixing the bar ends, the temperature stress will be produced in that bar.

8) Draw the Mohr's circle for the state of pure shear in a strained body and mark all salient points in it (Apr/ May 2015)





9) Derive a relation for change in length of a bar hanging freely under its own weight (May / June 2017) (Nov/ Dec 2014)



A bar of length - l (meter)

area – A (m^2)

Fixed at one end ρ – kg/m^3

Force acting down at CD= weight of bar CDEF = Aρy × 9.81

$$\sigma = \frac{\text{Force at CD}}{A} = \frac{A\, y\rho \times 9.81}{A}$$

$$\sigma = 9.81\rho y \ \ N/m^2$$

$$\sigma \ \alpha \ y \ [\text{stress is directly propotional to y}]$$

$$\text{Strain in length dy} = \frac{\sigma}{E} = \frac{9.81\rho\, y}{E}$$

$$\text{Elangation in dy} = \frac{9.81\ \rho y}{lE} dy$$

$$\text{Total elangation of bar}(Sl) = \int_0^{\ell} \frac{9.81\ \rho y}{lE} dy$$

$$= \frac{9.81\ \rho y}{lE} \frac{y^2}{2}\Big]_0^{\ell} = \frac{9.81\rho\ell^2}{2E}$$

10) Write the relationship between shear modulus & young's modulus of elasticity (Nov/ Dec 2014)

$$E = 2G\left(1+\frac{1}{m}\right) = 2G\left(1+\mu\right)$$

11) Define young's modulus (Nov/ Dec 2016)

When a body is stressed within elastic limit, the ratio of stress is constant and that constant is known as Young's modulus.

12) What do you mean by principal planes and principal stress? (Nov/ Dec 2016) (Nov/ Dec 2017) (Apr/ May 2018) (Apr/ May 2019)

Principal plane:
The plane which have no shear stress are known as principal plane
Principal Stress:
The magnitude of normal stress, acting on a principal plane are known as principal stress

13) Define Bulk – modulus. (Nov/ Dec 2017)

The ratio of direct stress to volumetric strain

K = Direct stress / Volumetric strain.

14) State Hooke's law

It states when a material is loaded, within its elastics limit, the stress is directly proportional to the strain.

15) Define strain energy

Whenever a body is strained, some amount of energy is absorbed in the body**.** The energy which is absorbed in the body due to straining effect is known as strain energy.

16) Define Poisson's ratio. (Nov/Dec 2018)

When a body is stressed, within it's elastic limit, the ratio of lateral strain to the longitudinal strain is constant for a given material. Poisson's ratio

$$\mu \text{ or } \frac{1}{m} = \text{Lateral strain } / \text{ Longitudinal strain}$$

17) What is compounds bar?

A composite bar composed of two or more different material joined together such that system is elongated or compressed in a single unit.

18) Define strain

When a body is subjected to an external force, there is some change of dimension in the body. Numerically the strain is equal to the ratio change in length to the original length of the body

Strain (e) = change in length / Original length $= \partial L / L$

19) Define stress

When an external forces acts on a body, it undergoes deformation. At the same time the body resists deformation. The magnitude of the resistance force is numerically equal to the applied force. This internal resistance force per unit area is called stress. Stress σ = Force/Area, P/A Unit N/mm^2.

20) Define shear stress and shear strain.

The two equal and opposite force act tangentially on any cross section plane of the body tending to slide one part of the body over the other part. The stress induced is called shear stress and corresponding strain is known as shear strain.

21) Define – Lateral strain.

The strain right to the direction of the applied load is called lateral strain.

22) Define – longitudinal strain

When a body is subjected to axis load P, the length of the body is increased. The axial deformation of the length of the body is called longitudinal strain.

23) A rod of diameter 30 mm and length 400 mm was found to eligible 0.35 mm. When it was subjected to a load of 65 KN. Compute the modulus of elasticity of material of this rod.

$$\delta l = \frac{P\ell}{AE} \Rightarrow E = \frac{P\ell}{A\delta l} = \frac{65\times10^3\times400}{\frac{\pi}{4}\times30^2\times0.35}$$

$$E = 105.09\times10^3 \ \ N/mm^2$$

24) The Young's modulus and the shear modulus of material are 120GPa and 45GPa respectively. What is it Bulk modulus?

$$E = 120\times10^9 \ N/m^2 \qquad G = 45\times10^9 \ N/m^2$$
$$= 120\times10^3 \ N/mm^2 \qquad = 45\times10^3 \ N/mm^2$$

$$E = \frac{9KG}{3K+G}$$
$$120\times10^3 = \frac{9K\times45\times10^3}{3K+45\times10^3}$$
$$120\times10^3\left[3K+45\times10^3\right] = 9K\times45\times10^3$$
$$3K+45\times10^3 = 3.375\ K$$
$$45\times10^3 = 0.375\ K$$
$$K = 120\times10^3 \ N/mm^2$$

PART – B

1) A steel rod of 3cm diameter and 5m long is connected to two grips and the rod is maintained at a temperature of 95^0C. Determine the stress and pull exerted when the temperature falls to 30^0C, if
(i) the ends do not yield and
(ii) the ends yield by 0.12cm. Take E=$2x10^5$MN/m^2 and $\alpha=12x10^{-6}/^0C$ (Apr/May 2019)

d=30mm

$A = \left(\pi/4\right) xd^2 = 225\pi \text{ mm}^2$

$L = 5000mm$

$T_1 = 95^0C$

$T_2 = 30^0C$

$T = T_1 - T_2 = 65^0C$

(i) when the ends do not yield

stress = αTE = 156N/mm^2

Pull in the rod = stress x area = 156x225π = 110269.9 N

(ii) When the ends yield by 0.12cm (δ=1.2mm)

stress = $\dfrac{(\alpha TL\text{-}\delta)}{L} xE$ = 108N/mm^2

Pull in the rod = stress x area = 108x225π = 76340.7 N

2) An elemental cube is subjected to tensile stresses of 30 N/mm^2 and 10 N/mm^2 acting on two mutually perpendicular planes and a shear stress of 10 N/mm^2 on these planes. Draw the Mohr's circle of stresses and hence or otherwise determine the magnitude and directions of principal stresses and also the greatest shear stress. (Apr/May 2019)

Major tensile stress (σ_1) = 30N/mm^2

Minor tensile stress (σ_2) = 10 N/mm^2

Shear stress (τ) = 10 N/mm^2

Location of principle planes,

θ = Angle, which one of the principle planes makes with the stress of 10 N/mm^2

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2} = \frac{2\times 10}{30-10} = 1$$
$$2\theta = \tan^{-1}(1) = 45° \text{ or } 225°$$
$$\theta = 22°5' \text{ or } 112° 5'$$

Principle stress

Major principle stress = $\dfrac{\sigma_1+\sigma_2}{2} + \sqrt{\left(\dfrac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$

$$= \frac{30+10}{2} + \sqrt{\left(\frac{30-10}{2}\right)^2 + 10^2}$$

$$= 20 + 14.14 = 34.14 \text{ N/mm}^2$$

Minor principle stress = $\frac{\sigma_1+\sigma_2}{2} - \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$

$$= \frac{30+10}{2} - \sqrt{\left(\frac{30-10}{2}\right)^2 + 10^2}$$

$$= 20 - 14.14 = 5.86 \text{ N/mm}^2$$

3) A reinforced short concrete column 250mm x 250mm in section is reinforced with 8 steel bars. The total area of steel bars is 2500 mm^2**. The column carries a load of 390kN. If the modulus of elasticity of steel is 15 times that of concrete. Find the stresses in concrete and steel. (Nov/Dec 2018)**

$E_s = 15E_c$

$A_s = 2500 \text{ mm}^2$

Area of concrete column = 250x250 = 62500mm^2

$A_c = 62500 - 2500 = 60000 \text{ mm}^2$

P = 390000N

i)

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_s = \sigma_c \times \frac{E_s}{E_c} = 15\sigma_c$$

$$\boxed{\sigma_s = 15\,\sigma_c} \qquad ...1$$

$$P = \sigma_s A_s + \sigma_c A_c$$

$$390000 = 15\sigma_c \times 2500 + 60000\sigma_c$$

$$390000 = 97500\,\sigma_c$$

$$\sigma_c = 4\text{N/mm}^2$$

$$\sigma_s = 60\text{N/mm}^2$$

4) The stresses at a point in a bar are 200 N/mm^2 (tensile) and 100 N/mm^2 (compressive). Determine the resultant stress in magnitude and direction on a plane inclined at 60^0 to the axis of major stress. Also determine the maximum shear stress in the material at the point. (Nov/Dec 2018)

Major Principal stress, σ_1=200 N/mm^2

Minor Principal stress, σ_2= - 100 N/mm^2

Angle inclined with major principal stress = 60^0

Angle inclined with minor principal stress θ = 90^0 - 60^0 = 30^0

Normal stress:

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta$$
$$\sigma_n = \frac{200 + (-100)}{2} + \frac{200 - (-100)}{2}\cos(2x30)$$
$$\sigma_n = 125 N/mm^2$$

Shear stress:

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta$$
$$\sigma_t = \frac{200 - (-100)}{2}\sin(2x30)$$
$$\sigma_t = 129.9 N/mm^2$$

Resultant stress:

$$\sigma_R = \sqrt{\left(\sigma_n^2 + \sigma_t^2\right)} = \sqrt{\left(125^2 + 129.9^2\right)} = 180.27 N/mm^2$$

Maximum shear stress:

$$(\sigma_t)_{max} = \frac{\sigma_1 - \sigma_2}{2}$$
$$(\sigma_t)_{max} = \frac{200 - (-100)}{2} = 150 N/mm^2$$
$$\tan\phi = \frac{\sigma_t}{\sigma_n} = 1.04$$
$$\phi = 46^0 6'$$

5) At a point in a strained material the principal stresses are 100 N/mm^2 (tensile) and 60 N/mm^2 (compressive). Determine the normal stress, shear stress and resultant stress on a plane inclined at 50^0 to the axis of major principal stress. Also determine the maximum shear stress at the point. (Nov/Dec 2017)

Major Principal stress, σ_1=100 N/mm^2

Minor Principal stress, σ_2= - 60 N/mm^2

Angle inclined with major principal stress = 50^0

Angle inclined with minor principal stress θ = 90^0 - 50^0 = 40^0

Normal stress:

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta$$
$$\sigma_n = \frac{100 + (-60)}{2} + \frac{100 - (-60)}{2}\cos(2x40)$$
$$\sigma_n = 33.89 N/mm^2$$

Shear stress:

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta$$

$$\sigma_t = \frac{100-(-60)}{2}\sin(2x40)$$

$$\sigma_t = 78.785 N/mm^2$$

Resultant stress:

$$\sigma_R = \sqrt{\left(\sigma_n{}^2 + \sigma_t{}^2\right)} = \sqrt{\left(33.89^2 + 78.785^2\right)} = 85.765 N/mm^2$$

Maximum shear stress:

$$(\sigma_t)_{max} = \frac{\sigma_1 - \sigma_2}{2}$$

$$(\sigma_t)_{max} = \frac{100-(-60)}{2} = 80 N/mm^2$$

6) A solid steel bar 40mm diameter 2m long passes centrally through a copper tube of internal diameter 40mm, thickness of metal 5mm and length 2m. The ends of the bar and tube are brazed together and a tensile load of 150kN is applied axially to the compound bar. Assume E_c= 100GN/m^2 and E_s= 200GN/m^2 Find the stresses and load sheared by the steel and copper section (Apr/May 2018)

$$d_s = 40\,mm$$
$$t = 5mm$$
$$d_c = 40\,mm$$
$$D_c = d_c + 2t = 40 + 2x5 = 50\,mm$$
$$P = 150000N$$
$$\ell = 2m$$

$$E_s = 2\times10^5\,N/mm^2$$
$$E_c = 1\times10^5\;N/mm^2$$

$$A_s = \frac{\pi}{4}d_s{}^2$$
$$= \frac{\pi}{4}\times40^2 = 1256.64\;mm^2$$

$$A_c = \frac{\pi}{4}\left(D_c{}^2 - d_c{}^2\right) = \frac{\pi}{4}\left(50^2 - 40^2\right)$$
$$= 706.86\;mm^2$$

i)

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_s = \sigma_c \times \frac{E_s}{E_c} = \frac{2\times10^5}{1\times10^5}\sigma_c$$

$$\boxed{\sigma_s = 2\;\sigma_c} \qquad ...1$$

$$P = \sigma_s A_s + \sigma_c A_c$$
$$150000 = 2\sigma_c \times 1256.64 + 706.86\sigma_c$$
$$150000 = 2120.58\sigma_c$$
$$\sigma_c = 70.74 N/mm^2$$
$$\sigma_s = 141.47 N/mm^2$$

7) At a point within a body subjected to two mutually perpendicular directions, the tensile stresses are 80N/mm^2 and 40N/mm^{2}respectively. Each stress is accompanied by shear stress of 60N/mm^2. Determine the normal stress, shear stress and resultant stress on an oblique plane inclined at an angle of 45^0 with the axis of minor tensile stress. (Apr/May 2018)

Major tensile stress σ_1=80N/mm^2

Minor tensile stress σ_2=40N/mm^2

Shear stress τ=60N/mm^2

Angle incline with minor axis (θ)= 45°

Normal Stress:

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta + \tau \sin 2\theta$$
$$\sigma_n = \frac{80+40}{2} + \frac{80-40}{2}\cos(2x45) + 60\sin(2x45)$$
$$\sigma_n = 120N/mm^2$$

Shear stress:

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta - \tau\cos 2\theta$$
$$\sigma_t = \frac{80-40}{2}\sin(2x45) - 60\cos(2\,x\,45)$$
$$\sigma_t = 20N/mm^2$$

Resultant stress:

$$\sigma_R = \sqrt{\left(\sigma_n{}^2 + \sigma_t{}^2\right)} = \sqrt{\left(120^2 + 20^2\right)} = 121.65N/mm^2$$

8) The bar shown in fig. Q.11(a) is subjected to a tensed load of 100 KN of the stress in middle portion is limited to 150N/mm^2. Determine the diameter of the middle portion. Find also the length of the middle portion if the total elongation of the bar is to be 0.2 mm young modules is 2.1 x 10^5 N/mm^2

(May / June 2017) 13-Marks

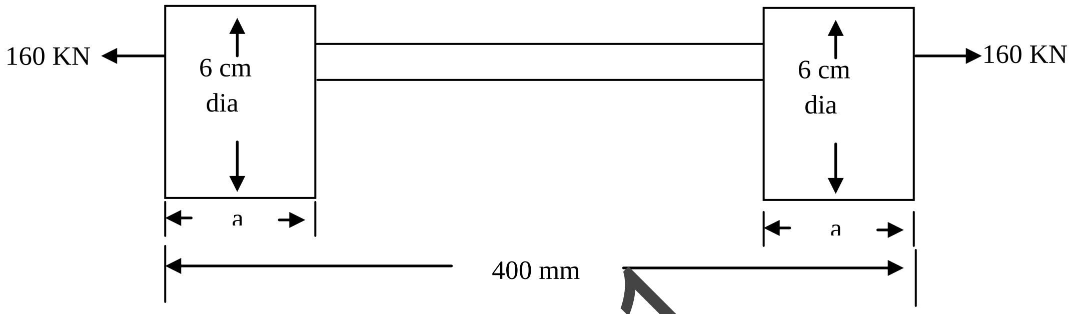


Fig.Q 11(a)

Given: $P = 100\ KN = 100 \times 10^3\ N$

Stress at middle portion, $\sigma_2 = 150\ N/mm^2$

Total elongation $\delta L = 0.2$ mm

Young modulus, $E = 2.$[illegible] $\times 10^5\ N/mm^2$

Total length, $L = 400$ mm

To find:

i) Diameter of the middle portion, D_2

ii) Length of the middle portion, L_2

Solution

Stress at the middle portion, $\sigma_2 = \frac{\text{Load}}{\text{Area}} = \frac{p}{A_2}$

$$150 = \frac{p}{\frac{\pi}{4} D_2^2} = \frac{100 \times 10^3}{\frac{\pi}{4} \times D_2^2}$$

Diameter of middle portion, $D_2 = 29.14$ mm

Let,

Length of first portion = L_1

Length of middle portion = L_2

Length of last portion = L_3

We know that

Total elongation, $\delta L = \frac{p}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right]$

$$= 0.2 = \frac{100\times10^3}{2.1\times10^5}\left[\frac{L_1}{\frac{\pi}{4}D_1^2}+\frac{L_2}{\frac{\pi}{4}D_2^2}+\frac{L_3}{\frac{\pi}{4}D_3^2}\right]$$

$$\Rightarrow 0.2 = \frac{100\times10^3}{2.1\times10^5}\left[\frac{L_1}{\frac{\pi}{4}(60)^2}+\frac{L_2}{\frac{\pi}{4}(29.14)^2}+\frac{L_3}{\frac{\pi}{4}(60)^2}\right]$$

$$\Rightarrow 0.2 = \frac{100\times10^3}{2.1\times10^5}\left[\frac{L_1}{2826}+\frac{L_2}{666.57}+\frac{L_3}{2826}\right]$$

$$\Rightarrow 0.2 = 0.476\left[\frac{L_1+L_3}{2826}+\frac{L_2}{666.57}\right]$$

$$0.2 = 0.476\left[\frac{(400-L_2)}{2826}+\frac{L_2}{666.57}\right]$$

$$0.2 = 0.476\left[\frac{400}{2826}-\frac{L_2}{2826}+\frac{L_2}{666.57}\right]$$

$$0.2 = 0.0673 - 1.684 \times 10^{-4} L_2 + 7.141 \times 10^{-4} L_2$$

$$0.2 = 0.0673 + 5.457 \times 10^{-4} L_2$$

$$\frac{0.2-0.0673}{5.457\times10^{-4}} = L_2$$

L_2 = 243.17 mm

Result

1) Diameter of middle portion, D_2 = 29.14 mm

2) Length of middle portion, L_2 = 243.17 mm

9) A bar of 30 mm diameter is subjected to a pull of 60KN. The measured extension on gauge length of 200 mm is 0.1 mm and change in diameter is 0.004 mm.

Calculate **(May 2017) 13 Marks**

(i) Young's modulus

(ii) Poisson's ratio and

(iii) Bulk modulus

Given:

Diameter, d = 30 mm
Pull, p = 60 KN = 60 x 10^3 N
Length, L = 200 mm
Change in Length, δL = 0.1 mm
Change in diameter, δd = 0.004 mm

To Find:

(i) Young's modulus
(ii) Poisson's ratio and
(iii) Bulk modulus

Solution: we know that

$$\text{Poisson's ratio} = \frac{1}{m} = \frac{\text{Lateral strain}}{\text{longitudinal strain}} = \frac{e_t}{e_\ell} \rightarrow (1)$$

$$\text{Lateral strain } = e_t = \frac{\delta b}{b} (\text{or}) \frac{\delta d}{d} (\text{or}) \frac{\delta t}{t}$$

$$e_t = \frac{\delta d}{d} = \frac{0.004}{30} = 1.333 \times 10^{-4}$$

$$\text{Longitudinal strain, } e_\ell = \frac{\delta L}{L} = \frac{0.1}{200} = 5 \times 10^{-4}$$

Substitute e_t and e_ℓ in equation (1)

$$\frac{1}{m} = \frac{1.333 \times 10^{-4}}{5 \times 10^{-4}} = 0.26$$

$$\boxed{\text{Poisson's ratio} = \frac{1}{m} = 0.26}$$

$$\text{young's modulus, } E = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{\sigma}{e_\ell}$$

$$\text{stress} = \sigma = \frac{\text{Load}}{\text{Area}} = \frac{p}{A}$$

$$E = \frac{60 \times 10^3}{\frac{\pi}{4} d^2 \times 5 \times 10^{-4}}$$

$$= \frac{60 \times 10^3}{\frac{\pi}{4} (50)^2 \times 5 \times 10^{-4}} = \frac{60 \times 10^3}{0.353}$$

$$\boxed{E = 1.69 \text{ x } 10^5 \text{ N / mm}^2}$$

We know that,

$$E = 3k\left(1 - \frac{2}{m}\right)$$

Young's modulus, $1.69 \times 10^5 = 3k\left[1 - 2(0.26)\right]$

$$\boxed{\text{Bulk modulus} = k = 1.17 \times 10^5 \text{ N / mm}^2}$$

Results:

(i) Poisson's ratio = $\frac{1}{m}$ =0.26

(ii) Young's modulus = E = 1.69 x 10^5 N/mm^2

(iii) Bulk modulus = k = 1.17 x 10^5 N/mm^2

10) A steel bar 20mm in diameter 2m long is subjected to an axial pull of 50 KN. If E= 2×10^5 N/mm^2 and m = 3. Calculate the change in the i) Length ii) diameter iii) Volume (8 mark)

(May / June 2016)

Given data:

$$d = 20 \text{ mm} \qquad \ell = 2\text{m} = 2000\,\text{mm} \qquad p = 50\text{KN} = 50\times10^3 \qquad E = 2\times10^5 \text{ N/mm}^2$$

$$m = 3$$

$$\text{i) } E = \frac{\sigma}{e} = \frac{P/A}{\delta\ell/l}$$

$$e = \frac{\sigma}{E}$$

$$\sigma = \frac{P}{A} = \frac{50\times10^3}{\pi/4\times20^2} = \frac{50\times10^3}{314.16} = 159.15 \text{ N/mm}^2$$

$$e = \frac{\sigma}{E} = \frac{159.15}{2\times10^5} = 7.957\times10^{-4}$$

$$e = \frac{\delta\ell}{\ell}$$

$$\delta\ell = 7.96\times10^{-4}\times2000 = 1.59\,\text{mm}$$

$$\text{Change in legnth } \boxed{\delta\ell = 1.59 \text{ mm}}$$

$$\Rightarrow \mu = \frac{1}{m} = \text{poisson's ratio} = \frac{1}{3} = 0.33$$

$$\mu = \frac{\text{Lateral strain}}{\text{Linear strain}} = \frac{(\delta d/d)}{(\delta\ell/\ell)}$$

$$0.33 = \frac{\delta d/d}{7.96\times10^{-4}}$$

$$\delta d/d = 2.6268\times10^{-4}$$

$$\delta d = 2.6268\times10^{-4}\times20 = 5.25\times10^{-3}\,\text{mm}$$

$$\text{change in dialmeter } \boxed{\delta d = 5.25\times10^{-3}}$$

$$\delta v/v = \delta\ell/\ell - 2\delta d/d$$

$$\frac{\delta v}{v} = 7.96\times10^{-4} - 2\times2.63\times10^{-4}$$

$$\frac{\delta v}{v} = 2.7\times10^{-4}$$

$$\delta V = 2.7\times10^{-4}\times\frac{\pi}{4}\times20^2\times2000$$

$$\text{change in volume } \boxed{\delta V = 169.65 \text{ mm}^3}$$

11) A mild steel bar 20 mm in diameter and 40 cm long is encase in a tube whose external diameter is 30 and internal diameter is 25 mm. The composite bar is heated through 80°C. Calculate the stress induced in each metal α for steel is 11.2 ×10^{-6} per °C; α for brass is 11.2 ×10^{-6} per °C. E for steel is 2 ×10^{5} N/mm^{2} and E for brass is 1 ×10^{5} N/mm^{2} (8mark)

(May /June 2016)

Given

$$d_s = 20mm \qquad \ell_s = 40\,cm = 400\,mm = \ell_b = \ell$$
$$D_b = 30\ mm \qquad d_b = 25mm \qquad \Delta t = 80°C$$
$$\alpha_s = 11.2\times10^{-6}\ /\ °C \qquad \alpha_b = 16.5\times10^{-6}\ /\ °C$$
$$E_s = 2\times10^5\ N/mm^2 \qquad E_b = 1\times10^5\ N/mm^2$$
$$A_s = \frac{\pi}{4}d_s{}^2 = 314.16\ mm^2$$
$$A_b = \frac{\pi}{4}\left(D_b{}^2 - d_b^2\right) = 215.98\ mm^2$$

Under equilibrium condition,

Compression in brass is equal to tension in steel i.e,

Load on brass (P_b) = load on steel (P_s)

$$\sigma_b A_b = \sigma_s A_s$$
$$\sigma_s = \sigma_b \times \frac{A_b}{A_s} = \sigma_b \times \frac{215.98}{314.16} = 0.687\sigma_b$$
$$\boxed{\sigma_s = 0.687\ \sigma_b} \qquad ...1$$

Actual expansion of steel = Actual expansion of brass

$$\alpha_s\ \Delta t\ \ell_s + \frac{\sigma_s}{E_s}\ell_s = \alpha_b\ \Delta t\ \ell_b - \frac{\sigma_b}{E_b}\ell_b$$
$$\ell_s\left(\alpha_s \Delta t + \frac{\sigma_s}{E_s}\right) = \ell_b\left(\alpha_b\ \Delta t - \frac{\sigma_b}{E_b}\right) \qquad \left(\ell_s = \ell_b = \ell\right)$$
$$\frac{\sigma_s}{E_s} + \frac{\sigma_b}{E_b} = \Delta t\left(\alpha_b - \alpha_s\right)$$
$$\frac{0.687\sigma_b}{2\times10^5} + \frac{\sigma_b}{1\times10^5} = \left(16.5\times10^{-5} - 11.2\times10^{-6}\right)\times 80°C$$
$$0.3435\sigma_b + \sigma_b = 5.3\times10^{-6}\times10^5\times80° = 42.4$$
$$1.3435\ \sigma_b = 42.4$$
$$\boxed{\sigma_b = 31.56\ N/mm^2} \qquad \text{substitute in equation 1,}$$
we get $\boxed{\sigma_s = 21.68\ N/mm^2}$

12) Two steel rods and one copper rod, each of 20 mm diameter together support a load 20 KN as shown in fig i) Find the stresses in the rods. Take E for steel = 210 KN/mm^2 and E for copper = 110 KN /mm^2

(May / June 2016)

$d_c = d_s = 20\,\text{mm}$

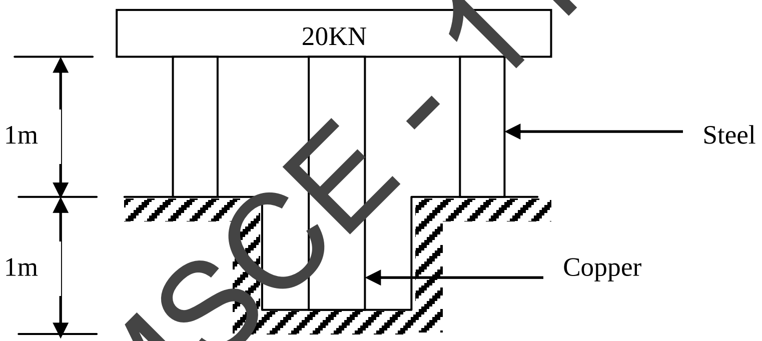
$P = 20\,\text{KN} = 20\times10^3\,\text{N}$

$E_s = 210\times10^3\ \text{N/mm}^2$

$E_c = 110\times10^3\ \text{N/mm}^2$

$\ell_s = 1\text{m} = 1000\ \text{mm}$

$\ell_c = 2\text{m} = 2000\ \text{mm}$



$A_s = A_c = \frac{\pi}{4}\times20^2 = 314.16\ \text{mm}^2$

$\text{contraction in steel}\left(\delta\ell_s\right) = \text{contraction in copper}\left(\delta\ell_c\right)$

$$\frac{P_s\ell_s}{A_sE_s} = \frac{\sigma_c\ell_c}{E_c}$$

$$\frac{\sigma_s\ell_s}{E_s} = \frac{\sigma_c\ell_c}{E_c}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{E_s}{E_c}\times\frac{\ell_c}{\ell_s}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{210\times10^{-3}}{110\times10^{-3}}\times\frac{2000}{1000}$$

$\boxed{\sigma_s = 3.82\ \sigma_c}$ 1

$P = \sigma_sA_x + \sigma_cA_c$

$20\times10^3 = 3.82\sigma_c\times2\times314.16\ \sigma_c$

$20\times10^3 = 2714.34\sigma_c$

$\boxed{\sigma_c = 7.37\ \text{N/mm}^2}$ sub in 1,

we get $\boxed{\sigma_s = 28.15\ \text{N/mm}^2}$

13) Direct stress of 140 N/mm^2 tensile and 100 N/mm^2 compression exist on two perpendicular planes at a certain point in a body, They are also accompanied by shear stress on the planes. The greatest principal stress at the point due to these is 160 N/mm^2

1) **What must be the magnitude of the shear stress on the two planes?**
2) **What will be the max. shear stress at the point?** **(May / June 2016)**

$\sigma_x = 140 N/mm^2$ $\sigma_y = 100 N/mm^2$ $\sigma_1 = 160 N/mm^2$

1 Shear stress(τ_{xy}):

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left[\frac{\sigma_x - \sigma_y}{2}\right]^2 + \tau_{xy}{}^2}$$

$$160 = \frac{140 + (-100)}{2} + \sqrt{\left[\frac{140 + (-100)}{2}\right]^2 + \tau_{xy}}$$

$$160 = 20 + \sqrt{120^2 + \tau_{xy}^2}$$

$$140 = \sqrt{120^2 + \tau_{xy}^2}$$

$$140^2 = 120^2 + \tau_{xy}^2$$

$$\boxed{\tau_{xy} = 72.11 N/mm^2}$$

Min. principal stress, $\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right) + \tau_{xy}^2}$

$$= 20 - \sqrt{120^2 + (72.11)^2}$$

$$= -119.99 \approx -120 N/mm^2$$

$$\sigma_2 = 120 N/mm^2 (comp)$$

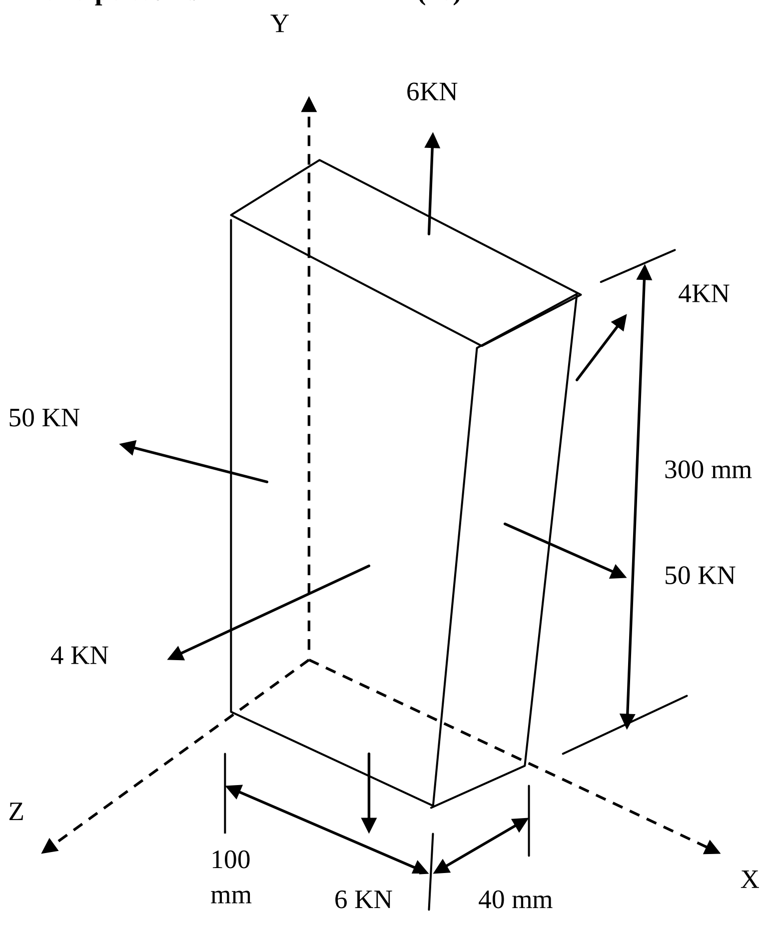
2. Max. shear stress(τ_{max}):

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{160 - (-120)}{2}$$

$$\tau_{max} = 140 N/mm^2$$

14) A metallic bar 300 mm × 100 mm×40 mm is subjected to a force of 50 KN (tensile), 6 KN (tensile), 4 KN (tensile) along x,y and z direction respectively. Determine the change in the volume of the block. Take E = 2×10^5 N/mm^2 and poisson's ratio = 0.25 (16)

(Nov / Dec 2015)



$x = 100\,\text{mm} \qquad y = 300\text{mm} \qquad z = 40\text{mm}$

$$\sigma_x = \frac{p_x}{A_{yz}} = \frac{50\times10^3}{300\times40} = 4.167\,\text{N}/\text{mm}^2$$

$$\sigma_y = \frac{p_y}{A_{zx}} = \frac{6\times10^3}{100\times40} = 1.5\,\text{N}/\text{mm}^2$$

$$\sigma_z = \frac{P_z}{A_{xy}} = \frac{4\times10^3}{100\times300} = 0.133\,\text{N}/\text{mm}^2$$

$$e_x = \frac{\sigma_x}{E} - \frac{\sigma_y}{mE} - \frac{\sigma_z}{mE}$$

$$= \frac{4.167}{2\times10^5} - \frac{-0.25\times1.5}{2\times10^5} - \frac{0.25\times0.133}{2\times10^5}$$

$$= \frac{1}{2\times10^5}\left(4.167 - 0.25\times1.5 - 0.25\times0.133\right)$$

$$= \frac{3.75875}{2\times10^5} = 1.879\times10^{-5}$$

$$e_y = \frac{\sigma_y}{E} - \frac{\sigma_x}{mE} - \frac{\sigma_z}{mE}$$

$$= \frac{1}{2\times10^5}\left(1.5 - 4.167\times0.25 - 0.133\times0.25\right)$$

$$= \frac{0.425}{2\times10^5} = 2.125\times10^{-6}$$

$$e_y = \frac{\sigma_z}{E} - \frac{\sigma_x}{mE} - \frac{\sigma_y}{mE}$$

$$= \frac{1}{2\times10^5}\left(0.133 - 4.167\times0.25 - 1.5\times0.25\right)$$

$$= \frac{-1.28375}{2\times10^5} = -6.418\times10^{-6}$$

$$e_v = \frac{\delta V}{V} = e_x + e_y + e_z = 1.4497\times10^{-5}$$

$$\delta V = 1.4497\times10^{-5}\times300\times100\times40$$

$$\boxed{\delta V = 17.396 \simeq 17.40\ \text{mm}^3}$$

15. A steel rod of 3 cm diameter is enclosed centrally in a hollow copper tube of external diameter 5 cm and internal diameter of 4 cm as shown fig. The composite bar is then subjected to axial pull of 45000 N. If the length of each bar is equal to 15 cm determine i) The stresses in the rod and the tube and ii) load carried by each bar Take E for steel = 2.1× 10^5 N/mm^2 and for copper = 1.1× 10^5 N/mm^2 (16)

(Nov /Dec 2015)

$d_s = 3\text{cm} = 30\,\text{mm}$

$D_c = 5\text{cm} = 50\,\text{mm}$

$d_c = 4\text{cm} = 40\,\text{mm}$

$\rho = 45000\,\text{N}$

$\ell = 15\,\text{cm}$

$E_s = 2.1\times10^5\, N/mm^2$

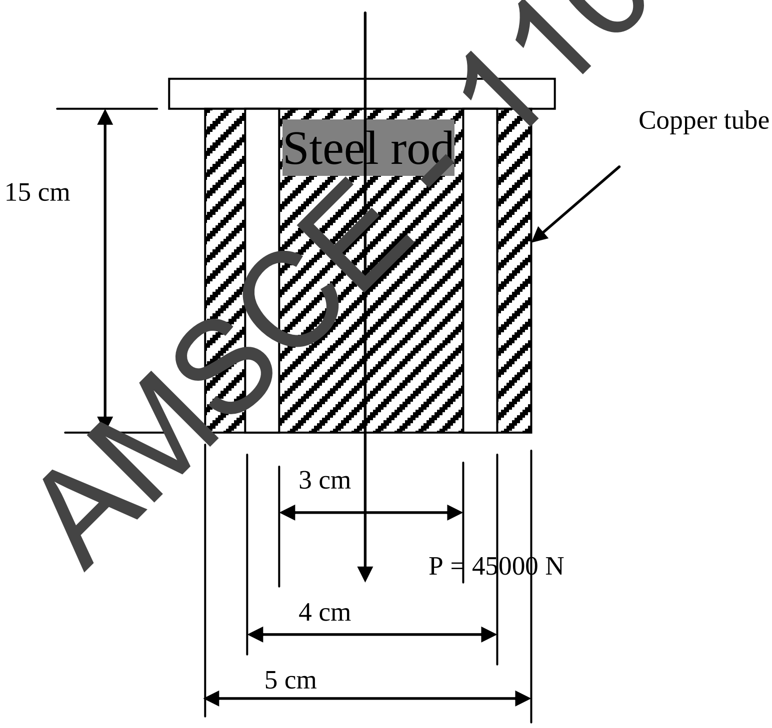
$E_c = 1.1\times10^5\ N/mm^2$

$A_s = \frac{\pi}{4} d_s^{\,2}$

$= \frac{\pi}{4}\times30^2 = 706.86\ mm^2$

$A_c = \frac{\pi}{4}\left(D_c^{\,2} - d_c^{\,2}\right)$

$= 706.86\ mm^2$



i)

$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$

$\sigma_s = \sigma_c \times \frac{E_s}{E_c} = \frac{2.1\times10^5}{1.1\times10^5}\sigma_c$

$\boxed{\sigma_s = 1.91\ \sigma_c}$ ...1

$P = \sigma_s A_s + \sigma_c A_c$

$4500 = 1.91\sigma_c \times 706.86 + 706.86\sigma_c$

$4500 = 2056.96\sigma_c$

$\boxed{\sigma_c = 21.88 N/mm^2}$ subs. (1), we get,

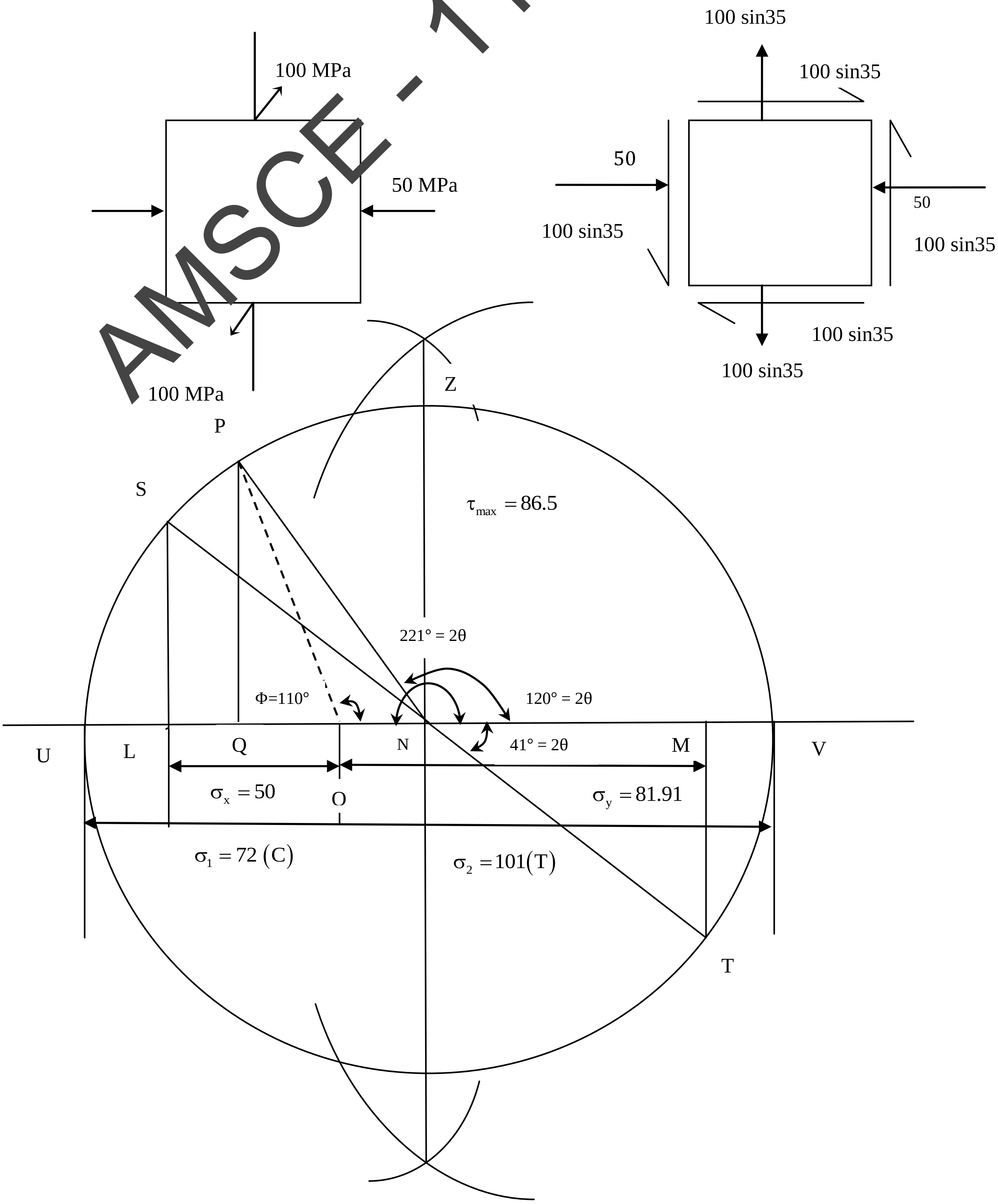
$\boxed{\sigma_s = 41.78 N/mm^2}$

ii)

$$P_c = \sigma_c A_c = 15466.09 \text{ N}$$
$$P_s = \sigma_s A_s = 29532.61 \text{ N}$$

16) At a point in a strained material the resultant intensity of stress across a vertical plane is 100MPa tensile inclined at 35° clockwise to its normal. The normal component of intensity of stress across the horizontal plane is 50 MPa compressive Determine graphically using Mohr's circle method
i) The position of principal planes and stresses across them and
ii) The normal and tangential stresses across a plane which is 60° clockwise to the vertical plane

(Apr/ May 2016)



i)

From

$\sigma_1 = OV = 72 \text{ MPa (Compressive)}$

$\sigma_2 = OV = 101 \text{ MPa (Tensile)}$

$\tau_{max} = NZ = 86.5 \text{ MPa(shear)}$

$2\theta = 41° \text{ or } 221°$

$\theta_1 = 20.5°$

$\theta_2 = 110.5°$

ii)

$\theta = 60°$

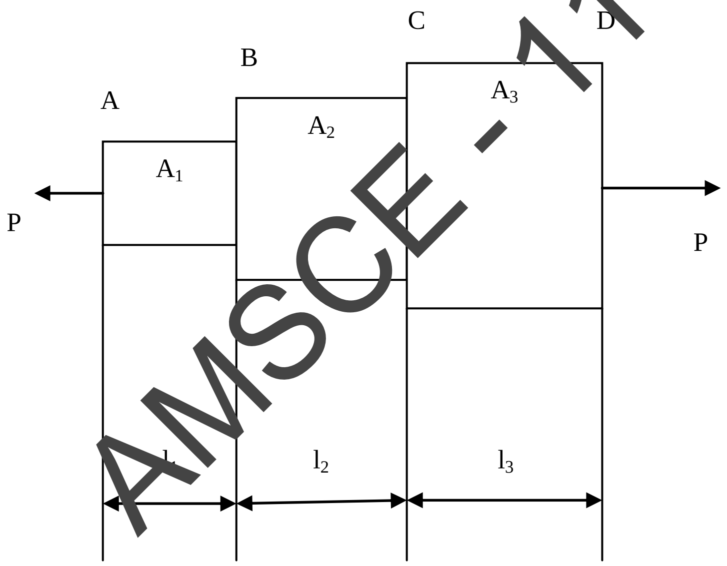
$\sigma_\eta = OQ = 28 \text{ MPa}$

$\tau = PQ = 75 \text{ MPa}$

$\sigma_r = OP = 80 \text{ MPa}$

$\phi = 110°$

17) Derive an expression for change in length of a circular bar with uniformly varying diameter and subjected to an axial tensile load P (8 mark) (Nov /Dec 2014)



Tensile stress in AB (σ_1) $= \dfrac{P}{A_1}$

Elongation in AB (δl_1) = $\dfrac{P\ell_1}{A_1 E}$

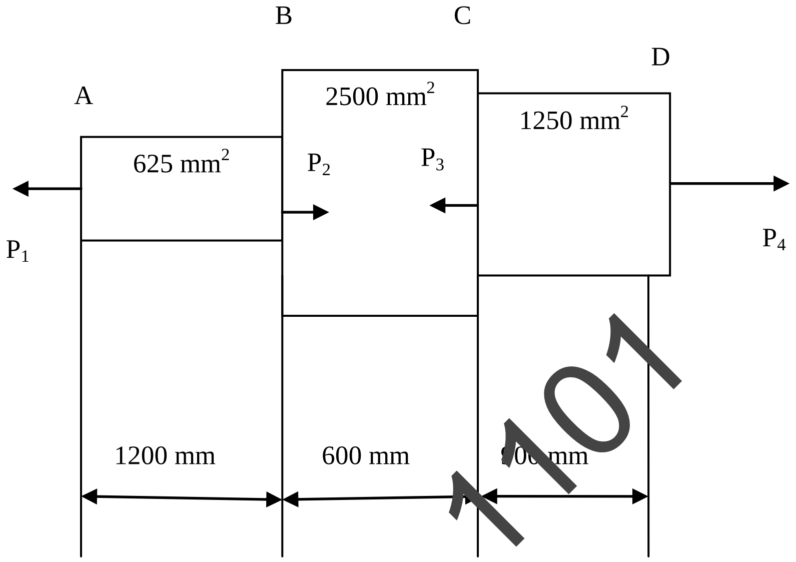
lll^{ly} for BC & CD

Total Elongation (δl) = $\delta l_1 + \delta l_2 + \delta l_2$

$$= \frac{Pl_1}{A_1E} + \frac{Pl_2}{A_2E} + \frac{Pl_3}{A_3E}$$

$$\boxed{\delta l = \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right]}$$

18) A member is subject to point load as shown in Fig Calculate the force P_2, necessary for equilibrium if P_1=45 KN; P_3=450 KN and P_4=130 KN. Determine the total elongation of the member, assuming the modulus of elasticity to be $E = 2.1\times10^5 \text{ N/mm}^2$ **(8 mark) (Nov /Dec 2014)**



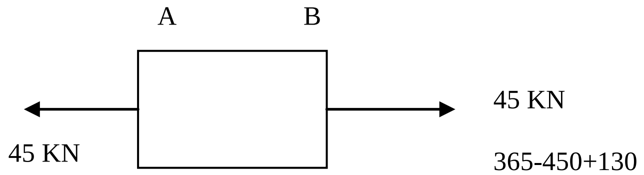
$P_1 = 45\,KN \qquad P_3 = 450\ KN \qquad P_4 = 130\ KN$

$-P_1 + P_2 - P_3 + P_4 = 0$

$-45 + P_2 - 450 + 130 = 0$

$P_2 = 365\ KN$

AB

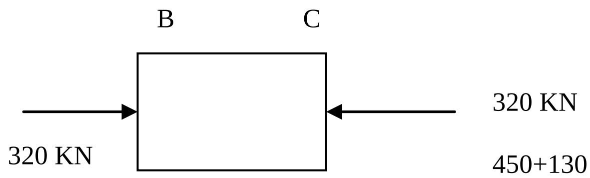


$$\delta l_{AB} = \frac{P_{AB} l_{AB}}{A_{AB} E}$$

$$= \frac{45 \times 10^3 \times 1200}{625 \times 2.1 \times 10^5}$$

$$\delta l_{AB} = 0.414\ \text{mm(Tensile)}$$

BC

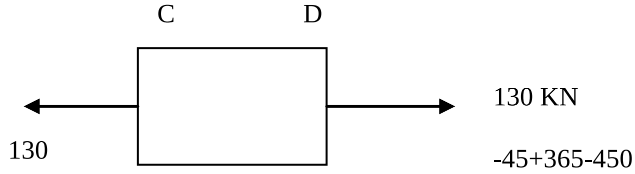


$$\delta l_{BC} = \frac{P_{BC} \times l_{BC}}{A_{BC} E}$$

$$= \frac{320 \times 10^3 \times 600}{2500 \times 2.1 \times 10^5}$$

$$\delta l_{BC} = 0.3657\ \text{mm (Comp)}$$

CD



$$\delta l_{CD} = \frac{P_{CD} l_{CD}}{A_{CD} E}$$

$$= \frac{130 \times 10^3 \times 900}{1250 \times 2.1 \times 10^5}$$

$$\delta l_{CD} = 0.4457\ \text{mm (Tansile)}$$

$$\boxed{\delta l = \delta l_{AB} - \delta l_{BC} + \delta l_{CD} = 0.4914\ \text{mm}}$$

19) A compound tube consists of a steel tube 140 mm internal diameter and 160 mm external diameter and an outer brass tube 160 mm internal diameter and 180 mm external diameter. The two tubes are of same length. The compound tube carries an axial compression load of 900 KN. Find the stress and the

load carried by each tube and the amount of it shorten. Length of each tube is 140 mm. Take E for steel as 2×10^{5}N/mm^2 & for brass is 1×10^{5}N/mm^2 (16 mark) (Nov /Dec 2016) (Nov /Dec 2017)

$D_s = 160$ $\quad D_b = 180$ mm $\quad P = 900\,KN$

$d_s = 140\,mm$ $\quad d_b = 160$ mm $\quad = 900\times10^3$

$\ell_s = \ell_b = \ell = 140$ mm $\quad E_s = 2\times10^5$ N / mm

$E_b = 1\times10^5$ N / mm

$A_s = \frac{\pi}{4}\left(D_s{}^2 - d_s{}^2\right)$ $\qquad A_b = \frac{\pi}{4}\left(D_b{}^2 - b_b{}^2\right)$

$= 4712.39mm^2$ $\qquad = 5340.71\ mm^2$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_b}{E_b}$$

$$\boxed{\sigma_s = \sigma_b \frac{E_s}{E_b} = 2\sigma_b} \qquad ...1$$

$P = \sigma_s A_s + \sigma_b A_b$

$900 = \sigma_b \times 4712.39 + 5340.71\ \sigma_b$

$\boxed{\sigma_b = 60.95\ N/mm^2}$ sub in 1, we get

$\boxed{\sigma_s = 121.91\ N/mm^2}$

$P_s = \sigma_s A_s = 574468.03\ N$

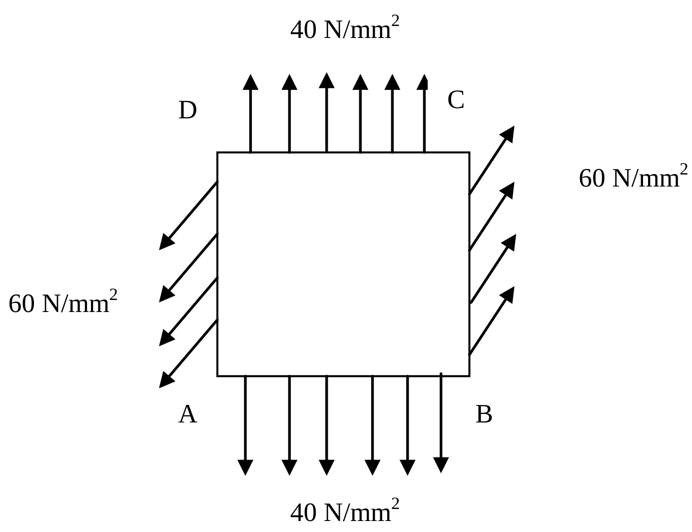
$P_b = \sigma_b A_b = 32516.27\ N$

20) Two members are connected to carry a tensile force of 80 KN by a lap joint with two number of 20 mm diameter bolt. Find the shear stress induced in the bolt (3)

(Nov / Dec 2016)

$$\tau = \frac{P}{A} = \frac{80\times10^3}{\frac{\pi}{4}\times20^2} = 254.65\ N/mm^2$$

21) A point in a strained material is subjected to the stress as shown in fig. Locate the principle phone and find the principle stress (7 marks) (Nov / Dec 2017)



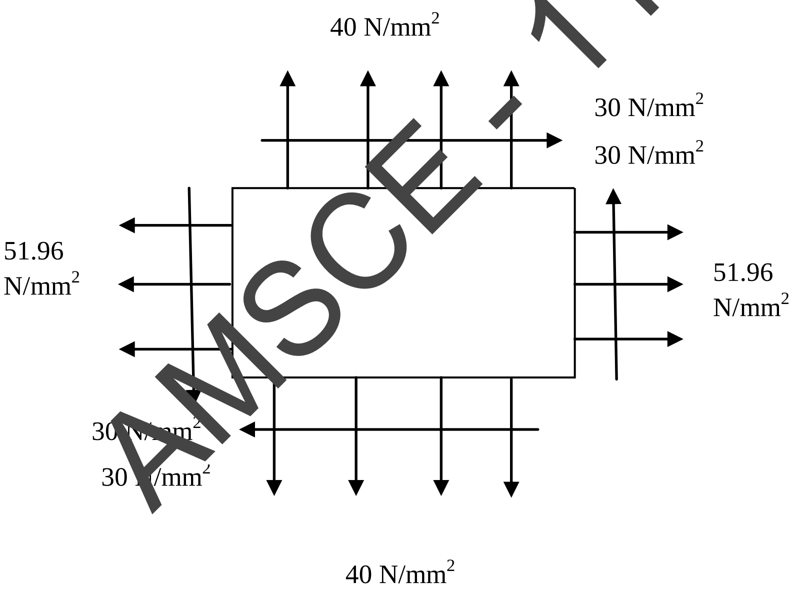
Stress on face AD & BC is not normal It is inclined at an angle 60° with face BC at AD stress can be resolved into two components

Stress normal to face (BC or AD) = 60 sin90°

$= 60 \times 0.866 = 51.96$ N/mm^2

Stress normal to face (BC or AD) = 60 cos90°

$= 60 \times 0.5 = 30$ N/mm^2



Major tensile stress (σ_1) = 51.9 N/mm^2

Minor tensile stress (σ_2) = 40 N/mm^2

Shear stress (τ) = 30 N/mm^2

Location of principle planes,

θ = Angle, which one of the principle planes makes with the stress of 40 N/mm^2

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2} = \frac{2 \times 30}{51.96 - 40} = 4.999$$
$$2\theta = \tan^{-1}(4.999) = 78°42' \text{ or } 258°42'$$
$$\theta = 39°21' \text{ or } 129°\ 21'$$

Principle stress

Major principle stress = $\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta$

$$= \frac{51.9+40}{2} + \sqrt{\left(\frac{51.9-40}{2}\right)^2 + 30^2}$$

$$= 45.98 + 30.6 = 76.58 \text{ N/mm}^2$$

Minor principle stress = $\frac{\sigma_1+\sigma_2}{2} - \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$

$$= \frac{51.9+40}{2} - \sqrt{\left(\frac{51.9-40}{2}\right)^2 + 30^2}$$

$$= 45.98 - 30.6 = 15.38 \text{ N/mm}^2$$

22) A steel rod of 20 mm diameter passes centrally through a copper tube of 50 mm external diameter and 40 mm internal diameter. The tube is closed at the end by rigid plates of negligible thickness. The nuts are tightened lightly on the projecting part of the rod. If the temperature of the assembly is raised by 50℃. Calculate the stresses developed in copper and steel. Take E for steel as $2\times10^5 \text{ N/mm}^2$ **and copper as** $1\times10^5 \text{ N/mm}^2$ **and as for steel and copper as** 12×10^{-6} °C & 18×10^{-6} °C **(6 mark)**

(Nov / Dec 2016)

$d_s = 20\text{mm}$, $D_c = 50\text{mm}$, $A_t = 50°\text{C}$,

$A_s = \frac{\pi}{4}\times 20^2 = 314.16 \text{ mm}^2$, $d_c = 40\text{mm}$,

$A_c = \frac{\pi}{4}\left(D_c^{\,2} - d_c^{\,2}\right) = A_c = 706.86 \text{ mm}^2$

$E_s = 2\times10^5 \text{ N/mm}^2$ $\alpha_s = 12\times10^{-6}/°\text{C}$

$E_c = 1\times10^5 \text{ N/mm}^2$ $\alpha_c = 18\times10^{-6}/°\text{C}$

$\sigma_s A_s = \sigma_c A_c$

$$\boxed{\sigma_s = \sigma_c \frac{A_c}{A_s} = 2.25\sigma_c} \qquad ...1$$

$$\frac{\sigma_c}{E_c} + \frac{\sigma_s}{E_s} = (\alpha_c - \alpha_s)\Delta t$$

$$\frac{\alpha_c}{1\times10^{-5}} + \frac{2.25\sigma_c}{2\times10^5} = 6\times10^{-6}\times50$$

$$2.215\sigma_c = 6\times10^{-6}\times50\times10^5 = 30$$

$$\boxed{\sigma_c = 14.11 \text{ N/mm}^2} \qquad \text{sub in 1}$$

$$\boxed{\sigma_s = 31.76 \text{ N/mm}^2}$$

23) A metallic bar 300 mm (x) × 100 mm(y) × 40 mm is subjected to a force of 5 KN tensile, 6KN (tensile) and 4 KN (tensile) along x,y,z direction respectively. Determine the change in volume of the block. Take E =2 × $\mathbf{10^5}$ **N/mm**^{2} **and Poisson's ratio = 0.25 (16 mark)**

(Nov /Dec 2014)

Solution

$x = 300\text{mm} \qquad y = 100\text{mm} \qquad z = 40\text{mm}$

$P_x = 5\,\text{KN} \qquad P_y = 6\,\text{KN} \qquad P_z = 4\,\text{KN}$

$$\sigma_x = \frac{P_x}{A_{yz}} = \frac{5\times10^3}{100\times40} = 1.25\ \text{N/mm}^2$$

$$\boxed{\sigma_x = 1.25\ \text{N/mm}^2}$$

$$\sigma_y = \frac{P_y}{A_{zx}} = \frac{6\times10^3}{30\times40} = 0.5\ \text{N/mm}^2$$

$$\boxed{\sigma_y = 0.5\ \text{N/mm}^2}$$

$$\sigma_z = \frac{P_z}{A_{xy}} = \frac{4\times10^3}{100\times300} = 0.133\ \text{N/mm}^2$$

$$\boxed{\sigma_z = 0.133\ \text{N/mm}^2}$$

$$e_x = \frac{\sigma_x}{E} - \frac{\sigma_y}{mE} - \frac{\sigma_z}{mE}$$

$$= \frac{1.25}{2\times10^5} - \frac{0.5\times0.25}{2\times10^5} - \frac{0.133\times0.25}{2\times10^5}$$

$$= \frac{1}{2\times10^5}\left[1.25 - 0.125 - 0.0332\right]$$

$$\boxed{e_x = 5.459\times10^{-6}}$$

$$e_y = \frac{\sigma_y}{E} - \frac{\sigma_x}{mE} - \frac{\sigma_z}{mE}$$

$$= \frac{0.5}{2\times10^5} - \frac{1.25\times0.25}{2\times10^5} - \frac{0.133\times0.25}{2\times10^5}$$

$$= \frac{1}{2\times10^5}\left[0.5 - 0.125 - 0.0332\right]$$

$$\boxed{e_y = 5.459\times10^{-6}}$$

$$e_z = \frac{\sigma_z}{E} - \frac{\sigma_x}{mE} - \frac{\sigma_y}{mE}$$

$$= \frac{0.133}{2\times10^5} - \frac{1.25\times0.25}{2\times10^5} - \frac{0.5\times0.25}{2\times10^5}$$

$$= \frac{1}{2\times10^5}\left[0.133 - 0.3125 - 0.125\right]$$

$$\boxed{e_z = -1.5225\times10^{-6}}$$

$$e_v = \frac{\delta V}{V} = e_x + e_y + e_z$$

$$\frac{\delta V}{V} = 5.459\times10^{-6}\times7.715\times10^{-7} - 1.5225\times10^{-6}$$

$$\delta V = 4.708\times10^{-6}\times300\times40\times40$$

$$\delta V = 5.6496\ \text{mm}^3$$

Part – C

1) (i) Draw stress strain curve for mild steel and explain the salient points on it. (7)

We have studied in chapter of simple stress and strain, that whenever some external system of forces acts on a body, it undergoes some deformation. If a body is stressed within its elastic limit, the deformation entirely disappears as soon as the forces are removed. It has been also found that beyond the elastic limit, the deformation does not disappear entirely, even after the removal of the forces and there remains some residual deformation. We study this phenomenon, in a greater detail by referring to a tensile test or stress-strain diagram) for a mild steel bar

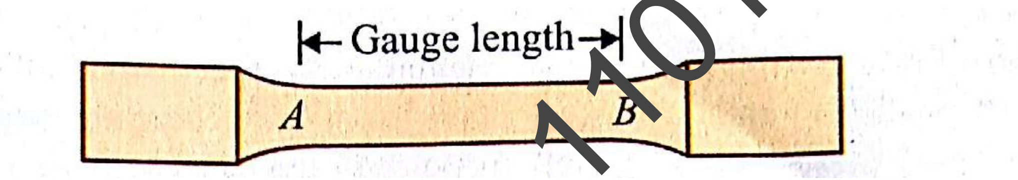


Fig. 11 a (i) Mild Steel Bar

Take a specimen of mild steel bar of uniform section as shown in Fig. 11 a (i). Let this bar be subjected to a gradually increasing pull (as applied by universal testing machine). If we plot the stresses along the vertical axis, and the corresponding strains along the horizontal axis and draw a curve passing through the vicinity of all such points, we shall obtain a graph as shown in Fig. **11 a (ii)**

We see from the graph, that

(1). From points O to A is a straight line, which represents that the stress is linearly proportional to strain.

(2). From A to B, the curve slightly deviates from the straight line but the material still shows behaviour until the curve reaches to point B, which is called elastic limit. Upto this point B if the load is removed the specimen will still come back to its original position. It is thus obvious, that the Hooke's law holds good only up to this limit. When the specimen is stressed beyond the elastic limit, the strain increases more quickly than the stress. This happens, because a sudden of the specimen takes place, without an appreciable increase in the stress (or load). This phenomenon is called yielding. The stress, corresponding to the point B is called the yield stress.

(3) After point B the material shows plastic behaviour. From points C to D the specimen shows perfectly plastic behaviour because specimen deforms without increase in the applied load. It may be noted, that if the load on the specimen is removed, then the elongation from points B to D will not disappear, but will remain as a permanent set.

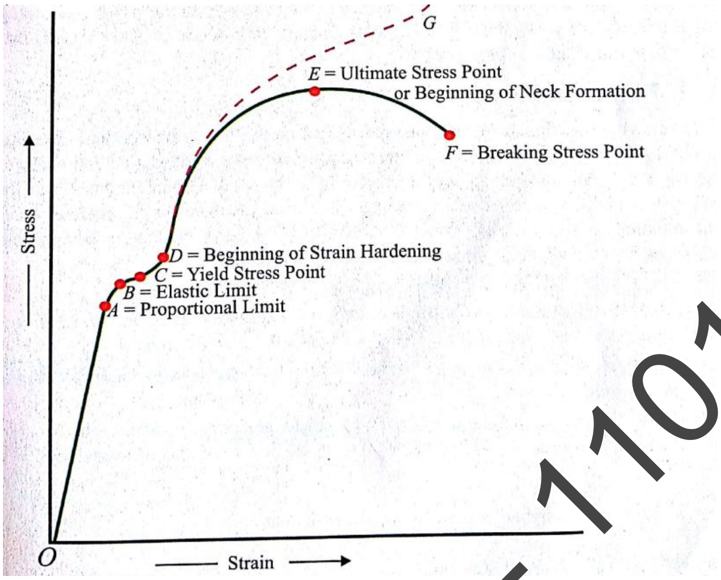


Fig. 11 a (ii) Stress-Strain Graph for a Mild Steel Bar

(4). At point D the specimen regains some strength and higher values of stresses are required, for higher strains. From points D to E is the region of strain hardening. During strain hardening the material undergoes the changes is crystalline structure, resulting in increased resistance of the material to further deformation.

(5). After point E the gradual increase in the length of the specimen is followed with the uniform reduction of its cross-sectional area. The work done during stretching the specimen, is transformed largely into heat and the specimen becomes hot. At point E, the stress attains its maximum value and is known as ultimate stress.

After the specimen has reached the ultimate stress, a neck is formed, which decreases the cross- sectional area of the specimen. From points E to F is the region of necking.

(6). A little consideration will show, that the stress (or load) necessary, to break away the specimen is less than the ultimate stress (or maximum load). The stress is therefore reduced until the specimen breaks away at the stress represented by the point F. At point F, the stress is known as the breaking stress.

Notes:

i) At this point, the elongation of a mild steel specimen is about 2%.

ii) The breaking stress (i e., stress at F which is less than that at E, appears to be somewhat misleading. As the formation of a neck takes place at E, which reduces the cross-sectional area. It causes the specimen suddenly to fail at F. If for each value of the strain between D and F the tensile load is divided by the reduced cross sectional area at the narrowest part of the neck, then the true stress-strain curve will follow the dotted line DG. However, it is an established practice, to calculate strains on the basis of original cross-sectional area of the specimen.

1) (ii) Derive a relation for change in length of a circular bar with uniformly varying diameter, subjected to an axial tensile load 'W' (8)

A bar of different lengths and of different diameters (and hence of different cross-sectional areas) is shown in Fig.12. Let this bar is subjected to an axial load P.

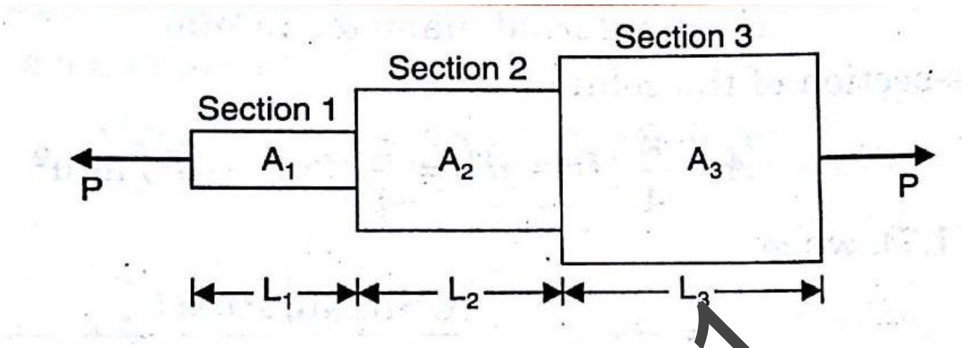


Fig.12

Though each section is subjected to the same axial load P, yet the stresses, strains and change in lengths will be different. The total change in length will be obtained by adding the changes in length of individual section.

Let P = Axial load acting on the bar,

L_1 = Length of section 1,

A_1 = Cross-sectional area of section 1,

L_2, A_2 = Length and cross-sectional area os section 2,

L_3, A_3 = Length and cross-sectional area of section 3, and

E = Young's modulus for the bar.

Then stress for thee section 1,

$$\sigma_1 = \frac{\text{Load}}{\text{Area of section 1}} = \frac{P}{A_1}$$

Similarly stresses for the section 2 and section 3 are given as,

$$\sigma_2 = \frac{P}{A_2} \text{ and } \sigma_3 = \frac{P}{A_3}$$

Using equations (1.5), the strains in different sections are obtained.

$$\therefore \text{ strain of section 1 , } e_1 = \frac{\sigma_1}{E} = \frac{P}{A_1 E} \qquad \left(\because \sigma_1 = \frac{P}{A_1}\right)$$

Similarly the strains of section 2 and section 3 are,

$$e_2 = \frac{\sigma_2}{E} = \frac{P}{A_2 E} \text{ and } e_3 = \frac{\sigma_3}{E} = \frac{P}{A_3 E}$$

But strain`in section 1= $\frac{\text{Change in length of section 1}}{\text{Length of section 1}}$

or $$e_1 = \frac{dL_1}{L_1}$$

where dL_1 = change in length of section 1.

∴ Change in length of section 1, $dL_1 = e_1L_1$

$$= \frac{PL_1}{A_1E} \qquad \left(\because e_1 = \frac{P}{A_1E}\right)$$

Similarly changes in length of section 2 and of section 3 are obtained as:

Change in length of section 2, $dL_2 = e_2L_2$

$$= \frac{PL_2}{A_2E} \qquad \left(\because e_2 = \frac{P}{A_2E}\right)$$

and change in length of section 3, $dL_3 = e_3L_3$

$$= \frac{PL_3}{A_3E} \qquad \left(\because e_3 = \frac{P}{A_3E}\right)$$

∴ Total change in the length of the bar,

$$dL = dL_1 + dL_2 + dL_3 = \frac{PL_1}{A_1E} + \frac{PL_2}{A_2E} + \frac{PL_3}{A_3E}$$

$$= \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right] \qquad ...(1.8)$$

Equation (1.8) is used when the young's modulus of different sections is same. If the Young's modulus of different sections is different , then total change in length of the bar is given by,

$$dL = P\left[\frac{L_1}{E_1A_1} + \frac{L_2}{E_2A_2} + \frac{L_3}{E_3A_3}\right] \qquad ...(1.9)$$