UNIT –II

Part – A

Transverse loading on Beans and stresses in Beam

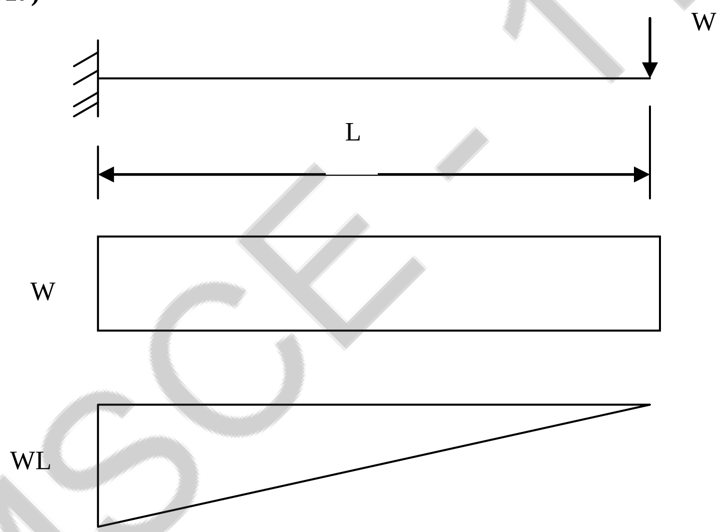
1) What is the ratio of maximum shear stress to the average shear stress in the case of solid circular section? (Apr/May 2019)

$$\frac{\tau_{\max}}{\tau_{avg}} = \frac{4}{3}$$

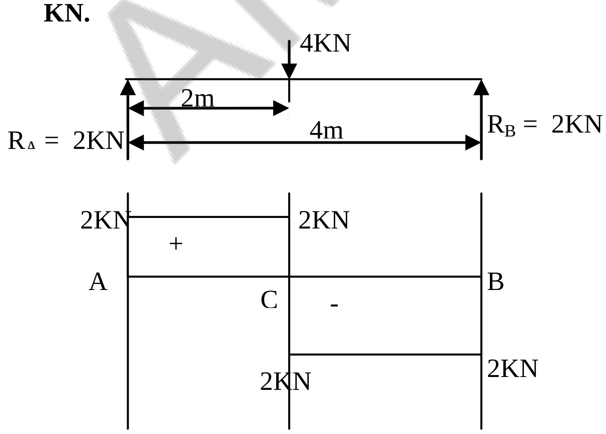
2) What is meant by shear stresses in beams? (Apr/May 2018)

When a beam is subjected to shear force and zero bending moment, then there will be only shear stresses in the beam. These stresses acting across the transverse section of the beam.

3) Draw the shear force and bending moment diagram for a cantilever of length L carrying a point load W at the free end. (Nov/Dec 2017)



4) Draw shear force diagram for a simply supported beam of length 4m carrying a central point load of 4 KN. (May/June 2017)



$$R_A + R_B = 4KN$$

$$M_A = 0 \Rightarrow 4R_B = 4KN \times 2$$

$$R_B = \frac{4 \times 2}{4} = 2KN$$

5) Prove that the shear stress distribution over a rectangular section due to shear force is parabolic.

(May/June 2017)

The figure shows a rectangular section of a beam of width b and depth d. Let F is the shear force acting at the section. Consider a level EF at a distance y from the neutral axis.

Shear stress , $\tau = F.\frac{A\bar{y}}{b \times \ell}$

Where A = Area of the section above y (i.e, shaded area ABFE) = $\left(\frac{d}{2} - y\right) \times b$

$\bar{y}$ = Distance of C.G of area A from neutral axis

$$= y + \frac{1}{2}\left(\frac{d}{2} - y\right) = y + \frac{d}{4} - \frac{y}{2} = \frac{y}{2} + \frac{d}{4}$$

$$= \frac{1}{2}\left(y + \frac{d}{2}\right)$$

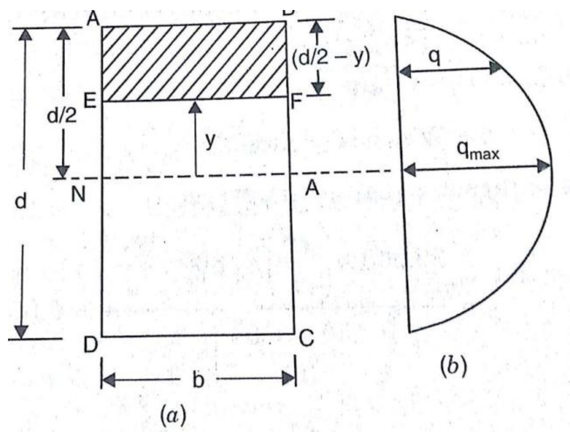
b = actual width of section at EF

I = M.O.I of whole section

Substituting the values in above equation

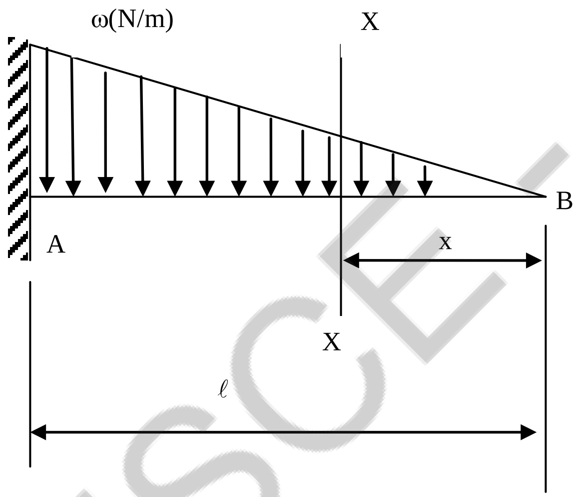
$$\tau = \frac{F\left(\frac{d}{2} - y\right) \times b \times \frac{1}{2}\left(y + \frac{d}{2}\right)}{b \times \ell}$$

$$= \frac{F}{2l}\left[\frac{d^2}{4} - y^2\right]$$ The variation of τ with respect y in parabola.



6) Draw the shear force diagram and bending moment diagram for the cantilever beam carriers uniformly varying load of zero initially at the freed end and w KN/m at the fixed end.

[Nov /Dec 2016]



SFD:

$$\text{SF at XX} = \frac{\omega x}{\ell} \times \frac{x}{2} = \frac{\omega x^2}{2\ell}$$

$$x = 0 \Rightarrow \text{SF at B} = 0$$

$$x = \ell \Rightarrow \text{SF at A} = \frac{\omega \ell}{2}$$

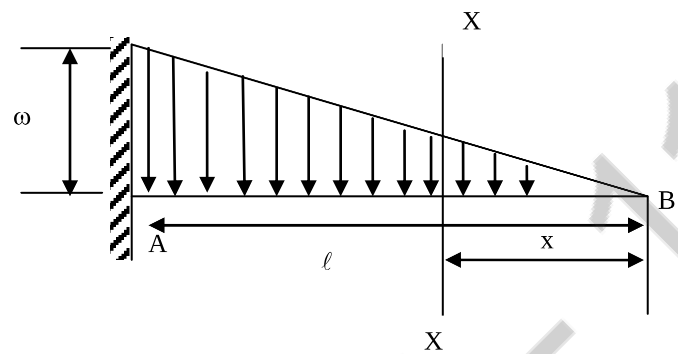
BMD:

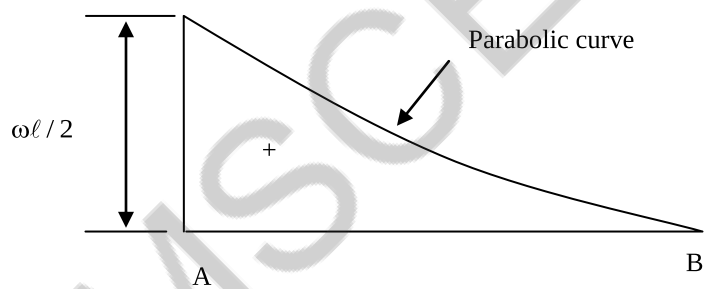
$$\text{BM at XX } = \frac{-\omega x}{\ell} \times \frac{x}{2} \times \frac{x}{3}$$

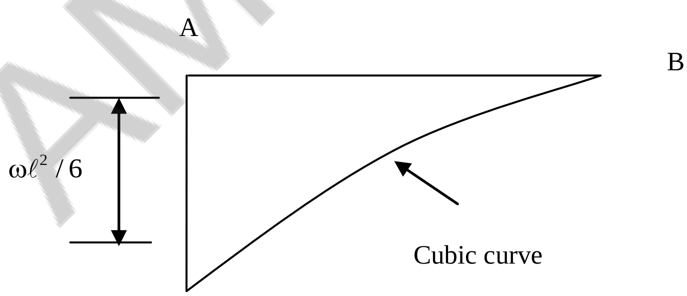
$$= \frac{-\omega x^3}{6\ell}$$

$x = 0 \Rightarrow \text{B.M at B} = 0$

$x = \ell \Rightarrow \text{B.M at A} = \frac{-\omega \ell^3}{6\ell} = \frac{-\omega \ell^2}{6}$







7) List out the assumption used to derive the simple bending equation[Nov/ Dec 2015, 2014, 2018]

1) The material is perfectly homogeneous and isotopic. It obeys hooks law.

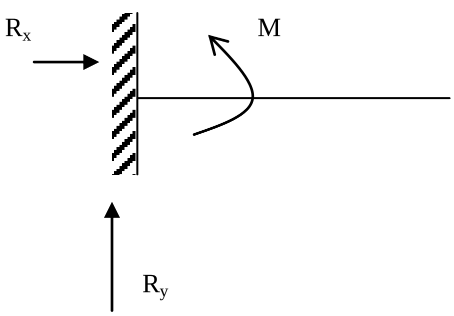
2) Transverse section, which are plane before bending, remains plane after bending

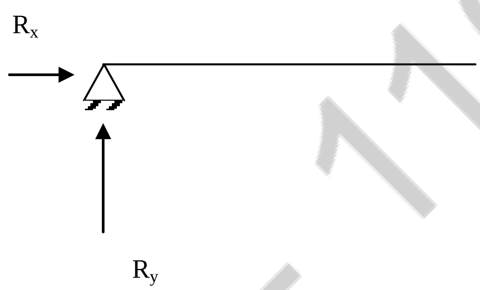
3) The radius of curvature of the beam is very large compared to the cross sectional dimension of the beam.

4) Each layer of the beam is free to expand or contract, independently of the layer above or below it.

8) Discuss the fixed and Hinged support

[May/ June 2016]





Resistance to the moment m = 0	No resistance to moment
Displacement at (x & y axis)	Resistance to Displacement x & y axis
$u_x = 0$	$u_x = 0$
$u_y = 0$	$u_y = 0$

9) What are the advantages of flitched beams [May /June 2016]

* It is used to strengthen the material. Ex. steel bars in concrete beam.

* Less space occupied.

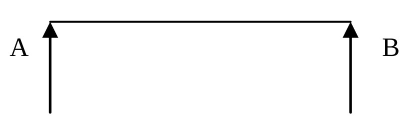
10) What is the type of beams? [Nov / Dec 2015]

a) Cantilever beams

A beam with on end free (B) fixed (A)

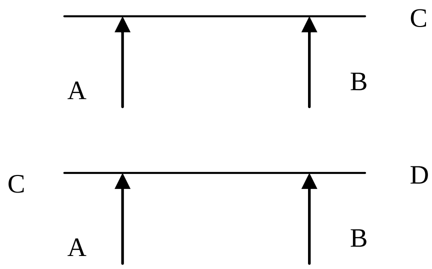
b) Simply supported beam (SSB)

A beam is resting freely on supports at is both ends (A&B)



C) Overhanging beam

One or both the end portion beyond the support

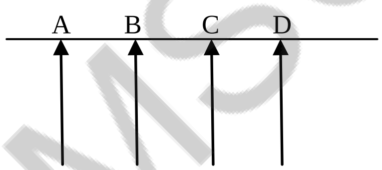


d) Fixed beam

A beam whose both ends are fixed

e) Continuous beam:

A beam which has more than two supports



11) Define a) sheer force b) bending moment [Apr /May 2015]

Sheer Force:

Algebraic sum of the forces acting on either right side or left side of the section

Bending moment

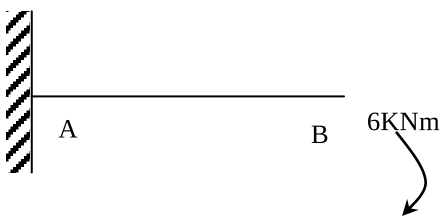
Algebraic sum of moment due to all forces acting on either right or left of the section

12) What is neutral axis of a beam under simple bending? [Apr/ May 2015]

The line of intersection of the neutral layer, with any normal cross section of a beam is known as neutral axis of that section.

To locate the neutral axis of a section, first find out the centroid of the section and then draw a line passing through this centroid and normal to the plane of bending. This line will be the neutral axis of the section.

13) Draw SFD for a 6m cantilever beam carrying a clockwise moment of 6KNm at free end [Nov/ Dec 2014]



SFD

No vertical force. So shear force is zero

14) What are flitched beams? (Nov/Dec 2017)

A beam which is constructed by two different materials is known as flitched or composite beam. It is used to reinforced the material and reduced the cost.

15) Mention the assumption made in the theory of simple bending?

Assumption made in the theory of pure Bending

- The material of the beam is homogeneous and isotropic.
- The value of young's modulus of elasticity is same in tension and compression
- The transverse section which were plane before bending remain plane after bending also
- The beam is initially straight and all longitudinal filaments bend into circular arcs with a common centre of curvature.
- The radius of curvature is large as compared to the dimension of the cross section
- Each layer of the beam is free to expand or contract, independently of the layer above or below it.

16) Define point of contra flexure? In which beam it occurs? (Apr/May 2018) (Nov/Dec 2018) (Apr/May 2019)

The point where the bending moments change its sign or zero is called point contra flexure. It occurs in overcharging beam.

17) Write the theory of simple bending equation?

$$\frac{M}{I} = \frac{F}{Y} = \frac{E}{R}$$

M – Maximum bending moments
I – Moments of inertia
F – Maximum stress induced
Y – Distance from the neutral axis

E – Young’s modules
R – Radius of curvature

18) Define beam?

BEAM is a structural member which is support along the length and subjected to external loads acting transversely (i.e) perpendicular to the centre line of the beam.

19) What is mean by transverse loading on beam?

If a load is acting on the beam perpendicular to the axis of the beam then it is called transverse loading.

20) What is mean by positive or sagging BM?

BM is said to positive if moment on left side of beam is clockwise or right side of the beam is counter clockwise.

21) What is mean by negative or hogging BM?

BM is said to negative if moment on left side of beam is counter clockwise or right side of the beam is clockwise.

22) When will bending moments is maximum?

BM will be maximum, when shear force change its sign.

23) What are the types of loads?

- Concentrated load or point load
- Uniform distributed load
- Uniform varying load.

24) Define “Section Modulus”

It is the ratio of moment of inertia to the distance of plane from the neutral axis.

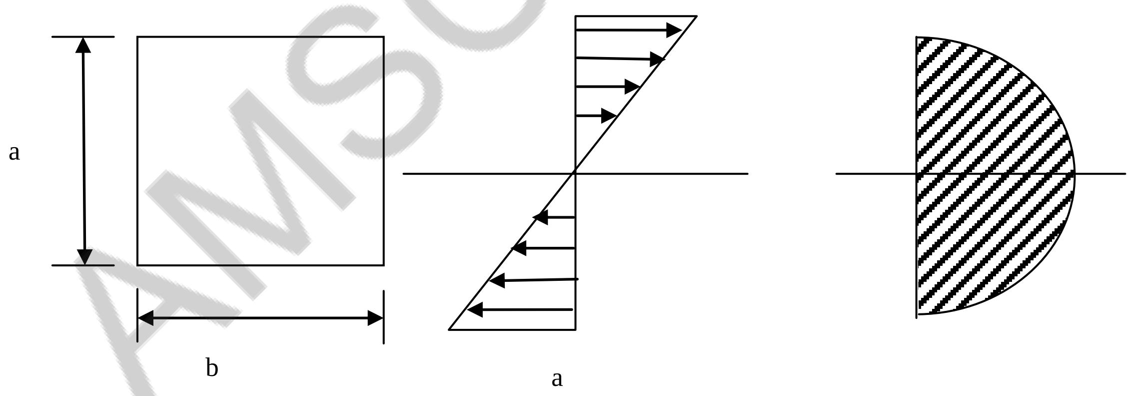
25) What is moment of resistance of the section?

It is product of the section modules and stress at that section

26) Define shear stress distribution

The variation of shear stress along the depth of the beam is called shear stress distribution.

27) Sketch a) the bending stress distribution b) shear stress distribution for a beam of rectangular cross section



28) A rectangular beam of 150 mm wide & 250 mm deep is subjected to a max. shear force of 30KN. Determine i) Avg. shear stress ii) max. shear stress iii) shear stress at a distance of 25 mm above the neutral axis

$$A = b \times d = 150 \times 250 = 37500\,mm^2$$

i) Avg shear stress :

$$q_{avg} = \frac{F}{A} = \frac{30 \times 10^3}{37500} = 0.8 N/mm^2$$

ii) Max shear stress :

$$q_{max} = 1.5 q_{avg} = 1.5 \times 0.8 = 1.2 N/mm^2$$

$$iii)\; q = \frac{F}{2I}\left(\frac{d^4}{4} - y^2\right)$$

$$I = \frac{bd^3}{12}$$

$$= \frac{150 \times 250^3}{12}$$

$$= 195312500\,mm^4$$

$$y = 25mm$$

$$q = \frac{30 \times 10^3}{2 \times 195312500}\left(\frac{250^2}{4} - 25^2\right)$$

$$q = 1.152\ N/mm^2$$

PART – B

1) Draw the shear force and bending moment diagram for the overhanging beam carrying uniformly distributed load of 2kN/m over the entire length and a point load of 2kN as shown in fig. Locate the point of contraflexure. (Apr/May 2019)

Sol. First calculate the reactions R_A and R_B.

Taking moments of all forces about A, we get

$$R_B \times 4 = 2 \times 6 \times 3 + 2 \times 6 = 36 + 12 = 48$$

$$\therefore \quad R_B = \frac{48}{4} = 12 \text{ kN}$$

and $$R_A = \text{Total load} - R_B = (2 \times 6 + 2) - 12 = 2 \text{ kN}$$

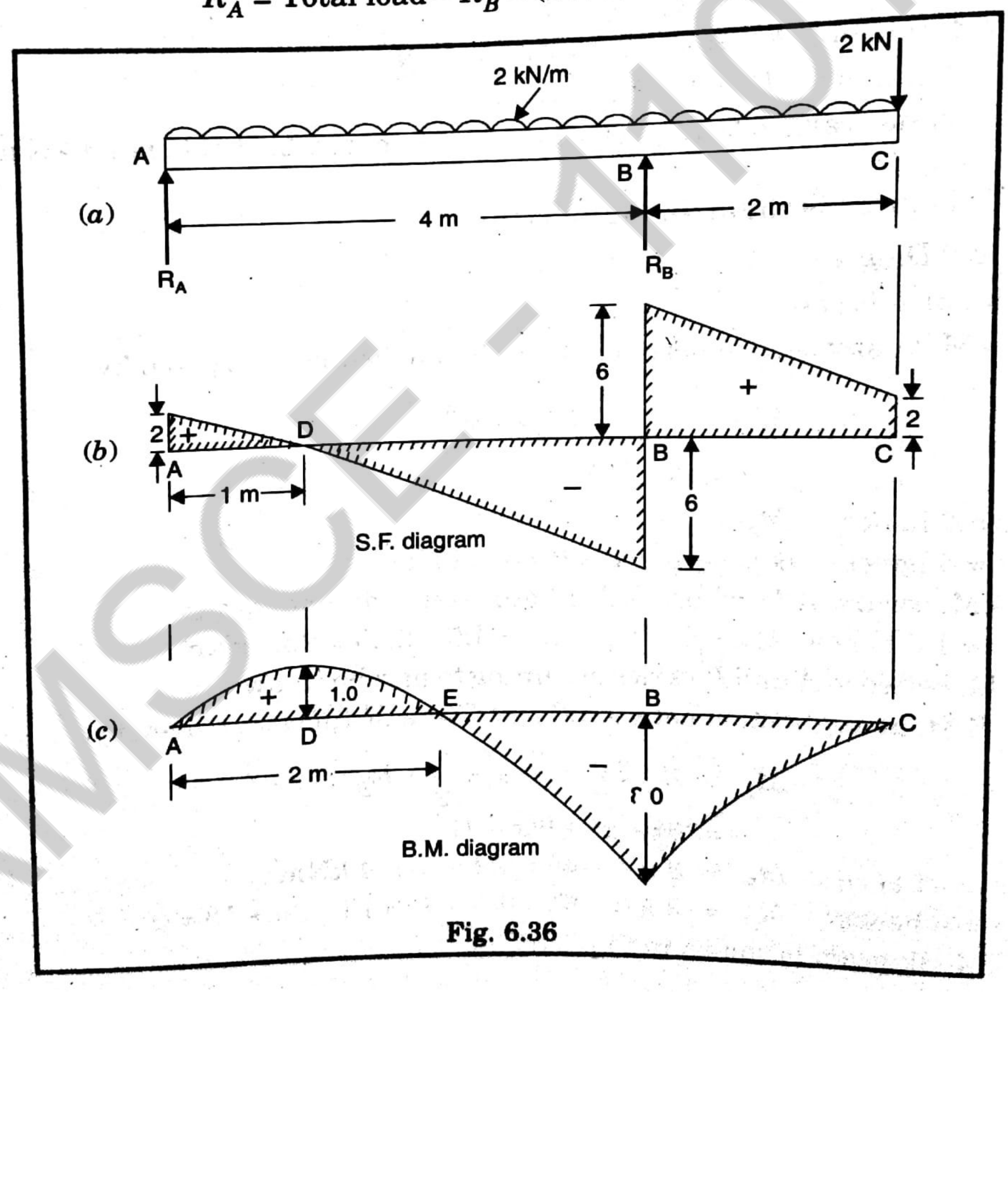


Fig. 6.36

S.F. Diagram

S.F. at $A = +R_A = +2$ kN

(*i*) The S.F. at any section between A and B at a distance x from A is given by,

$$F_x = +R_A - 2 \times x$$
$$= 2 - 2x \quad ...(i)$$

At A, $x = 0$ hence $F_A = 2 - 2 \times 0 = 2$ kN

At B, $x = 4$ hence $F_A = 2 - 2 \times 4 = -6$ kN

The S.F. between A and B varies according to straight line law. At A, S.F. is positive and at B, S.F. is negative. Hence between A and B, S.F. is zero. The point of zero S.F. is obtained by substituting $F_x = 0$ in equation (*i*).

$$\therefore \quad 0 = 2 - 2x \quad \text{or} \quad x = \frac{2}{2} = 1 \text{ m}$$

The S.F. is zero at point D. Hence distance of D from A is 1 m.

(*ii*) The S.F. at any section between B and C at a distance x from A is given by,

$$F_x = +R_A - 2 \times 4 + R_B - 2(x - 4)$$
$$= 2 - 8 + 12 - 2(x - 4) = 6 - 2(x - 4) \quad ...(ii)$$

At B, $x = 4$ hence $F_B = 6 - 2(4 - 4) = +6$ kN

At C, $x = 6$ hence $F_C = 6 - 2(6 - 4) = 6 - 4 = 2$ kN

The S.F. diagram is drawn as shown in Fig. 6.36 (*b*).

B.M. Diagram

B.M. at A is zero

(*i*) B.M. at any section between A and B at a distance x from A is given by,

$$M_x = R_A \times x - 2 \times x \times \frac{x}{2} = 2x - x^2 \quad ...(iii)$$

The above equation shows that the B.M. between A and B varies according to parabolic law.

At A, $x = 0$ hence $M_A = 0$

At B, $x = 4$ hence $M_B = 2 \times 4 - 4^2 = -8$ kNm

Max. B.M. is at D where S.F. is zero after changing sign

At D, $x = 1$ hence $M_D = 2 \times 1 - 1^2 = 1$ kNm

The B.M. at C is zero. The B.M. also varies between B and C according to parabolic law. Now the B.M. diagram is drawn as shown in Fig. 6.36 (*c*).

Point of Contraflexure

This point is at E between A and B, where B.M. is zero after changing its sign. The distance of E from A is obtained by putting $M_x = 0$ in equation (*iii*).

$$\therefore \quad 0 = 2x - x^2 = x(2 - x)$$
$$2 - x = 0$$

and $\quad x = $ **2 m. Ans.**

2) A timber beam 100mm wide and 200mm deep is to be reinforced by bolting on two steel flitches each 150mm by 12.5mm in section. Calculate the moment of resistance when flitches are attached symmetrically at the top and bottom. Allowable stress in timber is 6 N/mm^2. $E_s = 2\times 10^5$ N/mm^2 and $E_t = 1\times 10^4$ N/mm^2 (Apr/May 2019)

1st Case. *Flitches attached symmetrically at the top and bottom.*

(See Fig. 7.31).

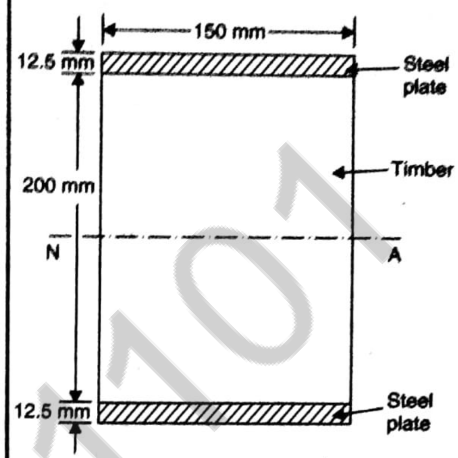


Fig. 7.31

Let suffix 1 represents steel and suffix 2 represents timber.

Width of steel, $b_1 = 150$ mm
Depth of steel, $d_1 = 12.5$ mm
Width of timber, $b_2 = 150$ mm
Depth of timber, $d_2 = 200$ mm
Number of steel plates $= 2$
Max. stress in timber, $\sigma_2 = 6$ N/mm^2
E for steel, $E_1 = E_s = 2 \times 10^5$ N/mm^2
E for timber, $E_2 = E_t = 1 \times 10^4$ N/mm^2

Distance of extreme fibre of timber from N.A.,

$y_2 = 100$ mm

Distance of extreme fibre of steel from N.A.,

$y_1 = 100 + 12.5 = 112.5$ mm.

Let $\sigma_1^* =$ Max. stress in steel

$\sigma_1 =$ Stress in steel at a distance of 100 mm from N.A.

Now we know that strain at the common surface is same. The strain at a common distance of 100 mm from N.A. is steel and wood would be same. Hence using equation (7.11), we get

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

$$\therefore \quad \sigma_1 = \frac{E_1}{E_2} \times \sigma_2 = \frac{2\times 10^5}{1\times 10^4} \times 6 = 120 \text{ N/mm}^2.$$

But σ_1 is the stress in steel at a distance of 100 mm from N.A. Maximum stress in steel would be at a distance of 112.5 mm from N.A. As bending stresses are proportional to the distance from N.A.

$$\text{Hence} \quad \frac{\sigma_1}{100} = \frac{\sigma_1^*}{112.5}$$

$$\therefore \quad \sigma_1^* = \frac{112.5}{100} \times \sigma_1 = \frac{112.5}{100} \times 120 = \mathbf{135\ N/mm^2}. \quad \textbf{Ans.}$$

Now moment of resistance of steel is given by

$$M_1 = \frac{\sigma_1^*}{y_1} \times I_1 \text{ (where } \sigma_1^* \text{ is the maximum stress in steel)}$$

$$= \frac{135}{112.5} \times I_1$$

where I_1 = M.O.I. of two steel plates about N.A.

= 2 × [M.O.I. one steel plate about its C.G. + Area of one steel plate × (Distance between its C.G. and N.A.)^2]

$$= 2 \times \left[\frac{b_1 d_1^3}{12} + b_1 d_1 \times \left(100 + \frac{d_1}{2}\right)^2\right]$$

$$= 2 \times \left[\frac{150 \times 12.5^3}{12} + 150 \times 12.5 \times \left(100 + \frac{12.5}{2}\right)^2\right]$$

$$= 2 \times [24414.06 + 21166992.18]$$

$$= 42382812.48 \text{ mm}^4$$

$$\therefore \quad M_1 = \frac{135}{112.5} \times 42382812.48$$

$$= 50859374.96 \text{ Nmm} = 50859.375 \text{ Nm}$$

Similarly, $M_2 = \frac{\sigma_2}{y_2} \times I_2$

$$= \frac{6}{100} \times \frac{150 \times 200^3}{12}$$

$$= 6000000 \text{ Nmm} = 6000 \text{ Nm}$$

∴ Total moment of resistance is given by,

$$M = M_1 + M_2$$

$= 50859.375 + 6000 =$ **56859.375 Nm. Ans.**

3) Draw a shear force and bending moment diagram for a simply supported beam of length 9m and carrying a uniformly distributed load of 10kN/m for a distance of 6m from the left end. Also calculate the maximum bending moment on the section. (Nov/Dec 2018)

Sol. First calculate reactions R_A and R_B.

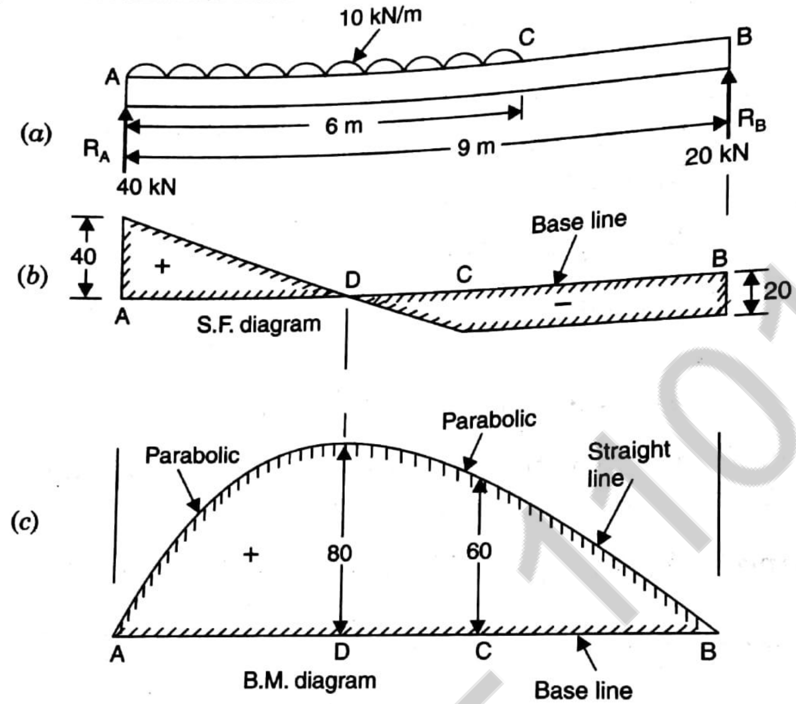


Fig. 6.28

Taking moments of the forces about A, we get

$$R_B \times 9 = 10 \times 6 \times \frac{6}{2} = 180$$

$$\therefore \quad R_B = \frac{180}{9} = 20 \text{ kN}$$

$$\therefore \quad R_A = \text{Total load on beam} - R_B = 10 \times 6 - 20 = 40 \text{ kN}.$$

Shear Force Diagram

Consider any section at a distance x from A between A and C. The shear force at the section is given by,

$$F_x = +R_A - 10\,x = +40 - 10\,x \qquad ...(i)$$

Equation (i) shows that shear force varies by a straight line law between A and C.

At A, $x = 0$ hence $\quad F_A = +40 - 0 = 40$ kN

At C, $x = 6$ m hence $\quad F_C = +40 - 10 \times 6 = -20$ kN

The shear force at A is + 40 kN and at C is – 20 kN. Also shear force between A and C varies by a straight line. This means that somewhere between A and C, the shear force is zero. Let the S.F. is zero at x metre from A. Then substituting the value of S.F. (*i.e.*, F_x) equal to zero in equation (i), we get

$$0 = 40 - 10x$$

$$\therefore \quad x = \frac{40}{10} = 4 \text{ m}$$

Hence shear force is zero at a distance 4 m from A.

The shear force is constant between C and B. This equal to – 20 kN.

Now the shear force diagram is drawn as shown in Fig. 6.28 (b). In the shear force diagram, distance AD = 4 m. The point D is at a distance 4 m from A.

B.M. Diagram

The B.M. at any section between A and C at a distance x from A is given by,

$$M_x = R_A \times x - 10\,.\,x\,.\,\frac{x}{2} = 40x - 5x^2 \qquad ...(ii)$$

Equation (ii) shows that B.M. varies according to parabolic law between A and C.

At A, $x = 0$ hence $\quad M_A = 40 \times 0 - 5 \times 0 = 0$

At C, $x = 6$ m hence $\quad M_C = 40 \times 6 - 5 \times 6^2 = 240 - 180 = +60$ kNm

At D, $x = 4$ m hence $\quad M_D = 40 \times 4 - 5 \times 4^2 = 160 - 80 = +80$ kNm

The bending moment between C and B varies according to linear law.

B.M. at B is zero whereas at C is 60 kNm.

The bending moment diagram is drawn as shown in Fig. 6.28 (c).

Maximum Bending Moment

The B.M. is maximum at a point where shear force changes sign. This means that the point where shear force becomes zero from positive value to the negative or *vice-versa,* the B.M. at that point will be maximum. From the shear force diagram, we know that at point D, the shear force is zero after changing its sign. Hence B.M. is maximum at point D. But the B.M. at D is + 80 kNm.

$\therefore$ **Max. B.M. = + 80 kN. Ans.**

4) A simply supported wooden beam of span 1.3m having a cross section 150mm wide by 250mm deep carries a point load W at the centre. The permissible stresses are 7 N/mm^2 in bending 1 N/mm^2 in shearing. Calculate the safe load W. (Nov/Dec 2018)

Sol. Given :

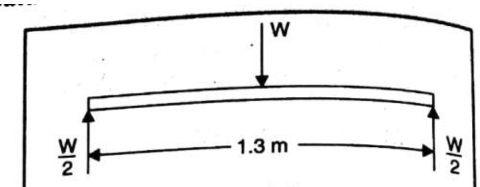


Fig. 8.6

Span, $L = 1.30$ mm

Width, $b = 150$ mm

Depth, $d = 250$ mm

Bending stress, $\sigma = 7$ N/mm^2

Shearing stress, $\tau = 1$ N/mm^2

Maximum B.M., $M = \dfrac{W \times L}{4} = \dfrac{W}{2} \times 1.3$

Nm

$$= \frac{W}{4} \times 1.3 \times 1000 \text{ Nmm} = 325\, W \text{ Nmm}$$

Maximum S.F. $= \dfrac{W}{2}$ N.

(i) Value of W for bending stress consideration

Using bending equation

$$\frac{M}{I} = \frac{\sigma}{y} \qquad ...(i)$$

where $M = 325\, W$ Nmm

$$I = \frac{bd^3}{12} = \frac{150 \times 250^3}{12} = 195312500 \text{ mm}^4$$

$\sigma = 7$ N/mm^2

and $y = \dfrac{d}{2} = \dfrac{205}{2} = 125.$

Substituting these values in the above equation (*i*), we get

$$\frac{325W}{195312500} = \frac{7}{125}$$

$\therefore$ $$W = \frac{7 \times 195312500}{325 \times 125} = 33653.8 \text{ N.}$$

(ii) Value of W for shear stress consideration

Average shear stress,

$$\tau_{avg} = \frac{\text{Shear force}}{\text{Area}} = \frac{\left(\frac{W}{2}\right)}{b \times d} = \frac{W}{2 \times 150 \times 250}$$

[redacted]

Maximum shear stress is given by equation (8.4)

$\therefore$ $$\tau_{max} = \frac{3}{2} \times \tau_{avg}$$

But $$\tau_{max} = 1 \text{ N/mm}^2$$

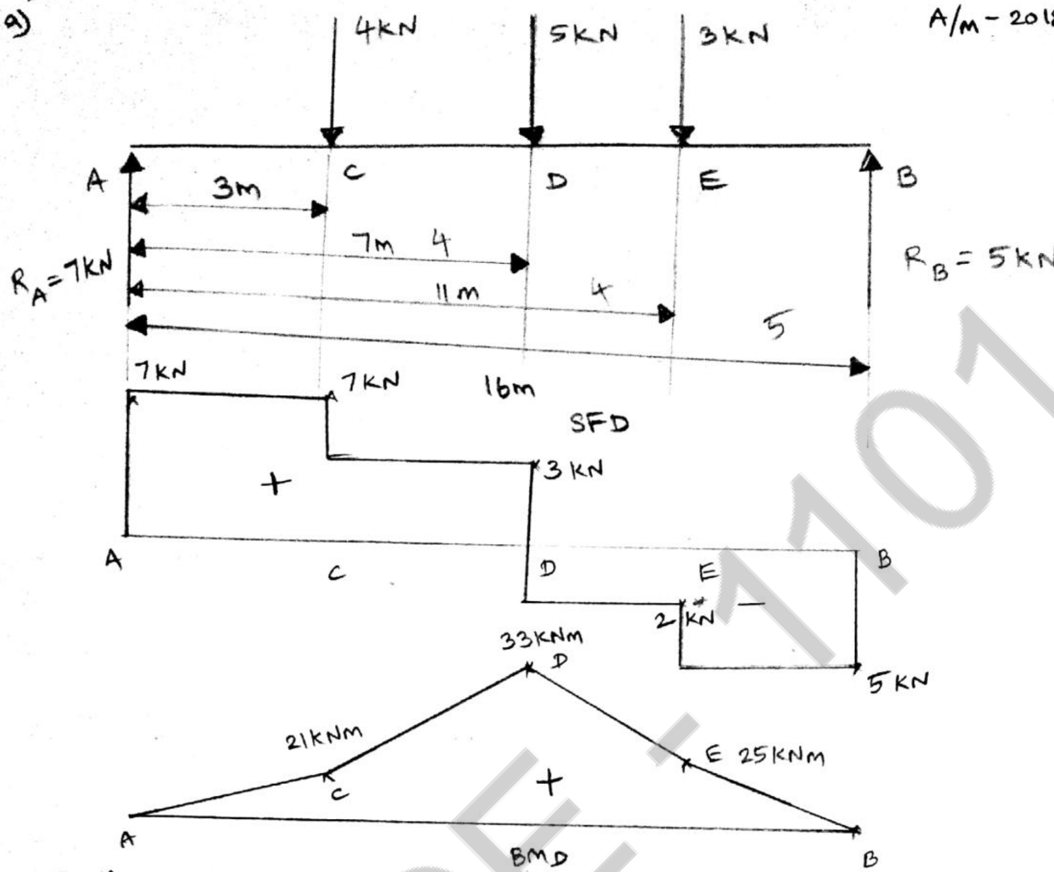
$\therefore$ $$1 = \frac{3}{2} \times \frac{W}{2 \times 150 \times 250}$$

or $$W = \frac{2 \times 2 \times 150 \times 250}{3} = 50000 \text{ N.}$$

Hence, the safe load is minimum of the two values (*i.e.*, 33653.8 and 50000 N) of *W*. Hence safe load is 33653.8 N. **Ans.**

5) A simply supported beam of 16m effective span carries the concentrated loads of 4kN, 5kN and 3kN at distances 3m, 7m and 11m respectively from the left end support. Calculate maximum shearing force and bending moment. Draw the S.F and B.M diagrams (Apr/May 2018)

12)
a) A/M - 2018



To find,

R_A & R_B, $R_A + R_B = 4+5+3 = 12\,KN \longrightarrow (1)$

$\Sigma M_A = 0 \Rightarrow \quad -4\times3 - 5\times7 - 3\times11 + R_B\times16 = 0$

$16R_B = 80$

$\boxed{R_B = 5KN}$ subs. in (1),

we get, $\boxed{R_A = 7KN}$

SF

SF at B $= -5KN$

L	R
+ ↑	↓
− ↓	↑

" " E $= -5+3$
$= -2KN$

" " D $= -2+5$
$= +3KN$

" " C $= +3+4$
$= +7KN$

" " A $= +7KN$

BM

$M_A = M_B = 0$ (End support)

$M_E = 5\times5 = 25\,KNm$

$M_D = 5\times9 - 3\times4 = 33\,KNm$

$M_C = 5\times13 - 3\times8 - 5\times4$
$= 21\,KNm$

6) A timber beam of rectangular section is support a load of 50kN uniformly distributed over a span of 4.8m when beam is simply supported. If the depth of section is to be twice the breath, and the stress in the timber is not to exceed 7 N/mm^2, find the dimensions of the cross section. (Apr/May 2018)

$$W = 50kN$$

$$l = 4.8m$$

$$d = 2b$$

$$\sigma_b = 7N/mm^2$$

$$Z = \frac{bd^2}{6} = \frac{b(2\text{b})^2}{6} = \frac{2b^3}{3}$$

we know that,

$$\text{SSB with UDL, M=}\frac{wl^2}{8} = \frac{Wl}{8}$$

$$\text{M=}\frac{50x10^3x4.8}{8} = 30000Nm = 30000000Nmm$$

$$\text{M=}\sigma_{\max}xZ$$

$$30000000=7x\frac{2b^3}{3} \Rightarrow \text{b=185.94} \approx \text{186mm}$$

d=2b=372mm

7) A cantilever of length 2m carries a uniformly distributed load of 2kN/m length over the whole length and a point load of 3kN at the free end. Draw the S.F and B.M diagram for the cantilever. (Nov/Dec 2017)

Sol. Given :

Length,	$L = 2.0$ m
U.D.L.,	$w = 2$ kN/m length
Point load at free end	= 3 kN

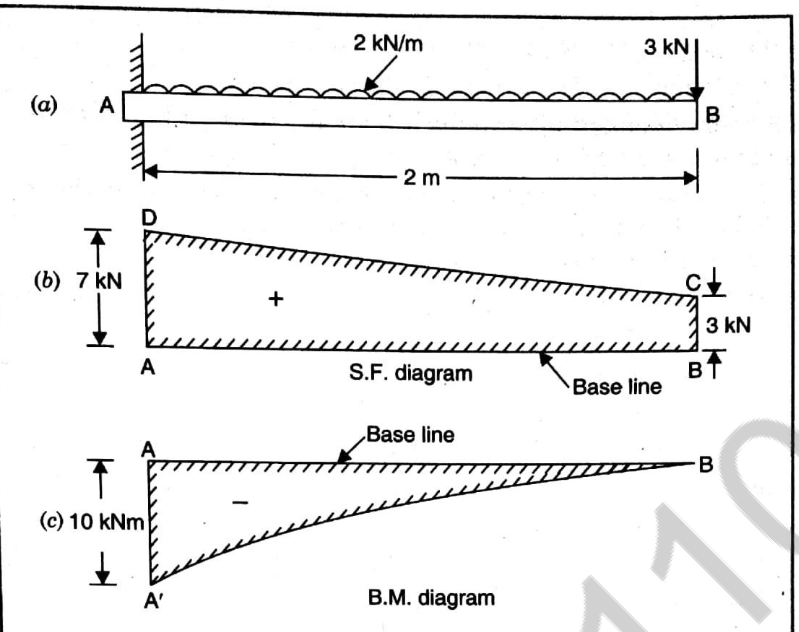


Fig. 6.18

Shear Force Diagram

The shear force at $B = 3$ kN

Consider any section at a distance x from the free end B. The shear force at the section is given by,

$$F_x = 3.0 + w.x \qquad \text{(+ve sign is due to downward force on right portion of the section)}$$

$$= 3.0 + 2 \times x \qquad (\because \quad w = 2 \text{ kN/m})$$

The above equation shows that shear force follows a straight line law.

At B, $x = 0$ hence $F_B = 3.0$ kN

At A, $x = 2$ m hence $F_A = 3 + 2 \times 2 = 7$ kN.

The shear force diagram is shown in Fig. 6.18 (b) in which $F_B = BC = 3$ kN and $F_A = AD = 7$ kN. The points C and D are joined by a straight line.

Bending Moment Diagram

The bending moment at any section at a distance x from the free end B is given by,

$$M_x = -\left(3x + wx \cdot \frac{x}{2}\right)$$

$$= -\left(3x + \frac{2x^2}{2}\right) \qquad (\because \quad w = 2 \text{ kN/m})$$

$$= -(3x + x^2) \qquad ...(i)$$

(The bending moment will be negative as for the right portion of the section, the moment of loads at x is clockwise).

Equation (*i*) shows that the B.M. varies according to the parabolic law. From equation (*i*), we have

At *B*, $x = 0$ hence $M_B = -(3 \times 0 + 0^2) = 0$

At *A*, $x = 2$ m hence $M_A = -(3 \times 2 + 2^2) = -10$ kN/m

Now the bending moment diagram is drawn as shown in Fig. 6.18 (*c*). In this diagram, *AA′* = 10 kNm and points *A′* and *B* are joined by a parabolic curve.

8) A beam is simply supported and carries a uniformly distributed load of 40kN/m run over the whole span. The section of the beam is rectangular having depth as 500mm. If the maximum stress in the material of the beam is 120 N/mm^2 and moment of inertia of the section is 7x10^{8}mm^4, find the span of the beam. (Nov/Dec 2017)

$$w = 40kN/m = 40000N/m$$

$$d = 500mm$$

$$\sigma_{max} = 120N/mm^2$$

$$I = 7x10^8 mm^4$$

$$Z = \frac{I}{y_{max}}$$

$$y_{max} = \frac{d}{2} = \frac{500}{2} = 250mm$$

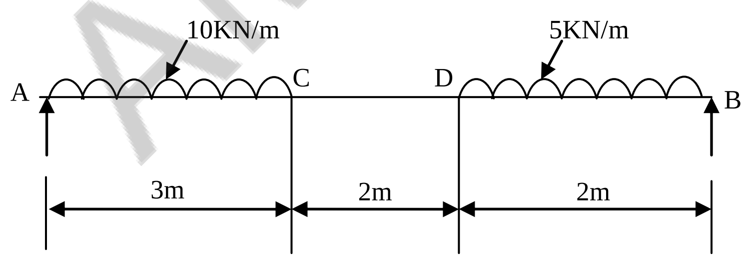
$$Z = \frac{I}{y_{max}} = \frac{7x10^8}{250} = 28x10^5 mm^3$$

$$M = \frac{wl^2}{8} = \frac{40000xl^2}{8} = 5000l^2 Nm = 5000l^2 x1000 Nmm$$

$$M = \sigma_{max} xZ$$

$$5000l^2 x1000 = 120x28x10^5 \quad \Rightarrow \quad \text{l=8.197mm} \approx 8.2\text{mm}$$

9) Draw shear force diagram and bending moment diagram for the beam given in fig.2 (May/June 2017)



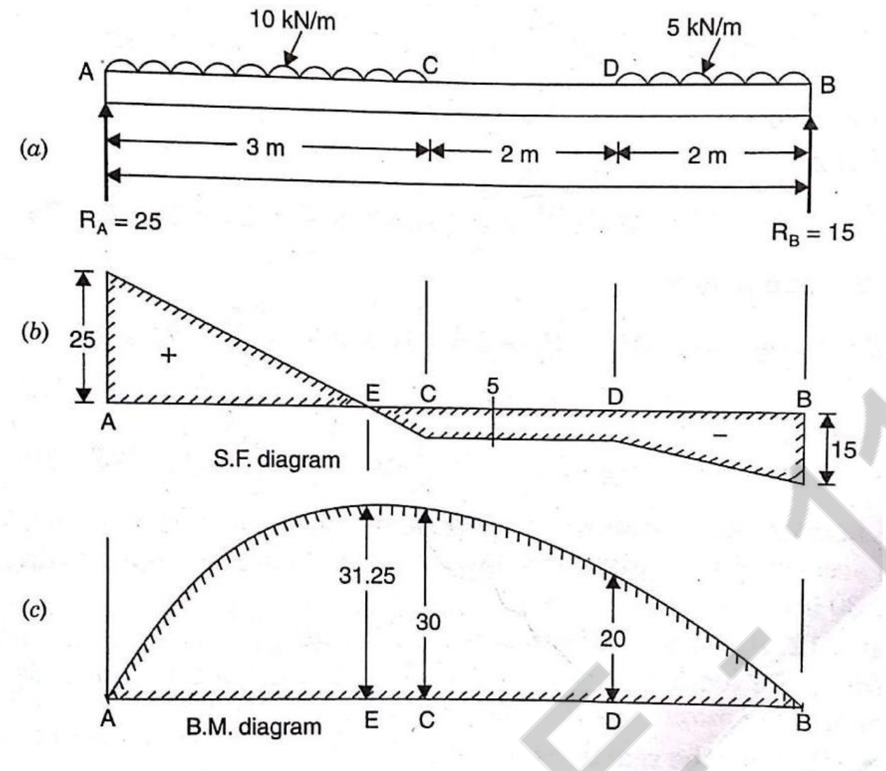


Fig. 2

Solution.

First calculate the reactions R_A and R_B

Taking moments of all forces about A, we get

$$R_B \times 7 = 10 \times 3 \times \frac{3}{2} + 5 \times 2 \times \left(3 + 2 + \frac{2}{2}\right) = 45 + 60 = 105$$

$$\therefore \qquad R_B = \frac{105}{7} = 15\text{kN}$$

and $\qquad R_A$ = Total load on beam-R_B

= (10 x 3 + 5 x 2) – 15 = 40 – 15 = 25 kN

S.F Diagram

The shear force At A is + 25kN

The shear force at C = R_A – 3 x 10 = +25 – 30 = -5kN

The shear force varies between A and C by a straight line law.

The shear force between C and D is constant and equal to -5kN

The shear force at B is -15kN

The shear force between D and B varies by a straight line law.

The shear force diagram is drawn as shown in Fig.2(b)

The shear force is zero at point E between A and C . Let us find the location of E from A. Let the point E at a be distance x from A.

The shear force at E = R_A – 10 x x= 25 – 10x

But shear force at E = 0

∴ 25 – 10x = 0 or 10x = 25

Or $x = \frac{25}{10} = 2.5\text{m}$

B.M.Diagram

B.M.. at A is zero

B.M. at B is zero

B.M. at C, $M_c = R_A \times 3 - 10 \times 3 \times \frac{3}{2} = 25 \times 3 - 45 = 75 - 45 = 30$ kNm

At E, x = 2.5 and hence

B.M. at E, $M_E = R_A \times 2.5 - 10 \times 2.5 \times \frac{2.5}{2} = 25 \times - 5 \times 6.25$

$= 62.5 - 31.25 = 31.25$ kNm

B.M. at D, $M_D = 25(3+2) - 10 \times 3 \times \left(\frac{3}{2}+2\right) = 125 - 105 = 20$ kNm

The B.M. between AC and between BD varies according to parabolic law. But B.M. between C and D varies according to straight line law. Now the bending moment diagram is drawn as shown in Fig.2.(c)

10) A beam of square section is used as a beam with one diagonal horizontal. The beam is subjected to a shear force F, at a section. Find the maximum shear in the cross section of the beam and draw shear stress distribution diagram for the section.

Solution:

Given : A square section with its diagonal horizontal.

The beam with horizontal diagonal is shown in Fig.2.(a)

Let 2b = Diagonal of the square, and

F = shear force at the section

Now consider the shaded strip AJK at a distance x from the corner A. From the geometry of the figure, we find that length JK = 2x

$\therefore$ Area of AJK, $A = \frac{1}{2} \times 2x.x = x^2$

and $\bar{y} = b - \frac{2x}{3}$

we know that moment of inertia of the section ABCD about the neutral axis,

$$I = 2 \times \frac{2b \times b^3}{12} = \frac{b^4}{3}$$

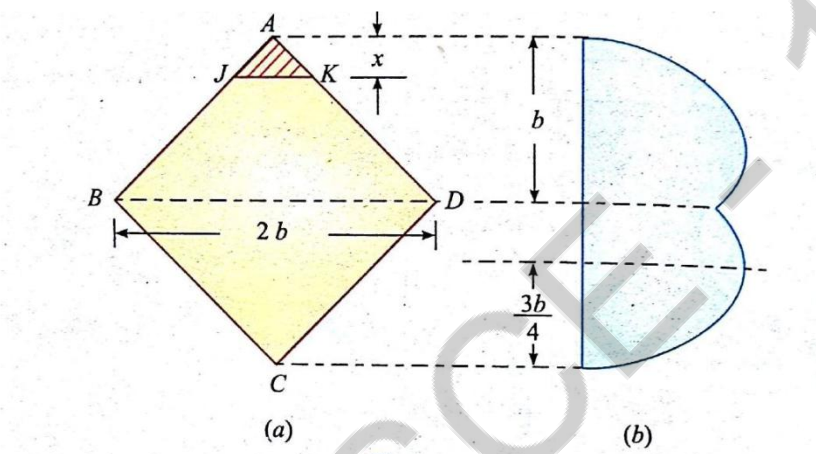


Fig. 2

and shearing stress at any point,

$$\tau = F \times \frac{A\bar{y}}{Ib} = F \times \frac{x^2\left(b - \frac{2x}{3}\right)}{\frac{b^4}{3} \times 2x} \qquad (\text{Here } b = JK = 2x)$$

$$= \frac{F}{2b^4}\left(3bx - 2x^2\right) \qquad ....(i)$$

We also know that when x = 0, τ = 0 and when x = b, then

$$\tau = \frac{F}{2b^2} = \frac{F}{\text{Area}} = \tau_{mean}$$

Now for maximum shear stress, differentiating the equation (i) and equating it to zero

$$\frac{d\tau}{dx} = \frac{d}{dx}\left[\frac{F}{2b^4}\left(3bx - 2x^2\right)\right] = 0$$

$$\therefore \quad 3b - 4x = 0 \text{ or } x = \frac{3b}{4}$$

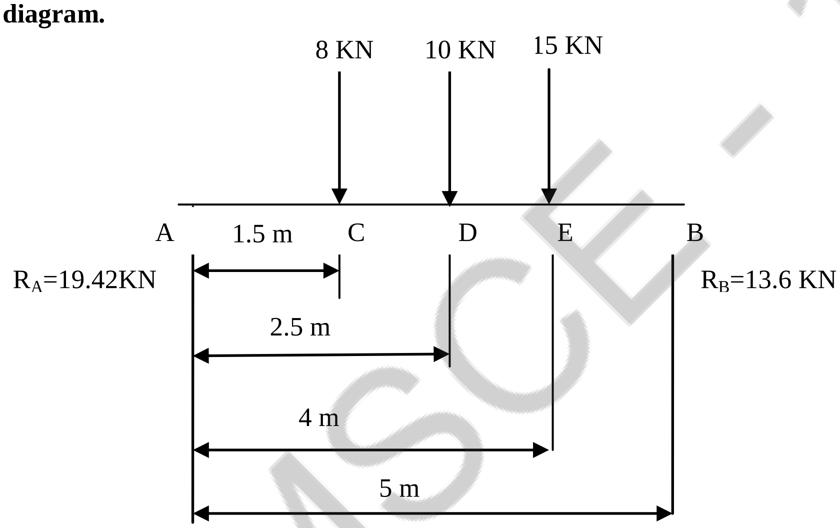
Substituting this value of x in equation (i),

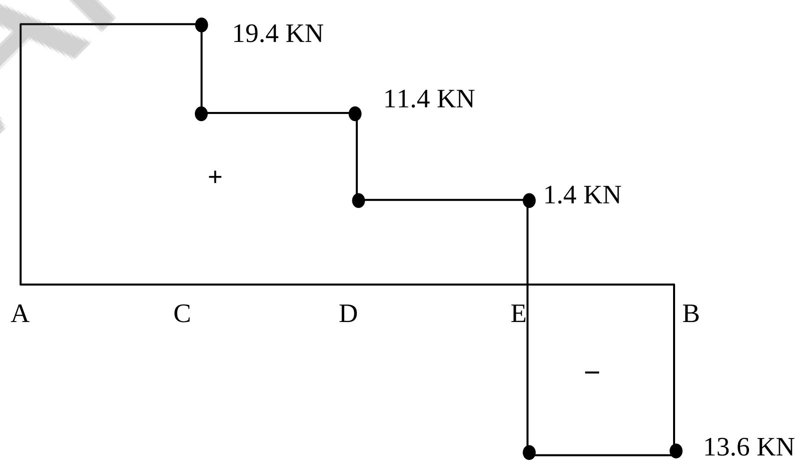
$$\tau_{max} = \frac{F}{2b^4}\left[3b \times \frac{3b}{4} - 2\left(\frac{3b}{4}\right)^2\right] = \frac{F}{2b^4} \times \frac{9b^2}{8}$$

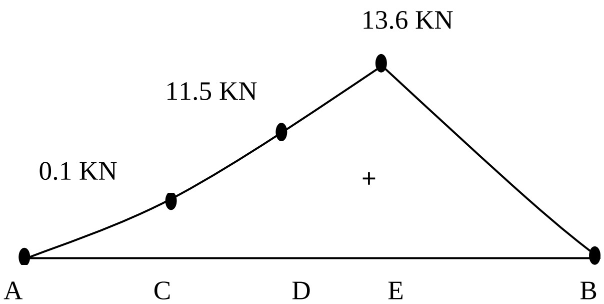
$$= \frac{9}{8} \times \frac{F}{2b^2} = \frac{9}{8} \times \frac{F}{Area} = \frac{9}{8} \times \tau_{mean}$$

Now complete the shear stress distribution diagram as shown in Fig.2(b)

11) A simply supported beam AB of length 5m carries point loads of 8 KN, 10 KN and 15 KN at 1.5 m, 2.50m and 4.0 m respectively from the left hand support. Draw the sheer force diagram and bending moment diagram. **(Nov / Dec 2016)**







To Find Reac

$$R_A + R_B = 8 + 10 + 15 = 33$$
$$R_A + R_B = 33\ KN \qquad ......1$$

$\sum M_A = 0 \Rightarrow \quad R_B \times 5 - 15 \times 4 - 10 \times 2.5 - 8 \times 1.5 = 0$

$5R_B = 97$

$\therefore \quad R_B = 19.4\,KN$ substitute in 1

$R_A = 13.6\,KN$

SFD

$SF\ at\ B = -13.6\ KN$

$SF\ at\ E = -13.6 + 15 = 1.4\,KN$

$SF\ at\ D = -13.6 + 15 + 10 = 11.4\,KN$

$SF\ at\ C = -13.6 + 15 + 10 + 8 = 19.4\ KN$

$SF\ at\ A = 19.4\,KN$

BMD

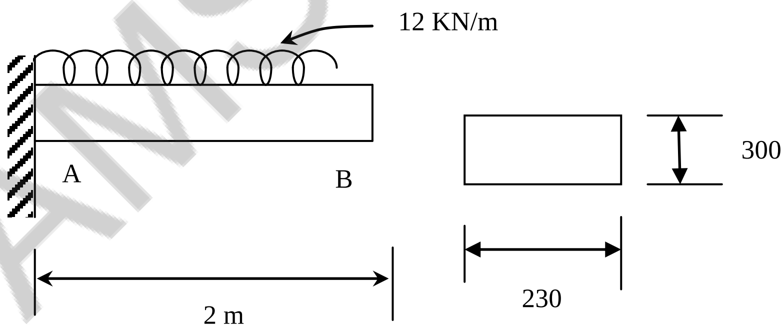
SSB at supports $M_A = M_B = 0$

$M_E = 13.6 \times 1 = 13.6\,KNm$

$M_D = 13.6 \times 2.5 - 15 \times 1.5 = 11.5\ KNm$

12) A cantilever beam AB of length 2m carries a uniformly distributed load of 12 KN/m over entire length. Find the shear stress and bending stress, if the size of the beam is 230mm $\times$ 300 mm. [5 mark]

[Nov/ Dec 2016]



Bending stress:

Shear stress:

$$\frac{\sigma_b}{y} = \frac{M}{I}$$

$$M = \frac{\omega \ell^2}{2}$$

$$M = \frac{12 \times 2^2}{2} = 24 \text{ KNm}$$

$$I = \frac{bd^3}{12} = \frac{230 \times 300^3}{12}$$

$$I = 517500000 \, mm^4$$

$$y = 150mm$$

$$\sigma_b = \frac{M}{I} y$$

$$= \frac{24 \times 10^3 \times 10^2 \times 150}{517500000}$$

$$\boxed{\sigma_b = 6.96 \text{ N/mm}^2}$$

$$F = \omega \ell = 24 \text{ KN}$$

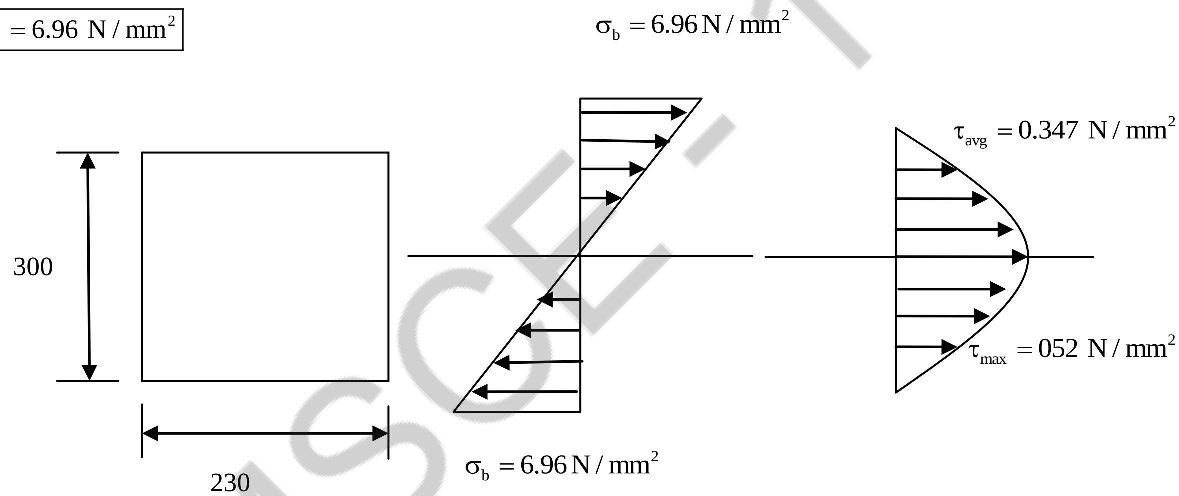
$$\tau_{max} = \frac{3}{2} \tau_{avg}$$

$$\tau_{avg} = \frac{F}{bd}$$

$$= \frac{24 \times 10^3}{230 \times 300}$$

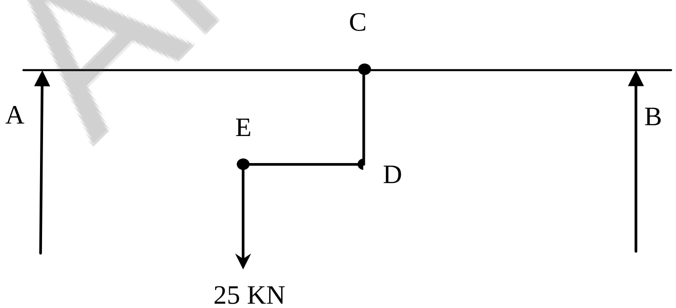
$$\boxed{\tau_{avg} = 0.347 \text{ N/mm}^2}$$

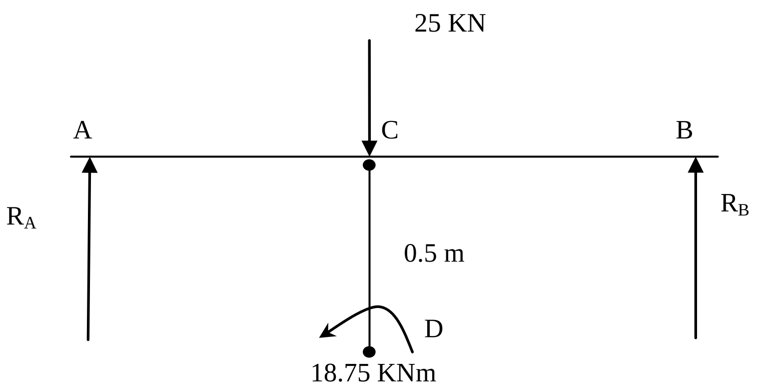
$$\boxed{\tau_{max} = 0.52 \text{ N/mm}^2}$$

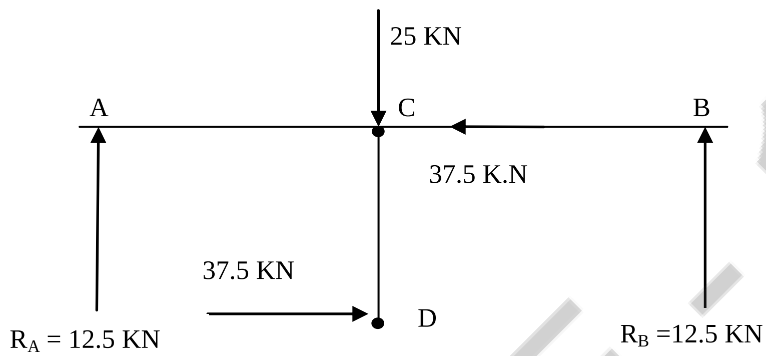


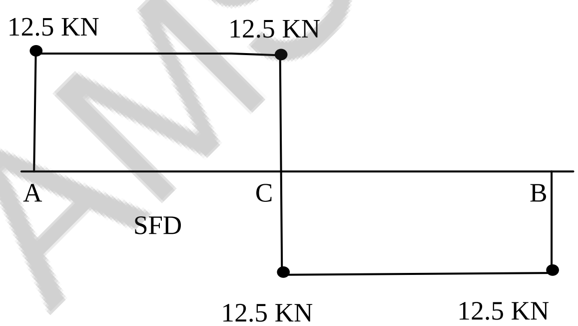
13) Construct the SFD & BMD for the beam as shown in fig (6 mark)

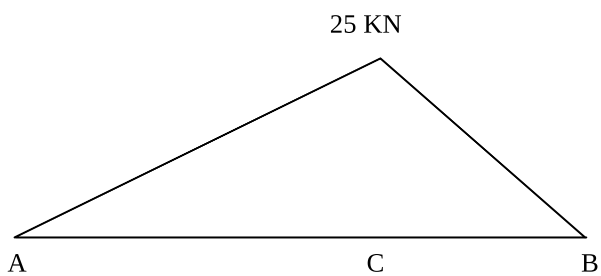
[Nov/ Dec 2016]




25 KN
A
C
B
R_A
R_B
0.5 m
D
18.75 KNm


25 KN
A
C
B
37.5 K.N
37.5 KN
D
R_A = 12.5 KN
R_B =12.5 KN


12.5 KN
12.5 KN
A
C
B
SFD
12.5 KN
12.5 KN


25 KN
A
C
B

$R_A + R_B = 25\,KN \qquad ...1$

$\sum M_A = 0$

$4R_B = 25 \times 2$

$R_B = 12.5\,KN$

$R_A = 12.5\,KN$

SFD

$S.F \text{ at } B = -12.5 \text{ KN}$

$S.F \text{ at } C = -12.5 + 25 \text{ KN}$

$= +12.5 \text{ KN}$

$S.F \text{ at } A = +12.5 \text{ KN}$

BMD

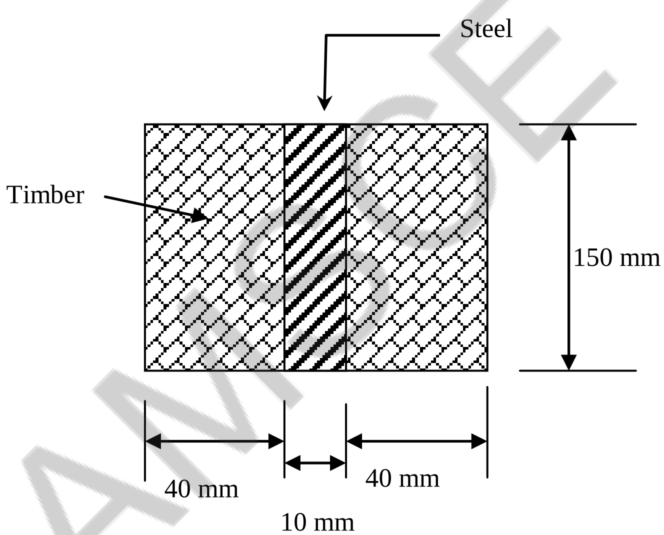
$m_A = m_B = 0$

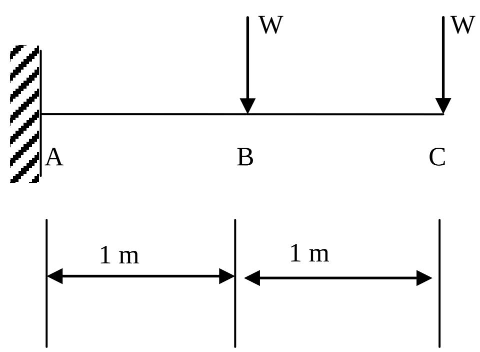
$m_C = 12.5 \times 2 = 25\,KNm$

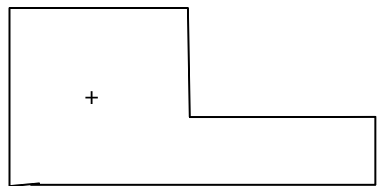
$SF = 0 \text{ at } C \Rightarrow (BM)_{max} = 25 \text{ KNm}$

14) Two timber joist are connected by a steel plate, are used as beam as show in fig find the load W if the permissible stress in steel and timber are 165 N/m^2 and 8.5 N/m^2 respectively (7 mark)

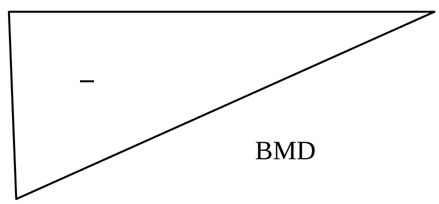
[Nov/ Dec 2016]







SFD



BMD

$\sigma_s = 165 \text{ N/mm}^2$ $\sigma_w = 8.5 \text{ N/mm}^2$

$$Z_w = \frac{bd^2}{6} = \frac{80 \times 150^2}{6}$$

$$= 300 \times 10^3 \text{ mm}^3$$

$$Z_s = \frac{bd^2}{6} = \frac{10 \times 150^2}{6}$$

$$= 37500 \text{ mm}^3$$

$m_w = \sigma_w Z_w$

$= 8.5 \times 300 \times 10^3$

$= 2.55 \times 10^6 \text{ N mm}$

$m_s = \sigma_s Z_s$

$= 165 \times 37500$

$= 6.18 \times 10^6 \text{ N mm}$

$m = m_w + m_s$

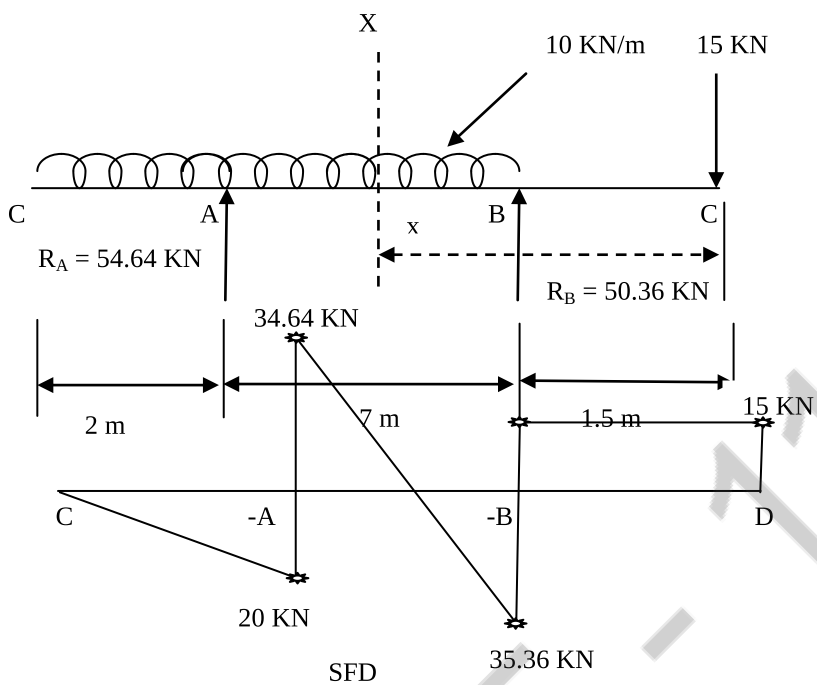
$3w = m = 8.7 \times 10^6 \text{ N mm}$

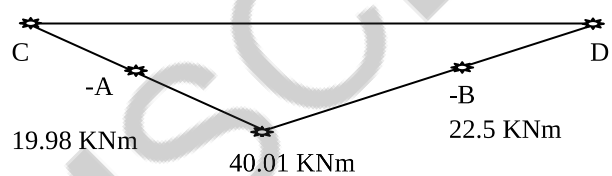
$W = m/3000 = 2.91 \times 10^3 \text{N}$

$\boxed{W = 2.91}$

15) Draw SRD & BDM and indicates the salient feature of beam loaded in fig

[May / June 2016]





R_A & R_B :

$\sum m_A = 0 \Rightarrow$

$$R_A + R_B = 10 \times (2+7) + 15 = 105\,KN \qquad ...1$$

$$R_B \times 7 - 5 \times 7 \times \frac{7}{2} - 15 \times 8.5 + 10 \times 2 \times 1 = 0$$

$$7R_B = 352.5\ KN$$

$$R_B = 50.36\ KN \qquad \text{sub in 1}$$

$$R_A = 54.64\ KN$$

SFD

$\text{SF at } D = 15\,\text{KN}$
$\text{SF at } B = +15 - 50.36 = -35.64$
$\text{SF at } A = +15 - 50.36 + 10 \times 7 = 34.64\ \text{KN}$
$\text{SF at } A = 34.64 - 54.64 = -20\ \text{KN}$
$\text{SF at } C = -20 + 10 \times 2 = 0$

BMD

$\text{BM at } D = 0$
$\text{BM at } B = -15 \times 1.5 = -22.5\ \text{KN}$

$$\text{BM at } A = -15 \times 8.5 + 50.36 \times 7 - 10 \times 7 \times \frac{7}{2}$$
$$= -19.98\ \text{KNm}$$

$$\text{BM at } C = -15 \times 10.5 + 50.36 \times 9 - 10 \times 9 \times \frac{9}{2} + 54.64 \times 2$$
$$= 0.02 \approx 0$$

$\text{SF at } XX = +15 - 50.36 + 10 \times (x - 1.5)$
$\text{BM at } XX = 15x - 50.6 \times (x - 1.5) + 10 \times (x - 1.5) \times (x - 1.5) \qquad ...2$
$\text{SF at } XX = 0$
$15 - 50.36 + 10(x - 1.5) = 0$
$10(x - 1.5) = 35.36$
$x - 1.5 = 3.536$
$\boxed{x = 5.036\,\text{m}}$ sub in equation 2
$\text{B.M at } (x = 5.036) = -40.01\ \text{KNm}$

16) Find the dimensions of a timber joist, spam 4m to carry a brickwork is 20 KN/m^3. Permissible bending stress in timber is 10N/mm^2. The depth of the joist twice the width (8)

[May/ June 2016]

$\ell = 4\text{m}$
$t = 230\ \text{mm} = 0.23\text{m}$
$h = 3\text{m}$
$\rho = 20\ \text{KN/m}^3$
$(\sigma_b)_{max} = 10\,\text{N/mm}^2$
$d = 2b$

Wt of bricks wall (W) = $\rho \times t \times h \times \ell$

SSB with UDL, $= 20 \times 0.23 \times 3 \times 4 = 55.2\,\text{KN}$

$$M = \frac{\omega \ell^2}{8} = \frac{\omega \ell}{8} = \frac{55.2 \times 4}{8}$$

$$M = 27.6 \text{ KNm} = 27.6 \times 10^6 \text{ Nmm}$$

$$I = \frac{bd^3}{12} = \frac{b \times (2b)^3}{12} = \frac{8b^4}{12}$$

$$y = \frac{d}{2} = \frac{2b}{2} = b$$

$$\text{section modulus}(Z) = \frac{I}{y} = \frac{8b^4}{12} \times \frac{1}{b} = \frac{8b^3}{12}$$

$$M = \sigma_b Z = 10 \times \frac{8b^3}{12}$$

$$10 \times \frac{8b^3}{12} = 27.6 \times 10^6$$

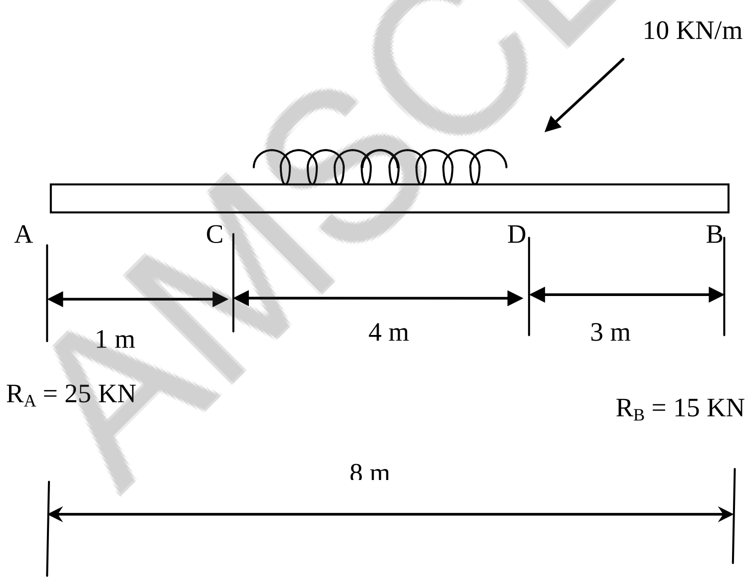
$$b^3 = \frac{27.6 \times 10^6 \times 12}{10 \times 8}$$

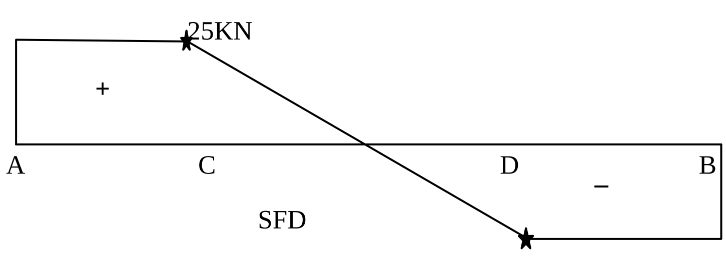
$$b = 160.57 \text{ mm}$$

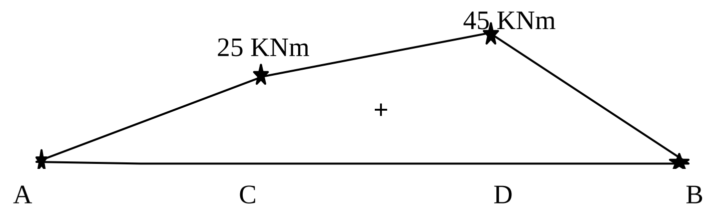
$$d = 2b = 321.14 \text{ mm}$$

17) Draw the sheer force & bending moment diagram for a simply supported beam of length 8m and carrying a UDL of 10KN/m for a distance of 4m as shown in fig (16)

[Nov/ Dec 2015]







R_A & R_B

$$R_A + R_B = 10 \times 4 = 40 \text{ KN} \qquad ...1$$

$$\sum M_A = 0 \Rightarrow \quad R_B \times 8 - 10 \times 4 \times 3 = 0$$

$$8R_B = 120$$

$$R_B = 15 \text{ KN} \text{ sub in } 1$$

$$R_A = 25 \text{ KN}$$

SFD

SF at B $= -15$ KN

SF at D $= -15$ KN

SF at C $= -15 + 10 \times 4 = 25$ KN

SF at A $= +25$ KN

BMD

$M_A = M_B = 0$ (At end of supporb)

BM at D $(M_D) = 15 \times 3 = 45$ KNm

BM at C $(M_C) = 15 \times 7 - 10 \times 4 \times 2$

$= 25$ KNm

$$10 \times \frac{8b^3}{12} = 27.6 \times 10^6$$

$$b^3 = \frac{27.6 \times 10^6 \times 12}{10 \times 8} = 4140000$$

$$b = 160.57$$

$$d = 2b = 321.14 \text{ mm}$$

18) A Steel plate at width 120 mm and of thickness 20 mm is bent into circular arc of radius 10m. Determine the maximum stress induced and the bending moment which will produce the maximum stress Take $E = 2 \times 10^5 \text{ N/mm}^2$ **(16)**

[Nov / Dec 2015]

$b = 120mm \qquad t = 20mm \qquad R = 10m$

$E = 2\times10^3 N/mm^2$

$I = \frac{bt^3}{12} = 80000\,mm^4$

$y_{max} = \frac{t}{2} = \frac{20}{2} = 10mm$

$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$

$(\sigma_b) = \frac{E}{R}\,y_{max}$

$$\boxed{(\sigma_b) = \frac{2\times10^5\times10^2}{10\times10^3}\times10 = 200\ N/mm^2}$$

$\frac{M}{I} = \frac{E}{R}$

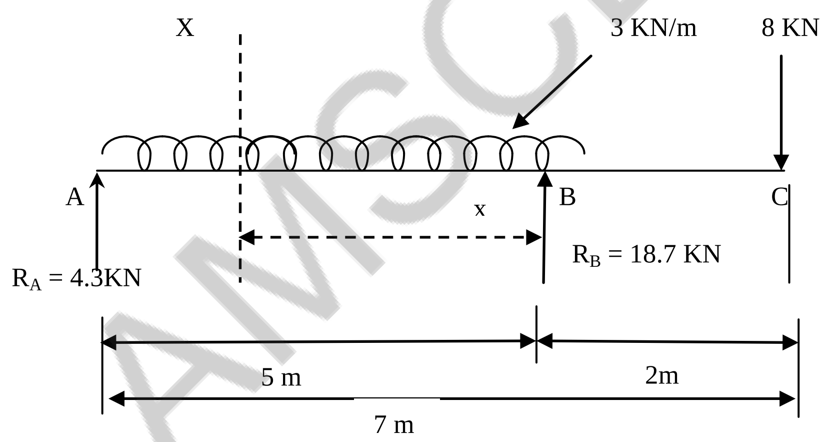
$M = \frac{E}{R}I = \frac{2\times10^5}{10\times10^3}\times80000$

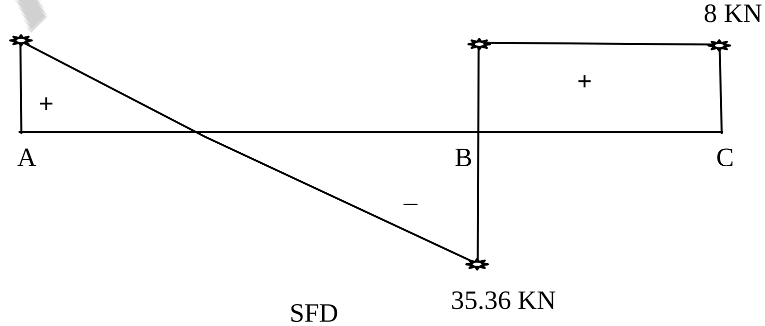
$$\boxed{M = 1.6\times10^6\,Nmm}$$

19) An overhanging beam ABC of length 7 m is simply supported at A & B over a span of 5m and the portion overhangs by 2m. Draw the shearing force & bending moments diagram and determine the point of contra flexure if it is subjected to UDL of 3 KN/ m over the portion AB and a concentrated load of 8 KN at C.(16)

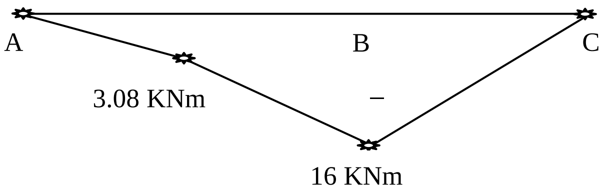
[Apr/ May 2015]

$R_A \,\&\, R_B$





SFD



$$R_A + R_B = 3\times5+8 = 23 \text{ KN} \qquad ...1$$

$$\sum M_A = 0 \Rightarrow \qquad R_B\times5-3\times5\times\frac{5}{2}-8\times7=0$$

$$5R_B = 93.5$$

$$\boxed{R_B = 18.7 \text{ KN}} \quad \text{sub in 1}$$

$$\boxed{R_A = 4.3 \text{ KN}}$$

SFD

SF at $C = +8$ KN

SF at $B = +8-18.7 = -10.7$ KN

SF at $A = -10.7+3\times5$

$= +4.3$ KN

BMD

$M_C = +4.3\times7-3\times5\times4.5+18.7\times2$

$M_B = -8\times2 = -16$ KNm

$M_A = -8\times7+18.7\times5-3\times5\times\frac{5}{2} = 0$

Assume

XX section at a distance of x from end B,

$$\text{SF at XX} = +8\text{KN}-18.7+3\text{X}x = 0$$

$$3\text{X}x = 10.7$$

$$x = 3.57\text{ m}$$

$$\text{BM at XX} = 8\times(x+2)-18.7\times x+3\times x\times\frac{x}{2}$$

$$(\text{BM})_{max} = -3.08 \text{ KNm}$$

$(x = 3.57)$ B.M at XX = 0

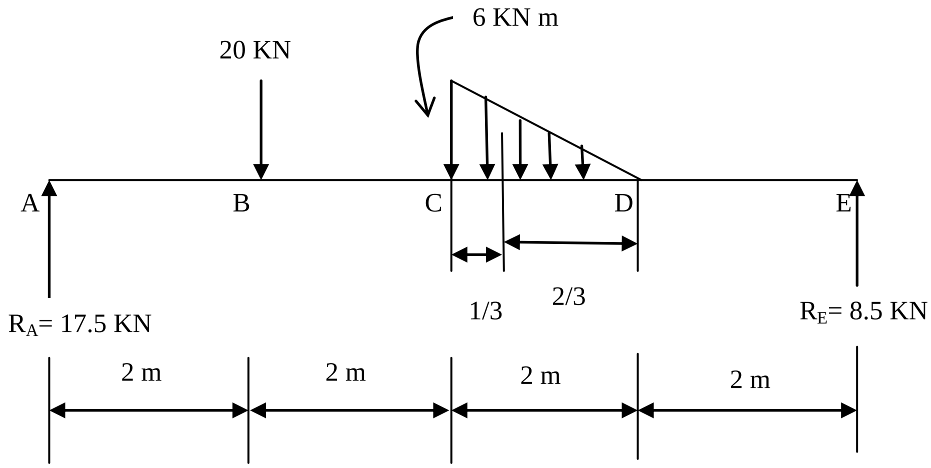
$$8x+16-18.7x+1.5x^2 = 0$$

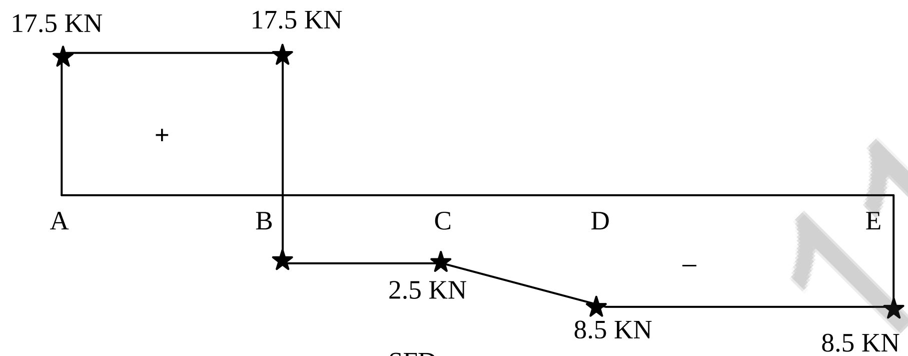
$$1.5x^2-10.7x+16=0$$

$$\boxed{x = 5\text{m}} \text{ or } \boxed{x = 2.13\text{m}}$$

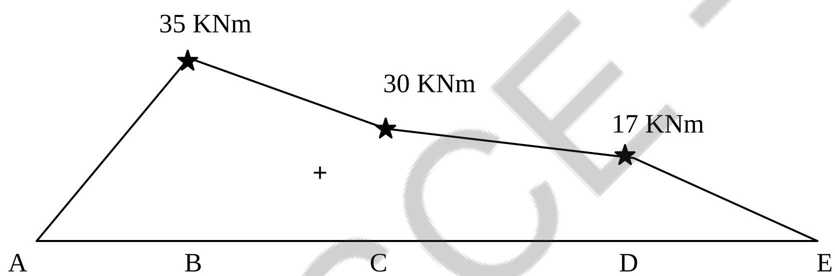
20) Draw SFD & BMD and find the max bending moment for the beam given in fig

[Nov/ Dec 2014]





SFD



R_A & R_E

$$R_A + R_E = 20 + \frac{1}{2}\times 2\times 6 = 26 \text{ KN} \qquad ...1$$

$$M_A = 0 \Rightarrow \quad R_E\times 8 - \frac{1}{2}\times 2\times 6\left(2 + 2\times\frac{1}{3}\times 2\right) - 20\times 2 = 0$$

$$8R_E = 68$$

$$\boxed{R_E = 8.5\text{KN}} \text{ sub in 1}$$

$$\boxed{R_A = 17.5 \text{ KN}}$$

SFD

$SF\ at\ E = -8.5\ KN$

$SF\ at\ D = -8.5\ KN$

$SF\ at\ C = -8.5 + \frac{1}{2} \times 2 \times 6 = 2.5\ KN$

$SF\ at\ B = -2.5 + 20 = 17.5\,KN$

$SF\ at\ A = +17.5\ KN$

BMD

$M_A = M_E = 0$

$B.M\ at\ D = 8.5 \times 2 = 17\ KNm$

$B.M\ at\ C = 8.5 \times 4 - \frac{1}{2} \times 2 \times 6 \times \frac{1}{3} \times 2$

$= 30\,KNm$

$B.M\ at\ B = 8.5 \times 6 - \frac{1}{2} \times 2 \times 6 \times \frac{1}{2} \times 2 + 2$

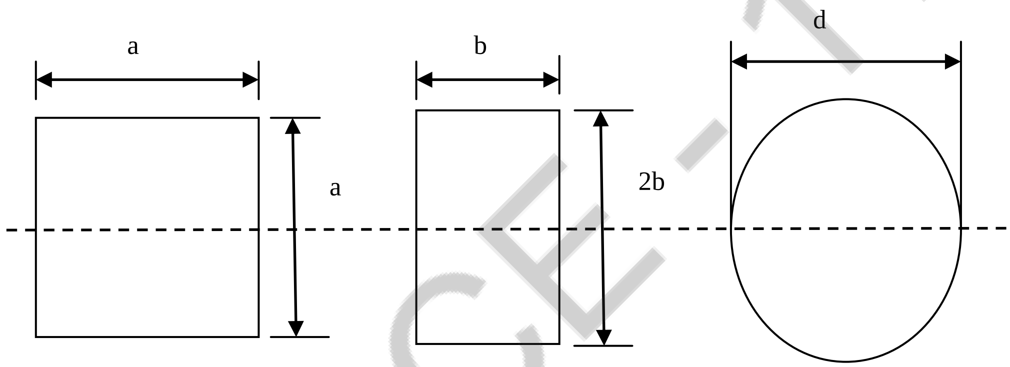
$= 35\,KNm$

$SF = 0 \Rightarrow (BM)_{max}$

$SF = 0\ at\ point\ B \Rightarrow (B.M)_{max}\ at\ B = 35KNm$

21) Three beams here the same length allowable stress and the same bending moment. The cross section of the beams are a square, a rectangular with depth twice the width and a circle. Find the ratio of weight of circular and the rectangular beam with respect to the square beam (16)

[Apr / May 2015]



A= side of square beam

Rectangular beam

b= Width

2b=Depth

d= disc of circular beam

Since all three beams here the same σ & M the modules of section of the three beams must be equal

Square beams

$Z_1 = \frac{bd^2}{6}$

$= \frac{a \times a^2}{6}$

$Z_1 = \frac{a^3}{6}$ ...1

Rectangular beam

$Z_2 = \frac{bd^2}{6}$

$= \frac{b(2b)^2}{6}$

$Z_2 = \frac{4b^2}{6} = \frac{2}{3}b^2$ ...2

Circular beam

$z_3 = \frac{\pi}{32} \times d^3$ ...3

Equating 1 & 2

$$\frac{a^3}{b} = \frac{2b^2}{3}$$

$$a^3 = 6 \times \frac{2}{3} b^2$$

$$a^3 = 4b^3$$

$$b = 0.63a \qquad ...4$$

Equating 1 & 3

$$\frac{a^3}{6} = \frac{\pi}{32} d^3$$

$$a^3 = 6 \times \frac{\pi}{32} \times d^3$$

$$d = 1.19\,a \qquad ...5$$

Weight of all the beams are proportional to the c/s area of their section,

$$\frac{\text{Weight of Square beam}}{\text{Weight of rectangular beam}} = \frac{\text{Area of square beam}}{\text{Area of rectangular beam}}$$

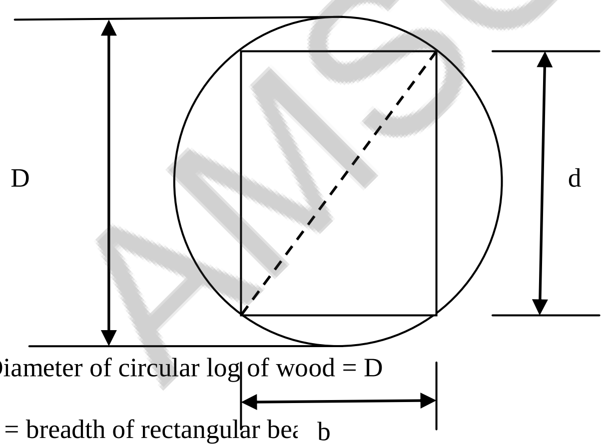
$$\frac{a^2}{2b^2} = \frac{a^2}{2 \times (0.63\,a)^2} = \frac{1}{0.79}$$

$$\frac{\text{Weight of Square beam}}{\text{Weight of circular beam}} = \frac{\text{Area of square beam}}{\text{Area of Circular beam}}$$

$$= \frac{a^2}{\frac{\pi}{4} d^2} = \frac{a^2}{\frac{\pi}{4} \times (1.19a)^2}$$

$$= \frac{1}{1.12}$$

22) Prove that the ratio of depth to width of the strongest beam that can be cut from a circular log of diameter d is 1.414. Hence calculate the depth and width of the strongest beam that can be cut of a cylindrical log of wood whose diameter is 300mm.



Diameter of circular log of wood = D

b = breadth of rectangular bea

d = depth of rectangular beam

$$\left.\begin{matrix}\text{section} \\ \text{modulus}\end{matrix}\right\} \qquad Z = \frac{bd^2}{6} \qquad ...1$$

From geometry of the fig ,

$$b^2 + d^2 = D^2$$

$$d^2 = D^2 - b^2 \qquad ..2\left(\text{substituting equ} 1\right)$$

$$Z = \frac{b \times \left(D^2 \times b^2\right)}{6}$$

$$= \frac{bD^2 - b^3}{6}$$

For strongest section, Differentiate the above equation and equate it to zero,

$$\frac{dz}{db} = \frac{d}{db}\left(\frac{bD^2 - b^3}{6}\right) = \frac{D^2 - 3b^2}{6}$$

$$\frac{D^2 - 3b^2}{6} = 0$$

$$D^2 - 3b^2 = 0$$

$$3b^2 = D^2$$

$$b = \frac{D}{\sqrt{3}} \qquad ...2 \text{ subtituting in equa} 1$$

$$d^2 = D^2 - \frac{D^2}{3} = \frac{2D^2}{3}$$

$$d = D\sqrt{\frac{2}{3}} \qquad ..4$$

$$\text{Equ } 4 \div \text{Equ } 3$$

$$\frac{d}{b} = \frac{D\sqrt{\frac{2}{3}}}{\frac{D}{\sqrt{3}}} = D\sqrt{\frac{2}{3}} \times \frac{\sqrt{3}}{D}$$

$$\frac{d}{b} = \sqrt{2} = 1.414 \left(\text{Hence it is proved}\right)$$

PART-C

23) A water main of 500mm internal diameter and 20mm thick is full. The water main is of cast iron and is supported at two points 10m apart. Find the maximum stress in the metal. The cast iron and water weigh 72000 N/m^3 and 10000 N/m^3 respectively. (May 2017 – 15 Marks)

Given:

Internal diameter, D_i = 500 mm = 0.5 m

Thickness of pipe, t = 20 mm

∴ outer dia, $D_0 = D_i + 2 \times t = 500 + 2 \times 20$

= 540 mm = 0.54 m

Weight density of cast iron = 72000 N/m^3

Weight density of water = 10000 N/m^3

Internal area of pipe = $\frac{\pi}{4}D_i^2 = \frac{\pi}{4}\times 0.5^2 = 0.1960\text{m}^2$

This is also equal to the area of water section

∴ Area of water section = 0.196 m^2

Outer area of pipe = $\frac{\pi}{4}D_0^2 = \frac{\pi}{4}\times 0.54^2\,\text{m}^2$

Area of pipe section = $\frac{\pi}{4}D_0^2 - \frac{\pi}{4}D_i^2$

$$= \frac{\pi}{4}\left[D_0^2 - D_i^2\right] = \frac{\pi}{4}\left[0.54^2 - 0.5^2\right] = 0.0327\text{m}^2$$

Moment of inertia of pipe section about neutral axis

$$I = \frac{\pi}{64}\left[D_0^4 - D_i^4\right] = \frac{\pi}{64}\left[540^4 - 500^4\right] = 1.105\times 10^9\ \text{mm}^4$$

Weight of pipe for one metre run = weight density of cast iron x volume of pipe

= 72000 x [area of pipe section x Length]

= 72000 x 0.0327 x 1

= 2354 N

Weight of water for me metre run = weight density of water x Volume of water

= 10000 x (Area of water section x length]

= 10000 x 0.196 x 1 = 1960 N

Total weight on the pipe for one metre run

= 2354 + 1960 = 4314 N

Hence the above weight is the U.D.L on pipe.

The maximum bending moment due to U.D.L is w x L^2/8, where w = Rate of U.D.L= 4314 N per metre run.

∴ Maximum bending moment due to U.D.L

$$M = \frac{w\times L^2}{8} = \frac{4314\times 10^2}{8} = 53925\,\text{Nm}$$

$$M = 53925\times 10^3\,\text{Nmm}$$

Now using $\frac{M}{I} = \frac{\sigma}{y}$

$$\sigma = \frac{M}{I} \times y$$

The stress is maximum when y is maximum

$$y = \frac{D_0}{2} = \frac{540}{2} = 270\,\text{mm}$$

$$y_{max} = 270 \text{ mm}$$

$\therefore$ maximum stress $\sigma_{max} = \frac{M}{I} \times y_{max}$

$$= \frac{53925 \times 10^3}{1.105 \times 10^9} \times 270$$
$$= 13.18\,\text{N/mm}^2$$