ME 8492 – KINEMATICS OF MACHINERY

QUESTION BANK

SOLVED ANNA UNIVERSITY QUESTION PAPERS

UNIT- I BASICS OF MECHANISMS

PART-A

1. Differentiate between Machine and Structure. (MAY/JUNE 2014)

Machine	Mechanism
Machine is a mechanism or collection of mechanism which transmits force from the source of power to the resistance (load) to overcome and thus performs useful mechanical work.	Combination of rigid or resistant bodies connected that they move upon each other with definite relative motion
E.g Lathe, Shaping Machine etc.	*E.g* single slider mechanism in IC engine
All machines are mechanism	All mechanisms are not Machine

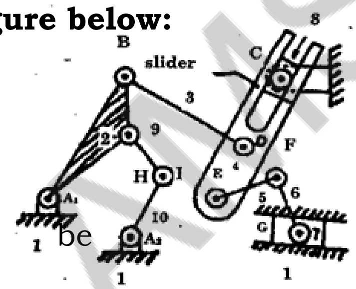
2. Classify the constrained motion.(MAY/JUNE 2014)

Constrained motions are classified into three types

- Completely constrained motion.
- Incompletely constrained motion.
- Successfully constrained motion.

3) Determine the number of freedom of the mechanism shown in the figure below:

(MAY/JUNE 2015)



<u>Using Grublers criterion:</u>

$F=3(N-1)-2P_1-P_2$

We can determine the degrees of freedom to be

F = 3(8-1) - (2x6) - 7

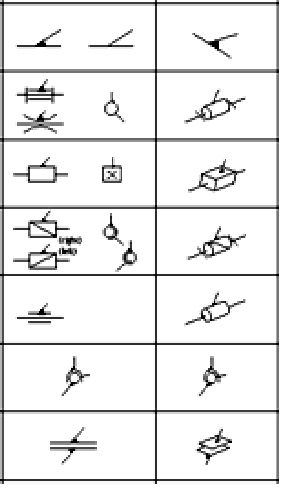
F = 2

4) Write a short notes on completed and incomplete constrains in lower and higher pairs, depict your answer with neat sketches.

(MAY/JUNE 2015)

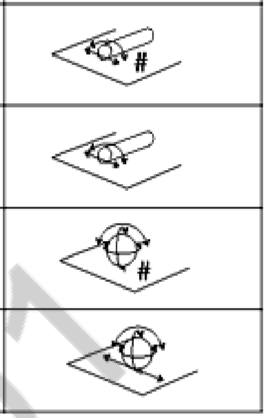
- A ***lower pair*** joint is one in which contact two rigid bodies occurs at every points of one or more surface segments.
- A ***higher pair*** joint is one which contact occurs only at isolated points or along a line segments.

Name	Relative motion	Degree of freedom (f_i)	Sketch symbol	Other view
Rigid joint	0 rotation 0 translation	0		
Revolute	1 rotation 0 translation	1		
Prismatic	0 rotation 1 translation	1		
Helical	1 rotation 1 translation	1		
Cylindrical	1 rotation 1 translation	2		
Spherical	3 rotations 0 translation	2		
Planar	1 rotation 2 translations	3		



Lower Pair

Description	Degree of freedom (f_i)	Typical form
Cylindrical surface on a plan without slipping	1	
Cylindrical surface on a plan with slipping	2	
Ball on a plan without slipping	3	
Point on a plan with slipping	5	



Higher Pair

5) Classify kinematics pair based on nature of contact. Give example. (MAY/JUNE 2016)

Kinematic Pair

I. **Depending upon the nature of relative motion between the links**	II. **Depending upon the nature of relative motion between the links**	III. **Depending upon the nature of mechanical arrangement between the links**
1. Sliding (or prismatic) pair 2. Turning (or revolute) pair 3. Screw (or helical) pair 4. Cylindrical pair 5. Spherical (or globular) pair 6. Rolling pair	1. Lower pair 2. Higher pair	1. Self-Closed Pair 2. Force Closed Pair

6) When does a linkage become mechanism? (MAY/JUNE 2016)

Linage**,** a system of solid, usually metallic, links (bars) connected to two or more other links by pin joints (hinges), sliding joints, or ball-and-socket joints, so as to form a closed chain or a Kinematic chains. When one of the links is fixed, the possible movement of the other links relative to the fixed link becomes a Mechanism.

7) Differentiate between Rigid bodies and Resistant Bodies. (NOV/DEC 2014)

A rigid body is defined as a body on which the distance between two points never changes whatever be the force applied on it. Such a body which does not deform under the influence of forces is known as a rigid body. Rigid body is ideal.

A body is said to be ***a resistant body***, if it does not deform for the purpose for which it is made. Resistant body is rigid for the purpose for which it is used, but is deforms which excess forces act on it. For example the chair, it does not deform if a person sits on it, but it will break if you put a load of 1000 kg on it.

8) The ratio between the width of front axle and that of wheel base of a steering mechanism is 0.44. At the instant when the front inner wheel is turned by 18^0, which should be the angle burned by the outer front wheel for perfect steering? (NOV/DEC 2014)

Using criterion of perfect steering,

$\cot \varphi - \cot \theta = 0.44$

$\cot \varphi - \cot 18^0 = 0.44$

$\boldsymbol{\varphi = 15.8^0}$

9) Define Grubler's criteria for the mechanism. (NOV/DEC 2015)

The following equation is used to describe mobility in 2D or planar systems:

$\mathbf{M = 3(n-1) - 2 f_1 - f_2}$

Where, n = total number of links, M = DOF.

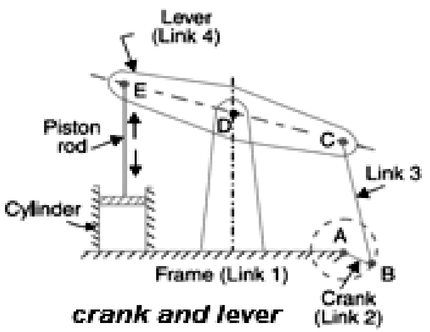
f_1 = number of 1DOF joints. f_2 = number of 2 DOF joints.

This is known as GRUBLER'S EQUATION and is for mobility of planar systems.

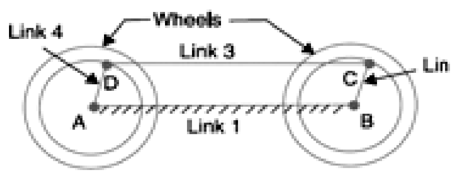
M = 1	Single input /monitoring necessary
M = 2	Double input/output necessary
M = -1	Statically indeterminate structure
M = 0	Motion impossible - statically determinate

10) Name any two inversions of the four bar mechanism. (NOV/DEC 2015)

- Beam engine (crank and lever mechanism).
- Coupling rod of a locomotive (Double crank mechanism).



crank and lever mechanism



Double Crank Mechanism

11) What is Kinematics?

Kinematics is the study of motion (position, velocity, acceleration). A major goal of understanding kinematics is to develop the ability to design a system that will satisfy specified motion requirements. This will be the emphasis of this class.

12) What is Kinetics?

Kinetics is the study of effect of forces on moving bodies. Good kinematic design should produce good kinetics.

13) Define Link.

A link is defined as a member or a combination of members of a mechanism connecting other members and having relative motion between them. The link may consist of one or more resistant bodies. A link may be called as kinematic link or element. E.g.: Reciprocating steam engine.

14) Define Kinematic Pair.

- Kinematic pair is a joint of two links having relative motion between them. The types of kinematic pair are classified according to
- Nature of contact (lower pair, higher pair)
- Nature of mechanical contact (Closed pair, unclosed pair)
- Nature of relative motion (Sliding pair, turning pair, rolling pair, screw pair, spherical pair)

15) Define Kinematic Chain.

When the kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definite motion it is called a kinematic chain.

E.g.: The crank shaft of an engine forms a kinematic pair with the bearings which are fixed in a pair, the connecting rod with the crank forms a second kinematic pair, the piston with the connecting rod forms a third pair and the piston with the cylinder forms the fourth pair. The total combination of these links is a kinematic chain. E.g.: Lawn mower.

16) Define Degrees of Freedom.

- It is defined as the number of input parameters which must be independently controlled in order to bring the mechanism in to useful engineering purposes.
- It is also defined as the number of independent relative motions, both translational and rotational, a pair can have.

17) Define Pantograph.

Pantograph is used to copy the curves in reduced or enlarged scales. Hence this mechanism finds its use in copying devices such as engraving or profiling machines.

18) What is meant by spatial mechanism?

Spatial mechanism have a geometric characteristics in that all revolute axes are parallel and perpendicular to the plane of motion and all prism lie in the plane of motion.

19) Classify the Constrained motion?

- Constrained motions are classified into three types
- Completely constrained motion.
- Incompletely constrained motion.
- Successfully constrained motion.

20) What is Toggle position?

It is the position of a mechanism at which the mechanical advantage is infinite and the sine of angle between the coupler and driving link is zero.

21) What are the important applications of a single slider crank mechanism?

- Rotary or Gnome engine.
- Crank and slotted lever mechanism.
- Oscillating cylinder engine.
- Bull engine and
- Hand pump.

22) Give some examples for kinematic pairs.

- Crank and connecting rod,
- Connecting rod and piston rod, and
- Piston and engine cylinder.

23) What is meant by transmission angle?

In a four bar chain mechanism, the angle between the coupler and the follower (driven) link is called as the transmission angle.

24) What are the applications of inversion of double slider crank chain mechanism?

It consists of two sliding pairs and two turning pairs. There are three important inversions of double slider crank chain. 1) Elliptical trammel.

2) Scotch yoke mechanism. 3) Oldham's coupling. Give some examples for kinematic pairs.

25) Write down the Grashof's law for a four bar mechanism?

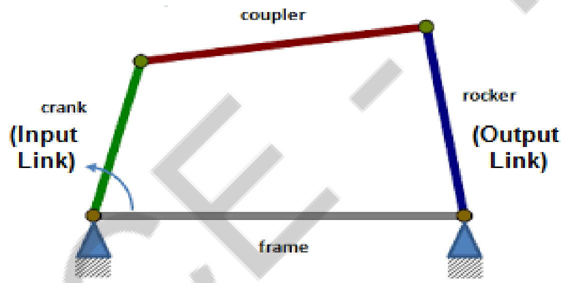
Grashof's law states that the sum of the shortest and longest links cannot be greater than the sum of the remaining two links lengths, if there is to be continuous relative motion between two members.

26) What type of kinematic pair exists between human shoulder and arm based on nature of contact and nature of relative motion? (A/M-2017)

Spherical or Globular Pair. (Ball & Socket Joint)

27) Sketch and define Transmission angle of a four bar mechanism. (A/M-2017)

In a four bar chain mechanism, the angle between the coupler and the follower (driven) link is called as the transmission angle.



28) Define the Grubler's criterion for plane mechanism with mathematical expression. (N/D-2017)

The following equation is used to describe mobility in 2D or planar systems:

$$\mathbf{M = 3(n-1) - 2\, f_1 - f_2}$$

Where, n = total number of links, M = DOF.

f_1 = number of 1DOF joints. f_2 = number of 2 DOF joints.

This is known as GRUBLER'S EQUATION and is for mobility of planar systems.

M = 1	Single input /monitoring necessary
M = 2	Double input/output necessary
M = -1	Statically indeterminate structure
M = 0	Motion impossible - statically determinate

29) Name any two inversions of single slider crank chain. (N/D-2017)

- Rotary or Gnome Engine
- Crank & Slotted lever mechanism

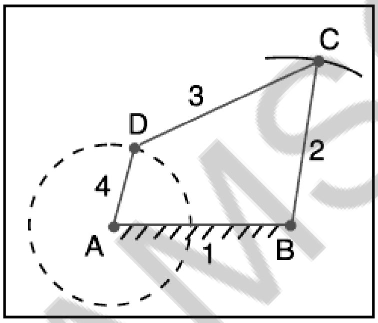
- Oscillating cylinder mechanism
- Bull Engine
- Hand Pump

30) Define kinematic pair and classify it according to the types of contact. (A/M 2018)

Kinematic Pair

I. **Depending upon the nature of relative motion between the links**	II. **Depending upon the nature of relative motion between the links**	III. **Depending upon the nature of mechanical arrangement between the links**
1. Sliding (or prismatic) pair 2. Turning (or revolute) pair 3. Screw (or helical) pair 4. Cylindrical pair 5. Spherical (or globular) pair 6. Rolling pair	1. Lower pair 2. Higher pair	1. Self-Closed Pair 2. Force Closed Pair

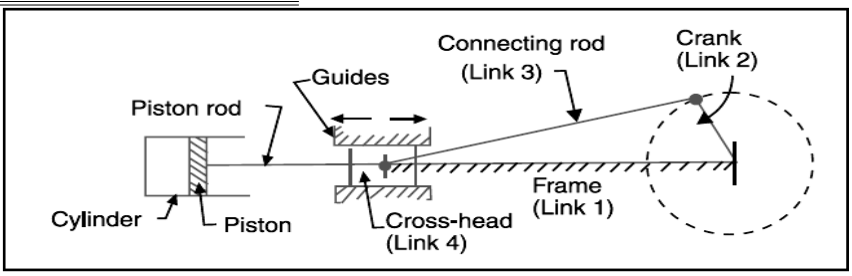
31) Sketch a crank-rocker mechanism and a slider crank mechanism indicating their input and output motions. (A/M 2018)



Crank – Rocker Mechanism

It consists of four links, each of them forms a turning pair at *A, B, C* and *D*. The four links may be of different lengths. In a four bar chain, one of the links, in particular the shortest link, will make a complete revolution relative to the other three links. Such a link is known as **Crank.** The link *BC* (link 2) which makes a partial rotation or oscillates is known as *lever* or ***Rocker* Or *Follower*** and the link *CD* (link 3) which connects the crank and lever is called ***Connecting Rod* or *Coupler***. The fixed link *AB* (link 1) is known as *frame* of the mechanism.

Slider Crank Mechanism



In a single slider crank chain, as shown in Fig. the links 1 and 2, links 2 and 3, and links 3 and 4 form three turning pairs while the links 4 and 1 form a sliding pair. The link 1 corresponds to the frame of the engine, which is fixed. The link 2 corresponds to the crank link 3 corresponds to the connecting rod and link 4 corresponds to cross-head. As the crank rotates, the cross-head reciprocates in the guides and thus the piston reciprocates in the cylinder.

32) Name the inversions of a double slider crank mechanism.
(N/D 2018) (A/M 2019)

Double Slider Crank Mechanism:

Inversion I	:	***Elliptical trammels***
Inversion II	:	***Scotch yoke mechanism***
Inversion III	:	***Oldham's coupling***

33) What is a pantograph? (N/D 2018)

Pantograph is used to copy the curves in reduced or enlarged scales. Hence this mechanism finds its use in copying devices such as engraving or profiling machines.

34) Define rigid link and give examples. (A/M 2019)

A Rigid link is the link which does not undergo any appreciable deformation while transmitting the motion and forces.
Ex: Connecting Rod, Crank, etc.

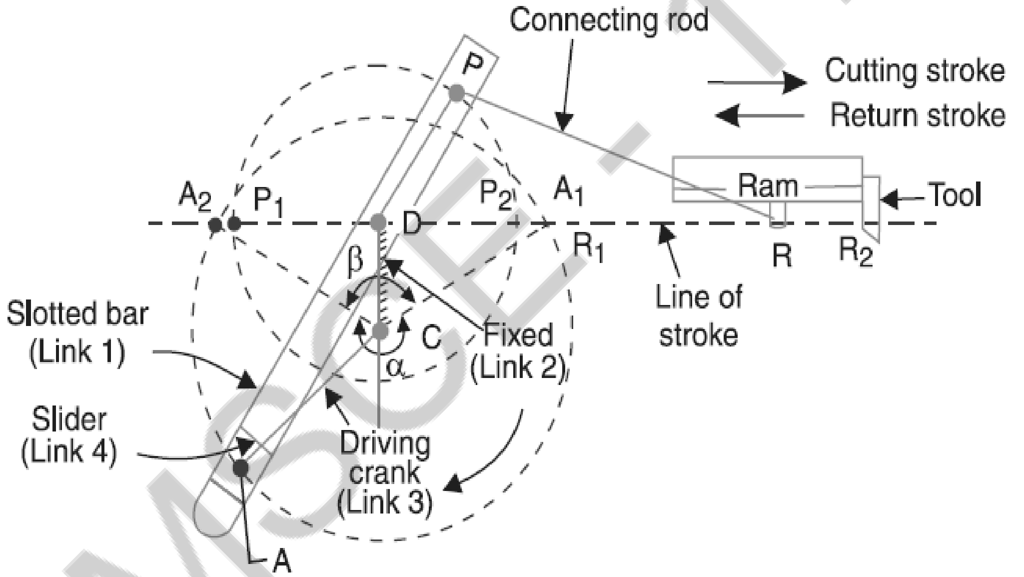
PART-B

1) Explain the working of two different types of Quick-Return Mechanisms. Derive an expression for the ratio of time taken in forward and return stroke for one of these mechanism. (16)

(MAY/JUNE 2014)

<u>WHITWORTH QUICK RETURN MOTION MECHANISM</u>

This mechanism is mostly used in shaping and slotting machines. In this mechanism, the link *CD* (link 2) forming the turning pair is fixed, as shown in Fig. The link 2 corresponds to a crank in a reciprocating steam engine. The driving crank *CA* (link 3) rotates at a uniform angular speed. The slider (link 4) attached to the crank pin at *A* slides along the slotted bar *PA* (link 1) which oscillates at a pivoted point *D*. The connecting rod *PR* carries the ram at *R* to which a cutting tool is fixed. The motion of the tool is constrained along the line *RD* produced, *i.e.* along a line passing through *D* and perpendicular to *CD*.



When the driving crank CA moves from the position CA_1 to CA_2 (or the link DP from the position DP_1 to DP_2) through an angle α in the clock wise direction, the tool moves from the left hand end its stroke to the hand end through a distance 2PD.

Now when the driving crank moves from the position CA_1 to CA_2 (or the link DP from DP_2 to DP_1) through an angle β in the clockwise direction, the tool moves back from right hand end of its stroke to the left hand end.

A little consideration will show that the time taken during the left to right movement of the ram(i.e during forward or cutting stroke) will be equal to the time taken by the driving crank to move from CA_1 to CA_2. Silimilarly, the time taken during the right to left movement of the ram (or during the idle or return stroke) will be equal to the time by the driving crank to mive from CA_2 to CA_1.

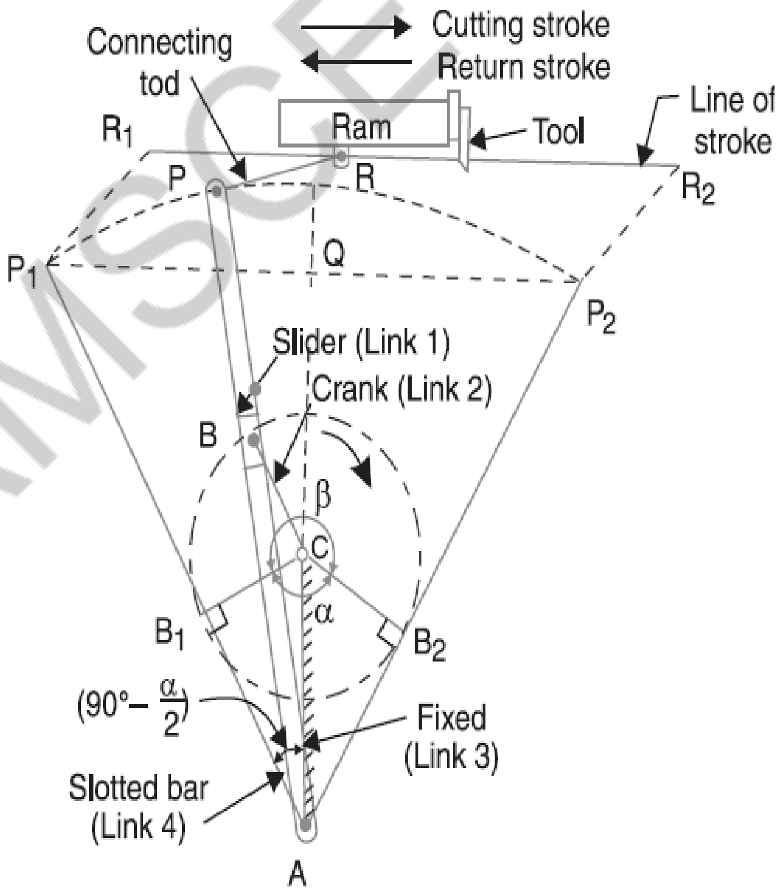
Since the crank link CA rotates at uniform angular velocity therefore time taken during the cutting stroke (or forward stroke) is more than time taken during the returns stroke. In other words, the mean speed of the ram during cutting stroke is less than the mean speed during the return stroke. The ratio between the time taken during the cutting and return strokes is given by

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360^\circ - \alpha} \text{ or } \frac{360^\circ - \beta}{\beta}$$

CRANK AND SLOTTED LEVER QUICK RETURN MOTION MECHANISM.

This mechanism is mostly used in shaping machines, slotting machines and in rotary internal combustion engines.

In this mechanism, the link AC (i.e. link 3) forming the turning pairs is first, as shown in fig. the link 3 corresponds to the connecting rod of a reciprocating steam engine. The driving crank CB revolves with uniform angular speed about the fixed centreC. A silding block attached to the craknk pin at B slides along the slotted bar AP and thus causes AP to oscillate about the pivotd point A. A short link PR transmits the motion from AP to the ram which carries the tool and reciprocates along the line of stroke R_1, R_2. The line of stroke of the ram (i.e. R_1 and R_2) is perpendicular to ACC produced.



In the extream positions, AP_1 and AP_2 are tangential to the circle and the cutting tool is at the end of the stroke. The forward or cutting stroke occurs when the crank rotates from the position CB_1 to CB_2 (or through an angle β) in the clock wise directions. The return stroke occurs when the crank rotates from the positions CB_2 to CB_1 (or through angle α) in the clock wise directions. Since the crank has uniform angular speed.

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360° - \beta} \text{ or } \frac{360° - \alpha}{\alpha}$$

Since the tool travels a distance of $R_1 R_2$ during cutting and return stroke, therfore travel of the tool or length of stroke

$$= R_1R_2 = P_1P_2 = 2P_1Q = 2AP_1 \sin < P_1AQ$$

$$= 2AP_1 \sin\left(90° - \frac{\alpha}{2}\right) = 2AP \cos\frac{\alpha}{2}$$

$$= 2AP \times \frac{CB_1}{AC}$$

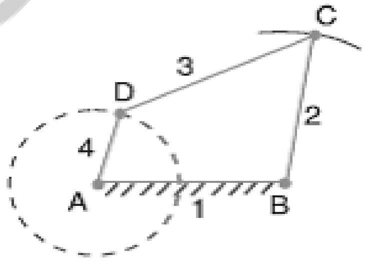
$$= 2AP \times \frac{CB}{AC}$$

From fig, we see that the angle β made by the forward or cutting stroke is greater than the angle α described by the return stroke. Since the crank rotates with uniform angular speed, therefore the returns stroke is completed within shorter time. Thus it is called quick return motion mechanism.

2) Sketch and explain the three inversions of four-bar chain. (16)
(MAY/JUNE 2014) (NOV/DEC 2015)

Four Bar Chain / Quadric Cycle Chain:

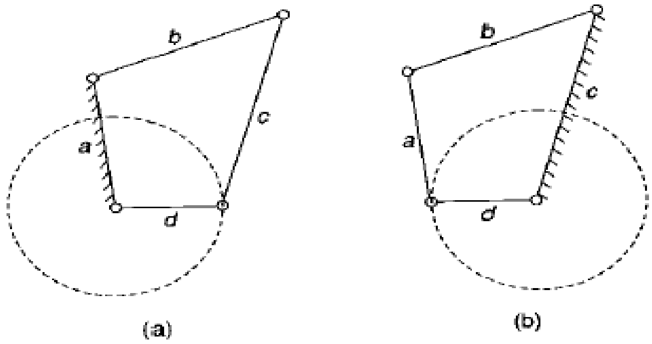
- It consists of four rigid links which are connected in the form of a quadrilateral by four pin-joints.



- A link makes complete revolution is called Crank (4).
- The link which is fixed is called fixed link (1).
- The link opposite to the fixed link is called Coupler (3).
- The fourth link is called Lever or Rocker (2).
- It is impossible to have a four bar linkage, if the length of one of the links is greater than the sum of the other three.
- Grashof's Law: For a four bar mechanism, the sum of the shortest and longest link lengths should not be greater than the sum of the remaining two link lengths.

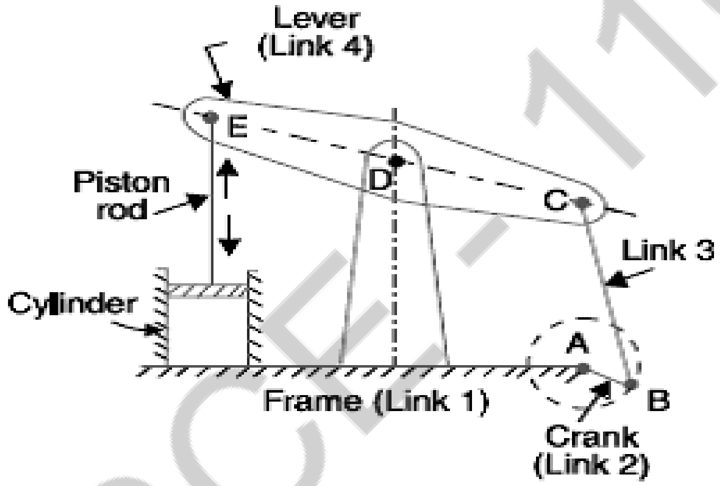
Inversions of Four Bar Chain:

1. First Inversion (Crank & Lever Mechanism):



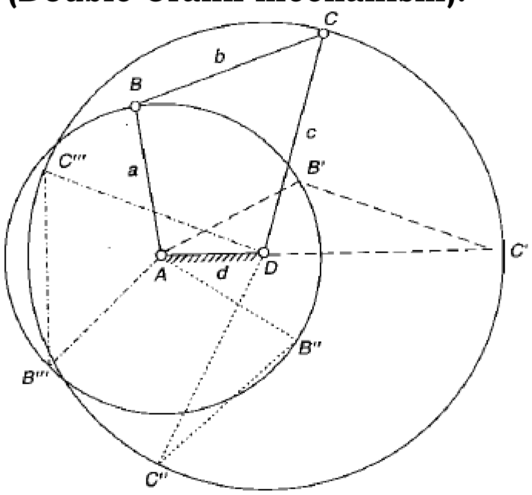
- If any of the adjacent links of link d, i.e., **link a or c** is fixed. The link d (crank) can have full revolution and the link (b) opposite to it oscillates.

Application: Beam Engine (Crank & Lever Mechanism)



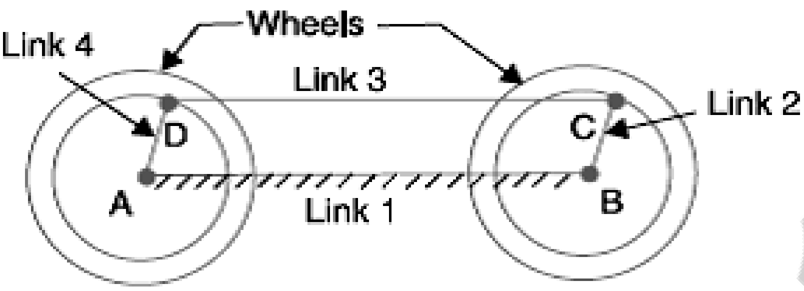
- When the crank rotates about the fixed centre A and the lever oscillates about a fixed centre D.
- The purpose of this mechanism is to convert rotary motion into reciprocating motion.

2. Second Inversion (Double Crank mechanism):



- If the shortest **link (d)** is fixed then the links a and c rotates full circle and link b also complete one revolution relative to fixed link d.

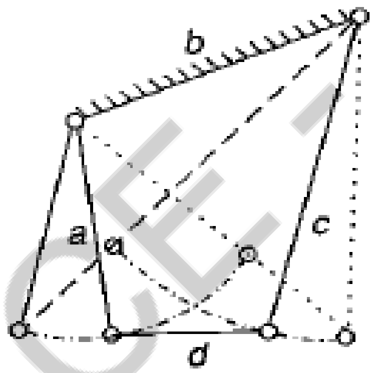
Application: Coupling Rod of a Locomotive (Double Crank Mechanism)



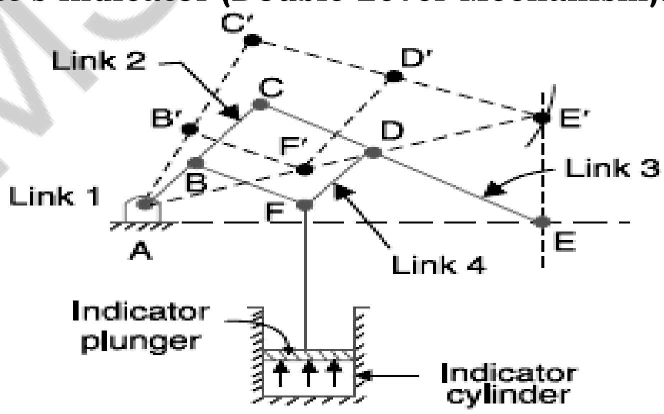
- It is meant for transmitting rotary motion from one wheel to the other wheel.

3. Third Inversion (Double Lever Mechanism):

- If the link opposite to the shortest link. i.e., **link b** is fixed and the two links a and c would oscillate.



<u>Application</u>: Watt's indicator (Double Lever Mechanism):



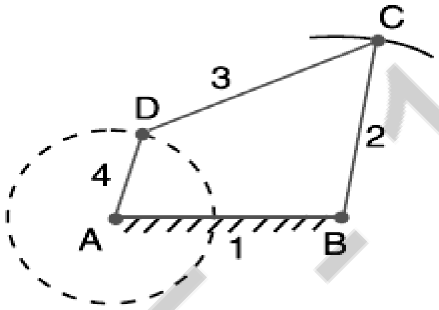
- It consists of four links which are: Fixed link at A, link AC, link CE and link BFD. The links CE and BFD act as lever.
- It is also called Watt's straight line mechanism and the dotted line shows the position of the mechanism.

3) What do you understand by inversion of kinematic chain? Describe the mechanisms obtained by inversion of the four-bar chain. (16)

(MAY/JUNE 2015)

<u>FOUR BAR CHAIN:</u>

The kinematic chain is a combination of four or more kinematic pairs, such that the relative motion between the links or elements is completely constrained. The simplest and the basic kinematic chain is a four bar chain or quadric cycle chain, as shown in Fig. It consists of four links, each of them forms a turning pair at *A, B, C* and *D*. The four links may be of different lengths. According to Grashoff 's lawfor a four bar mechanism, the sum of the shortest and longest link lengths should not be greater than the sum of the remaining two link lengths if there is to be continuous relative motion between the two links.

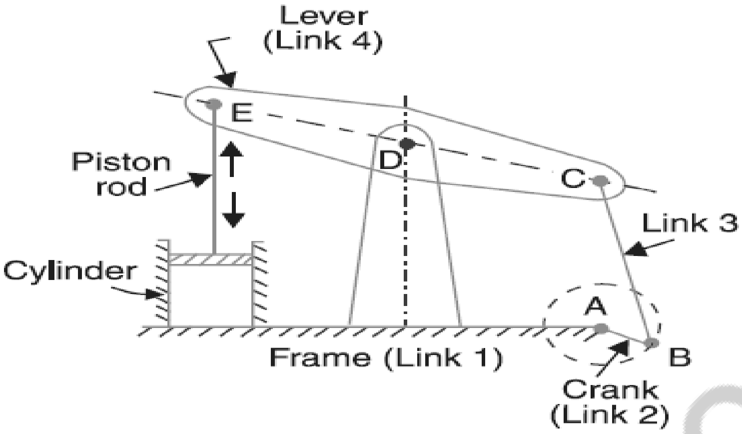


A very important consideration in designing a mechanism is to ensure that the input crank makes a complete revolution relative to the other links. The mechanism in which no link makes a complete revolution will not be useful. In a four bar chain, one of the links, in particular the shortest link, will make a complete revolution relative to the other three links, if it satisfies the Grashoff 's law. Such a link is known as crank or driver. In Fig., AD (link 4) is a crank. The link BC (link 2) which makes a partial rotation or oscillates is known as lever or rocker or follower and the link CD (link 3) which connects the crank and lever is called connecting rod or coupler. The fixed link AB (link 1) is known as frame of the mechanism. When the crank (link 4) is the driver, the mechanism is transforming rotary motion into oscillating motion.

Though there are many inversions of the four bar chain, yet the following are important from the subject point of view:

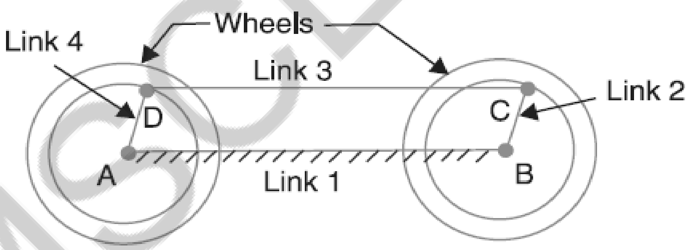
1. Beam engine (crank and lever mechanism).

A part of the mechanism of a beam engine (also known as crank and lever mechanism) which consists of four links, is shown in Fig. In this mechanism, when the crank rotates about the fixed centre A, the lever oscillates about a fixed centre D. The end E of the lever CDE is connected to a piston rod which reciprocates due to the rotation of the crank. In other words, the purpose of this mechanism is to convert rotary motion into reciprocating motion.



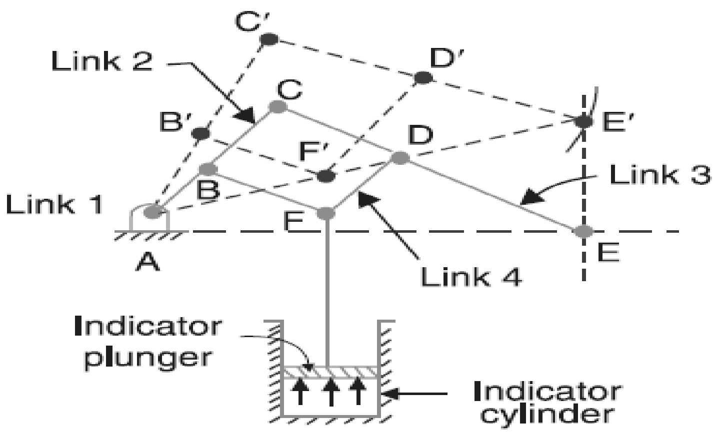
2. Coupling rod of a locomotive (Double crank mechanism):

The mechanism of a coupling rod of a locomotive (also known as double crank mechanism) which consists of four links, is shown in Fig. In this mechanism, the links AD and BC (having equal length) act as cranks and are connected to the respective wheels. The link CD acts as a coupling rod and the link AB is fixed in order to maintain a constant centre to centre distance between them. This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.



3. Watt's indicator mechanism (Double lever mechanism):

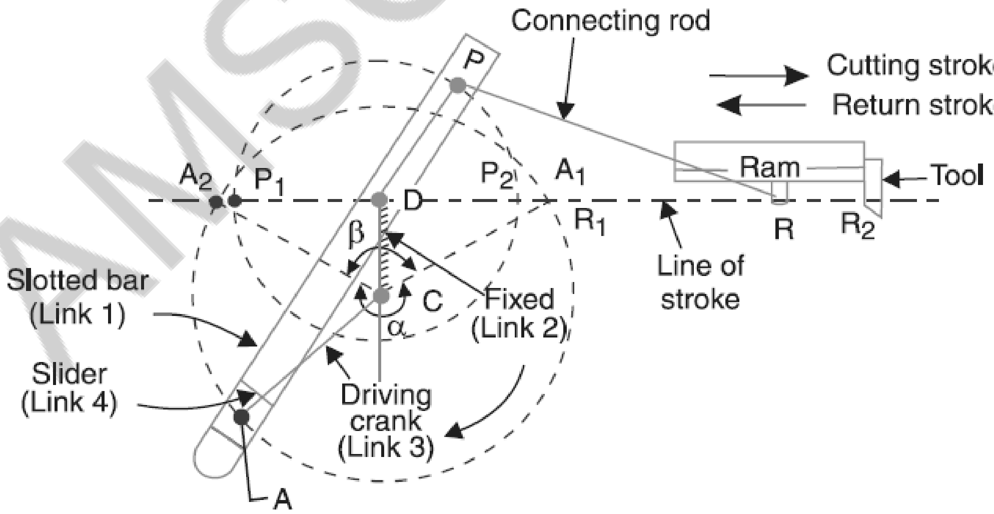
A *Watt's indicator mechanism (also known as Watt's straight line mechanism or double lever mechanism) which consists of four links, is shown in Fig. The four links are : fixed linkat *A*, link *AC*, link *CE* and link *BFD*. It may be noted that*BF* and *FD* form one link because these two parts have norelative motion between them. The links *CE* and *BFD* actas levers. The displacement of the link *BFD* is directlyproportional to the pressure of gas or steam which acts onthe indicator plunger. On any small displacement of themechanism, the tracing point *E* at the end of the link *CE*traces out approximately a straight line.The initial position of the mechanism is shown inFig. by full lines whereas the dotted lines show theposition of the mechanism when the gas or steam pressureacts on the indicator plunger.



4) Sketch and describe the working of two different types of quick return mechanisms. Give examples of their applications. Derive an expression for the ratio of times taken in forward and return stroke for one of these mechanisms. (MAY/JUNE 2015)

<u>WHITWORTH QUICK RETURN MOTION MECHANISM</u>

This mechanism is mostly used in shaping and slotting machines. In this mechanism, the link *CD* (link 2) forming the turning pair is fixed, as shown in Fig. The link 2 corresponds to a crank in a reciprocating steam engine. The driving crank *CA* (link 3) rotates at a uniform angular speed. The slider (link 4) attached to the crank pin at *A* slides along the slotted bar *PA* (link 1) which oscillates at a pivoted point *D*. The connecting rod *PR* carries the ram at *R* to which a cutting tool is fixed. The motion of the tool is constrained along the line *RD* produced, *i.e.* along a line passing through *D* and perpendicular to *CD*.



When the driving crank CA moves from the position CA_1 to CA_2 (or the link DP from the position DP_1 to DP_2) through an angle α in the clock wise direction, the tool moves from the left hand end its stroke to the hand end through a distance 2PD.

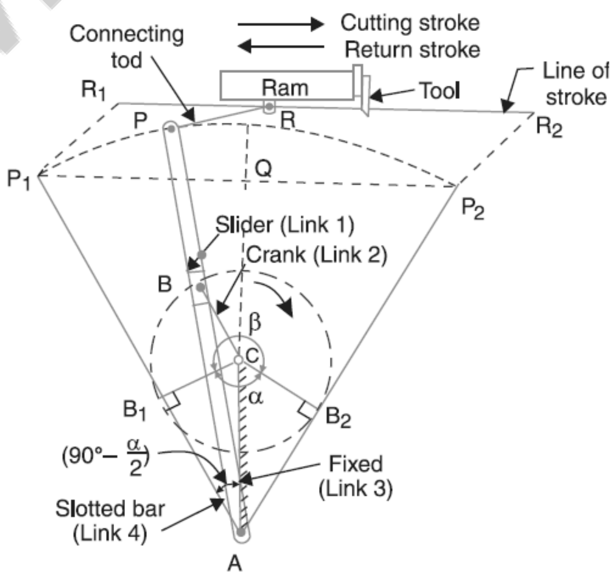
Now when the driving crank moves from the position CA_1 to CA_2 (or the link DP from DP_2 to DP_1) through an angle β in the clockwise direction, the tool moves back from right hand end of its stroke to the left hand end.
A little consideration will show that the time taken during the left to right movement of the ram(i.e during forward or cutting stroke) will be equal to the time taken by the driving crank to move from CA_1 to CA_2. Silimilarly, the time taken during the right to left movement of the ram (or during the idle or return stroke) will be equal to the time by the driving crank to mive from CA_2 to CA_1.

Since the crank link CA rotates at uniform angular velocity therefore time taken during the cutting stroke (or forward stroke) is more than time taken during the returns stroke. In other words, the mean speed of the ram during cutting stroke is less than the mean speed during the return stroke. The ratio between the time taken during the cutting and return strokes is given by

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360° - \alpha} \text{ or } \frac{360° - \beta}{\beta}$$

<u>CRANK AND SLOTTED LEVER QUICK RETURN MOTION MECHANISM.</u>

This mechanism is mostly used in shaping machines, slotting machines and in rotary internal combustion engines.
In this mechanism, the link AC (i.e. link 3) forming the turning pairs is first, as shown in fig. the link 3 corresponds to the connecting rod of a reciprocating steam engine. The driving crank CB revolves with uniform angular speed about the fixed centreC. A silding block attached to the craknk pin at B slides along the slotted bar AP and thus causes AP to oscillate about the pivotd point A. A short link PR transmits the motion from AP to the ram which carries the tool and reciprocates along the line of stroke R_1, R_2. The line of stroke of the ram (i.e. R_1 and R_2) is perpendicular to ACC produced.



In the extreme positions, AP_1 and AP_2 are tangential to the circle and the cutting tool is at the end of the stroke. The forward or cutting stroke occurs when the crank rotates from the position CB_1 to CB_2 (or through an angle β) in the clock wise directions. The return stroke occurs when the crank rotates from the positions CB_2 to CB_1 (or through angle α) in the clock wise directions. Since the crank has uniform angular speed.

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360° - \beta} \text{ or } \frac{360° - \alpha}{\alpha}$$

Since the tool travels a distance of R_1 R_2 during cutting and return stroke, therfore travel of the tool or length of stroke

$$= R_1R_2 = P_1P_2 = 2P_1Q = 2AP_1 \sin < P_1AQ$$

$$= 2AP_1 \sin\left(90° - \frac{\alpha}{2}\right) = 2AP \cos\frac{\alpha}{2}$$

$$= 2AP \times \frac{CB_1}{AC}$$

$$= 2AP \times \frac{CB}{AC}$$

From fig, we see that the angle β made by the forward or cutting stroke is greater than the angle α described by the return stroke. Since the crank rotates with uniform angular speed, therefore the returns stroke is completed within shorter time. Thus it is called quick return motion mechanism.

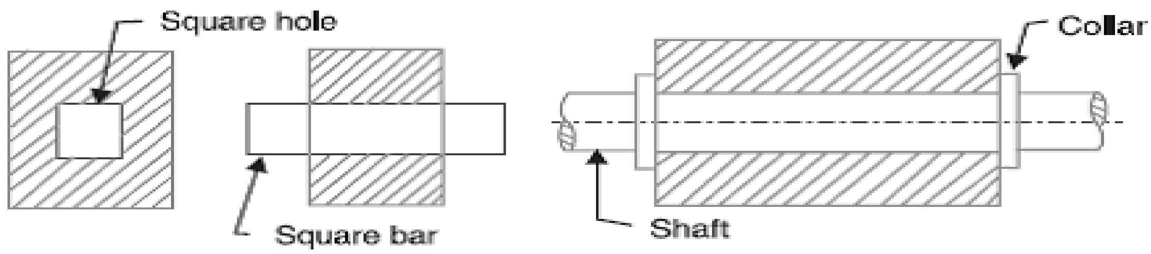
5). (a) (i) Classify kinematic pairs based on Degrees of Freedom. (10)
(MAY/JUNE 2016)

The two links or elements of a machine, when in contact with each other, are said to form a pair. If the relative motion between them is completely or successfully constrained (i.e. in a definite direction), the pair is known as kinematic pair.

<u>Types of Constrained Motions:</u>

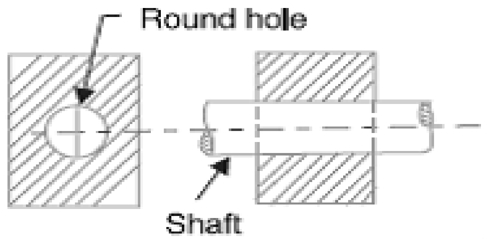
(i) Completely constrained motion:

When the motion between a pair is limited to a definite direction irrespective of the direction of force applied, then the motion is said to be a completely constrained motion. For example, the piston and cylinder (in a steam engine) form a pair and the motion of the piston is limited to a definite direction (i.e. it will only reciprocate) relative to the cylinder irrespective of the direction of motion of the crank, as shown in Fig.



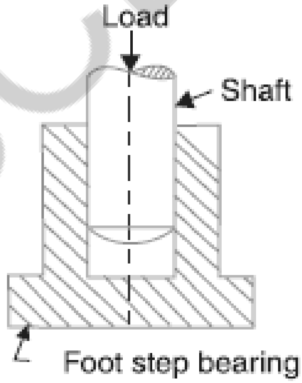
(ii) Incompletely constrained motion:

When the motion between a pair can take place in more than one direction, then the motion is called an incompletely constrained motion. The change in the direction of impressed force may alter the direction of relative motion between the pair. A circular bar or shaft in a circular hole, as shown in Fig., is an example of an incompletely constrained motion as it may either rotate or slide in a hole. These both motions have no relationship with the other.



(iii) Successfully / partially constrained motion:

When the motion between the elements, forming a pair, is such that the constrained motion is not completed by itself, but by some other means, then the motion is said to be successfully constrained motion. Consider a shaft in a foot-step bearing as shown in Fig. The shaft may rotate in a bearing or it may move upwards. This is a case of incompletely constrained motion. But if the load is placed on the shaft to prevent axial upward movement of the shaft, then the motion of the pair is said to be successfully constrained motion. The motion of an I.C. engine valve (these are kept on their seat by a spring) and the piston reciprocating inside an engine cylinder are also the examples of successfully constrained motion.



(ii) What is an inversion and list its properties. **(2+4)**

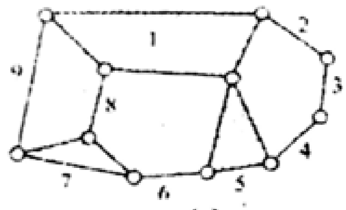
The method of obtaining different mechanisms by fixing different links in a kinematic chain is known as Inversion of the Mechanism.

Properties:

- It may be noted that the relative motions between the various links is not changed in any manner through the process of inversion.
- The absolute motions (those measured with respect to the fixed link) may be changed drastically.

- The part of a mechanism which initially moves with respect to the frame or fixed link is called driver and the part of the mechanism to which motion is transmitted is called follower.
- Most of the mechanisms are reversible, so that same link can play the role of a driver and follower at different times.

6). (b). (i) Find the degrees of freedom of the mechanism shown in fig. (10) (MAY/JUNE 2016)


(a)

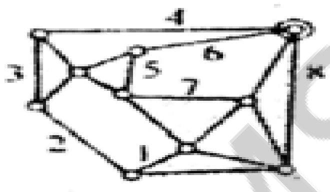
$$DOF = 3(n-1) - 2j - h \quad \Rightarrow \quad \begin{aligned} n &= \text{Total no. of links.} \\ j &= \text{Total no. of lower pairs.} \\ h &= \text{Total no. of higher pairs.} \end{aligned}$$

$n = 9$

$j = 11$

$h = 0$

$F = 3(9-1) - 2(11) - 0 = 2$

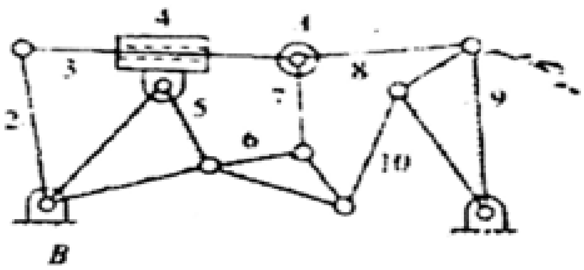

(b)

$n = 8$

$j = 11$

$h = 0$

$F = 3(8-1) - 2(11) - 0 = -1$

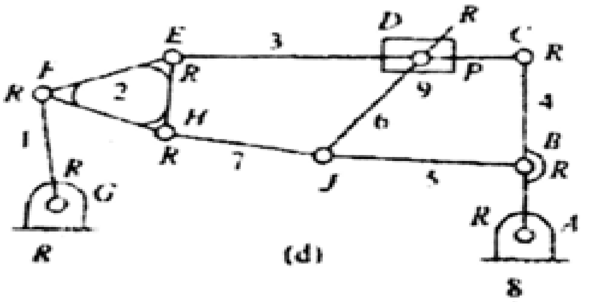

(c)

$n = 10$

$j = 12$

$h = 0$

$F = 3(10 - 1) - 2(12) - 0 = 3$

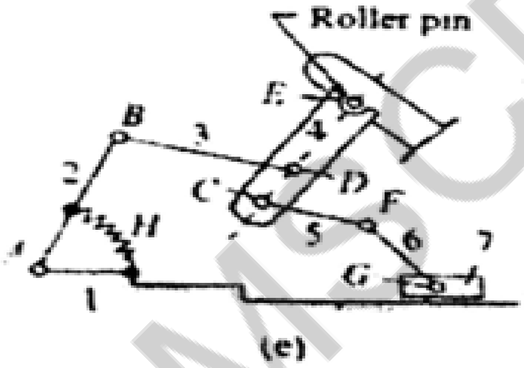


$n = 9$

$j = 10$

$h = 0$

$F = 3(9 - 1) - 2(10) - 0 = 4$



$n = 7$

$j = 8$

$h = 0$

$F = 3(7 - 1) - 2(8) - 0 = 2$

(ii) State the inconsistencies of Grubler's criterion. (6)

The Grubler's criterion only applies to mechanisms with only single degree of freedom joints where the overall movability of the mechanism is unity. Substituting n = 1 and h = 0 in Kutzbach equation, we have

$$1 = 3 (l - 1) - 2 j \quad \text{or} \quad 3l - 2j - 4 = 0$$

This equation is known as the Grubler's criterion for plane mechanisms with constrained motion.

A little consideration will show that a plane mechanism with a movability of 1 and only single degree of freedom joints cannot have odd number of links. The simplest possible mechanisms of this type are a four bar mechanism and a slider-crank mechanism in which $l = 4$ and $j = 4$.

7) Briefly explain the classification of Kinematic pair with neat sketches. (8) (NOV/DEC 2014)

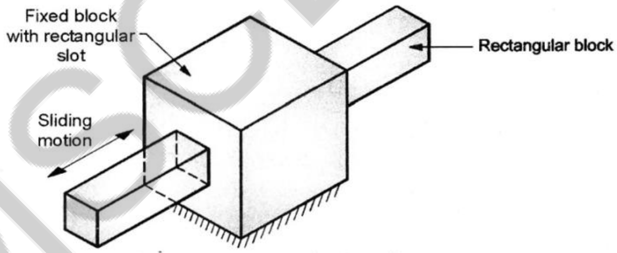
Classification of Kinematic Pairs:

The kinematic pairs may be classified according to the following considerations :

1. *According to the type of relative motion between the elements.*

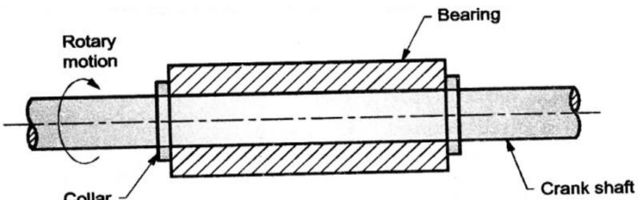
The kinematic pairs according to type of relative motion between the elements may be classified as discussed below:

(*a*) *Sliding pair.* When the two elements of a pair are connected in such a way that one can only slide relative to the other, the pair is known as a sliding pair. The piston and cylinder, cross-head and guides of a reciprocating steam engine, ram and its guides in shaper, tail stock on the lathe bed etc. are the examples of a sliding pair. A little consideration will show, that a sliding pair has a completely constrained motion.



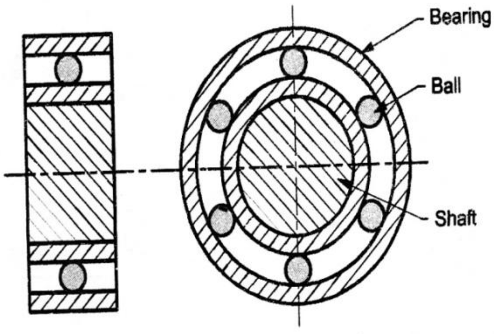
Completely constrained motion

(*b*) *Turning pair*. When the two elements of a pair are connected in such a way that one can only turn or revolve about a fixed axis of another link, the pair is known as turning pair. A shaft with collars at both ends fitted into a circular hole, the crankshaft in a journal bearing in an engine, lathe spindle supported in head stock, cycle wheels turning over their axles etc. are the examples of a turning pair. A turning pair also has a completely constrained motion.



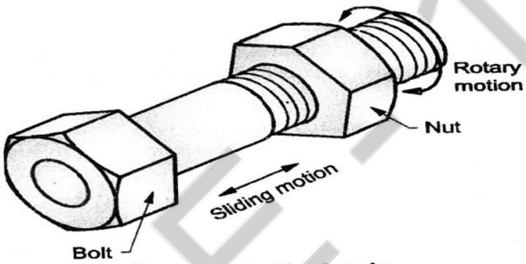
Turning or revolute pair

(*c*) *Rolling pair*. When the two elements of a pair are connected in such a way that one rolls over another fixed link, the pair is known as rolling pair. Ball and roller bearings are examples of rolling pair.



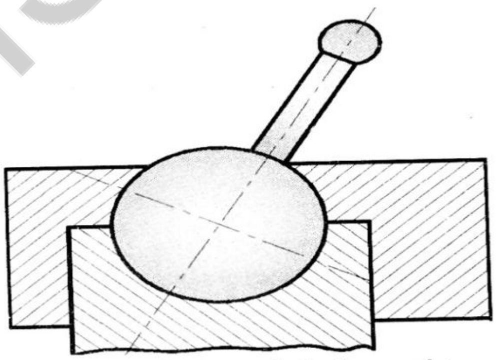
Rolling pair

(*d*) *Screw pair.* When the two elements of a pair are connected in such a way that one element can turn about the other by screw threads, the pair is known as screw pair. The lead screw of a lathe with nut, and bolt with a nut are examples of a screw pair.



Screw or helical pair

(*e*) *Spherical pair*. When the two elements of a pair are connected in such a way that one element (with spherical shape) turns or swivels about the other fixed element, the pair formed is called a spherical pair. The ball and socket joint, attachment of a car mirror, pen stand etc., are the examples of a spherical pair.



Spherical or globular pair

2. *According to the type of contact between the elements*.

The kinematic pairs according to the type of contact between the elements may be classified as discussed below :

(*a*) *Lower pair*. When the two elements of a pair have a surface contact when relative motion takes place and the surface of one element slides over

the surface of the other, the pair formed is known as lower pair. It will be seen that sliding pairs, turning pairs and screw pairs form lower pairs.
(*b*) ***Higher pair***. When the two elements of a pair have a line or point contact when relative motion takes place and the motion between the two elements is partly turning and partly sliding, then the pair is known as higher pair. A pair of friction discs, toothed gearing, belt and rope drives, ball and roller bearings and cam and follower are the examples of higher pairs.

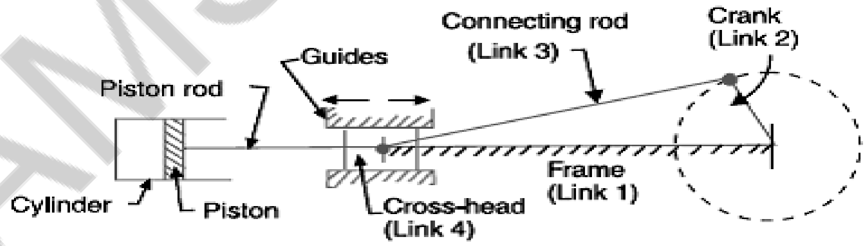
3. *According to the type of closure*.

The kinematic pairs according to the type of closure between the elements may be classified as discussed below :
(*a*) ***Self closed pair***. When the two elements of a pair are connected together mechanically in such a way that only required kind of relative motion occurs, it is then known as self closed pair. The lower pairs are self closed pair.
(*b*) ***Force - closed pair***. When the two elements of a pair are not connected mechanically but are kept in contact by the action of external forces, the pair is said to be a force-closed pair. The cam and follower is an example of force closed pair, as it is kept in contact by the forces exerted by spring and gravity.

8) Explain the inversions of Slider Crank Mechanisms with examples.(8) (NOV/DEC 2014)

Single Slider Crank Chain:

- When one of the turning pairs of a four bar chain is replaced by a sliding pair; it becomes a single slider crank chain.
- It consists of one sliding pair and three turning pairs.



- In a single slider crank chain as shown in the above figure, the links 1&2, links 2&3, and links 3&4 form three turning pairs while the links 4&1 form a slider pair.

<u>**Inversions of Single Slider Crank Chain:**</u>

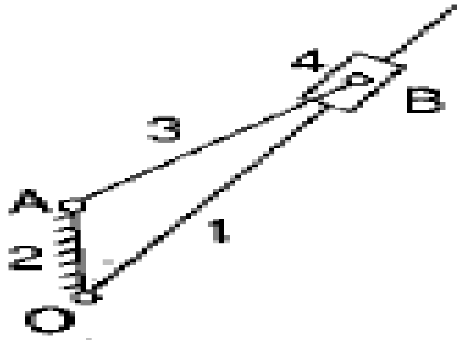
1. FIRST INVERSION :

➢ This inversion is obtained when **link 1** is fixed and links 2 & 4 are made the crank & the slider respectively.

Application:

1. Reciprocating Steam Engine: Link 4 (piston) is the driver.
2. Reciprocating Compressor. Link 2 (crank) is the driver.

2. SECOND INVERSION:



➢ This inversion is obtained when link 2 is fixed; link 3 along with the slider becomes crank and link 1 rotate about O along with the slider which also reciprocates on it.

Application:

➢ **Whitworth quick-return mechanism**

➢ **Rotary internal combustion engine / Gnome engine**

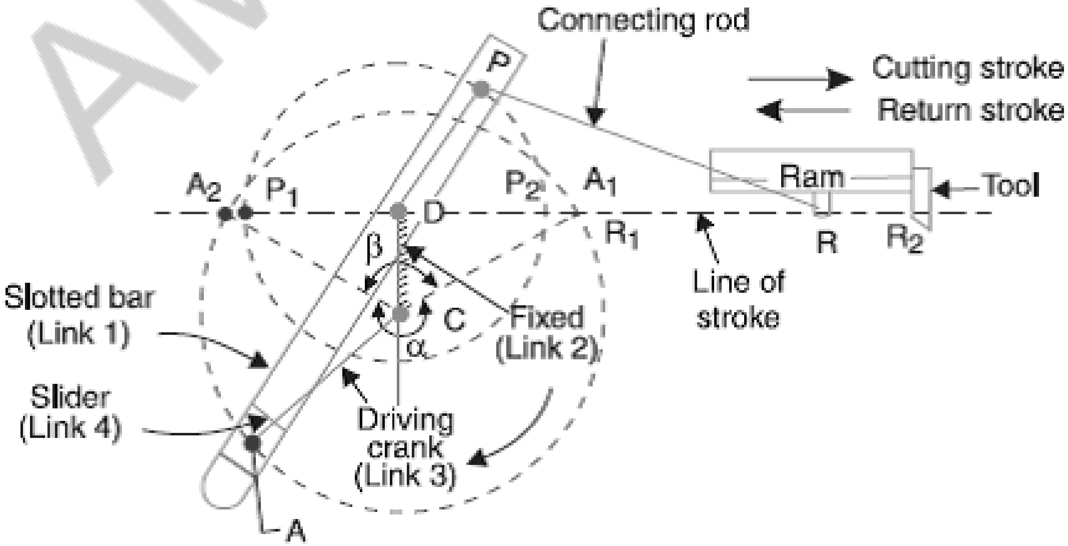
1. Whitworth Quick-Return Mechanism

➢ This mechanism used in shaping and slotting machines.

➢ In this mechanism the link CD (link 2) forming the turning pair is fixed; the driving crank CA (link 3) rotates at a uniform angular speed and the slider (link 4) attached to the crank pin at A slides along the slotted bar PA (link 1) which oscillates at D.

➢ The connecting rod PR carries the ram at R to which a cutting tool is fixed and the motion of the tool is constrained along the line RD produced.

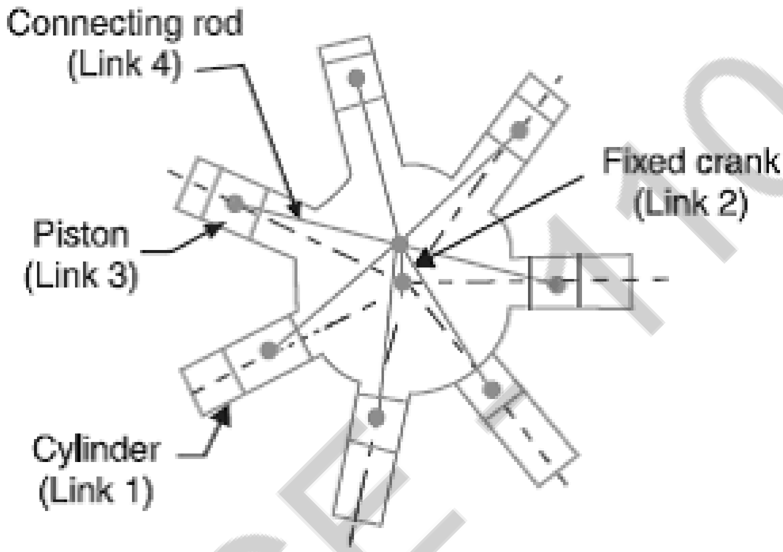
➢ The length of effective stroke = 2 PD. And mark $P_1R_1 = P_2R_2$ = PR.



$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360^\circ - \alpha} \quad \text{or} \quad \frac{360^\circ - \beta}{\beta}$$

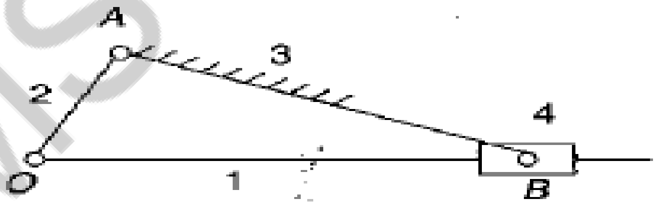
2. <u>Rotary Internal Combustion Engine / Gnome Engine:</u>

- This mechanism is used in aviation.
- It consists of seven cylinders in one plane and all revolves about fixed centre D.
- The crank 2 is fixed, connecting rod 4 rotates and the piston 3 reciprocates inside the cylinders forming link 1.



3. THIRD INVERSION:

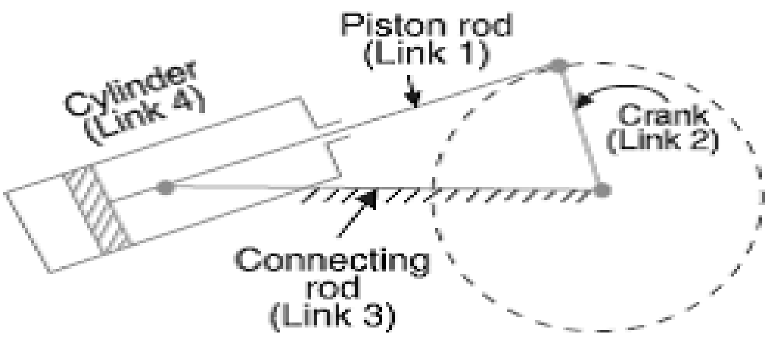
- This inversion is obtained when the link 3 is fixed, the link 2 acts as a crank and link 4 oscillates.



<u>Application:</u>

1. Oscillating Cylinder Engine:

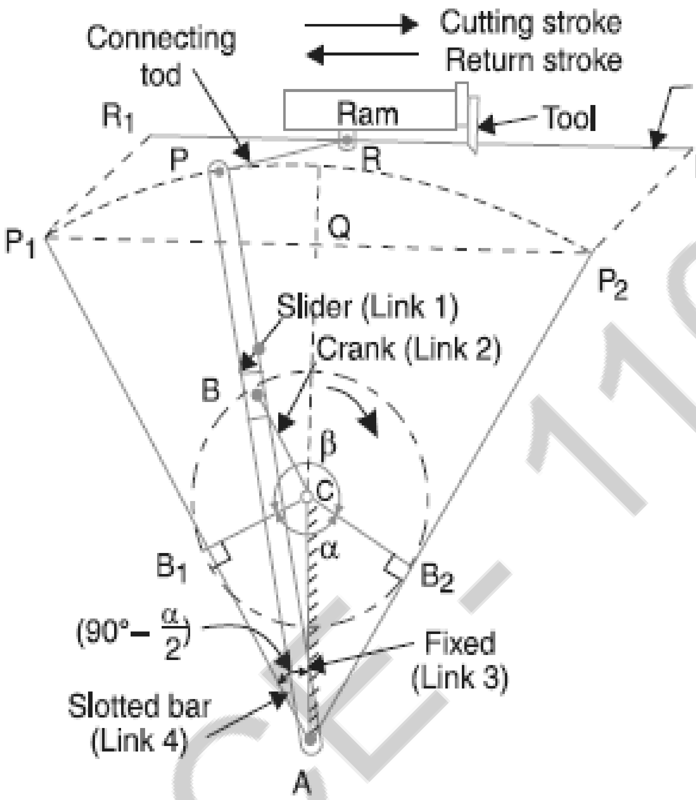
- It is used to convert reciprocating motion into rotary motion.



➢ In this mechanism link 3 is fixed, the crank 2 rotates, piston rod 1 reciprocates and cylinder 4 oscillates about A.

2. Crank & Slotted Lever Mechanism:

➢ This mechanism is used in shaping machines, slotting machines and in rotary internal combustion engine.



➢ In this mechanism link AC(3) corresponding to the connecting rod is fixed, the driving crank CB revolves about the fixed centre C and a sliding block attached to the crank pin at B slides along the slotted bar AP.

➢ AP oscillates about A and a short link PR transmits motion from AP to the arm which reciprocates along the line of stroke R_1R_2.

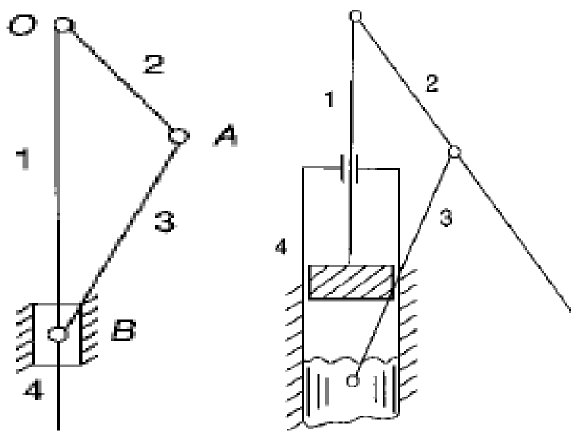
$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360^\circ - \beta} \text{ or } \frac{360^\circ - \alpha}{\alpha}$$

$$\text{Length of stroke} = R_1R_2 = P_1P_2 = 2P_1Q = 2AP\cos\frac{\alpha}{2} = 2AP \times \frac{CB_1}{AC} = 2AP \times \frac{CB}{AC}$$

4. FOURTH INVERSION:

This inversion is obtained when the link 4 is fixed, the link 3 oscillates about B on the link 4 and the end A of the link 2 is oscillates about B and the end O reciprocates along the fixed link 4.

<u>Application</u>: Hand-Pump



- The link 4 is made in the form of a cylinder and a plunger fixed to the link 1 reciprocates in it.

9) A Hooke's joint connects two shafts whose axes intersect at 18°. The driving shaft rotates at uniform speed of 210 rpm. The driven shaft with attached masses has a mass of 60 kg and the radius of gyration of 120 mm. Determine the torque required at the driving shaft if a steady torque is 45° and angle between the shafts at which the total fluctuation of speed of the driven shaft is limited to 18 rpm. (16)

(NOV/DEC 2014)

Given: $\alpha = 18°;\ N_1 = 210\text{rpm};\ m = 60\text{kg};\ k = 120\text{mm} = 0.12\text{m};\ \theta = 45°;$

$\text{Steady torque} = 180\text{N.m}$

Solution: $\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi \times 210}{60} = 21.99\,\text{rad/s}$

(i) Torque required at the driving shaft (T_1):

We know that the maximum angular acceleration of the driven shaft,

$$\alpha_2 = \frac{-\omega_1^2 \cdot \cos\alpha \cdot \sin 2\theta \cdot \sin^2\alpha}{\left(1-\cos^2\theta \cdot \sin^2\alpha\right)^2}$$

$$= \frac{-(21.99)^2 \cos 18° \cdot \sin(2\times 45°) \cdot \sin^2 18°}{\left(1-\cos^2 45° \cdot \sin^2 18°\right)^2}$$

$$= \frac{-43.91}{0.906} = -48.46\,\text{rad/s}^2$$

Accelerating torque required on the driven shaft

$$= I\alpha = mk^2 \times \alpha_2$$

$$= 60 \times (0.12)^2 \times (-48.46) = -41.87\text{N.m}$$

$\therefore$ Total torque required on the driven shaft is given by

$$T_2 = \text{Steady torque} + \text{Accelerating torque}$$

$$= 180 + (-41.87) = 138.13\text{N.m}$$

We know that $\mathbf{P = T_1\omega_1 = T_2\omega_2}$

or Torque required at the driving torque, $T_1 = T_2 \times \frac{\omega_2}{\omega_1}$

But angular velocity ratio, $\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1-\cos^2\theta\cdot\sin^2\alpha}$

$$= \frac{\cos 18°}{1-\cos^2 45°\cdot\sin^2 18°} = 0.998$$

$$T_1 = 138.13 \times 0.998 = 137.98 \text{N.m}$$

(ii) Angle between the shafts at which maximum fluctuation of speed of the driven shaft is 18 rpm:

We know that the maximum fluctuation of speed of the driven shaft

$$(\omega_2)_{max} - (\omega_2)_{min} = \omega\left(\frac{\sin^2\alpha}{\cos\alpha}\right)$$

$$18 = 210\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right)$$

On simplification, we get $\cos^2\alpha + 0.1\cos\alpha - 1 = 0$

or $\cos\alpha = \frac{-0.1 \pm \sqrt{(0.1)^2 - 4(1)(-1)}}{2(1)} = \frac{-0.1 \pm 2}{2} = 0.95 \text{ or } -1.05$

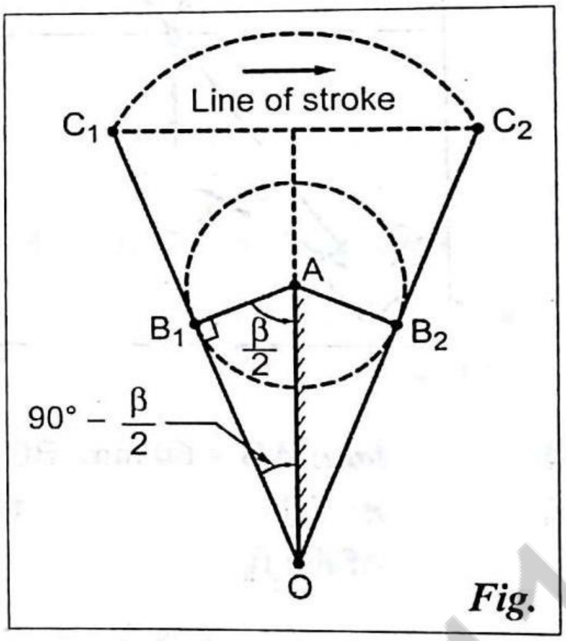
or $\alpha = \cos^{-1}(0.95) = 18.19°$

10) In a crank and slotted lever mechanism, the distance between the fixed centres is 240 mm and the length of the driving crank is 120 mm. Find the inclination of the slotted bar with the vertical in the extreme position and the time ratio of cutting stroke to return stroke. If the length of the slotted bar is 450 mm. find the length of the stroke, if the line of stroke passes through the extreme position of the free end of the lever. (16)

(NOV/DEC 2015) ***[NOV/DEC-2017]***

Given data: $OA = 240\text{mm}; \ AB_1 = AB_2 = 120\text{mm}$

Solution: The extreme positions of the crank in a crank and slotted lever quick-return motion mechanism is shown in Fig.



Case (i):

(i) Inclination of the slotted bar with the fixed link:

Let $\angle AOB_1$ = Inclination of the slotted bar with the vertical

We know that

$$\sin\angle AOB_1 = \sin\left(90° - \frac{\beta}{2}\right)$$

$$= \frac{AB_1}{AO} = \frac{120}{240} = 0.5$$

or $\angle AOB_1 = \sin^{-1}(0.5) = 30°$

(ii) Time ratio of cutting stroke to return stroke:

We know that $90° - \frac{\beta}{2} = 30°$

or $\beta = 120°$

$$\therefore \frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{360° - \beta}{\beta} = \frac{360° - 120°}{120°} = 2$$

Case (ii): Find the length of the stroke, If length of slotted bar is 450mm.

$$\text{Length of the stroke} = E_1 E_2 = 2 \times OE_1 \times \frac{AB_1}{OA}$$

$$= 2 \times 450 \times \frac{120}{240} = 450\text{mm}$$

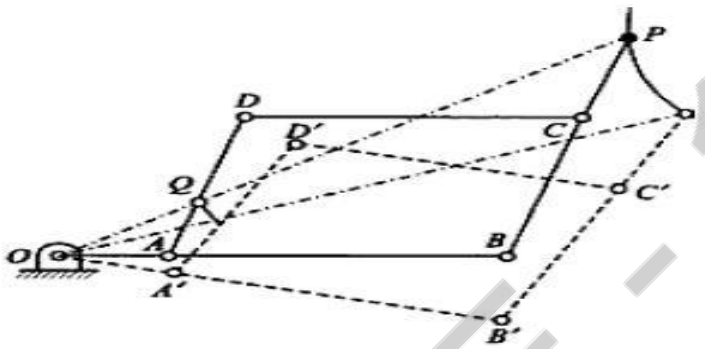
11) Explain Pantograph with neat diagram.

- A pantograph is an instrument used to reproduce to an enlarged or a reduced scale and as exactly as possible the path described by a given point.
- It is also used to guide cutting tools.
- A modified form of pantograph is used to collect power at the top of an electric locomotive.

Construction

- links OAR, RCP, CD and AD and pin-connected at A, R, C and D. Links are so proportioned that AR = CD and BC = AD with pins A, B, C. D constituting corners of

a parallelogram. Link OAR is pivoted to frame at 0.



- A point P on the extension of link BC and another point Q on link AD are so chosen that points 0, Q and P always lie on a common straight line.
- Since lines AD and BC are parallel and lines OQP and OAR cut them, triangles OAQ and ORP are similar.

$$\frac{OP}{OQ} = \frac{OB}{OA} = \frac{BP}{AQ} = k, \text{ say}$$

Let the point P be now displaced to position P' along the curved path PP'.

$$\frac{OB'}{OA'} = \frac{OB}{OA}$$

$$\frac{B'P'}{A'Q'} = \frac{BP}{AQ}$$

$$\frac{OB'}{OA'} = \frac{B'P'}{A'Q'} = \frac{OP}{OQ}$$

Since links $B'C'$ and $A'D'$ are parallel, triangles $OB'P'$ and $OA'Q'$ are similar.

we have from similar triangles $OA'Q'$ and $OB'P'$,

$$\frac{OP'}{OQ'} = \frac{OB'}{OA}$$

$$\frac{OP'}{OQ'} = \frac{OB}{OA} = k \qquad k = \frac{OB}{OA} = \frac{BP}{AQ}$$

12) Explain Inversion Double Slider crank Mechanism with neat diagram?

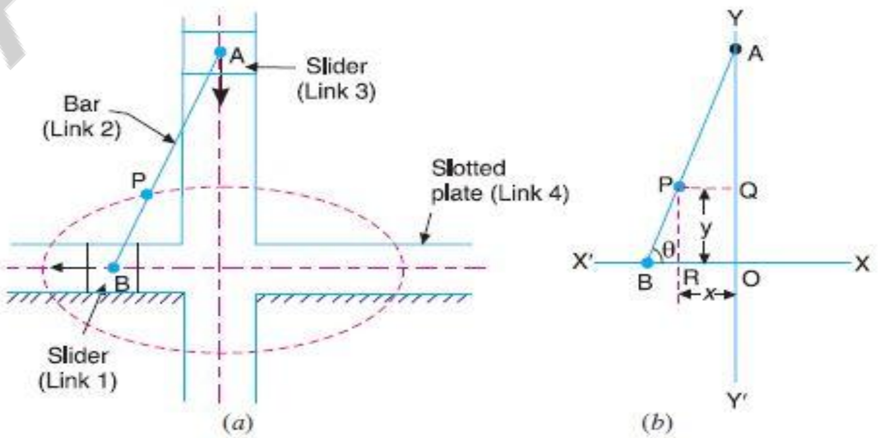
A kinematic chain which consists of two turning pairs and two sliding pairs is known as ***double slider crank chain***

Inversions of Double Slider Crank Chain

The following three inversions of a double slider crank chain are important from the subject point of view :

1. *Elliptical trammels*. It is an instrument used for drawing ellipses. This inversion is obtained by fixing the slotted plate (link 4), as shown in Fig. 5.34. The fixed plate or link 4 has two straight grooves cut in it, at right angles to each other. The link 1 and link 3, are known as sliders and form sliding pairs with link 4. The link *AB* (link 2) is a bar which forms turning pair with links 1 and 3.

When the links 1 and 3 slide along their respective grooves, any point on the link 2 such as *P* traces out an ellipse on the surface of link 4, as shown in Fig. 5.34 (*a*). A little consideration will show that *AP* and *BP* are the semi-major axis and semi-minor axis of the ellipse respectively. This can be proved as follows :



Let us take OX and OY as horizontal and vertical axes and let the link BA is inclined at an angle with the horizontal, as shown in Fig.(*b*). Now the co-ordinates of the point P on the link BA

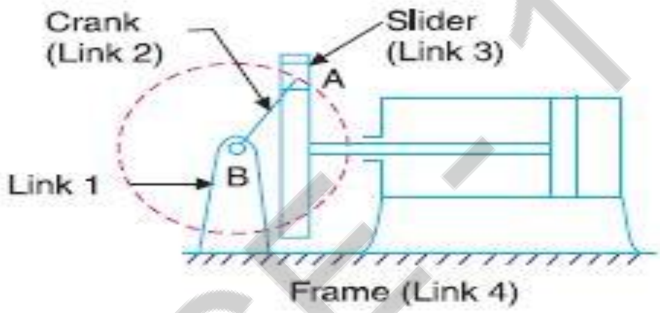
$$x = PQ = AP\cos\theta;\ \text{and}\ y = PR = BP\sin\theta$$

$$\frac{x}{AP} = \cos\theta;\ \text{and}\ \frac{y}{BP} = \sin\theta$$

Squaring and adding,

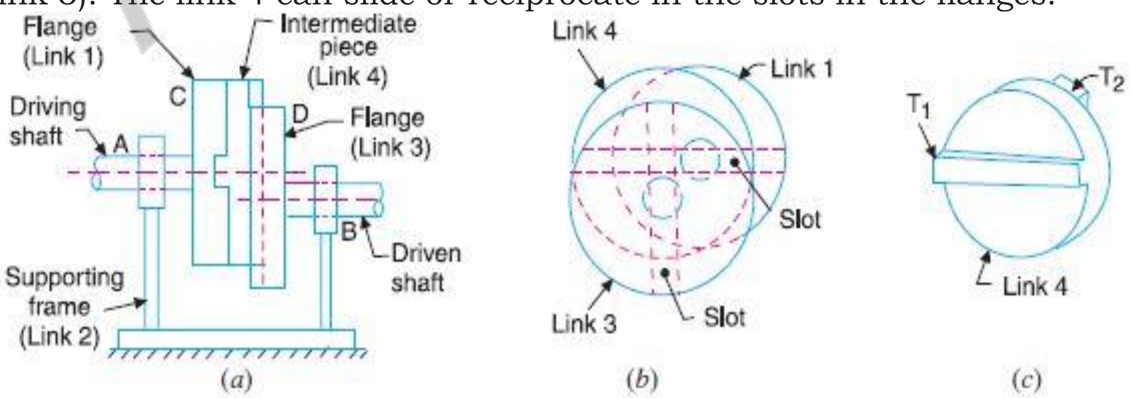
$$\frac{x^2}{(AP)^2} + \frac{y^2}{(BP)^2} = \cos^2\theta + \sin^2\theta = 1$$

2. Scotch yoke mechanism. This mechanism is used for converting rotary motion into a reciprocating motion. The inversion is obtained by fixing either the link 1 or link 3. link 1 is fixed. In this mechanism, when the link 2 (which corresponds to crank) rotates about B as centre, the link 4 (which corresponds to a frame) reciprocates. The fixed link 1 guides the frame.



3. Oldham's coupling. An oldham's coupling isused for connecting two parallel shafts whose axes are at a small distance apart. The shafts are coupled in such a way that if one shaft rotates, the other shaft also rotates at the same speed. This inversion is obtained by fixing the link 2, The shafts to be connected have two flanges (link 1 and link 3) rigidly fastened at their ends by forging.

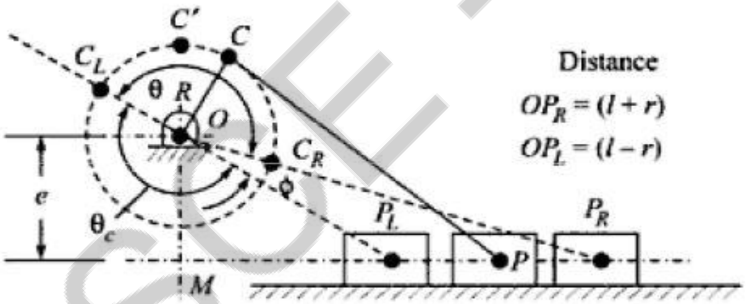
The link 1 and link 3 form turning pairs with link 2. These flanges have diametrical slots cut in their inner faces. The intermediate piece (link 4) which is a circular disc, have two tongues (*i.e.* diametrical projections) $T1$ and $T2$ on each face at right angles to each other, as shown in Fig. (*c*). The tongues on the link 4 closely fit into the slots in the two flanges (link 1 and link 3). The link 4 can slide or reciprocate in the slots in the flanges.



13) Explain Offset Slider crank Mechanism as slider crank Mechanism?

- In many situations, mechanisms are required to perform repetitive operations such as pushing parts along an assembly line and folding cardboard boxes in an automated packaging machine.
- Besides above there are applications like a shaper machine and punching/ riveting press in which working stroke is completed under load and must be executed slowly compared to the return stroke. This results in smaller work done per unit time.
- A quick return motion mechanism is useful in all such applications. A quick return motion mechanism is essentially a slider-crank mechanism in which the slider has different average velocities in forward and return stroke.
- Thus even if crank rotates uniformly, the slider completes one stroke quickly compared to the other stroke. An offset slider-crank mechanism can be used conveniently to achieve the above objective.

In Figure, the crank centre 0 is offset by an amount e with respect to the line of stroke of slider. If r be the radius of crank OC and l the length of connecting rod CP. the extreme right-hand position PR and extreme left-hand position PL of slider P occur when the crank .



$$OP_L = (C_L P_L) - OC_L = (l - r)$$

From right angled triangles $P_L\, OM$ and $P_R\, OM$ remembering that $\angle P_L O P_R = \phi$, we have

$$\cos\angle MOP_L = \frac{OM}{OP_L} = \frac{e}{(l-r)}$$

and
$$\cos\angle MOP_R = \frac{OM}{OP_R} = \frac{e}{(r+l)}$$

Hence
$$\phi = \cos^{-1}\left(\frac{e}{r+l}\right) - \cos^{-1}\left(\frac{e}{l-r}\right)$$

Thus, return stroke angle,

$$\theta_R = (180 - \phi)$$

and cutting stroke angle,

$$\theta_C = (180 + \phi)$$

$$Q = \frac{\text{Time of advance stroke}}{\text{Time of return stroke}}$$

When Q is greater than one, the mechanism is called *quick-return mechanism*.

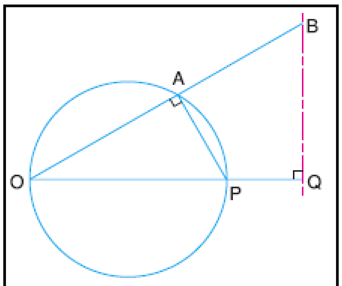
Assuming that the driving motor rotates at constant r.p.m. N, the time of advance stroke and return stroke can be obtained as

$$Q = \frac{\theta_c}{\theta_R}$$

Time of advance stroke = $\left(\frac{\theta_c}{2\pi N}\right)$, and time of return stroke = $\left(\frac{\theta_R}{2\pi N}\right)$

$$Q = \frac{\theta_c}{\theta_R}$$

14) Explain any two straight line motion mechanisms.



Exact straight line motion mechanism.

The principle adopted for a mathematically correct or exact straight line motion is described in Fig. Let O be a point on the circumference of a circle of diameter OP. Let OA be any chord and B is a point on OA produced, such that

$$OA \times OB = \text{constant}$$

Then the locus of a point B will be a straight line perpendicular to the diameter OP. This may be proved as follows:

Draw BQ perpendicular to OP produced. Join AP.

The triangles *OAP* and *OBQ* are similar.

$$\therefore \qquad \frac{OA}{OP} = \frac{OQ}{OB}$$

or $$OP \times OQ = OA \times OB$$

or $$OQ = \frac{OA \times OB}{OP}$$

But *OP* is constant as it is the diameter of a circle, therefore, if $OA \times OB$ is constant, then *OQ* will be constant. Hence the point *B* moves along the straight path *BQ* which is perpendicular to *OP*.

Following are the two well known types of exact straight line motion mechanisms made up of turning pairs.

1. ***Peaucellier mechanism***. It consists of a fixed link OO_1 and the other straight links O_1A, *OC*, *OD*, *AD*, *DB*, *BC* and *CA* are connected by turning pairs at their intersections, as shown in Fig.. The pin at *A* is constrained to move along the circumference of a circle with the fixed diameter *OP*, by means of the link O_1A. In Fig.

$$AC = CB = BD = DA \ ; \ OC = OD \ ; \text{ and } OO_1 = O_1A$$

It may be proved that the product $OA \times OB$ remains constant, when the link O_1A rotates. Join *CD* to bisect *AB* at *R*. Now from right angled triangles *ORC* and *BRC*, we have

$$OC^2 = OR^2 + RC^2 \qquad \text{...(i)}$$

and $$BC^2 = RB^2 + RC^2 \qquad \text{...(ii)}$$

Subtracting equation (*ii*) from (*i*), we have

$$OC^2 - BC^2 = OR^2 - RB^2$$

$$= (OR + RB)\,(OR - RB)$$

$$= OB \times OA$$

Since *OC* and *BC* are of constant length, therefore the product $OB \times OA$ remains constant. Hence the point *B* traces a straight path perpendicular to the diameter *OP*.

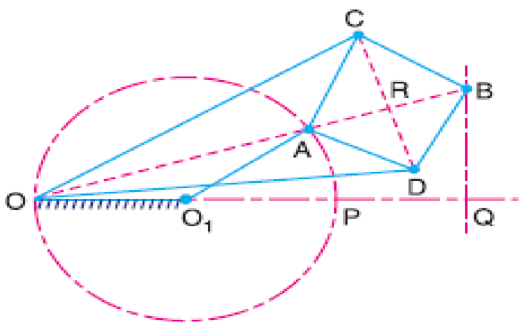


Fig. Peaucellier mechanism.

2. *Hart's mechanism*. This mechanism requires only six links as compared with the eight links required by the Peaucellier mechanism. It consists of a fixed link OO_1 and other straight links O_1A, FC, CD, DE and EF are connected by turning pairs at their points of intersection, as shown in Fig. The links FC and DE are equal in length and the lengths of the links CD and EF are also equal. The points O, A and B divide the links FC, CD and EF in the same ratio. A little consideration will show that $BOCE$ is a trapezium and OA and OB are respectively parallel to FD and CE.

Hence OAB is a straight line. It may be proved now that the product $OA \times OB$ is constant.

From similar triangles CFE and OFB,

$$\frac{CE}{FC} = \frac{OB}{OF} \quad \text{or} \quad OB = \frac{CE \times OF}{FC} \quad \text{...}(i)$$

and from similar triangles FCD and OCA

$$\frac{FD}{FC} = \frac{OA}{OC} \quad \text{or} \quad OA = \frac{FD \times OC}{FC} \quad \text{...}(ii)$$

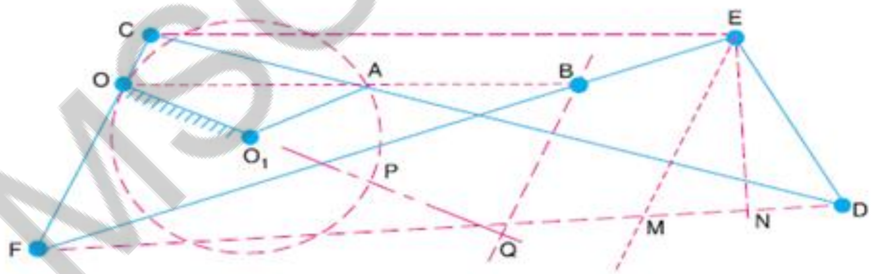


Fig. Hart's mechanism.

Multiplying equations (*i*) and (*ii*), we have

$$OA \times OB = \frac{FD \times OC}{FC} \times \frac{CE \times OF}{FC} = FD \times CE \times \frac{OC \times OF}{FC^2}$$

Since the lengths of OC, OF and FC are fixed, therefore

$$OA \times OB = FD \times CE \times \text{constant} \quad \text{...}(iii)$$

$$\text{...}\left(\text{substituting } \frac{OC \times OF}{FC^2} = \text{constant}\right)$$

Now from point E, draw EM parallel to CF and EN perpendicular to FD. Therefore

$$FD \times CE = FD \times FM \quad ...(\because CE = FM)$$
$$= (FN + ND)(FN - MN) = FN^2 - ND^2 \quad ...(\because MN = ND)$$
$$= (FE^2 - NE^2) - (ED^2 - NE^2)$$

...(From right angled triangles FEN and EDN)

From equations (*iii*) and (*iv*),

$$OA \times OB = \text{constant}$$

It therefore follows that if the mechanism is pivoted about O as a fixed point and the point A is constrained to move on a circle with centre O_1, then the point B will trace a straight line perpendicular to the diameter OP produced.

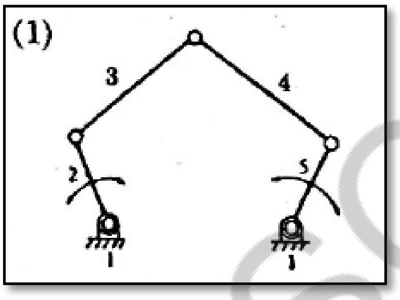
15. What is Kinematic Inversion? Discuss any three applications of Inversions of Slider Crank Mechanism with suitable sketches.

[APRIL/MAY-2017]

[REFER Q. No: 8 NOV/DEC 2014]

16. Find the degrees of freedom for the mechanisms shown in the fig.

[APRIL/MAY-2017]

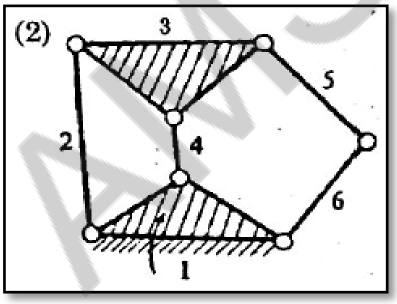


f = 3(n-1) – 2j - h

f = 3(5-1) – 2(5) – 0

f = 12 -10 – 0

f = 2

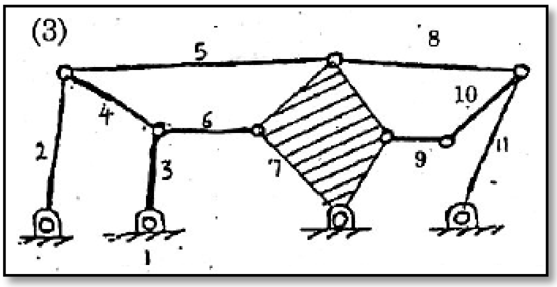


f = 3(n-1) – 2j - h

f = 3(6-1) – 2(7) – 0

f = 15 -14 – 0

f = 1



f = 3(n-1) – 2j - h

f = 3(11-1) – 2(12) – 0

f = 30 -24 – 0

f = 6

17. Explain the Mechanical Advantage & Transmission Angle related to Four Bar Mechanism. ***[APRIL/MAY-2017]***

MECHANICAL ADVANTAGE OF A MECHANISM

Because of the widespread use of the four bar mechanism, it is necessary to study some of the quality measures or indexes of merit of mechanisms which tell us whether a mechanism is a good one or a poor one. The mechanical advantage of a mechanism is one of the most important quality measure of all mechanisms.

Definition

The mechanical advantage is defined as the ratio of output torque to the input torque. It is also defined as the ratio of the load to the effort.

Let T_A = Torque exerted on the driving link (Driving torque),

T_B = Torque exerted by the driven link (Resisting torque),

ω_A = angular velocity of the driving link, and

ω_B = Angular velocity of the driven link.

∴ Ideal mechanical advantage, (*i.e.*, neglecting effect of friction)

$$M.A_{(ideal)} = \frac{T_B}{T_A} = \frac{\omega_A}{\omega_B} \quad \ldots (1)$$

If we consider the effect of friction, less resistance will be overcome with the given effort. Therefore actual mechanical advantage will be less.

Let η = Efficiency of the mechanism

∴ Actual mechanical advantage, (*i.e.*, considering the effect of friction)

$$M.A_{actual} = \eta \cdot \frac{T_B}{T_A} = \eta \times \frac{\omega_A}{\omega_B} \quad \ldots (2)$$

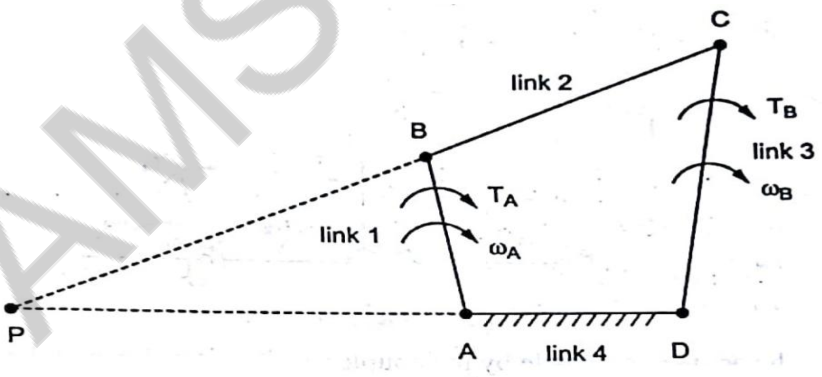


Fig.

Let P = Instant center common to links 2 and 4, obtained by extending the links 2 and link 4.

R_{PA} = Length of the segment PA, and

R_{PD} = Length of the segment PD.

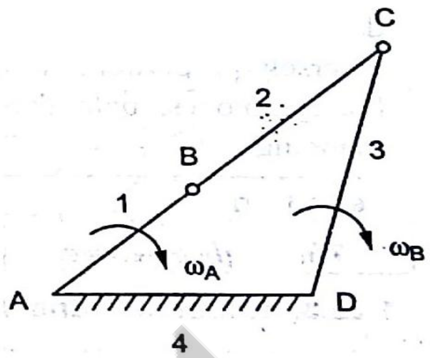
According to the angular velocity ratio theorem,

$$\frac{T_B}{T_A} = \frac{\omega_A}{\omega_B} = \frac{R_{PD}}{R_{PA}} \quad \ldots (3)$$

Four-bar linkage in toggle :

The mechanism is redrawn, as shown in Fig. At this position the links 2 and 3 are on the same straight line.

Values at this position, R_{PA} and ω_2 are zero. Hence an extreme value of the mechanical advantage (infinity) is obtained. A mechanism in this phase is said to be in *toggle*. Such ***toggle position are often used to produce a high mechanical advantage;*** e.g., clamping mechanism.



Transmission Angle and Mechanical Advantage :

Construct B′A and C′D perpendicular to the line PBC in a four-bar linkage as shown in Fig.

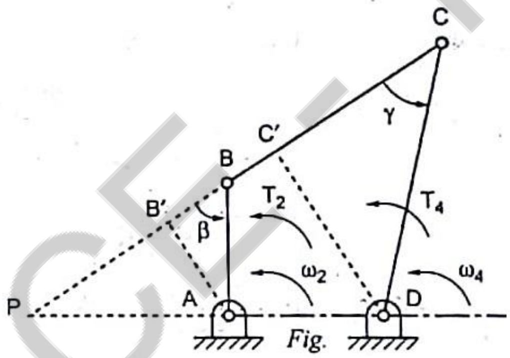


Fig.

Let β be the acute angle made by the coupler, or its extension and the driving link and γ be the acute angle made by the coupler and the driven link.

Then, by using the principle of similar triangles,

$$\frac{R_{PD}}{R_{PA}} = \frac{R_{C'D}}{R_{B'A}} = \frac{R_{CD}\sin\gamma}{R_{BA}\sin\beta}$$

Then, another equation for mechanical advantage is given by

$$\boxed{\frac{T_B}{T_A} = \frac{\omega_A}{\omega_B} = \frac{R_{CD}\sin\gamma}{R_{BA}\sin\beta}} \qquad \dots(4)$$

From the above equation, it is clear that the mechanical advantage of four-bar linkage is directly proportional to the sine of the angle γ between the coupler and the follower and inversely proportional to the sine of the angle β between the coupler and the driver. As the linkage moves, both these angles and hence the mechanical advantage is continuously changing.

18. Write in detail with neat Sketch any three inversions of double slider crank chain. ***[NOV/DEC-2017]***

[REFER Q.NO: 12]

19. Describe with neat sketch, the mechanisms obtained by the inversions of four-bar chain. ***[NOV/DEC-2017]***

[REFER Q.NO 3 MAY/JUNE 2015]

20. a) i) State and brief the Kutzbach criterion for planar mechanisms and using this criterion, determine the arrangement shown in Fig. 11 (a) as a structure or a constrained mechanism or an unconstrained mechanism. (6)

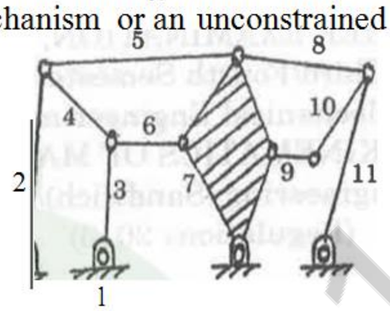


Fig. 11 (a)

[APRIL/MAY-2018]

ANS:

Expressing the number of degrees of freedom of a linkage in terms of the number of links and the number of pair connections of different types is known as *number synthesis*. *Degrees of freedom* of a mechanism in space can be determined as follows:

Let

N = total number of links in a mechanism

F = degrees of freedom

P_1 = number of pairs having one degree of freedom

P_2 = number of pairs having two degrees of freedom, and so on

In a mechanism, one link is fixed.

Therefore,

Number of movable links = $N - 1$

Number of degrees of freedom of $(N - 1)$ movable links = $6(N - 1)$

Each pair having one degree of freedom imposes 5 restraints on the mechanism, reducing its degrees of freedom by $5P_1$.

Each pair having two degrees of freedom will impose 4 restraints, reducing the degrees of freedom of the mechanism by $4P_2$.

Similarly, other pairs having 3, 4 and 5 degrees of freedom reduce the degrees of freedom of the mechanism. Thus,

$$F = 6(N - 1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - P_5 \quad (1.1)$$

The above criterion is hardly necessary to find the degrees of freedom, as space mechanisms, especially of the zero order are not practical. Most of the mechanisms are two-dimensional such as a four-link or a slider-crank mechanism in which displacement is possible along two axes (one restraint) and rotation about only one axis (two restraints). Thus, there are three general restraints.

Therefore, for plane mechanisms, the following relation may be used to find the degrees of freedom

$$F = 3(N - 1) - 2P_1 - 1P_2 \quad (1.2)$$

This is known as *Gruebler's criterion* for degrees of freedom of plane mechanisms in which each movable link possesses three degrees of freedom. Each pair with one degree of freedom imposes two further restraints on the mechanisms, thus reducing its degrees of freedom. Similarly, each pair with two degrees of freedom reduces the degrees of freedom of the mechanism at the rate of one restraint each.

Some authors mention the above relation as *Kutzback's criterion* and a simplified relation [$F = 3(N-1) - 2P_1$] which is applicable to linkages with a single degree of freedom only as Gruebler's criterion. However, many authors make no distinction between Kutzback's criterion and Gruebler's criterion.

Thus, for linkages with a single degree of freedom only, $P_2 = 0$

$$F = 3(N-1) - 2P_1 \quad (1.3)$$

Most of the linkages are expected to have one degree of freedom so that with one input to any of the links, a constrained motion of the others is obtained.

Then,

$$1 = 3(N-1) - 2P_1$$

or

$$2P_1 = 3N - 4 \quad (1.4)$$

As P_1 and N are to be whole numbers, the relation can be satisfied only if N is even. For possible linkages made of binary links only,

21. ii) Define transmission angle of a four bar mechanism and explain its Significance. Also, neatly sketch a Crank-Rocker mechanism in its minimum and maximum transmission angle positions. (7)

[APRIL/MAY-2018]

TRANSMISSION ANGLE

The angle μ between the output link and the coupler is known as *transmission angle*. In Fig. 1.43, if the link AB is the input link, the force applied to the output link DC is transmitted through the coupler BC. For a particular value of force in the coupler rod, the torque transmitted to the output link (about the point D) is maximum when the transmission angle μ is 90°. If links BC and DC become coincident, the *transmission angle* is zero and the mechanism would lock or jam. If μ deviates significantly from 90°, the torque on the output link decreases. Sometimes, it may not be sufficient to overcome the friction in the system and the mechanism may be locked or jammed. Hence μ is usually kept more than 45°. The best mechanisms, therefore, have a transmission angle that does not deviate much from 90°.

Applying cosine law to triangles ABD and BCD (Fig. 1.43),

$$a^2 + d^2 - 2ad\cos\theta = k^2 \quad \text{(i)}$$

and $$b^2 + c^2 - 2bc\cos\mu = k^2 \quad \text{(ii)}$$

From (i) and (ii),

$$a^2 + d^2 - 2ad\cos\theta = b^2 + c^2 - 2bc\cos\mu$$

or $$a^2 + d^2 - b^2 - c^2 - 2ad\cos\theta + 2bc\cos\mu = 0$$

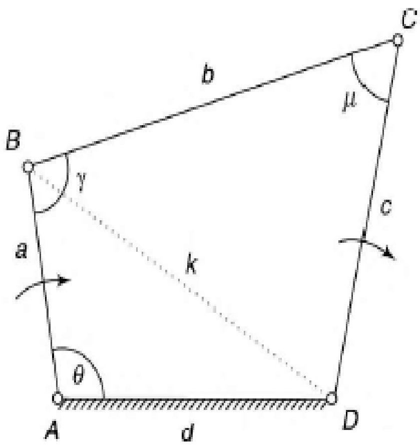


Fig. 1.43

The maximum or minimum values of the transmission angle can be found by putting $d\mu/d\theta$ equal to zero.

Differentiating the above equation with respect to θ,

$$2\,ad\sin\theta - 2bc\sin\mu.\frac{d\mu}{d\theta} = 0$$

or $$\frac{d\mu}{d\theta} = \frac{ad\sin\theta}{bc\sin\mu}$$

Thus, if $d\mu/d\theta$ is to be zero, the term $ad\sin\theta$ has to be zero which means θ is either 0° or 180°. It can be seen that μ is maximum when θ is 180° and minimum when θ is 0°. However, this would be applicable to the mechanisms in which the link a is able to assume these angles, i.e., in double-crank or crank-rocker mechanisms. Figures 1.44(a) and (b) show a crank-rocker mechanism indicating the positions of the maximum and the minimum transmission angles. Figures 1.45(a) and (b) show the maximum and the minimum transmission angles for a double-rocker mechanism.

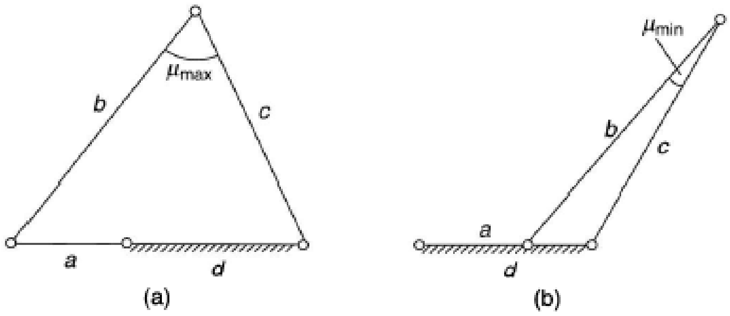


Fig. 1.44

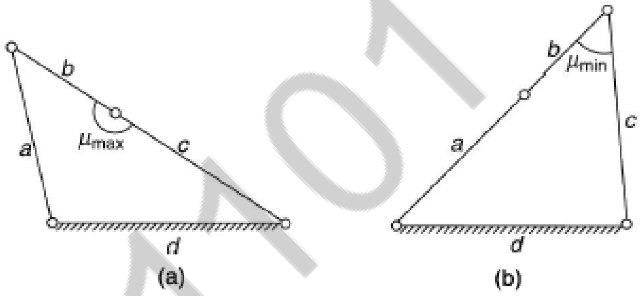


Fig. 1.45

Crank – Rocker Mechanism.

Thus, the necessary conditions for the link a to be a crank is

- the shortest link is fixed, and
- the sum of the shortest and the longest links is less than the sum of the other two links.

In a similar way, it can be shown that if the link c is to rotate through a full circle, i.e., if it is to be a crank then the conditions to be realised are the same as above. Also, it can be shown that if both the links a and c rotate through full circles, the link b also makes one complete revolution relative to the fixed link d.

The mechanism thus obtained is known as *crank-crank* or *double-crank* or *drag-crank mechanism* or *rotary-rotary converter*. Figure 1.35 shows all the three links a, b and c rotating through one complete revolution.

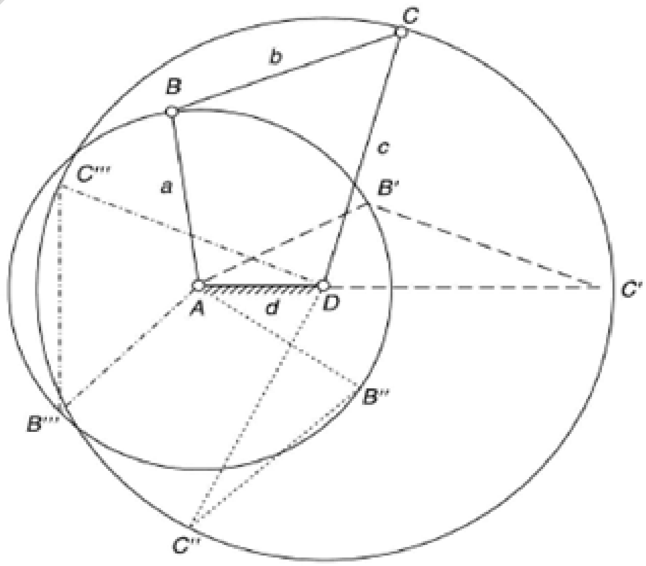
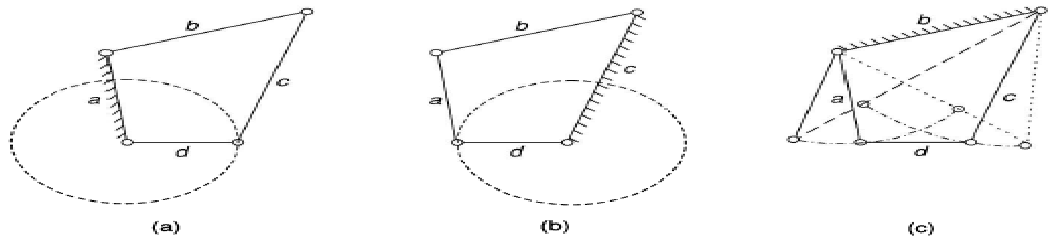


Fig. 1.35

In the above consideration, the rotation of the links is observed relative to the fixed link d. Now, consider the movement of b relative to either a or c. The complete rotation of b relative to a is possible if the angle $\angle ABC$ can be more than 180° and relative to c if the angle $\angle DCB$ more than 180°. From the positions of the links in Fig. 1.35(b) and (c), it is clear that these angles cannot become more than 180° for the above stated conditions.

Now, as the relative motion between two adjacent links remains the same irrespective of which link is fixed to the frame, different mechanisms (known as *inversions*) obtained by fixing different links of this kind of chain will be as follows:



1. If any of the adjacent links of link *d*, i.e., *a* or *c* is fixed, *d* can have a full revolution (crank) and the link opposite to it oscillates (rocks). In Fig. 1.36(a), *a* is fixed, *d* is the crank and *b* oscillates whereas in Fig. 1.36(b), *c* is fixed, *d* is the crank and *b* oscillates. The mechanism is known as *crank-rocker* or *crank-lever mechanism* or *rotary-oscillating converter.*
2. If the link opposite to the shortest link, i.e., link *b* is fixed and the shortest link *d* is made a coupler, the other two links *a* and *c* would oscillate [Fig. 1.36(c)]. The mechanism is known as a *rocker-rocker* or *double-rocker* or *double-lever mechanism* or *oscillating-oscillating converter*.

22. Sketch and describe the working of crank and slotted lever quick return mechanism. Derive an expression to find the length of the stroke for the quick return mechanism.

(13) *[NOV/DEC-2018]*

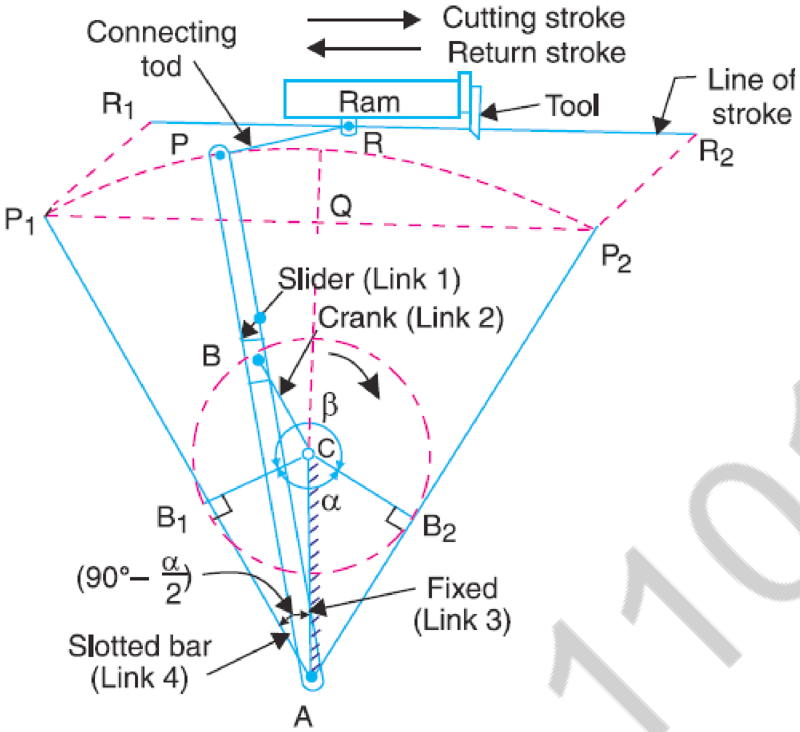
With neat sketch explain the crank and slotted lever quick return mechanism.

(13) *[APR/MAY-2019]*

CRANK AND SLOTTED LEVER QUICK RETURN MOTION MECHANISM.

This mechanism is mostly used in shaping machines, slotting machines and in rotary internal combustion engines.

In this mechanism, the link AC (i.e. link 3) forming the turning pairs is first, as shown in fig. the link 3 corresponds to the connecting rod of a reciprocating steam engine. The driving crank CB revolves with uniform angular speed about the fixed centre C. A sliding block attached to the crank pin at B slides along the slotted bar AP and thus causes AP to oscillate about the pivoted point A. A short link PR transmits the motion from AP to the ram which carries the tool and reciprocates along the line of stroke R1, R2. The line of stroke of the ram (i.e. R1 and R2) is perpendicular to ACC produced.



In the extreme positions, AP_1 and AP_2 are tangential to the circle and the cutting tool is at the end of the stroke. The forward or cutting stroke occurs when the crank rotates from the position CB_1 to CB_2 (or through an angle β) in the clock wise directions. The return stroke occurs when the crank rotates from the positions CB_2 to CB_1 (or through angle α) in the clock wise directions. Since the crank has uniform angular speed.

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360^\circ - \beta} \text{ or } \frac{360^\circ - \alpha}{\alpha}$$

Since the tool travels a distance of $R_1 R_2$ during cutting and return stroke, therefore travel of the tool or length of stroke.

$$= R_1R_2 = P_1P_2 = 2P_1Q = 2AP_1 \sin < P_1AQ$$

$$= 2AP_1 \sin\left(90^\circ - \frac{\alpha}{2}\right) = 2AP \cos\frac{\alpha}{2}$$

$$= 2AP \times \frac{CB_1}{AC}$$

$$= 2AP \times \frac{CB}{AC}$$

From fig, we see that the angle β made by the forward or cutting stroke is greater than the angle α described by the return stroke. Since the crank rotates with uniform angular speed, therefore the returns stroke is completed within shorter time. Thus it is called quick return motion mechanism.

23. Describe the watts parallel mechanism for straight line motion and derive the condition under which the straight line is traced.

(13) *[NOV/DEC-2018]*

The approximate straight line motion mechanisms are the modifications of the four-bar chain mechanisms. Following mechanisms to give approximate straight line motion, are important from the subject point of view :

1. *Watt's mechanism.* It is a crossed four bar chain mechanism and was used by Watt for his early steam engines to guide the piston rod in a cylinder to have an approximate straight line motion.

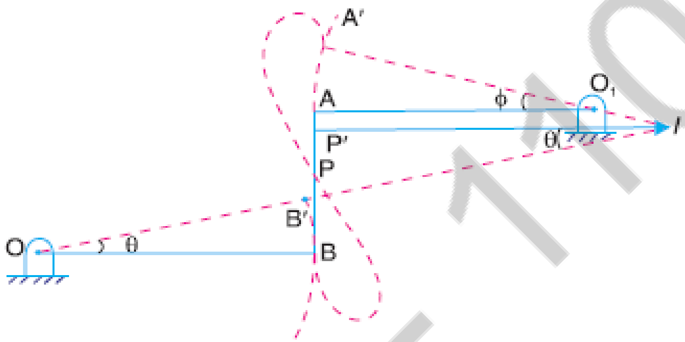


Fig. 9.6. Watt's mechanism.

In Fig. 9.6, $OBAO_1$ is a crossed four bar chain in which O and O_1 are fixed. In the mean position of the mechanism, links OB and O_1A are parallel and the coupling rod AB is perpendicular to O_1A and OB. The tracing point P traces out an approximate straight line over certain positions of its movement, if $PB/PA = O_1A/OB$. This may be proved as follows :

A little consideration will show that in the initial mean position of the mechanism, the instantaneous centre of the link BA lies at infinity. Therefore the motion of the point P is along the vertical line BA . Let $OB'A'O_1$ be the new position of the mechanism after the links OB and O_1A are displaced through an angle θ and ϕ respectively. The instantaneous centre now lies at I. Since the angles θ and ϕ are very small, therefore

$$\text{arc } BB' = \text{arc } AA' \quad \text{or} \quad OB \times \theta = O_1A \times \phi \qquad ...(i)$$

$\therefore \quad OB / O_1A = \phi / \theta$

Also $\quad A'P' = IP' \times \phi$, and $B'P' = IP' \times \theta$

$$\therefore \quad A'P' / B'P' = \phi / \theta \qquad ...(ii)$$

From equations (*i*) and (*ii*),

$$\frac{OB}{O_1A} = \frac{A'P'}{B'P'} = \frac{AP}{BP} \quad \text{or} \quad \frac{O_1A}{OB} = \frac{PB}{PA}$$

Thus, the point P divides the link AB into two parts whose lengths are inversely proportional to the lengths of the adjacent links.

24. Explain with neat sketch kinematic inversions of four bar chain.
(13) *[APR/MAY-2019]*

Though there are many inversions of the four bar chain, yet the following are important from the subject point of view :

1. *Beam engine* (*crank and lever mechanism*).

A part of the mechanism of a beam engine (also known as crank and lever mechanism) which consists of four links, is shown in Fig. 5.19. In this mechanism, when the crank rotates about the fixed centre *A*, the lever oscillates about a fixed centre *D*. The end *E* of the lever *CDE* is connected to a piston rod which reciprocates due to the rotation of the crank. In other words, the purpose of this mechanism is to convert rotary motion into reciprocating motion.

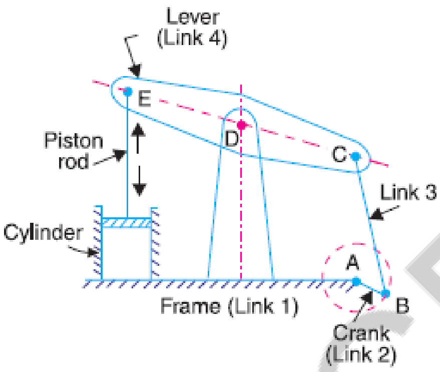


Fig. 5.19. Beam engine.

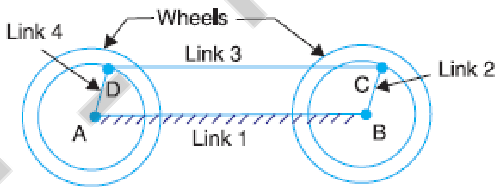


Fig. 5.20. Coupling rod of a locomotive.

2. *Coupling rod of a locomotive* (*Double crank mechanism*).

The mechanism of a coupling rod of a locomotive (also known as double crank mechanism) which consists of four links, is shown in Fig. 5.20. In this mechanism, the links *AD* and *BC* (having equal length) act as cranks and are connected to the respective wheels. The link *CD* acts as a coupling rod and the link *AB* is fixed in order to maintain a constant centre to centre distance between them. This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.

3. *Watt's indicator mechanism* (*Double lever mechanism*).

A Watt's indicator mechanism (also known as Watt's straight line mechanism or double lever mechanism) which consists of four links, is shown in Fig. The four links are : fixed link at *A*, link *AC*, link *CE* and link *BFD*. It may be noted that *BF* and *FD* form one link because these two parts have no relative motion between them. The links *CE* and *BFD* act as levers. The displacement of the link *BFD* is directly proportional to the pressure of gas or steam which acts on the indicator plunger. On any small displacement of the mechanism, the tracing point *E* at the end of the link *CE* traces out approximately a straight line. The initial position of the mechanism is shown in Fig. by full lines whereas the dotted lines show the position of the mechanism when the gas or steam pressure acts on the indicator plunger.

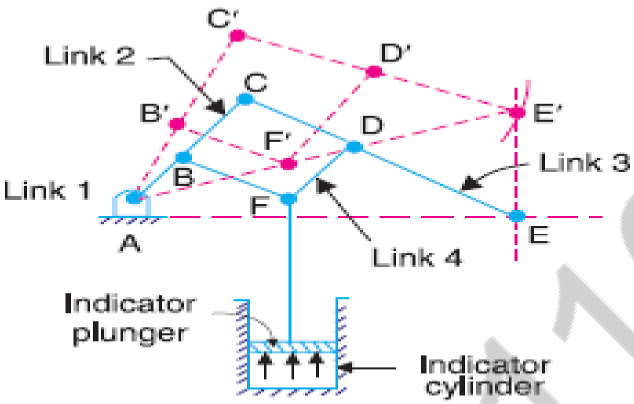


Fig. Watt's indicator mechanism.

UNIT- II KINEMATICS OF LINKAGE MECHANISMS

PART-A

1) Define the instantaneous centre. (MAY/JUNE 2014)

The combined motion of translation and rotation of the link may be assumed to be a motion of pure rotation about a centre known as Virtual centre or Instantaneous centre.

2) What is the expression for Coriolis component of acceleration? (MAY/JUNE 2014)

$a^c = 2v^s\omega$

where, $\boldsymbol{a^c}$ *= Coriolis component of acceleration,*

$\boldsymbol{v^s}$ *= velocity of sliding, &* $\boldsymbol{\omega}$ *= Angular velocity of OA.*

3) Write the relation between the number of instantaneous centre and the number of links in a mechanism. (MAY/JUNE 2015)

Instantaneous center is a point in common between two members where the velocities are equal, both in direction and magnitude. The number of instantaneous centers in a considered kinematic chain is equal to number of combinations of two links: If N is the number of instantaneous centers and n is the number of links

Then number of instantaneous centre is $\mathbf{N} = \frac{\mathbf{n(n+1)}}{\mathbf{2}}$

4) Depict all the direction of coriolis component of acceleration that arises in a completed cycle of quick return motion of the crank mechanism? (MAY/JUNE 2015)

When a point on one link is sliding along another rotating link, such as in quick return motion mechanism, then the coriolis component of the acceleration must be calculated.

Consider a link OA and a slider B as shown in Fig. The slider B moves along the link OA. The point C is the coincident point on the link OA.

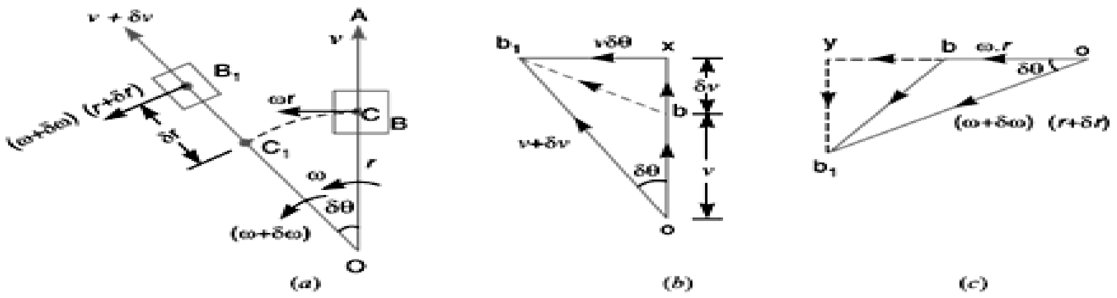
Let ω = Angular velocity of the link OA at time t seconds.

v = Velocity of the slider B along the link OA at time t seconds.

$\omega.r$ = Velocity of the slider B with respect to O (perpendicular to the link OA) at time t seconds, and

$(\omega + \delta\omega), (v + \delta v)$ and $(\omega + \delta\omega), (r + \delta r)$

$=$ Corresponding values at time $(t+\delta t)$ seconds.



5) What is relative pole, with respect to velocity analysis?

(MAY/JUNE 2016)

The relative pole is the centre of rotation of the connecting rod relative to the crank rotation and the corresponding slider displacement.

6) What are the different methods used for finding the velocity?

(MAY/JUNE 2016)

Important methods for determining the velocity of a body are:

- Graphical method: i) Relative velocity method
  ii) Instantaneous centre method
- Analytical method.

7) What is the need of finding acceleration of linkage in a mechanism?

(NOV/DEC 2014)

The Dynamic forces are functions of acceleration (i.e. F = ma). Therefore, the determination of acceleration of various links becomes the integral part in the design of any mechanism.

8) Name the two mechanisms having Coriolis component.

(NOV/DEC 2014)

- Slider Crank Mechanism
- Quick Return Mechanism

9) What is the total number of instantaneous centre that are possible for a mechanism consisting 'n' links? (NOV/DEC 2015)

The combination of rotation and translation of a link in the mechanism may be assumed to be a motion of pure rotation about some centre I, known as instantaneous centre of rotation or virtual centre of rotation.

The number of instantaneous centre is $N = \frac{n(n+1)}{2}$

n = Number of links

10) Name the mechanism in which Coriolis component of acceleration is taken into account. (NOV/DEC 2015)

- Slotted lever mechanism
- Withworth quick return mechanism
- Oscillating cylinder mechanism
- Swivelling joint mechanism

11) What are the important concepts in velocity analysis?

- ► The absolute velocity of any point on a mechanism is the velocity of that point with reference to ground.
- ► Relative velocity describes how one point on a mechanism moves relative to another point on the mechanism.

12) Define Instantaneous centre.

Instantaneous centre of a moving body may be defined as that centre which goes on changing from one instant to another.

13) Define Instantaneous Axis.

Instantaneous axis is a line drawn through an instantaneous centre and perpendicular to the plane of motion.

14) How to represent the direction of linear velocity of any point on a link with respect to another point on the same link?

The direction of linear velocity of any point on a link with respect to another point on the same link is perpendicular to the line joining the points.

15) Define Kennedy's theorem.

The Kennedy's theorem states that if three bodies move relatively to each other, they have three instantaneous centers and lie on a straight line.

16) Define displacement.

It may be defined as the distance moved by a body with respect to a fixed certain fixed point. When there is no displacement in a body it is said to be at rest and when it is being displaced, it is said to be in motion.

17) What are the types of motions?

1. Rectilinear motion.
2. Curvilinear motion.
3. Circular motion.

18) What are the methods for determining the velocity of a body?

Important methods for determining the velocity of a body are:

Graphical method: i) Relative velocity method

ii) Instantaneous centre method

Analytical method.

19) Define velocity.

Velocity may be defined as the rate of change of displacement of a body with respect to the time. Since the velocity has both magnitude and direction, therefore it is a vector quantity.

20) Define speed.

Speed may be defined as the rate of change of linear displacement of a body with respect to the time. Since the speed is irrespective of its direction, therefore it is a scalar quantity.

21) What is deceleration?

The negative acceleration is also known as deceleration or retardation.

22) Define Acceleration.

The rate of change of velocity with respect to time is known as acceleration.

23) Define coincident points.

When a point on one link is sliding along another rotating link, then the point is known as coincident point.

24) Define centrode.

The locus of all instantaneous centre's (i.e., I_1, I_2,...) is known as centrode.

25) Define Axode.

The locus of all instantaneous axis is known as axode.

26) Define Body centrode.

The locus of all instantaneous centre relative to the body itself is called the body centrode.

27) Find the resultant acceleration of an 80mm radius crank rotating at a constant angular velocity of 10 rad/s, at the crank — pin position.

(A/M-2017)

Given: Length of Crank = 80 mm;

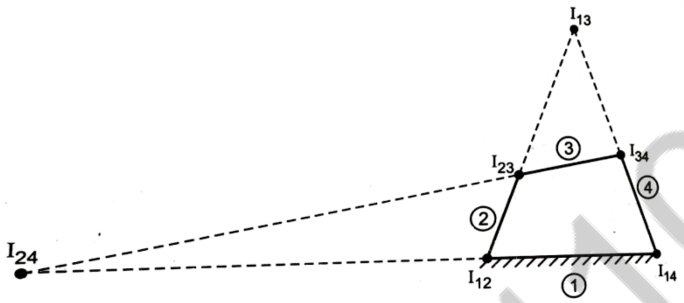
ω = 10 rad/s

Soln: acceleration of crank, a = ω^2 X length of Crank

= 10^2 X 0.08

a = 8 rad/s²

28) Illustrate the instantaneous centres of a typical four bar mechanism. (A/M-2017)



Number of instantaneous centre = n(n-1)/2 = 4(4-1)/2 = 6

Fixed Instantaneous centre = I_{12} & I_{14}

Permanent Instantaneous Centre = I_{23} & I_{34}

Neither Fixed nor Permanent Instantaneous centre = I_{13} & I_{24}

29) Define Coriolis component of acceleration. (N/D-2017)

When the sliding pair itself revolves, its acceleration will include Coriolis component of acceleration due to change in relative distance between two points.

$$a_c = 2v_s\omega$$

where, a_c = *Coriolis component of acceleration,*

v_s = *velocity of sliding, &* ω = *Angular velocity of OA.*

30) State the Arnold Kennedy theorem. (N/D-2017)

Kennedy theorem states that, *"if three bodies move relative to each other, then they have three instantaneous centres and lie on a straight line".*

It is used for locating ICR's in IC engine mechanism, four bar mechanism and Quick return mechanism.

31) How will you find out the total acceleration from its normal and tangential components? (A/M 2018)

To draw acceleration diagram for a rigid link use following procedure :

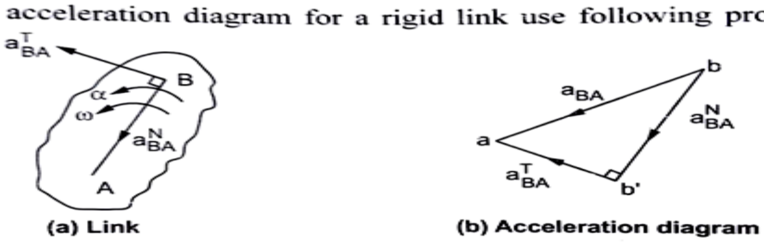


Fig. : Acceleration of a rigid link

- From point b draw vector bb′ parallel to BA to represent normal component of acceleration of B with respect to A i.e. a_{BA}^{N}.

$$a_{BA}^{N} = \frac{V_{BA}^{2}}{\text{length of BA}} = \frac{V_{BA}^{2}}{AB}$$

- From point b′ draw vector b′a perpendicular to BA to represent tangential acceleration of B with respect to A i.e. a_{BA}^{T}.

$$a_{BA}^{T} = \text{Length of BA} \times \alpha_{BA} = AB \times \alpha_{BA}$$

- Now join ab which represents total acceleration of link BA.

  Total acceleration is given by,

$$\bar{a}_{BA} = \overline{a_{BA}^{N}} + \overline{a_{BA}^{T}}$$

32) Mention any two motives for doing acceleration analysis of mechanisms or machines. (A/M 2018)

- To understand the Dynamics of each force in a link/Mechanism.
- To understand the Relative acceleration of one point relative to another point.

33) Define Instantaneous centre. (N/D 2018)

The combined motion of translation and rotation of the link may be assumed to be a motion of pure rotation about a centre known as Virtual centre or Instantaneous centre.

34) What is meant by Coriolis component of acceleration? (N/D 2018)

When a point on one link is sliding along another rotating link, such as in quick return motion mechanism, then the coriolis component of the acceleration must be calculated.

$$a_c = 2 v_s \omega$$

where, a_c = *Coriolis component of acceleration,*

v_s = *velocity of sliding, &* ω = *Angular velocity of OA.*

35) What is meant by normal component of acceleration? (A/M 2019)

The acceleration, which is perpendicular to the velocity of the particle at the given instant is called Radial or Normal Component of acceleration.

36) Define Centrode. (A/M 2019)

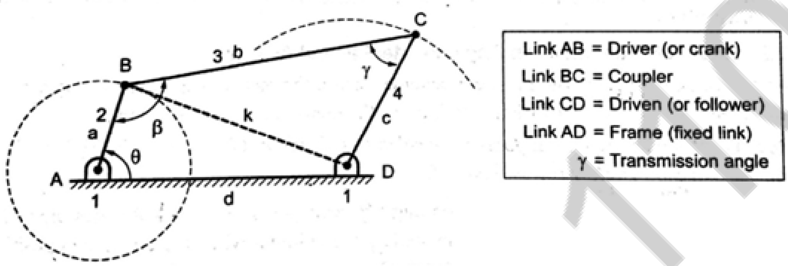
The locus of all instantaneous centre's (i.e., I_1, I_2,…) is known as Centrode.

PART-B

1) (a) (i) Derive an expression for the relationship between the angular velocities of links in terms of known link-lengths, angular position of links and angular velocity of input link for a four bar linkage. (6)
(MAY/JUNE 2014)

Transmission Angle and Mechanical Advantage of Four – Bar Mechanism

- ✓ Consider a four – bar mechanism shown in Fig. In this, link AB is the driver link, BC is the coupler, link CD is the driven link (or follower), and link DA is the frame (i.e. fixed link).



Transmission angle (γ): The angle between the coupler link and the driven link (or follower) is known as transmission angle. It is denoted by 'γ'.
Equation for mechanical advantage:
Let θ = Crank angle (i.e. angle between the driver link and the fixed link),
γ = Transmission angle (i.e, angle between the coupler link and the driven link),
β = Angle between the coupler link and the driver link,
A, b, c, d = Length of links AB, BC, CD and DA respectively.
The equation for mechanical advantage in terms of angle γ and β can be derived and it is given by

$$MA = \frac{T_{CD}}{T_{AB}} = \frac{\omega_{AB}}{\omega_{CD}} = \frac{c\sin\gamma}{a\sin\beta}$$

From the above equation, the following points can be observed.

The mechanical advantage of the four-bar mechanism is directly proportional to the sine of angle γ between the coupler and the follower.

The mechanical advantage of the four-bar mechanism is inversely proportional to the sine of angle β between the coupler and the driver.

As the linkage moves, the mechanical advantage is continuously changing (because values of both angles γ and β keep changing as linkage moves).

Note: 1. Applying cosine law to triangles BAD and BCD (Fig.1.60), we get

$$a^2 + d^2 - 2ad\cos\theta = k^2 \quad \text{(i)}$$

And $$b^2 + c^2 - 2bc\cos\gamma = k^2$$
(ii)

From (i) and (ii), we can write

$$a^2 + d^2 - 2ad\cos\theta = b^2 + c^2 - 2bc\cos\gamma$$

Using equation (1.12), when the link lengths and input crank angle are known, the transmission angle (γ) can be determined.

2. In equation (1.12), the transmission angle (γ) is maximum when $\theta = 180°$ and μ is minimum when $\theta = 0°$. However, these conditions are applicable only to the double-crank or crank-rocker mechanisms.

(ii) In a slider-crank mechanism, the length of crank OB and connecting rod AB are 125 mm and 500 mm respectively. The centre of gravity G of connecting rod is 275 mm from slider A. The crank speed is 600 rpm clockwise. When the crank has turned 45⁰ from inner-dead centre position, determine velocity of slider A, velocity of point G and angular velocity of the connecting rod AB. (10)

(MAY/JUNE 2014)

Given:

$OB = 125mm$

$AB = 500mm$

$AG = 275mm$ SLIDER – CRANK MECHANISM

$N = 600rpm$

Solution:

$$\omega_{Bo} = \frac{2\pi N}{60} = \frac{2\pi \times 600}{60} = 62.83 \text{rad/sec}$$

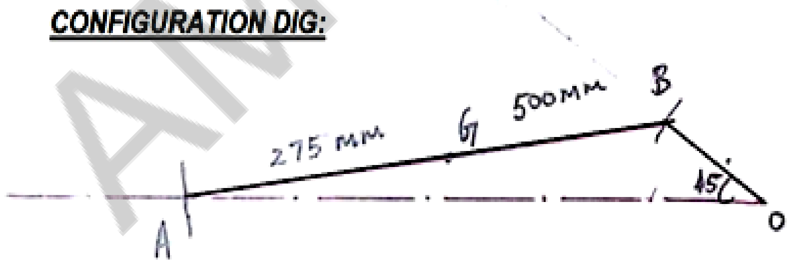
$\theta = 45°$ (From Inner Dead Centre Position)

$$V_{Bo} = \omega_{Bo} \times \text{Length of OB}$$

$$= 62.83 \times 125$$

$$V_{Bo} = 7853.75 \text{mm/sec}$$

CONFIGURATION DIG:

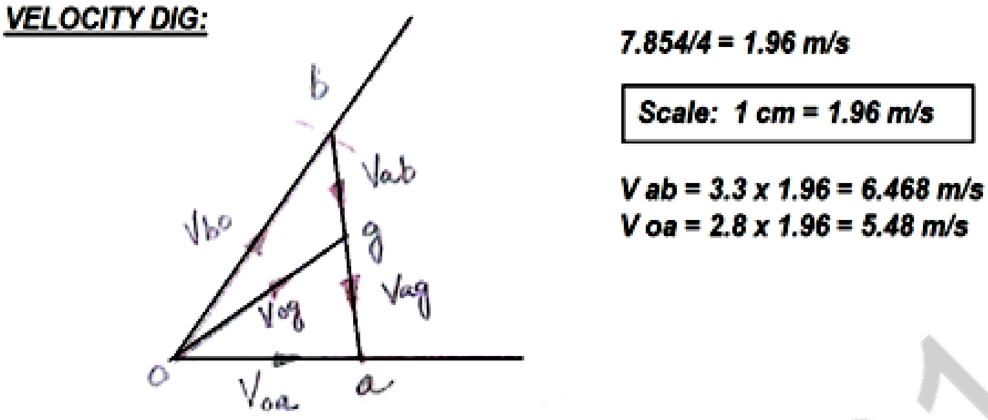


Scale: 1 cm = 62.5 mm

OB = 125 mm = 2 cm
AB = 500 mm = 8 cm
AG = 275 mm = 4.4 cm

V_{BO} = 7853.75 mm/sec
(or)
V_{BO} = 7.854 m/sec

Velocity diagram:



$7.854/4 = 1.96\,\text{m/s}$

To locate pt 'G'

$$\frac{V_{ag}}{V_{ab}} = \frac{AG}{AB} \Rightarrow V_{ag} = \frac{AG}{AB} \times V_{ab}$$

$$= \frac{275}{500} \times 6.468$$

$$V_{ag} = 3.55\,\text{m/sec}$$

From Velocity Diagram:

$$V_{ab} = 3.3 \times 1.96 = 6.468\,\text{m/sec}$$

$$V_{oa} = 2.8 \times 1.96 = 5.586\,\text{m/sec}$$

$$V_{ag} = 3.55\,\text{m/s}$$

$$V_{go} = 3.2 \times 1.96 = 6.27\,\text{m/sec}$$

Result:

(i) Velocity of slider A (i.e) $V_{oa} = 5.586\,\text{m/sec}$

(ii) Velocity of centre G (i.e) $V_{og} = 6.27\,\text{m/sec}$

Angular Velocity of Connecting Rod:

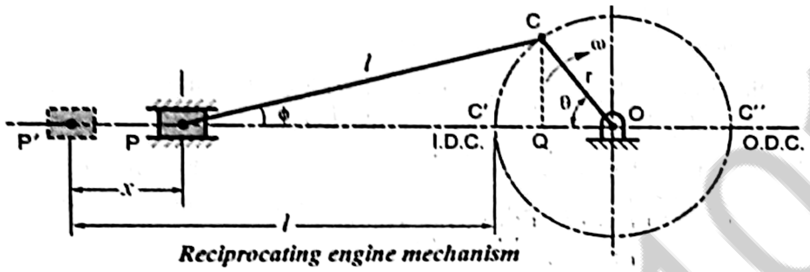
$$\omega_{AB} = \frac{V_{ab}}{\text{Length of AB}} = \frac{6.468}{0.5}$$

$$\omega_{AB} = 12.9\,\text{rad/sec}$$

2) By analytical method, derive the velocity and acceleration for the reciprocating steam engine mechanism. (16)

(MAY/JUNE 2015)

Consider a reciprocating steam engine mechanism OCP as shown in Fig. Let crank OC rotates with angular velocity ω rad/s and the connecting rod PC makes angle ϕ with the line of stroke PO. Let x be the displacement of piston from initial point P' to P, when the crank turns through an angle θ from I.D.C.



Reciprocating engine mechanism

Let

$r =$ Crank radius,

$l =$ Length of the connecting rod,

$\theta =$ Angle made crank with I.D.C.,

$\phi =$ Inclination of connecting rod to the line of stroke PO, and

$n = \frac{l}{r} =$ Ratio of length of connecting rod to the radius of crank, also known as obliquity ratio.

Velocity of the Piston (v_P)

From the geometry of Fig. Displacement of the piston is given by

$$x = P'P = OP' - OP = (P'C' + C'O) - (PQ + QO)$$

From $\Delta CPQ,\ PQ = l\cos\phi$ and from $\Delta COQ,\ QO = r\cos\theta$

$$\therefore\ x = (l + r) - (l\cos\phi + r\cos\phi)$$

$$= r(1-\cos\theta) + l(1-\cos\phi) = r\left[(1-\cos\theta) + \frac{l}{r}(1-\cos\phi)\right]$$

$$x = r\left[(1-\cos\theta) + n(1-\cos\phi)\right]$$

From $\Delta CPQ,\ \frac{CQ}{l} = \sin\phi$ or $CQ = l\sin\phi$

and from $\Delta CPQ,\ \frac{CQ}{r} = \sin\theta$ or $CQ = r\sin\theta$

Thus, $l\sin\phi = r\sin\theta$ or $\sin\phi = \frac{r}{l}\sin\theta = \frac{\sin\theta}{n}$ (ii)

We know that $\cos\phi = (1-\sin^2\phi)^{1/2} = \left[1 - \frac{\sin^2\theta}{n^2}\right]^{\frac{1}{2}} = \frac{1}{n}(n^2 - \sin^2\theta)^{1/2}$

By expanding the above expression by binomial theorem, we get

$$\cos\phi = 1 - \frac{1}{2}\times\frac{\sin^2\theta}{n^2} + \ldots \qquad \text{[Neglecting higher order terms]}$$

Or $$1-\cos\phi=\frac{\sin^2\theta}{2n^2} \qquad \text{(iii)}$$

Substituting equation (iii) in equation (i), we get

$$x=r\left[(1-\cos\theta)+\frac{\sin^2\theta}{2n}\right]$$

Differentiating equation (2.20) with respect to θ, we get

$$\frac{dx}{d\theta}=r\left[\sin\theta+\frac{1}{2n}\times 2\sin\theta\cos\theta\right]=r\left[\sin\theta+\frac{\sin 2\theta}{2n}\right] \qquad \text{(iv)}$$

Therefore, velocity of the piston or velocity of P with respect to O,

$$v_{PO}=v_P=\frac{dx}{dt}=\frac{dx}{d\theta}\times\frac{d\theta}{dt}=\frac{dx}{d\theta}\times\omega \qquad \left[\because \frac{d\theta}{dt}=\times\omega\right]$$

$$\therefore v_{PO}=v_P=r\omega\left[\sin\theta+\frac{\sin 2\theta}{2n}\right]$$

Acceleration of the Piston (a_P)

We know that acceleration is the rate of change of velocity. So, acceleration of the piston P is given by

$$a_P=\frac{dv_P}{dt}=\frac{dv_P}{d\theta}\times\frac{d\theta}{dt}=\frac{dv_P}{d\theta}\times\omega$$

Differentiating equation (2.21) with respect to θ, we get

$$\frac{dv_P}{d\theta}=\omega\cdot r\left[\cos\theta+\frac{\cos 2\theta\times 2}{2n}\right]=\omega\cdot r\left[\cos\theta+\frac{\cos 2\theta}{n}\right]$$

Substituting the value of $\frac{dv_P}{d\theta}$ in the above equation, we get

$$a_P=\omega^2\cdot r\left[\cos\theta+\frac{\cos 2\theta}{n}\right]$$

Note: 1. If the value of n is very large, then $a_P=\omega^2 r\cos\theta$, as in case of SHM.

2. When $\theta=0°$, i.e., at IDC position, $a_P=\omega^2 r\left(1+\frac{1}{n}\right)$

3. When $\theta=180°$, i.e., at ODC position, $a_P=\omega^2 r\left(-1+\frac{1}{n}\right)$

As the direction of motion is reversed at the outer dead centre (ODC) position, therefore, changing the sign of the above expression, we get

$$a_P=\omega^2 r\left(1-\frac{1}{n}\right)$$

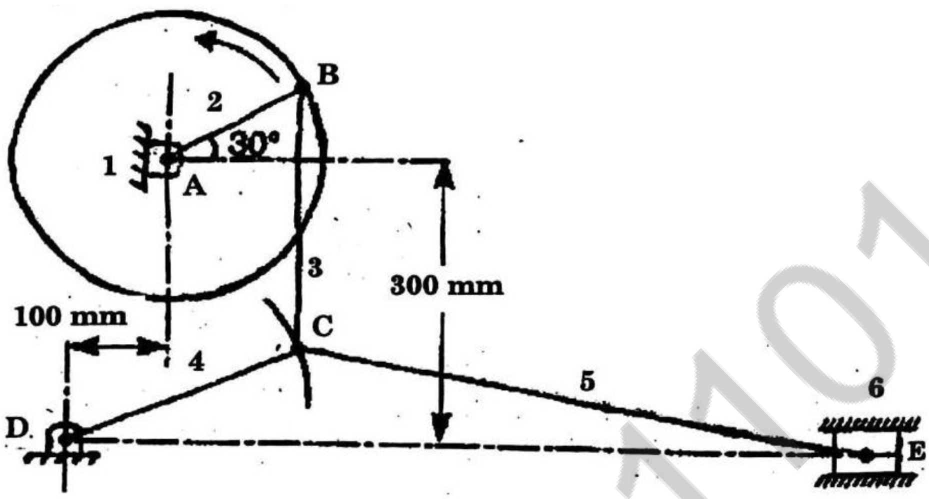
3) Locate all the instantaneous centres of the mechanism as shown in Fig. shown below. The lengths of various links are: AB = 150 mm; BC = 300 mm; CD = 225 mm; and CE = 500 mm. When the crank AB rotates in the anticlockwise direction at a uniform speed of 240 r.p.m.

Find

(i) Velocity of the slider E, and

(ii) Angular velocity of the links BC and CE. **(16)**

(MAY/JUNE 2015)



Given:

$AB = 150mm$

$BC = 300mm$

$CD = 225mm$

$CE = 500mm$

$N_{AB} = 240rpm$

To find: (i) Velocity of slider E

(ii) Angular Velocity of links BC and CE

Solution:

Speed of link $N_{ab} = 240rpm$

$$\omega_{ab} = \frac{2\pi N_{ab}}{60} = \frac{2\pi \times 240}{60} = 25.13 rad/sec$$

Velocity of AB $\omega_{ab} \times$ length of AB

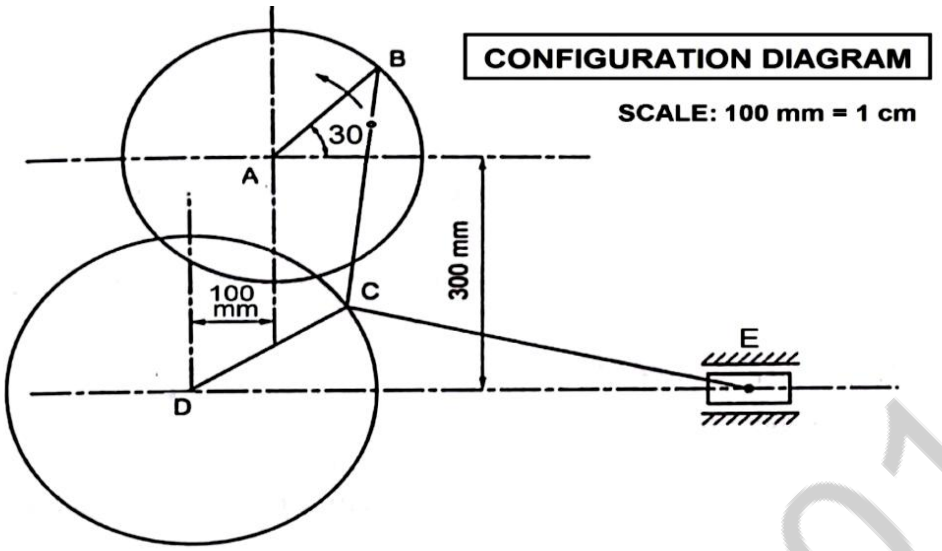
$= 25 \cdot 13 \times 0.150$

$V_{ab} = 3.77 m/sec$

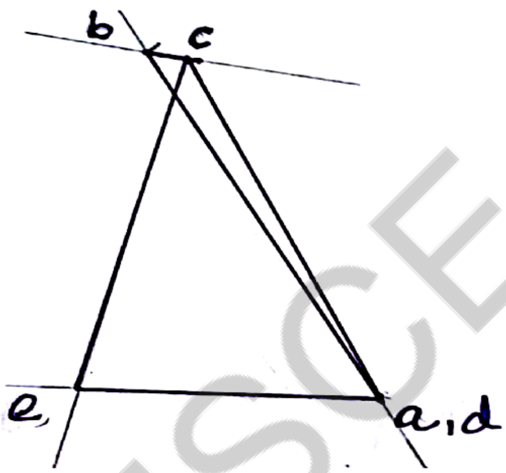
Velocity of slider E:

$V_E = V_{DE} = (\text{Measurement from velocity})$

$V_E = 2.262 m/sec$



VELOCITY DIAGRAM:



3.77/5 = 0.754 m/s

SCALE:
0.754 m/s = 1 cm

bc = 0.4 cm
ce = 4.5 cm
de = 3 cm

V_{bc} = 0.4 x 0.754 = 0.3016 m/s
V_{ce} = 4.5 x 0.754 = 3.393 m/s

V_e = 3 x 0.754 = 2.262 m/s

Angular velocity of BC and CE:

$$\omega_{BC} = \frac{\text{velocity of BC}}{\text{length of BC}} = \frac{0.3016}{0.3}$$

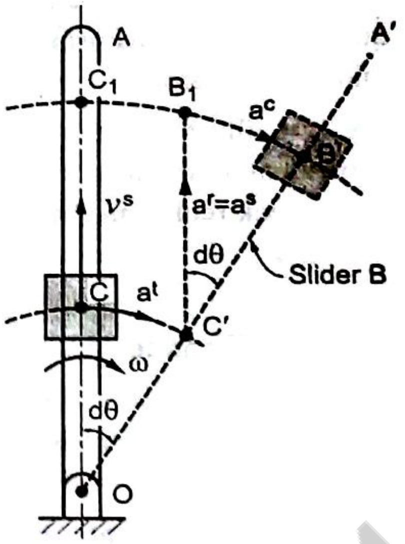
$$\omega_{BC} = 1.005 \text{ rad/sec}$$

$$\omega_{CE} = \frac{\text{velocity of CE}}{\text{length of CE}} = \frac{3.393}{0.5}$$

$$\omega_{CE} = 6.786 \text{ rad/sec}$$

4) Derive the expression for Coriolis component of acceleration with neat sketch and give its direction for various conditions. (16)

(MAY/JUNE 2016)



Consider a link OA which has a slider B which is free to slide, as shown in Fig. With O as centre, let the link OA move, with a uniform angular velocity ω, to its new position OA' such that it is displaced $d\theta$ in time dt. The slider B moves outwards with sliding velocity v^s on link OA and occupies the position B' in the same interval of time.

The point C is the coincident point with slider on link OA.

The motion of slider can be explained in the following three stages:

(i) Motion from C to C' due to rotation of link OA. It is caused by tangential component of acceleration a^t.

(ii) Motion from C' to B_1 due to outward motion along the link OA. It is caused by radial component a^r (or sliding component, a^s) of acceleration.

(iii) Motion from B_1 to B' is caused by Coriolis component of acceleration a^c.

From the geometry of Fig.

$$\text{Arc}B_1B' = \text{Arc}C_1B' - \text{Arc}C_1B_1$$

$$= \text{Arc}C_1B' - \text{Arc}CC'$$

$$= OC_1(d\theta) - OC(d\theta) = (OC_1 - OC)d\theta$$

$$= CC_1 \times d\theta = C'B_1 \times d\theta \qquad \textbf{(i)}$$

We know that $C'B_1 = \text{Motion of the slider} \times \text{Time} = v \times dt$

and $\dfrac{d\theta}{dt} = \omega$ or $d\theta = \omega \cdot dt$

Now equation (i) can be written as

$$\text{Arc}B_1B' = v^s \cdot d\theta \cdot \omega \cdot dt = v^s\omega(dt)^2 \qquad \textbf{(ii)}$$

Considering the acceleration of the slider (a^c) as constant, we get

$$\text{Arc}B_1B' = \frac{1}{2}a^c(dt)^2 \qquad \left[\because s = ut + \frac{1}{2}at^2 \text{ and } u = 0\right]$$

For small value of $d\theta$,

$$\text{Arc}B_1B' = B_1B'$$

$$\therefore \qquad B_1B' = \frac{1}{2}a^c(dt)^2 \qquad \textbf{(iii)}$$

From equations (ii) and (iii), we get

$$v^s \cdot \omega(dt)^2 = \frac{1}{2}a^c(dt)^2$$

Or Coriolis component of acceleration, $\quad a^c = 2 \cdot v^s \cdot \omega \qquad$ (A)

Where $\quad v^s$ = Velocity of sliding, and

ω = Angular velocity of link OA.

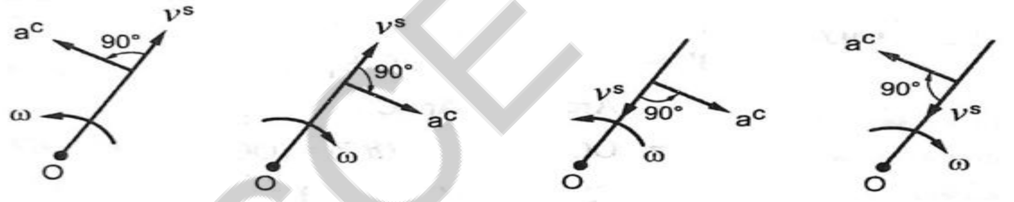
Direction of Coriolis Component of Acceleration

The direction of Coriolis component of acceleration a^c is to rotate the sliding velocity vector v^s in the same sense as the angular velocity of OA.

The direction of Coriolis component of acceleration is obtained by rotating the velocity of sliding vector v^s through 90° in the direction of rotation of angular velocity, ω.

Since the direction of a^c depends on the direction of v^s and ω, therefore there are four possible cases of direction of a^c, as shown in Fig.

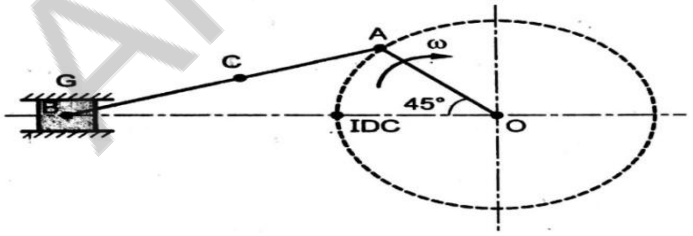
It may be noted that in equation A, the outward direction of velocity of sliding v^s is assumed as positive and the counter clockwise direction of ω is assumed as positive.



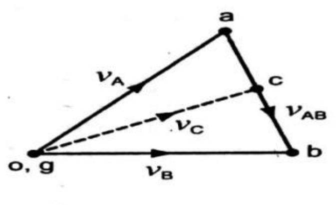
(a) *(b)* *(c)* *(d)*

5) Deduce an expression to find out the velocity and acceleration of a piston in a reciprocating machine. (16)

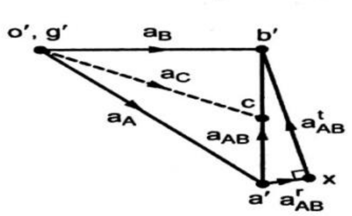
(NOV/DEC 2014)



(a) Configuration diagram



(b) Velocity diagram



(c) Acceleration diagram

VELOCITY OF SLIDER-CRANK MECHANISM:

Step 1: Configuration diagram: First of all, draw the configuration diagram, to some suitable scale, as shown in Fig.

Step 2: Velocity of input link: When length of input link OA and its angular velocity ω_{OA} are known, then the velocity of the input link (i.e., crank OA) is given by

$$v_{AO} = v_A = \omega_{AO} \times OA \text{ (clockwise about O)}$$

Step 3: Velocity diagram: Now draw the velocity diagram, as shown in Fig, using the procedure given below.

1. From any arbitrary point o, draw vector oa perpendicular to the link OA, to some suitable scale, such that $oa = v_{AO} = \omega \cdot OA.$
2. Since AB is a rigid link, therefore the velocity of B with respect to A is perpendicular to AB. Now from point a, draw vector ab perpendicular to AB to represent the velocity of B with respect to A $(\text{i.e., } v_{BA})$
3. Since the velocity of slider B relative to O $(\text{i.e., } v_{BO})$ is along the line of stroke OB, so from point o, draw vector ob parallel to OB.
4. The vectors ab and ob intersect at point b.

Step 4: Velocity of various links:

By measurement, from the velocity diagram, we get

Velocity of the connecting rod, $v_{BA} = \text{vector ab}$

And velocity of the slider, $v_{BO} = \text{vector ob}$

The angular velocity of the connecting rod AB (ω_{AB}) can be determined using the relation

$$\omega_{AB} = \frac{v_{BA}}{AB} \quad \text{(counter clockwise about B)}$$

The direction of vector ba determines the sense of ω_{BA} which shows that it is counter clockwise.

ACCELERATION OF SLIDER-CRANK MECHANISM:

Now using the known values of magnitude and direction of acceleration components, the acceleration diagram can be constructed, as shown in Fig. to some suitable scale, using the procedure given below.

1. Since the link AG is fixed, therefore take o' and g' as one point.
2. From point o', draw vector o' a' such that $o'a' = a'_{AO} = \frac{v_{OA}^2}{OA}$ in the direction parallel to OA to represent the radial component of acceleration of link OA $(\text{i.e., } a'_{AO})$. Since $a'_{AO} = 0$, therefore $a_{AO} = a'_{AO}$.
3. From point a' draw vector $a'x$ such that $a'x = a'_{BA} = v_{BA}^2/AB$ in the direction parallel to BA to represent the radial component of acceleration of link AB

$(\text{i.e., } a'_{BA})$. Now from point x, draw vector xb' perpendicular to AB to represent the tangential component of acceleration of link AB $(\text{i.e., } a'_{BA})$. whose magnitude is unknown.

4. We know that the acceleration of slider B acts in the direction parallel to the line of motion of slider B. So from point o', draw a vector $o'b'$ parallel to BO, intersecting the vector xb' at b'

5. Join points a' and b'.

Step 6: Acceleration of various links:

Now by measurement from the acceleration diagram, the various components of acceleration of links can be found.

Acceleration of slider, $a_{BO} = \text{vector } b'o'$

Radial component of acceleration of the connecting rod, $a^r_{BA} = \text{vector dx}$

Tangential component of acceleration of the connecting rod, $a^t_{BA} = \text{vector b'x}$

Total acceleration of the connecting rod, $a_{BA} = \text{vector b'a'}$

Also the angular acceleration of the connecting rod can be determined as

$$\alpha_{BA} = \frac{a^t_{BA}}{AB} \text{ (clockwise about B)}$$

6) In a four bar chain ABCD, AD is fixed and is 15 cm long. The crank AB is 4 cm long and rotates at 120 rpm clockwise, while the link CD (whose length is 8 cm) oscillates about D. BC and AD are of equal length. Find the angular velocity of link CD when angle BAD = 60°. (16) (NOV/DEC 2015)

Given: $AD = 15\text{cm}(\text{fixed})$

$AB = 4\text{cm}$

$N_{AB} = 120\text{rpm}(\text{clockwise})$

$CD = 8\text{cm}$

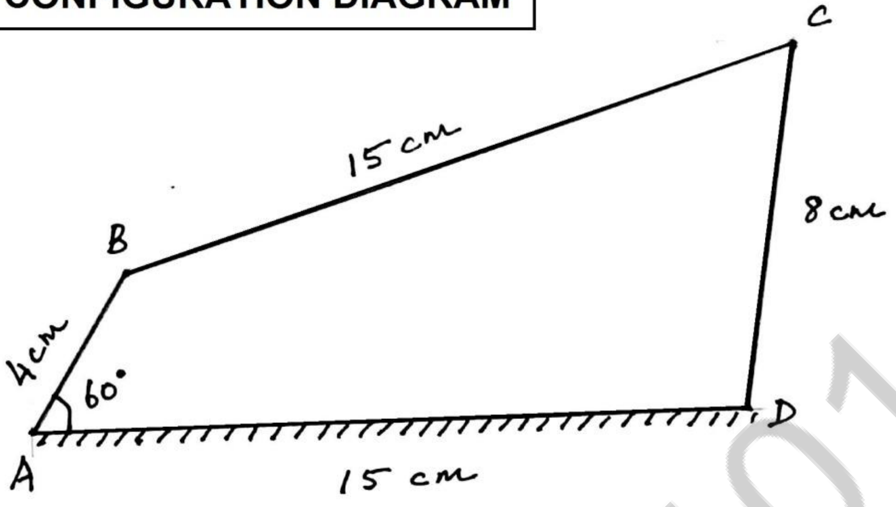
$BC = AD = 15\text{cm}$

$\angle BAD = 60°$

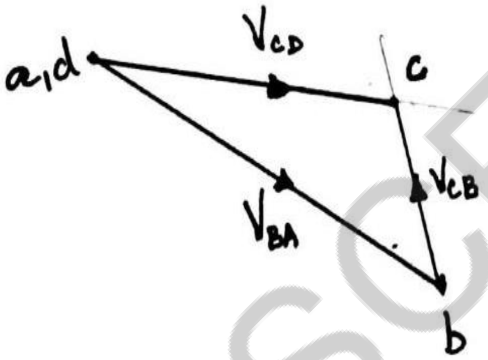
To find: (i) Angular velocity of link CD

Solution:

CONFIGURATION DIAGRAM



VELOCITY DIAGRAM



SCALE:

0.10048 m/s = 1 cm

V_{CD} = 3.9 x 0.10048 = 0.39 m/s

Velocity of Input Link:

Speed of Input Link, $N_{AB} = 120\text{rpm}$

$$\omega_{AB} = \frac{2\pi \times N_{AB}}{60} = \frac{2\pi \times 120}{60} = 12.56\,\text{rad/s}$$

Velocity of the link $\Rightarrow V_{AB} = \omega_{AB} \times \text{length of AB}$

$$= 12.56 \times 0.04$$

$$V_{AB} = 0.5024\,\text{m/s}$$

Angular velocity of link CD:

$$\omega_{CD} = \frac{V_{CD}}{CD} = \frac{0.39}{0.08}$$

$$\omega_{CD} = 4.875\,\text{rad/sec}$$

7) The crank of a slider crank mechanism is 15 cm and the connecting rod is 60 cm long. The crank makes 300 rpm in the clockwise direction. When it has turned 45° from the inner dead centre position, determine (i) acceleration of the mid-point of the connecting rod and (ii) angular acceleration of the connecting rod. (16)

(NOV/DEC 2015)

Given:

$OA = 15cm = 0.150m$

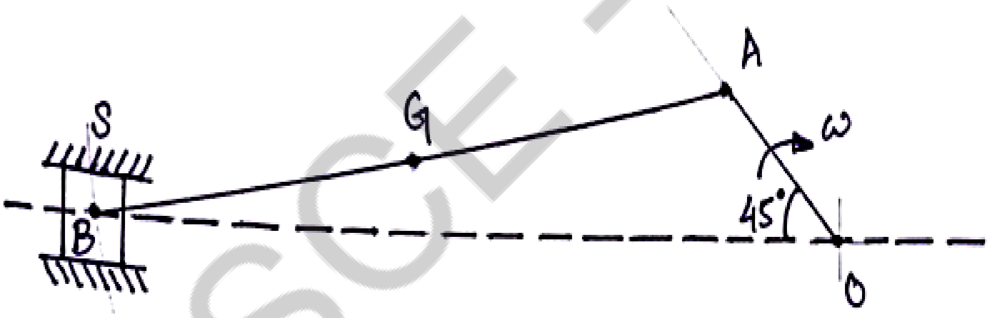
$AB = 60cm = 0.600m$

$\omega_{OA} = 300rpm = 300/60 = 5\,rad/sec$

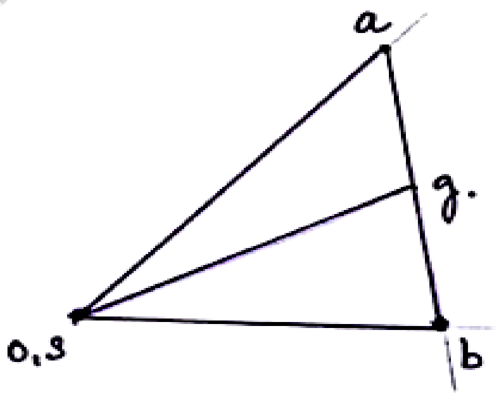
$\angle BOA = 45°$

To find: (i) Acceleration of the mid-point of the connecting rod.
(ii) Angular Acceleration of Connecting rod.

Configuration diagram



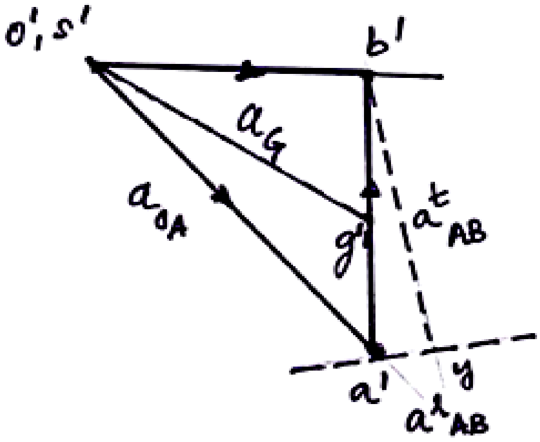
Velocity diagram



Scale: $0.15\,m/s = 1cm$

$V_{ab} = 3.6 \times 0.15 = 0.54\,m/s$

Acceleration diagram



Scale: $0.75\,m/s^2 = 1cm$

$a_{a'b'} = 3.5 \times 0.75 = 2.625\,m/s^2$

$a_{g'} = 3.9 \times 0.75 = 2.925\,m/s^2$

$a^t_{AB} = 3.6 \times 0.75 = 2.7\,m/s^2$

Solution:

Velocity of Crank OA

$$V_{OA} = \omega_{OA} \times \text{length of OA}$$

$$= 5 \times 0.150$$

$$V_{OA} = 0.75\,m/s$$

Radial Component of acceleration:

$$a^r_{OA} = \frac{(V_{OA})^2}{\text{Length of OA}} = \frac{(0.75)^2}{0.150}$$

$$a^r_{OA} = 3.75\,m/s^2$$

$$a^r_{AB} = \frac{(V_{AB})^2}{\text{Length of AB}} = \frac{(0.54)^2}{0.600}$$

$$a^r_{AB} = 0.486\,m/s^2$$

The position of 'g' on connecting rod can be obtained as

$$\frac{AG}{AB} = \frac{aa'g'}{aa'b'}$$

$$a'g' = \frac{AG}{AB} = a'b'$$

$$= \frac{0.300}{0.600} \times 2.625$$

$$aa'g' = 1.3125 \, m/s^2$$

(i) Acceleration at the mid point of Connecting Rod.

$$ag' = \text{vector } o'g'$$

$$= 3.9 \times 0.75$$

$$ag' = 2.925 \, m/s^2$$

(ii) Angular Acceleration of Connecting rod:

$$\alpha_{AB} = \frac{a^t_{AB}}{AB} = \frac{3.6 \times 0.75}{0.6}$$

$$\alpha_{AB} = 4.5 \, rad/sec^2$$

8) Locate all the instantaneous centres of the slider crank mechanism as shown in Fig. The lengths of crank OB and connecting rod AB are 100 mm and 400 mm respectively. If the crank rotates clockwise with an angular velocity of 10 rad/s,

find: 1. Velocity of the slider A, and

2. Angular velocity of the connecting rod AB.

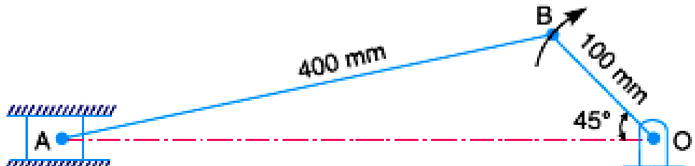


Fig.

Solution. Given : $\omega_{OB} = 10$ rad/ s; $OB = 100$ mm $= 0.1$ m

We know that linear velocity of the crank OB,

$$v_{OB} = v_B = \omega_{OB} \times OB = 10 \times 0.1 = 1 \text{ m/s}$$

Location of instantaneous centres

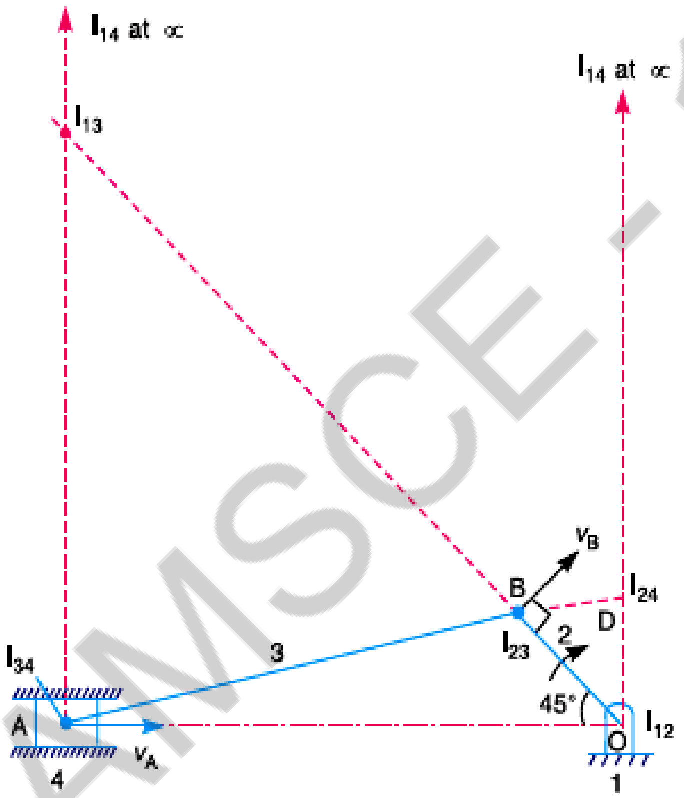
The instantaneous centres in a slider crank mechanism are located as discussed below:

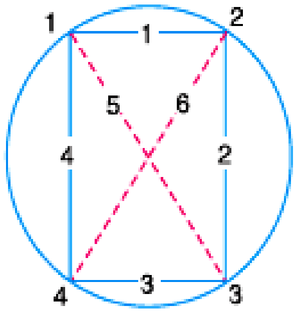
1. Since there are four links (*i.e.* $n = 4$), therefore the number of instantaneous centres,

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

2 . Locate the fixed and permanent instantaneous centres by inspection. These centres are I_{12}, I_{23} and I_{34} as shown in Fig. 6.13. Since the slider (link 4) moves on a straight surface (link 1), therefore the instantaneous centre I_{14} will be at infinity.

4. Locate the other two remaining neither fixed nor permanent instantaneous centres, by Aronhold Kennedy's theorem. This is done by circle diagram as shown in Fig. 6.14. Mark four points 1, 2, 3 and 4 (equal to the number of links in a mechanism) on the circle to indicate I_{12}, I_{23}, I_{34} and I_{14}.





5. Join 1 to 3 to form two triangles 1 2 3 and 3 4 1 in the circle diagram. The side 1 3, common to both triangles, is responsible for completing the two triangles. Therefore the centre I_{13} will lie on the intersection of I_{12} I_{23} and I_{14} I_{34}, produced if necessary. Thus centre I_{13} is located. Join 1 to 3 by a dotted line and mark number 5 on it.

6. Join 2 to 4 by a dotted line to form two triangles 2 3 4 and 1 2 4. The side 2 4, common to both triangles, is responsible for completing the two triangles. Therefore the centre I_{24} lies on the intersection of I_{23} I_{34} and I_{12} I_{14}. Join 2 to 4 by a dotted line on the circle diagram and mark number 6 on it. Thus all the six instantaneous centres are located.

1. *Velocity of the slider A*

Let v_A = Velocity of the slider A.

We know that $\frac{v_A}{I_{13}A} = \frac{v_B}{I_{13}B}$

or $v_A = v_B \times \frac{I_{13}A}{I_{13}B} = 1 \times \frac{0.46}{0.56} = 0.82$ m/s **Ans.**

2. *Angular velocity of the connecting rod AB*

Let ω_{AB} = Angular velocity of the connecting rod AB.

We know that $\frac{v_A}{I_{13}A} = \frac{v_B}{I_{13}B} = \omega_{AB}$

$\therefore \omega_{AB} = \frac{v_B}{I_{13}B} = \frac{1}{0.56} = 1.78$ rad/s **Ans.**

The velocity of the slider A and angular velocity of the connecting rod AB may also be determined as follows :

From similar triangles $I_{13}I_{23}I_{34}$ and $I_{12}I_{23}I_{24}$,

$$\frac{I_{12}I_{23}}{I_{13}I_{23}} = \frac{I_{23}I_{24}}{I_{23}I_{34}} \quad \text{...}(i)$$

and

$$\frac{I_{13}I_{34}}{I_{34}I_{23}} = \frac{I_{12}I_{24}}{I_{23}I_{24}} \quad \text{...}(ii)$$

We know that $\omega_{AB} = \frac{v_B}{I_{13}B} = \frac{\omega_{OB} \times OB}{I_{13}B}$ ...($\because v_B = \omega_{OB} \times OB$)

$= \omega_{OB} \times \frac{I_{12}I_{23}}{I_{13}I_{23}} = \omega_{OB} \times \frac{I_{23}I_{24}}{I_{23}I_{34}}$ ...[From equation (*i*)] ...(*iii*)

Also $v_A = \omega_{AB} \times I_{13}A = \omega_{OB} \times \frac{I_{23}I_{24}}{I_{23}I_{34}} \times I_{13}I_{34}$. ...[From equation (*iii*)]

$= \omega_{OB} \times I_{12}I_{24} = \omega_{OB} \times OD$ ...[From equation (*ii*)]

9) A mechanism, as shown in Fig. has the following dimensions:
OA = 200 mm; AB = 1.5 m; BC = 600 mm; CD = 500 mm and BE = 400 mm. Locate all the instantaneous centres.
If crank OA rotates uniformly at 120 r.p.m. clockwise,
find 1. The velocity of B, C and D,
2. the angular velocity of the links AB, BC and CD.

Solution. Given : $N_{OA} = 120$ r.p.m. or $\omega_{OA} = 2\pi \times 120/60 = 12.57$ rad/s

Since the length of crank $OA = 200$ mm $= 0.2$ m, therefore linear velocity of crank OA,

$$v_{OA} = v_A = \omega_{OA} \times OA = 12.57 \times 0.2 = 2.514 \text{ m/s}$$

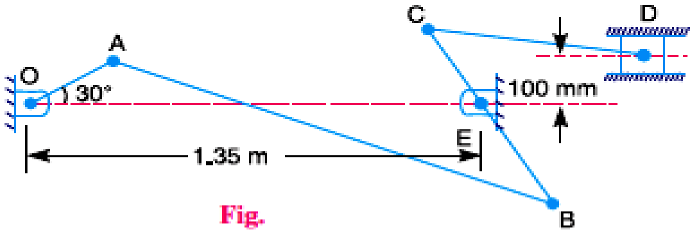


Fig.

Location of instantaneous centres

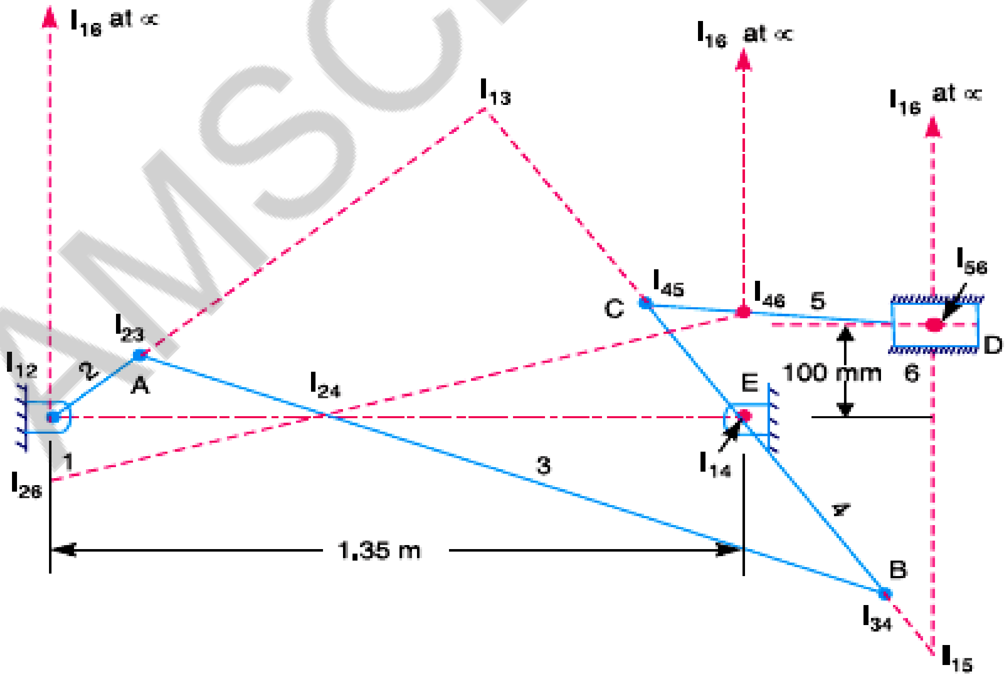
The instantaneous centres are located as discussed below:

1. Since the mechanism consists of six links (*i.e.* $n = 6$), therefore the number of instantaneous centres,

$$N = \frac{n(n-1)}{2} = \frac{6(6-1)}{2} = 15$$

2. Make a list of all the instantaneous centres in a mechanism. Since the mechanism has 15 instantaneous centres, therefore these centres are listed in the following book keeping table.

Links	1	2	3	4	5	6
Instantaneous centres (15 in number)	12 13 14 15 16	23 24 25 26	34 35 36	45 46	56	



3. Locate the fixed and permanent instantaneous centres by inspection. These centres are I_{12} I_{23}, I_{34}, I_{45}, I_{56}, I_{16} and I_{14} as shown in Fig.

4. Locate the remaining neither fixed nor permanent instantaneous centres by Aronhold Kennedy's theorem. Draw a circle and mark points equal to the number of links such as 1, 2, 3, 4, 5 and 6 as shown in Fig. Join the points 12, 23, 34, 45, 56, 61 and 14 to indicate the centres I_{12}, I_{23}, I_{34}, I_{45}, I_{56}, I_{16} and I_{14} respectively.

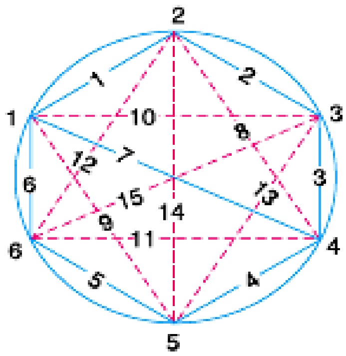


Fig.

5. Join point 2 to 4 by a dotted line to form the triangles 1 2 4 and 2 3 4. The side 2 4, common to both triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{24} lies on the intersection of I_{12} I_{14} and I_{23} I_{34} produced if necessary. Thus centre I_{24} is located. Mark number 8 on the dotted line 24 (because seven centres have already been located).

6. Now join point 1 to 5 by a dotted line to form the triangles 1 4 5 and 1 5 6. The side 1 5, common to both triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{15} lies on the intersection of I_{14} I_{45} and I_{56} I_{16} produced if necessary. Thus centre I_{15} is located. Mark number 9 on the dotted line 1 5.

7. Join point 1 to 3 by a dotted line to form the triangles 1 2 3 and 1 3 4. The side 1 3, common to both triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{13} lies on the intersection I_{12} I_{23} and I_{34} I_{14} produced if necessary. Thus centre I_{13} is located. Mark number 10 on the dotted line 1 3.

8. Join point 4 to 6 by a dotted line to form the triangles 4 5 6 and 1 4 6. The side 4 6, common to both triangles, is responsible for completing the two triangles. Therefore, centre I_{46} lies on the intersection of I_{45} I_{56} and I_{14} I_{16}. Thus centre I_{46} is located. Mark number 11 on the dotted line 4 6.

9. Join point 2 to 6 by a dotted line to form the triangles 1 2 6 and 2 4 6. The side 2 6, common to both triangles, is responsible for completing the two triangles. Therefore, centre I_{26} lies on the intersection of lines joining the points I_{12} I_{16} and I_{24} I_{46}. Thus centre I_{26} is located. Mark number 12 on the dotted line 2 6.

10. In the similar way the thirteenth, fourteenth and fifteenth instantaneous centre (*i.e.* I_{35}, I_{25} and I_{36}) may be located by joining the point 3 to 5, 2 to 5 and 3 to 6 respectively.

By measurement, we find that

$I_{13}A = 840$ mm $= 0.84$ m ; $I_{13}B = 1070$ mm $= 1.07$ m ; $I_{14}B = 400$ mm $= 0.4$ m ;

$I_{14}C = 200$ mm $= 0.2$ m ; $I_{15}C = 740$ mm $= 0.74$ m ; $I_{15}D = 500$ mm $= 0.5$ m

1. *Velocity of points B, C and D*

Let v_B, v_C and v_D = Velocity of the points *B*, *C* and *D* respectively.

We know that $\dfrac{v_A}{I_{13}A} = \dfrac{v_B}{I_{13}B}$ ...(Considering centre I_{13})

$\therefore$ $$v_B = \frac{v_A}{I_{13}A} \times I_{13}B = \frac{2.514}{0.84} \times 1.07 = 3.2 \text{ m/s}$$ **Ans.**

Again, $\dfrac{v_B}{I_{14}B} = \dfrac{v_C}{I_{14}C}$ ...(Considering centre I_{14})

$\therefore \quad v_C = \frac{v_B}{I_{14} B} \times I_{14} C = \frac{3.2}{0.4} \times 0.2 = 1.6 \text{ m/s}$ **Ans.**

Similarly, $\frac{v_C}{I_{15} C} = \frac{v_D}{I_{15} D}$ ...(Considering centre I_{15})

$\therefore \quad v_D = \frac{v_C}{I_{15} C} \times I_5 D = \frac{1.6}{0.74} \times 0.5 = 1.08 \text{ m/s}$ **Ans.**

2. Angular velocity of the links AB, BC and CD

Let ω_{AB}, ω_{BC} and ω_{CD} = Angular velocity of the links AB, BC and CD respectively.

We know that $\omega_{AB} = \frac{v_A}{I_{13} A} = \frac{2.514}{0.84} = 2.99 \text{ rad/s}$ **Ans.**

$$\omega_{BC} = \frac{v_B}{I_{14} B} = \frac{3.2}{0.4} = 8 \text{ rad/s} \quad \textbf{Ans.}$$

and $\omega_{CD} = \frac{v_C}{I_{15} C} = \frac{1.6}{0.74} = 2.16 \text{ rad/s}$ **Ans.**

10) The mechanism of a wrapping machine, as shown in Fig. has the following dimensions :
O_1A = 100 mm; AC = 700 mm; BC = 200 mm; O_3C = 200 mm; O_2E = 400 mm; O_2D = 200 mm and BD = 150 mm.
The crank O_1A rotates at a uniform speed of 100 rad/s. Find the velocity of the point E of the bell crank lever by instantaneous centre method.

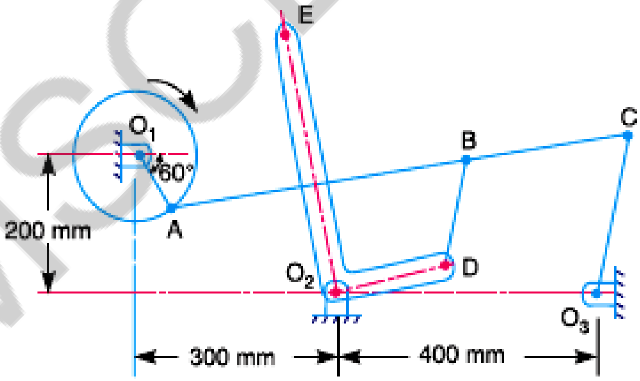


Fig.

Solution. Given : ω_{O1A} = 100 rad/s ; O_1A = 100 mm = 0.1 m

We know that the linear velocity of crank O_1A,

$$v_{O1A} = v_A = \omega_{O1A} \times O_1A = 100 \times 0.1 = 10 \text{ m/s}$$

Now let us locate the required instantaneous centres as discussed below :

1. Since the mechanism consists of six links (*i.e.* $n = 6$), therefore number of instantaneous centres,

$$N = \frac{n(n-1)}{2} = \frac{6(6-1)}{2} = 15$$

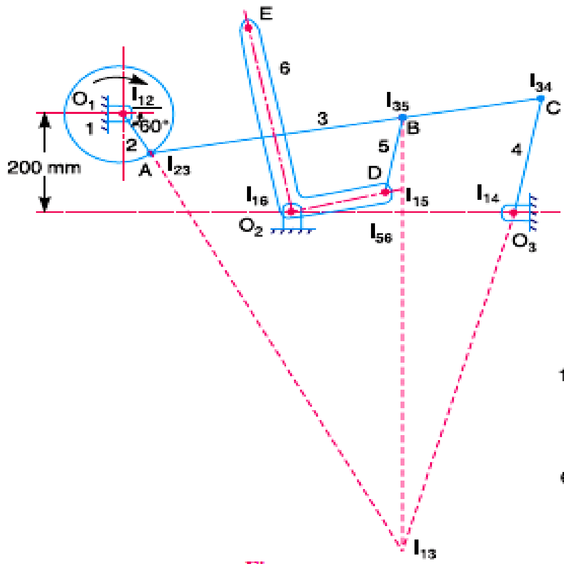


Fig.

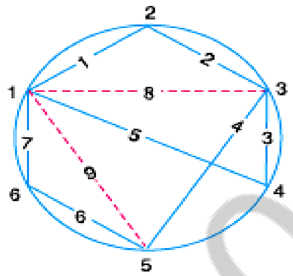


Fig.

3. Locate the fixed and the permanent instantaneous centres by inspection. These centres are I_{12}, I_{23}, I_{34}, I_{35}, I_{14}, I_{56} and I_{16} as shown in Fig.

4. Locate the remaining neither fixed nor permanent instantaneous centres by Aronhold Kennedy's theorem. This is done by circle diagram as shown in Fig. Mark six points on the circle (*i.e.* equal to the number of links in a mechanism), and join 1 to 2, 2 to 3, 3 to 4, 3 to 5, 4 to 1, 5 to 6, and 6 to 1, to indicate the fixed and permanent instantaneous centres *i.e.* I_{12}, I_{23}, I_{34}, I_{35}, I_{14}, I_{56}, and I_{16} respectively.

5. Join 1 to 3 by a dotted line to form two triangles 1 2 3 and 1 3 4. The side 1 3, common to both triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{13} lies on the intersection of the lines joining the points I_{12} I_{23} and I_{14} I_{34} produced if necessary. Thus centre I_{13} is located. Mark number 8 (because seven centres have already been located) on the dotted line 1 3.

6. Join 1 to 5 by a dotted line to form two triangles 1 5 6 and 1 3 5. The side 1 5, common to both triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{15} lies on the intersection of the lines joining the points I_{16} I_{56} and I_{13} I_{35} produced if necessary. Thus centre I_{15} is located. Mark number 9 on the dotted line 1 5.

By measurement, we find that

$$I_{13}A = 910 \text{ mm} = 0.91 \text{ m} \; ; \; I_{13}B = 820 \text{ mm} = 0.82 \text{ m} \; ; \; I_{15}B = 130 \text{ mm} = 0.13 \text{ m} \; ;$$

$$I_{15}D = 50 \text{ mm} = 0.05 \text{ m} \; ; \; I_{16}D = 200 \text{ mm} = 0.2 \text{ m} \; ; \; I_{16}E = 400 \text{ mm} = 0.4 \text{ m}$$

Velocity of point E on the bell crank lever

Let v_E = Velocity of point *E* on the bell crank lever,

v_B = Velocity of point *B*, and

v_D = Velocity of point *D*.

We know that $$\frac{v_A}{I_{13}A} = \frac{v_B}{I_{13}B} \qquad \text{...(Considering centre } I_{13})$$

$$\therefore \quad v_B = \frac{v_A}{I_{13}A} \times I_{13}B = \frac{10}{0.91} \times 0.82 = 9.01 \text{ m/s} \quad \textbf{Ans.}$$

and $$\frac{v_B}{I_{15}B} = \frac{v_D}{I_{15}D} \qquad \text{...(Considering centre } I_{15})$$

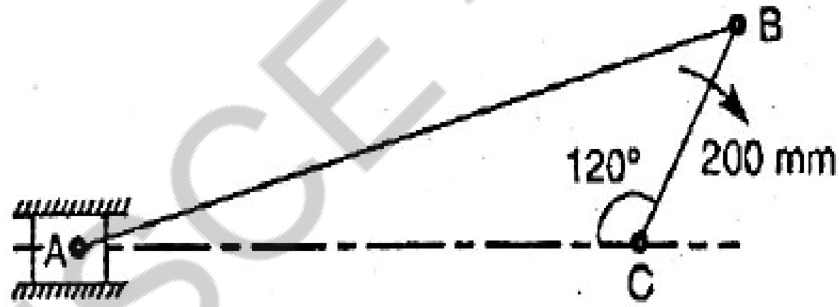
$$\therefore \quad v_D = \frac{v_B}{I_{15}B} \times I_{15}D = \frac{9.01}{0.13} \times 0.05 = 3.46 \text{ m/s} \quad \textbf{Ans.}$$

Similarly, $$\frac{v_D}{I_{16}D} = \frac{v_E}{I_{16}E} \qquad \text{...(Considering centre } I_{16})$$

$$\therefore \quad v_E = \frac{v_D}{I_{16}D} \times I_{16}E = \frac{3.46}{0.2} \times 0.4 = 6.92 \text{ m/s} \quad \textbf{Ans.}$$

11) An engine mechanism is shown in Fig. The crank CB = 200 mm and the connecting rod BA = 600 mm. In the position shown, the crankshaft has a speed of 50 rad/s and an angular acceleration of 800 rad/s². Find: (i) angular velocity of AB and (ii) angular acceleration of AB. ***[APRIL/MAY-2017]***

Fig.



Given:

Crank CB = 200 mm

Connecting Rod BA = 600 mm

ω_{cb} = 50 rad/s

α_{cb} = 800 rad/s^2

To Find:

(i) Angular velocity of AB (ω_{ab}) and

(ii) Angular acceleration of AB (α_{ab})

Soln:

Velocity Dig:

Velocity of Crank: (CB)

$V_{CB} = \omega_{CB} \times \text{length of CB}$

$= 50 \times 0.2$

$\boxed{V_{CB} = 10\,M/S}$

Velocity of connecting rod AB

$V_{ba} = 2.2\text{cm} \times 2\,M/S$

$\boxed{V_{ba} = 4.4\,M/S}$

Angular velocity fo crank AB $(\omega_{AB}) = \dfrac{\gamma}{\text{lengthAB}} = 7.3\,\text{rod/sec}$

Acceleration Dig:

Angular Acceleration of crank (CB)

$\alpha_{CB} = 800\,\text{rad/s}^2$ (given)

Radial component of acceleration:

$$a^r_{CB} = \frac{v^2_{CB}}{\text{Length}0 + CB} = \frac{10^2}{0.2} = 500\,m/s^2$$

$$a^r_{CB} = \alpha_{CB} \times \text{length of CB} = 800 \times 0.2 = 160\,m/s^2$$

Angular acceleration of connecting

$$a^r_{AB} = \frac{(V_{AB})^2}{\text{Length of AB}} = \frac{(4.4)^2}{0.6}$$

$\boxed{a^r_{AB} = 32.26\,m/s^2}$

WKT

$$a^r_{AB} = \alpha_{AB} \times \text{length of AB}$$

where, $\alpha_{AB} \rightarrow$ Angular acceleration of AB

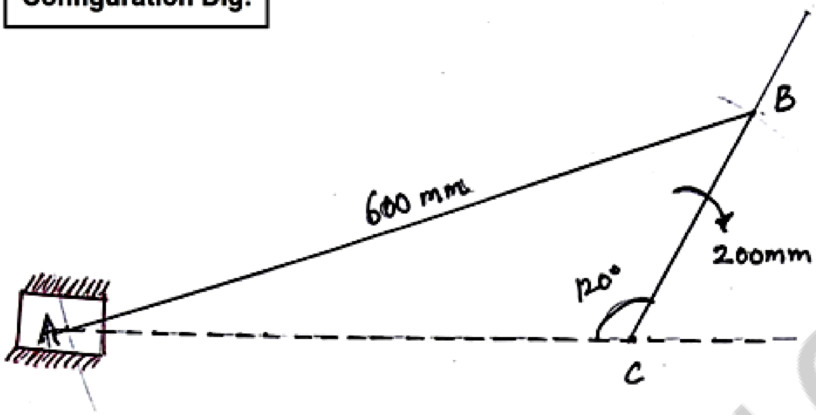
$$\alpha_{AB} = \frac{a^t_{AB}}{\text{Length of AB}}$$

$$= \frac{5.5 \times 100}{0.6}$$

$\boxed{\alpha_{AB} = 916.66\,\text{rad/sec}^2}$

Configuration Dig:

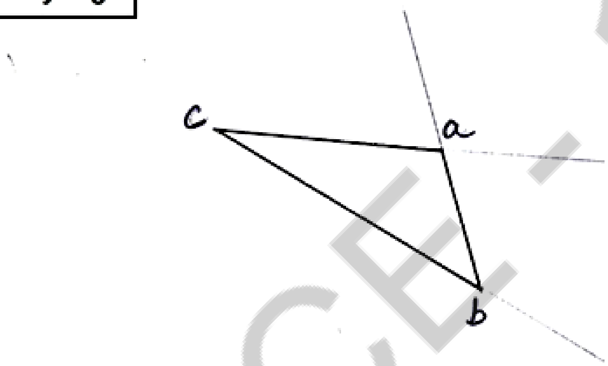
Scale:
50 mm = 1 cm



Velocity Dig:

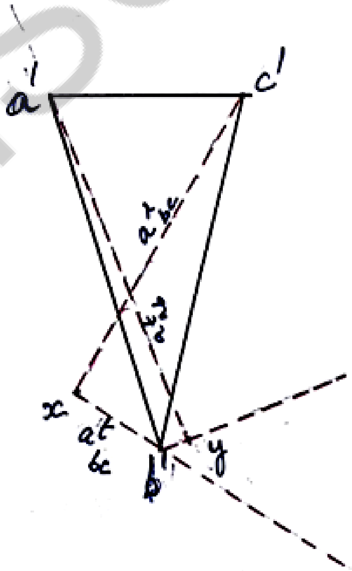
Scale
2 m/s = 1 cm

V_{ba} = 2.2 cm
V_{ac} = 3.7 cm



Acceleration Dig:

Scale:
100 m/s^2 = 1 cm
a^t_{ba} = 5.5 cm



Result:

(i) Angular velocity of AB = 7.3 rad/sec

(ii) Angular acceleration of AB = 916.66rad/sec^2

12) Locate all the instantaneous centres of the slider crank mechanism as shown in Fig. The lengths of crank OB and connecting rod AB are 100 mm and 400 mm respectively. If the crank rotates clockwise with an angular velocity of 10 rad/s, find: (i) Velocity of the slider A, and (ii) Angular velocity of the connecting rod AB.

[APRIL/MAY-2017]

Fig.

Given:

Crank OB = 100 mm

Connecting rod AB = 400 mm

ω_{ob} = 10 rad/s

To find:

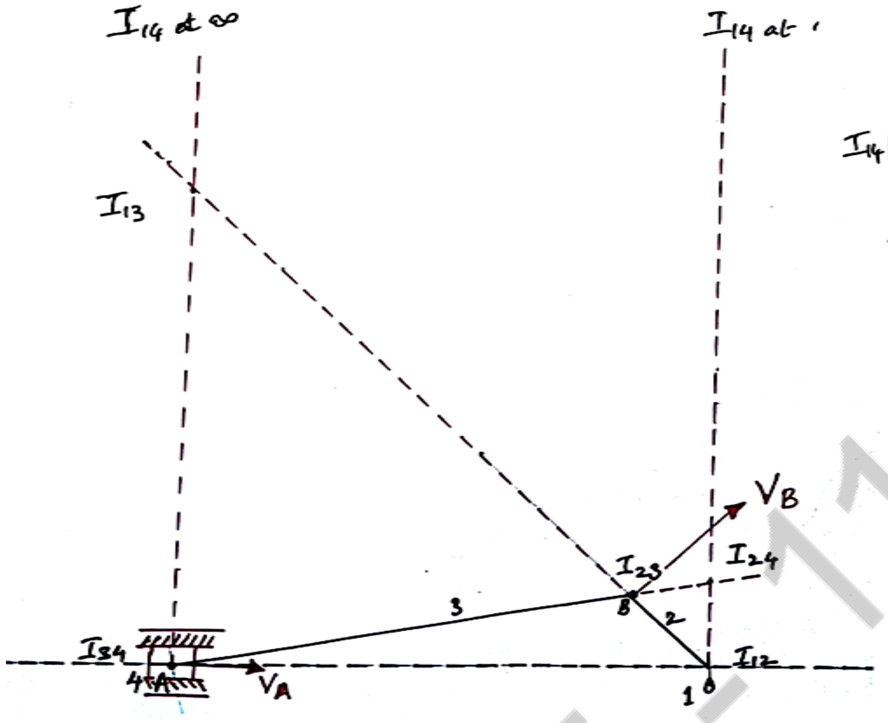
(i) Velocity of the slider A, (V_a) and

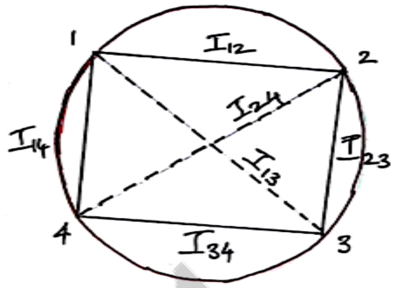
(ii) Angular velocity of the connecting rod AB (ω_{ab}).

Soln:

Configuration Dig:

Scale:
50 mm = 1 cm





Velocity of Input link:

$V_{OB} = \omega_{OB} \times \text{length of OB}$

$V_{OB} = 10 \times 0.1 = 1 m/s$

No. Of instantaneous canties:

$$N = \frac{n(n-1)}{2}$$

$$= \frac{4(4-1)}{2} = 6$$

Determination of velocity of various

(i) Velocity of piston $V_A = V_4 = \omega_2 . R_{I_{12}I_{24}}$

$R_{I_{12}I_{24}}$ =Distance between I_{12} & I_{24}

$$R_{I_{12}I_{24}} = 1.6cn = 0.016 m/s$$

$$\boxed{V_A = 10 \times 0.016 = 0.16 m/s}$$

(ii) Angular velocity of connecting rod (AB)

$V_{AB} \Rightarrow V_3 = \omega_3 \times R_{I_{13}I_{23}}$

$R_{I_{13}I_{23}} = 11.3cm = 0.113m$

$$\boxed{\Rightarrow V_3 = V_{OB} = \omega_3 . R_{I_{13}I_{23}}}$$

$1 = \omega_3 = 0.113$

$\omega_{AB} = \omega_3 = 8.85$ rad/s

Result:

(i) Velocity of piston $V_A = 0.16$ m/s

(ii) Angular Velocity of Connecting Rod $\omega_{AB} = 8.85$rad/s

13) Locate all the instantaneous centres of the slider crank mechanism. The crank (OA) is 160 mm and the connecting rod (AB) is 470 mm long. If the crank rotates clockwise with an angular velocity of 12 rad/s, Determine 1. Linear velocity of slider (B) 2. Angular velocity of the connecting rod (AB), at a crank angle of 30° from inner dead centre position using instantaneous centre method.

[NOV/DEC-2017]

[REFER QB, Q. NO 8]

14) a) i) A four-bar mechanism A_oABB_o has the following lengths:

Fixed link, A_oB_o- 60 mm; Input link, A_oA- 30 mm; Coupler, AB - 45 mm Output link, BB_o - 50mm.

Pivot A_o is left of pivot Bo and both pin joints A and B are above the horizontal fixed link. A point C is on the straight extension of the coupler, such that BC = 25 mm. Input link rotates at a constant speed of 20 rpm clockwise. Determine the linear velocities of points Band C separately, and angular velocities of the coupler and the output link, when the input link is 60° counter clockwise from the fixed link. (9)

ii) What is Kinematic Synthesis? Name the three phases of kinematic synthesis and classify the linkage synthesis problems. (4)

[APRIL/MAY-2018]

(i)

☹ ***Given data:*** A_0A = 30 mm; AB = 45 mm; B_0B = 50 mm; A_0B_0 = 60 mm; $\angle AA_0B_0$ = 60 °; ω_{AA_0} = 20 rpm

☺ ***Solution:*** Relative velocity method

Procedure:

Step 1: Configuration diagram: First of all, draw the configuration diagram, to some suitable scale (say, 1 cm = 20 mm), as shown in Fig

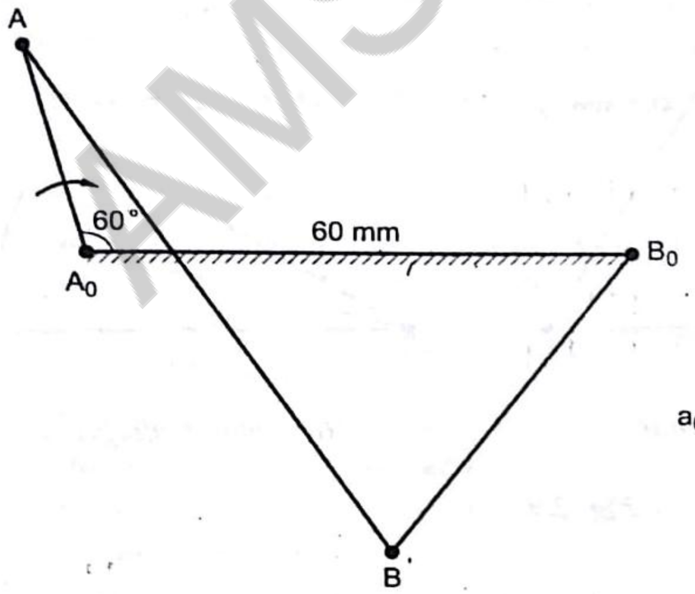
Step 2: Velocity of input link:

Angular velocity of input link, ω_{AA_0} = 20 rpm (given)

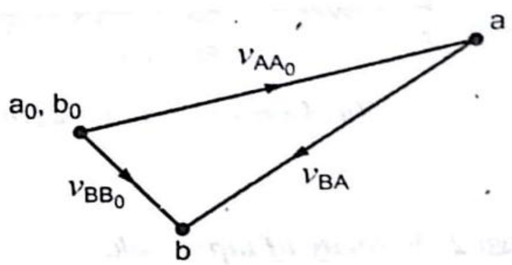
∴ Velocity of the input link AB is given by

$$v_{AA_0} = \omega_{AA_0} \times AA_0 = 20 \times 0.030 = 0.6 \text{ m/s}$$

Step 3: Velocity diagram: Now draw the velocity diagram, as shown in Fig some suitable scale (say, 1 cm = 0.6 m/s)



(a) Configuration diagram



(b) Velocity diagram

Step 4: Velocities of various links:

By measurement from velocity diagram, we get

Velocity of link BB_0, v_{BB_0} = vector bb_0 = 0.018 m/s **Ans.**

The angular velocity of link BB_0 is given by

$$\omega_{BB_0} = \frac{v_{BB_0}}{BB_0} = \frac{0.018}{0.045} = 0.4 \text{ rad/s (counter clockwise about } B_0) \quad \textbf{Ans.}$$

(ii)

Kinematic Synthesis:

➢ To design or create a mechanism to yield prescribed motion characteristics (kinematic parameters) satisfying the various constraints and under specified input motions.

➢ The general steps in kinematic synthesis are:

- Type Synthesis refers to the kind of mechanism selected (e.g., linkage, cams, gear trains and so on.
- Number Synthesis deals with the number of links or joints (pairs) to obtain certain mobility.
- Dimensional Synthesis is the detailed design process, in which the dimensions (*say length*) of individual links are decided.

15) b) i) State and prove the Aronhold-Kennedy theorem related to instantaneous centres. (6)

ii) Explain in detail, the concept of Coriolis component of acceleration with neat sketches and equations. (7)

[APRIL/MAY-2018]

(i)

Aronhold Kennedy (or Three Centres in Line) Theorem

The Aronhold Kennedy's theorem states that ***if three bodies move relatively to each other, they have three instantaneous centres and lie on a straight line.***

Consider three kinematic links A, B and C having relative plane motion. The number of instantaneous centres (N) is given by

$$N = \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3$$

where n = Number of links = 3

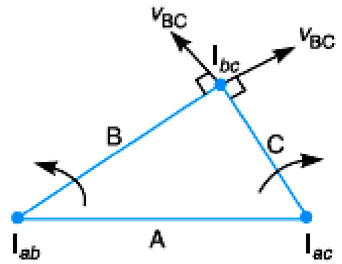


Fig. 6.7. Aronhold Kennedy's theorem.

The two instantaneous centres at the pin joints of B with A, and C with A (*i.e.* I_{ab} and I_{ac}) are the permanent instantaneous centres. According to Aronhold Kennedy's theorem, the third instantaneous centre I_{bc} must lie on the line joining I_{ab} and I_{ac}. In order to prove this, let us consider that the instantaneous centre I_{bc} lies outside the line joining I_{ab} and I_{ac} as shown in Fig. 6.7. The point I_{bc} belongs to both the links B and C. Let us consider the point I_{bc} on the link B. Its velocity v_{BC} must be perpendicular to the line joining I_{ab} and I_{bc}. Now consider the point I_{bc} on the link C. Its velocity v_{BC} must be perpendicular to the line joining I_{ac} and I_{bc}.

We have already discussed in Art. 6.5, that the velocity of the instantaneous centre is same whether it is regarded as a point on the first link or as a point on the second link. Therefore, the velocity of the point I_{bc} cannot be perpendicular to both lines $I_{ab} I_{bc}$ and $I_{ac} I_{bc}$ unless the point I_{bc} lies on the line joining the points I_{ab} and I_{ac}. Thus the three instantaneous centres (I_{ab}, I_{ac} and I_{bc}) must lie on the same straight line. The exact location of I_{bc} on line $I_{ab} I_{ac}$ depends upon the directions and magnitudes of the angular velocities of B and C relative to A.

Method of Locating Instantaneous Centres in a Mechanism

Consider a pin jointed four bar mechanism as shown in Fig. 6.8 (*a*). The following procedure is adopted for locating instantaneous centres.

1. First of all, determine the number of instantaneous centres (N) by using the relation

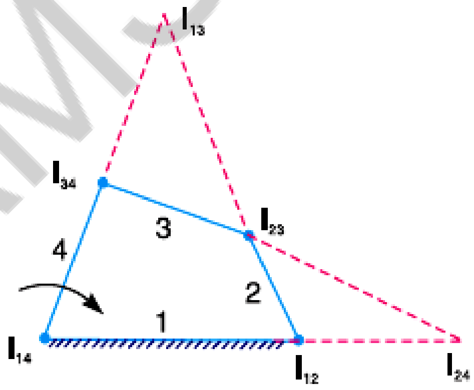
$$N = \frac{n(n-1)}{2}, \quad \text{where } n = \text{Number of links.}$$

In the present case, $N = \frac{4(4-1)}{2} = 6$ ...($\because n = 4$)

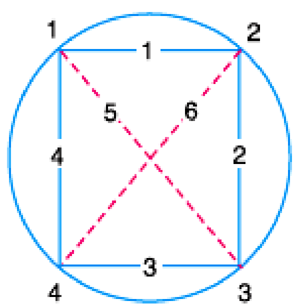
2. Make a list of all the instantaneous centres in a mechanism. Since for a four bar mechanism, there are six instantaneous centres, therefore these centres are listed as shown in the following table (known as book-keeping table).

Links	1	2	3	4
Instantaneous centres (6 in number)	12 13 14	23 24	34	–

3. Locate the fixed and permanent instantaneous centres by inspection. In Fig. 6.8 (*a*), I_{12} and I_{14} are fixed instantaneous centres and I_{23} and I_{34} are permanent instantaneous centres.



(*a*) Four bar mechanism.



(*b*) Circle diagram.

Fig. 6.8. Method of locating instantaneous centres.

4. Locate the remaining neither fixed nor permanent instantaneous centres (or secondary centres) by Kennedy's theorem. This is done by circle diagram as shown in Fig. 6.8 (*b*). Mark points on a circle equal to the number of links in a mechanism. In the present case, mark 1, 2, 3, and 4 on the circle.

5. Join the points by solid lines to show that these centres are already found. In the circle diagram [Fig. 6.8 (*b*)] these lines are 12, 23, 34 and 14 to indicate the centres I_{12}, I_{23}, I_{34} and I_{14}.

6. In order to find the other two instantaneous centres, join two such points that the line joining them forms two adjacent triangles in the circle diagram. The line which is responsible for completing two triangles, should be a common side to the two triangles. In Fig. 6.8 (*b*), join 1 and 3 to form the triangles 123 and 341 and the instantaneous centre* I_{13} will lie on the intersection of I_{12} I_{23} and I_{14} I_{34}, produced if necessary, on the mechanism. Thus the instantaneous centre I_{13} is located. Join 1 and 3 by a dotted line on the circle diagram and mark number 5 on it. Similarly the instantaneous centre I_{24} will lie on the intersection of I_{12} I_{14} and I_{23} I_{34}, produced if necessary, on the mechanism. Thus I_{24} is located. Join 2 and 4 by a dotted line on the circle diagram and mark 6 on it. Hence all the six instantaneous centres are located.

(ii)

Coriolis Component of Acceleration

When a point on one link is sliding along another rotating link, such as in quick return motion mechanism, then the coriolis component of the acceleration must be calculated.

Consider a link *OA* and a slider *B* as shown in Fig. 8.26 (*a*). The slider *B* moves along the link *OA*. The point *C* is the coincident point on the link *OA*.

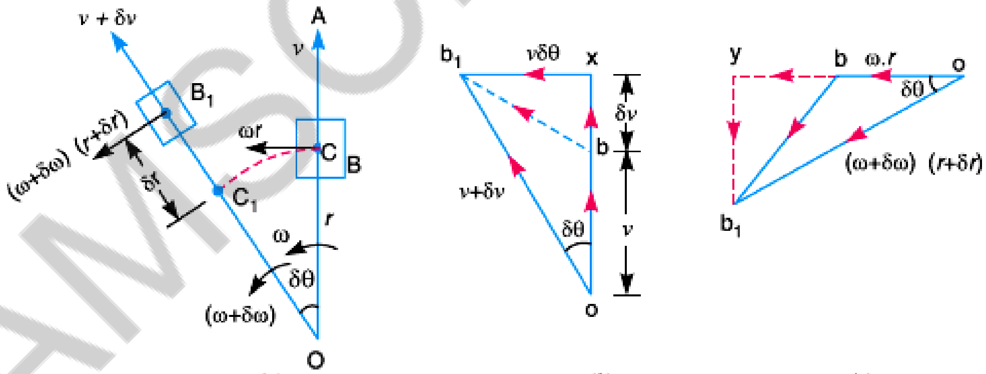
Let ω = Angular velocity of the link *OA* at time *t* seconds.

v = Velocity of the slider *B* along the link *OA* at time *t* seconds.

$\omega.r$ = Velocity of the slider *B* with respect to *O* (perpendicular to the link *OA*) at time *t* seconds, and

$(\omega + \delta\omega)$, $(v + \delta v)$ and $(\omega + \delta\omega)(r + \delta r)$

= Corresponding values at time $(t + \delta t)$ seconds.



(*a*) (*b*) (*c*)

Let us now find out the acceleration of the slider *B* with respect to *O* and with respect to its coincident point *C* lying on the link *OA*. Fig. (*b*) shows the velocity diagram when their velocities v and $(v + \partial v)$ are considered. In this diagram, the vector bb_1 represents the change in velocity in time *t* sec ; the vector *bx* represents the component of change of velocity bb_1 along *OA* (*i.e.* along radial direction) and vector xb_1 represents the component of change of velocity bb_1 in a direction perpendicular to *OA* (*i.e.* in tangential direction). Therefore

$$bx = ox - ob = (v + \delta v)\cos\delta\theta - v\uparrow$$

Since $\delta\theta$ is very small, therefore substituting $\cos\delta\theta = 1$, we have

$$bx = (v + \delta v - v)\uparrow = \delta v\uparrow$$

...(Acting radially outwards)

and $$xb_1 = (v + \delta v)\sin\delta\theta$$

Since $\delta\theta$ is very small, therefore substituting $\sin\delta\theta = \delta\theta$, we have

$$xb_1 = (v + \delta v)\,\delta\theta = v.\delta\theta + \delta v.\delta\theta$$

Neglecting $\delta v.\delta\theta$ being very small, therefore

$$xb_1 = v.\overleftarrow{\delta\theta}$$

Fig. shows the velocity diagram when the velocities $\omega.r$ and $(\omega + \delta\omega)(r + \delta r)$ are considered. In this diagram, vector bb_1 represents the change in velocity ; vector yb_1 represents the component of change of velocity bb_1 along OA (*i.e.* along radial direction) and vector by represents the component of change of velocity bb_1 in a direction perpendicular to OA (*i.e.* in a tangential direction). Therefore

$$yb_1 = (\omega + \delta\omega)(r + \delta r)\sin\delta\theta\downarrow$$

$$= (\omega r + \omega\delta r + \delta\omega r + \delta\omega\delta r)\sin\delta\theta$$

Since $\delta\theta$ is very small, therefore substituting $\sin\delta\theta = \delta\theta$ in the above expression, we have

$$yb_1 = \omega.r.\delta\theta + \omega.\delta r.\delta\theta + \delta\omega.r.\delta\theta + \delta\omega.\delta r.\delta\theta$$

$$= \omega.r.\delta\theta\downarrow\text{, acting radially inwards}$$ ...(Neglecting all other quantities)

and $$by = oy - ob = (\omega + \delta\omega)(r + \delta r)\cos\delta\theta - \omega.r$$

$$= (\omega.r + \omega.\delta r + \delta\omega.r + \delta\omega.\delta r)\cos\delta\theta - \omega.r$$

Since $\delta\theta$ is small, therefore substituting $\cos\delta\theta = 1$, we have

$$by = \omega.r + \omega.\delta r + \delta\omega.r + \delta\omega.\delta r - \omega.r = \omega.\delta r + r.\delta\omega$$ ...(Neglecting $\delta\omega.\delta r$)

...(Perpendicular to OA and towards left)

Therefore, total component of change of velocity along radial direction

$$= bx - yb_1 = (\delta v - \omega.r.\delta\theta)\uparrow$$ ...(Acting radially outwards from O to A)

$\therefore$ Radial component of the acceleration of the slider B with respect to O on the link OA, acting radially outwards from O to A,

$$a^r_{BO} = \text{Lt}\frac{\delta v - \omega.r.\delta\theta}{\delta t} = \frac{dv}{dt} - \omega.r \times \frac{d\theta}{dt} = \frac{dv}{dt} - \omega^2.r\uparrow \qquad ...(i)$$

...($\because d\theta/dt = \omega$)

Also, the total component of change of velocity along tangential direction,

$$= xb_1 + by = v.\overleftarrow{\delta\theta} + (\omega.\delta r \overleftarrow{+} r.\delta\omega)$$

...(Perpendicular to OA and towards left)

∴ Tangential component of acceleration of the slider B with respect to O on the link OA, acting perpendicular to OA and towards left,

$$a^t_{BO} = \text{Lt} \frac{v.\delta\theta + (\omega.\delta r + r.\delta\omega)}{\delta t} = v\frac{d\theta}{dt} + \omega\frac{dr}{dt} + r\frac{d\omega}{dt}$$

$$= v.\omega + \omega.v + r.\alpha = (2\,v.\omega + r.\alpha)\overleftarrow{} \qquad ...(ii)$$

...($\because$ $dr/dt = v$, and $d\omega/dt = \alpha$)

Now radial component of acceleration of the coincident point C with respect to O, acting in a direction from C to O,

$$a^r_{CO} = \omega^2.r \uparrow \qquad ...(iii)$$

and tangential component of acceleraiton of the coincident point C with respect to O, acting in a direction perpendicular to CO and towards left,

$$a^t_{CO} = \overleftarrow{\alpha.r} \uparrow \qquad ...(iv)$$

Radial component of the slider B with respect to the coincident point C on the link OA, acting radially outwards,

$$a^r_{BC} = a^r_{BO} - a^r_{CO} = \left(\frac{dv}{dt} - \omega^2.r\right) - \left(-\omega^2.r\right) = \frac{dv}{dt} \uparrow$$

and tangential component of the slider B with respect to the coincident point C on the link OA acting in a direction perpendicular to OA and towards left,

$$a^t_{BC} = a^t_{BO} - a^t_{CO} = (2\omega v + \alpha.r) - \alpha.r = \overleftarrow{2\omega v}$$

This tangential component of acceleration of the slider B with respect to the coincident point C on the link is known as ***coriolis component of acceleration*** and is always perpendicualr to the link.

∴ Coriolis component of the acceleration of B with respect of C,

$$a^c_{BC} = a^t_{BC} = 2\omega.v$$

where ω = Angular velocity of the link OA, and

v = Velocity of slider B with respect to coincident point C.

In the above discussion, the anticlockwise direction for ω and the radially outward direction for v are taken as ***positive***. It may be noted that the direction of coriolis component of acceleration changes sign, if either ω or v is reversed in direction. But the direction of coriolis component of acceleration will not be changed in sign if both ω and v are reversed in direction. It is concluded that the direction of coriolis component of acceleration is obtained by rotating v, at 90°, about its origin in the same direction as that of ω.

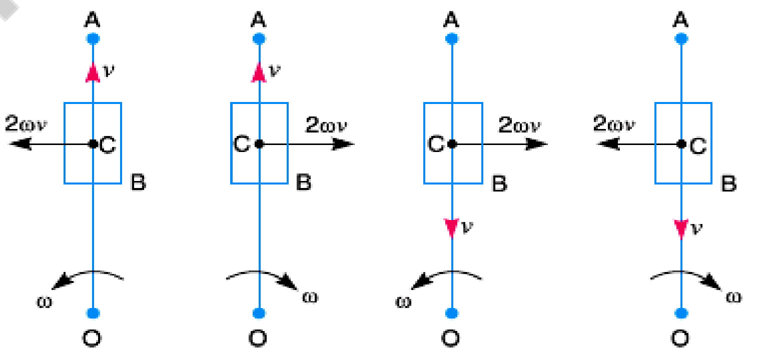


Fig. Direction of coriolis component of acceleration.

The direction of coriolis component of acceleration ($2\,\omega.v$) for all four possible cases, is shown in Fig. The directions of ω and v are given.

16). *PQRS is a four bar chain with link PS fixed. The lengths of the links are PQ = 62.5 mm; QR = 175 mm; RS = 112.5 mm; and PS = 200 mm. The crank PQ rotates at 10 rad/s clockwise. Draw the velocity and acceleration diagram when angle QPS = 60° and Q and R lie on the same side of PS. Find the angular velocity and angular acceleration of links QR and RS.* **(13)**

[NOV/DEC-2018]

Solution. Given : $\omega_{QP} = 10$ rad/s; $PQ = 62.5$ mm $= 0.0625$ m ; $QR = 175$ mm $= 0.175$ m ; $RS = 112.5$ mm $= 0.1125$ m ; $PS = 200$ mm $= 0.2$ m

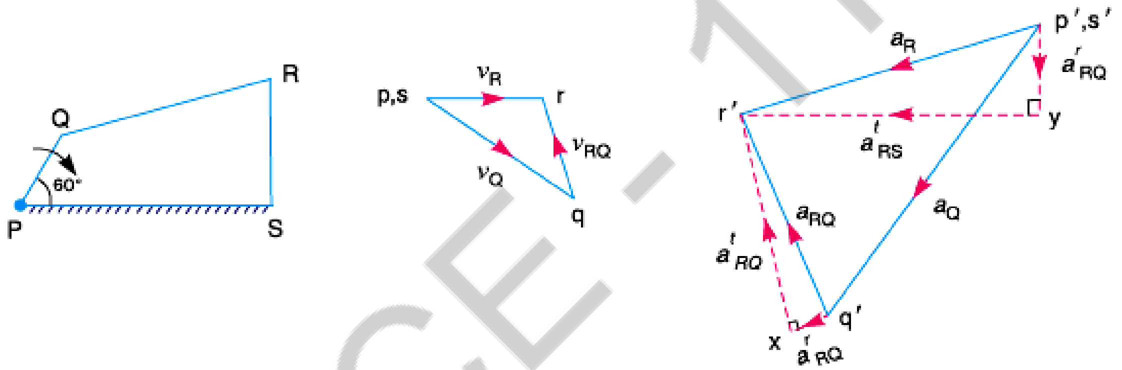
We know that velocity of Q with respect to P or velocity of Q,

$$v_{QP} = v_Q = \omega_{QP} \times PQ = 10 \times 0.0625 = 0.625 \text{ m/s}$$

...(Perpendicular to PQ)

Angular velocity of links QR and RS

First of all, draw the space diagram of a four bar chain, to some suitable scale, as shown in Fig. 8.9 (*a*). Now the velocity diagram as shown in Fig. 8.9 (*b*), is drawn as discussed below:



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

Fig.

1. Since P and S are fixed points, therefore these points lie at one place in velocity diagram. Draw vector pq perpendicular to PQ, to some suitable scale, to represent the velocity of Q with respect to P or velocity of Q *i.e.* v_{QP} or v_Q such that

$$\text{vector } pq = v_{QP} = v_Q = 0.625 \text{ m/s}$$

2. From point q, draw vector qr perpendicular to QR to represent the velocity of R with respect to Q (*i.e.* v_{RQ}) and from point s, draw vector sr perpendicular to SR to represent the velocity of R with respect to S or velocity of R (*i.e.* v_{RS} or v_R). The vectors qr and sr intersect at r. By measurement, we find that

$$v_{RQ} = \text{vector } qr = 0.333 \text{ m/s, and } v_{RS} = v_R = \text{vector } sr = 0.426 \text{ m/s}$$

We know that angular velocity of link QR,

$$\omega_{QR} = \frac{v_{RQ}}{RQ} = \frac{0.333}{0.175} = 1.9 \text{ rad/s (Anticlockwise)}$$ **Ans.**

and angular velocity of link RS,

$$\omega_{RS} = \frac{v_{RS}}{SR} = \frac{0.426}{0.1125} = 3.78 \text{ rad/s (Clockwise)}$$ **Ans.**

Angular acceleration of links QR and RS

Since the angular acceleration of the crank PQ is not given, therefore there will be no tangential component of the acceleration of Q with respect to P.

We know that radial component of the acceleration of Q with respect to P (or the acceleration of Q),

$$a^r_{QP} = a_{QP} = a_Q = \frac{v^2_{QP}}{PQ} = \frac{(0.625)^2}{0.0625} = 6.25 \text{ m/s}^2$$

Radial component of the acceleration of R with respect to Q,

$$a^r_{RQ} = \frac{v^2_{RQ}}{QR} = \frac{(0.333)^2}{0.175} = 0.634 \text{ m/s}^2$$

and radial component of the acceleration of R with respect to S (or the acceleration of R),

$$a^r_{RS} = a_{RS} = a_R = \frac{v^2_{RS}}{SR} = \frac{(0.426)^2}{0.1125} = 1.613 \text{ m/s}^2$$

The acceleration diagram, as shown in Fig. is drawn as follows :

1. Since P and S are fixed points, therefore these points lie at one place in the acceleration diagram. Draw vector $p'q'$ parallel to PQ, to some suitable scale, to represent the radial component of acceleration of Q with respect to P or acceleration of Q *i.e* a^r_{QP} or a_Q such that

$$\text{vector } p'q' = a^r_{QP} = a_Q = 6.25 \text{ m/s}^2$$

2. From point q', draw vector $q'x$ parallel to QR to represent the radial component of acceleration of R with respect to Q *i.e.* a^r_{RQ} such that

$$\text{vector } q'x = a^r_{RQ} = 0.634 \text{ m/s}^2$$

3. From point x, draw vector xr' perpendicular to QR to represent the tangential component of acceleration of R with respect to Q *i.e* a^t_{RQ} whose magnitude is not yet known.

4. Now from point s', draw vector $s'y$ parallel to SR to represent the radial component of the acceleration of R with respect to S *i.e.* a^r_{RS} such that

$$\text{vector } s'y = a^r_{RS} = 1.613 \text{ m/s}^2$$

5. From point y, draw vector yr' perpendicular to SR to represent the tangential component of acceleration of R with respect to S *i.e.* a^t_{RS}.

6. The vectors xr' and yr' intersect at r'. Join $p'r$ and $q'r'$. By measurement, we find that

$$a^t_{RQ} = \text{vector } xr' = 4.1 \text{ m/s}^2 \text{ and } a^t_{RS} = \text{vector } yr' = 5.3 \text{ m/s}^2$$

We know that angular acceleration of link QR,

$$\alpha_{QR} = \frac{a^t_{RQ}}{QR} = \frac{4.1}{0.175} = 23.43 \text{ rad/s}^2 \text{ (Anticlockwise)}$$ **Ans.**

and angular acceleration of link RS,

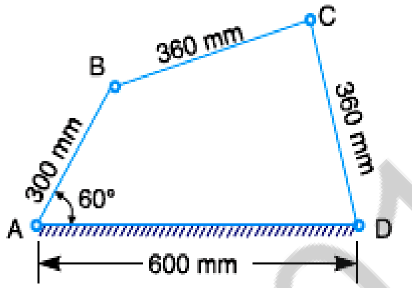
$$\alpha_{RS} = \frac{a^t_{RS}}{SR} = \frac{5.3}{0.1125} = 47.1 \text{ rad/s}^2 \text{ (Anticlockwise)}$$ **Ans.**

17) In a pin jointed four bar mechanism ABCD, length of links AB = 300 mm, BC = CD = 360 mm and AD = 600 mm. The angle BAD = 60°. The crank AB rotates uniformly at 100 rpm. Locate the entire instantaneous centre and find the angular velocity of link BC. (13)

[NOV/DEC-2018]

Solution. Given : $N_{AB} = 100$ r.p.m or

$\omega_{AB} = 2\pi \times 100/60 = 10.47$ rad/s



Since the length of crank $AB = 300$ mm $= 0.3$ m, therefore velocity of point B on link AB,

$$v_B = \omega_{AB} \times AB = 10.47 \times 0.3 = 3.141 \text{ m/s}$$

Location of instantaneous centres

The instantaneous centres are located as discussed below:

1. Since the mechanism consists of four links (*i.e.* $n = 4$), therefore number of instantaneous centres,

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

2. For a four bar mechanism, the book keeping table may be drawn as discussed in Art. 6.10.

3. Locate the fixed and permanent instantaneous centres by inspection. These centres are I_{12}, I_{23}, I_{34} and I_{14}, as shown in Fig. 6.10.

4. Locate the remaining neither fixed nor permanent instantaneous centres by Aronhold Kennedy's theorem. This is done by circle diagram as shown in Fig. 6.11. Mark four points (equal to the number of links in a mechanism) 1, 2, 3, and 4 on the circle.

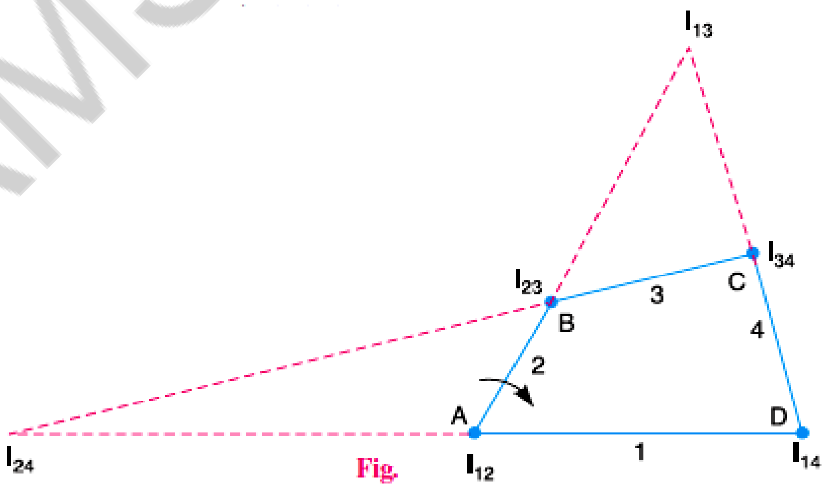


Fig.

5. Join points 1 to 2, 2 to 3, 3 to 4 and 4 to 1 to indicate the instantaneous centres already located *i.e.* I_{12}, I_{23}, I_{34} and I_{14}.

6. Join 1 to 3 to form two triangles 1 2 3 and 3 4 1. The side 13, common to both triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{13} lies on the intersection of the lines joining the points I_{12} I_{23} and I_{34} I_{14} as shown in Fig. 6.10. Thus centre I_{13} is located. Mark number 5 (because four instantaneous centres have already been located) on the dotted line 1 3.

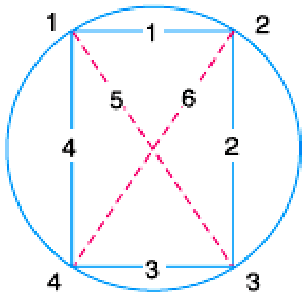


Fig.

7. Now join 2 to 4 to complete two triangles 2 3 4 and 1 2 4. The side 2 4, common to both triangles, is responsible for completing the two triangles. Therefore centre I_{24} lies on the intersection of the lines joining the points I_{23} I_{34} and I_{12} I_{14} as shown in Fig. 6.10. Thus centre I_{24} is located. Mark number 6 on the dotted line 2 4. Thus all the six instantaneous centres are located.

Angular velocity of the link BC

Let ω_{BC} = Angular velocity of the link BC.

Since B is also a point on link BC, therefore velocity of point B on link BC,

$$v_B = \omega_{BC} \times I_{13} B$$

By measurement, we find that $I_{13} B$ = 500 mm = 0.5 m

$$\therefore \quad \omega_{BC} = \frac{v_B}{I_{13} B} = \frac{3.141}{0.5} = 6.282 \text{ rad/s} \quad \textbf{Ans.}$$

18) (i) State and prove the Kennedy's theorem of three instantaneous centres. (5)

(ii) Give the types of instantaneous centres with examples. (4)

(iii) Give the procedure to be followed for locating instantaneous centres for four bar mechanism. (4)

(13) *[APR/MAY-2019]*

(i) State and prove the Kennedy's theorem of three instantaneous centres. *(REFER PART-B, Q. NO 15)*

(ii) Give the types of instantaneous centres with examples.

Types of Instantaneous Centres

The instantaneous centres for a mechanism are of the following three types :

1. Fixed instantaneous centres, 2. Permanent instantaneous centres, and 3. Neither fixed nor permanent instantaneous centres.

The first two types *i.e.* fixed and permanent instantaneous centres are together known as *primary instantaneous centres* and the third type is known as *secondary instantaneous centres*.

Consider a four bar mechanism $ABCD$ as shown in Fig. 6.5. The number of instantaneous centres (N) in a four bar mechanism is given by

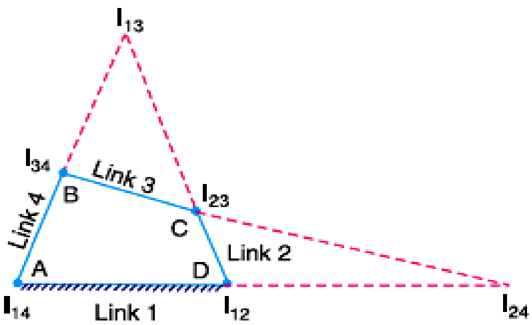


Fig. Types of instantaneous centres.

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6 \quad \ldots (\because n = 4)$$

The instantaneous centres I_{12} and I_{14} are called the ***fixed instantaneous centres*** as they remain in the same place for all configurations of the mechanism. The instantaneous centres I_{23} and I_{34} are the ***permanent instantaneous centres*** as they move when the mechanism moves, but the joints are of permanent nature. The instantaneous centres I_{13} and I_{24} are ***neither fixed nor permanent instantaneous centres*** as they vary with the configuration of the mechanism.

(iii) Give the procedure to be followed for locating instantaneous centres for four bar mechanism.

(REFER PART-B, Q. NO 15)

19) The crank of slider crank mechanism rotates clockwise at a constant speed of 300 rpm. The crank is 150 mm and the connecting rod is 600 mm long. Determine:
(i) Linear velocity and acceleration at the midpoint of the connecting rod.
(ii) Angular velocity and angular acceleration of the connecting rod, at a crank angle of 45^0 from inner dead centre position. (13)

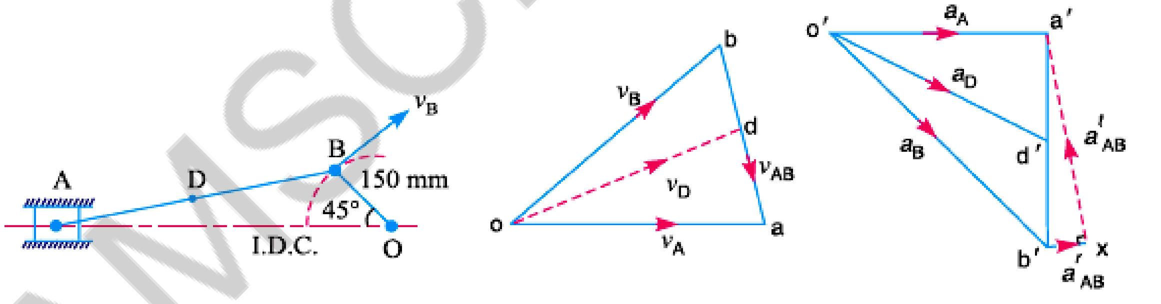
[APR/MAY-2019]

Solution. Given : N_{BO} = 300 r.p.m. or $\omega_{BO} = 2\pi \times 300/60 = 31.42$ rad/s; OB = 150 mm = 0.15 m ; BA = 600 mm = 0.6 m

We know that linear velocity of B with respect to O or velocity of B,

$$v_{BO} = v_B = \omega_{BO} \times OB = 31.42 \times 0.15 = 4.713 \text{ m/s}$$

...(Perpendicular to BO)



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

Fig.

1. *Linear velocity of the midpoint of the connecting rod*

First of all draw the space diagram, to some suitable scale; as shown in Fig. 8.4 (*a*). Now the velocity diagram, as shown in Fig. 8.4 (*b*), is drawn as discussed below:

1. Draw vector ob perpendicular to BO, to some suitable scale, to represent the velocity of B with respect to O or simply velocity of B *i.e.* v_{BO} or v_B, such that

$$\text{vector } ob = v_{BO} = v_B = 4.713 \text{ m/s}$$

2. From point b, draw vector ba perpendicular to BA to represent the velocity of A with respect to B *i.e.* v_{AB}, and from point o draw vector oa parallel to the motion of A (which is along AO) to represent the velocity of A *i.e.* v_A. The vectors ba and oa intersect at a.

By measurement, we find that velocity of A with respect to B,

$$v_{AB} = \text{vector } ba = 3.4 \text{ m/s}$$

and Velocity of A, $v_A = \text{vector } oa = 4 \text{ m/s}$

3. In order to find the velocity of the midpoint D of the connecting rod AB, divide the vector ba at d in the same ratio as D divides AB, in the space diagram. In other words,

$$bd / ba = BD/BA$$

Note: Since D is the midpoint of AB, therefore d is also midpoint of vector ba.

4. Join od. Now the vector od represents the velocity of the midpoint D of the connecting rod *i.e.* v_D.

By measurement, we find that

$$v_D = \text{vector } od = 4.1 \text{ m/s} \textbf{ Ans.}$$

Acceleration of the midpoint of the connecting rod

We know that the radial component of the acceleration of B with respect to O or the acceleration of B,

$$a_{BO}^r = a_B = \frac{v_{BO}^2}{OB} = \frac{(4.713)^2}{0.15} = 148.1 \text{ m/s}^2$$

and the radial component of the acceleraiton of A with respect to B,

$$a_{AB}^r = \frac{v_{AB}^2}{BA} = \frac{(3.4)^2}{0.6} = 19.3 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. 8.4 (*c*) is drawn as discussed below:

1. Draw vector $o'b'$ parallel to BO, to some suitable scale, to represent the radial component of the acceleration of B with respect to O or simply acceleration of B *i.e.* a_{BO}^r or a_B, such that

$$\text{vector } o'b' = a_{BO}^r = a_B = 148.1 \text{ m/s}^2$$

2. The acceleration of A with respect to B has the following two components:

(*a*) The radial component of the acceleration of A with respect to B *i.e.* a_{AB}^r, and

(*b*) The tangential component of the acceleration of A with respect to B *i.e.* a_{AB}^t. These two components are mutually perpendicular.

Therefore from point b', draw vector $b'x$ parallel to AB to represent $a_{AB}^r = 19.3 \text{ m/s}^2$ and from point x draw vector xa' perpendicular to vector $b'x$ whose magnitude is yet unknown.

3. Now from o', draw vector $o'a'$ parallel to the path of motion of A (which is along AO) to represent the acceleration of A *i.e.* a_A. The vectors xa' and $o'a'$ intersect at a'. Join $a'b'$.

4. In order to find the acceleration of the midpoint D of the connecting rod AB, divide the vector $a'b'$ at d' in the same ratio as D divides AB. In other words

$$b'd'/b'a' = BD/BA$$

5. Join $o'd'$. The vector $o'd'$ represents the acceleration of midpoint D of the connecting rod *i.e.* a_D.

By measurement, we find that

$$a_D = \text{vector } o'd' = 117 \text{ m/s}^2 \textbf{ Ans.}$$

2. *Angular velocity of the connecting rod*

We know that angular velocity of the connecting rod AB,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{3.4}{0.6} = 5.67 \text{ rad/s}^2 \text{ (Anticlockwise about } B) \textbf{ Ans.}$$

Angular acceleration of the connecting rod

From the acceleration diagram, we find that

$$a^t_{AB} = 103 \text{ m/s}^2 \quad \text{...(By measurement)}$$

We know that angular acceleration of the connecting rod AB,

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{103}{0.6} = 171.67 \text{ rad/s}^2 \text{ (Clockwise about } B\text{)} \textbf{ Ans.}$$

$$a^r_{AB} = \frac{v^2_{AB}}{BA} = \frac{(3.4)^2}{0.6} = 19.3 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. 8.4 (c) is drawn as discussed below:

1. Draw vector $o'b'$ parallel to BO, to some suitable scale, to represent the radial component of the acceleration of B with respect to O or simply acceleration of B *i.e.* a^r_{BO} or a_B, such that

$$\text{vector } o'b' = a^r_{BO} = a_B = 148.1 \text{ m/s}^2$$

PART-A

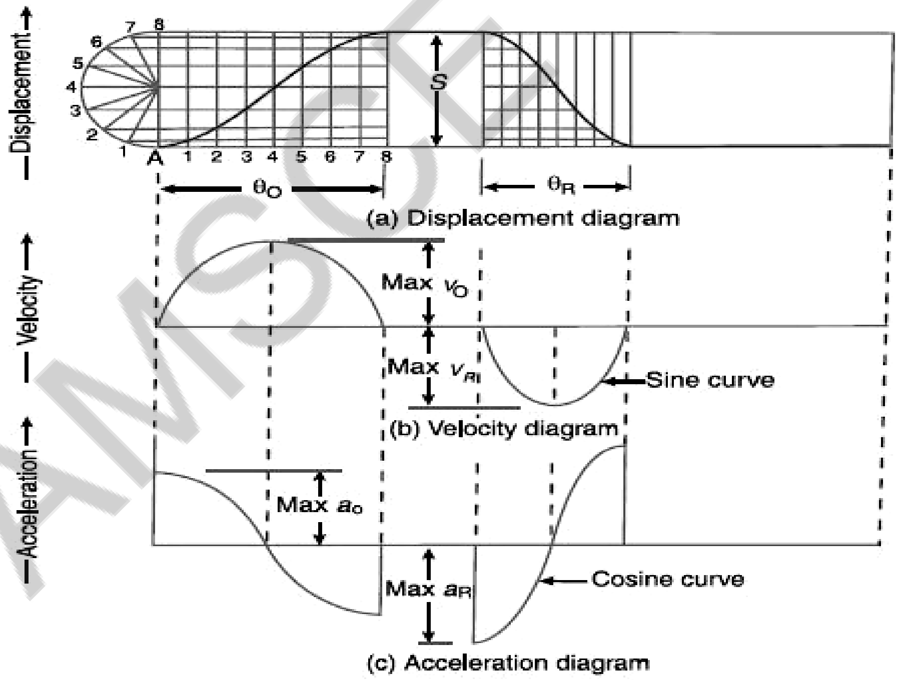
1) Define Tangent cam. **(MAY/JUNE 2014)**

When the flanks of the cam are straight and tangential to the base circle and nose circle, the cam is known as Tangent Cam.

2) What are the different motions of the follower? **(MAY/JUNE 2014)**

- Uniform motion,
- Simple harmonic motion,
- Uniform acceleration and retardation, and
- Cycloidal motion.

3) Draw the displacement, velocity and acceleration diagram for a follower when it moves with simple harmonic motion?
(MAY/JUNE 2015)



4) Why a roller follower is preferred to that of a knife-edged follower?
(MAY/JUNE 2015)

The knife-edge follower is rarely used because of excessive wear due to small area of contact.

5) Define trace point of a cam?

It is a reference point on the follower to trace the cam profile. In case of a knife edge follower, the knife edge itself is a tracing point and in roller follower, the centre of the roller is the tracing point.

6) Define tangent cam.

When the flanks of the cam are straight and tangential to the base circle and nose circle, then the cam is known as a tangent cam.

7) Write down the classification of cams according to the manner of constraint of the follower.

(i) *Pre-loaded Spring Cam:* A pre-loaded compression spring is used for the purpose of keeping the contact between the cam and the follower.

(ii). *Positive-drive Cam:* In this type, constant touch between the cam and the follower is maintained by a roller follower operating in the groove of a cam. The follower cannot go out of this groove under the normal working operations. A constrained or positive drive is also obtained by the use of a conjugate cam.

(iii). *Gravity Cam:* If the rise of the cam is achieved by the rising surface of the cam and the return by the force of gravity or due to the weight of the cam, the cam is known as a gravity cam. However, these cams arc not preferred due to their uncertain behavior.

8) Define the term jump speed of a cam.

Follower looses contact with cam surface when the cam rotates beyond particular speed due to inertia forces. The speed is known as Jump Speed of cam.

9) Define the following with respect to cam and follower mechanism

Pressure angle: It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by the standard pressure angles are 14° and 20°.

Pitch circle: It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

10) State the reason for providing offset in a cam follower mechanism

Offset is generally preferred in cam and follower

- To reduce the pressure angle during ascent of the follower
- To reduce surface wear of cam

11) What is cam?

A cam is a rotating machine element which gives reciprocating (or) oscillating motion to another element known as follower.

12) Define tangent cam?

When the flanks of the cam are straight and tangential to the base circle and nose circle, the cam is known as tangent cam.

13) Distinguish radial and cylindrical cams.

Radial cam	Cylindrical cam
In this cam, the follower reciprocates (or) oscillates in a direction perpendicular to the axis.	In this the follower reciprocates (or) oscillates in a direction parallel to the cam axis.

14) What are the different motions of the follower?

- Uniform motion,
- Simple harmonic motion,
- Uniform acceleration and retardation, and
- Cycloidal motion.

15) Compare Roller and mushroom follower of a cam.

S.No	Roller Follower	Mushroom Follower
1.	Roller followers are extensively used where more space is available.	The mushroom followers are generally used where space is limited.
2.	It is used in stationary gas engines, oil engines and aircraft valves in engines.	It is used in cams which operate the valves in automobile engines.

16) Explain offset follower.

When the motion of the follower is along an axis away from the axis of the cam centre, it is called offset follower.

17) Define trace point in the study of cams.

It is a reference point on the follower and is used to generate the pitch curve. In case of knife edge follower the knife edge represents the trace point and the pitch curve corresponds to the cam profile. In a roller follower the centre of the roller represents the trace point.

18) Define pressure angle with respect to cams.

It is the angle between the direction of the follower motion and a normal to the pitch curve. This angle is very important in designing a cam profile.

If the pressure angle is too large, a reciprocating follower will jam in its bearings.

19) Define Lift (or) Stroke in cam.

It is the maximum travel of the follower from its lowest position to the topmost position.

20) Define undercutting in cam. How is occurs?

The cam profile must be continuous curve without any loop. If the curvature of the pitch curve is too sharp, then the part of the cam shape would be lost and thereafter the intended cam motion would not be achieved. Such a cam is said to be undercut. Undercutting occurs in the cam because of attempting to achieve too great a follower lift with very small cam rotation with a smaller cam.

21) What do you know about Nomogram?

In Nomogram, by knowing the values of total lift of the follower (L) and the cam rotation angle (β) for each segment of the displacement diagram, we can read directly the maximum pressure angle occurring n the segment for a particular choice of prime circle radius (R_0).

22) What are the classifications of cam based on the follower movement?

- Rise-Return-Rise (R-R-R) cams,
- Dwell-Rise-Return-Dwell (D-R-R-D) cams,
- Dwell-Rise-Dwell-Return-Dwell (D-R-D-R-D) cams,
- Dwell-Rise-Dwell (D-R-D) cams.

23) What are the different types of cams?

- Wedge (or) flat cams
- Radial (or) Disc cams
- Spiral cams
- Cylindrical (or) Barrel (or) Drum Cams
- Conjugate cams
- Globoidal cams
- Spherical cams

24) What do you know about gravity cam?

In this type, the rise of the cam is achieved by the rising surface of the cam and the return by the force of gravity of die to the weight of the cam.

25) Define Trace point.

It is a reference point on the follower to trace the cam profile. In case of a knife edge follower, the knife edge itself is a tracing point and in roller follower, the centre of the roller is the tracing point.

26) Define pressure angle.

It is the angle between the direction of the follower motion and a normal to the pitch curve. This is very important in cam design as it represents steepness of the cam profile. If the pressure angle is too large, a reciprocating follower will jam in its bearings.

27) Define Prime circle.

The smallest circle drawn tangent to the pitch curve is known as the prime circle.

28) Define Angle of Ascent.

The angle of rotation of cam from the position when the follower begins to rise till it reaches its highest position is known as angle of ascent. It is also known as out stroke and is denoted by θ_0.

29) What is meant by Simple Harmonic Motion?

When a body rotates on a circular path with uniform angular velocity, its projection on the diameter will have simple harmonic motion. The velocity of the projection will be maximum at the centre of and zero at the ends of the diameter. In case of acceleration and retardation, the values will be zero at the centre and maximum at the ends of diameter.

30) What are the different shapes of high speed cams?

1. Circular Arc cam with flat faced follower
2. Tangent cam with reciprocating roller follower

31) Define cam angle.

It is the angle of rotation of the cam for a definite displacement of the follower.

32) What are the classifications of follower based on the follower movement?

a. Reciprocating (or) translating follower.
b. Oscillating (or) rotating follower.

33) Define Pitch curve.

The locus of the tracing point is known as the pitch curve. For the purpose of laying out the cam profiles, it is assumed that the cam is fixed and the follower rotates around it.

34) What are the classifications of the follower based on the path of motion of the follower?

- Radial follower.
- Offset follower.

35) What are the classifications of cam base on the constraint of the follower?

- Pre-loaded spring cams.
- Positive drive cams.
- Gravity cams.

36) Which type of cam follower motion is preferred for high speed engines? Why? (A/M-2017)

Roller or Spherical follower is preferred for High speed engines, because of its ability to withstand high dynamic loads and low wear rate.

37) Give any two applications of cam mechanism. (A/M-2017)

- IC Engine
- Machine tools
- Printing control mechanism

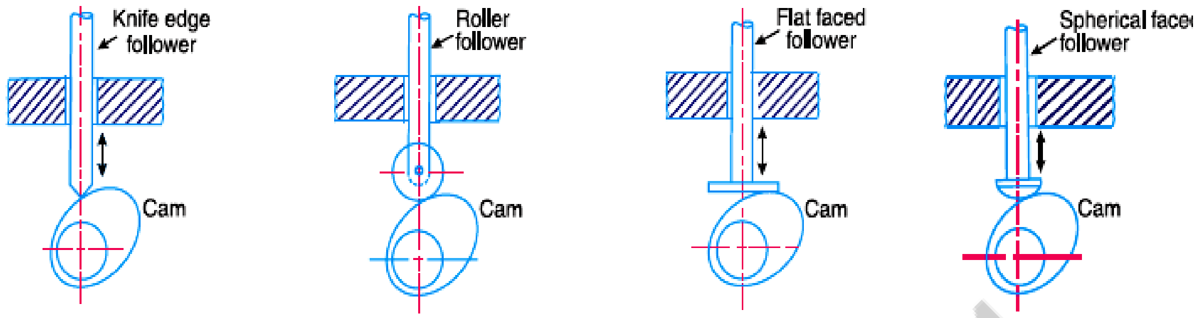
38) Differentiate between radial cam and cylindrical cam. (N/D-2017)

S.No	RADIAL CAMS	CYLINDRICAL CAMS
1	The follower moves radially along with the center of rotation, is known as radial cam.	Cylindrical cams are also known as drum cams or barrel cams.
2	Due to the simplicity and compactness the radial cams are very popular.	In the cam, the cylinder consists of a circumferential contour cut in the surface and the cylinder rotates about its axis.
3	The follower oscillates about an axis parallel to the axis of rotation of the cam.	The follower reciprocates about an axis parallel to the axis of rotation of the cam.

39) Name the cam follower extensively used in air-craft engines. (N/D-2017)

Roller Follower.

40) Classify and sketch the translating cam followers based on their position. (A/M 2018)

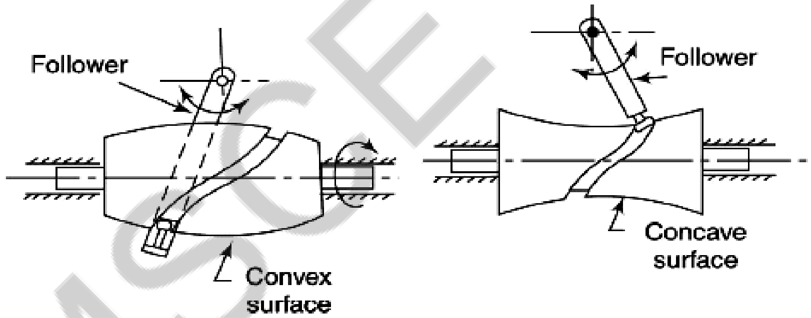


(*a*) Cam with knife edge follower. (*b*) Cam with roller follower. (*c*) Cam with flat faced follower. (*d*) Cam with spherical faced follower.

Classification of Translating cam Follower

41) Sketch and name a specified contour cam, stating its advantage. (A/M 2018)

Globoidal Cam - A circumferential contour is cut on the surface of rotation of the Cam to impart motion to the follower which has an Oscillatory motion. The advantage of such Cams is limited to moderate speeds where the angle of Oscillation of the Follower is large.



42) Define prime circle of cam. (N/D 2018)

Prime Circle. It is the smallest circle that can be drawn from the centre of the cam and tangent to the pitch curve. For a knife edge and a flat face follower, the prime circle and the base circle are identical. For a roller follower, the prime circle is larger than the base circle by the radius of the roller.

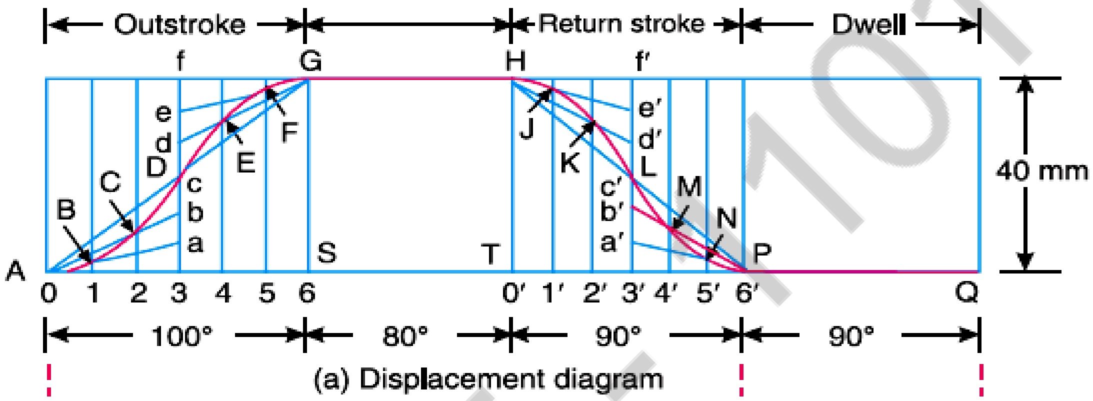
43) What is meant by Tangent cam? What are its applications? (N/D 2018)

When the flanks of the cam are straight and tangential to the base circle and nose circle, the cam is known as Tangent Cam. These cams are usually symmetrical about the centre line of the cam shaft. Such type of cams is used for operating the inlet and exhaust valves of internal combustion engines.

44) Differentiate between radial cam and cylindrical cam. (A/M 2019)

Radial cam	Cylindrical cam
n this cam, the follower reciprocates (or) oscillates in a direction perpendicular to the axis.	n this the follower reciprocates (or) oscillates in a direction parallel to the cam axis.

45) Draw the displacement diagram for a follower when it moves with uniform acceleration and retardation. (A/M 2019)

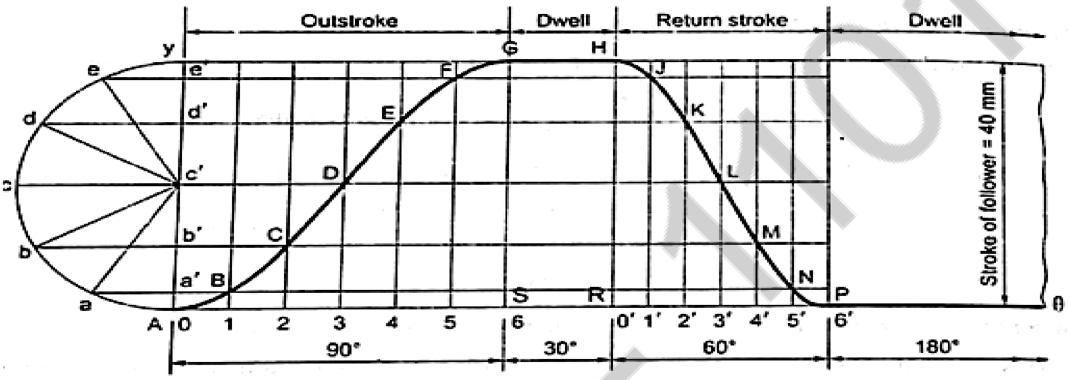


(a) Displacement diagram

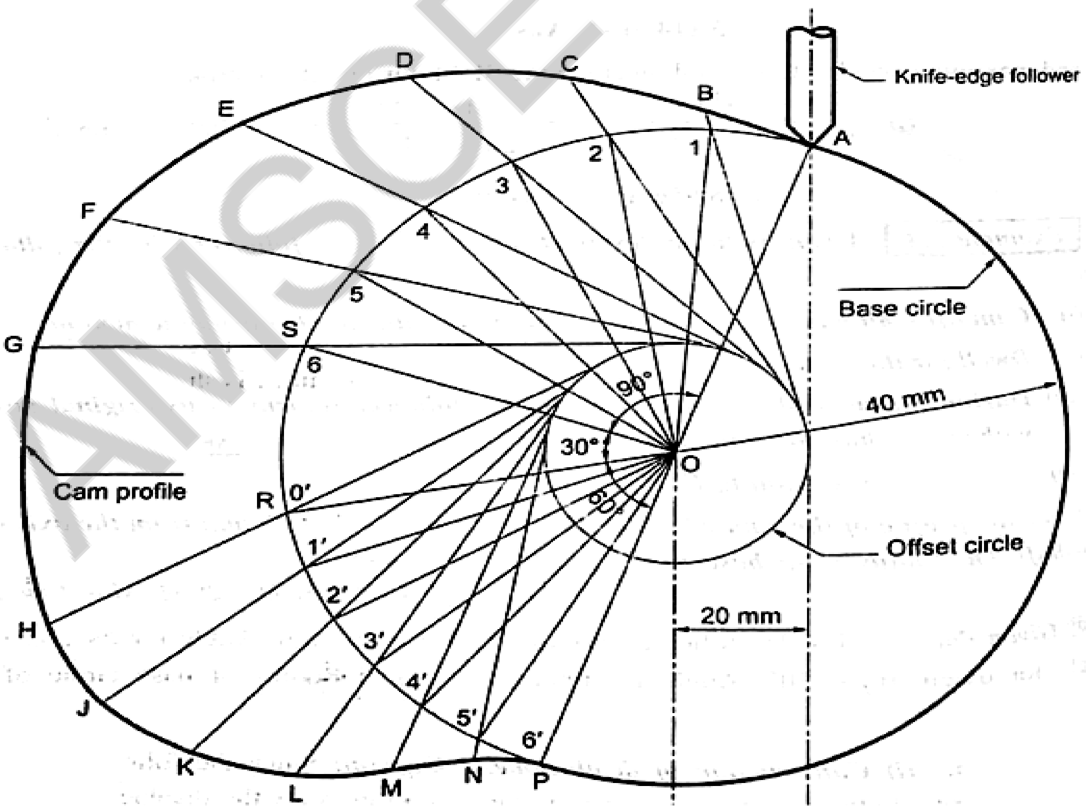
PART-B

1) A cam is designed for a knife edge follower with following data:

- **Cam lift = 40 mm during 90^0 of cam rotation with SHM.**
- **Dwell for next 30^0.**
- **During the next 60^0 of cam rotation, the cam follower returns to original position with SHM.**
- **Dwell for the remaining 180^0.**
- **Draw the profile of the cam when the line of stroke is offset 20 mm from the axis of the camshaft. Radius of base circle of Cam is 40 mm. (16) (MAY/JUNE 2014)**

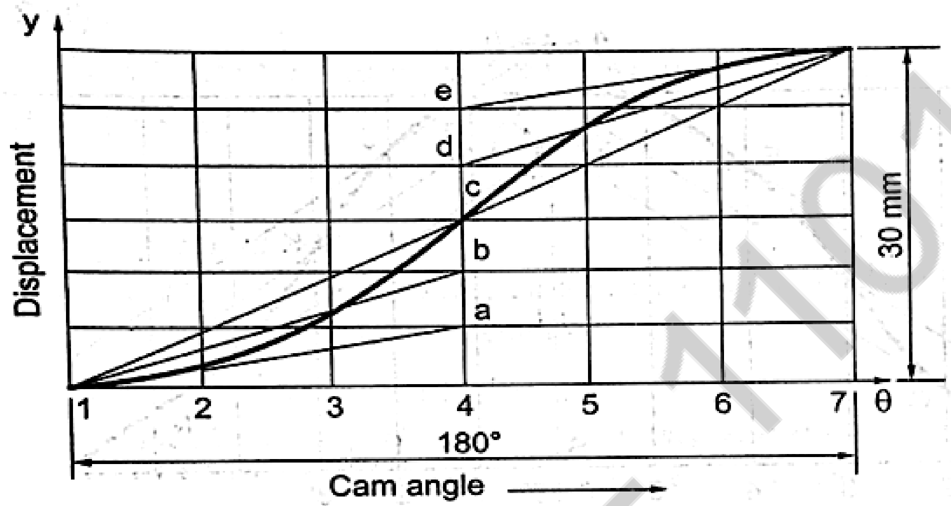


Displacement diagram

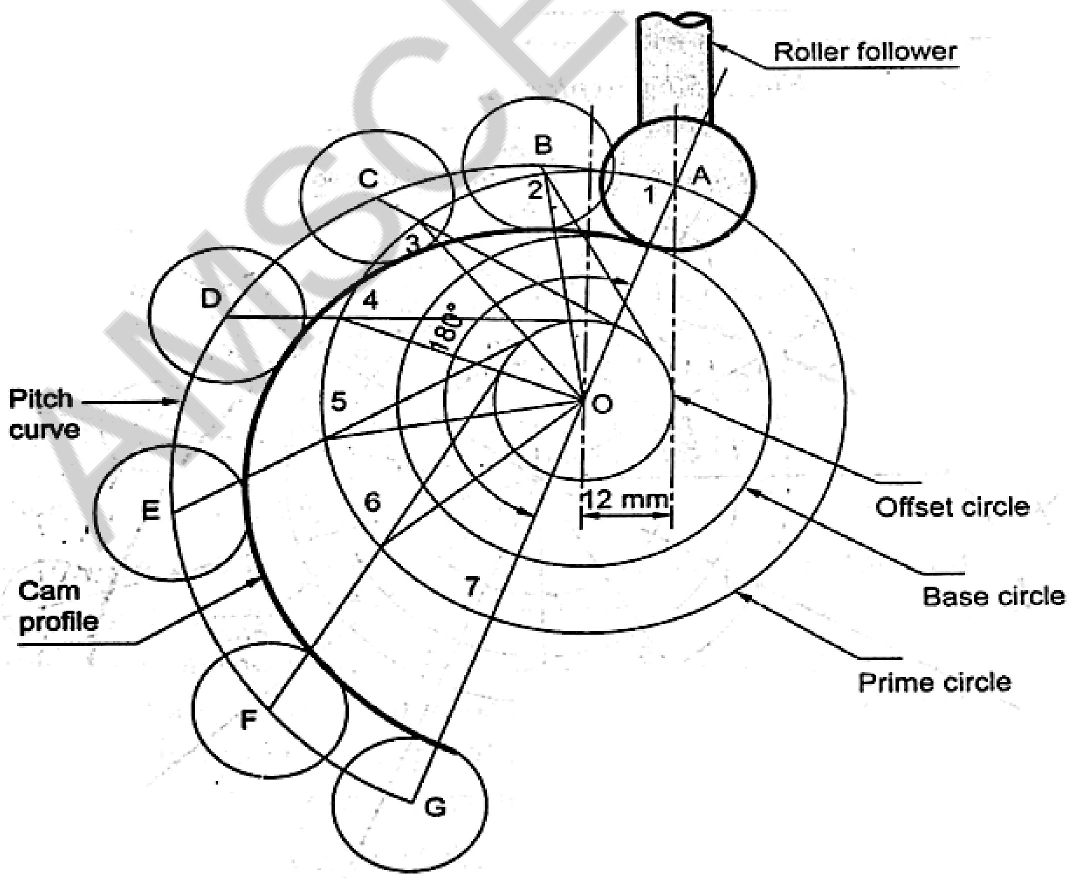


Cam profile

2) In a cam with translating roller follower, the follower axis is right of the cam hinge by 12 mm. The roller radius is 10 mm and the cam rotates in counter-clockwise direction. Layout the rise portion of the cam profile to meet the following specification. Rise takes place during 180⁰ of cam rotation of which for which the first 90⁰, the rise is with constant acceleration and the rest is with constant retardation. Take seven station points only. The lift of the cam is 30 mm and the least radius of the cam is 25 mm. (16) (MAY/JUNE 2014)



Displacement diagram



3) The following particulars relate to a symmetrical circular cam operating a flat faced follower:

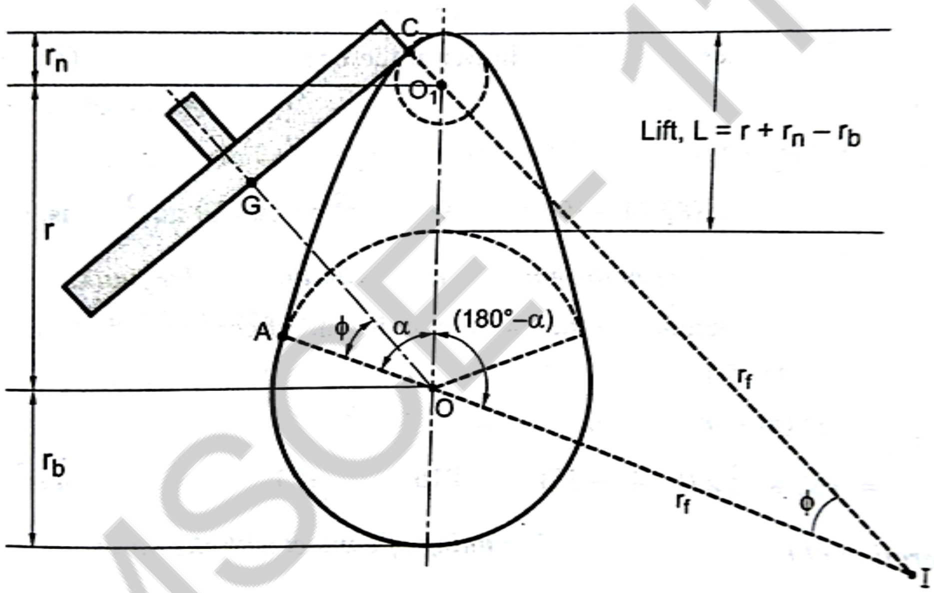
Least radius = 16 mm, nose radius = 3.2 mm, distance between cam shaft centre and nose centre = 25 mm, angle of action of cam = 150°, and cam shaft speed = 600 r.p.m. Assuming that there is no dwell between ascent or descent, determine the lift of the valve, the flank radius and the acceleration and retardation of the follower at a point where circular nose merges into circular flank. (16) (MAY/JUNE 2015)

Given data: Circular arc cam;

$r_b = 16\text{mm};\ r_n = 3.2\text{mm};\ r = 25\text{mm};$ Angle of action, $2\alpha = 150°$ or $\alpha = 75°;\ N = 600\text{rpm}.$

Solution:

(i) Lift of the follower: From the geometry of Fig.



Follower lift, $L = r + r_n - r_b$

$$L = 2.5 + 3.2 - 16 = 12.2\text{mm}$$

(ii) Flank radius: From triangle IOO_1, in Fig. we get

$$(IO_1)^2 = (IO)^2 + (OO_1)^2 - 2 \times IO \times OO_1 \times \cos(180° - \alpha)$$

$$(r_f - r_n)^2 = (r_f - r_b)^2 + r^2 - 2(r_f - r_b) \times r \times \cos(180° - \alpha)$$

$$(r_f - 3.2)^2 = (r_f - 16)^2 + 25^2 - 2(r_f - 16) \times 25 \times \cos(180° - 75°)$$

$$12.66 r_f = 663.71$$

Flank radius, $r_f = 52.42\text{mm}$

(iii) Acceleration and retardation of follower at a point where circular nose merges into circular flank:

First let us find angle of action ϕ . Consider a triangle IOO_1 in Fig.. Applying sine rule to triangle IOO_1 , we get

$$\frac{OO_1}{\sin\phi} = \frac{IO_1}{\sin(180° - \alpha)}$$

$$\frac{r}{\sin\phi} = \frac{r_f - r_n}{\sin(180° - \alpha)}$$

$$\sin\phi = \frac{r \times \sin(180° - \alpha)}{r_f - r_n} = \frac{25 \times \sin(180° - 75°)}{52.42 - 3.2} = 0.49$$

Angle of action, $\phi = 29.38°$

Angular velocity of cam, $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 600}{60} = 62.83\text{rad/s}$

We know that the acceleration of the follower at the point where circular nose merges into circular flank (i.e., at point C) when $\theta = \phi$,

$$a_{min} = \omega^2 (r_f - r_b)\cos\phi$$

$$= (62.83)^2 (52.42 - 16)\cos 29.38° = 125280.7\,\text{mm/s}^2 = 125.25\,\text{m/s}^2$$

Also, we know that the retardation of the follower at point C when $\theta = \phi$,

$$a_{min} = -\omega^2 r\cos(\alpha - \phi)$$

$$= -(62.83)^2 \times 25 \times \cos(75° - 29.38°) = -69025.3\,\text{mm/s}^2$$

$$a_{min} = 69.02\,\text{m/s}^2\,(\text{retardation})$$

4) A cam with 30 mm as minimum diameter is rotating clockwise at a uniform speed of 1200 r.p.m. and has to give the following motion to a roller follower 10 mm in diameter:

(i) Follower to complete outward stroke of 25 mm during 120° of cam rotation with equal uniform acceleration and retardation;

(ii) Follower to dwell for 60° of cam rotation;

(iii) Follower to return to its initial position during 90° of cam rotation with equal uniform acceleration and retardation;

(iv) Follower to dwell for the remaining 90° of cam rotation.

Draw the cam profile if the axis of the roller follower passes through the axis of the cam. Determine the maximum velocity of the follower during the outstroke and return stroke and also the uniform acceleration of the follower on the out stroke and the return stroke.

(16) (MAY/JUNE 2015)

Solution: Given data: $r_b = \frac{30}{2} = 15\text{mm},\ N = 1200\text{r.p.m},$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1200}{60} = 125.663\text{rad/s},$$

Given:

diameter of roller $= 10\text{mm},\ \theta_O = 120°,\ \theta_D = 60°,\ \theta_R = 90°,\ S = 25\text{mm}$

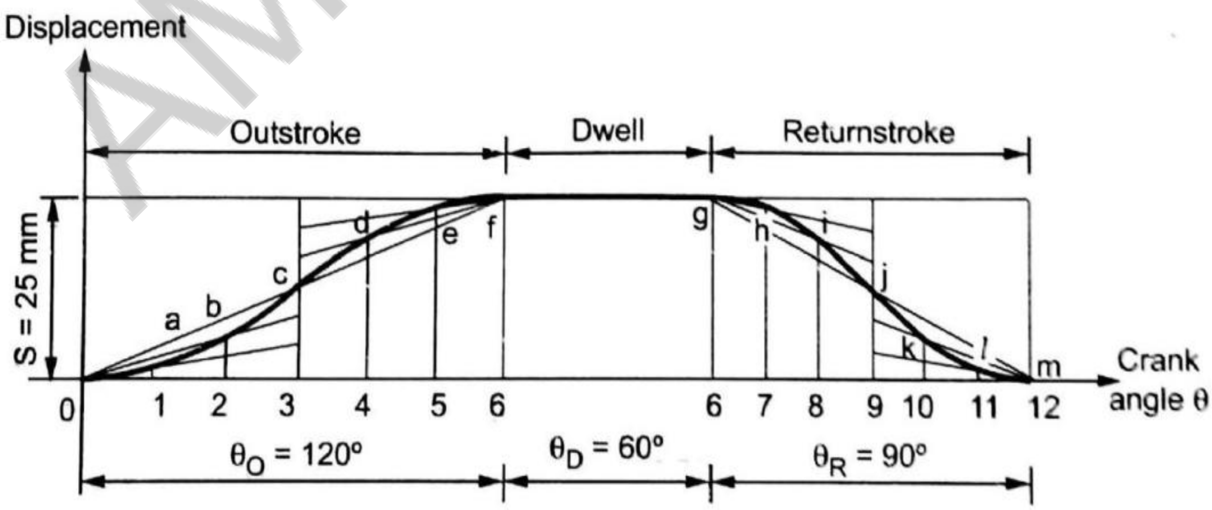
Motion type = Uniform acceleration and retardation for both outstroke and return stroke.

To find: i) Draw the cam profile

ii) Maximum velocity and acceleration of follower during return stroke.

Soln:

Step – 1: Draw the displacement diagram and required cam profile.



Step – 2: Draw a cam profile

Step – 3: Calculate the maximum velocity and acceleration of follower during return stroke.

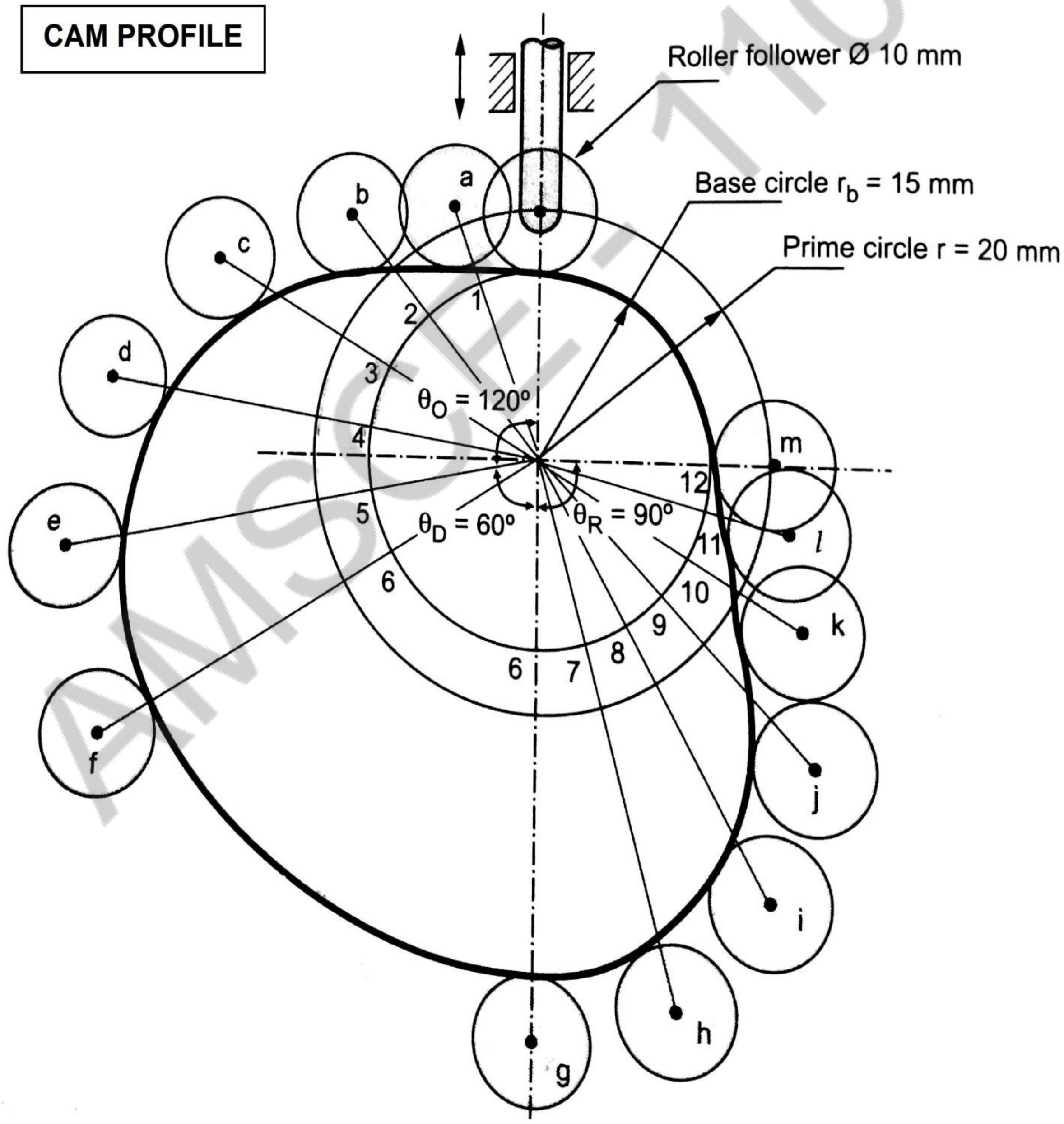
Follower is moving return with uniform acceleration and retardation

Maximum velocity of follower is,

$$V_{R(max)} = \frac{2S}{\theta_R} \times \omega = -\frac{2 \times 25 \times 10^{-3}}{90 \times \frac{\pi}{180}} \times 125.663 = -3.999\,m/s$$

Maximum acceleration of follower is,

$$a_{R(max)} = \mp \frac{4S}{\theta_R^2} \times \omega^2 = \mp \frac{4 \times 25 \times 10^{-3}}{\left(90 \times \frac{\pi}{180}\right)^2} \times 125.663^2 = 639.992\,m/s^2$$



5) Draw the displacement, velocity and acceleration curves, when the follower moves with simple harmonic motion and derive the expression for maximum velocity and maximum acceleration. (10)

(MAY/JUNE 2016)

Maximum velocity of follower during outward and return strokes:

Since the variation in velocity is a sine curve, therefore the maximum velocity of follower during outward stroke occurs at $\theta = \frac{\theta_0}{2}$

Therefore, substituting $\theta = \frac{\theta_0}{2}$ in equation, we get

$$(v_0)_{max} = \frac{\pi L \omega}{2\theta_0}\left[\sin \frac{\pi}{\theta_0} \times \frac{\theta_0}{2}\right] = \frac{\pi L \omega}{2\theta_0} \times \sin \frac{\pi}{2}$$

$$(v_0)_{max} = \frac{\pi L \omega}{2\theta_0} \qquad \left[\because \sin \frac{\pi}{2} = 1\right]$$

Similarly, the maximum velocity of follower during return stroke occurs at $\theta = \frac{\theta_r}{2}$ and is given by

$$(v_r)_{max} = \frac{\pi L \omega}{2\theta_r}$$

Since the variation in acceleration is a cosine curve, therefore the maximum acceleration of follower during outward stroke occurs at $\theta = 0$ and $\theta = \theta_0$.

Therefore, substituting $\theta = 0$ and $\theta = \theta_0$ in equation, we get

$$(a_o)_{max} = \frac{\pi^2 L \omega^2}{2\theta_0^2}\left[\cos \frac{\pi}{\theta_0} \times 0\right] = \frac{\pi^2 L \omega^2}{2\theta_0^2} \qquad [\because \cos 0 = 1]$$

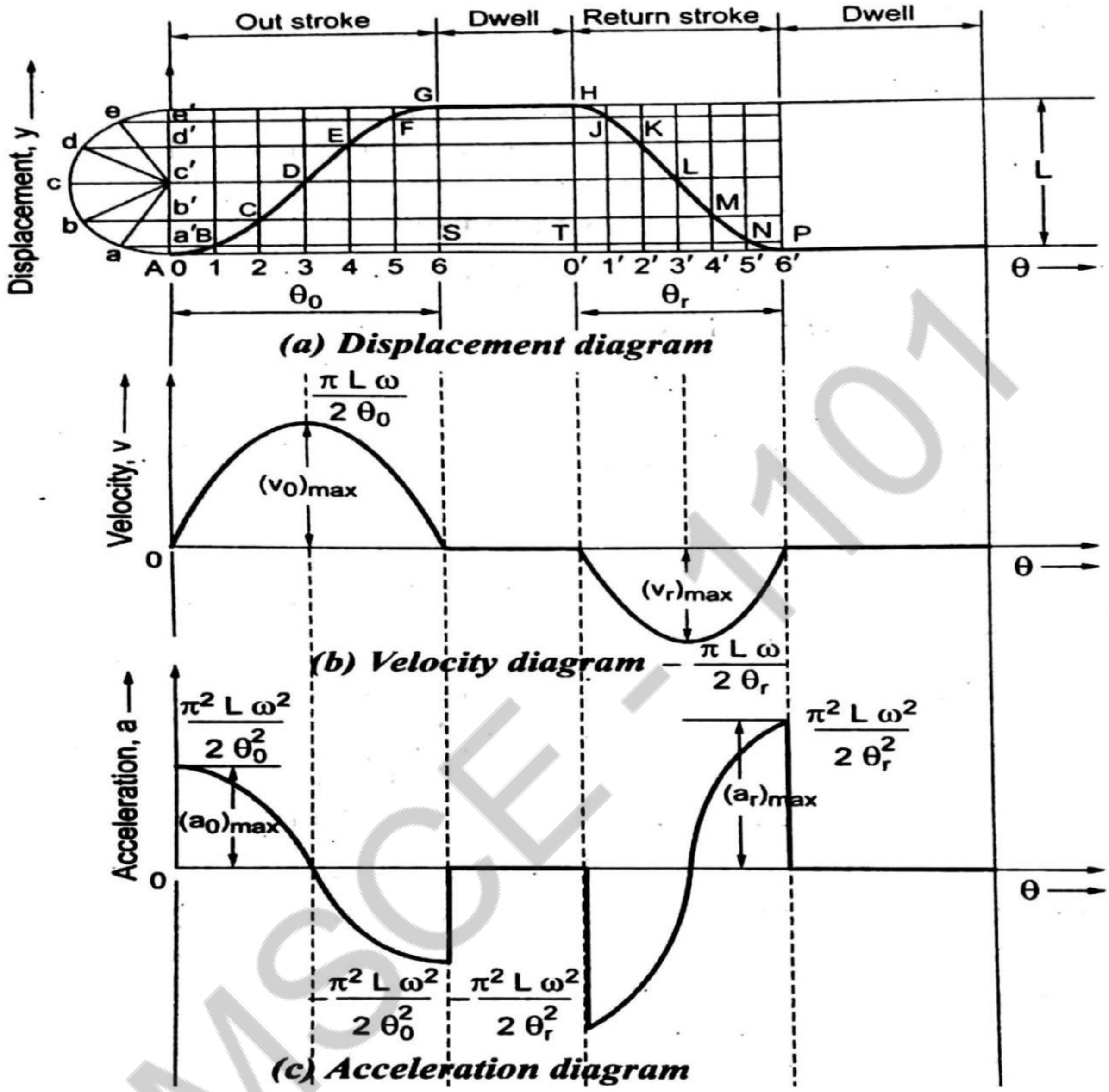
and $$(a_o)_{max} = \frac{\pi^2 L \omega^2}{2\theta_0^2}\left[\cos \frac{\pi}{\theta_0} \times 0\right] = \frac{-\pi^2 L \omega^2}{2\theta_0^2} \qquad [\because \cos \pi = -1]$$

$$\therefore \quad (a_o)_{max} = \pm \frac{\pi^2 L \omega^2}{2\theta_0^2}$$

Similarly, the maximum velocity of follower during return stroke occurs at $\theta = 0$ and $\theta = \theta_r$, and is given by

$$(a_r)_{max} = \mp \frac{\pi^2 L \omega^2}{2\theta_r^2}$$

DISPLACEMENT, VELOCITY AND ACCELERATION DIAGRAM (SHM):

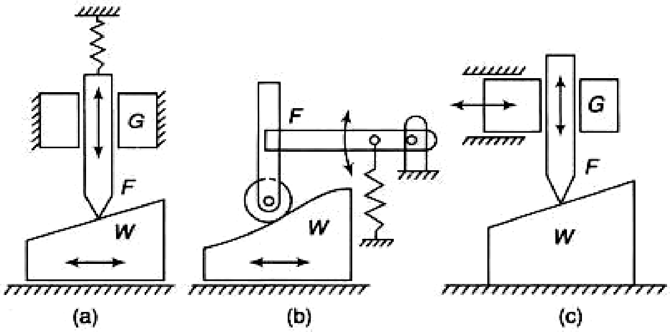


(ii) Depict the types of Cams. **(6)**

(MAY/JUNE 2016)

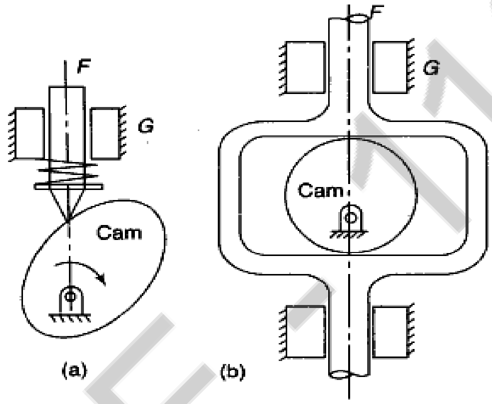
► **Wedge and Flat Cams:**

- A wedge cam has a wedge W which, in general, has a translational motion
- The follower F can either translate [Fig.(a)] or oscillate [Fig.(b)]
- A spring is, usually, used to maintain the contact between the cam and the follower
- In Fig.(c), the cam is stationary and the follower constraint or guide G causes the relative motion of the cam and the follower



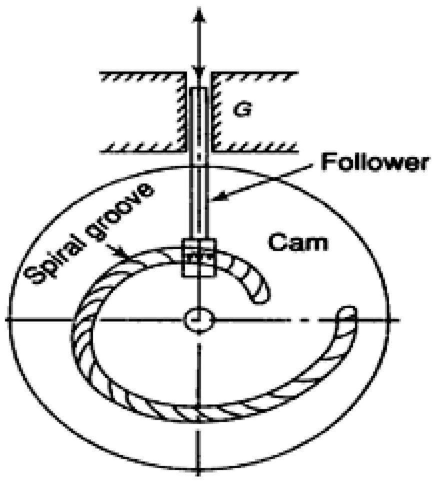
► **Radial or Disc Cams:**

- A cam in which the follower moves radially from the centre of rotation of the cam is known as a radial or a disc cam (Fig. (a) and (b)]
- Radial cams are very popular due to their simplicity and compactness



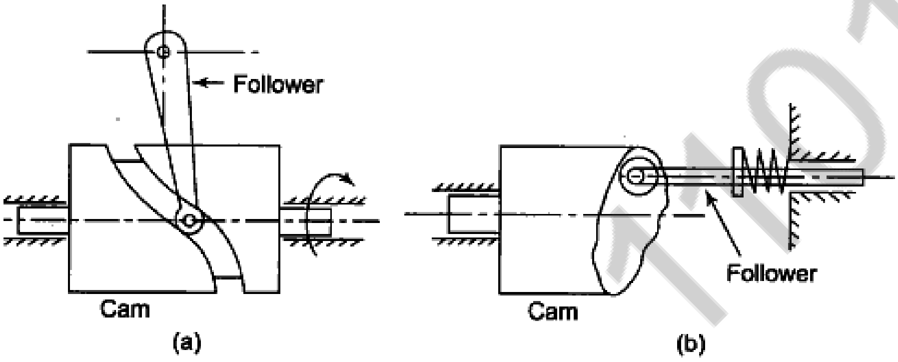
► **Spiral Cams:**

- A spiral cam is a face cam in which a groove is cut in the form of a spiral as shown in Fig.
- The spiral groove consists of teeth which mesh with a pin gear follower
- The velocity of the follower is proportional to the radial distance of the groove from the axis of the cam
- The use of such a cam is limited as the cam has to reverse the direction to reset the position of the follower. It finds its use in computers



► **<u>Cylindrical Cams</u>:**

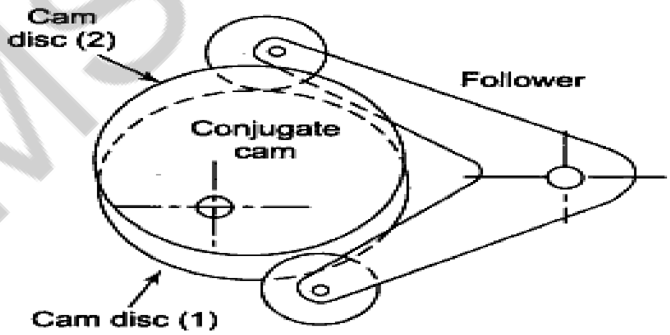
- In a cylindrical cam, a cylinder which has a circumferential contour cut in the surface, rotates about its axis
- The follower motion can be of two types as follows: In the first type, a groove is cut on the surface of the cam and a roller follower has a constrained (or positive) oscillating motion [Fig.(a)]
- Another type is an end cam in which the end of the cylinder is the working surface (b)
- A spring-loaded follower translates along or parallel to the axis of the rotating cylinder



► **<u>Conjugate Cams</u>:**

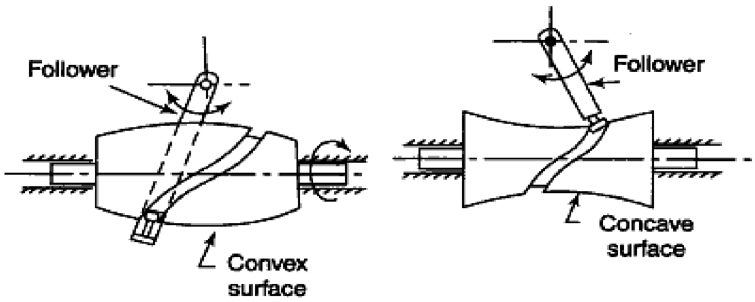
- A conjugate cam is a double-disc cam, the two discs being keyed together and are in constant touch with the two rollers of a follower (shown in Fig.)
- Thus, the follower has a positive constraint

Such a type of cam is preferred when the requirements are low wear, low noise, better control of the follower, high speed, high dynamic loads, etc.



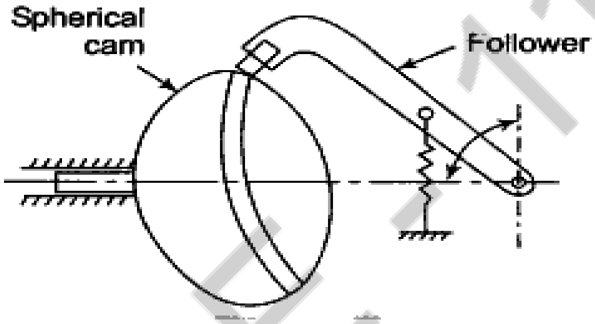
► **Globoidal Cams:**

- A globoidal cam can have two types of surfaces, convex or concave
- A circumferential contour is cut on the surface of rotation of the cam to impart motion to the follower which has an oscillatory motion (Fig.)
- The application of such cams is limited to moderate speeds and where the angle of oscillation of the follower is large



► **Spherical Cams:**

- In a spherical cam, the follower oscillates about an axis perpendicular to the axis surface of rotation of the cam
- Note that in a disc cam, the follower oscillates about an axis parallel to the axis of rotation of the cam
- A spherical cam is in the form of a spherical surface which transmits motion to the follower (Fig.)



6) Follower type = roller follower, lift = 25 mm; base circle radius = 20 mm; roller radius = 5 mm; out stroke with UARM, for 120° cam rotation; dwell for 60° cam rotation; return stroke with UARM, 90° cam rotation; dwell for the remaining period. Determine max. velocity and acceleration during out stroke and return stroke if the cam rotates at 1200 rpm in counter clockwise direction.
Draw the cam profile for conditions with follower off set to right of cam center by 5 mm. (16)

(MAY/JUNE 2016)

Given:

Lift $L = 25mm = 0.025m$

$\theta_0 = 120° = 120 \times \pi/180 = 2.094 rad.$

$\theta_r = 90° = 90 \times \pi/180 = 1.571 rad.$

$N = 1200 rpm (CCW)$

Solution:

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1200}{60} = 125.66\,\text{rad/sec}$$

Maximum velocity during outward and return stroke:

$$(V_0)_{max} = \frac{2L\omega}{\theta_0} = \frac{2 \times 0.025 \times 125.66}{2.094} = 3\,\text{m/s}$$

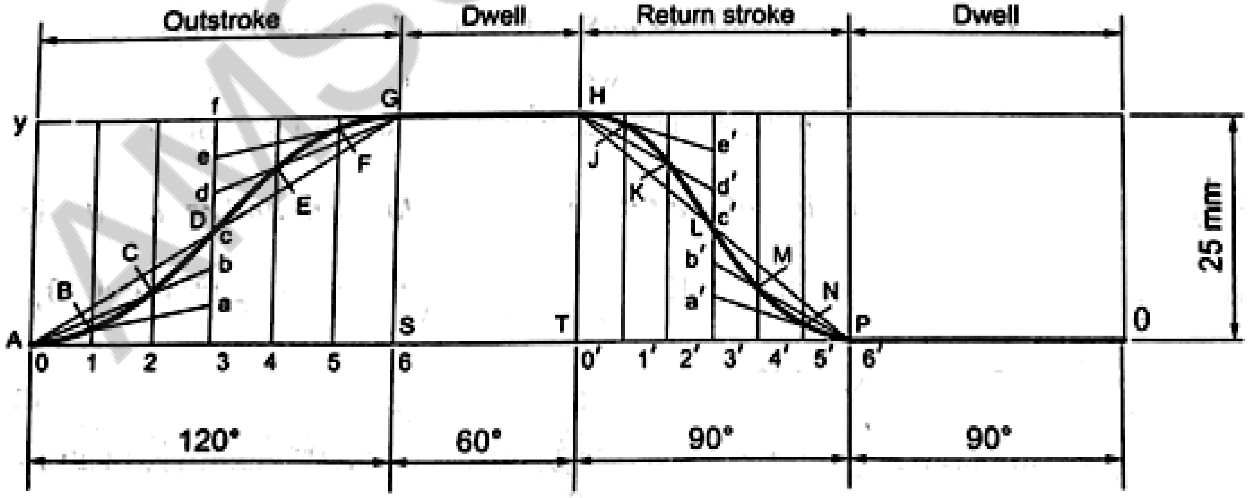
$$(V_r)_{max} = \frac{-2L\omega}{\theta_r} = \frac{-2 \times 0.025 \times 125.66}{1.571} = -4\,\text{m/s}$$

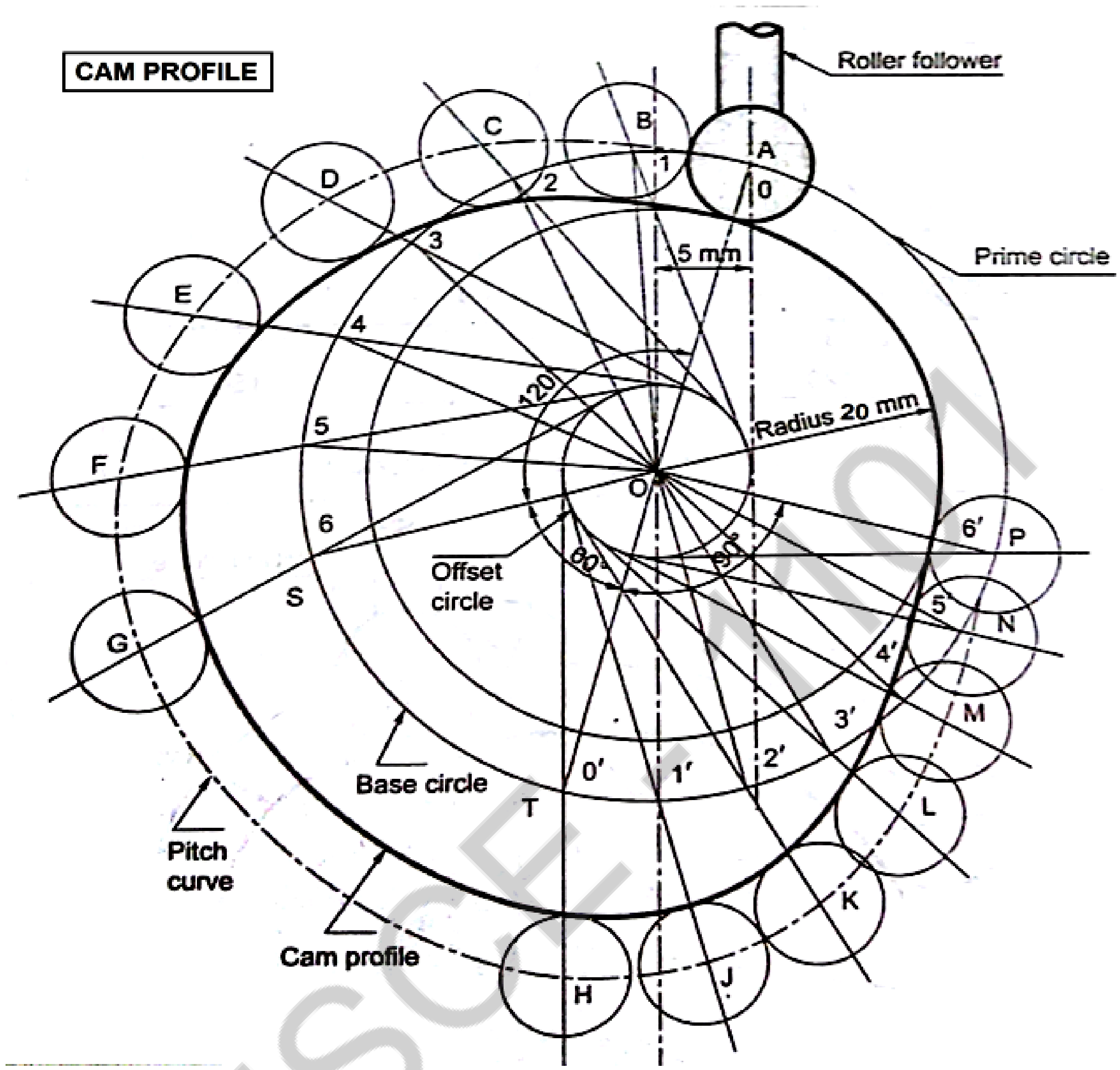
Maximum acceleration during outward and return stroke:

$$(a_0)_{max} = \frac{\pm 4L\omega^2}{(\theta_0)^2} = \pm\frac{4 \times 0.025 \times (125.66)^2}{(2.094)^2} = \pm 360.11\,\text{m/s}^2$$

$$(a_r)_{max} = \pm\frac{4L\omega^2}{(\theta_r)^2} = \pm\frac{4 \times 0.025 \times (125.66)^2}{(1.571)^2} = \pm 639.79\,\text{m/s}^2$$

DISPLACEMENT DIAGRAM

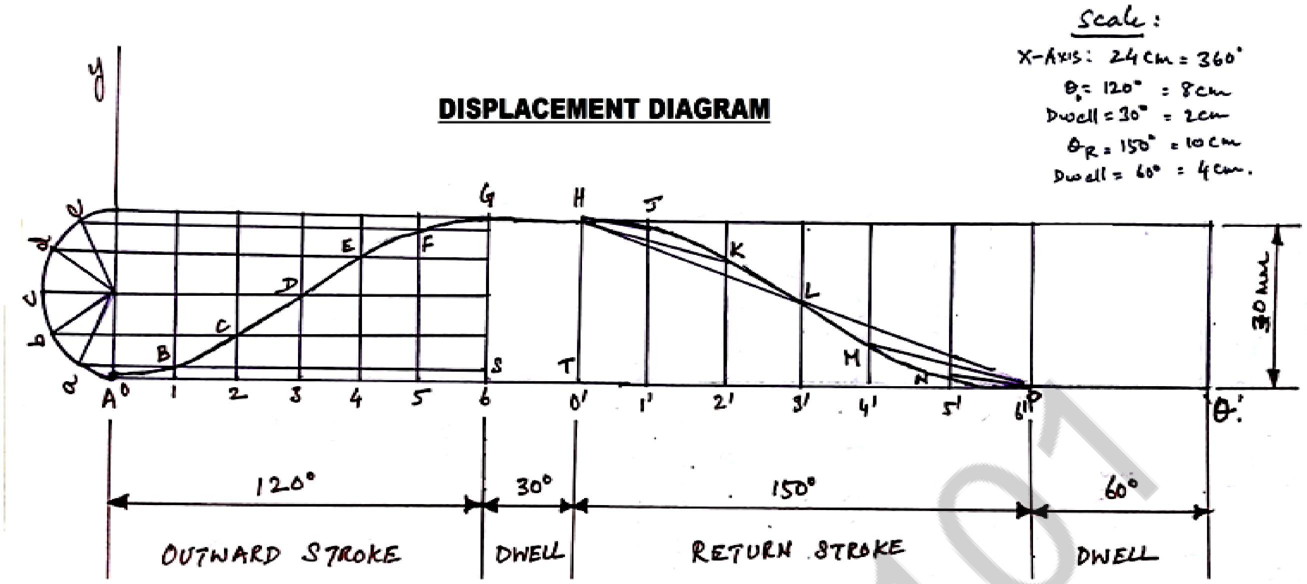




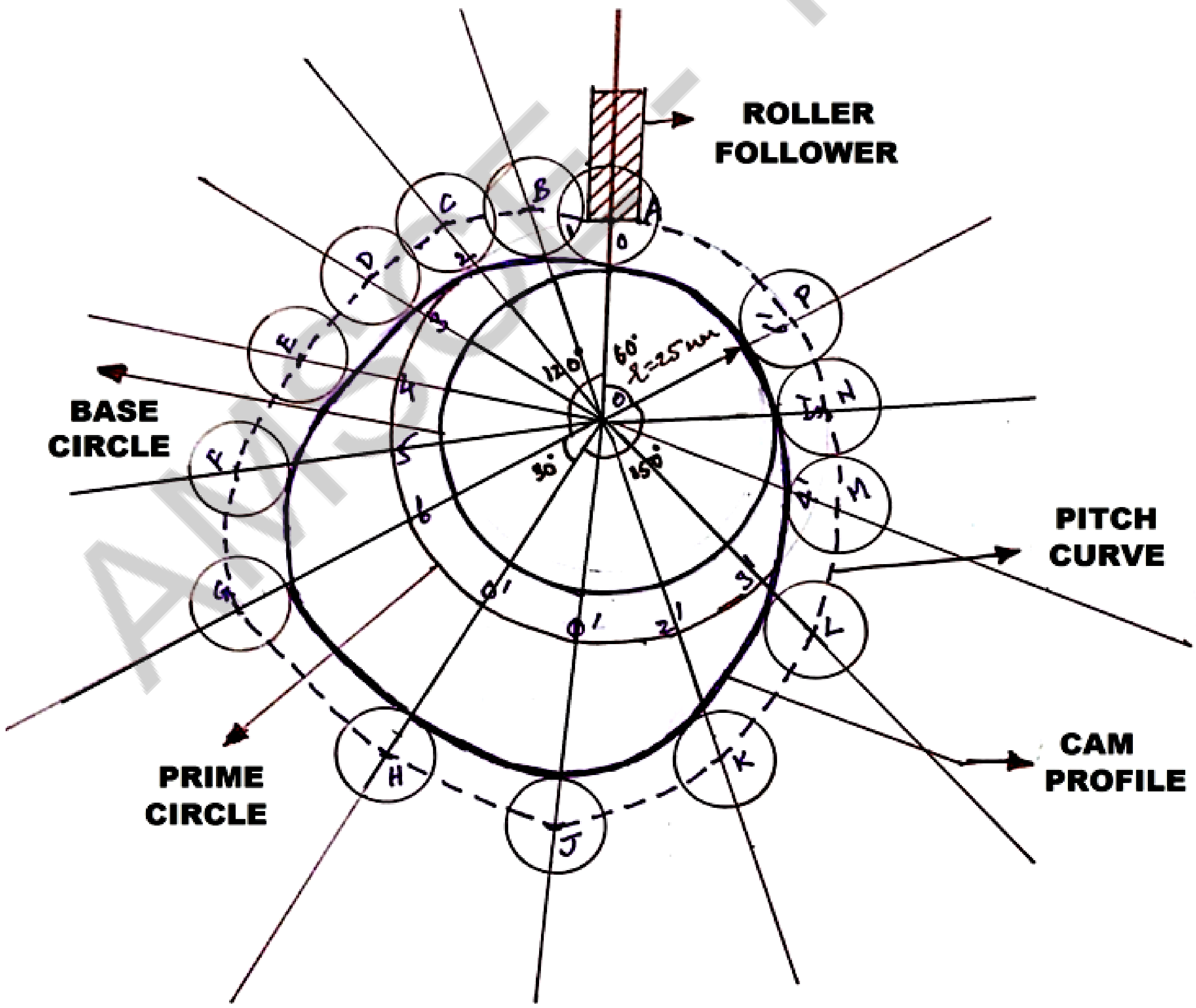
7) Draw the profile of a cam operating a roller reciprocating follower with the following data: Minimum radius of cam = 25 mm, Lift = 30 mm and Roller diameter = 15 mm. The cam lifts the follower for 120° with SHM followed by a dwell period of 30°. Then the follower lowers down during 150° of the cam rotation with uniform acceleration and deceleration followed by a dwell period. If the cam rotates at a uniform speed of 150 rpm. Calculate the maximum velocity and acceleration of the follower during the descent period. (16)

(NOV/DEC 2014)

DISPLACEMENT DIAGRAM



CAM PROFILE



8) The following data relate to a cam profile in which the follower moves with uniform acceleration and deceleration during ascent and descent. Minimum radius of cam = 25 mm, roller diameter = 7.5 mm, lift = 28 mm, offset of follower axis = 120 mm towards right, angle of ascent = 60°, angle of descent = 90°, angle of dwell between ascent and descent = 45° and speed of the cam = 200 rpm. Draw the profile of the cam and determine the maximum velocity and the uniform acceleration of the follower during the out stroke and the return stroke. (16)

(NOV/DEC 2014)

Given: $L = 28\text{mm} = 0.028\text{m};\ \theta_o = 60° \times (\pi/180°) = 1.047\text{rad};$

$\theta_r = 90° \times (\pi/180°) = 1.571\text{rad};\ N = 200\text{rpm}.$

Solution:

We know that angular velocity of the cam,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 200}{60} = 20.94\,\text{rad/s}$$

Maxium velocity of follower during outward and return strokes:

We know that maximum velocity of follower havng UARM during outward stroke,

$$(v_o)_{max} = \frac{2L\omega}{\theta_o} = \frac{2 \times 0.028 \times 20.94}{1.047} = 1.12\,\text{m/s}$$

and maximum velocity of follower having UARM during return stroke,

$$(v_r)_{max} = -\frac{2L\omega}{\theta_r} = -\frac{2 \times 0.028 \times 20.94}{1.571} = -0.746\,\text{m/s}$$

Maximum acceleration of follower during outward and return strokes:

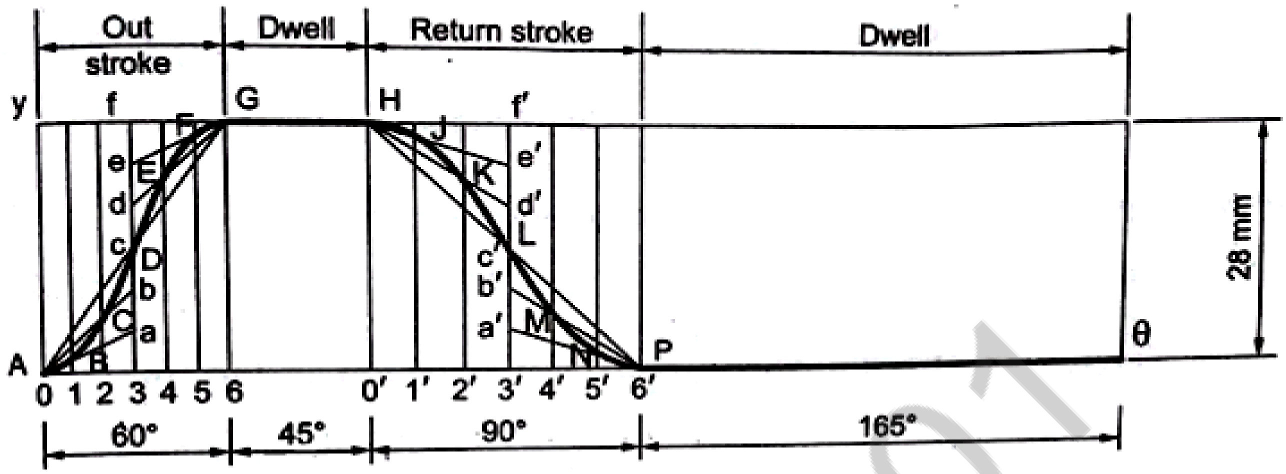
We know that maximum acceleration of follower having UARM during outward strokes,

$$(a_0)_{max} = \pm\frac{4L\omega^2}{\theta_o^2} = \pm\frac{4 \times 0.028 \times (20.94)^2}{(1.047)^2} = \pm 44.8\,\text{m/s}^2$$

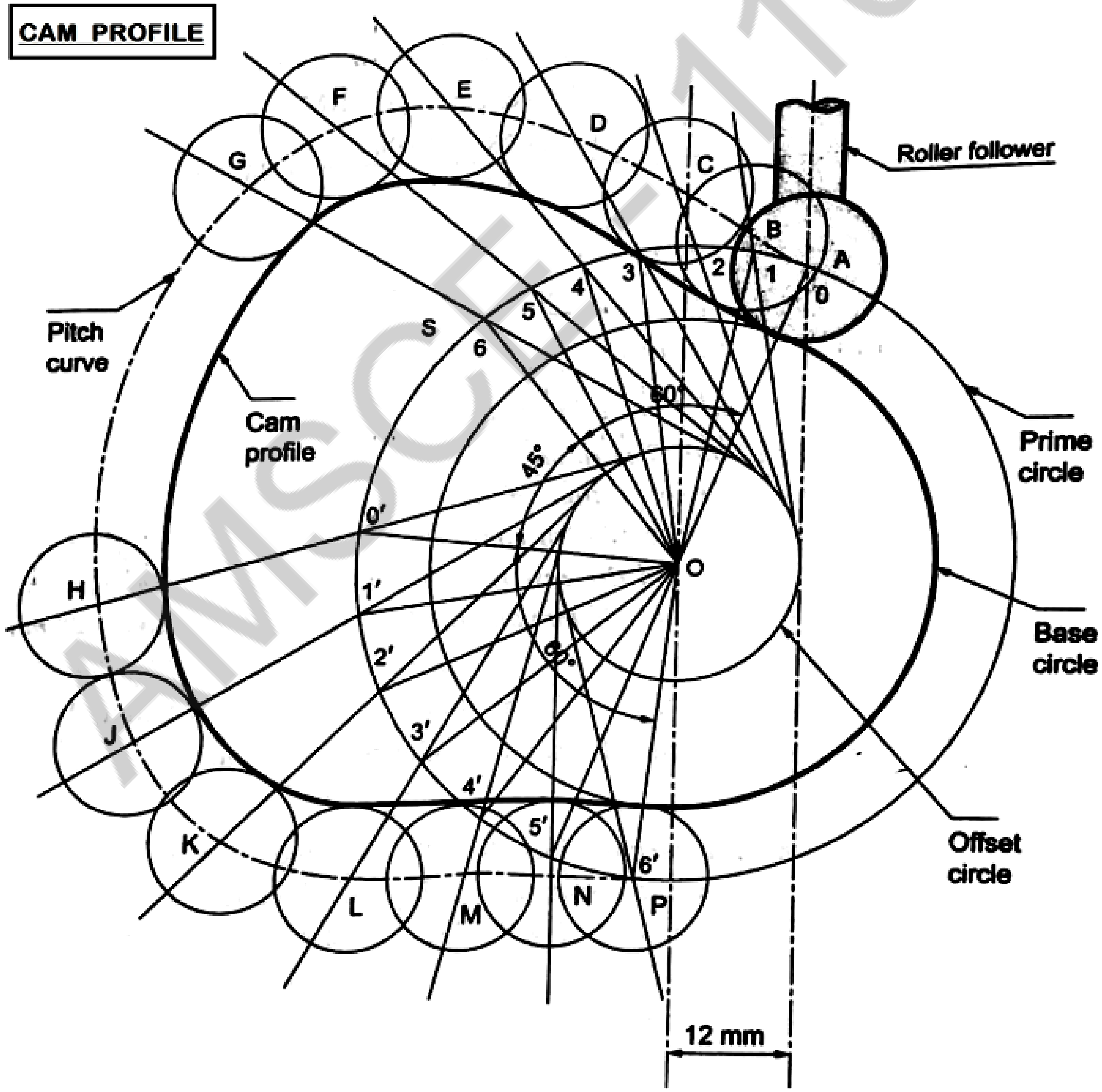
and maximum acceleration of follower having UARM during return stroke,

$$(a_r)_{max} = \pm\frac{4L\omega^2}{\theta_r^2} = \mp\frac{4 \times 0.028 \times (20.94)^2}{(1.571)^2} = \mp 31.28\,\text{m/s}^2$$

DISPLACEMENT DIAGRAM



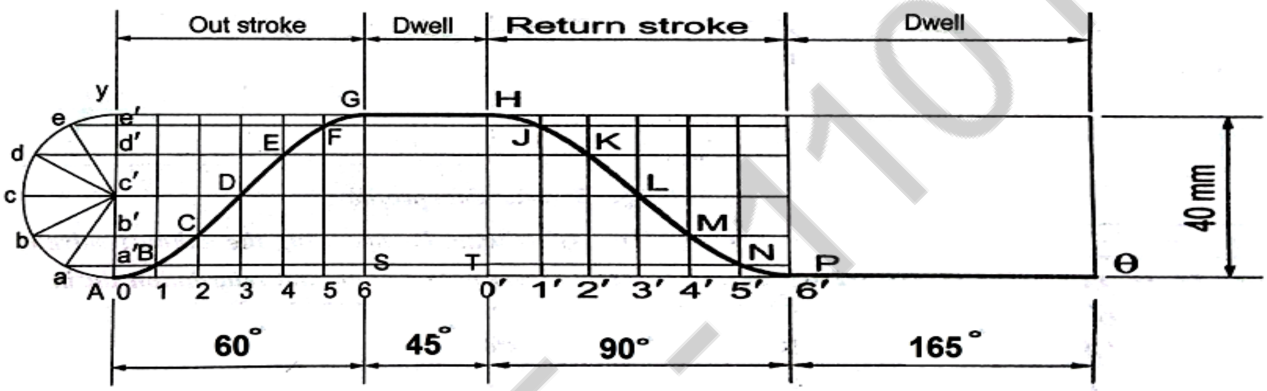
CAM PROFILE



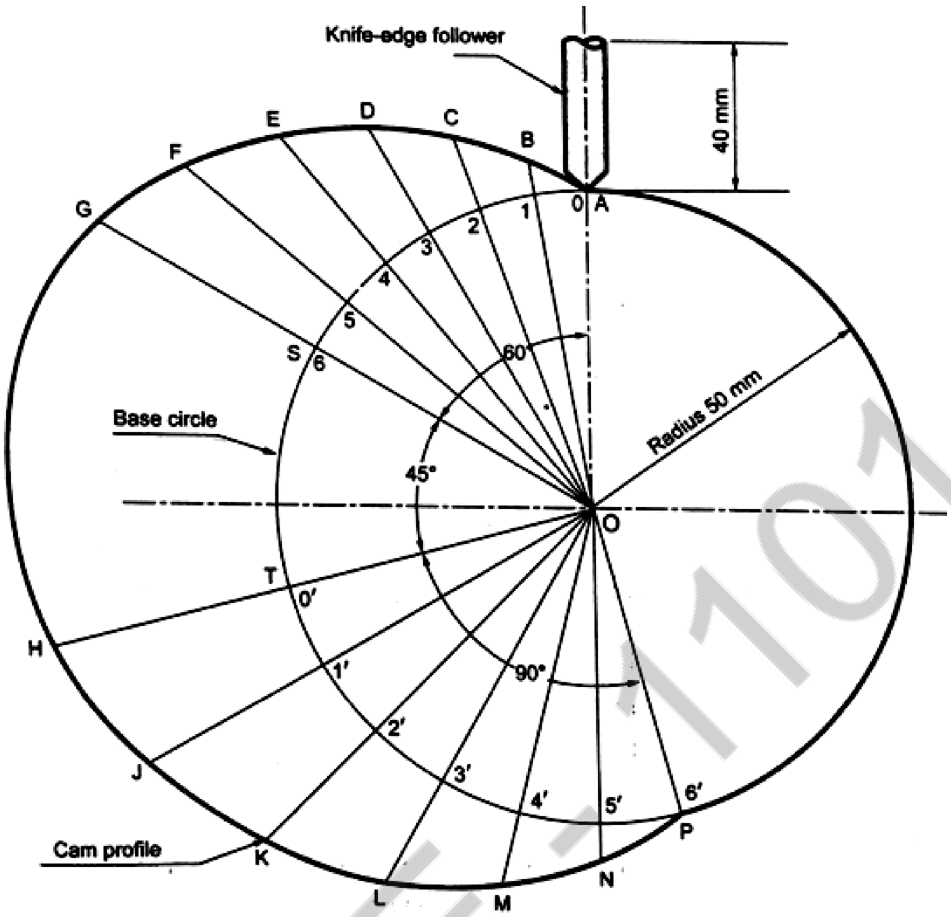
9) Draw the profile of a cam operating a knife edge follower when the axis of the follower passes through the axis of cam shaft from the following data:

(i) Follower to move outwards through 40 mm during 60° of cam rotation, (ii) Follower to dwell for the next 45°, (iii) Follower to return to its original position during next 90°, (iv) Follower to dwell for the rest of the cam rotation. The displacement of the follower is to take place with simple harmonic motion during both the outward and return strokes. The least radius of cam is 50 mm. (16) (NOV/DEC 2015)

DISPLACEMENT DIAGRAM



CAM PROFILE:



10) Draw the profile of a cam operating a knife-edge follower (when the axis of the follower passes through the axis of cam shaft) from the following data:

(i) Follower to move outward through 30 mm with Simple Harmonic motion during 120° of cam rotation,

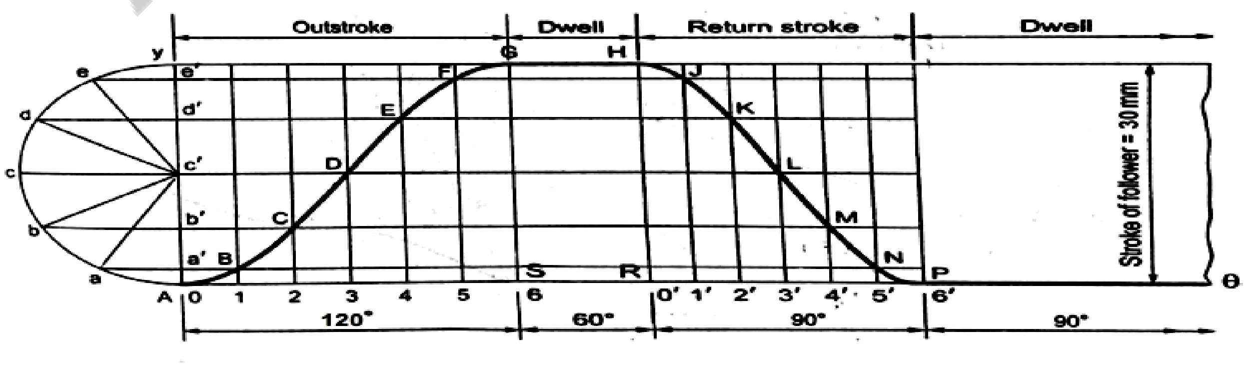
(ii) Follower to dwell for the next 60°,

(iii) Follower to return to its original position with uniform velocity during 90° of cam rotation.

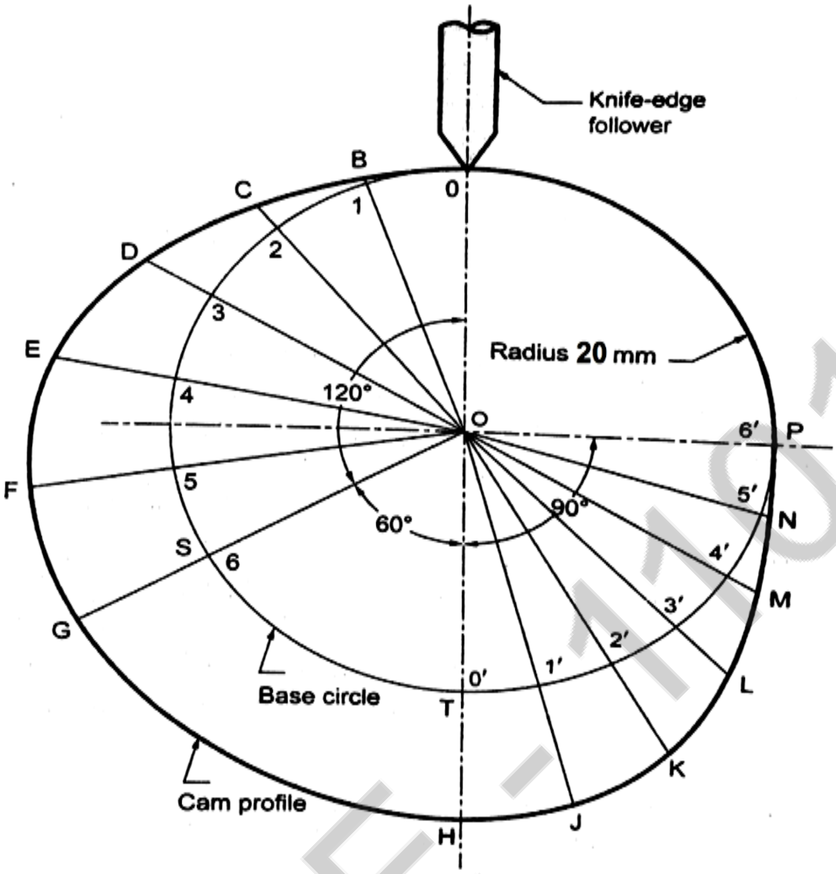
(iv) Follower to dwell for the rest of the cam rotation. The least radius of cam is 20 mm and the cam rotates at 240 rpm. (16)

(NOV/DEC 2015)

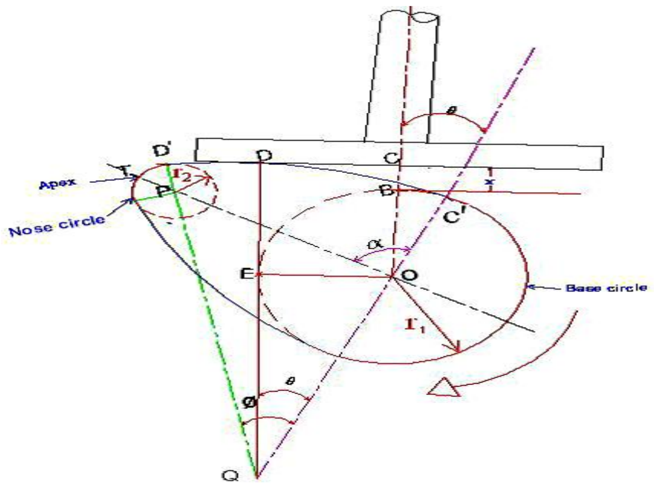
DISPLACEMENT DIAGRAM:



CAM PROFILE:



11) Expression for determining the displacement, velocity acceleration, of the follower when the flat face of the follower has conduct on the circular flank?



Let

r_1=OB=Least base circle radius
r_2=Nose circle radius
R=QD= Flank circle radius
d=Distance between the centres of cam and nose circles
α= Angle of ascent
ϕ=Angle of contact on circular flank

Displacement:

$$X = BC = OC - OB = DE - r_1$$
$$= (QD\text{-}QE) - r_1$$
$$= (R - OQ\cos\theta) - r_1$$
$$= R-(R - r_1)\cos\theta - r_1$$

$$x = (R-r_1)(1-\cos\theta)$$

Velocity:

$$v = \frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt}$$
$$= (R-r_1)(\sin\theta)\,\omega$$

$$V = \omega\,(R-r_1)\sin\theta$$

Acceleration:

$$a = \frac{dv}{dt} = \frac{dv}{d\theta} \times \frac{d\theta}{dt}$$

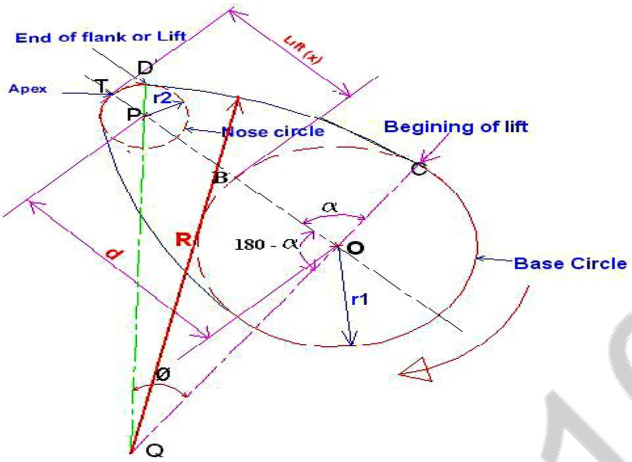
$$a = \omega^2 (R - r_1)\cos\theta$$

It is obvious from the above equation that, at the beginning of the ascent when θ=0, acceleration is maximum and it goes on decreasing and is maximum when θ=φ

$$a_{max} = \omega^2 (R - r_1)$$

$$a_{min} = \omega^2 (R - r_1)\cos\varphi$$

12) The following particulars relate to a symmetrical circular cam operating a flat faced follower.
Least radius = 16 mm; Nose radius = 3.2 mm; Distance between cam shaft centre and nose centre = 25 mm; Angle of action of cam = 150° and cam shaft speed = 600 rpm.
Assuming that, there is no dwell between the ascent and descent, determine the lift of the valve, the flank radius and the acceleration and retardation of the follower at a point where circular nose merges into circular flank.
Solution :
r_1 = 16 mm, r_2 = 3.2 mm , OP = d = 25mm, 2α = 150°, α = 75°, N = 600 rpm



We know, (i)Lift = BT = OT- OB

= OP+PT – OT

= $d + r_2 - r_1$

= 25 + 3.2 -16

Lift =x = 12.2 mm

(ii) Flank radius, R,

$$R = \frac{r_1^2 - r_2^2 + d^2 - 2r_1.d.Cos\,\alpha}{2(r_1 - r_2 - d\cos\alpha)}$$

$$R = \frac{16^2 - 3.2^2 + 25^2 - 2x16x25x\cos 75}{2(16 - 3.2 - 25\cos 75)}$$

R = 52.82 mm

(iii) Flank angle, ϕ

We have, from triangle OQP,

$$\frac{PO}{\sin\phi} = \frac{PQ}{\sin(180-\alpha)}$$

$$\sin\phi = \frac{\sin(180-75)\,25}{(52.82-3.2)}$$

$\phi = 29°.6'$

(iv) Acceleration at the end of the contract with flank, when $\theta = \phi = 29°.6'$

$a = \omega^2 (R - r_1)\cos\varphi$

$$= \left(\frac{2\pi x 600}{60}\right)^2 (52.82 - 16)10^{-3} x \cos 29.6$$

$= 129.39\ m/s^2$

(v)Retardation at the beginning of contact with nose,

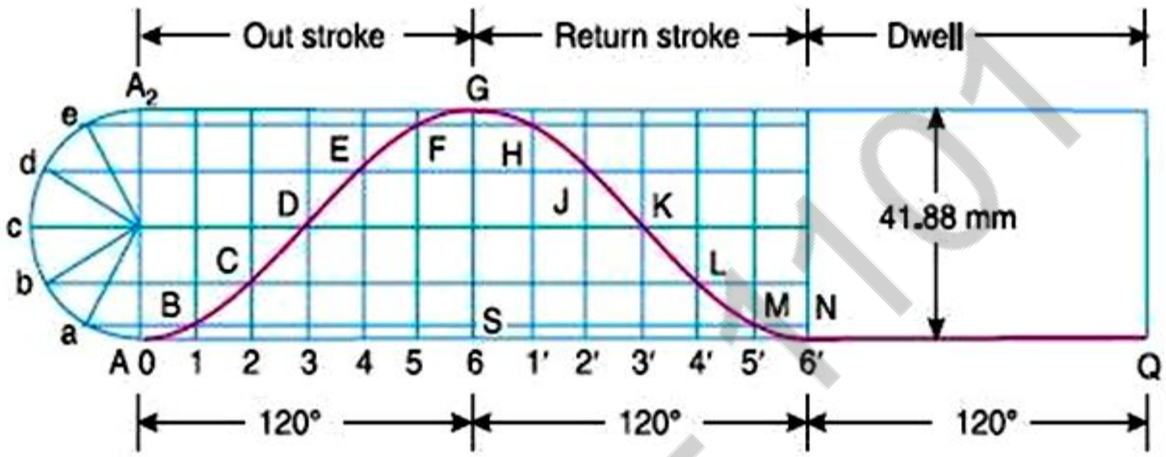
Acceleration, $a = -\omega^2 d\cos\varphi$

$$= -\left(\frac{2\pi x 600}{60}\right)^2 (25)10^{-3} x \cos 29.6$$

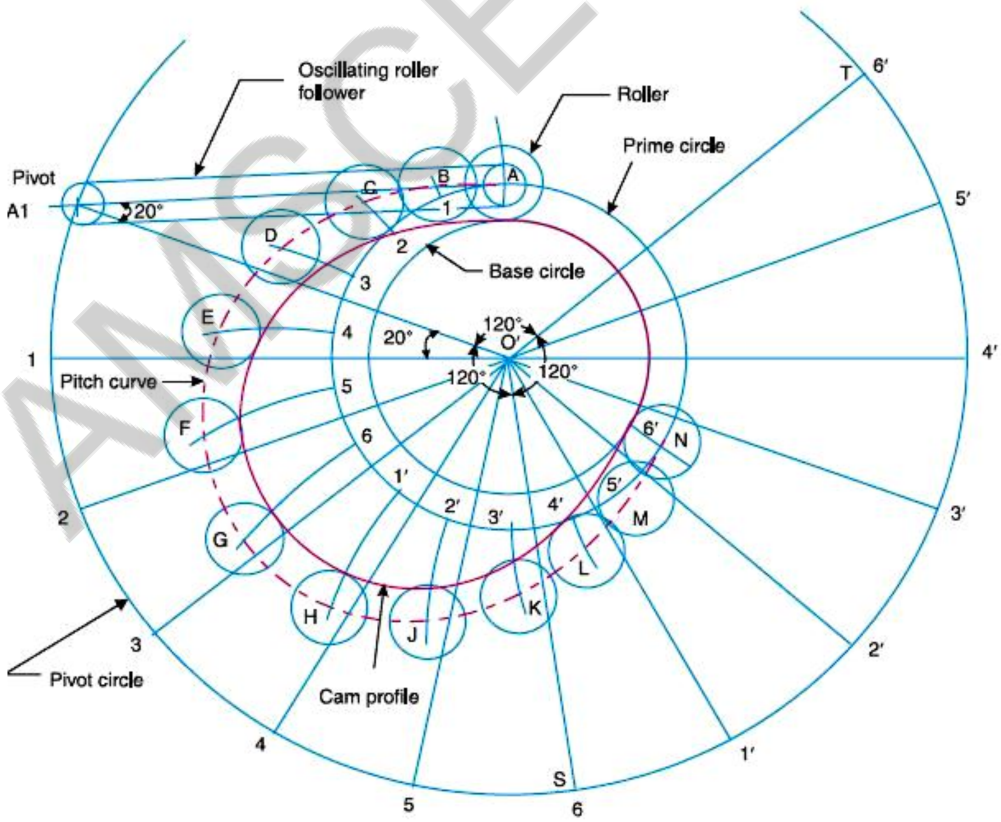
$= -85.81\ m/s^2$

13) Draw a cam profile to drive an oscillating roller follower to the specifications given below :

(a) **Follower to move outwards through an angular displacement of 20° during the first 120° rotation of the cam ;**
(b) Follower to return to its initial position during next 120° rotation of the cam;
(c) Follower to dwell during the next 120° of cam rotation.
The distance between pivot centre and roller centre = 120 mm ; distance between pivot centre and cam axis = 130 mm ; minimum radius of cam = 40 mm ; radius of roller = 10 mm ; inward and outward strokes take place with simple harmonic motion.

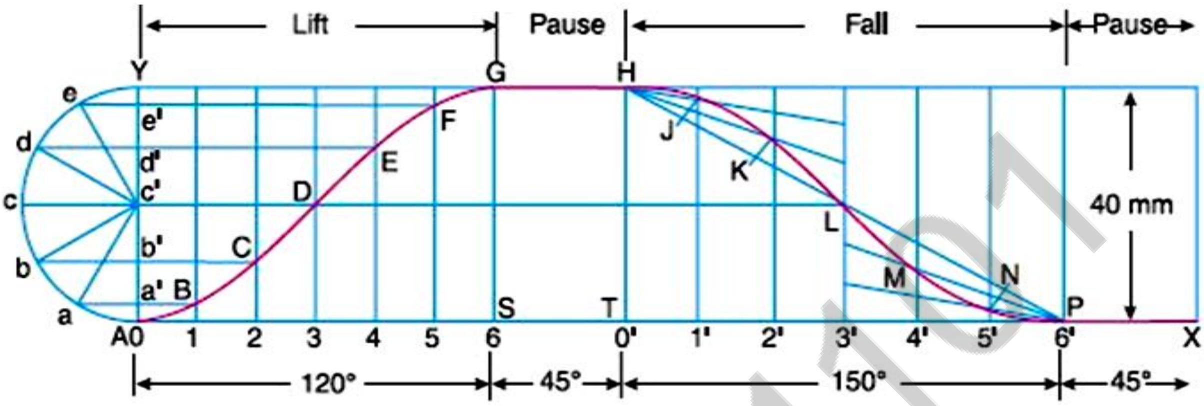


<u>**CAM PROFILE:**</u>

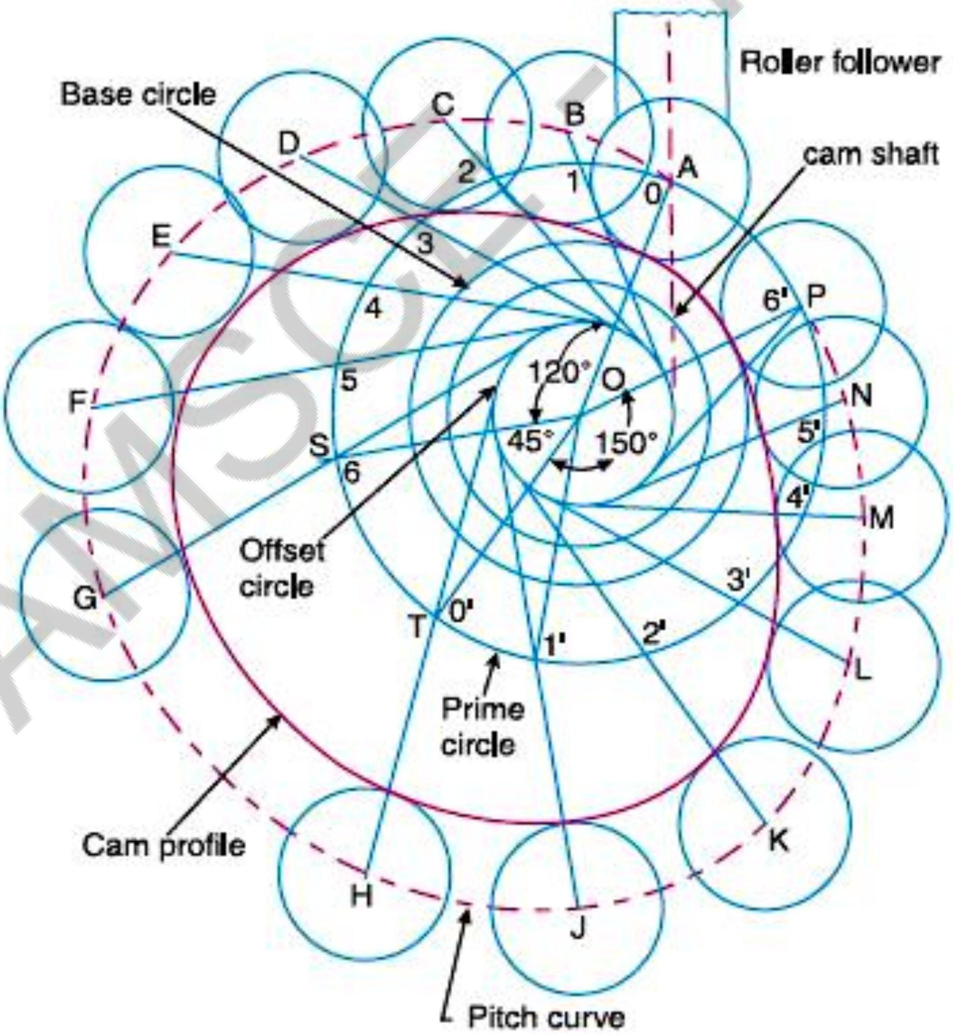


14) Construct the profile of a cam to suit the following specifications:

Cam shaft diameter = 40 mm ; Least radius of cam = 25 mm ; Diameter of roller = 25 mm; Angle of lift = 120° ; Angle of fall = 150° ; Lift of the follower = 40 mm ; Number of pauses are two of equal interval between motions. During the lift, the motion is S.H.M. During the fall the motion is uniform acceleration and deceleration. The speed of the cam shaft is uniform. The line of stroke of the follower is off-set 12.5 mm from the centre of the cam.



CAM PROFILE:



15) Draw the profile of a cam operating a knife edge follower having a

lift of 30 mm. The cam raises the follower with SHM for 150° of the rotation followed by a period of dwell for 60°. The follower descends for the next 100° rotation of the cam with uniform velocity, again followed by a dwell period. The cam rotates at a uniform Velocity of 120 rpm and has a least radius of 20 mm. What will be the maximum velocity and acceleration of the follower during the lift and the return?

[APRIL/MAY-2017]

Solution :

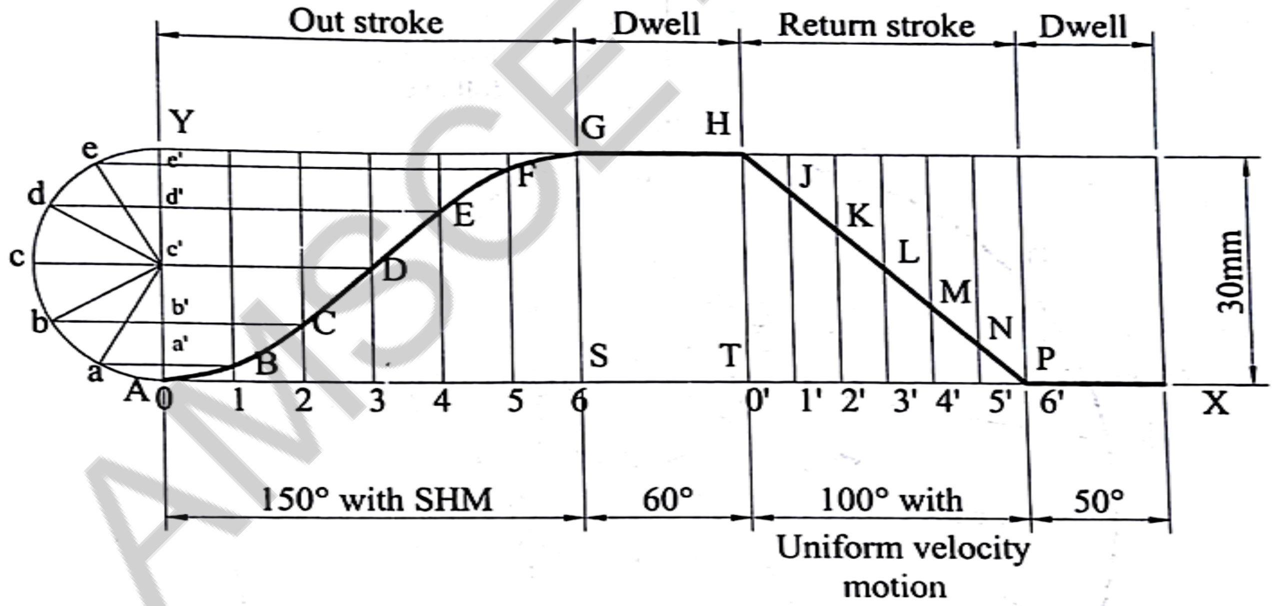
Given Data : $S = 30 \text{ mm} = 0.03 \text{ m};$

$$\theta_0 = 150° = 150 \times \frac{\pi}{180} = 2.618 \text{ rad};$$

$$\theta_R = 100° = 100 \times \frac{\pi}{180} = 1.745 \text{ rad};$$

$$N = 120 \text{ rpm}$$

Displacement Dig:



CAM PROFILE:

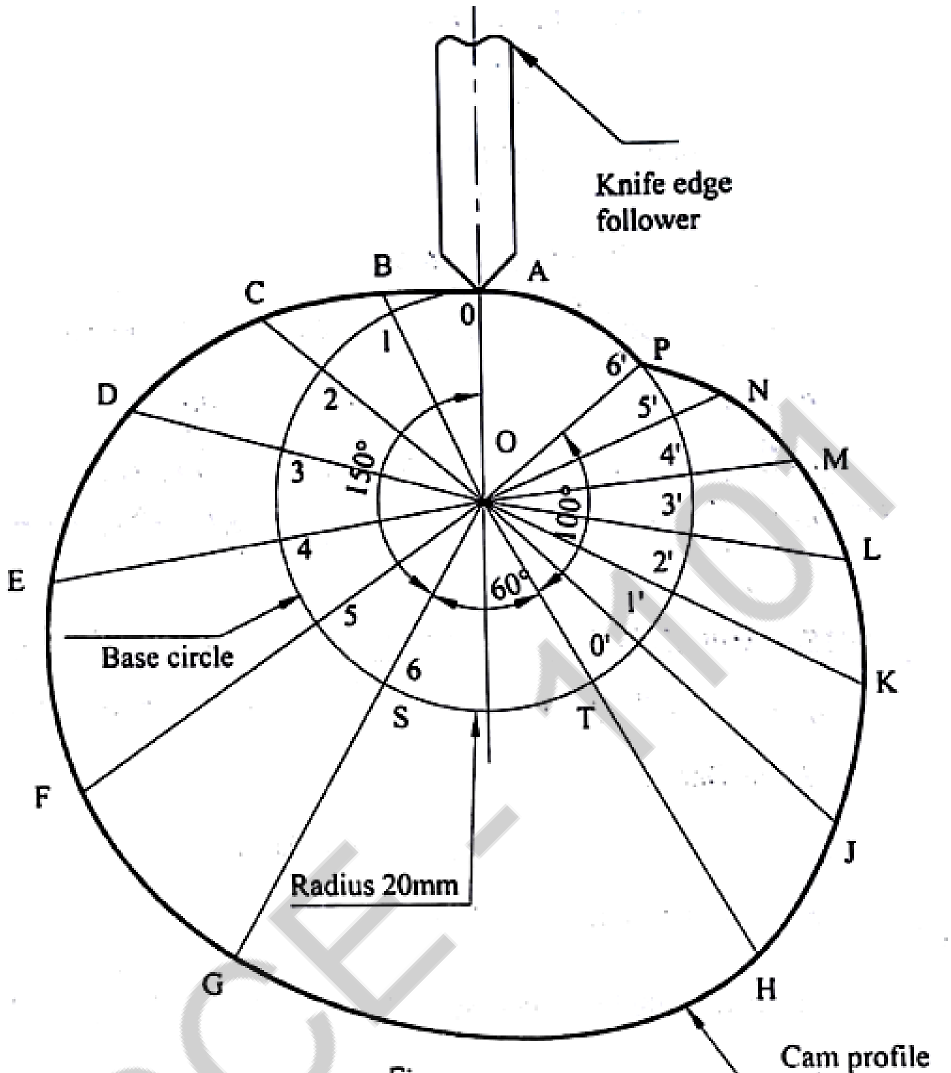


Fig.

Maximum velocity and Acceleration of the follower during the lift :

We know that, the angular velocity of the cam shaft,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 120}{60} = 12.57 \text{ rad/s}$$

Maximum velocity of follower during lift,

$$v_0 = \frac{\pi\,\omega\,S}{2\,\theta_0} = \frac{\pi \times 12.57 \times 0.03}{2 \times 2.618} = \mathbf{0.226\ m/s} \quad \textbf{Ans}$$

and Maximum acceleration of follower during lift,

$$a_0 = \frac{\pi^2\,\omega^2\,S}{2\,(\theta_0)^2} = \frac{\pi^2 \times (12.57)^2 \times 0.03}{2 \times (2.618)^2} = \mathbf{3.413\ m/s^2} \quad \textbf{Ans.}$$

16) In a symmetrical tangent cam operating a roller follower, the least radius of the cam is 30 mm and roller radius is 17.5 mm. The angle of

ascent 75° and the total lift is 17.5 mm. The speed of the cam shaft is 600 rpm. Calculate: (i) The principal dimensions of the cam; (ii) the accelerations of the follower at the beginning of the lift, where straight flank merges into the circular nose and at the apex of the Circular nose; (iii) Draw the profile of the cam. Assume that there is no dwell between ascent and descent. ***[APRIL/MAY-2017]***

Solution : Given data : r_b = 30 mm, d_r = 17.5 mm, α = 75°, total lift of follower = 17.5 mm, N = 600 rpm, $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 600}{60}$ = 62.8318 rad/s

Assume there is no dwell between ascent and descent.

To find : i) Principal dimensions of cam

ii) Acceleration of follower at the begining of lift.

Step - 1 : Calculate the dimensions of the cam.

From Fig. we can write,

$$OE = OI + IE = OO_1 \cos\alpha + IE$$

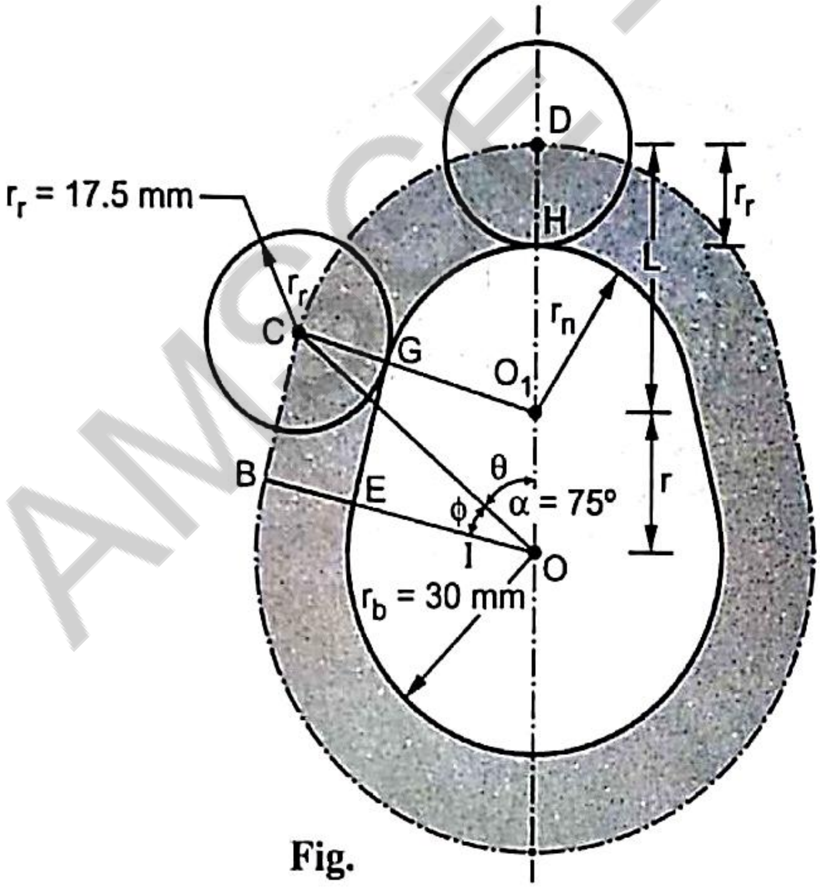


Fig.

$$\therefore \quad r_b = r\cos\alpha + r_n \quad \text{... (i)}$$

but total lift of follower is,

$$\text{Total lift} = r + r_n - r_b$$

$$17.5 = r + r_n - 30 \qquad \therefore r + r_n = 47.5$$

or

$$r = 47.5 - r_n$$

Substituting in equation (i), we get

$$r_b = (47.5 - r_n)\cos(75) + r_n$$

$$\therefore \quad 30 = 12.2939 - 0.2588\, r_n + r_n$$

$$\therefore \quad \mathbf{r_n = 23.8884\ mm}$$

$$\therefore \quad r = 47.5 - 23.8884 = \mathbf{23.6116\ mm}$$

Consider ΔBOC,

$$\tan\phi = \frac{BC}{BO} = \frac{O_1I}{r_b + r_r} = \frac{OO_1 \cdot \sin\alpha}{r_b + r_r}$$

$$\therefore \quad \tan\phi = \frac{23.6116 \times \sin(75)}{30 + 17.5}$$

$$\therefore \quad \phi = 25.6479^\circ \quad \text{... Ans.}$$

Step - 2 : Calculate acceleration of follower at the begining of lift.

Acceleration of follower at the begining of lift is,

$$a = (r_b + r_r)\left(\frac{2 - \cos^2\theta}{\cos^3\theta}\right) \cdot \omega^2$$

But at the begining of lift $\theta = 0$

$$\therefore \quad a = (r_b + r_r) \cdot \omega^2$$

$$\therefore \quad a = (30 + 17.5) \times 62.8318^2$$

$$\therefore \quad a = 187.5221 \times 10^3\ mm/s^2 = 187.5221\ m/s^2 \quad \text{... Ans.}$$

17) Design a cam for operating the exhaust valve of an oil engine. It is required to give equal uniform acceleration and, retardation during opening and closing of the valve each of which corresponds to 60° of cam rotation. The valve must remain in the fully open position for 20° of cam rotation. The lift of the valve is 37.5 mm and the least radius of the cam is 40 mm. The follower is provided with a roller of radius 20 mm and its line of stroke passes through the axis of the cam.

[NOV/DEC 2017]

Given:

$\theta_0 = 60^0$

$\theta_R = 60^0$

Dwell = 30^0

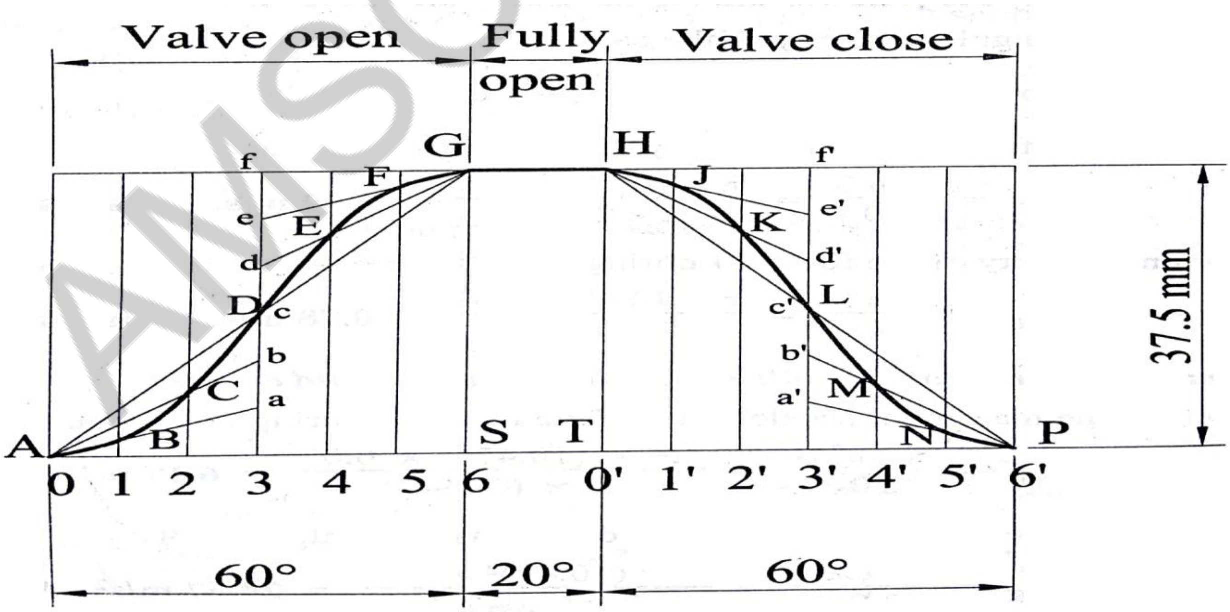
Stroke Length S = 37.5 mm

Base Circle radius = 40 mm

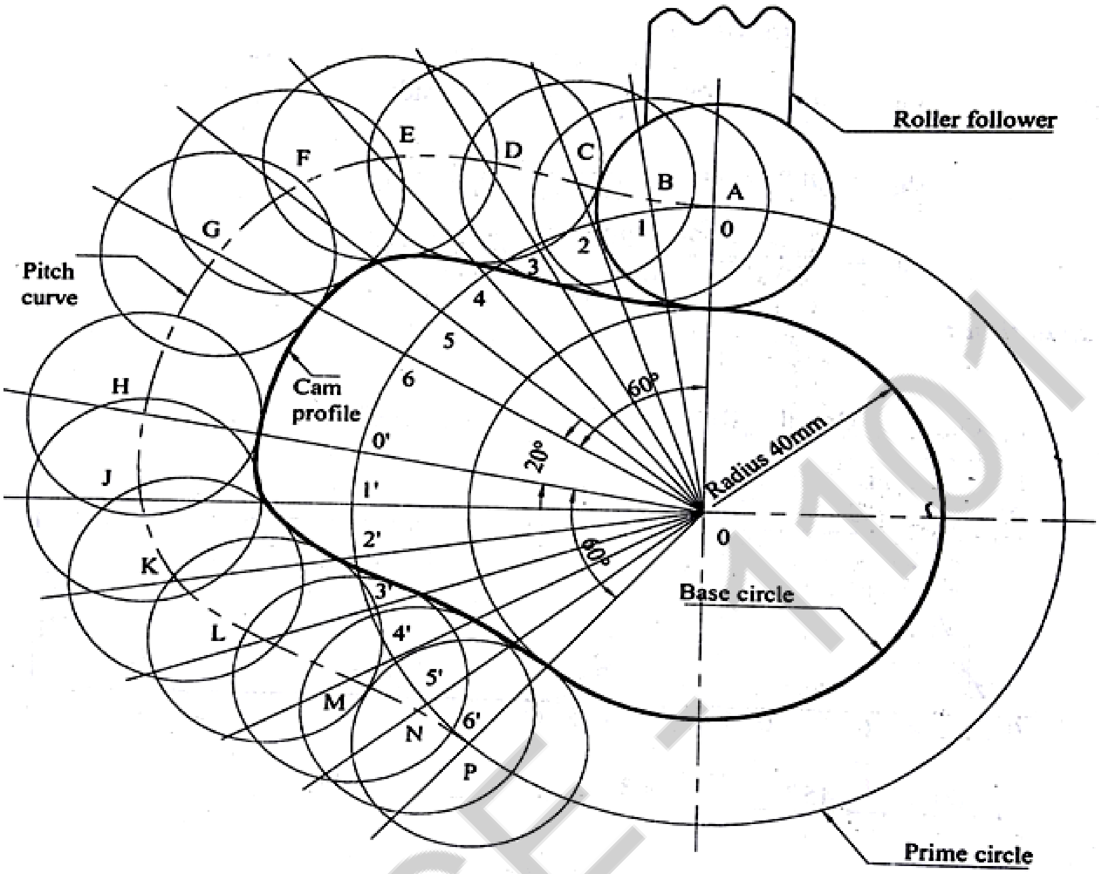
Roller radius = 20 mm

Soln:

DISPLACEMENT DIAGRAM:



CAM PROFILE:



18) Design a cam for operating exhaust Valve of an oil engine. It is required~ to give simple harmonic motion during opening of valve with 120° of cam rotation and simple harmonic motion during closing of the valve with 60° of cam rotation. The valve must remain in the fully open position for 30° of cam rotation. The lift of the valve is 50 mm and the least radius of the cam is 25 mm. The follower is provided with a roller of radius 10 mm and its line of stroke passes through the axis of the cam. ***[NOV/DEC 2017]***

Given Data

Type of Follower : Roller follower

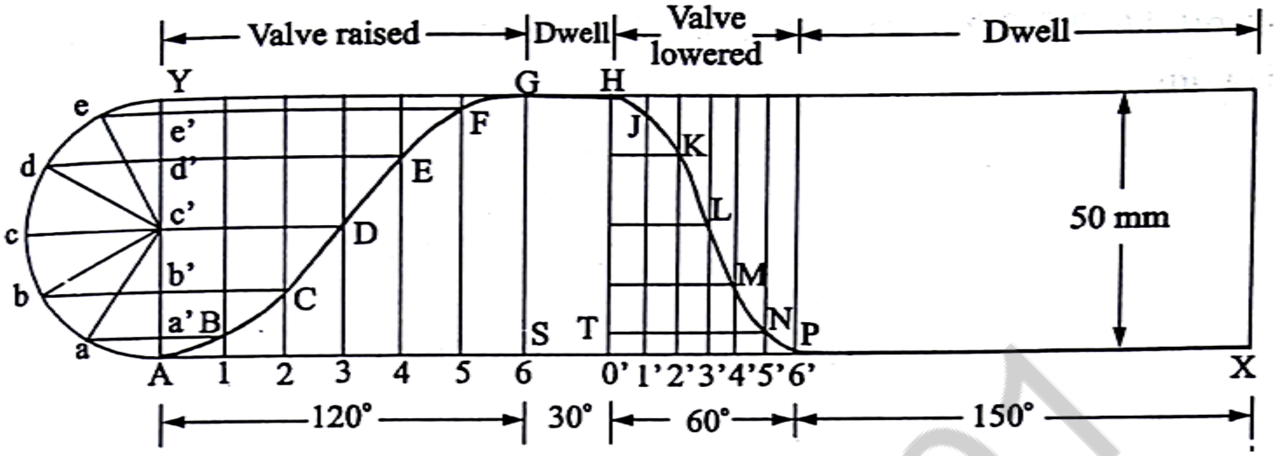
Type of Motion : SHM

$S = 50$ mm $= 0.05$ m ;

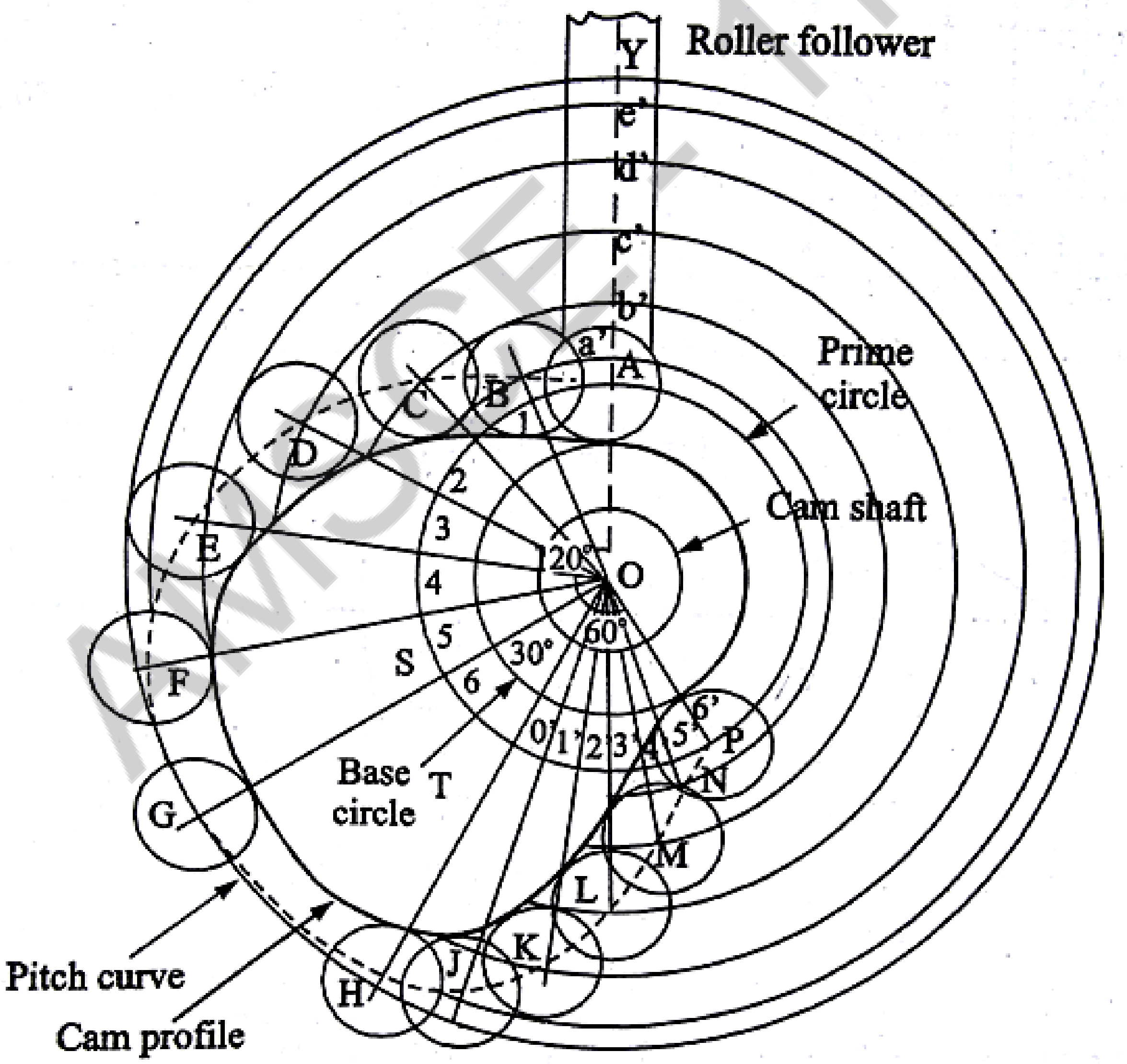
$\theta_O = 120° = 2\pi/3$ rad $= 2.1$ rad ;

$\theta_R = 60° = \pi/3$ rad $= 1.047$ rad ;

DISPLACEMENT DIAGRAM:



CAM PROFILE:



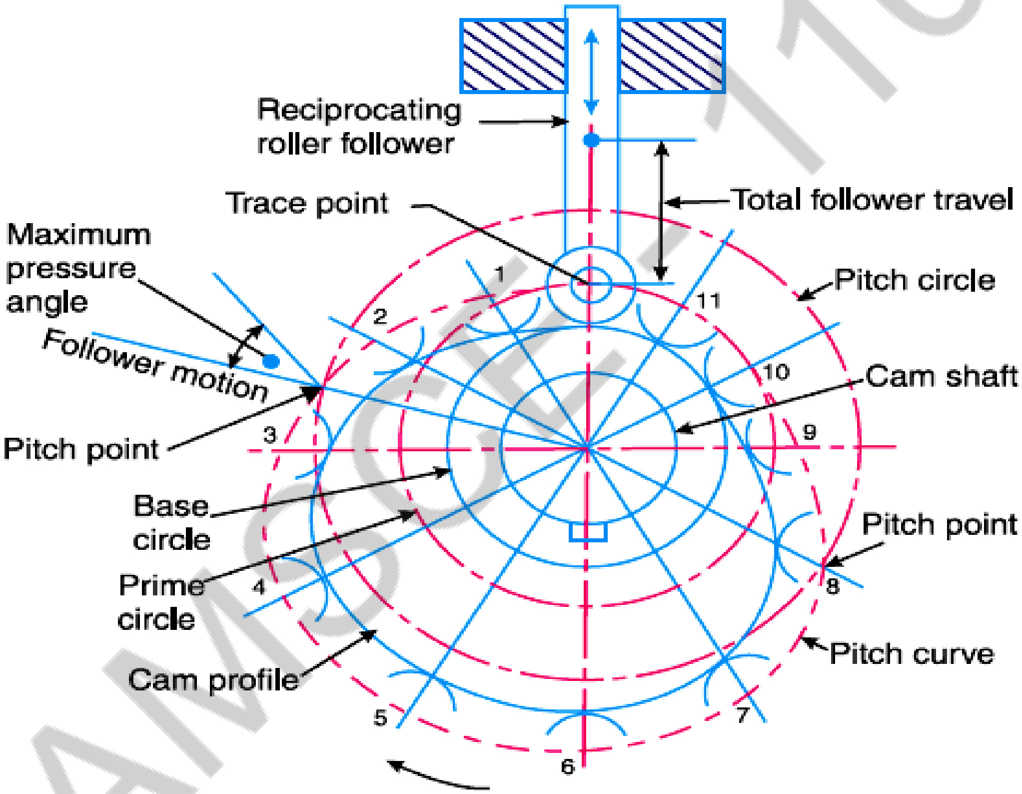
19) a) i) Neatly sketch a cam mechanism with roller follower and indicate the following in the sketch and brief them : Cam profile, Base circle, Prime circle and Pressure angle. (6)

ii) In a cam follower mechanism, 40 mm lift of the follower has to be made in the first 120° rotation of the cam. Draw the displacement diagrams for the following types of motions, separately for each, taking atleast 8 equal divisions of 120° : (7)

a) Simple harmonic motion

b) Cycloidal motion. ***[APRIL/MAY-2018]***

(i)

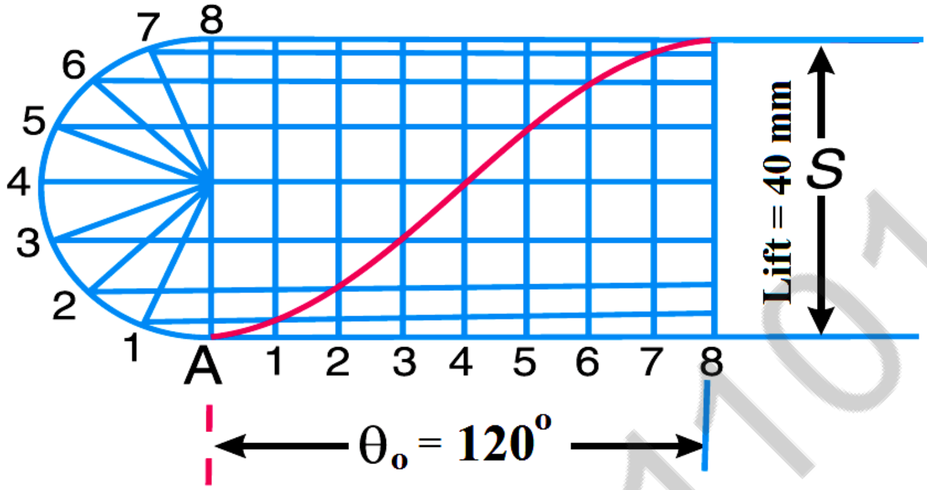


Base circle. It is the smallest circle that can be drawn to the cam profile.

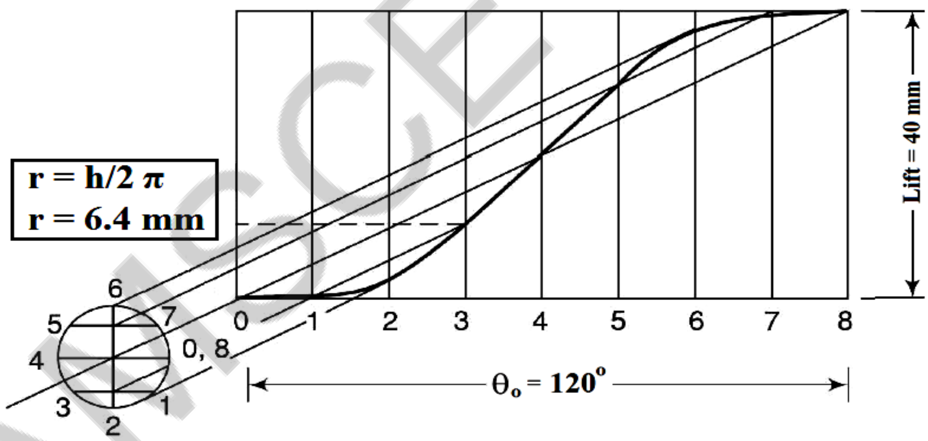
Prime circle. It is the smallest circle that can be drawn from the centre of the cam and tangent to the pitch curve. For a knife edge and a flat face follower, the prime circle and the base circle are identical. For a roller follower, the prime circle is larger than the base circle by the radius of the roller.

Pressure angle. It is the angle between the direction of the follower motion and a normal to the pitch curve. This angle is very important in designing a cam profile. If the pressure angle is too large, a reciprocating follower will jam in its bearings.

(ii) (a) SIMPLE HARMONIC MOTION:



(b) CYCLOIDAL MOTION:



20) b) Draw the cam profile of an offset knife edge follower cam which rotates in clockwise direction, with both rise and return have Uniform Acceleration and retardation motions, for the following data:

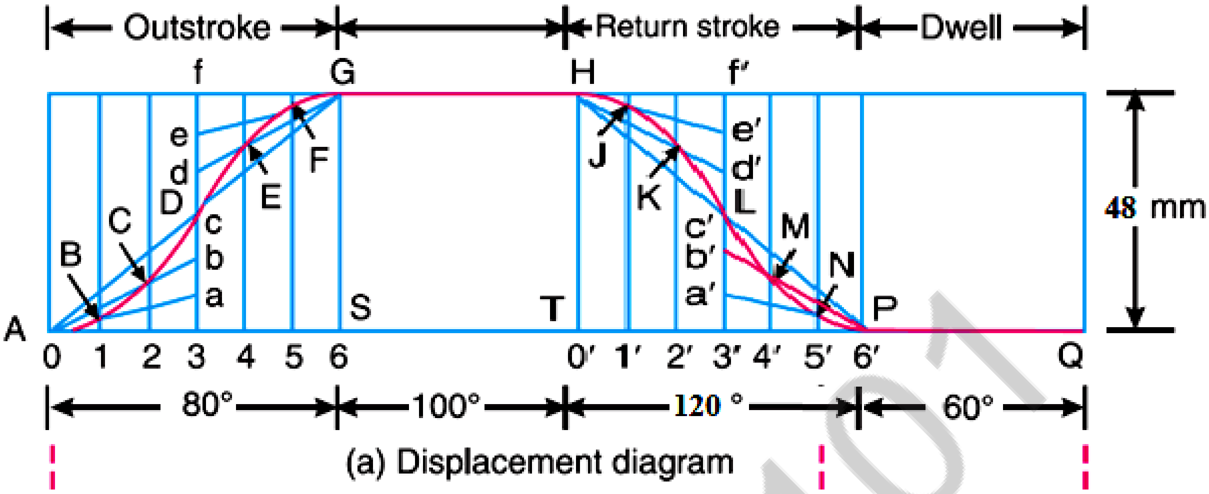
Base circle Diameter of the cam = 50 mm,

Lift of the follower = 48mm

Offset of follower = 10 mm to the right of cam rotation centre

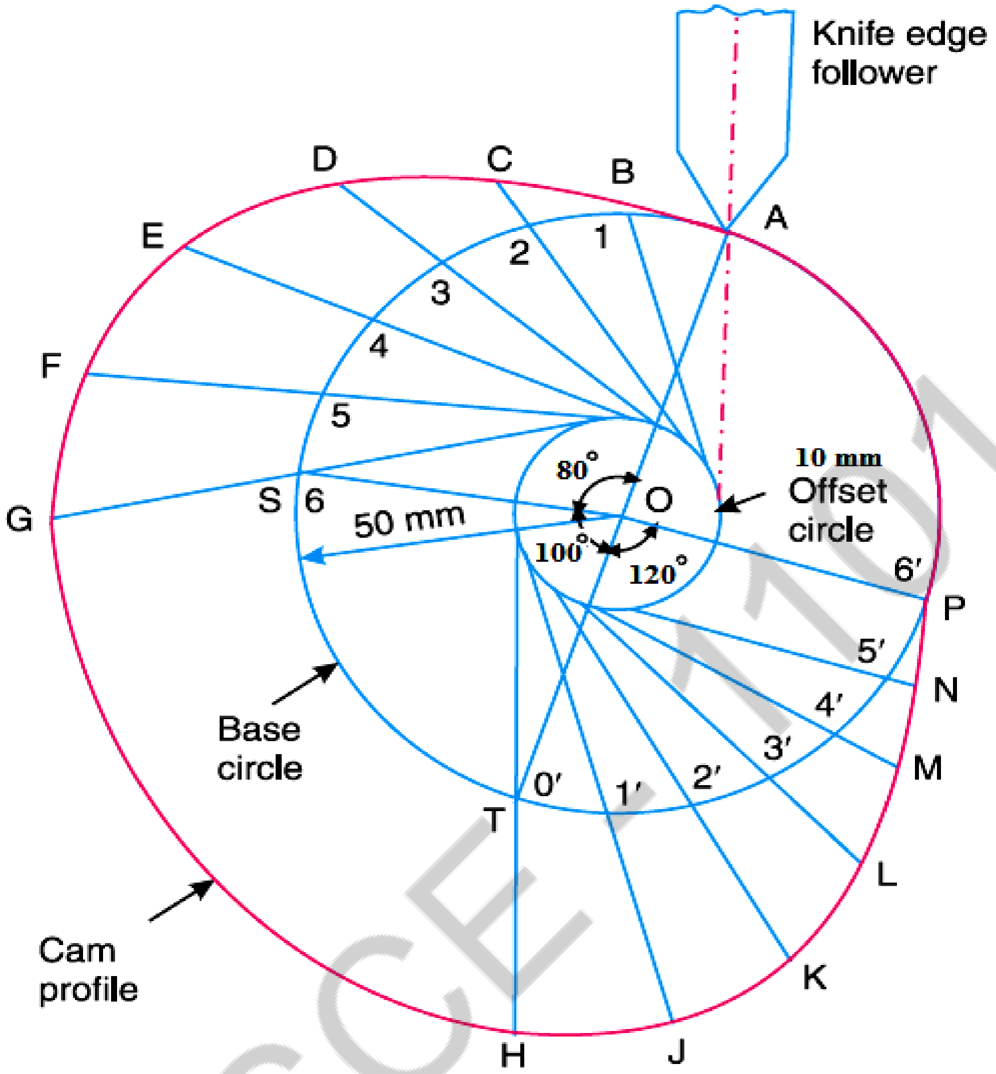
Cam rotation angles for the follower motions are:

Rise = 80° First Dwell = 100°, Return = 120° and Second dwell = 60°. Assume the length' of the displacement diagram as 180 mm (x-axis) and divide the rise and return rectangles into at least 6 equal divisions each. (13)



(a) Displacement diagram

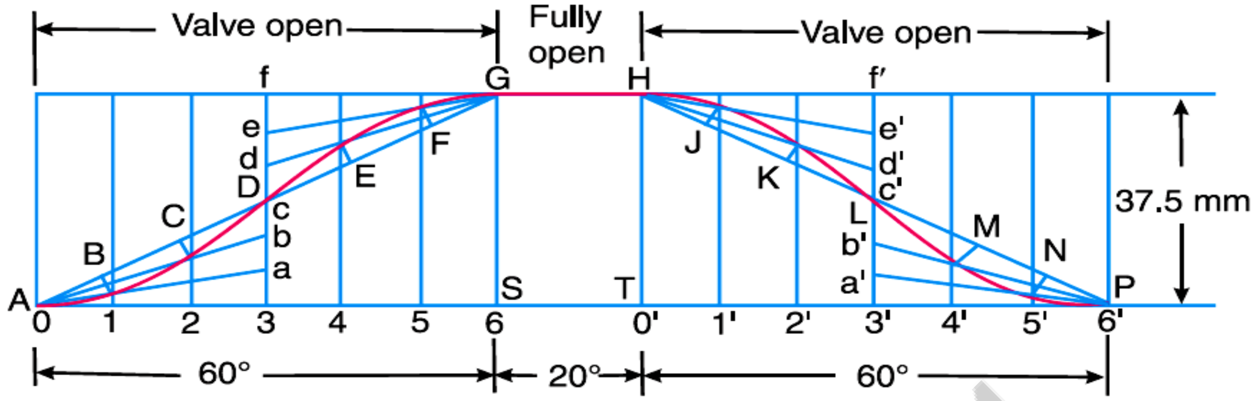
CAM PROFILE:



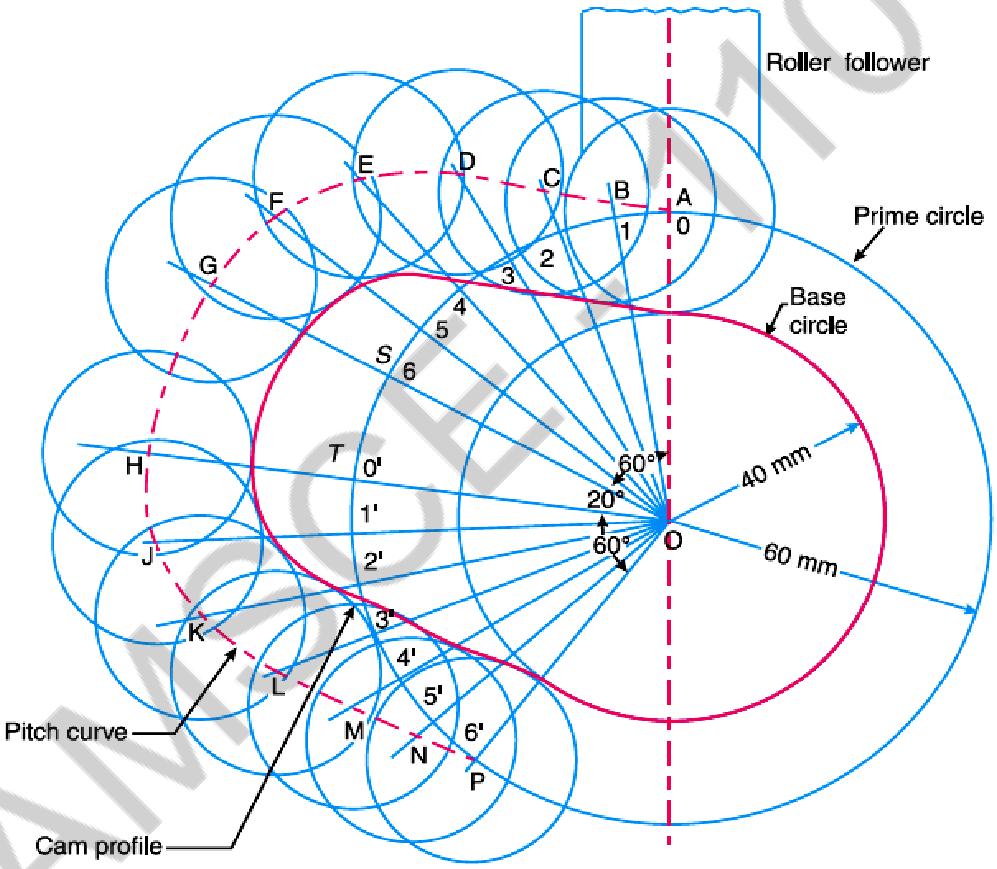
21) Draw the profile of a cam for operating the exhaust valve of an oil engine. It is required to give equal uniform acceleration and retardation during opening and closing- of the valve each of which corresponds to 60° of cam rotation. The valve must remain in the fully open position for 20° of cam rotation. The lift of the valve is 37.5 mm and the least radius of cam is 40 mm. The follower provided with a roller radius of 20 mm and its line of stroke passes through the axis of the cam. (13)

[NOV/DEC-2018]

Displacement Diagram:



CAM PROFILE:



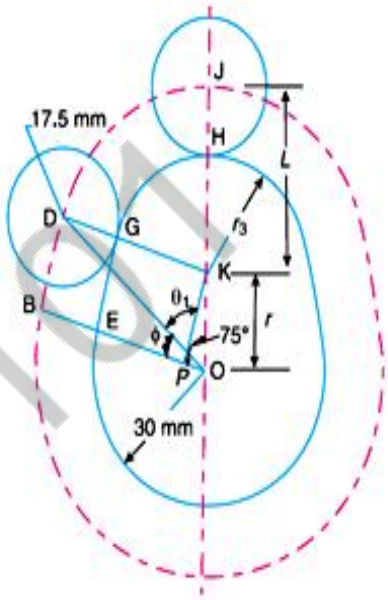
22) In a symmetrical tangent earn operating a roller follower, the least radius of the cam is 30 mm and roller radius is 17.5 mm. The angle of ascent is 75° and the total lift is 17.5 mm. The speed of the cam shaft is 600 rpm. Calculate

(i) The principal dimensions of the cam.

(ii) The acceleration of the follower at the beginning of the lift, where straight flank merges into the circular nose and at the apex of the circular nose. Assume that there is no dwell between ascent and descent. (13)

Solution. Given : r_1 = 30 mm ; r_2 = 17.5 mm ; α = 75° ; Total lift = 17.5 mm ; N = 600 r.p.m. or $\omega = 2\pi \times 600/60 = 62.84$ rad/s

1. *Principal dimensions of the cam*



Let $r = OK$ = Distance between cam centre and nose c

r_3 = Nose radius, and

ϕ = Angle of contact of cam with straight flanks.

From the geometry of Fig. 20.42,

$$r + r_3 = r_1 + \text{Total lift}$$

$$= 30 + 17.5 = 47.5 \text{ mm}$$

$\therefore$ $$r = 47.5 - r_3 \quad \ldots (i)$$

Also, $$OE = OP + PE \quad \text{or} \quad r_1 = OP + r_3$$

$\therefore$ $$OP = r_1 - r_3 = 30 - r_3 \quad \ldots (ii)$$

Now from right angled triangle OKP,

$$OP = OK \times \cos\alpha \quad \ldots (\because \cos\alpha = OP/OK)$$

or $$30 - r_3 = (47.5 - r_3)\cos 75° = (47.5 - r_3)0.2588 = 12.3 - 0.2588\, r_3$$

$$\ldots (\because OK = r)$$

$\therefore$ r_3 = 23.88 mm **Ans.**

and $r = OK = 47.5 - r_3 = 47.5 - 23.88 = 23.62$ mm **Ans.**

Again, from right angled triangle ODB,

$$\tan\phi = \frac{DB}{OB} = \frac{KP}{OB} = \frac{OK\sin\alpha}{r_1 + r_2} = \frac{23.62\sin 75°}{30 + 17.5} = 0.4803$$

$\therefore \qquad \phi = 25.6°$ **Ans.**

2. Acceleration of the follower at the beginning of the lift

We know that acceleration of the follower at the beginning of the lift, *i.e.* when the roller has contact at E on the straight flank,

$$a_{min} = \omega^2 (r_1 + r_2) = (62.84)^2 (30 + 17.5)^2 = 187\ 600\ \text{mm/s}^2$$

$$= 187.6\ \text{m/s}^2 \textbf{ Ans.}$$

Acceleration of the follower where straight flank merges into a circular nose

We know that acceleration of the follower where straight flank merges into a circular nose *i.e.* when the roller just leaves contact at G,

$$a_{max} = \omega^2 (r_1 + r_2)\left[\frac{2 - \cos^2\phi}{\cos^3\phi}\right] = (62.84)^2 (30 + 17.5)\left(\frac{2 - \cos^2 25.6°}{\cos^3 25.6°}\right)$$

$$= 187\ 600\left(\frac{2 - 0.813}{0.733}\right) = 303\ 800\ \text{mm/s}^2 = 303.8\ \text{m/s}^2 \textbf{ Ans.}$$

Acceleration of the follower at the apex of the circular nose

We know that acceleration of the follower for contact with the circular nose,

$$a = \omega^2 . r\left[\cos\theta_1 + \frac{L^2 . r\cos 2\theta_1 + r^3 \sin^4\theta_1}{(L^2 - r^2\sin^2\theta_1)^{3/2}}\right]$$

Since θ_1 is measured from the top position of the follower, therefore for the follower to have contact at the apex of the circular nose (*i.e.* at point H), $\theta_1 = 0$.

$\therefore$ Acceleration of the follower at the apex of the circular nose,

$$a = \omega^2 . r\left(1 + \frac{L^2 . r}{L^3}\right) = \omega^2 . r\left(1 + \frac{r}{L}\right) = \omega^2 . r\left(1 + \frac{r}{r_2 + r_3}\right)$$

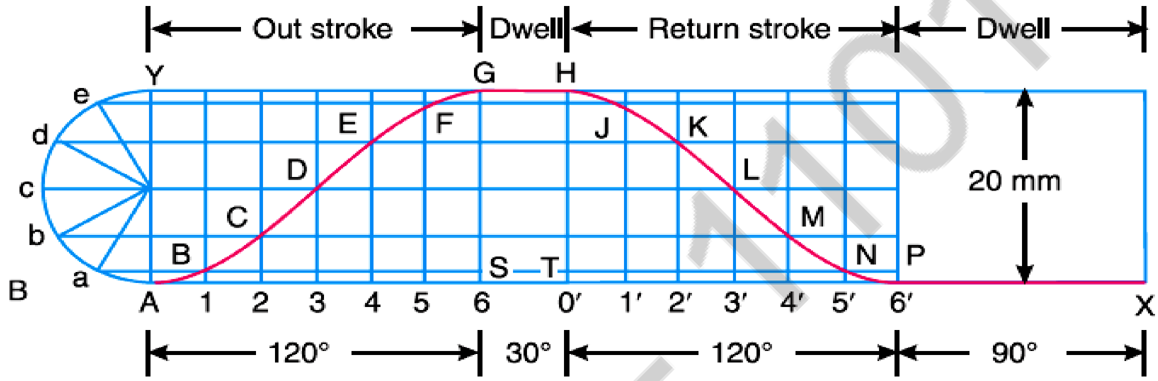
$$= (62.84)^2 23.62\left(1 + \frac{23.62}{17.5 + 23.88}\right) = 146\ 530\ \text{mm/s}^2 \qquad \ldots (\because L = r_2 + r_3)$$

$$= 146.53\ \text{m/s}^2 \textbf{ Ans.}$$

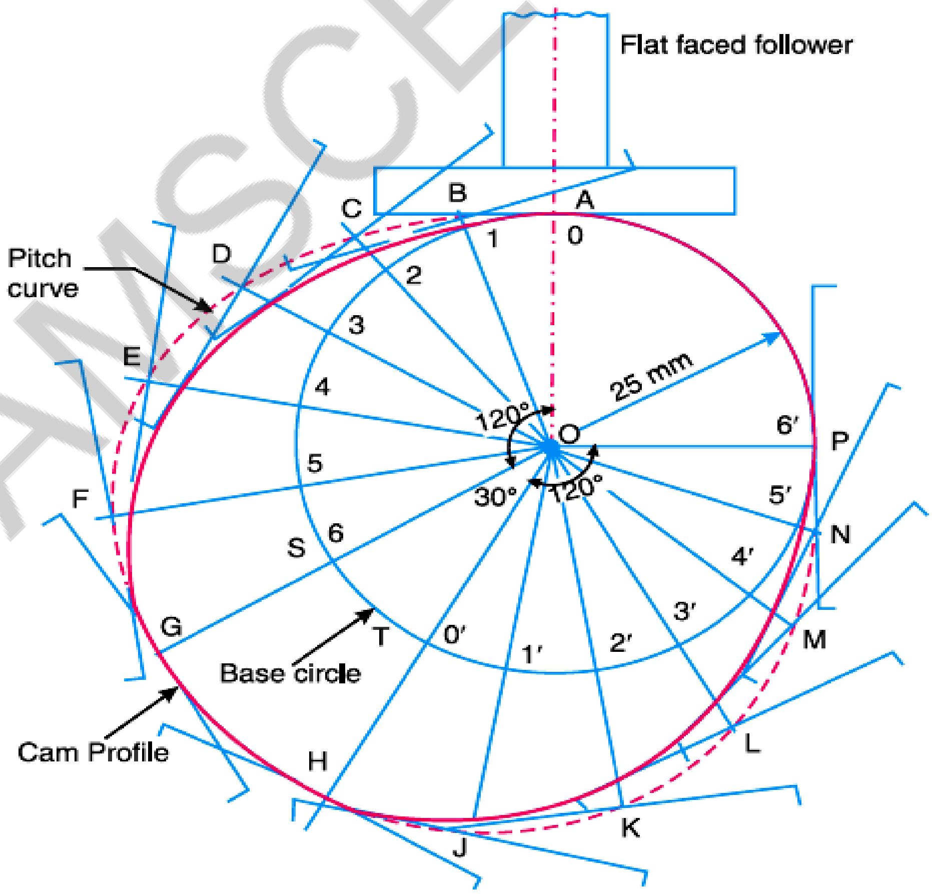
23) It is required to set out the profile of a cam to give the following motion to the reciprocating follower with a flat mushroom contact face: (i) Follower to have a stroke of 20 mm during 120⁰ of cam rotation. (ii) Follower to dwell for 30° of cam rotation. (iii) Follower to return to its initial position during 120° of cam rotation; and (iv) Follower to dwell for remaining 90° of cam rotation. The minimum radius of the cam is 25 mm. The motion of the follower is to take place with simple harmonic motion during out stroke and return stroke. (13)

[APR/MAY-2019]

Displacement Diagram:



CAM PROFILE:



24) A flat ended valve tappet is operated by a symmetrical cam with circular arc for flank and nose. The straight line path of the tappet passes through the cam axis. Total angle of action 150°, lift 6mm. Base circle diameter 30mm, period of acceleration is the half the period of retardation during the lift the cam rotates at 1250 rpm. Find {i) Flank and nose radii (ii) Maximum acceleration and retardation during the lift. (13)

[APR/MAY-2019]

Solution. Given : $2\alpha = 150°$ or $\alpha = 75°$; $x = JK = 6$ mm ; $d_1 = 30$ mm or $r_1 = OE = OJ = 15$ mm ; $N = 1250$ r.p.m. or $\omega = 2\pi \times 1250/60 = 131$ rad/s

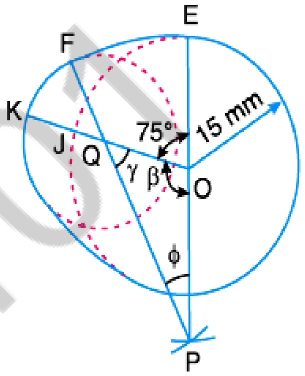


Fig.

1. *Flank and nose radii*

The circular arc cam operating a flat ended valve tappet is shown in Fig. 20.49.

Let $R = PE$ = Flank radius, and

$r_2 = QF = QK$ = Nose radius.

First of all, let us find the values of OP, OQ and PQ. The acceleration takes place while the follower is on the flank and retardation while the follower is on nose. Since the period of acceleration is half the period of retardation during the lift, therefore

$$\phi = \frac{1}{2}\gamma \quad \ldots (i)$$

We know that $\beta = 180° - \alpha = 180° - 75° = 105°$

$$\therefore \quad \phi + \gamma = 75° = 180° - \beta = 180° - 105° = 75° \quad \ldots (ii)$$

From equations (*i*) and (*ii*),

$$\phi = 25°, \quad \text{and} \quad \gamma = 50°$$

Now from the geometry of Fig. 20.49,

$$OQ = OJ + JK - QK = r_1 + x - r_2 = 15 + 6 - r_2 = 21 - r_2 \quad \ldots (iii)$$

and

$$PQ = PF - FQ = PE - FQ = (OP + OE) - FQ = OP + 15 - r_2 \quad \ldots (iv)$$

Now from triangle OPQ,

$$\frac{OP}{\sin\gamma} = \frac{OQ}{\sin\phi} = \frac{PQ}{\sin\beta}$$

or

$$\frac{OP}{\sin 50°} = \frac{21 - r_2}{\sin 25°} = \frac{OP + 15 - r_2}{\sin 105°}$$

$$\therefore \quad OP = \frac{21 - r_2}{\sin 25°} \times \sin 50° = \frac{21 - r_2}{0.4226} \times 0.766 = 38 - 1.8 r_2 \quad \ldots (v)$$

Also $$OP = \frac{OP + 15 - r_2}{\sin 105°} \times \sin 50° = \frac{OP + 15 - r_2}{0.966} \times 0.766$$

$$= 0.793 \times OP + 11.9 - 0.793\, r_2$$

$$\therefore \quad 0.207\, OP = 11.9 - 0.793\, r_2 \quad \text{or} \quad OP = 57.5 - 3.83\, r_2 \quad \ldots (vi)$$

From equations (*v*) and (*vi*),

$$38 - 1.8\, r_2 = 57.5 - 3.83\, r_2 \quad \text{or} \quad 2.03\, r_2 = 19.5$$

$$\therefore \quad r_2 = 9.6 \text{ mm}$$ **Ans.**

We know that $OP = 38 - 1.8 r_2 = 38 - 1.8 \times 9.6 = 20.7$ mm ... [From equation (*v*)]

$$\therefore \quad R = PE = OP + OE = 20.7 + 15 = 35.7 \text{ mm}$$ **Ans.**

2. ***Maximum acceleration and retardation during the lift***

We know that maximum acceleration

$$= \omega^2 (R - r_1) = \omega^2 \times OP = (131)^2\, 20.7 = 355230 \text{ mm/s}^2$$

$$= 355.23 \text{ m/s}^2$$ **Ans.**

and maximum retardation, $= \omega^2 \times OQ = \omega^2 (21 - r_2)$ ... [From equation (*iii*)]

$$= (131)^2 (21 - 9.6) = 195\ 640 \text{ mm/s}^2 = 195.64 \text{ m/s}^2$$ **Ans.**

25) A tangent cam with straight working faces tangential to a base circle of 120 mm diameter has a roller follower of 48mm diameter. The line of stroke of the roller follower passes through the axis of the cam. The nose circle radius of the cam is 12 mm and the angle between the tangential faces of the cam 90°. If the speed of the cam is 180 rpm. Determine the acceleration of the follower, when

(i) During the lift, the roller just leaves the straight flank

(ii) The roller is at the outer end of its lift. i.e. at the top of the nose.

(15) *[APR/MAY-2019]*

Solution:

$r_c = 60$ mm $\qquad r_n = 12$ mm

$r_r = 24$ mm $\qquad N = 180$ rpm

$$\alpha = (180° - 45° - 90°) = 45°$$

$$OA = OP + PA$$

$$= OQ \cos \alpha + QK$$

$$r_c = r \cos \alpha + r_n$$

$$60 = r \cos 45° + 12$$

$$r = 67.9 \text{ mm}$$

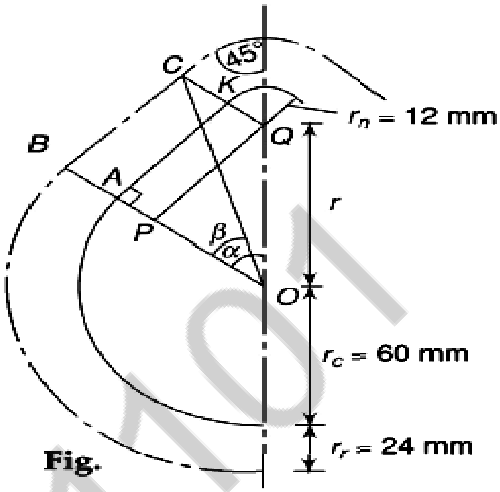


Fig.

$$\tan \beta = \frac{BC}{OB} = \frac{PQ}{OB}$$

$$= \frac{r \sin \alpha}{r_c + r_r}$$

$$= \frac{67.9 \sin 45°}{60 + 24}$$

$$= 0.571$$

$$\omega = \frac{2\pi \times 180}{60} = 6\pi \text{ rad/s}$$

$$\therefore \quad \beta = 29.74°$$

(i) Acceleration when the roller just leaves the straight flank,

$$f = \frac{\omega^2 (r_c + r_r) \quad (2 - \cos^2 \theta)}{\cos^3 \theta}$$

$$= \frac{(6\pi)^2 \quad (0.06 + 0.024) \quad (2 - \cos^2 29.74°)}{\cos^3 29.74°}$$

$$= (6\pi)^2 \times 0.084 \times 1.9035 = \underline{56.8 \text{ m/s}^2}$$

(ii) Acceleration when the roller is at the outer end of its lift, i.e., at the top of the nose,

$\theta = \alpha$

$$f = \omega^2 r \left[-\cos(\alpha - \theta) - \frac{r^3 \sin^2 2(\alpha - \theta)}{4\left[l^2 - r^2 \sin^2 (\alpha - \theta)\right]^{3/2}} - \frac{r \cos 2(\alpha - \theta)}{\sqrt{l^2 - r^2 \sin^2 (\alpha - \theta)}} \right]$$

$$= \omega^2 r \left[-1 - \frac{r}{l} \right]$$

$$= (6\pi)^2 \times 0.0679 \left[-1 - \frac{0.0679}{0.036} \right]$$

$$(l = r_r + r_n = 24 + 12 = 36 \text{ mm})$$

$$= \underline{69.6 \text{ m/s}^2}$$

UNIT- IV GEARS AND GEAR TRAINS

PART-A

1) State the law of Gearing. (MAY/JUNE 2014)

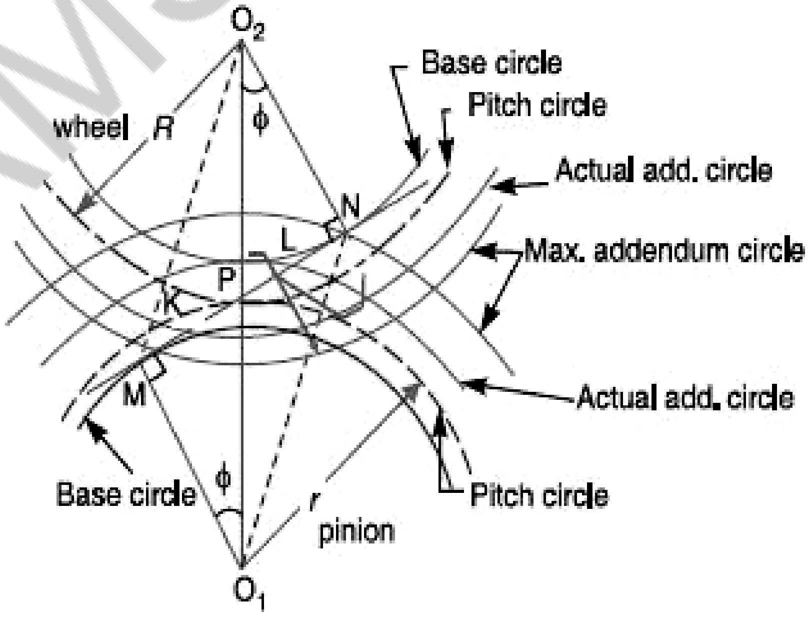
The law of gearing states that for obtaining a constant velocity ratio, at any instant of teeth the common normal at each point of contact should always pass through a pitch point, situated on the line joining the centre of rotation of the pair of mating gears.

2) What are the methods to avoid interference? (MAY/JUNE 2014)

- By modifying addendum of Gear teeth
- By increasing the pressure angle
- By modifying tooth profile or profile shifting
- By increasing the centre distance

3) What do you understand by the term 'interference' as applied to gear? (MAY/JUNE 2015)

A little consideration will show that if the radius of the addendum circle of pinion is increased to ON, the point of contact L will move from L to N. When this radius is further increased, the point of contact L will be on the inside of base circle of wheel and not on the involute profile of tooth on wheel. The tip of tooth on the pinion will then undercut the tooth on the wheel at the root and remove part of the involute profile of tooth on the wheel. This effect is known as interference, and occurs when the teeth are being cut. In brief, the phenomenon when the tip of tooth undercuts theRoot on its mating gear is known as interference.



4) What is the special advantage of epicyclic gear trains?

(MAY/JUNE 2015)

The special advantages of epicyclic gear trains over simple or compound gear train is that it can achieve high speed reductions within a very a limited space.

5) Define normal and axial pitch in helical gear? (MAY/JUNE 2016)

Normal pitch is the distance between similar face of adjacent teeth, along a helix on the pitch cylinder normal to the teeth.

Axial pitch is the distance measured parallel to the axis between similar faces of a adjacent teeth.

6) What is the advantage when arc recess is equal to arc approach in meshing gears? (MAY/JUNE 2016)

When arc of recess equal to arc of approach, the work wasted by friction is minimum and the efficiency of drive is maximum.

7) What is mean by interference of gear? How can it be avoided?

(NOV/DEC 2014)

The phenomenon when the tip of tooth undercuts the roots on its mating gear is known as interference.

It can be avoided by:

- The height of the teeth may be reduced
- The pressure angle may be increased
- The radial flank of the pinion may be cut back
- The face of the gear tooth may be relieved

8) Differentiate between Involute profile and Cycloidal profile.

(NOV/DEC 2014)

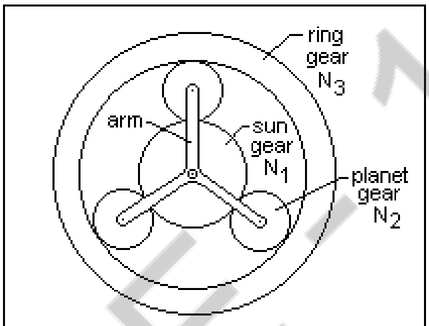
Involute profile	Cycloidal profile
▪ Pressure angle remains same throughout the operation. ▪ Teeth's are weaker. ▪ It is easier to manufacture due to convex surface. ▪ The velocity is not affected due to variation in centre distance. ▪ Interference takes place. ▪ More wear and tear as contact takes place between convex surfaces.	▪ Pressure angle keeps on changing during the operation. ▪ Teeth's are stronger. ▪ It is difficult to manufacture due to requirement of hypocycloid and epicycloids. ▪ The centre distance should remain the same. ▪ There is no interference. ▪ Less wear and tear as concave flank makes contact with convex flank.

9) State the law of gearing? (NOV/DEC 2015)

The law of gearing states that for obtaining a constant velocity ratio, at any instant of teeth the common normal at each point of contact should always pass through a pitch point, situated on the line joining the centre of rotation of the pair of mating gears.

10) How the epicyclic gear train works? (NOV/DEC 2015)

An epicyclic gear train consists of two gears mounted so that the center of one gear revolves around the center of the other. A carrier connects the centers of the two gears and rotates to carry one gear, called the planet gear, around the other, called the sun gear. The planet and sun gears mesh so that their pitch circles roll without slip. A point on the pitch circle of the planet gear traces an epicycloid curve. In this simplified case, the sun gear is fixed and the planetary gear(s) roll around the sun gear.



Sun-and-Planet Epicyclic Train

11) State law of Gearing.

The law of gearing states that for obtaining a constant velocity ratio, at any instant of teeth the common normal at each point of contact should always pass through a pitch point, situated on the line joining the centre of rotation of the pair of mating gears.

12) Define normal and axial pitch in helical gears.

Normal pitch is the distance between similar face of adjacent teeth, along a helix on the pitch cylinder normal to the teeth.

Axial pitch is the distance measured parallel to the axis between similar faces of a adjacent teeth.

13) What is the maximum efficiency in worm and worm gear?

$\eta_{max} = 1 - \sin\varphi / 1 + \sin\varphi$

14) What are the advantages and limitations of gear drive? Write any two.

Advantages:

- Since there is no slip, so exact velocity ratio is obtained.
- It is more efficient and effective means of power transmission.

Limitations:

- Manufacture of gear is complicated.
- The error in cutting teeth may cause vibration and noise during operation.

15) Define interference.

The phenomenon when the tip of tooth undercuts the roots on its mating gear is known as interference.

16) Define cycloidal tooth profile and involute tooth profile.

A cycloid is the curve traced by a point on the circumference of a circle which rolls without slipping on a fixed straight line.

Involute profile is defined as the locus of a point on a straight line which rolls without slipping on the circumference of a circle.

17) Define circular pitch and diametral pitch in spur gears.

Circular pitch (p_c) : It is the distance measured along the circumference of the pitch circle from a point on one teeth to the corresponding point on the adjacent tooth.

$$p_c = \pi D / T$$

Diametral pitch (p_D) : It is the ratio of number of teeth to the pitch circle diameter.

$$P_D = T/D = \pi / p_c$$

18) Define Backslash.

It is the difference between the tooth space and the tooth thickness along the pitch circle.

Backslash = Tooth space – Tooth thickness

19) What is gear train of train of wheels?

Two or more gears re made to mesh with each other to transmit power from one shaft to another. Such a combination is called a gear train or train of wheels.

20) Write velocity ratio in compound train of wheels?

Speed of last follower - Product of teeth on drivers
Speed of first driver – Product of teeth on followers.

21) Define simple gear train.

When there is only one gear on each shaft, it is known as simple gear train.

22) What is reverted gear train?

When the axes of the first and last wheels are co-axial, the train is known as reverted gear train.

23) Where the epicyclic gear trains are used?

The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, pulley blocks, wrist watches, etc.

24) Write down the difference between involute and cycloidal tooth profile.

S.No	Involute Tooth Profile	Cycloidal Tooth Profile
1.	Variation in centre distance does not affect the velocity ratio.	Centre distance should not vary.
2.	Pressure angle remains constant throughout the teeth.	Pressure angle varies. It is zero at the pitch point and maximum at the start and end of engagement.
3.	Interference occurs.	No interference occurs.
4.	Weaker teeth.	Stronger teeth.

25) Define Contact Ratio.

It is the ratio of the length of arc contact to the circular pitch is known as contact ratio. The value gives the number of pairs of teeth in contact.

26) What is an angle of obliquity in gears?

It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is called as pressure angle.

27) What is bevel gearing? Mention its types.

When the non-parallel (or) intersecting but coplanar shafts connected by gears, they are called bevel gears and the arrangement is bevel gearing. It is of two types namely skew bevel gearing and spiral gearing.

28) What are the methods to avoid interference?

- The height of the teeth may be reduced.
- The pressure angle may be increased.

- The radial flank of the pinion may be cut back (undercutting).

29) What is the advantage when arc of recess is equal to arc of approach in meshing gears?

When arc of recess equal to arc of approach, the work wasted by friction is minimum and efficiency of drive is maximum.

30) What do you know about tumbler gear?

Tumbler gears are those which are used in lathes for reversing the direction of rotation of driven gears.

31) What you meant by non-standard gear teeth?

The gear tooth obtained by modifying the standard proportions of gear teeth parameters is known as non- standard gear teeth.

32) What is meant by compound gear train?

When there are more than one gear on shaft, it is called a compound gear train.

33) What is the advantage of a compound gear train over a simple gear train?

The advantage of a compound gear train over a simple gear train is that a much larger speed reduction from the first shaft to the last shaft can be obtained with small gears.

34) State the methods to find the velocity ratio of epicyclic gear train.

Two methods are:

- Tabulation method.
- Algebraic method.

35) What is the externally applied torques used to keep the gear train in equilibrium?

- Impart torque on the driving member.
- Resisting or holding torque on the driven member.
- Holding or braking torque on the fixed member.

36) State law of gearing. (A/M-2017)

The law of gearing states that for obtaining a constant velocity ratio, at any instant of teeth the common normal at each point of contact should always pass through a pitch point, situated on the line joining the centre of rotation of the pair of mating gears.

37) What type of gear arrangement is used to traverse the carriage in lathe machine? (A/M-2017)

Rack and Pinion arrangement

38) What is meant by crossed belt drive? (N/D-2017)

In a two pulley system, the belt can either drive the pulleys normally in one direction (the same if on parallel shafts), or the belt may be crossed, so that the direction of the driven shaft is reversed (the opposite direction to the driver if on parallel shafts).

Driving shaft and driven shaft rotates in the opposite direction.

39) Write the conditions for the maximum power transmission by a belt from one pulley to another. (N/D-2017)

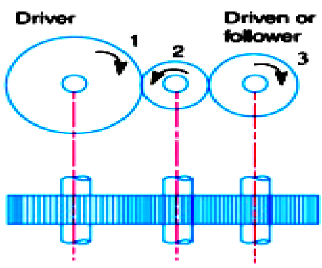
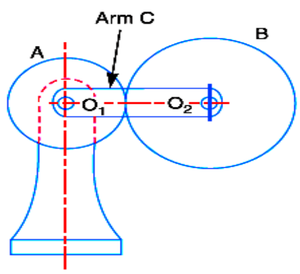
$$V_{max} = \sqrt{\frac{T}{3m}} \quad \text{and } T = 3\, T_C$$

The power transmitted shall be maximum, when the centrifugal tension (T_C) is one third the Maximum belt tension (T)

40) State the two important similarities of a spur gear pair and helical gear pair. (A/M 2018)

- They both have the same efficiency.
- They are designed to transmit power to parallel shaft.

41) Sketch an ordinary gear train and an Epicyclic gear train stating their important difference. (A/M 2018)

Ordinary Gear Train	Epicyclic Gear Train
When there is only one gear on each shaft, as shown in Fig. it is known as simple gear train. The gears are represented by their pitch circles.	When gear *A* is fixed and the arm is rotated about the axis of gear *A* (*i.e.* *O*1), then the gear *B* is forced to rotate *upon* and *around* gear *A*. Such a motion is called Epicyclic and the gear train arranged in such a manner is known as Epicyclic gear trains.
	

42) What are the advantages of Cycloidal gears? (N/D 2018) (A/M 2019)

- Having a wider flank as compared to Involute gears they are considered to have more strength and hence can withstand further load and stress.
- The contact in case of Cycloidal gears is between the concave surface and the convex flank. This results in less wear and tear.
- No interference occurs in these types of gears.

43) Define Train value of a gear train. (N/D 2018)

It may be noted that ratio of the speed of the driven or follower to the speed of the driver is known as train value of the gear train. Mathematically,

$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

44) Define module of gears and its relation to circular pitch. (A/M 2019)

Module: It is the ratio of the pitch circle diameter in millimetres to the number of teeth. It is usually denoted by *m*.

Module, m = D/T

Circular Pitch: It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by p_c.

Circular pitch, $p_c = \pi D/T$

where D = Diameter of the pitch circle, and

T = Number of teeth on the wheel.

Circular pitch, $p_c = \pi m$ *(Where m = module.)*

PART-B

1) Two gear wheels mesh externally to give a velocity ratio of 3:1. The involute teeth have 6 mm module and 200 pressure angle. The addendum has 1 module. The pinion rotates at 90 rpm. Determine:

- **a) Number of teeth an pinion to avoid interference and the corresponding number on the Wheel and Arc.**
- **b) The length of Path and Arc of contact.**
- **c) Contact Ratio.**
- **d) The maximum velocity of sliding.** **(16)**

(MAY/JUNE 2014)

Given data: $\frac{\omega_P}{\omega_G} = \frac{T_G}{T_P} = 3;\ m = 6\,\text{mm};\ a_P = a_W = 1\text{module};\ \phi = 20°;\ N_P = 90\text{rpm}$

Solution: We know that, $a_W = A_W \cdot m$

Given that, $a_W = 1 \times \text{Module}$

$\therefore$ Addendum coefficient, $A_W = A_P = 1$

(i) Number of teeth on each wheel so that interference is just avoided:
We know that minimum number of teeth required on wheel to avoid interference,

$$T_{G(min)} = \frac{2A_W}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}$$

$$= \frac{2\times 1}{\sqrt{1+\frac{1}{3}\left(\frac{1}{3}+2\right)\sin^2 20°} - 1} = 44.94 \text{ say } 45 \text{ Ans.}$$

Now the number of teeth on pinion is given by

$$T_P = \frac{T_G}{G} = \frac{45}{3} = 15 \text{ Ans.}$$

(ii) Length of path of contact:
Pitch circle radii of pinion and gear wheel are given by

$$r = \frac{mT_P}{2} = \frac{6\times 15}{2} = 45\text{mm}$$

and $$R = \frac{mT_G}{2} = \frac{6\times 45}{2} = 135\text{mm}$$

Addendum circle radii of pinion and gear wheel are given by

$$r_A = r + \text{Addendum} = 45 + 6 = 51\text{mm}$$

and $$R_A = R + \text{Addendum} = 135 + 6 = 141\text{mm}$$

Length of path of approach, $KP = \sqrt{{R_A}^2 - R^2\cos^2\phi} - R\sin\phi$

$$= \sqrt{(141)^2 - (135)^2\cos^2 20^\circ} - 135\sin 20^\circ$$

$$= 15.37\text{mm}$$

Length of path of recess, $PL = \sqrt{{r_A}^2 - r^2\cos^2\phi} - r\sin\phi$

$$= \sqrt{(51)^2 - (45)^2\cos^2 20^\circ} - 45\sin 20^\circ = 13.12\text{mm}$$

$\therefore$ Length of path of contact, $KL = KP + PL$

$$= 15.37 + 13.12 = 28.49\text{mm} \quad \text{Ans.}$$

(iii) Contact ratio

$$\therefore \text{ Contact ratio} = \frac{\text{Length of path of contact}}{\pi \times m}$$

$$= \frac{28.49}{\pi \times 6} = 1.51 \approx 2\text{Pairs}$$

(iv) Maximum velocity of sliding between the teeth:

Angular velocity of pinion, $\omega_P = \frac{2\pi N_P}{60} = \frac{2\pi \times 90}{60} = 9.42\,\text{rad/s}$

Given that, Velocity ratio $= \frac{\omega_P}{\omega_G} = \frac{T_G}{T_P} = 3$

or $$\omega_G = \frac{\omega_P}{3} = \frac{9.42}{3} = 3.14\,\text{rad/s}$$

We know that maximum velocity of sliding between teeth,

$$v_S = (\omega_P + \omega_G) \times KP \quad \text{as } KP > PL$$

$$= (9.42 + 3.14) \times 15.37$$

$$= 193.05\text{mm/s or } 0.193\text{m/s Ans.}$$

2) (i) Derive an expression to determine the length of path of contact between two spur gear of different size. (10)

(MAY/JUNE 2014)

Length of Path of Contact:

- ✓ Fig. Shows two involute gears i.e., pinion and wheel in mesh.
- ✓ When the pinion (driver) rotates in clockwise direction, the contact between a pair of teeth begins at point K and ends at point L. Therefore the length of path of contact is KL.
- ✓ Point K is located on the flank near the base circle of pinion or the outer end of the tooth face on the wheel. Similarly, point L is on the flank near the base circle of pinion. MN is the common tangent.

- ✓ The point K is the intersection of the addendum circle of wheel and the common tangent.
- ✓ The point L is the intersection of the addendum circle of pinion and the common tangent.

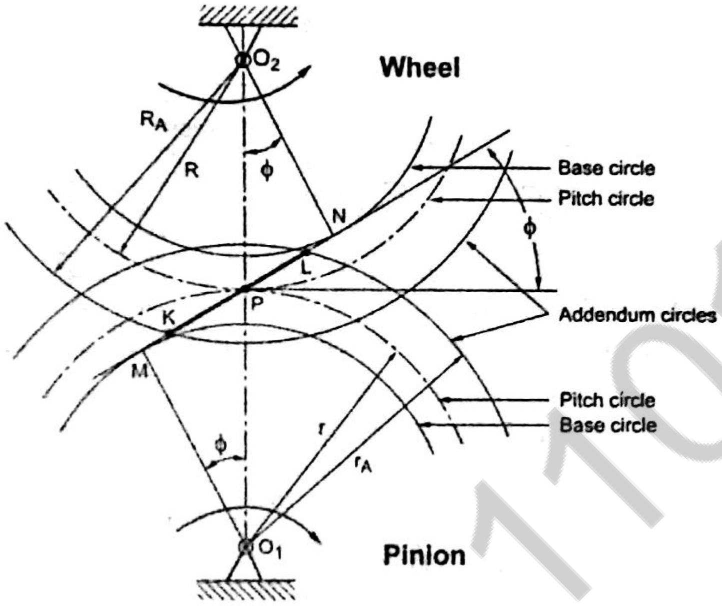


Fig. Length of path of contact

The lengths KP and PL are known as the path of approach and path of recess respectively. The total length KL is called the path of contact.

Let $r = O_1P =$ Pitch circle radius of pinion,

$R = O_2P =$ Pitch circle radius of wheel,

$r_A = O_1L =$ Addendum circle radius of pinion, and

$R_A = O_2K =$ Addendum circle radius of wheel.

From Fig., the radius of the base circle of pinion is given by

$$O_1M = O_1P\cos\phi = r\cos\phi$$

and radius of the base circle of wheel,

$$O_2N = O_2P\cos\phi = R\cos\phi$$

From right angled triangle O_2KN,

$$KN = \sqrt{(O_2K)^2 - (O_2N)^2} = \sqrt{(R_A)^2 - R^2\cos^2\phi}$$

and $PN = Q_2P\sin\phi = R\sin\phi$

$\therefore$ Length of path of approach is given by

$$KP = KN - PN$$

or $KP = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$ ....**(1)**

Similarly from right angled triangle O_1ML,

$$ML = \sqrt{(O_1L)^2 - (O_1M)^2} = \sqrt{(r_A)^2 - r^2\cos\phi}$$

and $MP = O_1P\sin\phi = r\sin\phi$

$\therefore$ Length of path of recess is given by

$$PL = ML - MP$$

or $$PL = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$ ....(2)

Then, length of the path of contact is given by

$$KL = KP + PL$$

Substituting the values of KP and PL from equations (1) and (2), we get

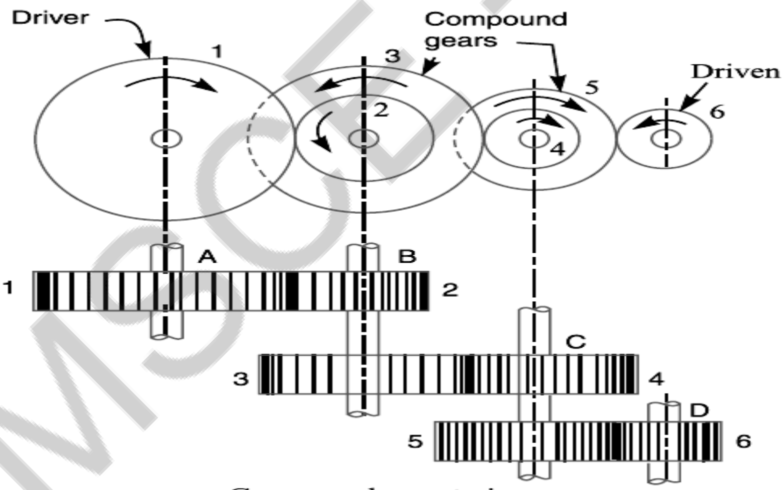
$$KL = \left[\sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi\right] + \left[\sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi\right]$$

or $$KL = \sqrt{(R_A)^2 - R^2\cos^2\phi} + \sqrt{(r_A)^2 - r^2\cos^2\phi} - (R + r)\sin\phi$$

(ii) Briefly explain the sub classification of compound gear train with neat sketches. **(6)**

(MAY/JUNE 2014)

COMPOUND GEAR TRAIN



Compound gear train.

Compound gears which are mounted on shaft C and the gear 6 is the driven gear mounted on shaft D.

Let N_1 = Speed of driving gear 1,

T_1 = Number of teeth on driving gear 1,

$N_2, N_3 ..., N_6$ = Speed of respective gears in r.p.m., and

$T_2, T_3 ..., T_6$ = Number of teeth on respective gears.

Since gear 1 is in mesh with gear 2, therefore its speed ratio is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1}$$ ....(i)

Similarly, for gears 3 and 4, speed ratio is

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \qquad \textbf{(ii)}$$

and for gears 5 and 6, speed ratio is

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \qquad \textbf{(iii)}$$

The speed ratio of compound gear train is obtained by multiplying the equations (i),(ii) and (iii),

$$\therefore \qquad \frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5} \quad \text{or} \quad \frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

REVERTED – COMPOUND GEAR TRAIN

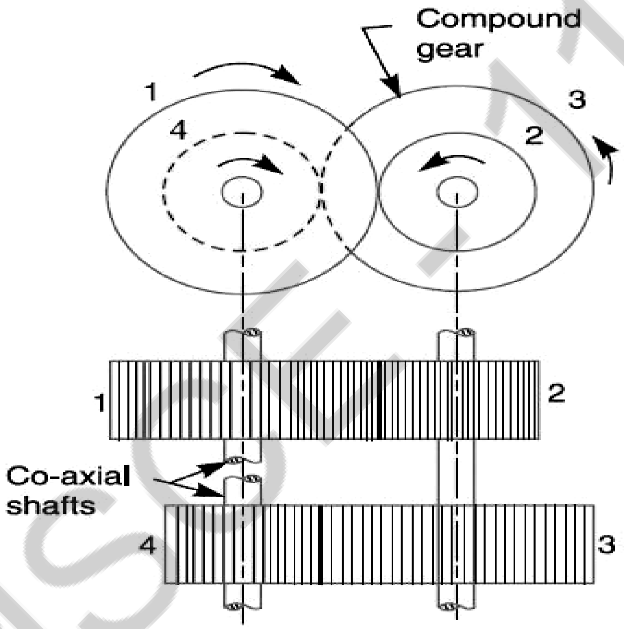


Fig. Reverted gear train.

When the axes of the first gear (i.e. first driver) and the last gear (i.e. last driven or follower) are co-axial, then the gear train is known as reverted gear train as shown in Fig.

We see that gear 1 (i.e. first driver) drives the gear 2 (i.e. first driven or follower) in the opposite direction. Since the gears 2 and 3 are mounted on the same shaft, therefore they form a compound gear and the gear 3 will rotate in the same direction as that of gear 2. The gear 3 (which is now the second driver) drives the gear 4 (i.e. the last driven or follower) in the same direction as that of gear 1. Thus we see that in a reverted gear train, the motion of the first gear and the last gear is like.

Let

T_1 = Number of teeth on gear 1,

r_1 = Pitch circle radius of gear 1, and

$N_1 = \text{Speed of gear 1 in r.p.m.}$

Similarly,

$T_2, T_3, T_4 = \text{Number of teeth on respective gears,}$

$r_2, r_3, r_4 = \text{Pitch circle radii of respective gears, and}$

$N_2, N_3, N_4 = \text{Speed of respective gears in r.p.m.}$

Since the distance between the centres of the shafts of gears 1 and 2 as well as gears 3 and 4 is same, therefore

$$r_1 + r_2 = r_3 + r_4 \qquad \textbf{(i)}$$

Also, the circular pitch or module of all the gears is assumed to be same, therefore number of teeth on each gear is directly proportional to its circumference or radius.

$$\therefore \qquad T_1 + T_2 = T_3 + T_4 \qquad \textbf{(ii)}$$

and $$\text{Speed ratio} = \frac{\text{Product of number of teeth on drivens}}{\text{Product of number of teeth on drivers}}$$

or $$\frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3} \qquad \textbf{(iii)}$$

EPICYCLIC – COMPOUND GEAR TRAIN

A compound epicyclic gear train is shown in Fig. It consists of two co-axial shafts *S*1 and *S*2, an annulus gear *A* which is fixed, the compound gear (or planet gear) *B*-*C*, the sun gear *D* and the arm *H*. The annulus gear has internal teeth and the compound gear is carried by the arm and revolves freely on a pin of the arm *H*. The sun gear is co-axial with the annulus gear and the arm but independent of them.

The annulus gear *A*meshes with the gear *B* and thesun gear *D* meshes with the gear *C*. It may be noted that when theannulus gear is fixed, the sun gearprovides the drive and when thesun gear is fixed, the annulus gear provides the drive. In both cases, the arm acts as a follower.

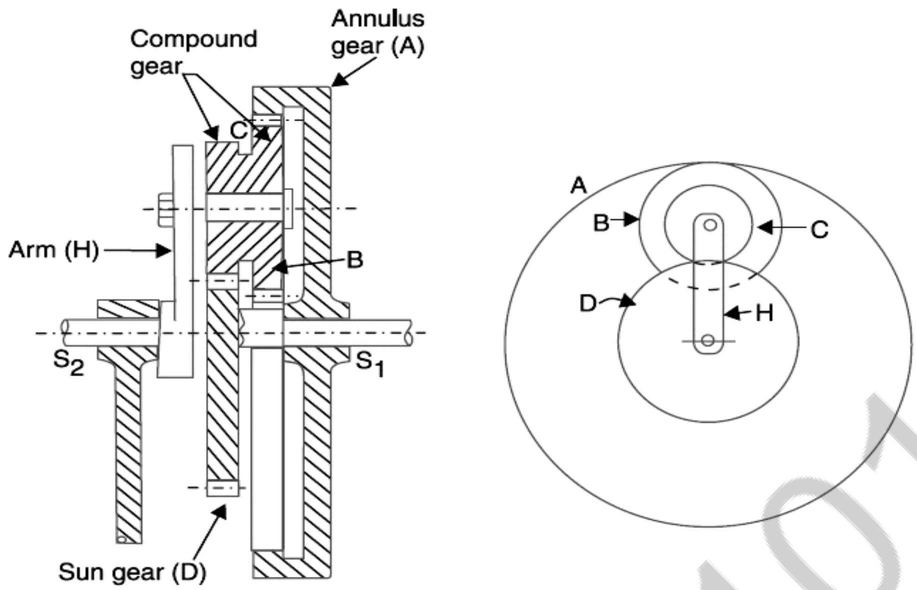


Fig. Compound epicyclic gear train.

3) Calculate:
(i) Length of path of contact
(ii) Arc of contact and
(iii) The contact ratio when a pinion having 23 teeth drives a gear having teeth 57. The profile of the gears is involute with pressure angle 20°, module 8mm and addendum equal to one module. (16)
(MAY/JUNE 2015) (MAY/JUNE 2014) (NOV/DEC 2015)

Given: $m = 8mm$

$\phi = 20°$

$T_p = 23$

$T_G = 57$

$a_p = a_w = 1 \text{module} = 8mm$

To find: (i) Contact Ratio
(ii) Angle of action of Pinion and Gear Wheel.
(iii) Ratio of sliding to Rolling velocity at beginning of contact, Pitch point, End of contact.

Solution:

(i) Contact Ratio:

Pitch circle Radii of Pinion and Gear Wheel are given by

$$r = \frac{m \cdot T_p}{2} = \frac{8 \times 23}{2} = 92mm$$

$$R = \frac{m \cdot T_G}{2} = \frac{8 \times 57}{2} = 228mm$$

Addendum radii of Pinion and Wheel

$$r_A = r + a_p = 92 + 8 = 100\text{mm}$$

$$R_A = R + a_w = 228 + 8 = 236\text{mm}$$

Length of Path of Approach:

$$KP = \sqrt{R_A^2 - R^2 \cdot \cos^2\phi} - R\sin\phi$$

$$= \sqrt{(236)^2 - (228)^2 \cdot \cos^2 20°} - 228\sin 20°$$

$$KP = 20.97\text{mm}$$

Length of path of Recess:

$$PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$$

$$= \sqrt{(100)^2 - (92)^2\cos^2 20°} - 92\sin 20°$$

$$PL = 18.79\text{mm}$$

Length of Path of Contact

$$KL = KP + PL$$

$$20.97 + 18.79$$

$$KL = 39.76\text{mm}$$

$$\text{Length of Arc of contact} \Rightarrow \frac{\text{Length of Path of contact}}{\cos\phi}$$

$$= \frac{39.76}{\cos 20°} = 42.31\text{mm}$$

$\therefore$ Contact Ratio:

$$\text{Contact Ratio} = \frac{\text{Length of arc of Contact}}{\pi \times m}$$

$$= \frac{42.31}{\pi \times 8} = 1.68 \text{ pairs}$$

(ii) Angle of Action of Pinion and Gear Wheel:

(a) Angle turned through by pinion $= \dfrac{\text{Length of arc. of contact}}{\text{Circumference of Pinion}} \times 360°$

$$= \frac{42.31}{2\pi \times 92} \times 360° = 26.34°$$

(b) Angle turned through by wheel $= \dfrac{\text{Length of arc. of contact}}{\text{Circumference of Wheel}} \times 360°$

$$= \frac{42.31}{2\pi \times 228} \times 360° = 10.63°$$

(iii) Ratio of Sliding to Rolling Motion:

$$\text{Gear Ratio: } \frac{\omega_p}{\omega_G} = \frac{T_G}{T_p}$$

$$\therefore \ \omega_p = \frac{T_G}{T_p} \times \omega_G = \frac{57}{23} \times \omega_G$$

$$\omega_p = 2.48\omega_G \text{ rad/sec}$$

Rolling Velocity:

$$V_r = \omega_p \cdot r = \omega_G \cdot R$$

$$= (2.48\omega_G) \cdot 92$$

$$V_r = 228.16\omega_G \cdot \text{mm/sec}$$

(a) At Beginning of Contact:

Sliding velocity $V_{s1} = (\omega_p + \omega_G) \times KP$

$$= (2.48\omega_G + \omega_G) \times 20.97$$

$$V_{s1} = 72.97\omega_G \text{ mm/sec}$$

Ratio of Sliding to Rolling Velocity $\left.\right\} \dfrac{V_{s1}}{V_r} = \dfrac{72.97\omega_G}{228.16\omega_G}$

$$\frac{V_{s1}}{V_r} = 0.319$$

(b) At Pitch Point:

At pitch point, the length of path of contact is zero.

$$\therefore \quad \frac{V_{s2}}{V_r} = 0$$

(c) At End of Contact:

Sliding Velocity $V_{s3} = (\omega_p + \omega_G) \times PL$

$$= (2.48\omega_G + \omega_G) \times 18.79$$

$$V_{s3} = 65.38\omega_G \text{ mm/sec}.$$

Velocity of Sliding to Rolling Velocity $\left.\right\} \dfrac{V_{s3}}{V_r} = \dfrac{65.38\omega_G}{228.16\omega_G}$

Result: (i) Contact Ratio: 1.68 pairs

(ii) Angle turned through by Pinion $= 26.34°$

Angle turned through by wheel $= 10.63°$

(iii) Ratio of sliding to Rolling Velocity

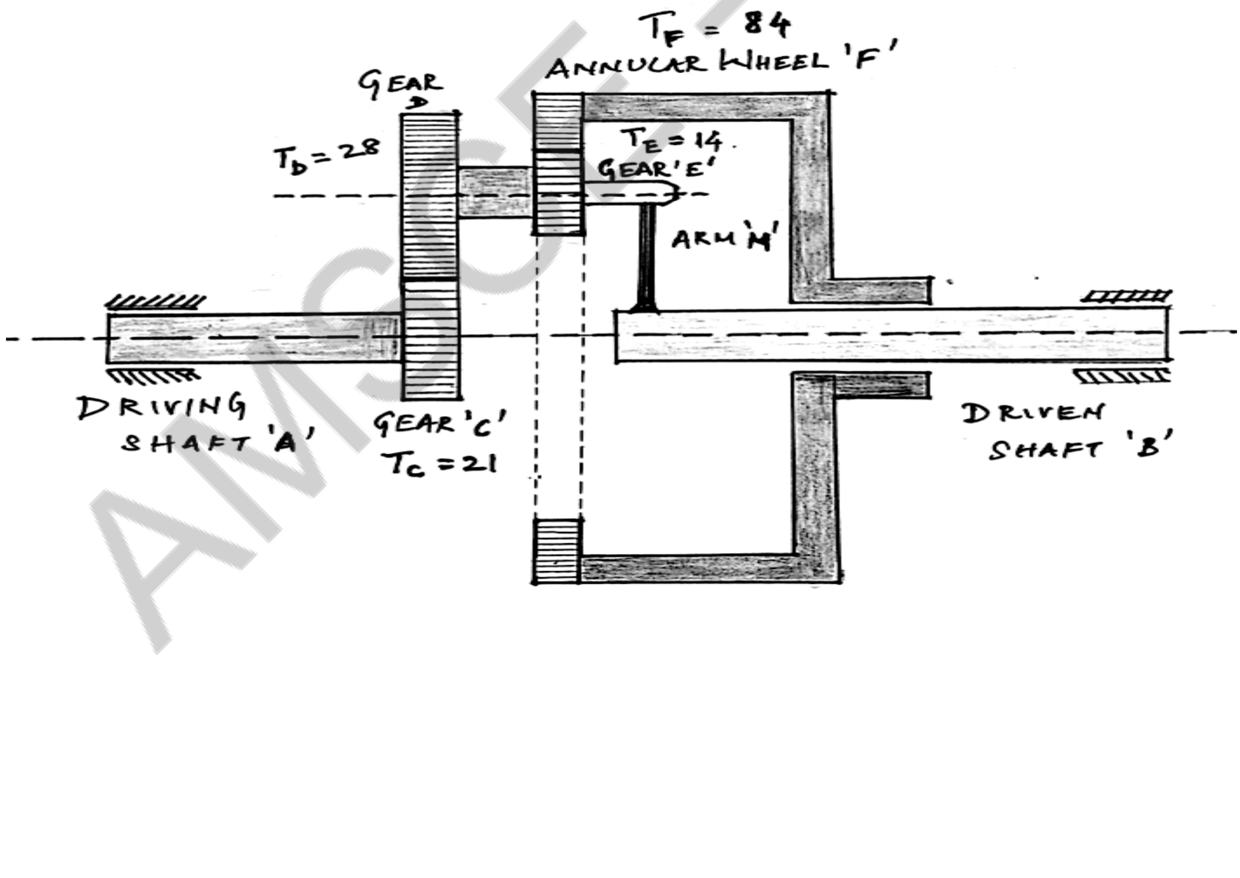
(a) At Beginning $= 0.319$

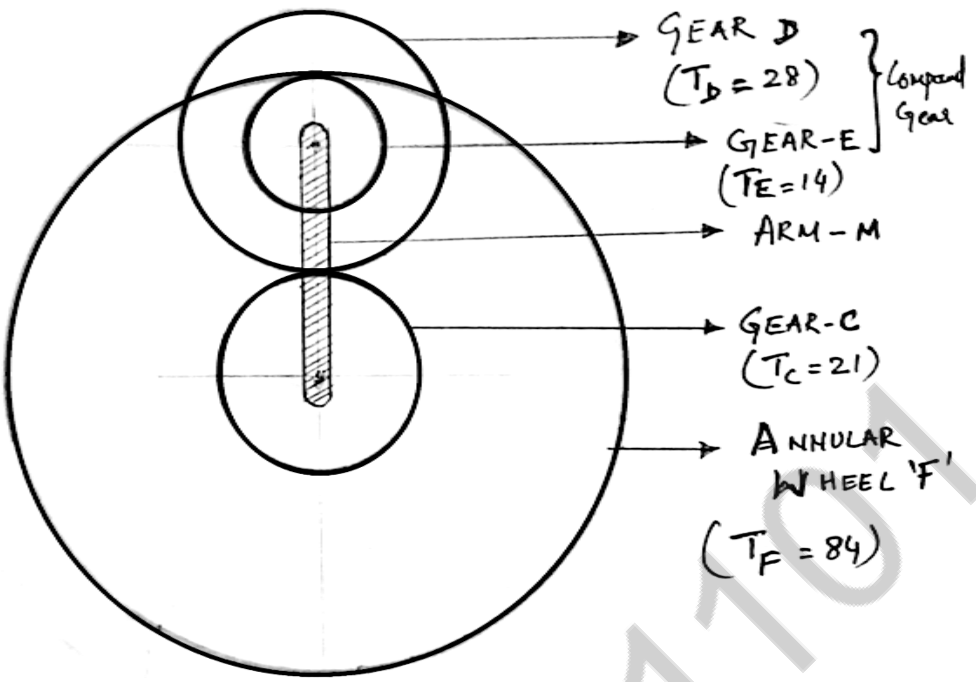
(b) At Pitch point $= 0$

(c) At End of Contact $= 0.286$

4) In an epicyclic gear train a gear C is keyed to the driving shaft A which rotates at 900 rpm. Gears D and E are fixed together and rotate freely on a pin carried by the arm M which is keyed to the driven shaft B. Gear D is in mesh with gear C while the gear E is in mesh with a fixed annular wheel F. The annular wheel is concentric with the driven shaft B. if the shafts A and B are collinear and number of teeth on gears C, D, E and F are respectively 21, 28, 14 and 84. Determine the speed and sense of rotation of the driven shaft B. (16)

(MAY/JUNE 2015)





Given:

Speed of shaft A (i.e Driving shaft) = 900 rpm

$T_C = 21$

$T_D = 28$

$T_E = 14$

$T_F = 84$

To find: Speed and sense of Rotation of Driver shaft 'B'

Solution:

S.NO	OPERATIONS	REVOLUTIONS OF ELEMENTS			
		ARM	GEAR C	COMPOUND GEAR D-E	ANNULAR WHEEL F
1	Arm fixed, Gear C rotates through (+1) revolution anti-clockwise	0	+1	$-\frac{T_D}{T_B}$	$\frac{T_D}{T_B} \times \frac{T_A}{T_F}$
2	Arm fixed, Gear C rotates through (+x) revolution anti-clockwise.	0	+ x	$-x\frac{T_D}{T_B}$	$x\left[\frac{T_D}{T_B} \times \frac{T_A}{T_F}\right]$
3	Add +y is all elements	+y	+y	+y	+y

4	**Total Motion**	**+y**	**x + y**	$y - x\frac{T_D}{T_B}$	$y - x\left[\frac{T_D}{T_B} \times \frac{T_A}{T_F}\right]$

Shaft A rotates at 900 rpm
(i.e) Gear C rotates at 900 rpm

$$x + y = 900 \quad \textbf{(1)}$$

Speed at F:

$$y - x\left[\frac{T_C}{T_D} \times \frac{T_E}{T_F}\right] = 0$$

$$y - x\left[\frac{21}{28} \times \frac{14}{84}\right] = 0$$

$$y - 0.125x = 0 \quad \textbf{(2)}$$

Substituting equation (1) and (2), we get…

$$x = 800 \quad \text{and} \quad y = 100$$

Result:

$$\left.\begin{array}{l}\text{The speed of Driven shaft 'B'}\\ (\text{i.e. Speed of the Arm})\end{array}\right\} = 100\text{rpm.}(\text{CCW})$$

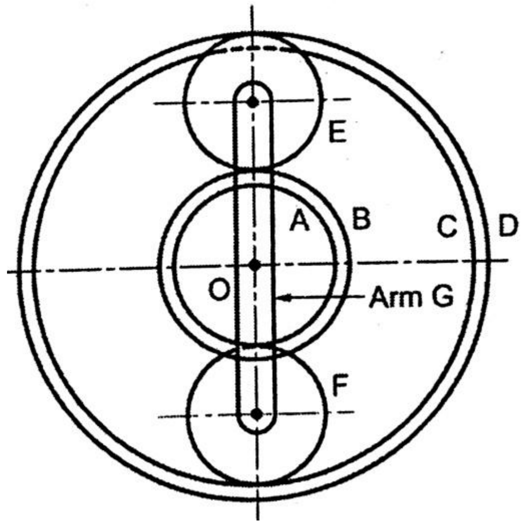
5) In the epicyclic gear train shown in Fig 14 (b) (i) the compound wheels A and B as well as interval wheels C and D rotate independently about the axis O. The wheels E and F rotate on the pins fixed to the arm a. All the wheels are of the same module. The number of teeth on the wheels are $T_A = 52,\ T_B = 56,\ T_E = T_F = 36.$

Determine:

(1) The speed of C if the wheel D fixed and arm a rotates at 200 rpm clockwise.

(2) The speed of C, if the wheel D rotates at 200 rpm counter clockwise and the arm a rotates 20 rpm couter clockwise. (8)

(NOV/DEC 2014)



Given:

$$T_A = 52$$

$$T_B = 56$$

$$T_E = T_F = 36$$

To find:

(i) Speed of C, if the wheel D is fixed and Arm 'G' rotates at 200 rpm clockwise.

(ii) Speed of C, if the wheel D rotates at 200 rpm counter-clockwise and arm 'G' rotates 20 rpm counter-clockwise.

Solution:

Number of Teeth on wheels C and D

$$d_c = d_A + 2d_F$$

$$d_D = d_B + 2d_E$$

Similarly we can write

$$T_C = T_A + 2T_F$$

$$T_D = T_B + 2T_E$$

$$T_C = 52 + 2(36) = 124$$

$$T_D = 56 + 2(36) = 128$$

S.NO	OPERATIONS	REVOLUTION OF ELEMENTS (N)					
		ARM G	WHEEL D	WHEEL E	COMPOUND WHEEL A-B	WHEEL F	WHEEL C

1	Arm fixed and Annular Wheel D rotates through + 1 Rev CCW	0	+1	$\frac{T_D}{T_E}$	$-\frac{T_D}{T_B}$	$\frac{T_D}{T_B}\times\frac{T_A}{T_F}$	$\frac{T_D}{T_B}\times\frac{T_A}{T_C}$
2	Arm fixed and Annular Wheel D rotates through + x Rev CCW	0	+ x	$+x\frac{T_D}{T_E}$	$-x\frac{T_D}{T_B}$	$x\left[\frac{T_D}{T_B}\times\frac{T_A}{T_F}\right]$	$x\left[\frac{T_D}{T_B}\times\frac{T_A}{T_C}\right]$
3	Add + y to all elements	+y	+y	+y	+y	+y	+y
4	Total Motion	**+y**	**x + y**	$\boldsymbol{y+x\frac{T_D}{T_E}}$	$\boldsymbol{y-x\frac{T_D}{T_B}}$	$\boldsymbol{y - x\left[\frac{T_D}{T_B}\times\frac{T_A}{T_F}\right]}$	$\boldsymbol{y - x\left[\frac{T_D}{T_B}\times\frac{T_A}{T_C}\right]}$

Case (i): Speed of C, if the wheel D is fixed and Arm 'G' rotates 200 rpm clockwise.

(i) Wheel D is fixed.

x + y = 0

(ii) Arm 'G' rotates at 200 rpm clockwise.

$$+y = -200\text{rpm}$$

x – 200 = 0

x = 200 rpm

Speed of C:

$Nc = y - x\left[\frac{T_D}{T_B}\times\frac{T_A}{T_C}\right]$ = - 391.70 rpm (clockwise)

Case (ii): Speed of C, if the wheel of D rotates at 200 rpm counter-clockwise and Arm 'G' rotates at 20 rpm counter-clockwise.

Arm 'G' rotates at 20 rpm. (Counter-clockwise)

$$+y = 20$$

Wheel 'D' rotates at 200 rpm (Counter-clockwise)

x +y = +200

x + 20 = 200

x = 180

Speed of C:

$Nc = y - x\left[\frac{T_D}{T_B}\times\frac{T_A}{T_C}\right]$ = **- 152.53 rpm (clockwise)**

(ii) With a neat sketch explain the working principle of differential. (8) (NOV/DEC 2014)

Differential gear of an automobile.

The differential gear used in the rear drive of an automobile. Its function is

- ► to transmit motion from the engine shaft to the rear driving wheels, and
- ► to rotate the rear wheels at different speeds while the automobile is taking a turn.

As long as the automobile is running on a straight path, the rear wheels are driven directly by the engine and speed of both the wheels is same. But when the automobile is taking a turn, the outer wheel will run faster than theinner wheel because at that time the outer rear wheel has to cover more distance than the inner rear wheel. This is achieved by epicyclic gear train with bevel gears as shown in Fig.

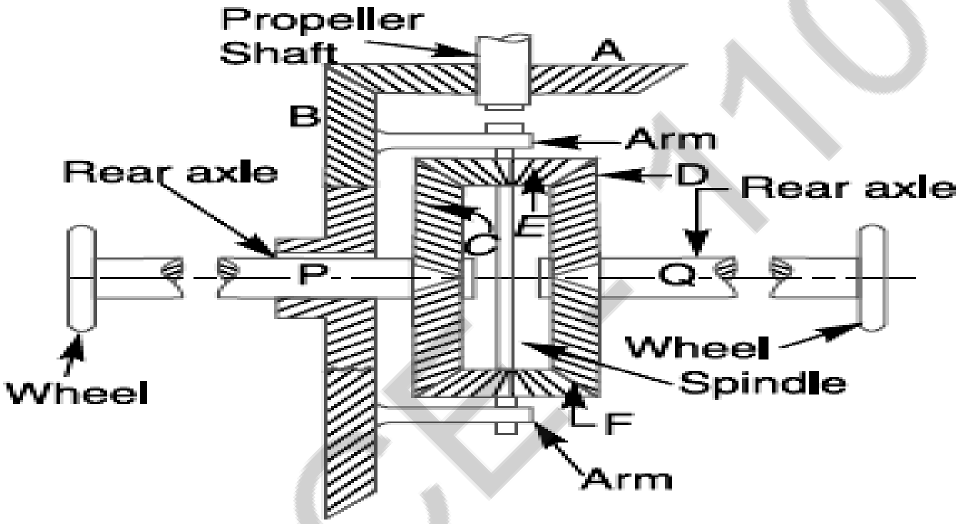


Fig: Differential

The bevel gear *A*(known as pinion) is keyed to the propeller shaft driven from the engine shaft through universal coupling. This gear *A* drives the gear *B* (known as crown gear) which rotates freely on the axle *P*. Two equal gears *C* and *D* are mounted on two separate parts *P* and *Q* of the rear axles respectively. These gears, in turn, mesh with equal pinions *E* and *F* which can rotate freely on the spindle provided on the arm attached to gear *B*. When the automobile runs on a straight path, the gears *C* and *D* must rotate together. These gears are rotated through the spindle on the gear *B*. The gears *E* and *F* do not rotate on the spindle. But when the automobile is taking a turn, the inner rear wheel should have lesser speed than the outer rear wheel and due to relative speed of the inner and outer gears *D* and *C*, the gears *E* and *F* start rotating about the spindle axis and at the same time revolve about the axle axis.

6) The arm of an epicyclic gear train rotates at 100 rpm in the anti-clockwise direction. The arm carries two wheels A and B having 36 and 45 teeth respectively. The wheel A is fixed and the arm rotates about

the centre of wheel A. Find the speed of wheel B. What will be the speed of B, if the wheel A instead of being fixed, makes 200 rpm clockwise. **(16)**

(NOV/DEC 2015)

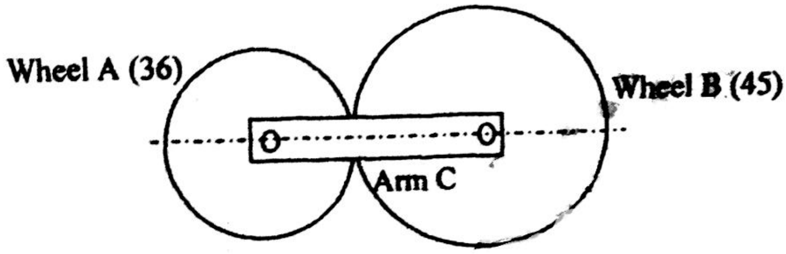


Fig 14(b)

Given

$T_A = 36$

$T_B = 45$

$N_C = 100 rpm (CCW)$

To find: (i) Speed of Gear B

(ii) Speed of B, if Gear A rotates at 200rpm clockwise.

Solution:

S.No	Condition of Motion	Revolution of Elements		
		Arm 'C'	**Gear'A'**	**Gear'B'**
1.	Arm fixed, Gear A rotates through (+1) revolution anti-clockwise	0	+1	$-\frac{T_A}{T_B}$
2.	Arm fixed, Gear A rotates through (+x) revolution anti-clockwise	0	+x	$-x \cdot \frac{T_A}{T_B}$
3.	Add +y is all elements	+y	+y	+y
4	**Total Motion**	**y**	**x+y**	$y - x \cdot \frac{T_A}{T_B}$

(i) Speed of Gear B, when Gear A is fixed:

Speed of the arm = 100 rpm (CCW)

Gear A is fixed

$$x + y = 0$$

$$x = -y = -100$$

Speed of Gear B:

$$N_B = y - x\left[\frac{T_A}{T_B}\right]$$

$$= y - x\left[\frac{36}{45}\right]$$

$$N_B = 100 + 100\left(\frac{36}{45}\right)$$

$$N_B = 180\text{rpm}[CCW]$$

(ii) Speed of Gear B, when Gear A makes 200 rpm clockwise:

$$x + y = -200$$

$$x + 100 = -200$$

$$x = -300\text{rpm}$$

Speed of Gear B:

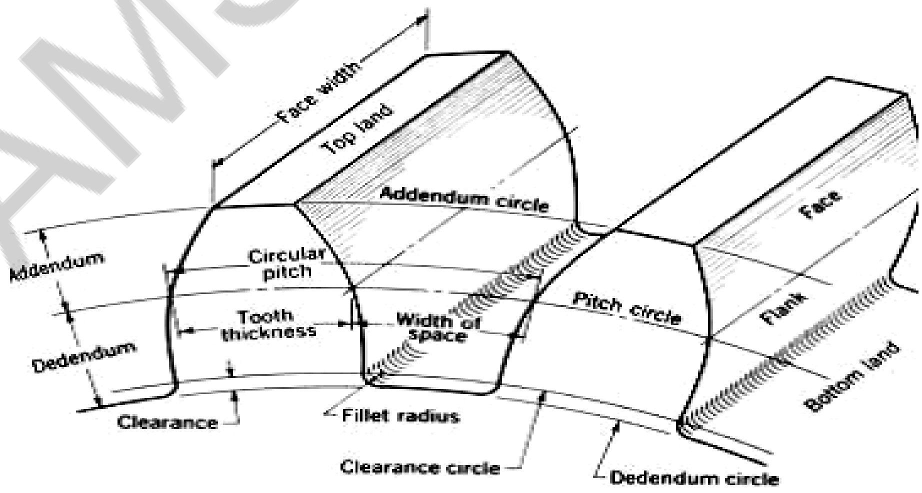
$$N_B = y - x\left[\frac{T_A}{T_B}\right]$$

$$= 100 + 300\left(\frac{36}{45}\right)$$

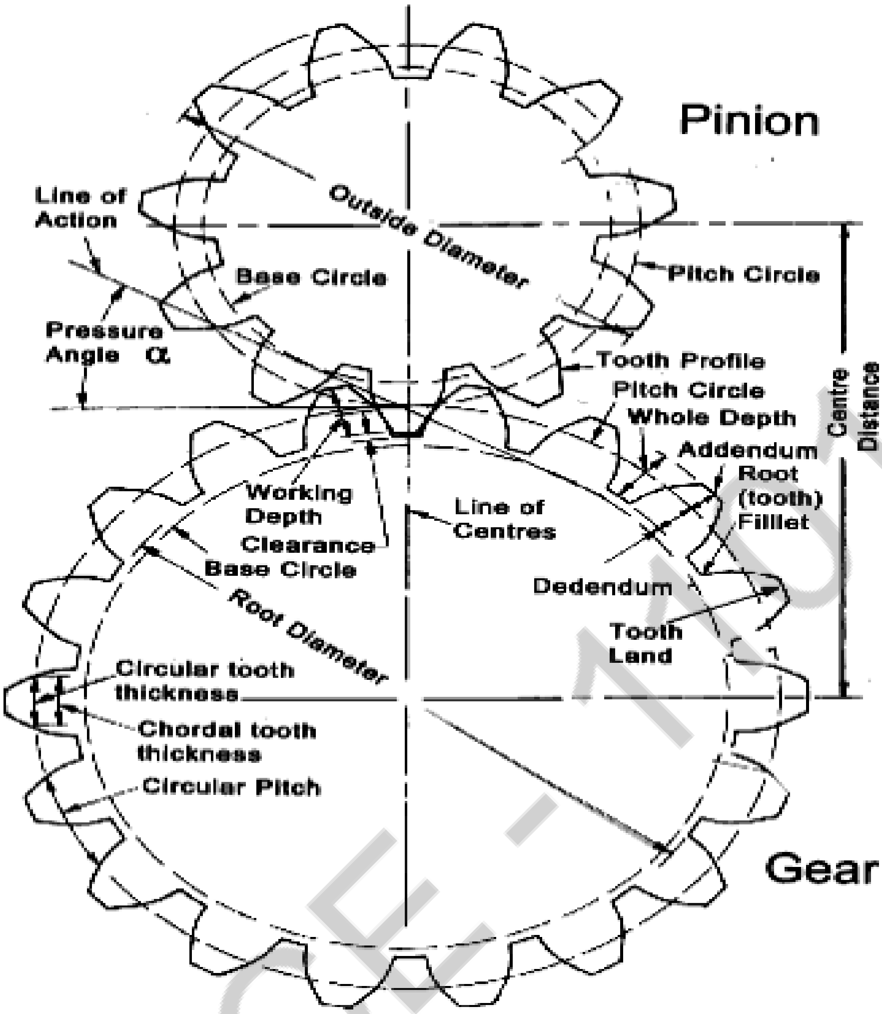
$$N_B = 340\text{rpm}(CCW)$$

7) Explain Gear nomenclature with a neat diagram and define all the salient terms pertaining to Gears. (16)

(MAY/JUNE 2016)



<u>Gear Terminology:</u>



- Pitch circle. It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.
- Pitch circle diameter. It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as pitch diameter.
- Pitch point. It is a common point of contact between two pitch circles.
- Pitch surface. It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.
- Pressure angle or angle of obliquity. It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point.
- Addendum. It is the radial distance of a tooth from the pitch circle to the top of the tooth.
- Dedendum. It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.
- Addendum circle. It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

- Dedendum circle. It is the circle drawn through the bottom of the teeth. It is also called root circle.
- Circular pitch. It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth.

$$p_c = \frac{\pi D}{z}$$

Where, D = Diameter of the pitch circle, and
z = Number of teeth on the wheel.

- Diametral pitch. It is the ratio of number of teeth to the pitch circle diameter in millimetres.

$$p_d = \frac{z}{D} = \frac{\pi}{p_c}$$

- Module. It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by m.

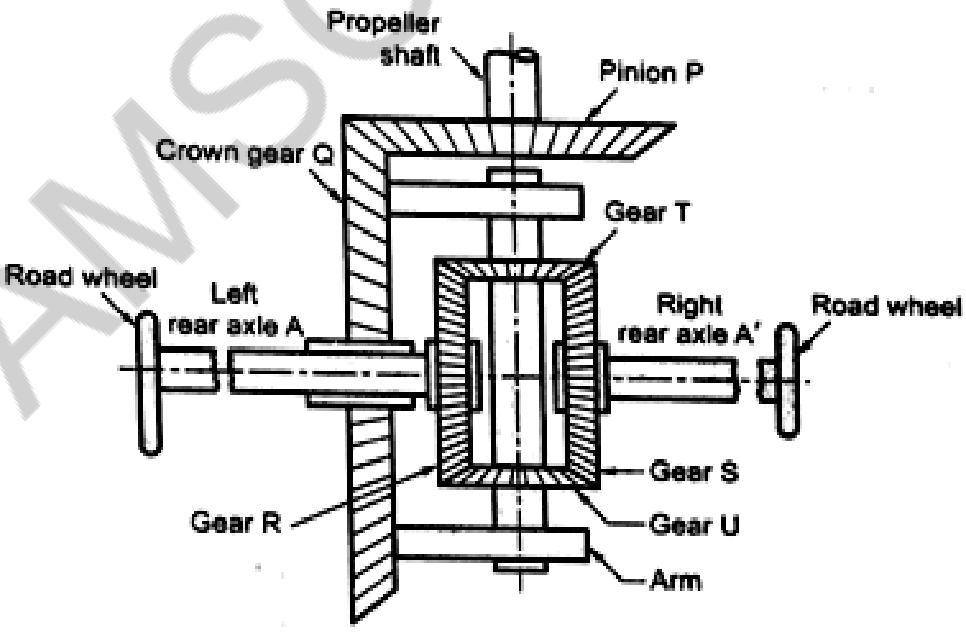
Module, $$m = \frac{D}{z}$$

- Clearance. It is the radial distance from the top of the tooth to the bottom of the tooth, in a meshing gear. A circle passing through the top of the meshing gear is known as clearance circle.
- Total depth. It is the radial distance between the addendum and the dedendum circles of a gear. It is equal to the sum of the addendum and dedendum.
- Working depth. It is the radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.
- Tooth thickness. It is the width of the tooth measured along the pitch circle.
- Tooth space. It is the width of space between the two adjacent teeth measured along the pitch circle.
- Backlash. It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle. Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.
- Face of tooth. It is the surface of the gear tooth above the pitch surface.
- Flank of tooth. It is the surface of the gear tooth below the pitch surface.
- Top land. It is the surface of the top of the tooth.

- ✪ Face width. It is the width of the gear tooth measured parallel to its axis.
- ✪ Profile. It is the curve formed by the face and flank of the tooth.
- ✪ Fillet radius. It is the radius that connects the root circle to the profile of the tooth.
- ✪ Path of contact. It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.
- ✪ Length of the path of contact. It is the length of the common normal cut-off by the addendum circles of the wheel and pinion.
- ✪ Arc of contact. It is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. The arc of contact consists of two parts, i.e.
    - o Arc of approach. It is the portion of the path of contact from the beginning of the engagement to the pitch point.
    - o Arc of recess. It is the portion of the path of contact from the pitch point to the end of the engagement of a pair of teeth.

8) Fig. shows a differential gear used in motor car. The pinion A of the propeller shaft has 12 teeth and gears with crown Gear B which has 60 teeth. The shaft P and Q form the rear axle to which the road wheels are attached. If the propeller shaft rotates at 1000 rpm and the rod wheels attached to Axle Q has a speed of 210 rpm, while taking a turn. Find the speed of road wheel attached to axle P. (16)

(MAY/JUNE 2016)



Given data: $T_P = 12;\ T_Q = 60;\ N_P = 1000\text{rpm};\ N_A' = N_S = 210\text{rpm}$

Solution: The differential epicyclic gear train⁺ is shown in Fig.
We know that velocity ratio for pinion and crown gear meshing,

$$\frac{N_Q}{N_P} = \frac{T_P}{T_Q}$$

$$\therefore \text{ Speed of crown gear Q, } N_Q = \frac{T_P}{T_Q \times} N_P = \frac{12}{60} \times 1000 = 200\text{rpm}$$

Step No.	Operations	Revolutions of elements (N)			
		Gear Q	Gear R	Gear T	Gear S
1.	Fix the gear Q and give gear R +1 revolution (CCW)	0	+1	$+\frac{T_R}{T_T}$	$-\frac{T_R}{T_T} \times \frac{T_T}{T_S} = -1$ $[\because T_R = T_S]$
2.	Multiply by x	0	+x	$+x\frac{T_R}{T_T}$	-x
3.	Add +y revolutions to all elements	+y	+y	+y	+y
4.	**Total motion**	**y**	**x + y**	$y + x\frac{T_R}{T_T}$	**y - x**

Given conditions are:

(i) Gear Q rotates at 200 rpm, so

$y = +200\text{rpm}$ (i)

(ii) Road wheel attached to axle A'**i.e., the gear D rotates at 210 rpm, so**

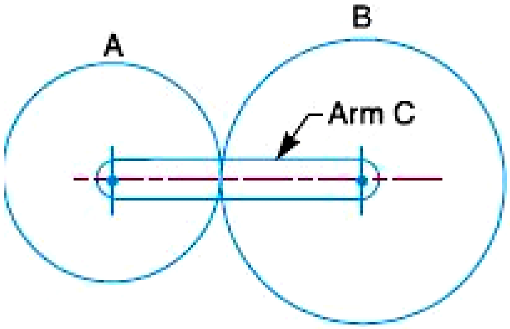
$y - x - 210$ (ii)

Solving equations (i) and (ii), we get

$x = -10\text{rpm and } y = +200\text{rpm}$

$$\therefore \text{ Speed of road wheel attached to axle A} = \text{Speed of gear R} = x + y$$
$$= -10 + 200 = +190\text{rpm}$$

9) In an epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 r.p.m. in the anticlockwise direction about the centre of the gear A which is fixed, determine the speed of gear B. If the gear A instead of being fixed makes 300 r.p.m. in the clockwise direction, what will be the speed of gear B?



Given

T_A = 36 ;

T_B = 45 ;

N_C = 150 r.p.m.(anticlockwise)

1. Tabular method

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear *A* rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear *A* rotates through + *x* revolutions	0	+ *x*	$-x \times \frac{T_A}{T_B}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y - x \times \frac{T_A}{T_B}$

Speed of gear B when gear A is fixed

Since the speed of arm is 150 r.p.m. anticlockwise, therefore from the fourth row of the table,

$$y = +\ 150 \text{ r.p.m.}$$

$$y = +\ 150 \text{ r.p.m.}$$

Also the gear *A* is fixed, therefore

$$x + y = 0 \text{ or } x = -y = -150 \text{ r.p.m.}$$

$$\text{Speed of gear } B,\quad N_B = y - x \times \frac{T_A}{T_B} = 150 + 150 \times \frac{36}{45} = +\ 270 \text{ r.p.m.}$$

Speed of gear B when gear A makes 300 r.p.m. clockwise

Since the gear A makes 300 r.p.m. clockwise, therefore from the fourth row of the table,

x + y = – 300
or x = – 300 – y = – 300 – 150 = – 450 r.p.m.

Speed of gear B

$$N_B = y - x \times \frac{T_A}{T_B} = 150 + 450 \times \frac{36}{45} = +510 \text{ r.p.m.}$$

2. *Algebraic method*
Let N_A = Speed of gear *A*.

N_B = Speed of gear *B*, and

N_C = Speed of arm *C*.

Assuming the arm *C* to be fixed, speed of gear *A* relative to arm $C = N_A - N_C$

and speed of gear *B* relative to arm $C = N_B - N_C$

Since the gears *A* and *B* revolve in ***opposite*** directions, therefore

$$\frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B}$$ ..

Speed of gear B when gear A is fixed

When gear *A* is fixed, the arm rotates at 150 r.p.m. in the anticlockwise direction, *i.e.*

$$N_A = 0, \quad \text{and} \quad N_C = +150 \text{ r.p.m.}$$

$$\therefore \quad \frac{N_B - 150}{0 - 150} = -\frac{36}{45} = -0.8$$ ...[From equation

or $$N_B = -150 \times -0.8 + 150 = 120 + 150 = 270 \text{ r.p.m.}$$ **Ans.**

Speed of gear B when gear A makes 300 r.p.m. clockwise

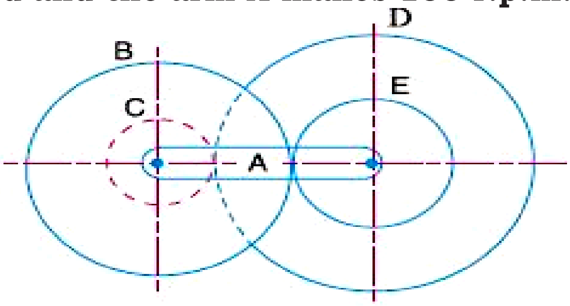
Since the gear *A* makes 300 r.p.m. clockwise, therefore

$$N_A = -300 \text{ r.p.m.}$$

$$\therefore \quad \frac{N_B - 150}{-300 - 150} = -\frac{36}{45} = -0.8$$

or $$N_B = -450 \times -0.8 + 150 = 360 + 150 = 510 \text{ r.p.m.}$$ **Ans.**

10) In a reverted epicyclic gear train, the arm A carries two gears B and C and a compound gear D - E. The gear B meshes with gear E and the gear C meshes with gear D. The number of teeth on gears B, C and D are 75, 30 and 90 respectively. Find the speed and direction of gear C when gear B is fixed and the arm A makes 100 r.p.m. clockwise.



Given :

$T_B = 75$;

$T_C = 30$;

$T_D = 90$;

$N_A = 100$ r.p.m. (clockwise)

Solution:

The reverted epicyclic gear train is shown in Fig. First of all, let us find the number of teeth on gear E (T_E).

Let d_B ,d_C , d_D and d_E be the pitch circle diameters of gears B, C, D and E respectively. From the geometry of the figure,

$d_B + d_E = d_C + d_D$

Since the number of teeth on each gear, for the same module, are proportional to their pitch circle diameters, therefore $T_B + T_E = T_C + T_D$

$T_E = T_C + T_D - T_B = 30 + 90 - 75 = 45$

The table of motions is drawn as

Step No.	Conditions of motion	Revolutions of elements			
		Arm A	Compound gear D-E	Gear B	Gear C
1.	Arm fixed-compound gear *D-E* rotated through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+1	$-\frac{T_E}{T_B}$	$-\frac{T_D}{T_C}$
2.	Arm fixed-compound gear *D-E* rotated through $+x$ revolutions	0	$+x$	$-x\times\frac{T_E}{T_B}$	$-x\times\frac{T_D}{T_C}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y-x\times\frac{T_E}{T_B}$	$y-x\times\frac{T_D}{T_C}$

Since the gear B is fixed, therefore from the fourth row of the table,

$$y - x \times \frac{T_E}{T_B} = 0 \quad \text{or} \quad y - x \times \frac{45}{75} = 0$$

$\therefore$ $$y - 0.6 = 0 \qquad ...(i)$$

Also the arm A makes 100 r.p.m. clockwise, therefore

$$y = -100 \qquad ...(ii)$$

Substituting $y = -100$ in equation (i), we get

$$-100 - 0.6x = 0 \quad \text{or} \quad x = -100/0.6 = -166.67$$

From the fourth row of the table, speed of gear C,

$$N_C = y - x \times \frac{T_D}{T_C} = -100 + 166.67 \times \frac{90}{30} = +400 \text{ r.p.m.}$$

$$= 400 \text{ r.p.m. (anticlockwise)}$$ **Ans.**

11) An epicyclic gear consists of three gears A, B and C as shown in Figure. The gear A has 72 internal teeth and gear C has 32 external teeth. The gear B meshes with both A and C and is carried on an arm EF which rotates about the centre of A at 18 r.p.m.. If the gear A is fixed, determine the speed of gears B and C.

Given :
$T_A = 72$;
$T_C = 32$;
Speed of arm EF = 18 r.p.m.
Considering the relative motion of rotation

Solution:

Step No.	Conditions of motion	Revolutions of elements			
		Arm EF	Gear C	Gear B	Gear A
1.	Arm fixed-gear C rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_C}{T_B}$	$-\frac{T_C}{T_B}\times\frac{T_B}{T_A}=-\frac{T_C}{T_A}$
2.	Arm fixed-gear C rotates through $+x$ revolutions	0	$+x$	$-x\times\frac{T_C}{T_B}$	$-x\times\frac{T_C}{T_A}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y-x\times\frac{T_C}{T_B}$	$y-x\times\frac{T_C}{T_A}$

Speed of gear C

We know that the speed of the arm is 18 r.p.m. therefore,

$$y = 18 \text{ r.p.m.}$$

and the gear A is fixed, therefore

$$y - x\times\frac{T_C}{T_A} = 0 \quad \text{or} \quad 18 - x\times\frac{32}{72} = 0$$

$\therefore$ $x = 18 \times 72 / 32 = 40.5$

$\therefore$ Speed of gear C $= x + y = 40.5 + 18$

$= + 58.5$ r.p.m.

$= 58.5$ r.p.m. in the direction of arm. **Ans.**

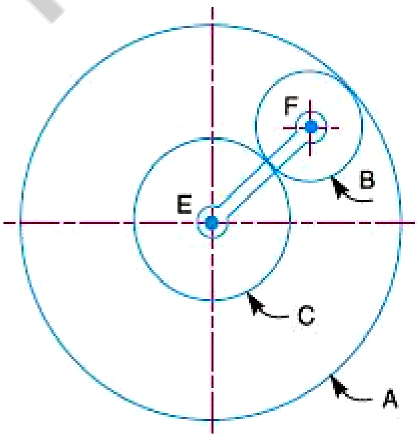


Fig.

Speed of gear B

Let d_A, d_B and d_C be the pitch circle diameters of gears A, B and C respectively. Therefore, from the geometry of Fig.

$$d_B + \frac{d_C}{2} = \frac{d_A}{2} \quad \text{or} \quad 2\,d_B + d_C = d_A$$

Since the number of teeth are proportional to their pitch circle diameters, therefore

$$2\,T_B + T_C = T_A \quad \text{or} \quad 2\,T_B + 32 = 72 \quad \text{or} \quad T_B = 20$$

$\therefore$ Speed of gear B $= y - x\times\frac{T_C}{T_B} = 18 - 40.5\times\frac{32}{20} = -46.8$ r.p.m.

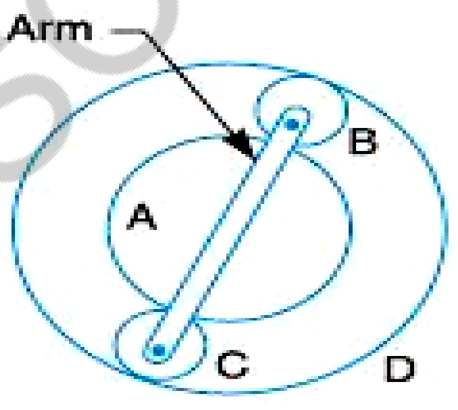
$= 46.8$ r.p.m. in the opposite direction of arm. **Ans.**

12) An epicyclic train of gears is arranged as shown in Figure. How many revolutions does the arm, to which the pinions B and C are attached, make?
1. When A makes one revolution clockwise and D makes half a revolution anticlockwise, and
2. When A makes one revolution clockwise and D is stationary? The number of teeth on the gears A and D are 40 and 90 respectively.

Given :
$T_A = 40$;
$T_D = 90$

Solution:
First of all, let us find the number of teeth on gears B and C (i.e. T_B and T_C).
Let d_A, d_B, d_C and d_D be the pitch circle diameters of gears A, B, C and D respectively.

Therefore from the geometry of the figure,
$d_A + d_B + d_C = d_D$ or
$d_A + 2\,d_B = d_D$
Since the number of teeth are proportional to their pitch circle diameters, therefore,
$T_A + 2\,T_B = T_D$ or $40 + 2\,T_B = 90$
$\boldsymbol{T_B = 25}$**, and** $\boldsymbol{T_C = 25}$



Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear A	Compound gear B-C	Gear D
1.	Arm fixed, gear A rotates through – 1 revolution (*i.e.* 1 rev. clockwise)	0	– 1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B}\times\frac{T_B}{T_D}=+\frac{T_A}{T_D}$
2.	Arm fixed, gear A rotates through $-x$ revolutions	0	$-x$	$+x\times\frac{T_A}{T_B}$	$+x\times\frac{T_A}{T_D}$
3.	Add $-y$ revolutions to all elements	$-y$	$-y$	$-y$	$-y$
4.	Total motion	$-y$	$-x-y$	$x\times\frac{T_A}{T_B}-y$	$x\times\frac{T_A}{T_D}-y$

1. ***Speed of arm when A makes 1 revolution clockwise and D makes half revolution anticlockwise***

Since the gear A makes 1 revolution clockwise, therefore from the fourth row of the table,

$$-x-y=-1 \quad \text{or} \quad x+y=1 \qquad ...(i)$$

Also, the gear D makes half revolution anticlockwise, therefore

$$x\times\frac{T_A}{T_D}-y=\frac{1}{2} \quad \text{or} \quad x\times\frac{40}{90}-y=\frac{1}{2}$$

$$\therefore \quad 40x-90y=45 \quad \text{or} \quad x-2.25y=1.125 \qquad ...(ii)$$

From equations *(i)* and *(ii)*, $x = 1.04$ and $y = -0.04$

$$\therefore \quad \text{Speed of arm} = -y = -(-0.04) = +0.04$$

= 0.04 revolution anticlockwise **Ans.**

2. ***Speed of arm when A makes 1 revolution clockwise and D is stationary***

Since the gear A makes 1 revolution clockwise, therefore from the fourth row of the table,

$$-x-y=-1 \quad \text{or} \quad x+y=1 \qquad ...(iii)$$

Also the gear D is stationary, therefore

$$x\times\frac{T_A}{T_D}-y=0 \quad \text{or} \quad x\times\frac{40}{90}-y=0$$

$$\therefore \quad 40x-90y=0 \quad \text{or} \quad x-2.25y=0 \qquad ...(iv)$$

From equations *(iii)* and *(iv)*,

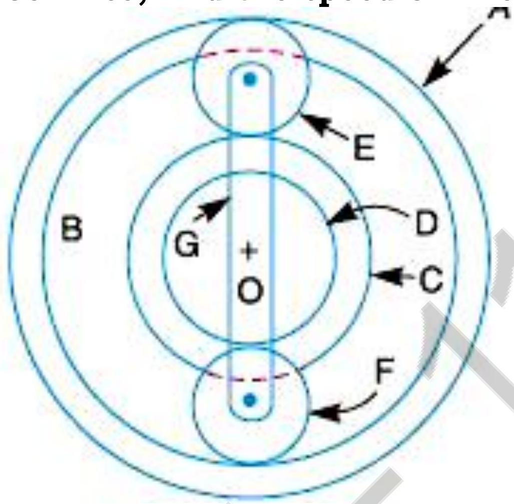
$$x = 0.692 \quad \text{and} \quad y = 0.308$$

$\therefore$ Speed of arm $= -y = -0.308$ = 0.308 revolution clockwise **Ans.**

13) In an epicyclic gear train, the internal wheels A and B and compound wheels C and D rotate independently about axis O. The

wheels E and F rotate on pins fixed to the arm G. E gears with A and C and F gears with B and D. All the wheels have the same module and the number of teeth are : T_C = 28; T_D = 26; T_E = T_F = 18.

i. **Sketch the arrangement;**
ii. **Find the number of teeth on A and B;**
iii. **If the arm G makes 100 r.p.m. clockwise and A is fixed, find the speed of B; and**
iv. **If the arm G makes 100 r.p.m. clockwise and wheel A makes 10 r.p.m. counter clockwise; find the speed of wheel B.**



Given :

T_C = 28;

T_D = 26;

T_E = T_F = 18

2. *Number of teeth on wheels A and B*

Let T_A = Number of teeth on wheel A, and

T_B = Number of teeth on wheel B.

If d_A, d_B, d_C, d_D, d_E and d_F are the pitch circle diameters of wheels A, B, C, D, E and F respectively, then from the geometry of Fig. 13.12,

$$d_A = d_C + 2\,d_E$$

and

$$d_B = d_D + 2\,d_F$$

Since the number of teeth are proportional to their pitch circle diameters, for the same module, therefore

$$T_A = T_C + 2\,T_E = 28 + 2 \times 18 = 64 \qquad \textbf{Ans.}$$

and

$$T_B = T_D + 2\,T_F = 26 + 2 \times 18 = 62 \qquad \textbf{Ans.}$$

3. *Speed of wheel B when arm G makes 100 r.p.m. clockwise and wheel A is fixed*

First of all, the table of motions is drawn as given below :

Step No.	Conditions of motion	Revolutions of elements					
		Arm G	Wheel A	Wheel E	Compound wheel C-D	Wheel F	Wheel B
1.	Arm fixed- wheel A rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$+\frac{T_A}{T_E}$	$-\frac{T_A}{T_E}\times\frac{T_E}{T_C}$ $=-\frac{T_A}{T_C}$	$+\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$+\frac{T_A}{T_C}\times\frac{T_D}{T_F}\times\frac{T_F}{T_B}$ $=+\frac{T_A}{T_C}\times\frac{T_D}{T_B}$
2.	Arm fixed-wheel A rotates through $+x$ revolutions	0	$+x$	$+x\times\frac{T_A}{T_E}$	$-x\times\frac{T_A}{T_C}$	$+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_B}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y+x\times\frac{T_A}{T_E}$	$y-x\times\frac{T_A}{T_C}$	$y+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$y+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_B}$

Since the arm G makes 100 r.p.m. clockwise, therefore from the fourth row of the table,

$$y = -100 \quad ...(i)$$

Also, the wheel A is fixed, therefore from the fourth row of the table,

$$x + y = 0 \quad \text{or} \quad x = -y = 100 \quad ...(ii)$$

$$\therefore \text{ Speed of wheel } B = y + x\times\frac{T_A}{T_C}\times\frac{T_D}{T_B} = -100 + 100\times\frac{64}{28}\times\frac{26}{62} = -100 + 95.8 \text{ r.p.m.}$$

$$= -4.2 \text{ r.p.m.} = 4.2 \text{ r.p.m. clockwise} \quad \textbf{Ans.}$$

4. *Speed of wheel B when arm G makes 100 r.p.m. clockwise and wheel A makes 10 r.p.m. counter clockwise*

Since the arm G makes 100 r.p.m. clockwise, therefore from the fourth row of the table

$$y = -100 \quad ...(iii)$$

Also the wheel A makes 10 r.p.m. counter clockwise, therefore from the fourth row of the table,

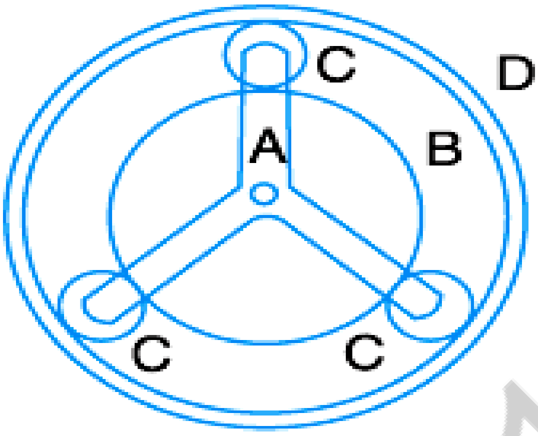
$$x + y = 10 \quad \text{or} \quad x = 10 - y = 10 + 100 = 110 \quad ...(iv)$$

$$\therefore \text{ Speed of wheel } B = y + x\times\frac{T_A}{T_C}\times\frac{T_D}{T_B} = -100 + 110\times\frac{64}{28}\times\frac{26}{62} = -100 + 105.4 \text{ r.p.m.}$$

$$= +5.4 \text{ r.p.m.} = 5.4 \text{ r.p.m. counter clockwise} \quad \textbf{Ans.}$$

14) In an epicyclic gear of the 'sun and planet' type shown in Figure the pitch circle diameter of the internally toothed ring is to be 224 mm and the module 4 mm. When the ring D is stationary, the spider A,

which carries three planet wheels C of equal size, is to make one revolution in the same sense as the sunwheel B for every five revolutions of the driving spindle carrying the sunwheel B. Determine suitable numbers of teeth for all the wheels.



Given;

d_D = 224 mm ;

m = 4 mm;

Solution:

$N_A = N_B / 5$

Let T_B , T_C and T_D be the number of teeth on the sun wheel *B*, planet wheels *C* and the internally toothed ring *D*.

The table of motions is given below:

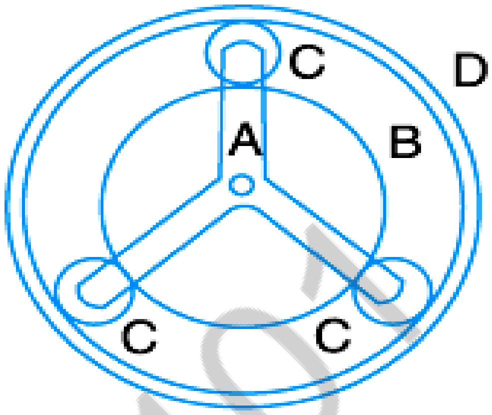
Step No.	Conditions of motion	Revolutions of elements			
		Spider A	Sun wheel B	Planet wheel C	Internal gear D
1.	Spider *A* fixed, sun wheel *B* rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+1	$-\frac{T_B}{T_C}$	$-\frac{T_B}{T_C}\times\frac{T_C}{T_D}=-\frac{T_B}{T_D}$
2.	Spider *A* fixed, sun wheel *B* rotates through + *x* revolutions	0	$+x$	$-x\times\frac{T_B}{T_C}$	$-x\times\frac{T_B}{T_D}$
3.	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y-x\times\frac{T_B}{T_C}$	$y-x\times\frac{T_B}{T_D}$

We know that when the sun wheel B makes + 5 revolutions, the spider A makes + 1 revolution. Therefore from the fourth row of the table,

$$y = +1\ ; \text{ and } x + y = +5$$

$$\therefore \quad x = 5 - y = 5 - 1 = 4$$

Since the internally toothed ring D is stationary, therefore from the fourth row of the table,



$$y - x \times \frac{T_B}{T_D} = 0$$

or $$1 - 4 \times \frac{T_B}{T_D} = 0$$

$$\therefore \quad \frac{T_B}{T_D} = \frac{1}{4} \quad \text{or} \quad T_D = 4\,T_B \qquad ...(i)$$

We know that $T_D = d_D / m = 224 / 4 = 56$ **Ans.**

$\therefore$ $T_B = T_D / 4 = 56 / 4 = 14$ **Ans.** ...[From equation (*i*)]

Let d_B, d_C and d_D be the pitch circle diameters of sun wheel B, planet wheels C and internally toothed ring D respectively. Assuming the pitch of all the gears to be same, therefore from the geometry of Fig. 13.13,

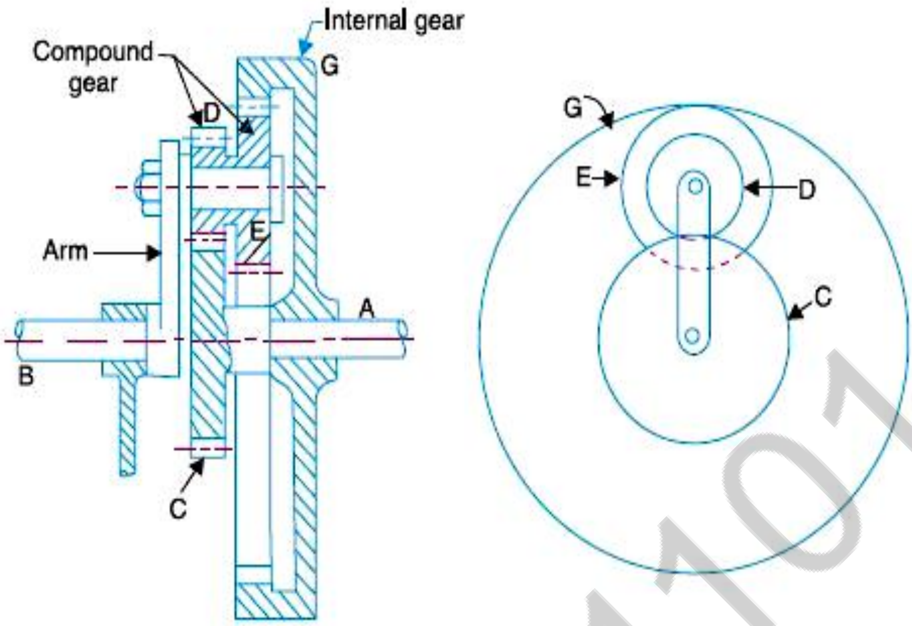
$$d_B + 2\,d_C = d_D$$

Since the number of teeth are proportional to their pitch circle diameters, therefore

$$T_B + 2\,T_C = T_D \quad \text{or} \quad 14 + 2\,T_C = 56$$

$$\therefore \quad T_C = 21 \textbf{ Ans.}$$

15) Two shafts A and B are co-axial. A gear C (50 teeth) is rigidly mounted on shaft A. A compound gear D-E gears with C and an internal gear G. D has 20 teeth and gears with C and E has 35 teeth and gears with an internal gear G. The gear G is fixed and is concentric with the shaft axis. The compound gear D-E is mounted on a pin which projects from an arm keyed to the shaft B. Sketch the arrangement and find the number of teeth on internal gear G assuming that all gears have the same module. If the shaft A rotates at 110 r.p.m., find the speed of shaft B.



Given:
$T_C = 50$;
$T_D = 20$;
$T_E = 35$
$N_A = 110$ r.p.m.

Solution:

Number of teeth on internal gear G

Let d_C, d_D, d_E and d_G be the pitch circle diameters of gears *C*, *D*, *E* and *G* respectively. From the geometry of the figure,

$$\frac{d_G}{2} = \frac{d_C}{2} + \frac{d_D}{2} + \frac{d_E}{2}$$

or $$d_G = d_C + d_D + d_E$$

Let T_C, T_D, T_E and T_G be the number of teeth on gears *C*, *D*, *E* and *G* respectively. Since all the gears have the same module, therefore number of teeth are proportional to their pitch circle diameters.

$\therefore$ $$T_G = T_C + T_D + T_E = 50 + 20 + 35 = 105 \text{ Ans.}$$

Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear C (or shaft A)	Compound gear D-E	Gear G
1.	Arm fixed - gear C rotates through + 1 revolution	0	+ 1	$-\frac{T_C}{T_D}$	$-\frac{T_C}{T_D}\times\frac{T_E}{T_G}$
2.	Arm fixed - gear C rotates through $+x$ revolutions	0	$+x$	$-x\times\frac{T_C}{T_D}$	$-x\times\frac{T_C}{T_D}\times\frac{T_E}{T_G}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y-x\times\frac{T_C}{T_D}$	$y-x\times\frac{T_C}{T_D}\times\frac{T_E}{T_G}$

Since the gear G is fixed, therefore from the fourth row of the table,

$$y - x\times\frac{T_C}{T_D}\times\frac{T_E}{T_G} = 0 \qquad \text{or} \qquad y - x\times\frac{50}{20}\times\frac{35}{105} = 0$$

$$\therefore \qquad y - \frac{5}{6}x = 0 \qquad \text{...}(i)$$

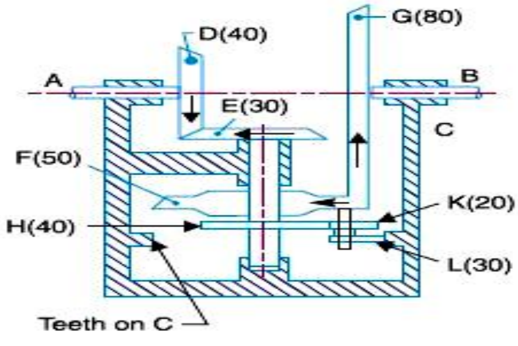
Since the gear C is rigidly mounted on shaft A, therefore speed of gear C and shaft A is same. We know that speed of shaft A is 110 r.p.m., therefore from the fourth row of the table,

$$x + y = 100 \qquad \text{...}(ii)$$

From equations (i) and (ii), $x = 60$, and $y = 50$

$\therefore$ Speed of shaft B = Speed of arm = $+y$ = 50 r.p.m. anticlockwise **Ans.**

16) In the gear drive as shown in Figure, the driving shaft A rotates at 300 r.p.m. in the clockwise direction, when seen from left hand. The shaft B is the driven shaft. The casing C is held stationary. The wheels E and H are keyed to the central vertical spindle and wheel F can rotate freely on this spindle. The wheels K and L are rigidly fixed to each other and rotate together freely on a pin fitted on the underside of F. The wheel L meshes with internal teeth on the casing C. The numbers of teeth on the different wheels are indicated within brackets in Fig. 13.18. Find the number of teeth on wheel C and the speed and direction of rotation of shaft B.



Given :
N_A = 300 r.p.m. (clockwise) ;
T_D = 40 ;
T_B = 30 ;
T_F = 50 ;
T_G = 80 ;
T_H = 40 ;
T_K = 20 ;
T_L = 30

Solution:
In the arrangement the wheel D and G are auxiliary gears and does not form a part of the epicyclic gear train.

$$\text{Speed of wheel } E,\ N_E = N_A \times \frac{T_D}{T_E} = 300 \times \frac{40}{30} = 400 \text{ r.p.m. (clockwise)}$$

Number of teeth on wheel C

Let T_C = Number of teeth on wheel *C*.

Assuming the same module for all teeth and since the pitch circle diameter is proportional to the number of teeth ; therefore from the geometry of Fig.

$$T_C = T_H + T_K + T_L = 40 + 20 + 30 = 90 \text{ Ans.}$$

Speed and direction of rotation of shaft B

The table of motions is given below. The wheel *F* acts as an arm.

Step No.	Conditions of motion	Revolutions of elements				
		Arm or wheel F	Wheel E	Wheel H	Compound wheel K-L	Wheel C
1.	Arm fixed-wheel *E* rotated through – 1 revolution (*i.e.* 1 revolution clockwise)	0	– 1	– 1(∵ *E* and *H* are on the same shaft)	$+\frac{T_H}{T_K}$	$+\frac{T_H}{T_K}\times\frac{T_L}{T_C}$
2.	Arm fixed-wheel *E* rotated through – *x* revolutions	0	$-x$	$-x$	$+x\times\frac{T_H}{T_K}$	$+x\times\frac{T_H}{T_K}\times\frac{T_L}{T_C}$
3.	Add – *y* revolutions to all elements	$-y$	$-y$	$-y$	$-y$	$-y$
4.	Total motion	$-y$	$-x-y$	$-x-y$	$x\times\frac{T_H}{T_K}-y$	$x\times\frac{T_H}{T_K}\times\frac{T_L}{T_C}-y$

Since the speed of wheel E is 400 r.p.m. (clockwise), therefore from the fourth row of the table,

$$-x - y = -400 \quad \text{or} \quad x + y = 400 \qquad ...(i)$$

Also the wheel C is fixed, therefore

$$x \times \frac{T_H}{T_K} \times \frac{T_L}{T_C} - y = 0$$

or

$$x \times \frac{40}{20} \times \frac{30}{90} - y = 0$$

$$\therefore \quad \frac{2x}{3} - y = 0 \qquad ...(ii)$$

From equations (*i*) and (*ii*),

$$x = 240 \quad \text{and} \quad y = 160$$

$\therefore$ Speed of wheel F, $N_F = -y = -160$ r.p.m.

Since the wheel F is in mesh with wheel G, therefore speed of wheel G or speed of shaft B

$$= -N_F \times \frac{T_F}{T_G} = -\left(-160 \times \frac{50}{80}\right) = 100 \text{ r.p.m.}$$

...($\because$ Wheel G will rotate in opposite direction to that of wheel F.)

= 100 r.p.m. anticlockwise *i.e.* in opposite direction of shaft A. **Ans.**

17) The following data relate to a pair of 20° involute gears in mesh: Module = 6 mm, Number of teeth on pinion = 17, Number of teeth on gear = 49; Addenda on pinion and gear wheel = 1 module.

Find:

(i) The number of pairs of teeth in contact

(ii) The angle turned through by the pinion and the gear wheel when one pair of teeth 15 in contact, and

(iii) The ratio of sliding to rolling motion when the tip of a tooth on the larger wheel
(1) Is just making contact,
(2) Is just leaving contact with its mating tooth, and
(3) Is at the pitch point.

[APRIL/MAY-2017]

Given Data : $\phi = 20°$; $m = 6$ mm ; $T_P = 17$; $T_G = 49$;

Addenda on pinion and gear wheel = 1 module = 6 mm.

☺ ***Solution : (i) Number of pairs of teeth in contact :***

Pitch circle radii of the pinion and the gear wheel is given by

$$r = \frac{m\,T_P}{2} = \frac{6 \times 17}{2} = 51 \text{ mm}$$

and

$$R = \frac{m\,T_G}{2} = \frac{6 \times 49}{2} = 147 \text{ mm}$$

Addendum radius of pinion, $r_A = r + \text{addendum} = 51 + 6 = 57$ mm

Addendum radius of gear wheel, $R_A = R + \text{Addendum}$

$$= 147 + 6 = 153 \text{ mm}$$

Length of path of approach, $KP = \sqrt{R_A^2 - R^2 \cos^2\phi} - R\sin\phi$

$$= \sqrt{(153)^2 - (147)^2 \cos^2 20°} - 147 \sin 20°$$

$$= 15.5 \text{ mm}$$

Length of path of recess, $PL = \sqrt{r_A^2 - r^2 \cos^2\phi} - r\sin\phi$

$$= \sqrt{(57)^2 - (51)^2 \cos^2 20°} - 51 \sin 20°$$

$$= 13.41 \text{ mm}$$

Length of path of contact, $KL = KP + PL$

$$= 15.5 + 13.41 = 28.91 \text{ mm}$$

$$\text{Length of arc of contact} = \frac{\text{Length of path of contact}}{\cos\phi}$$

$$= \frac{28.91}{\cos 20°} = 30.8 \text{ mm}$$

∴ Number of pairs of teeth in contact, *i.e.*,

$$\text{Contact ratio} = \frac{\text{Length of arc of contact}}{\text{Circular pitch } (p_c)}$$

$$= \frac{30.8}{\pi m} = \frac{30.8}{\pi \times 6} = 1.6 \text{ say } 2 \quad \textbf{Ans.}$$

2. ***Angle turned through by the pinion and gear wheel when one pair of teeth is in contact :***

$$\text{Angle turned through by the pinion,} = \frac{\text{Length of arc of contact}}{\text{Circumference of pinion}} \times 360°$$

$$= \frac{30.8}{2\pi \times 51} \times 360° = 34.6° \quad \textbf{Ans.}$$

Angle turned through by the gear wheel

$$= \frac{\text{Length of arc of contact}}{\text{Circumference of gear wheel}} \times 360°$$

$$= \frac{30.8}{2\pi \times 147} \times 360° = 12° \quad \textbf{Ans.}$$

3. Ratio of sliding to rolling motion :

Let ω_P and ω_G be the angular velocities of pinion and gear wheel respectively.

$$V_R = \text{Rolling velocity}$$

We know that, gear ratio $\frac{\omega_P}{\omega_G} = \frac{T_G}{T_P}$ or

$$\omega_G = \omega_P \times \frac{T_P}{T_G} = \omega_P \times \frac{17}{49} = 0.347\ \omega_P$$

and Rolling velocity, $v_R = \omega_P \cdot r$

$$= \omega_G \cdot R = \omega_P \times 51 = 51\ \omega_P \text{ mm/s}$$

(i) At the instant when the tip of a tooth on the larger wheel is just making contact with its mating teeth :

The sliding velocity $= (\omega_P + \omega_G)$ Length of path of approach

$$= (\omega_P + \omega_G)\ KP$$

$$v_S = (\omega_P + 0.347\ \omega_P)\ 15.5 = 20.88\ \omega_P \text{ mm/s}$$

∴ Ratio of sliding velocity to rolling velocity,

$$\frac{v_S}{v_R} = \frac{20.88\ \omega_P}{51\ \omega_P} = \mathbf{0.41} \quad \textbf{Ans.}$$

(ii) At the instant when the tip of a tooth on the larger wheel is just leaving contact with its mating teeth :

The sliding velocity, $v_S = (\omega_P + \omega_G) \times$ length of path of recess

$$= (\omega_P + \omega_G) \times PL$$

$$v_S = (\omega_P + 0.347\ \omega_P) \times 13.41 = 18.1\ \omega_1 \text{ mm/s}$$

∴ Ratio of sliding velocity to rolling velocity,

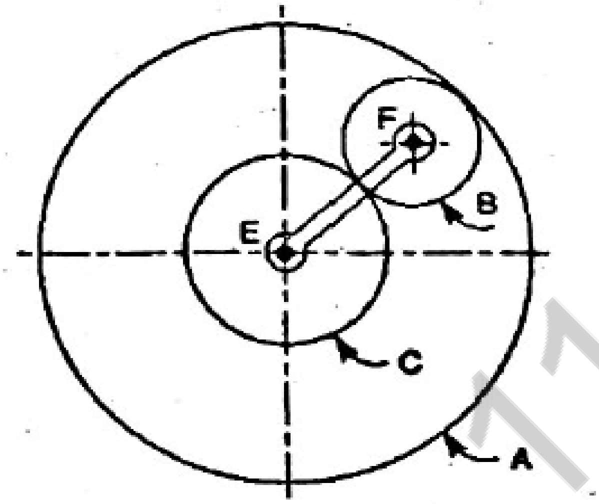
$$\frac{v_S}{v_R} = \frac{18.1\ \omega_P}{51\ \omega_P} = \mathbf{0.355} \quad \textbf{Ans.}$$

(iii) At the instant when the tip of a tooth on the larger wheel is at the pitch point :

At the pitch point, the length of path of contact is zero. So the sliding velocity will be zero. ***Therefore the ratio of sliding velocity to rolling velocity is zero.*** **Ans.**

18) An epicyclic gear consists of three gears A, B and C as shown in Fig. The gear A has 72 internal teeth and gear C has 32 external teeth. The gear B meshes with both A and C and is carried on an arm EF which rotates about the centre of A at 18 r.p.m.. If the gear A is fixed, determine the speed of gears B and C.

[APRIL/MAY-2017]



Given:

T_A = 72

T_C = 32

N_{EF} = 18 r.p.m.

To find:

(i) Determine the speed of gears B and C.

Soln:

Step No.	Operations	Revolutions of elements (N)			
		Arm EF	Gear C (T_C = 32)	Gear B	Gear A (T_A = 72)
1.	Fix the arm P and give gear C +1 revolution (CCW)	0	+1	$-\frac{T_C}{T_B}$	$-\frac{T_C}{T_B} \times \frac{T_B}{T_C} = -\frac{T_C}{T_A}$
2.	Multiply by x	0	$+x$	$-x\frac{T_C}{T_B}$	$-x\frac{T_C}{T_A}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	y	$x+y$	$y-x\frac{T_C}{T_B}$	$y-x\frac{T_C}{T_A}$

WKT,

$d_C + 2d_B = d_A$

similarly,

$T_C + 2T_B = T_A$

$2T_B = T_A - T_C$

$2T_B = 72 - 32$

$2T_B = 40$

$\mathbf{T_B = 20}$

Gear A is fixed (Given);

$$y - x\left[\frac{T_C}{T_A}\right] = 0$$

Arm rotates at 18 r.p.m. (i.e.) **y = 18 r.p.m.**

$$18 - x\left[\frac{32}{72}\right] = 0$$

$-0.44\,x = -18$

x = 40.5

Speed of Gear B:

$$N_B = y - x\left[\frac{T_C}{T_B}\right]$$

$$N_B = 18 - 40.5\left[\frac{32}{20}\right]$$

N_B = -46.8 r.p.m. (or) 46.8 r.p.m. (Clockwise)

Speed of Gear C:

$N_C = x + y$

N_C = 40.5 + 18 = 58.5 r.p.m. (Counter-Clockwise)

19) A pinion having 24 teeth drives a gear having 60 teeth. The profile of the gears is involute with 20° pressure angle, 10 mm module and 10 mm addendum. Find the length of path of contact, are of contact and the contact ratio. ***[NOV/DEC 2017]***

Given:

T_P=24

T_G=60

Φ=20°

Module m = 10 mm

Addendum = 10mm

To find:

(i) Length of path of contact

(ii) Length of are of contact

(iii) Contact ratio

Solution:

(i) Length of path of contact

$$R = \frac{mT_G}{2} = \frac{10 \times 60}{2} = 300mm$$

$$r = \frac{mT_P}{2} = \frac{10 \times 24}{2} = 120mm$$

Radius of addendum circle of pinion:

r_A= r+ addendum

r_A= 120+10= 130mm

Radius of addendum circle of gear wheel:

R_A=R-Addendum of Wheel

R_A= 300-10=290mm

Length of path approach: (KP)

$$KP = R\sin\phi - \sqrt{R_A^2 - R^2\cos^2\phi}$$

$$= 300\sin 20^\circ - \sqrt{(290)^2 - (300)^2(\cos 20^\circ)^2}$$

$$KP = 34.6mm$$

Length of path of recess (PL)

$$PL = \sqrt{r_1^2 - r^2\cos^2\phi} - r\sin\phi$$

$$= \sqrt{(130)^2 - (120)^2\cos^2 20} - 120\sin 20^\circ$$

$$PL = 23.6mm$$

Length of path of contact KL=KP+PL

=34.6+23.6

$\boxed{KL = 58.2mm}$

Length of Arc of Contact:

$$\text{Length of Acc. Of contact} = \frac{\text{Length of path of contact}}{\cos\phi}$$

$$=> \frac{58.2}{\cos 20^\circ} = \boxed{61.93mm}$$

Contact Ratio:

$$\text{Contact Ratio} = \frac{\text{Length of are of contact}}{\pi.m}$$

$$=> \frac{61.93}{\pi \times 10} = \boxed{1.97 \simeq 1}$$

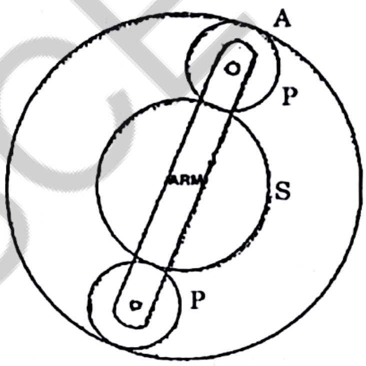
RESULT:

(i) Length of path of contact KL = 58.2mm

(ii) Length of are of contact = 61.93mm

(iii) Contact Ratio = 2

20) An epicyclic train of gears is arranged as shown in Fig. 14 (b). How many revolutions does the arm, to which the pinions P are attached, when S makes 300 rpm counter clockwise and A is stationary. The number of teeth on the gears S and A are 30 and 130 respectively. ***[NOV/DEC 2017]***



Given: $T_S = 30$; $T_A = 130$

Solution:

WKT,

$d_s + 2d_p = d_A$

111ly

$T_S + 2T_P = T_A$

$2T_P = T_A - T_S$

$2T_P = 130 - 30$

$2T_P = 100$

$T_P = 50$

Given

'S' makes 300 rpm (CCW)

$X + y = +300 \quad \rightarrow (1)$

Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear S	Gear P	Gear A
1.	Arm fixed; Gear S rotates through +1 revolution. (*i.e.*, 1 rev. anticlockwise)	0	+1	$-\frac{T_S}{T_P}$	$-\frac{T_S}{T_P} \times \frac{T_P}{T_A} = \frac{-T_S}{T_A}$
2.	Arm fixed; Gear S rotates through $+x$ revolutions.	0	$+x$	$-x\frac{T_S}{T_P}$	$-x\frac{T_S}{T_A}$
3.	Add $+y$ revolutions to all elements.	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x + y$	$y - x\frac{T_S}{T_P}$	$y - x\frac{T_S}{T_A}$

'A' is stationary

$$y - x\left[\frac{T_S}{T_A}\right] = 0$$

$$y - x\left[\frac{30}{130}\right] = 0$$

$$y - 0.230x = 0$$

$$0.230x - y = 0 \rightarrow 2$$

$$\boxed{x = 243.90} \ \& \ \boxed{y = 56.09}$$

Speed of the Arm (y)

$$\boxed{y = 56.09\text{rpm}}$$

21. a) i) State the fundamental law of gearing. Prove this law, by considering and neatly sketching two moving curved surfaces in contact. (10)

ii) Name the two types of tooth profiles satisfying the law of gearing and brief any one of them. (3)

[APRIL/MAY-2018]

(a) (i) Law of Gearing:

Consider the portions of the two teeth, one on the wheel 1 (or pinion) and the other on the wheel 2, as shown by thick line curves in Fig. 12.6. Let the two teeth come in contact at point Q, and the wheels rotate in the directions as shown in the figure.

Let T T be the common tangent and MN be the common normal to the curves at the point of contact Q. From the centres O_1 and O_2, draw O_1M and O_2N perpendicular to MN. A little consideration will show that the point Q moves in the direction QC, when considered as a point on wheel 1, and in the direction QD when considered as a point on wheel 2.

Let v_1 and v_2 be the velocities of the point Q on the wheels 1 and 2 respectively. If the teeth are to remain in contact, then the components of these velocities along the common normal MN must be equal.

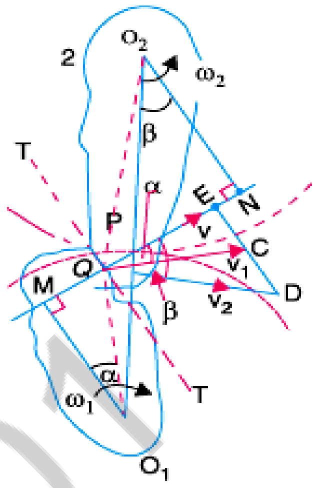


Fig. Law of gearing.

$$\therefore \quad v_1 \cos\alpha = v_2 \cos\beta$$

or
$$(\omega_1 \times O_1Q)\cos\alpha = (\omega_2 \times O_2Q)\cos\beta$$

$$(\omega_1 \times O_1Q)\frac{O_1M}{O_1Q} = (\omega_2 \times O_2Q)\frac{O_2N}{O_2Q} \quad \text{or} \quad \omega_1 \times O_1M = \omega_2 \times O_2N$$

$$\therefore \quad \frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} \qquad \ldots(i)$$

Also from similar triangles O_1MP and O_2NP,

$$\frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} \qquad \ldots(ii)$$

Combining equations *(i)* and *(ii)*, we have

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} \qquad \ldots(iii)$$

From above, we see that the angular velocity ratio is inversely proportional to the ratio of the istances of the point P from the centres O_1 and O_2, or the common normal to the two surfaces at the oint of contact Q intersects the line of centres at point P which divides the centre distance inversely s the ratio of angular velocities.

Therefore in order to have a constant angular velocity ratio for all positions of the wheels, the oint P must be the fixed point (called pitch point) for the two wheels. In other words, *the common ormal at the point of contact between a pair of teeth must always pass through the pitch point.* 'his is the fundamental condition which must be satisfied while designing the profiles for the teeth of ear wheels. It is also known as *law of gearing.*

(ii)

Therefore, in actual practice following are the two types of teeth commonly used:

- ➔ ***Cycloidal* teeth**
- ➔ **Involute teeth**

Involute Teeth

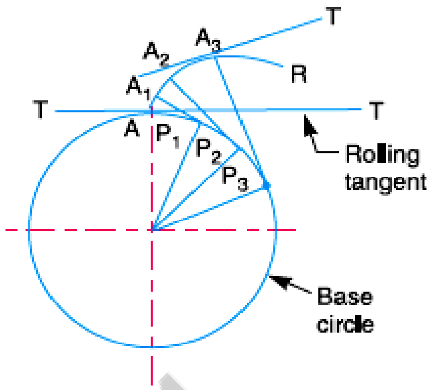
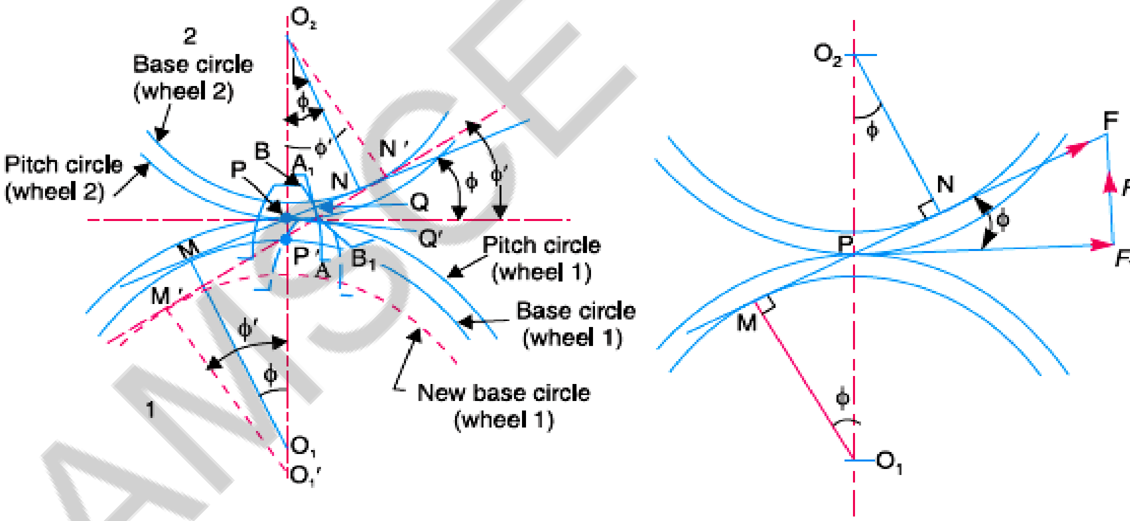


Fig. Construction of involute.

An involute of a circle is a plane curve generated by a point on a tangent, which rolls on the circle without slipping or by a point on a taut string which in unwrapped from a reel as shown in Fig. 12.9. In connection with toothed wheels, the circle is known as base circle. The involute is traced as follows :

Let A be the starting point of the involute. The base circle is divided into equal number of parts *e.g.* AP_1, P_1P_2, P_2P_3 etc. The tangents at P_1, P_2, P_3 etc. are drawn and the length P_1A_1, P_2A_2, P_3A_3 equal to the arcs AP_1, AP_2 and AP_3 are set off. Joining the points A, A_1, A_2, A_3 etc. we obtain the involute curve AR. A little consideration will show that at any instant A_3, the tangent A_3T to the involute is perpendicular to P_3A_3 and P_3A_3 is the normal to the involute. In other words, ***normal at any point of an involute is a tangent to the circle.***

Now, let O_1 and O_2 be the fixed centres of the two base circles as shown in Fig. 12.10 (*a*). Let the corresponding involutes AB and A_1B_1 be in contact at point Q. MQ and NQ are normals to the involutes at Q and are tangents to base circles. Since the normal of an involute at a given point is the tangent drawn from that point to the base circle, therefore the common normal MN at Q is also the common tangent to the two base circles. We see that the common normal MN intersects the line of centres O_1O_2 at the fixed point P (called pitch point). Therefore the involute teeth satisfy the fundamental condition of constant velocity ratio.



(*a*) (*b*)

Fig. 12.10. Involute teeth.

From similar triangles O_2NP and O_1MP,

$$\frac{O_1M}{O_2N} = \frac{O_1P}{O_2P} = \frac{\omega_2}{\omega_1} \quad \dots (i)$$

which determines the ratio of the radii of the two base circles. The radii of the base circles is given by

$$O_1M = O_1P\cos\phi, \quad \text{and} \quad O_2N = O_2P\cos\phi$$

Also the centre distance between the base circles,

$$O_1O_2 = O_1P + O_2P = \frac{O_1M}{\cos\phi} + \frac{O_2N}{\cos\phi} = \frac{O_1M + O_2N}{\cos\phi}$$

where ϕ is the pressure angle or the angle of obliquity. It is the angle which the common normal to the base circles (*i.e.* MN) makes with the common tangent to the pitch circles.

When the power is being transmitted, the maximum tooth pressure (neglecting friction at the teeth) is exerted along the common normal through the pitch point. This force may be resolved into tangential and radial or normal components. These components act along and at right angles to the common tangent to the pitch circles.

If F is the maximum tooth pressure as shown in Fig. 12.10 (*b*), then

Tangential force, $F_T = F \cos \phi$

and radial or normal force, $F_R = F \sin \phi$.

$\therefore$ Torque exerted on the gear shaft

$= F_T \times r$, where r is the pitch circle radius of the gear.

22) b) i) Explain with neat sketches various classifications of gear trains. (7)

ii) Neatly sketch the gear train called as Fergusson's Paradox. Explain and prove why is it called Paradox, by assuming suitable number of teeth for the gears of this train. (6)

[APRIL/MAY-2018]

(i) <u>Classification of Gear Trains:</u>

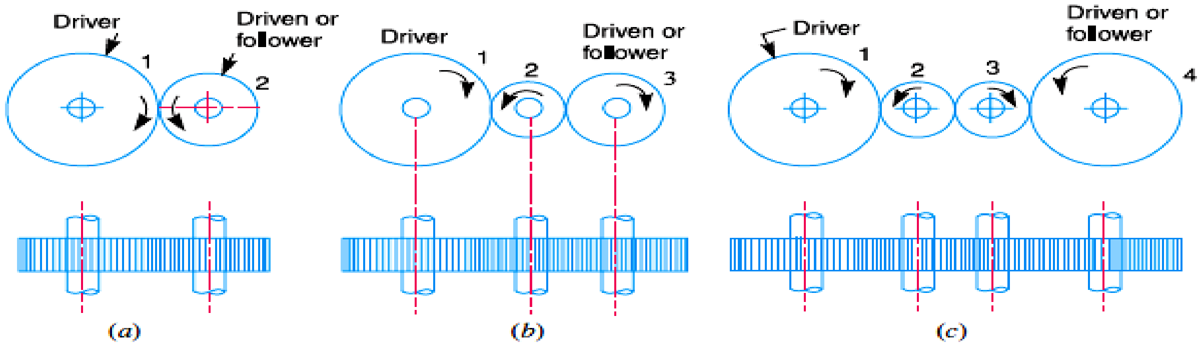
Depending upon the arrangement of wheels, Gear Trains are classified as.

1. **Simple Gear Train,**
2. **Compound Gear Train,**
3. **Reverted Gear Train, And**
4. **Epicyclic Gear Train.**

In the first three types of gear trains, the axes of the shafts over which the gears are mounted are fixed relative to each other. But in case of Epicyclic gear trains, the axes of the shafts on which the gears are mounted may move relative to a fixed axis.

Simple Gear Train:

When there is only one gear on each shaft, as shown in Fig. it is known as *simple gear train*. The gears are represented by their pitch circles. When the distance between the two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other, as shown in Fig. Since the gear 1 drives the gear 2, therefore gear 1 is called the ***driver*** and the gear 2 is called the ***driven*** or ***follower.*** It may be noted that the motion of the driven gear is opposite to the motion of driving gear.



Let N_1 = Speed of gear 1(or driver) in r.p.m.,

N_2 = Speed of gear 2 (or driven or follower) in r.p.m.,

T_1 = Number of teeth on gear 1, and

T_2 = Number of teeth on gear 2.

Since the speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth, therefore

$$\text{Speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

It may be noted that ratio of the speed of the driven or follower to the speed of the driver is known as ***train value*** of the gear train. Mathematically,

$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

Compound Gear Train

When there are more than one gear on a shaft, as shown in Fig. 13.2, it is called a ***compound train of gear.***

We have seen in Art. 13.3 that the idle gears, in a simple train of gears do not effect the speed ratio of the system. But these gears are useful in bridging over the space between the driver and the driven.

But whenever the distance between the driver and the driven or follower has to be bridged over by intermediate gears and at the same time a great (or much less) speed ratio is required, then the advantage of intermediate gears is intensified by providing compound gears on intermediate shafts. In this case, each intermediate shaft has two gears rigidly fixed to it so that they may have the same speed. One of these two gears meshes with the driver and the other with the driven or follower attached to the next shaft as shown in Fig.13.2.

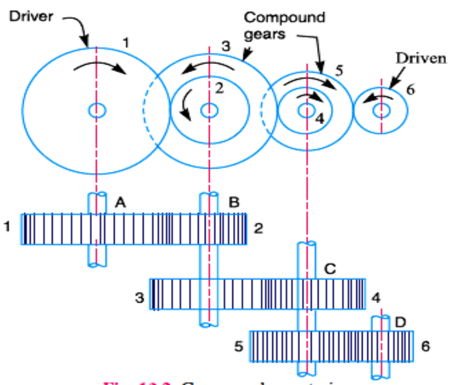


Fig. 13.2. Compound gear train.

In a compound train of gears, as shown in Fig. 13.2, the gear 1 is the driving gear mounted on shaft A, gears 2 and 3 are compound gears which are mounted on shaft B. The gears 4 and 5 are also compound gears which are mounted on shaft C and the gear 6 is the driven gear mounted on shaft D.

Let N_1 = Speed of driving gear 1,

T_1 = Number of teeth on driving gear 1,

$N_2, N_3 ..., N_6$ = Speed of respective gears in r.p.m., and

$T_2, T_3 ..., T_6$ = Number of teeth on respective gears.

Since gear 1 is in mesh with gear 2, therefore its speed ratio is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad ...(i)$$

Similarly, for gears 3 and 4, speed ratio is

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \quad ...(ii)$$

and for gears 5 and 6, speed ratio is

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \quad ...(iii)$$

The speed ratio of compound gear train is obtained by multiplying the equations (*i*), (*ii*) and (*iii*),

$$\therefore \quad \frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5} \quad \text{or} \quad {}^*\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

Reverted Gear Train

When the axes of the first gear (*i.e.* first driver) and the last gear (*i.e.* last driven or follower) are co-axial, then the gear train is known as ***reverted gear* train** as shown in Fig. 13.4.

We see that gear 1 (*i.e.* first driver) drives the gear 2 (*i.e.* first driven or follower) in the opposite direction. Since the gears 2 and 3 are mounted on the same shaft, therefore they form a compound gear and the gear 3 will rotate in the same direction as that of gear 2. The gear 3 (which is now the second driver) drives the gear 4 (*i.e.* the last driven or follower) in the same direction as that of gear 1. Thus we see that in a reverted gear train, the motion of the first gear and the last gear is ***like.***

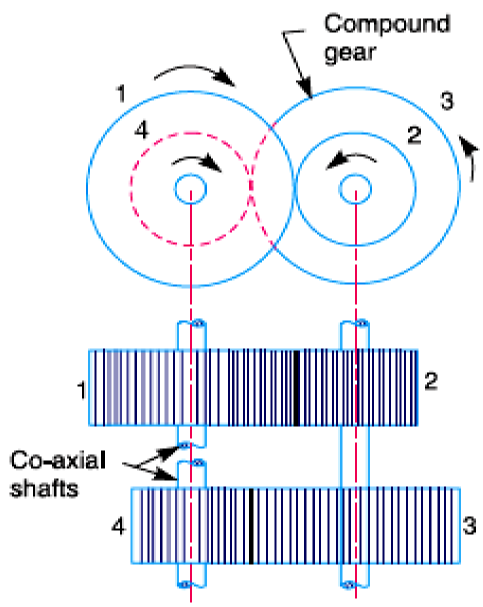


Fig. Reverted gear train.

Let T_1 = Number of teeth on gear 1,

r_1 = Pitch circle radius of gear 1, and

N_1 = Speed of gear 1 in r.p.m.

Similarly,

T_2, T_3, T_4 = Number of teeth on respective gears,

r_2, r_3, r_4 = Pitch circle radii of respective gears, and

N_2, N_3, N_4 = Speed of respective gears in r.p.m.

Epicyclic Gear Train

We have already discussed that in an epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis. A simple epicyclic gear train is shown in Fig. 13.6, where a gear A and the arm C have a common axis at O_1 about which they can rotate. The gear B meshes with gear A and has its axis on the arm at O_2, about which the gear B can rotate. If the arm is fixed, the gear train is simple and gear A can drive gear B

arm is fixed, the gear train is simple and gear A can drive gear B or ***vice- versa***, but if gear A is fixed and the arm is rotated about the axis of gear A (*i.e.* O_1), then the gear B is forced to rotate ***upon*** and ***around*** gear A. Such a motion is called **epicyclic** and the gear trains arranged in such a manner that one or more of their members move upon and around another member are known as ***epicyclic gear trains*** (***epi.*** means upon and ***cyclic*** means around). The epicyclic gear trains may be ***simple*** or ***compound.***

The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space. The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.

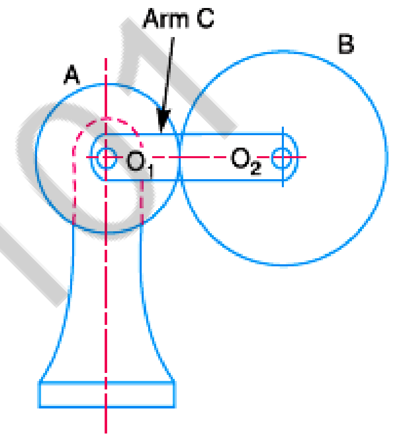
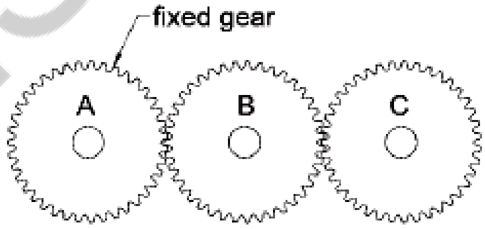


Fig. Epicyclic gear train.

(ii) <u>Fergusson Paradox:</u>

"Three wheels on the same axis mesh with one thick wheel. Turn the thick wheel. One of the thin wheels goes forward, one backwards, and one goes no way at all!"

What Ferguson was really doing with his Mechanical Paradox was emphasizing the importance of defining “rotation” with respect to a fixed frame of reference.



The “paradox” arises when in a train of gears – A, B, and C – gear A is fixed and gears B and C has Epicyclic motion around it. Gear A is the gear under the sun and is fixed to the base. When all three gears have the same number of teeth, gear B rotates twice for each rotation and gear C maintains its orientation to a fixed frame of reference. That keeps the Earth’s axis pointed in the same direction. When gear C has fewer teeth than gears A and B, it turns in the direction opposite the mechanism, in this case illustrating the regression of the nodes. When gear C has a few more teeth, it will slowly turn in the same direction as that of the mechanism, illustrating the advancement of the apogee of the Moon’s orbit.

23) Derive an expression to find the minimum number of teeth on the pinion to avoid interference of gears. (13)

[NOV/DEC-2018]

Interference in Involute Gears

Fig. shows a pinion with centre O_1, in mesh with wheel or gear with centre O_2. MN is the common tangent to the base circles and KL is the path of contact between the two mating teeth.

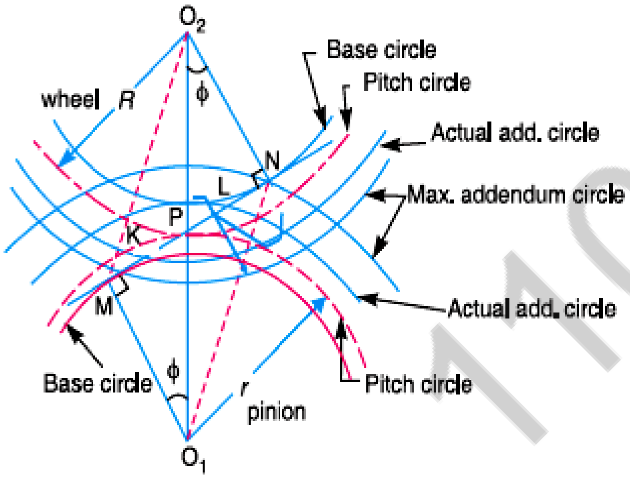


Fig. Interference in involute gears.

A little consideration will show, that if the radius of the addendum circle of pinion is increased to O_1N, the point of contact L will move from L to N. When this radius is further increased, the point of contact L will be on the inside of base circle of wheel and not on the involute profile of tooth on wheel. The tip of tooth on the pinion will then undercut the tooth on the wheel at the root and remove part of the involute profile of tooth on the wheel. This effect is known as ***interference***, and occurs when the teeth are being cut. In brief, ***the phenomenon when the tip of tooth undercuts the root on its mating gear is known as interference.***

Similarly, if the radius of the addendum circle of the wheel increases beyond O_2M, then the tip of tooth on wheel will cause interference with the tooth on pinion. The points M and N are called ***interference points***. Obviously, interference may be avoided if the path of contact does not extend beyond interference points. The limiting value of the radius of the addendum circle of the pinion is *O_1N and of the wheel is O_2M.

* From Fig. we see that

$$O_1N = \sqrt{(O_1M)^2 + (MN)^2} = \sqrt{(r_b)^2 + [r + R)\sin\phi]^2}$$

where r_b = Radius of base circle of pinion = $O_1P\cos\phi = r\cos\phi$

and $$O_2M = \sqrt{(O_2N)^2 + (MN)^2} = \sqrt{(R_b)^2 + [r + R)\sin\phi]^2}$$

where R_b = Radius of base circle of wheel = $O_2P\cos\phi = R\cos\phi$

words, ***interference may only be prevented, if the addendum circles of the two mating gears cut the common tangent to the base circles between the points of tangency.***

When interference is just avoided, the maximum length of path of contact is MN when the maximum addendum circles for pinion and wheel pass through the points of tangency N and M respectively as shown in Fig.   . In such a case,

Maximum length of path of approach,

$$MP = r \sin \phi$$

and maximum length of path of recess,

$$PN = R \sin \phi$$

∴ Maximum length of path of contact,

$$MN = MP + PN = r \sin \phi + R \sin \phi = (r + R) \sin \phi$$

and maximum length of arc of contact

$$= \frac{(r + R) \sin \phi}{\cos \phi} = (r + R) \tan \phi$$

Note : In case the addenda on pinion and wheel is such that the path of approach and path of recess are half of their maximum possible values, then

Path of approach, $KP = \frac{1}{2} MP$

or $$\sqrt{(R_A)^2 - R^2 \cos^2 \phi} - R \sin \phi = \frac{r \sin \phi}{2}$$

and path of recess, $PL = \frac{1}{2} PN$

or $$\sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin \phi = \frac{R \sin \phi}{2}$$

∴ Length of the path of contact

$$= KP + PL = \frac{1}{2} MP + \frac{1}{2} PN = \frac{(r + R) \sin \phi}{2}$$

24) An internal wheel B with 80 teeth is keyed to a shaft F. A fixed internal wheel C with 82 teeth is concentric with B. A compound wheel D-E gears with the two internal wheels. D has 28 teeth and gears with C while E gears with B. The compound wheels revolve freely on a pin which projects from a disc keyed to a shaft A co-axial with F. If the wheels have the same pitch and the shaft A makes 800 rpm. What is the speed of the shaft F? (13)

[NOV/DEC-2018]

Solution. Given : $T_B = 80$; $T_C = 82$; $T_D = 28$; $N_A = 800$ r.p.m.

The arrangement is shown in Fig.

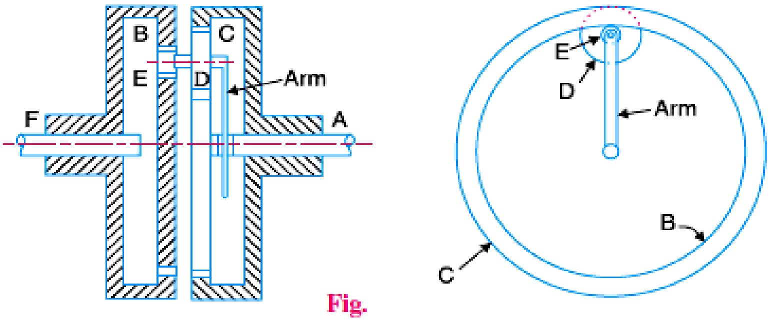


Fig.

First of all, let us find out the number of teeth on wheel E (T_E). Let d_B, d_C, d_D and d_E be the pitch circle diameter of wheels B, C, D and E respectively. From the geometry of the figure,

$$d_B = d_C - (d_D - d_E)$$

or

$$d_E = d_B + d_D - d_C$$

Since the number of teeth are proportional to their pitch circle diameters for the same pitch, therefore

$$T_E = T_B + T_D - T_C = 80 + 28 - 82 = 26$$

The table of motions is given below :

Table of motions.

Step No.	*Conditions of motion*	*Revolutions of elements*			
		Arm (or shaft A)	*Wheel B (or shaft F)*	*Compound gear D-E*	*Wheel C*
1.	Arm fixed - wheel B rotated through + 1 revolution (*i.e.* 1 revolution anticlockwise)	0	+ 1	$+\frac{T_B}{T_E}$	$+\frac{T_B}{T_E} \times \frac{T_D}{T_C}$
2.	Arm fixed - wheel B rotated through $+x$ revolutions	0	$+x$	$+x \times \frac{T_B}{T_E}$	$+x \times \frac{T_B}{T_E} \times \frac{T_D}{T_C}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y + x \times \frac{T_B}{T_E}$	$y + x \times \frac{T_B}{T_E} \times \frac{T_D}{T_C}$

Since the wheel C is fixed, therefore from the fourth row of the table,

$$y + x \times \frac{T_B}{T_E} \times \frac{T_D}{T_C} = 0 \quad \text{or} \quad y + x \times \frac{80}{26} \times \frac{28}{82} = 0$$

$\therefore$ $y + 1.05\,x = 0$ ...(*i*)

Also, the shaft A (or the arm) makes 800 r.p.m., therefore from the fourth row of the table,

$y = 800$ ...(*ii*)

From equations (*i*) and (*ii*),

$$x = -762$$

$\therefore$ Speed of shaft F = Speed of wheel $B = x + y = -762 + 800 = +38$ r.p.m.

= 38 r.p.m. (anticlockwise) **Ans.**

25) A pair of 20° full depth involute spur gears having 30 and 50 teeth respectively of module 4 mm is in mesh. The smaller gear rotates at 1000 r.p.m. Determine:

(i) Sliding velocities at engagement and disengagement of pair of a teeth and

(ii) Contact ratio. **(13)**

[APR/MAY-2019]

Solution. Given: $\phi = 20°$; $t = 30$; $T = 50$; $m = 4$; $N_1 = 1000$ r.p.m. or $\omega_1 = 2\pi \times 1000/60 = 104.7$ rad/s

1. ***Sliding velocities at engagement and at disengagement of pair of a teeth***

First of all, let us find the radius of addendum circles of the smaller gear and the larger gear. We know that

Addendum of the smaller gear,

$$= \frac{m.t}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

$$= \frac{4 \times 30}{2}\left[\sqrt{1 + \frac{50}{30}\left(\frac{50}{30} + 2\right)\sin^2 20°} - 1\right]$$

$$= 60(1.31 - 1) = 18.6 \text{ mm}$$

and addendum of the larger gear,

$$= \frac{m.T}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right]$$

$$= \frac{4 \times 50}{2}\left[\sqrt{1 + \frac{30}{50}\left(\frac{30}{50} + 2\right)\sin^2 20°} - 1\right]$$

$$= 100(1.09 - 1) = 9 \text{ mm}$$

Pitch circle radius of the smaller gear,

$$r = m.t/2 = 4 \times 30/2 = 60 \text{ mm}$$

$\therefore$ Radius of addendum circle of the smaller gear,

$$r_A = r + \text{Addendum of the smaller gear} = 60 + 18.6 = 78.6 \text{ mm}$$

Pitch circle radius of the larger gear,

$$R = m.T/2 = 4 \times 50/2 = 100 \text{ mm}$$

$\therefore$ Radius of addendum circle of the larger gear,

$$R_A = R + \text{Addendum of the larger gear} = 100 + 9 = 109 \text{ mm}$$

We know that the path of approach (*i.e.* path of contact when engagement occurs),

$$KP = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$ ...(Refer Fig. 12.11)

$$= \sqrt{(109)^2 - (100)^2\cos^2 20°} - 100\sin 20° = 55.2 - 34.2 = 21 \text{ mm}$$

and the path of recess (*i.e.* path of contact when disengagement occurs),

$$PL = \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin \phi$$

$$= \sqrt{(78.6)^2 - (60)^2 \cos^2 20°} - 60 \sin 20° = 54.76 - 20.52 = 34.24 \text{ mm}$$

Let ω_2 = Angular speed of the larger gear in rad/s.

We know that $\frac{\omega_1}{\omega_2} = \frac{T}{t}$ or $\omega_2 = \frac{\omega_1 \times t}{T} = \frac{10.47 \times 30}{50} = 62.82$ rad/s

∴ Sliding velocity at engagement of a pair of teeth

$$= (\omega_1 + \omega_2) KP = (104.7 + 62.82) 21 = 3518 \text{ mm/s}$$

$$= 3.518 \text{ m/s}$$ **Ans.**

and sliding velocity at disengagement of a pair of teeth

$$= (\omega_1 + \omega_2) PL = (104.7 + 62.82) 34.24 = 5736 \text{ mm/s}$$

$$= 5.736 \text{ m/s}$$ **Ans.**

2. Contact ratio

We know that the length of the arc of contact

$$= \frac{\text{Length of the path of contact}}{\cos \phi} = \frac{KP + PL}{\cos \phi} = \frac{21 + 34.24}{\cos 20°}$$

$$= 58.78 \text{ mm}$$

and Circular pitch $= \pi \times m = 3.142 \times 4 = 12.568$ mm

$$\therefore \quad \text{Contact ratio} = \frac{\text{Length of arc of contact}}{\text{Circular pitch}} = \frac{58.78}{12.568} = 4.67 \text{ say } 5$$ **Ans.**

26) In an Epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 r.p.m. in the anticlockwise direction about the centre of the gear A which is fixed, determine the speed of gear B. If the gear A instead of being fixed makes 300 r.p.m. in the clockwise direction, what will he the speed of gear B? (13)

[APR/MAY-2019]

Solution. Given : $T_A = 36$; $T_B = 45$; $N_C = 150$ r.p.m (anticlockwise)

The gear train is shown in Fig.

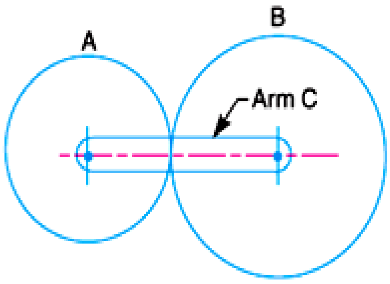


Fig.

We shall solve this example, first by tabular method and then by algebraic method.

1. Tabular method

First of all prepare the table of motions as given below :

Table of motions.

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear *A* rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear *A* rotates through + *x* revolutions	0	+ *x*	$-x \times \frac{T_A}{T_B}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y - x \times \frac{T_A}{T_B}$

Speed of gear B when gear A is fixed

Since the speed of arm is 150 r.p.m. anticlockwise, therefore from the fourth row of the table,

$$y = +150 \text{ r.p.m.}$$

Also the gear *A* is fixed, therefore

$$x + y = 0 \quad \text{or} \quad x = -y = -150 \text{ r.p.m.}$$

$$\therefore \text{ Speed of gear } B,\quad N_B = y - x \times \frac{T_A}{T_B} = 150 + 150 \times \frac{36}{45} = +270 \text{ r.p.m.}$$

$$= 270 \text{ r.p.m. (anticlockwise)} \quad \textbf{Ans.}$$

Speed of gear B when gear A makes 300 r.p.m. clockwise

Since the gear *A* makes 300 r.p.m.clockwise, therefore from the fourth row of the table,

$$x + y = -300 \quad \text{or} \quad x = -300 - y = -300 - 150 = -450 \text{ r.p.m.}$$

$\therefore$ Speed of gear *B*,

$$N_B = y - x \times \frac{T_A}{T_B} = 150 + 450 \times \frac{36}{45} = +510 \text{ r.p.m.}$$

$$= 510 \text{ r.p.m. (anticlockwise)} \quad \textbf{Ans.}$$

UNIT- V FRICTION IN MACHINE ELEMENTS

PART-A

1) Define Velocity ratio. **(MAY/JUNE 2014)**

Velocity ratio is the ratio of speed of driving gear to the speed of driven gear.

$$\frac{V_1}{V_2} = \frac{N_2}{N_1}$$

2) What is the maximum efficiency of the Screw jack? **(MAY/JUNE 2014)**

$$\eta_{max} = \frac{1-\sin\varphi}{1+\sin\varphi}$$

3) What is centrifugal tension in a belt? How does it affect the power transmitted? **(MAY/JUNE 2015)**

The belt continuously runs over the pulleys, therefore, some centrifugal force is caused, whose effect is to increase the tension on both, tight as well as the slack sides. The tension caused by centrifugal force is called centrifugal tension.

The Centrifugal tension on both sides will be increased, but at the same time normal reaction force in between the belt and pulley surface will be decreased and ultimately the power transmission efficiency will decrease.

4) Distinguish between breaks and dynamometers. **(MAY/JUNE 2015)**

BRAKES: Device which to stop or to retard the motion of the vehicle without skidding and converting kinetic energy into heat dissipation.

DYNAMOMETER: It is the device which measures the power developed by the vehicle or torque developed by the vehicle. This is done by utilizing the amount of heat developed during the breaking of the vehicle.

5) What are self energizing brakes? **(MAY/JUNE 2016)**

When moments of efforts applied on the brake drum and frictional force are in the same direction, the braking torque becomes maximum (frictional force aids the braking action). In such a case the brake is said to be partially self-actuating or self energising brake.

6) Why self locking screws have lesser efficiency? (MAY/JUNE 2016)

Self locking needs some friction on the thread surface of the screw and nut hence it needs higher effort to lift a body and hence automatically the efficiency decreases.

7) What are laws of solid dry friction? (NOV/DEC 2014)

- The total amount of friction which can be developed is independent of the magnitude of the area of contact.
- The total friction force which can be developed is proportional to the normal force transmitted across the surface of contact.
- For low velocities, the total amount of friction which can be developed is practically independent of velocity. However, it is less than the frictional force corresponding to impending motion.

8) What is meant by crowning of pulleys in flat belt drives? Also write its purpose. (NOV/DEC 2014)

The key to keeping the belts on tracking centered on the pulleys is the use of "crowned pulleys". A crowned pulley is a pulley that has a slight hump in the middle, tapering off ever so slightly towards either edge. The purpose is to prevent the slippage of belt from the belt from the pulley.

9) Write the expression for the maximum efficiency of a screw jack? (NOV/DEC 2015)

The efficiency of a screw jack may be defined as the ratio of ideal effort (i.e, the tangential force required to move the load neglecting friction) to the actual effort (i.e., the tangential force required to move the load with friction).

The efficiency of screw jack to raise load is given by

$$\eta_{\text{screwjack(up)}} = \frac{\tan\alpha}{\tan(\alpha+\phi)}$$

The efficiency of screw jack to lower the load is given by

$$\eta_{\text{screwjack(down)}} = \frac{\tan(\alpha-\phi)}{\tan\alpha}$$

The maximum efficiency of a screw jack for raising (or lowering) a load is given by

$$\eta_{max} = \frac{1-\sin\phi}{1+\sin\phi}$$

10) Write the mathematical expression for the length of belt required for two pulleys of diameter d_1 and d_2 and at distance x apart are connected by means of an open belt drive. (NOV/DEC 2014)

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x} \quad \textbf{(or)}$$

$$L = \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x}$$

11) .What is meant by slope of a thread?

It is the inclination of the thread with horizontal.

Slope of thread = $\tan^{-1}$ [Lead screw/Circumference of screw]

12) What are the effects of limiting angle of friction?

- If limiting angle of friction (φ) is equal to $\tan^{-1}\mu$, then the body will move over the plane irrespective of the magnitude of the force (F) (Limiting force of friction).
- If $\varphi<\tan^{-1}\mu$, then no motion of body on plane is possible irrespective of how large the magnitude of F may be.

13) Define co-efficient of friction (μ).

It is defined as the ratio of the limiting friction (F) to the normal reaction (R_N) between the two bodies.

μ = Limiting force of friction/Normal reaction = F/R_N

14) Differentiate coefficient of friction in square thread and V-thread.

(a) In square thread, $\mu = F/R_N$

(b) In V thread, $\mu_1 = \mu/\cos\beta$

Where F = Limiting force of friction,

R_N = Normal reaction, and

2β = Angle of 'V' in a 'V' thread.

15) What is the efficiency of inclined plane?

The efficiency of an inclined plane is defined as the ratio between effort without friction (P_0) and the effort with friction (P).

16) Why self- locking screws have lesser efficiency?

Self locking needs some friction on the thread surface of the screw and nut hence it needs higher effort to lift a body and hence automatically the efficiency decreases.

17) What are the functions of clutches?

- It supplies power to the transmission system.
- It stops the vehicle by disconnecting the engine from transmission system.
- It is used to change the gear and idling the engine.
- It gives gradual increment of speed to the wheels.

18) What is the difference between cone clutch and centrifugal clutch?

Cone clutch works on the principle of friction alone. But centrifugal clutch uses principle of centrifugal force in addition with it.

19) Why friction is called as 'necessary evil'?

Friction is the important factor in engineering and physical applications such as belt and ropes, jibs, clutches and brakes, nut and bolts, so it is the necessary one. If the friction exceeds certain value it will cause heat, damage and wear when applied. So it is called 'necessary evil'.

20) What are the belt materials?

- Leather,
- Cotton or fabric,
- Rubber,
- Balata, and Nylon.

21) State the law of belting?

Law of belting states that the centre line of the belt as it approaches the pulley must lie in a plane perpendicular to the axis of the pulley or must lie in the plane of the pulley, otherwise the belt will runoff the pulley.

22) What you meant by 'Crowing in pulley"?

The process of increasing the frictional resistance on the pulley surface is known as crowning. It is done in order to avoid slipping of the belt.

23) What is meant by initial tension in belts?

In order to increase the frictional grip between the belt and pulleys, the belts is tightened up. Due to this the belt gets subjected to some tension even when the pulleys are stationary. This tension in the belts is called initial tension (T_0).

24) List out the commonly used breaks.

- Hydraulic brakes: e.g., Pumps or hydrodynamic brake and fluid agitator.

- Electric brakes: e.g., Eddy current brakes.
- Mechanical brakes: e.g., Radial brakes and axial brakes

25) What do you mean by a brake?

Brake is a device by means of which motion of a body is retarded for slowing down (or) to bring it to rest which works on the principle of frictional force, it acts against the driving force.

26) Explain velocity ratio.

It is defined as the ratio between velocity of the driver and the follower (or) driven.

27) State the law of belting?

Law of belting states that the centre line of the belt as it approaches the pulley must lie in a plane perpendicular to the axis of the pulley or must lie in the plane of the pulley, otherwise the belt will runoff the pulley.

28) What is the centrifugal effect on belts?

During operation, as the belt passes over a pulley the centrifugal effect due to its weight tends to lift the belt from the pulley surface. This reduces the normal reaction and hence the frictional resistance. The centrifugal force produces additional tension in the belt.

29) Write down the disadvantage of V-belt drive over flat belt?

- V belt cannot be used in large distance.
- It is not as durable as flat belt.
- Since the V belt subjected to certain amount of creep therefore it is not suitable for constant speed applications such as synchronous machines, and timing devices.
- It is a costlier system.

30) When is the cross belt used instead of open belt?

- Cross belt is used where the direction of rotation of driven pulley is opposite to driving pulley.
- Where we need more power transmission there we can use cross belt drive.

31) Why lubrication reduces friction?

In practical all the mating surfaces are having roughness with it. It causes friction. If the surfaces are smooth then friction is very less. Lubrication smoothens the mating surface by introducing oil film between it.

The fluids are having high smoothness then solids and thus lubrication reduces friction.

32) What you meant by ‘crowning in pulley’?

The process of increasing the frictional resistance on the pulley surface is known as crowning. It is done in order to avoid slipping of the belt.

33) What is meant by initial tension in belts?

In order to increase the frictional grip between the belt and pulleys, the belt is tightened up. Due to this belt gets subjected to some tension even when the pulleys are stationary. This tension in the belt is called initial tension (T_0).

34) Where does the P.I.V. drive system used?

P.I.V. (Positive Infinitely variable) drive is used in an infinitely varying speed system.

35) When the intensity of pressure acting brake shoe is assumed to uniform?

The intensity of pressure is assumed to be constant when the break shoe has small angle of contact. For large angle of contact, it is assumed that the rate of wear of the shoe remains constant.

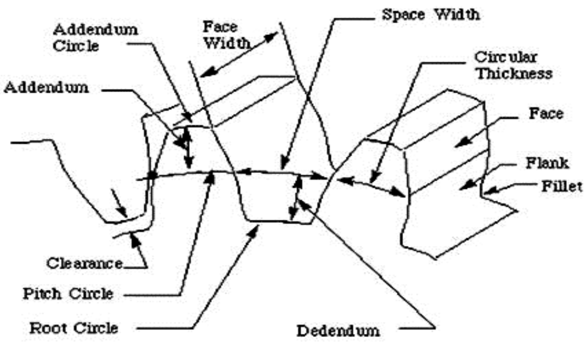
36) What kind of friction acts between the tyre and road in an automobile? (A/M 2017)

Static friction

37) State the functional difference between a clutch and a brake. (A/M-2017)

- **A Clutch** is a transmission and control device that provides for energy transfer from the driver to the driven shaft.
- **A Brake** is a transmission and control device that stops a moving load, regulates movement, or holds a load at rest by transforming kinetic energy into heat.

38) Give the classification of gears based on position of teeth on the Wheel. (N/D-2017)



Addendum: The radial distance between the Pitch Circle and the top of the teeth.

Dedendum: The radial distance between the bottom of the tooth to pitch circle.

Face Width: The width of the tooth measured parallel to the gear axis.

Flank: The working surface of a gear tooth, located between the pitch diameter and the bottom of the teeth

Gear: The larger of two meshed gears. If both gears are the same size, they are both called "gears".

Pinion: The smaller of two meshed gears.

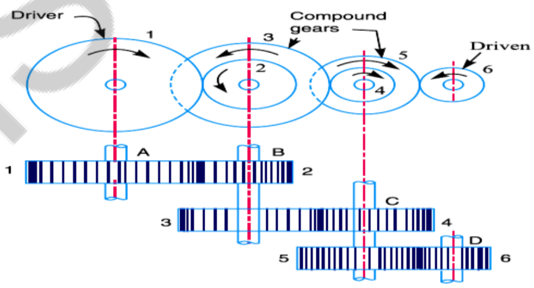
Module: Millimeter of Pitch Diameter to Teeth.

Pitch Circle: The circle, the radius of which is equal to the distance from the center of the gear to the pitch point.

39) Draw the compound gear train and write its Speed ratio.

(N/D-2017)

When there are more than one gear on a shaft, as shown in Fig it is called a compound train of gear.



$$\text{Speed ratio} = \frac{\text{Speed of the first driver}}{\text{Speed of the last driven or follower}}$$

$$= \frac{\text{Product of the number of teeth on the drivens}}{\text{Product of the number of teeth on the drivers}}$$

$$\text{Train value} = \frac{\text{Speed of the last driven or follower}}{\text{Speed of the first driver}}$$

$$= \frac{\text{Product of the number of teeth on the drivers}}{\text{Product of the number of teeth on the drivens}}$$

$$*\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

40) In an open belt drive of horizontal type, the slack side of belt should be kept on the top side of pulleys. Why? (A/M 2018)

The slack side of the belt is preferably placed on the top side because, the slack side of the belt, due to its self-weight, will sag. For this reason the angle of contact between the belt and the pulleys will increase.

41) What are the advantages of using friction clutches? (A/M 2018)

- Its engagement is smooth.
- No heat generation unless the operation requires frequent starts and stops.
- Once engaged there is no slip.
- In some cases it works as safety devices because it gets disengaged when torque crosses safety limit.

42) What are the characteristics of Brake lining material? (N/D 2018)

- The brake lining creates friction between brake shoe and rotor to slow them down.
- The brake lining protects the rotors from coming into direct contact with the metal backing.
- This prevents (Brake shoe) from wearing out quickly and needing frequent replacing.

43) Define slip and Creep in a belt drive. (N/D 2018)

Slip --- Slip is defined as insufficient frictional grip between pulley (driver/driven) and belt. Slip is the difference between the linear velocities of pulley (driver/driven) and belt.

Creep ----- When the belt passes from the slack side to the tight side, a certain portion of the belt extends and it contracts again when the belt passes from the tight side to slack side. Due to these changes of length, there is a relative motion between the belt and the pulley surfaces. This relative motion is termed as **Creep.**

44) What is the effect of centrifugal tension in belt drives? (A/M 2019)

Since the belt continuously runs over the pulleys, therefore, some centrifugal force is caused, whose effect is to increase the tension on both, tight as well as the slack sides. The tension caused by centrifugal force is called *centrifugal tension*.

45) What are the advantages of hydraulic brake over other brakes?

(A/M 2019)

- They transmit uniform pressure. (Due to hydrostatic pressure being equal in all directions, Pascal law)
- They help in multiplying the driver's effort more times than that of Mechanical Brakes. (Hydraulic leverage ratio, hydraulic advantage)
- The brake fluid also acts as lubricant and reduces the frictional losses at high speed braking.
- They are simpler in construction and lighter in weight.
- Thermal stresses generated are much lower in hydraulic brakes than Mechanical Brakes.
- Disc brakes fade in longer time than drum brakes. Hydraulic Disc Brakes are more wear resistant.
- Hydraulic Disc Brakes provide more braking power than hydraulic drum brakes. Mechanical efficiency is more due to greater surface area of the contact.

PART-B

1) Two pulleys one 450 mm diameter and the other 200 mm diameter are in parallel shafts 1.95 m apart and are connected by a crossed belt. Find the length of the belt required and the angle of contact between the belt and each pulley. What power can be transmitted by the belt when the larger pulley rotates at 200 rpm, if the maximum permissible tension in the belt is 1 KN and co-efficient of friction between the belt and pulley is 0.25? (16)

(MAY/JUNE 2014)

Given:

$$d_1 = 450mm = 0.45m \Rightarrow r_1 = 0.225m$$

$$d_2 = 200mm = 0.2m \Rightarrow r_2 = 0.1m$$

$$x = 1.95m$$

$$N_1 = 200rpm$$

$$T_1 = 1KN$$

$$\mu = 0.25$$

Drive: Cross-Belt Drive.

To find:

(i) Length of the Belt (L)

(ii) Angle of contact (θ)

(iii) Power Transmitted (P)

Solution:

(i) Length of the Belt Required:

$$L = \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x}$$

$$= \frac{\pi}{2}(0.45 + 0.2) + (2 \times 1.95) + \frac{(0.45 + 0.2)^2}{4 \times 1.95}$$

$$L = 4.97m$$

(ii) Angle of contact $'\theta'$

$$\theta = (180° + 2\alpha) \times \frac{\pi}{180°} rad$$

Where, $\sin\alpha = \frac{r_1 + r_2}{x} = \frac{0.225 + 0.1}{1.95}$

$$\alpha = \sin^{-1}(0.166) = 9.5° rad.$$

$$\theta = (180° + 2\alpha) \cdot \frac{\pi}{180°} = (180° + 2 \times 9.5°) \times \frac{\pi}{180°}$$

$$\theta = 3.47 rad.$$

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\frac{1000}{T_2} = e^{0.25\times3.47}$$

$$T_2 = \frac{1000}{2.358} = 424.08N$$

(iii) Power Transmitted 'P'

$$P = (T_1 - T_2)v$$

$$v = \frac{\pi d_1 N_1}{60} = \frac{\pi\times0.45\times200}{60} = 4.71m/s$$

$$P = (1000 - 424.08)\times4.71$$

$$P = 2712.58W \ (or) \ 2.712KW$$

Result:

(i) Length of the Belt $= 4.97m$

(ii) Angle of contact, $\theta = 3.47rad$

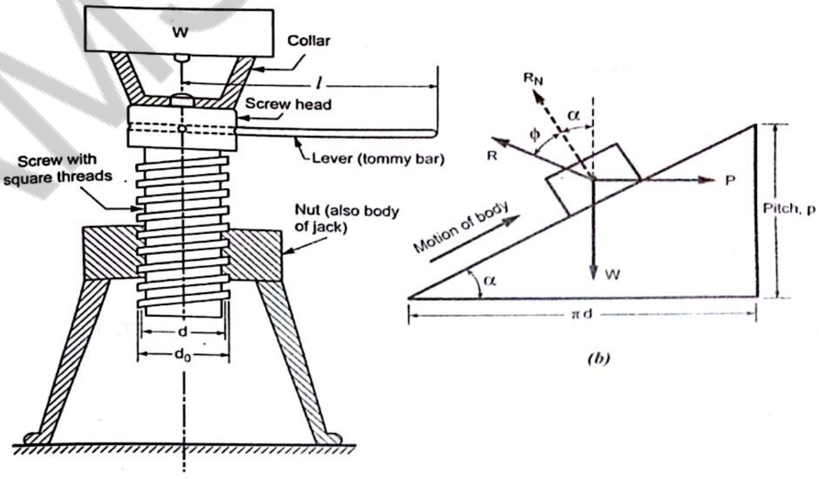
(iii) Power Transmitted, $P = 2.712KW$

2) (i) Derive an expression for the effort required to raise the load with screw jack taking friction into consideration. (8)

(MAY/JUNE 2014)

Screw jack with square threads

A screw jack, with its spindle having square threads is shown in Fig. The load to be raised or lowered, is placed on the square threaded rod which is rotated by the application of an effort at the end of the tommy bar (lever). As we have already discussed that motion of nut on the screw is analogous to sliding along an inclined plane.



(a) Screw jack

Let W = Load to be lifted,

P = Effort (i.e., horizontal force) applied at the screw tangentially,

l = Horizontal distance between central axis of the screw and the end of the bar,

$\mu = \tan\phi$, coefficient of friction between the screw and nut,

ϕ = Friction angle,

α = Inclination of thread or helix angle,

p = Pitch of the screw, and

d = Mean diameter of the screw.

Torque Required to raise the load W

If the nut is rotated so that the screw moves against the axial load W, then it is treated as body is moving upwards on the inclined plane. All the forces acting on the screw are shown in Fig.

$$P = W\tan(\alpha + \phi)$$

The turning moment or torque required to overcome friction between screw and nut is

$$T_1 = P \cdot \frac{d}{2} = W\tan(\alpha + \phi) \times \frac{d}{2}$$

Torque required to overcome the collar friction

When the axial load is taken up by a thrust collar as shown in Fig., then some torque is required to overcome friction at the collar also.
The torque required to overcome collar friction is given by

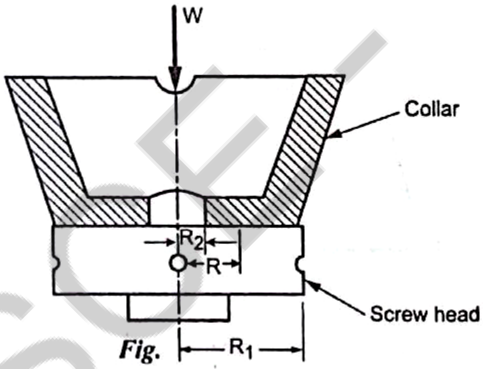


Fig.

$$T_2 = \mu_1 WR$$

Where μ_1 = Coefficient of friction of the collar,

R_1 and R_2 = Outside and inside radius of the collar, and

$$R = \text{Mean radius of the collar} = \frac{R_1 + R_2}{2}$$

Total Torque Required

Now total torque required to overcome friction i.e., to rotate the screw is given by

$$T = T_1 + T_2 = P \times \frac{d}{2} + \mu_1 WR$$

If effort P is applied tangentially at the end of a tommy bar/lever, then the total torque required to overcome friction must be equal to torque due to force P applied at the end of the lever.

$$\therefore \quad T = P \cdot \frac{d}{2} + \mu_1 WR = Pl \text{ or } P = \frac{T}{l}$$

(ii) A 150 mm diameter value, against a steam pressure of 2 MN/m² is acting, is closed by means of a square threaded screw 50 mm in external diameter with 6 mm pitch. If the co-efficient of friction is 0.12, find the torque required to turn the handle. (8)

(MAY/JUNE 2014)

Given data: $D = 150mm;\ p_{steam} = 2MN/m^2 = 2\times10^6 N/m^2;\ d_0 = 50mm = 0.05m;$
$p = 6mm = 0.006m;\ \mu = \tan\phi = 0.12.$

Solution:

Mean diameter of the screw, $d = d_0 - \frac{p}{2} = 0.05 - \frac{0.006}{2} = 0.04m$

Then, $\tan\alpha = \frac{p}{\pi d} = \frac{0.006}{\pi\times0.047} = 0.0406$ or $\alpha = \tan^{-1}(0.0406) = 2.33°$

and $\mu = \tan\phi = 0.12$ or $\phi = \tan^{-1}(0.12) = 6.84°$

Load on the valve, $W = \text{Pressure}\times\text{Area} = p_{steam}\times\frac{\pi}{4}D^2$

$$= 2\times10^6\left[\frac{\pi}{4}(0.15)^2\right] = 35343N$$

We know that force required to turn the handle,

$$P = W\tan(\alpha+\phi) = 35343\tan(2.33°+6.84°) = 5705.33N$$

$\therefore$ Torque required to turn the handle,

$$T = P\times\frac{d}{2} = 5705.33\times\left(\frac{47\times10^{-3}}{2}\right) = 134.07N.m$$

3) A flat belt, 8 mm thick and 100 mm wide transmits power between two pulleys, running at 1600 m/min. The mass of the belt is 0.9 kg/m length. The angle of lap in the smaller pulley is 165° and the coefficient of friction between the belt and pulley is 0.3. if the maximum permissible stress in the belt is 2 MN/m²,

Find:

(i) Maximum power transmitted; and

(ii) Initial tension in the belt. (16)

(MAY/JUNE 2015)

Given:

Thickness $t = 8mm = 0.008m$

Width $'b' = 100mm = 0.1m$

$$V = 1600\,m/min = \frac{1600}{60} = 26.67\,m/sec$$

$m = 0.9\,Kg/m$

$$\theta = 165^\circ = 165 \times \frac{\pi}{180} = 2.879^\circ$$

$$\mu = 0.3$$

$$\sigma = 2\,MN/m^2 = 2 \times 10^6\ N/m^2$$

To find:

(i) Maximum Power Transmitted 'P'

(ii) Initial tension in the belt 'T_0'

Solution:

(i) Maximum Power Transmitted: 'P'

$$P = (T_1 - T_2) \cdot v$$

WKT, $T_{max} = T_1 + T_c$.

$$T_{max} = \sigma \times b \times t$$

$$= 2 \times 10^6 \times 0.1 \times 0.008$$

$$T_{max} = 1600\,N$$

$$T_c = m.v^2$$

$$= 0.9 \times (26.67)^2$$

$$T_c = 640.16\,N$$

$$T_{max} = T_1 + T_c$$

$$T_1 = T_{max} - T_c$$

$$= 1600 - 640.16$$

$$T_1 = 959.84\,N$$

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\frac{T_1}{T_2} = e^{0.3 \times 2.879}$$

$$\frac{T_1}{T_2} = 2.37$$

$$T_2 = \frac{959.84}{2.37}$$

$$T_2 = 404.99\,N$$

Power $P=(T_1-T_2)\cdot v$

$$=(959.84-404.99)\times 26.67$$

$$P=14797.84W \text{ (or) } 14.79KW$$

(ii) Initial Tension $'T_0'$

$$T_0=\frac{T_1+T_2+2T_c}{2}$$

$$=\frac{959.84+404.99+2(640.16)}{2}$$

$$T_0=1322.57N$$

Result:

(i) Maximum Power Transmitted $P=14.79KW$

(ii) Initial Tension in Belt $T_0=1322.57N$

4) (i) The spindle of a screw jack has single start square threads with an outside diameter of 45 mm and a pitch of 10 mm. The spindle moves in a fixed nut. The load is carried on a swivel head but is not free to rotate. The bearing surface of the swivel head has a mean diameter of 60 mm. The coefficient of friction between the nut and screw is 0.12 and that between the swivel head and the spindle is 0.10. Calculate the load which can be raised by efforts of 100 N each applied at the end of two levers each of effective length of 350 mm. Also determine the velocity ratio and the efficiency of the lifting arrangement. (16)

(MAY/JUNE 2015)

Given:

$$d_0=45mm=0.045m$$

$$p=10mm$$

$$D=60mm\Rightarrow R=30mm=30\times10^{-3}m$$

$$\mu=0.12$$

$$\mu_1=0.10$$

Force to be applied at the lever $F=100N$

Effective length , $l=350mm=0.35m$

To find:

(i) Load that can be raised 'W'.

(ii) Efficiency. 'η'.

Solution:

Mean diameter, $d = d_0 - \frac{p}{2}$

$= 45 - \frac{10}{2}$

$d = 40mm \ (or) \ 0.04m$

$\tan\alpha = \left(\frac{p}{\pi d}\right)$

$\alpha = \tan^{-1}\left(\frac{p}{\pi d}\right)$

$= \tan^{-1}\left(\frac{10\times10^{-3}}{\pi\times0.04}\right)$

$\alpha = 4.55°$

$\mu = \tan\phi$

$\phi = \tan^{-1}(\mu)$

$= \tan^{-1}(0.12)$

$\phi = 6.84°$

Torque $T = F\times l$

$= 100\times0.35$

$T = 35N - m$

Torque $T = p\cdot\frac{d}{2} + \mu_1\cdot W\cdot R$

$T = W\cdot\tan(\alpha+\phi)\cdot\frac{d}{2} + \mu_1\cdot W\cdot R$

$= W\tan(4.55+6.84)\cdot\frac{0.04}{2} + 0.10\times W\times30\times10^{-3}$

$T = 7.03\times10^{-3}W$

$35 = 7.03\times10^{-3}W$

$$W = 4978.66N \ (or) \ 4.97KN$$

(ii) Efficiency:

$$\eta = \frac{\tan \propto}{\tan(\propto + \phi)}$$

$$= \frac{\tan 4.55}{\tan(4.55 + 6.84)}$$

$$\eta = 0.395 \ (or) \ 39.5\%$$

Result:

(i) Load that can be raised $W = 4.97KN$

(ii) Efficiency $\eta = 39.5\%$

5) The cutter of a broaching machine is pulled by square threaded screw of 55 mm external diameter and 10 mm pitch. The operating nut takes the axial load of 400 N on a flat surface of 60 mm internal diameter and 90 mm external diameter. If the coefficient of friction is 0.15 for all contact surfaces on the nut. Determine the power required torotate the operating nut, when the cutting speed is 6 m/min. (16) (MAY/JUNE 2016)

Solution: Mean diameter of the screw,

$$d = d_0 - \frac{P}{2} = 55 \times 10^{-3} - \frac{10 \times 10^{-3}}{2} = 50 \times 10^{-3} m$$

$$\tan \alpha = \frac{p}{\pi d} = \frac{10 \times 10^{-3}}{\pi \times 50 \times 10^{-3}} = 0.0637$$

and force required at the circumference of the screw is given by

$$P = W \tan(\alpha + \phi) = W\left[\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \cdot \tan \phi}\right]$$

$$= 500\left[\frac{0.0637 + 0.15}{1 - 0.0637 \times 0.15}\right] = 108N$$

The mean radius of the flat surface is given by

$$R = \frac{R_1 + R_2}{2} = \frac{45 + 30}{2} = 37.5mm$$

$\therefore$ Total torque required, $T = P \times \frac{d}{2} + \mu_1 WR$ $\quad$ [Here $\mu_1 = \mu$]

$$= 108 \times \frac{50 \times 10^{-3}}{2} + 0.15 \times 500 \times 37.5 \times 10^{-3} = 5.5125N.m$$

Since the cutting speed is 6 m/min, therefore speed of the screw

$$N = \frac{\text{Cutting speed}}{\text{Pitch}} = \frac{6}{0.01} = 600\text{rpm}$$

And $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 600}{60} = 62.84\,\text{rad/s}$

We know that the power required to operate the nut,

$$P = T \cdot \omega = 4.41 \times 62.84 = 277\text{W} = 0.277\text{kW}$$

6) Following data is given for a rope pulley transmitting 23.628kW. Dia of pulley = 40 cm; speed = 110 rpm, angle of groove = 45°; angle of lap = 60°, co efficient of friction = 0.28, No. of ropes = 10. Mass in kg/m length of ropes = 0.0053 × C^2 and working tension is limited 12.2 C^2 N where C = girth of rope in cm. Find (i) initial tension, and (ii) diameter of each rope. (16)

(MAY/JUNE 2016)

Given:

$P = 23.628\text{KW}$

$d = 40\text{cm} = 0.400\text{m}$

$N = 110\text{rpm}$

$2\beta = 45° \Rightarrow \beta = 22.5°$

$\theta = 60° = 60 \times \pi/180 = 1.047\text{rad}$

$\mu = 0.28$

No. of ropes $'n' = 10$

$m = 0.0053c^2\ \text{kg/m}$

$T = 12.2C^2\text{N}$

$C \rightarrow \text{girth of rope, in cm}$

To find: (i) Initial Tension (T_0)

(ii) Diameter of each rope (d_r)

Solution:

(i) Initial Tension in the rope: (T_0)

WKT $T_0 = \frac{T_1 + T_2}{2}$

$P = (T_1 - T_2) \cdot v$

$\frac{T_1}{T_2} = e^{\mu\theta \cdot \text{cosec}\beta}$

$$\text{Power Transmitted per rope} = \frac{\text{Total Power Transmitted}}{\text{No. of ropes.}}$$

$$= \frac{23.628 \times 10^3}{10} = 2362.8\text{W}$$

$$\frac{T_1}{T_2} = e^{\mu\theta\cdot\operatorname{cosec}\beta}$$

$$= e^{0.28\times1.04\times\operatorname{cosec}22.5^\circ}$$

$$\frac{T_1}{T_2} = 2.15 \quad \text{(or)} \quad T_1 = 2.15T_2$$

$$P = (T_1 - T_2)\cdot v$$

Veloctiy $v = \frac{\pi dN}{60} = \frac{\pi\times0.4\times110}{60}$

$$v = 2.30\,m/s$$

$$P = (2.15T_2 - T_2)\cdot 2.30$$

$$2362.8 = 2.645T_2$$

$$T_2 = 893.30N$$

$$T_1 = 1920.59N$$

$$T_0 = \frac{T_1 + T_2}{2}$$

$$= \frac{893.30 + 1920.59}{2}$$

$$T_0 = 1406.94N$$

(ii) Diameter of the rope (d_r):

Let $c \rightarrow$ Circumference of the girth

Centrifugal Tension $T_c = m\cdot v^2$

$$= 0.0053c^2\times(2.30)^2$$

$$T_c = 0.028037c^2$$

Maximum safe tension $T_{max} = T_1 + T_c$

$$12.2C^2 = 1920.59 + 0.0280C^2$$

$$12.2C^2 - 0.0280C^2 = 1920.59$$

$$12.17C^2 = 1920.59$$

$$C^2 = 157.79$$

$$C = 12.56cm\,(or)\,0.125m$$

$\therefore$ Circumference of Belt $(c) = \pi\times$ diameter of rope (d_r)

$$(d_r) = \frac{12.56}{\pi} = 3.99 \simeq 4cm.$$

$d_r \simeq 4\text{cm}$

Result:

(i) Initial tension in rope 1406.94M

(ii) Diameter of rope $(d_r) \rightarrow 4\text{cm}.$

7) In a screw jack, the diameter of the threaded screw is 40 mm and the pitch is 8 mm. The load is 20KN and it does not rotate with the screw but is carried on a swivel head having a bearing diameter of 70 mm, The coefficient of friction between the swivel head and the spindle is 0.08 and between the screw and nut is 0.1. Determine the total torque required to raise the load and efficiency. (8)

(NOV/DEC 2014)

Given

$d = 40\text{mm} = 40 \times 10^{-3}\text{m}$

$p = 8\text{mm} = 8 \times 10^{-3}\text{m}$

$W = 20\text{KN} = 20 \times 10^{3}\text{N}$

$D = 70\text{mm} = 70 \times 10^{-3}\text{m}$

$R = 35\text{mm} = 35 \times 10^{-3}\text{m}$

$\mu = 0.1$

$\mu_1 = 0.08$

To find: (i) Total Torque required to raise the load

(ii) Efficiency.

Solution:

(i) Total Torque Required:

$$T = p \times \frac{d}{2} + \mu_1 \cdot W \cdot R$$

$$P = W\tan(\alpha + \phi) \qquad \begin{Bmatrix} \text{required to Raise} \\ \text{the load} \end{Bmatrix}$$

$$\tan\alpha = \frac{\phi}{\pi d} = \frac{8 \times 10^{-3}}{\pi \times 40 \times 10^{-3}} = 0.0636$$

$$\alpha = 3.64^\circ$$

$$\mu = \tan\phi \qquad \phi = \tan^{-1}(\mu)$$

$$\phi = \tan^{-1}(0.1)$$

$$\phi = 5.71$$

$P = 20\times10^3 \tan(5.71+3.64)$

$P = 3293.04N$

$T = 3293.04\times\frac{40\times10^{-3}}{2} + (0.08 * 20\times10^3\times35\times10^{-3})$

$T = 121.86N-m$

Let $'T_o'$ be the Torque required to lift the load neglecting friction

$T_o = P_o\times\frac{d}{2}$

$= W\tan(\alpha+\phi)\cdot d/2$ $\quad [\because \phi = 0]$ $\quad \mu = \tan\phi$ $\quad \phi = \tan^{-1}(\mu)$ $\quad \therefore \phi = 0$

$= W\tan\alpha\cdot d/2$

$= 20\times10^3\times\tan3.64\times\frac{40\times10^{-3}}{2}$

$T_o = 25.45N-m.$

(ii) Efficiency:

$$\text{Efficiency} = \frac{T_o}{T} \Rightarrow \frac{25.45}{121.86}\times100 = 20.9\%$$

Result:

(i) Torque required to Lower the load $= 121.86N-m$

(ii) Efficiency $= 20.9\%$

8) A single plate clutch transmits 20KW 900 rpm. The maximum pressure intensity between plates is 85 KN/m². The outer diameter of the plate is 360 mm. Both the sides of the plate are effective and the coefficient of friction is 0.25. Determine the inner radius of the plate and axial force to engage the clutch. (8)

(NOV/DEC 2014)

Given:

$P = 20KW$

$N = 900rpm$

$p_{max} = 85KN/m^2 = 85\times10^3 N/m^2$

$n = 2$

$d_1 = 360mm = 0.360m$ (or) $r_1 = 0.180m$

$\mu = 0.25$

To find (i) Inner Radius of the plate $'r_2'$

(ii) Axial force required to engage the clucth.

Solution:

(i) Inner Radius of the plate $'r_2'$.

$$P = \frac{2\pi NT}{60} = \frac{2\pi \times 900 \times T}{60}$$

$$20 \times 10^3 = \frac{2\pi \times 960 \times T}{60}$$

$$T = 212.20 N - m$$

$$p_{max} \cdot r_2 = C$$

$$C = 85 \times 10^3 \cdot r_2 \ N/m$$

Axial Thrust Transmitted:

$$W = 2\pi C(r_1 - r_2)$$

$$= 2\pi \times 85 \times 10^3 \cdot r_2 \times (0.180 - r_2)$$

$$T = n \cdot \mu \cdot W \cdot R$$

Where, $R = \frac{r_1 + r_2}{2} = \frac{0.180 + r_2}{2}$

$$T = 2 \times 0.25 \times (2\pi \times 85 \times 10^3 \cdot r_2) \cdot (0.180 - r_2) \cdot \frac{(180 + r_2)}{2}$$

$$T = 2 \times \frac{1}{2} \times 0.25 \times 2\pi \times 85 \times 10^3 r_2 (0.180^2 - r_2^2)$$

$$T = 133517.68 r_2 (0.180^2 - r_2^2)$$

$$212.20 = 133517.68 \cdot r_2 (0.0324 - r_2^2)$$

$$1.589 \times 10^{-3} = 0.0324 r_2 - r_2^3$$

(or) $\Rightarrow r_2^3 - 0.0324 r_2 + 1.589 \times 10^{-3} = 0$

$$r_2 = 0.146 m \ (or) \ 146 mm$$

(ii) Axial force required to engage the clutch:

$$W = 2\pi c(r_1 - r_2)$$

$$= 2\pi \times (85 \times 10^3 \times 0.146)(0.180 - 0.146)$$

$$W = 2651.12 N$$

Result:

(i) Inner Radius of the plate $r_2 = 0.146m$

(ii) Axial force required to engage the clutch $= 2651.12N$

9) (i) Two parallel shafts that are 3.5 m apart are connected by a flat belt running between two pulleys of 1000 mm and 400 mm diameters, the larger pulley being the driver runs at 220 rpm. The belt weighs 1.2 kg per metre length. The maximum tension in the belt is not to exceed 1.8 KN. The co-efficient of friction is 0.28. Owing to slip on one of the pulleys, the velocity of the driven shaft is 520 rpm only. Determine the torque on each shaft, power transmitted, power lost in friction and efficiency of the belt drive. (10)

(NOV/DEC 2014)

Given: $x = 3.5m$

$d_1 = 1000mm = 1m \text{ (or) } r_1 = 0.5m$

$d_2 = 400mm = 0.4m \text{ (or) } r_2 = 0.2m$

$N_1 = 220rpm$

$m = 1.2 kg/m$

$T_{max} = 1.8KN = 1.8 \times 10^3 N$

$N_2 = 520rpm$

$\mu = 0.28$

To find (i) Torque on each shaft
(ii) Power Transmitted
(iii) Power lost in friction
(iv) Efficiency of the drive

Solution:

(i) Torque on each pulley:

Torque on larger pulley $\Rightarrow T_{LP} = (T_1 - T_2) \cdot r_1$

Torque on smaller pulley $\Rightarrow T_{SP} = (T_1 - T_2) \cdot r_2$

WKT, $T_{max} = T_1 + T_c$

$\therefore T_1 = T_{max} - T_c$

$T_c = mv^2$

Where,(Velocity of driver) $v = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 1 \times 220}{60}$

$$v = 11.52 m/s$$

$T_c = 1.2 \times (11.52)^2 = 159.25$

$T_1 = 1.8 \times 10^3 - 159.25$

$T_1 = 1640.75N$

WKT, $\frac{T_1}{T_2} = e^{\mu\theta}$

and $'\theta' = \text{Angle of Contact}$

$\theta = (180 - 2\alpha) \times \pi/180°$

$\sin\alpha = \frac{r_1 - r_2}{x}$

$\sin\alpha = \frac{0.5 - 0.2}{3.5} = 0.085$

$\alpha = \sin^{-1}(0.085) = 4.87°$

$\theta = (180 - 2 \times 4.87) \times \pi/180°$

$\theta = 2.971 \text{ rad.}$

$\frac{T_1}{T_2} = e^{\mu\theta}$

$T_2 = \frac{T_1}{e^{\mu\theta}} = \frac{1640.75}{e^{0.28 \times 2.971}}$

$T_2 = \frac{1640.75}{2.29}$

$T_2 = 716.48N$

$\therefore \quad T_{LP} = (T_1 - T_2) \cdot r_1 = (1640.75 - 716.48) \times 0.5$

$T_{LP} = 462.13N - m$

$T_{SP} = (T_1 - T_2) \cdot r_2 = (1640.75 - 716.48) \times 0.2$

$T_{SP} = 184.85N - m$

(ii) Power Transmitted:

$P = (T_1 - T_2) \cdot v$

$= (1640.75 - 716.48) \cdot 11.52$

$P = 10.64KW$

(iii) Power lost in friction:

Input power $P_I = \frac{2\pi N_1 \times T_{LP}}{60}$

$$P_I = \frac{2\pi \times 220 \times 462.13}{60} = 10.646KW$$

Output power, $P_O = \frac{2\pi N_2 \times T_{SP}}{60}$

$$P_O = \frac{2\pi \times 520 \times 184.85}{60}$$

$$P_O = 10.065KW$$

Power loss due to friction $= P_I - P_O$

$$= 10.646 - 10.065$$

$$\text{Power loss} = 0.581KW$$

(iv) Efficiency:

$$\eta = \frac{\text{Output Power}}{\text{Input Power}} = \frac{P_O}{P_I}$$

$$\Rightarrow \frac{10.065}{10.646} = 0.945$$

$$\eta = 94.5\%$$

Result:

(i) Torque on each pulley $T_{LP} = 462.13N-m$

$T_{SP} = 184.85N-m$

(ii) Power Transmitted $P = 10.64KW$

(iii) Power loss due to friction $= 0.581KW$

(iv) Efficiency of Belt Drive $= 94.5\%$

(ii) A bicycle and rider, travelling at 12 km/hr on a level road, have a mass of 105 kg. A brake is applied to the rear wheel which is 80 mm in diameter. The pressure on the brake is 80 N and the coefficient of friction is 0.06. Find the distance covered by the bicycle and number of turns on its wheels before coming to rest. (6)

(NOV/DEC 2014)

Given:

$v = 12 km/hr = 3.333 m/s$

$m = 105kg$

$D = 80mm = 0.08m$

$R_N = 80N$

$\mu = 0.06$

To find: (i) Distance covered by bi-cycle

(ii) No. of turns by wheel before it comes to rest.

Solution:

(i) Distance covered by Bi-cycle:

Kinetic energy is given by $KE = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 105 \times (3.33)^2$

$KE = 583.21N - m$

Let 'x' be the distance travelled by the bi-cycle before it comes to rest.

$\left.\begin{matrix}\text{Work}\\\text{Done}\end{matrix}\right\} W = \mu \cdot R_m \cdot x$

$= 0.06 \times 80 \times x$

$= 4.8x$

$583.21 = 4.8x \qquad [\because KE = WD]$

$x = 121.50m$

(ii) Number of revolution made by bicycle before it comes to rest.

Let 'N' be the no. of revolutions.

$x = \pi DN$

$121.50 = \pi \times 0.08 \times N$

$N = 483.43 \text{ (or) } 484 \text{ rev.}$

Result:

(i) Distance covered by Bi-cycle $= 121.50m$

(ii) No. of revolution made by the Bi-cycle before it comes to rest $= 484rev$

10) The external and internal radii of a friction plate of a single clutch are 120mm and 60mm respectively. The total axial thrust with which the friction surfaces are held together is 1500 N. For uniform wear, find the maximum, minimum and average pressure on the contact surfaces. (16) (NOV/DEC 2015)

Solution:

Maximum pressure:

Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p_{max} \times r_2 = C \quad \text{or} \quad C = 0.06 p_{max}$$

Axial force exerted on the contact surface (W) is given by

$$W = 2\pi C(r_1 - r_2)$$

$$1500 = 2\pi \times 0.06 p_{max}(0.12 - 0.06) \quad \text{or} \quad p_{max} = 66.314 k\, N/m^2$$

Minimum pressure:

Since the intensity of pressure is minimum at the outer radius (r_1), therefore

$$p_{min} \times r_1 = C \quad \text{or} \quad C = 0.1 p_{min}$$

Axial force exerted on the contact surface (W) is given by

$$W = 2\pi C(r_1 - r_2)$$

$$1500 = 2\pi \times 0.1 p_{min}(0.1 - 0.06) \quad \text{or} \quad p_{min} = 33.157 k\, N/m^2$$

11) Determine the maximum power that can be transmited using a belt of 100 mm × 10 mm with an angle of lap 160°. The density of the belt is 1000 kg/m³ and the co-efficient of friction may taken as 0.25. The tension in the belt should not exceed 1.5 N/mm². (16)

(NOV/DEC 2015)

Given:

$b = 100mm = 0.1m$

$t = 10mm = 0.01m$

$\theta = 160° = 160 \times \pi/180 = 2.792 rad$

$P_{belt} = 1000 kg/m^3$

$\mu = 0.25$

$\sigma = 1.5 N/mm^2 = 1.5 \times 10^{-6} N/m^2$

To find: (i) Maximum power that can be transmitted. 'P'

Solution:

$$P = (T_1 - T_2) \cdot v$$

$$T_{max} = \sigma \times b \times t$$

$= 1.5 \times 10^{-6} \times 0.1 \times 0.01$

$T_{max} = 1500N$

Mass 'm' = Area × Length × Density

$= (0.1 \times 0.01) \times .1 \times 1000$

$'m' = 1kg/m$ (per metre length)

Speed of maximum power:

$$V = \sqrt{\frac{T_{max}}{3m}} = \sqrt{\frac{1500}{3 \times 1}}$$

$V = 22.36\,m/s$

For Maximum Power Centrifugal Tension is given by

$$T_c = \frac{T_{max}}{3} = \frac{1500}{3} = 500N$$

Tension in the Belt:

$T_1 = T_{max} - T_c = 1500 - 500 = 1000N$

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.25 \times 2.792}$$

$$T_2 = \frac{1000}{2.009} = 497.76N$$

$\therefore$ Maximum Power that can be transmitted

$P = (T_1 - T_2)v$

$= (1000 - 497.76) \times 22.36$

$P = 11230.08W\,(or)$

$P = 11.23KW$

12) A turnbuckle, with right and left hand single start threads, is used to couple two wagons. Its thread pitch is 12 mm and mean diameter 40 mm. The coefficient of friction between the nut and screw is 0.16.1. Determine the work done in drawing the wagons together a distance of 240 mm, against a steady load of 2500 N. 2. If the load increases from 2500 N to 6000 N over the distance of 240 mm, what is the work to be done?

Given :

p = 12 mm ;

d = 40 mm ;

$\mu = \tan \phi = 0.16$;

$W = 2500$ N

Solution:

1. *Work done in drawing the wagons together against a steady load of 2500 N*

We know that $\tan \alpha = \frac{p}{\pi d} = \frac{12}{\pi \times 40} = 0.0955$

$\therefore$ Effort required at the circumference of the screw,

$$P = W \tan(\alpha + \phi) = W\left[\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha . \tan \phi}\right]$$

$$= 2500\left[\frac{0.0955 + 0.16}{1 - 0.0955 \times 0.16}\right] = 648.7 \text{ N}$$

and torque required to overcome friction between the screw and nut,

$$T = P \times d/2 = 648.7 \times 40/2 = 12\,947 \text{ N-mm} = 12.974 \text{ N-m}$$

A little consideration will show that for one complete revolution of the screwed rod, the wagons are drawn together through a distance equal to $2p$, *i.e.* $2 \times 12 = 24$ mm. Therefore in order to draw the wagons together through a distance of 240 mm, the number of turns required are given by

$$N = 240/24 = 10$$

$\therefore$ Work done $= T \times 2\pi N = 12.974 \times 2\pi \times 10 = 815.36$ N-m **Ans.**

2. *Work done in drawing the wagons together when load increases from 2500 N to 6000 N*

For an increase in load from 2500 *N* to 6000 *N*,

$$\text{Work done} = \frac{815.3(6000 - 2500)}{2500} = 114.4 \text{ N-m}$$ **Ans.**

13) A 150 mm diameter valve, against which a steam pressure of 2 MN/m2 is acting, is closed by means of a square threaded screw 50 mm in external diameter with 6 mm pitch. If the coefficient of friction is 0.12 ; find the torque required to turn the handle.

Solution. Given : $D = 150$ mm = 0.15 mm = 0.15 m ; $Ps = 2$ MN/m^2 = 2×10^6 N/m^2 ; $d_0 = 50$ mm ; $p = 6$ mm ; $\mu = \tan\phi = 0.12$

We know that load on the valve,

$$W = \text{Pressure} \times \text{Area} = p_S \times \frac{\pi}{4} D^2 = 2 \times 10^6 \times \frac{\pi}{4}(0.15)^2 \text{ N}$$
$$= 35\,400 \text{ N}$$

Mean diameter of the screw,

$$d = d_0 - p/2 = 50 - 6/2 = 47 \text{ mm} = 0.047 \text{ m}$$

$$\therefore \qquad \tan\alpha = \frac{p}{\pi d} = \frac{6}{\pi \times 47} = 0.0406$$

We know that force required to turn the handle,

$$P = W\ \tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha . \tan\phi}\right]$$

$$= 35400\left[\frac{0.0406 + 12}{1 - 0.0406 \times 0.12}\right] = 5713 \text{ N}$$

$\therefore$ Torque required to turn the handle,

$$T = P \times d/2 = 5713 \times 0.047/2 = 134.2 \text{ N-m} \quad \textbf{Ans.}$$

14) A square threaded bolt of root diameter 22.5 mm and pitch 5 mm is tightened by screwing a nut whose mean diameter of bearing surface is 50 mm. If coefficient of friction for nut and bolt is 0.1 and for nut and bearing surface 0.16, find the force required at the end of a spanner 500 mm long when the load on the bolt is 10 kN.

Given :
d_c = 22.5 mm ;
p = 5 mm ;
D = 50 mm or R = 25 mm ;
$\mu = \tan\phi = 0.1$;
$\mu_1 = 0.16$;
l = 500 mm ;
W = 10 kN = 10×10^3 N
Let P_1 = Force required at the end of a spanner in newtons.

Solution:

We know that mean diameter of the screw,

$$d = d_c + p/2 = 22.5 + 5/2 = 25 \text{ mm}$$

$$\therefore \qquad \tan \alpha = \frac{p}{\pi d} = \frac{5}{\pi \times 25} = 0.0636$$

Force requred at the circumference of the screw,

$$P = W \tan(\alpha + \phi) = W\left[\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi}\right]$$

$$= 10 \times 10^3 \left[\frac{0.0636 + 0.1}{1 - 0.06363 \times 0.1}\right] = 1646 \text{ N}$$

We know that total torque required,

$$T = P \times \frac{d}{2} + \mu_1.W.R. = 1646 \times \frac{25}{2} + 0.16 \times 10 \times 10^3 \times 25$$

$$= 60575 \text{ N - mm} \qquad ..(i)$$

We also know that torque required at the end of a spanner,

$$T = P_1 \times l = P_1 \times 500 = 500\, P_1 \text{ N-mm} \qquad ...(ii)$$

Equating equations (*i*) and (*ii*),

$$P_1 = 60575/500 = 121.15 \text{ N} \quad \textbf{Ans.}$$

15) The mean diameter of a square threaded screw jack is 50 mm. The pitch of the thread is 10 mm. The coefficient of friction is 0.15. What force must be applied at the end of a 0.7 m long lever, which is perpendicular to the longitudinal axis of the screw to raise a load of 20 kN and to lower it?

Solution. Given : $d = 50$ mm $= 0.05$ m ; $p = 10$ mm ; $\mu = \tan\phi = 0.15$; $l = 0.7$ m ; $W = 20$ kN $= 20 \times 10^3$ N

We know that $\tan\alpha = \dfrac{p}{\pi d} = \dfrac{10}{\pi \times 50} = 0.0637$

Let P_1 = Force required at the end of the lever.

Force required to raise the load

We know that force required at the circumference of the screw,

$$P = W\tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha . \tan\phi}\right]$$

$$= 20 \times 10^3 \left[\frac{0.0637 + 0.15}{1 - 0.0637 \times 0.15}\right] = 4314 \text{ N}$$

Now the force required at the end of the lever may be found out by the relation,

$$P_1 \times l = P \times d/2$$

$$\therefore \quad P_1 = \frac{P \times d}{2\,l} = \frac{4314 \times 0.05}{2 \times 0.7} = 154 \text{ N} \quad \textbf{Ans.}$$

Force required to lower the load

We know that the force required at the circumference of the screw,

$$P = W\tan(\phi - \alpha) = W\left[\frac{\tan\phi - \tan\alpha}{1 + \tan\phi . \tan\alpha}\right]$$

$$= 20 \times 10^3 \left[\frac{0.15 - 0.0637}{1 + 0.15 \times 0.0637}\right] = 1710 \text{ N}$$

Now the force required at the end of the lever may be found out by the relation,

$$P_1 \times l = P \times \frac{d}{2} \quad \text{or} \quad P_1 = \frac{P \times d}{2l} = \frac{1710 \times 0.05}{2 \times 0.7} = 61 \text{ N} \quad \textbf{Ans.}$$

16) The mean diameter of the screw jack having pitch of 10 mm is 50 mm. A load of 20 kN is lifted through a distance of 170 mm. Find the work done in lifting the load and efficiency of the screw jack when
1. The load rotates with the screw, and
2. The load rests on the loose head which does not rotate with the screw.
The external and internal diameter of the bearing surface of the loose head are 60 mm and 10 mm respectively. The coefficient of friction for the screw as well as the bearing surface may be taken as 0.08.

Given :
$p = 10$ mm ;
$d = 50$ mm ;
$W = 20$ kN $= 20 \times 10^3$ N ;
$D_2 = 60$ mm or
$R_2 = 30$ mm ;

D_1 = 10 mm or
R_1 = 5 mm ;
μ = = μ_1 = 0.08

Solution. Given : p = 10 mm ; d = 50 mm ; W = 20 kN = 20 × 10^3 N ; D_2 = 60 mm or R_2 = 30 mm ; D_1 = 10 mm or R_1 = 5 mm ; $\mu = \tan\phi = \mu_1 = 0.08$

We know that $\tan\alpha = \dfrac{p}{\pi d} = \dfrac{10}{\pi \times 50} = 0.0637$

∴ Force required at the circumference of the screw to lift the load,

$$P = W\tan(\alpha+\phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha.\tan\phi}\right]$$

$$= 20 \times 10^3\left[\frac{0.0637 + 0.08}{1 - 0.0637 \times 0.08}\right] = 2890 \text{ N}$$

and torque required to overcome friction at the screw,

$$T = P \times d/2 = 2890 \times 50/2 = 72250 \text{ N-mm} = 72.25 \text{ N-m}$$

Since the load is lifted through a vertical distance of 170 mm and the distance moved by the screw in one rotation is 10 mm (equal to pitch), therefore number of rotations made by the screw,

$$N = 170/10 = 17$$

1. *When the load rotates with the screw*

We know that work done in lifting the load

$$= T \times 2\pi N = 72.25 \times 2\pi \times 17 = 7718 \text{ N-m} \quad \textbf{Ans.}$$

and efficiency of the screw jack,

$$\eta = \frac{\tan\alpha}{\tan(\alpha+\phi)} = \frac{\tan\alpha(1 - \tan\alpha.\tan\phi)}{\tan\alpha + \tan\alpha}$$

$$= \frac{0.0637(1 - 0.0637 \times 0.08)}{0.0637 + 0.08} = 0.441 \text{ or } 44.1\% \quad \textbf{Ans.}$$

2. *When the load does not rotate with the screw*

We know that mean radius of the bearing surface,

$$R = \frac{R_1 + R_2}{2} = \frac{30 + 5}{2} = 17.5 \text{ mm}$$

and torque required to overcome friction at the screw and the collar,

$$T = P \times d/2 + \mu_1 W.R$$

$$= 2890 \times 50/2 + 0.08 \times 20{\times}10^3 \times 17.5 = 100\ 250 \text{ N-mm}$$

$$= 100.25 \text{ N-m}$$

∴ Work done by the torque in lifting the load

$$= T \times 2\pi N = 100.25 \times 2\pi \times 17 = 10\ 710 \text{ N-m} \quad \textbf{Ans.}$$

We know that the torque required to lift the load, neglecting friction,

$$T_0 = P_0 \times d/2 = W \tan\alpha \times d/2 \qquad \ldots(\because P_0 = W \tan\alpha)$$

$$= 20 \times 10^3 \times 0.0637 \times 50/2 = 31\ 850 \text{ N-mm} = 31.85 \text{ N-m}$$

∴ Efficiency of the screw jack,

$$\eta = T_0/T = 31.85/100.25 = 0.318 \text{ or } 31.8\% \quad \textbf{Ans.}$$

17) A load of 10 kN is raised by means of a screw jack, having a square threaded screw of 12 mm pitch and of mean diameter 50 mm. If a force of 100 N is applied at the end of a lever to raise the load, what should be the length of the lever used? Take coefficient of friction = 0.15. What is the mechanical advantage obtained? State whether the screw is self locking.

Given :

W = 10 kN = 10 × 10^3 N ;
p = 12 mm ;
d = 50 mm ;
P_1 = 100 N ;
μ = 0.15

Solution:

Length of the lever

Let l = Length of the lever.

We know that $\tan\alpha = \frac{p}{\pi d} = \frac{12}{\pi \times 50} = 0.0764$

∴ Effort required at the circumference of the screw to raise the load,

$$P = W \tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha \tan\phi}\right]$$

$$= 10 \times 10^3\left[\frac{0.0764 + 0.15}{1 - 0.0764 \times 0.15}\right] = 2290 \text{ N}$$

and torque required to overcome friction,

$$T = P \times d/2 = 2290 \times 50/2 = 57\ 250 \text{ N-mm} \quad ...(i)$$

We know that torque applied at the end of the lever,

$$T = P_1 \times l = 100 \times l \text{ N-mm} \quad ..(ii)$$

Equating equations (*i*) and (*ii*)

$$l = 57\ 250/100 = 572.5 \text{ mm} \quad \textbf{Ans.}$$

Mechanical advantage

We know that mechanical advantage,

$$M.A. = \frac{W}{P_1} = \frac{10 \times 10^3}{100} = 100 \quad \textbf{Ans.}$$

Self locking of the screw

We know that efficiency of the screw jack,

$$\eta = \frac{\tan\alpha}{\tan(\alpha + \phi)} = \frac{\tan\alpha(1 - \tan\alpha.\tan\phi)}{\tan\alpha + \tan\phi}$$

$$= \frac{0.0764(1 - 0.0764 \times 0.15)}{0.0764 + 0.15} = \frac{0.0755}{0.2264} = 0.3335 \text{ or } 33.35\%$$

Since the efficiency of the screw jack is less than 50%, therefore the screw is a self locking screw. **Ans.**

18) The thrust of a propeller shaft in a marine engine is taken up by a number of collars integral with the shaft which is 300 mm in diameter. The thrust on the shaft is 200 kN and the speed is 75 r.p.m. Taking μ constant and equal to 0.05 and assuming intensity of pressure as uniform and equal to 0.3 N/mm2, find the external diameter of the collars and the number of collars required, if the power lost in friction is not to exceed 16 kW.

Solution. Given : $d_2 = 300$ mm or $r_2 = 150$ mm $= 0.15$ m ; $W = 200$ kN $= 200 \times 10^3$ N ; $N = 75$ r.p.m. or $\omega = 2\pi \times 75/60 = 7.86$ rad/s ; $\mu = 0.05$; $p = 0.3$ N/mm^2 ; $P = 16$ kW $= 16 \times 10^3$ W

Let T = Total frictional torque transmitted in N-m.

We know that power lost in friction (P),

$$16 \times 10^3 = T.\omega = T \times 7.86 \text{ or } T = 16 \times 10^3/7.86 = 2036 \text{ N-m}$$

External diameter of the collar

Let d_1 = External diameter of the collar in metres = $2\ r_1$.

We know that for uniform pressure, total frictional torque transmitted (T),

$$2036 = \frac{2}{3} \times \mu.W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \frac{2}{3} \times \mu \times W\left[\frac{(r_1)^2 + (r_2)^2 + r_1.r_2}{r_1 + r_2}\right]^*$$

$$= \frac{2}{3} \times 0.05 \times 200 \times 10^3\left[\frac{(r_1)^2 + (0.15)^2 + r_1 \times 0.15}{r_1 + 0.15}\right]$$

$$2036 \times 3(r_1 + 0.15) = 20 \times 10^3[(r_1)^2 + 0.15\,r_1 + 0.0225]$$

Dividing throughout by 20×10^3,

$$0.305\,(r_1 + 0.15) = (r_1)^2 + 0.15\,r_1 + 0.0225$$

$$(r_1)^2 - 0.155\,r_1 - 0.0233 = 0$$

Solving this as a quadratic equation,

$$r_1 = \frac{0.155 \pm \sqrt{(0.155)^2 + 4 \times 0.0233}}{2} = \frac{0.155 \pm 0.342}{2}$$

$$= 0.2485 \text{ m} = 248.5 \text{ mm} \qquad \text{...(Taking + ve sign)}$$

$\therefore$ $d_1 = 2\,r_1 = 2 \times 248.5 = 497$ mm **Ans.**

Number of collars

Let n = Number of collars.

We know that intensity of pressure (p),

$$0.3 = \frac{W}{n\pi[r_1)^2 - (r_2)^2]} = \frac{200 \times 10^3}{n\pi[(248.5)^2 - (150)^2]} = \frac{1.62}{n}$$

$\therefore$ $n = 1.62 / 0.3 = 5.4$ or 6 **Ans.**

19) Determine the maximum, minimum and average pressure in plate clutch when the axial force is 4 kN. The inside radius of the contact surface is 50 mm and the outside radius 100 mm. Assume uniform wear.

Solution. Given : $W = 4$ kN $= 4 \times 10^3$ N ; $r_2 = 50$ mm ; $r_1 = 100$ mm

Maximum pressure

Let p_{max} = Maximum pressure.

Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p_{max} \times r_2 = C \quad \text{or} \quad C = 50\,p_{max}$$

We know that the total force on the contact surface (W),

$$4 \times 10^3 = 2\,\pi\,C\,(r_1 - r_2) = 2\,\pi \times 50\,p_{max}\,(100 - 50) = 15\,710\,p_{max}$$

$\therefore$ $p_{max} = 4 \times 10^3 / 15\,710 = 0.2546$ N/mm² **Ans.**

Minimum pressure

Let p_{min} = Minimum pressure.

Since the intensity of pressure is minimum at the outer radius (r_1), therefore

$$p_{min} \times r_1 = C \quad \text{or} \quad C = 100\,p_{min}$$

We know that the total force on the contact surface (W),

$$4 \times 10^3 = 2\,\pi\,C\,(r_1 - r_2) = 2\pi \times 100\,p_{min}\,(100 - 50) = 31\,420\,p_{min}$$

$\therefore$ $p_{min} = 4 \times 10^3 / 31\,420 = 0.1273$ N/mm² **Ans.**

Average pressure

We know that average pressure,

$$p_{av} = \frac{\text{Total normal force on contact surface}}{\text{Cross-sectional area of contact surfaces}}$$

$$= \frac{W}{\pi[(r_1)^2 - (r_2)^2]} = \frac{4 \times 10^3}{\pi[(100)^2 - (50)^2]} = 0.17 \text{ N/mm}^2 \text{ Ans.}$$

20) A single plate clutch, with both sides effective, has outer and inner diameters 300 mm and 200 mm respectively. The maximum intensity of pressure at any point in thecontact surface is not to exceed 0.1 N/mm2. If the coefficient of friction is 0.3, determine the power transmitted by a clutch at a speed 2500 r.p.m.

Solution. Given : $d_1 = 300$ mm or $r_1 = 150$ mm ; $d_2 = 200$ mm or $r_2 = 100$ mm ; $p = 0.1$ N/mm^2 ; $\mu = 0.3$; $N = 2500$ r.p.m. or $\omega = 2\pi \times 2500/60 = 261.8$ rad/s

Since the intensity of pressure (p) is maximum at the inner radius (r_2), therefore for uniform wear,

$$p.r_2 = C \quad \text{or} \quad C = 0.1 \times 100 = 10 \text{ N/mm}$$

We know that the axial thrust,

$$W = 2\pi C(r_1 - r_2) = 2\pi \times 10(150 - 100) = 3142 \text{ N}$$

and mean radius of the friction surfaces for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{150 + 100}{2} = 125 \text{ mm} = 0.125 \text{ m}$$

We know that torque transmitted,

$$T = n.\mu.W.R = 2 \times 0.3 \times 3142 \times 0.125 = 235.65 \text{ N-m}$$

...($\because$ $n = 2$, for both sides of plate effective)

$\therefore$ Power transmitted by a clutch,

$$P = T.\omega = 235.65 \times 261.8 = 61\,693 \text{ W} = 61.693 \text{ kW Ans.}$$

21) A single plate clutch, effective on both sides, is required to transmit 25 kW at 3000 r.p.m. Determine the outer and inner radii of frictional surface if the coefficient of friction is 0.255, the ratio of radii is 1.25 and the maximum pressure is not to exceed 0.1 N/mm2. Also determine the axial thrust to be provided by springs. Assume the theory of uniform wear.

Solution. Given: $n = 2$; $P = 25$ kW $= 25 \times 10^3$ W ; $N = 3000$ r.p.m. or $\omega = 2\pi \times 3000/60$ $= 314.2$ rad/s ; $\mu = 0.255$; $r_1/r_2 = 1.25$; $p = 0.1$ N/mm^2

Outer and inner radii of frictional surface

Let r_1 and r_2 = Outer and inner radii of frictional surfaces, and

T = Torque transmitted.

Since the ratio of radii (r_1/r_2) is 1.25, therefore

$$r_1 = 1.25\, r_2$$

We know that the power transmitted (P),

$$25 \times 10^3 = T.\omega = T \times 314.2$$

$$\therefore \quad T = 25 \times 10^3/314.2 = 79.6 \text{ N-m} = 79.6 \times 10^3 \text{ N-mm}$$

Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p.r_2 = C \quad \text{or} \quad C = 0.1\, r_2 \text{ N/mm}$$

and the axial thrust transmitted to the frictional surface,

$$W = 2\pi C(r_1 - r_2) = 2\pi \times 0.1\, r_2 (1.25\, r_2 - r_2) = 0.157\,(r_2)^2 \qquad ...(i)$$

We know that mean radius of the frictional surface for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{1.25\, r_2 + r_2}{2} = 1.125\, r_2$$

We know that torque transmitted (T),

$$79.6 \times 10^3 = n.\mu.W.R = 2 \times 0.255 \times 0.157\,(r_2)^2 \times 1.125\, r_2 = 0.09\,(r_2)^3$$

$$\therefore \quad (r_2)^3 = 79.6 \times 10^3/0.09 = 884 \times 10^3 \quad \text{or} \quad r_2 = 96 \text{ mm} \quad \textbf{Ans.}$$

and $$r_1 = 1.25\, r_2 = 1.25 \times 96 = 120 \text{ mm} \quad \textbf{Ans.}$$

Axial thrust to be provided by springs

We know that axial thrust to be provided by springs,

$$W = 2\pi C(r_1 - r_2) = 0.157\,(r_2)^2 \qquad \text{...[From equation } (i)]$$

$$= 0.157\,(96)^2 = 1447 \text{ N} \quad \textbf{Ans.}$$

22) A single dry plate clutch transmits 7.5 kW at 900 r.p.m. The axial pressure is limited to 0.07 N/mm2. If the coefficient of friction is 0.25, find 1. Mean radius and face width of the friction lining assuming the ratio of the mean radius to the face width as 4, and 2. Outer and inner radii of the clutch plate.

Solution. Given : $P = 7.5$ kW $= 7.5 \times 10^3$ W ; $N = 900$ r.p.m or $\omega = 2\pi \times 900/60 = 94.26$ rad/s ; $p = 0.07$ N/mm^2 ; $\mu = 0.25$

1. *Mean radius and face width of the friction lining*

Let R = Mean radius of the friction lining in mm, and

w = Face width of the friction lining in mm,

Ratio of mean radius to the face width,

$$R/w = 4 \quad \text{...(Given)}$$

We know that the area of friction faces,

$$A = 2\pi R.w$$

∴ Normal or the axial force acting on the friction faces,

$$W = A \times p = 2\pi R.w.p$$

We know that torque transmitted (considering uniform wear),

$$T = n.\mu W.R = n\mu\,(2\pi R.w.p)R$$

$$= n\mu\left(2\pi R \times \frac{R}{4} \times p\right)R = \frac{\pi}{2} \times n\mu.p.R^3 \quad \text{...}(\because w = R/4)$$

$$= \frac{\pi}{2} \times 2 \times 0.25 \times 0.07\ R^3 = 0.055\ R^3 \text{ N-mm} \quad \text{...(i)}$$

...($\because$ $n = 2$, for single plate clutch)

We also know that power transmitted (P),

$$7.5 \times 10^3 = T.\omega = T \times 94.26$$

$$\therefore \quad T = 7.5 \times 10^3/94.26 = 79.56 \text{ N-m} = 79.56 \times 10^3 \text{ N-mm} \quad \text{...(ii)}$$

From equations (*i*) and (*ii*),

$$R^3 = 79.56 \times 10^3/0.055 = 1446.5 \times 10^3 \text{ or } R = 113 \text{ mm}$$ **Ans.**

and $w = R/4 = 113/4 = 28.25$mm **Ans.**

2. *Outer and inner radii of the clutch plate*

Let r_1 and r_2 = Outer and inner radii of the clutch plate respectively.

Since the width of the clutch plate is equal to the difference of the outer and inner radii, therefore

$$w = r_1 - r_2 = 28.25 \text{ mm} \quad \text{...(iii)}$$

Also for uniform wear, the mean radius of the clutch plate,

$$R = \frac{r_1 + r_2}{2} \quad \text{or} \quad r_1 + r_2 = 2R = 2 \times 113 = 226 \text{ mm} \quad \text{...(iv)}$$

From equations (*iii*) and (*iv*),

$r_1 = 127.125$ mm ; and $r_2 = 98.875$ **Ans.**

23) A dry single plate clutch is to be designed for an automotive vehicle whose engine is rated to give 100 kW at 2400 r.p.m. and maximum torque 500 N-m. The outer radius of friction plate is 25% more than the inner radius. The intensity of pressure between the plate is not to exceed 0.07 N/mm2. The coefficient of friction may be assumed equal to 0.3. The helical springs required by this clutch to provide axial force necessary to engage the clutch are eight. If each spring has stiffness equal to 40 N /mm, determine the initial compression in the springs and dimensions of the friction plate.

Solution. Given : $P = 100$ kW $= 100 \times 10^3$ W ; $T = 500$ N-m $= 500 \times 10^3$ N-mm ; $p = 0.07$ N/mm^2 ; $\mu = 0.3$; Number of springs = 8 ; Stiffness = 40 N/mm

Dimensions of the friction plate

Let r_1 and r_2 = Outer and inner radii of the friction plate respectively.

Since the outer radius of the friction plate is 25% more than the inner radius, therefore

$$r_1 = 1.25\, r_2$$

We know that, for uniform wear,

$$p.r_2 = C \quad \text{or} \quad C = 0.07\, r_2 \text{ N/mm}$$

and load transmitted to the friction plate,

$$W = 2\pi C (r_1 - r_2) = 2\pi \times 0.07\, r_2 (1.125\, r_2 - r_2) = 0.11\, (r_2)^2 \text{ N} \quad ...(i)$$

We know that mean radius of the plate for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{1.25\, r_2 + r_2}{2} = 1.125\, r_2$$

$\therefore$ Torque transmitted (T),

$$500 \times 10^3 = n.\mu.W.R = 2 \times 0.3 \times 0.11\, (r_2)^2 \times 1.125\, r_2 = 0.074\, (r_2)^3 \quad ...(\because n = 2)$$

$$\therefore \quad (r_2)^3 = 500 \times 10^3 / 0.074 = 6757 \times 10^3 \quad \text{or} \quad r_2 = 190 \text{ mm} \quad \textbf{Ans.}$$

and

$$r_1 = 1.25\, r_2 = 1.25 \times 190 = 273.5 \text{ mm} \quad \textbf{Ans.}$$

Initial compression of the springs

We know that total stiffness of the springs,

$$s = \text{Stiffness per spring} \times \text{No. of springs} = 40 \times 8 = 320 \text{ N/mm}$$

Axial force required to engage the clutch,

$$W = 0.11\, (r_2)^2 = 0.11\, (190)^2 = 3970 \text{ N} \quad ...[\text{From equation } (i)]$$

$\therefore$ Initial compression in the springs

$$= W/s = 3970/320 = 12.5 \text{ mm} \quad \textbf{Ans.}$$

24) A rotor is driven by a co-axial motor through a single plate clutch, both sides of the plate being effective. The external and internal diameters of the plate are respectively 220 mm and 160 mm and the total spring load pressing the plates together is 570 N. The motor armature and shaft has a mass of 800 kg with an effective radius of gyration of 200 mm. The rotor has a mass of 1300 kg with an effective radius of gyration of 180 mm. The coefficient of friction for the clutch is 0.35. The driving motor is brought up to a speed of 1250 r.p.m. when the current is switched off and the clutch suddenly engaged. Determine

- **a. The final speed of motor and rotor,**
- **b. The time to reach this speed, and**
- **c. The kinetic energy lost during the period of slipping.**

How long would slipping continue if it is assumed that a constant resisting torque of 60 N-m were present? If instead of a resisting torque, it is assumed that a constant driving torque of 60 N-m is maintained on the armature shaft, what would then be slipping time?

Solution. Given : $d_1 = 220$ mm or $r_1 = 110$ mm ; $d_2 = 160$ mm or $r_2 = 80$ mm ; $W = 570$ N ; $m_1 = 800$ kg ; $k_1 = 200$ mm $= 0.2$ m ; $m_2 = 1300$ kg ; $k_2 = 180$ mm $= 0.18$ m ; $\mu = 0.35$; $N_1 = 1250$ r.p.m. or $\omega_1 = \pi \times 1250/60 = 131$ rad/s

1. Final speed of the motor and rotor

Let ω_3 = Final speed of the motor and rotor in rad/s.

We know that moment of inertia for the motor armature and shaft,

$$I_1 = m_1 (k_1)^2 = 800\,(0.2)^2 = 32 \text{ kg-m}^2$$

and moment of inertia for the rotor,

$$I_2 = m_2 (k_2)^2 = 1300\,(0.18)^2 = 42.12 \text{ kg-m}^2$$

Since the angular momentum before slipping is equal to the angular momentum after slipping, therefore

$$I_1.\omega_1 + I_2.\omega_2 = (I_1 + I_2)\,\omega_3$$

$$32 \times 131 + I_2 \times 0 = (32 + 42.12)\,\omega_3 = 74.12\,\omega_3 \qquad \ldots(\because \omega_2 = 0)$$

$$\therefore \quad \omega_3 = 32 \times 131 / 74.12 = 56.56 \text{ rad/s } \textbf{Ans.}$$

2. Time to reach this speed

Let t = Time to reach this speed *i.e.* 56.56 rad/s.

We know that mean radius of the friction plate,

$$R = \frac{r_1 + r_2}{2} = \frac{110 + 80}{2} = 95 \text{ mm} = 0.095 \text{ m}$$

and total frictional torque,

$$T = n.\mu.W.R = 2 \times 0.35 \times 570 \times 0.095 = 37.9 \text{ N-m} \qquad ...(\because n = 2)$$

Considering the rotor, let α_2, ω_1 and ω_F be the angular acceleration, initial angular speed and the final angular speed of the rotor respectively.

We know that the torque (T),

$$37.9 = I_2.\alpha_2 = 42.12\ \alpha_2 \quad \text{or} \quad \alpha_2 = 37.9/42.12 = 0.9 \text{ rad/s}^2$$

Since the angular acceleration is the rate of change of angular speed, therefore

$$\alpha_2 = \frac{\omega_F - \omega_1}{t} \quad \text{or} \quad t = \frac{\omega_F - \omega_1}{\alpha_2} = \frac{56.56 - 0}{0.9} = 62.8 \text{ s} \quad \textbf{Ans.}$$

$$...(\because \omega_F = \omega_3 = 56.56 \text{ rad/s, and } \omega_1 = 0)$$

3. ***Kinetic energy lost during the period of slipping***

We know that angular kinetic energy before impact,

$$E_1 = \frac{1}{2} I_1 (\omega_1)^2 + \frac{1}{2} I_2 (\omega_2)^2 = \frac{1}{2} I_1 (\omega_1)^2 \qquad ...(\because \omega_2 = 0)$$

$$= \frac{1}{2} \times 32 (131)^2 = 274\ 576 \text{ N-m}$$

and angular kinetic energy after impact,

$$E_2 = \frac{1}{2}(I_1 + I_2)(\omega_3)^2 = \frac{1}{2}(32 + 42.12)(56.56)^2 = 118\ 556 \text{ N-m}$$

$\therefore$ Kinetic energy lost during the period of slipping,

$$= E_1 - E_2 = 274\ 576 - 118\ 556 = 156\ 020 \text{ N-m} \quad \textbf{Ans.}$$

Time of slipping assuming constant resisting torque

Let t_1 = Time of slipping, and

ω_3 = Common angular speed of armature and rotor shaft = 56.56 rad/s

When slipping has ceased and there is exerted a constant torque of 60 N-m on the armature shaft, then

Torque on armature shaft,

$$T_1 = -60 - 37.9 = -97.9 \text{ N-m}$$

Torque on rotor shaft,

$$T_2 = T = 37.9 \text{ N-m}$$

Considering armature shaft,

$$\omega_3 = \omega_1 + \alpha_1 t_1 = \omega_1 + \frac{T_1}{I_1} \times t_1 = 131 - \frac{97.9}{32} \times t_1 = 131 - 3.06\ t_1 \qquad ...(i)$$

Considering rotor shaft,

$$\omega_3 = \alpha_2.t_1 = \frac{T_2}{I_2} \times t_1 = \frac{37.9}{42.12} \times t_1 = 0.9\ t_1 \qquad ...(ii)$$

From equations *(i)* and *(ii)*,

$$131 - 3.06\ t_1 = 0.9\ t_1 \quad \text{or} \quad 3.96\ t_1 = 131$$

$$\therefore \qquad t_1 = 131/3.96 = 33.1 \text{ s} \quad \textbf{Ans.}$$

Time of slipping assuming constant driving torque of 60 N-m

In this case, $T_1 = 60 - 37.9 = 22.1$ N-m

Since $\omega_1 + \frac{T_1}{I_1} \times t_1 = \frac{T_2}{I_2} \times t_1$, therefore

$$131 + \frac{22.1}{32} \times t_1 = \frac{37.9}{42.12} \times t_1 \quad \text{or} \quad 131 + 0.69\,t_1 = 0.9\,t_1$$

$\therefore$ $0.9\,t_1 - 0.69\,t_1 = 131$ or $t_1 = 624$ s **Ans.**

25) A plate clutch has three discs on the driving shaft and two discs on the driven shaft, providing four pairs of contact surfaces. The outside diameter of the contact surfaces is 240 mm and inside diameter 120 mm. Assuming uniform pressure and μ = 0.3; find the total spring load pressing the plates together to transmit 25 kW at 1575 r.p.m. If there are 6 springs each of stiffness 13 kN/m and each of the contact surfaces has worn away by 1.25 mm, find the maximum power that can be transmitted, assuming uniform wear.

Solution. Given : $n_1 = 3$; $n_2 = 2$; $n = 4$; $d_1 = 240$ mm or $r_1 = 120$ mm ; $d_2 = 120$ mm or $r_2 = 60$ mm ; $\mu = 0.3$; $P = 25$ kW $= 25 \times 10^3$ W ; $N = 1575$ r.p.m. or $\omega = 2\pi \times 1575/60 = 165$ rad/s

Total spring load

Let W = Total spring load, and

T = Torque transmitted.

We know that power transmitted (P),

$$25 \times 10^3 = T.\omega = T \times 165 \quad \text{or} \quad T = 25 \times 10^3/165 = 151.5 \text{ N-m}$$

Mean radius of the contact surface, for uniform pressure,

$$R = \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \frac{2}{3}\left[\frac{(120)^3 - (60)^3}{(120)^2 - (60)^2}\right] = 93.3 \text{ mm} = 0.0933 \text{ m}$$

and torque transmitted (T),

$$151.5 = n.\mu.W.R = 4 \times 0.3\,W \times 0.0933 = 0.112\,W$$

$\therefore$ $W = 151.5/0.112 = 1353$ N **Ans.**

Maximum power transmitted

Given : No of springs = 6

$\therefore$ Contact surfaces of the spring

= 8

Wear on each contact surface

= 1.25 mm

$\therefore$ Total wear $= 8 \times 1.25 = 10$ mm $= 0.01$ m

Stiffness of each spring $= 13$ kN/m $= 13 \times 10^3$ N/m

$\therefore$ Reduction in spring force

= Total wear × Stiffness per spring × No. of springs

$= 0.01 \times 13 \times 10^3 \times 6 = 780$ N

∴ New axial load, $W = 1353 - 780 = 573$ N

We know that mean radius of the contact surfaces for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{120 + 60}{2} = 90 \text{ mm} = 0.09 \text{ m}$$

∴ Torque transmitted,

$$T = n.\mu.W.R. = 4 \times 0.3 \times 573 \times 0.09 = 62 \text{ N-m}$$

and maximum power transmitted,

$$P = T.\,\omega = 62 \times 155 = 10\,230 \text{ W} = 10.23 \text{ kW} \quad \textbf{Ans.}$$

26) The following data relate to a screw jack:
Pitch of the threaded screw = 8 mm
Diameter of the threaded screw = 40 mm
Coefficient of friction between screw and nut = 0.1
Load = 20kN
Assuming that the load rotates with the screw, determine the
(i) Ratio of torques required to raise and lower the load
(ii) Efficiency of the machine. ***[APRIL/MAY-2017]***

Given Data :

$$p = 8 \text{ mm} = 8 \times 10^{-3} \text{ m};$$
$$d = 40 \text{ mm} = 40 \times 10^{-3} \text{ m};$$
$$\mu = \tan\phi = 0.1;$$
$$W = 20 \text{ KN} = 20 \times 10^3 \text{ N}$$

☺ ***Solution :*** Helix angle is given by

$$\tan\alpha = \frac{p}{\pi d} = \frac{8 \times 10^{-3}}{\pi \times 40 \times 10^{-3}} = 0.0637$$

$$\alpha = 3.64°$$

Friction angle ϕ is given by

$$\mu = \tan\phi = 0.1$$
$$\phi = 5.71°$$

(i) Ratio of torques required to raise and lower the load :

Torque required to raise the load $T_1 = W\tan(\alpha + \phi)\cdot\frac{d}{2}$

$$= 20 \times 10^{-3}\tan(3.64° + 5.71°)\cdot\frac{40 \times 10^{-3}}{2}$$

$$T_1 = 65.861 \text{ N-m}$$

Torque required to lower the load, $T_2 = W\tan(\phi - \alpha)\,\frac{d}{2}$

$$= 20 \times 10^3 \tan(5.71° - 3.64°) \times \frac{40 \times 10^{-3}}{2}$$

$$T_2 = 14.457 \text{ N-m}$$

Ratio of torques, $\frac{T_1}{T_2} = \frac{65.861}{14.457} = 4.556$ **Ans.**

(ii) Efficiency of the machine :

$$\eta_{screwjack} = \frac{\tan \alpha}{\tan(\alpha + \phi)} = \frac{\tan 3.64°}{\tan(3.64° + 5.71°)}$$

$$\eta = 0.386 \text{ or } 38.6\% \text{ Ans.}$$

27) A single plate clutch transmits 25 kw at 900 rpm. The maximum pressure intensity between the plates is 85 kN/m². The outer diameter of the plate is 360 mm. Both the sides of the plate are effective and the coefficient of friction is 0.25. Determine the (i) Inner radius of the plate (ii) Axial force to engage the clutch. ***[APRIL/MAY-2017]***

Given Data:

$P = 25KW = 25 \times 10^3 W$

$N = 900rpm$

$P_{max} = 85KN/m^2 = 85 \times 10^3 N/m^2$

$d_1 = 360mm \Rightarrow r_1 = 180mm = 0.180\ m$

$n = 2$

$\mu = 0.25$

To find:

(i) Innu radius of plate (r_1);

(ii) Axial force required to engage the dutch "W".

Solution:

(i) Innu Radius of plate 'r'

$$\text{Power } P = \frac{2\pi NT}{60}$$

$$25 \times 10^3 = \frac{2\pi \times 900 \times T}{60}$$

$$\boxed{T = 265.26 N\text{-}m}$$

The maximum pressure in tensity is at inner radius 'r_2'.

$P_{max}.\ r_2 = c$

$C = 85 \times 10^3.\ r_2\ N/mm$

Axial Force W:

$W = 2\pi C(r_1 - r_2)$

$= 2\pi \times 85 \times 10^3 \times r_2 \times (0.180 - r_2)$

$W = 534.07 \times 10^3 r_2(0.180 - r_2)$

Torque Transmitted 'T':

$T = n.\mu.W.R$

$265.26=2\times0.25\times534.07\times10^3 r_2(0.180-r_2).\left(\frac{180+r_2}{2}\right)$

$\boxed{r_2 = 0.321m(or) = 132mm}$

(ii) Arial force required to engage the cluetch:

$W = 2\pi C(r_1 - r_2)$

$= 2 \times \pi \times 85 \times 10^3 \times 0.132 \times 90.180 - 0.132)$

$\boxed{W = 3383.87N}$

Result:

1. Inner redius of plate : $r_2 = 132$ mm
2. Arial Force Required to engage the cluteh: (W)=3383.87N

28) A cross belt running over two pulleys 600 mm and 300 mm diameter connects two parallel shafts 4 meters apart and transmits 7.5 kW from the larger pulley that rotates at 225 rpm. Coefficient of friction between the belt and the pulley is 0.35 and the safe working tension is 25 N per mm width. Determine 1. Minimum width of the belt 2. Initial belt tension and 3. Length of the belt required.

[NOV/DEC 2017]

Given :

d_1=600mm = 0.6m

d_2=300mm = 0.3m

x=4m

P= 7.5KW=7500W

N_1=225 rpm

μ=0.35

T_1=25N/mm width

Determine:

1. Minimum width of the belt (b)
2. Initial Belt Transision(T_0)
3. Length of the Belt required (L)

Solution:

(i) Minimum Width of the Belt (b)

$$V = \frac{\pi . d_1 . N_1}{60} = \frac{\pi \times 0.6 \times 225}{60}$$

$$\boxed{V = 2.25 m/s}$$

$$P = (T_1 - T_2).V$$

$$7500 = (T_1 - T_2) 2.25$$

$$T_1 - T_2 = 3333.33N \qquad \rightarrow 1$$

WKT, for given belt drive

$$\sin\alpha = \frac{r_1 + r_2}{x} = \frac{d_1 + d_2}{2x}$$

$$= \frac{0.6 + 0.3}{2(4)} = 0.1125$$

$$\alpha = \sin^{-1}(0.1125) => \boxed{\alpha = 6.45^o}$$

∴ Angle of Contact:

$$\theta = (180^o + 2\alpha)\frac{\pi}{180}$$

$$= [180 + 2 \times (6.45)] \times \frac{\pi}{180}$$

$$\boxed{\theta = 3.36 rad}$$

Tansion Ratio

$$\frac{T_1}{T_2} = e^{\mu . \theta}$$

$$\frac{T_1}{T_2} = e^{0.35 \times 3.36}$$

$$\frac{T_1}{T_2} = 3.24 => \boxed{T_1 = 3.24 T_2} \qquad \rightarrow 2$$

Solving 1 and 2

$$\boxed{T_1 = 4821.42N} \& \boxed{T_2 = 1488.09N}$$

Since, the safe working tension is 25N/mm width

$$b = \frac{4821.42}{25} = 192.85mm$$

(ii) Initial Belt Tension (T_0):

$$T_0 = \frac{T_1 + T_2}{2} = \frac{4821.42 + 1488.09}{2}$$

$$\boxed{T_0 = 3204.75N}$$

(iii) Length of the Belt required (L)

$$L = \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x}$$

$$= \frac{\pi}{2}(0.6 + 0.3) + 2 \times 4 + \frac{(0.6 + 0.3)^2}{16}$$

$$\boxed{L = 9.46m}$$

Result :

1. Width of the Belt b= 192.85mm
2. Initial Belt Tension, T_o=3204.75N
3. Length of Belt required, L=9.46m

29) An electric motor driven power screw moves a nut in a horizontal plane against a force of 75 kN at a speed of 300 mm/min. The screw has a single square thread Of 6 mm pitch on a major diameter of 40 mm. The coefficient of friction at the screw threads is 0.1. Estimate power of the motor. ***[NOV/DEC 2017]***

Given data:

Load to be lifted	$W = 75$ kN
	$= 75 \times 10^3$ N
Speed	$v = 300$ mm/min
Pitch of the screw	$P = 6$ mm
Major diameter	$d_0 = 40$ mm
Co-efficient of friction	$\mu = \tan\phi = 0.1$

To Find

Power of the motor

☛ **Solution**

We know that mean diameter of the screw

$$d = d_0 - \frac{P}{2}$$

$$d = 40 - \frac{6}{2}$$

$$\boxed{d = 37 \text{ mm} \quad 0.037 \text{ m}}$$

$$\tan\alpha = \frac{P}{\pi d}$$

$$\tan\alpha = \frac{6}{\pi \times 37}$$

$$\boxed{\tan\alpha = 0.0516}$$

Force required at the circumference of the screw

$$P = W\tan(\alpha + \phi)$$

(Or)

$$P = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha \cdot \tan\phi}\right]$$

$$P = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha \cdot \tan\phi}\right]$$

$$P = 75 \times 10^3\left[\frac{0.0516 + 0.1}{1 - 0.0516 \times 0.1}\right]$$

$$\boxed{P = 11.43 \times 10^3 \text{ N}}$$

Torque required to overcome friction

$$T = P \times \frac{d}{2}$$

$$T = 11.43 \times 10^3 \times \frac{0.0372}{2}$$

$$\boxed{T = 211.45 \text{ N} - \text{m}}$$

Speed of the screw

$$N = \frac{\text{Speed of the nut}}{\text{Pitch of the screw}}$$

$$N = \frac{300}{6}$$

$$\boxed{N = 50 \text{ r.p.m}}$$

Angular speed

$$\omega = \frac{2 \times \pi \times N}{60}$$

$$\omega = \frac{2 \times \pi \times 50}{60}$$

$$\boxed{\omega = 5.24 \text{ rad/sec}}$$

Power of the motor

Power $= T \times \omega$

211.45×5.24

$$\boxed{\text{Power} = 1108 \text{ w} = 1.108 \text{ kw}}$$

Result

Power of the motor $P = 1.108$ kw

30. a) i) State and prove the relationship between angle of friction and co-efficient of friction with suitable sketches. (5)

ii) An open belt running over two pulleys of diameters 600 mm and 200 mm connects two parallel shafts which are 2.5 m apart. The smaller pulley transmits 7.5 kW at 300 rpm. The co-efficient of friction between the pulley and the belt is 0.3. Determine the ratio of tension on tight side, T1 with tension on slack side, T2 and the initial tension on the belt. (8)

[APRIL/MAY-2018]

(a) (i) Relationship between Angle of Friction (φ) and Co-efficient of Friction (μ):

Limiting Angle of Friction

Consider that a body *A* of weight (*W*) is resting on a horizontal plane *B*, as shown in Fig. If a horizontal force *P* is applied to the body, no relative motion will take place until the applied force *P* is equal to the force of friction *F*, acting opposite to the direction of motion. The magnitude of this force of friction is $F = \mu . W = \mu . R_N$, where R_N is the normal reaction. In the limiting case, when the motion just begins, the body will be in equilibrium under the action of the following three forces :

1. Weight of the body (*W*),
2. Applied horizontal force (*P*), and
3. Reaction (*R*) between the body *A* and the plane *B*.

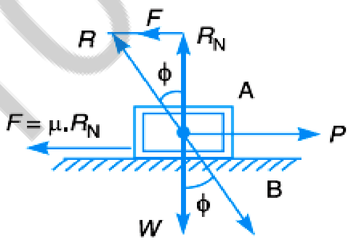


Fig. Limiting angle of friction.

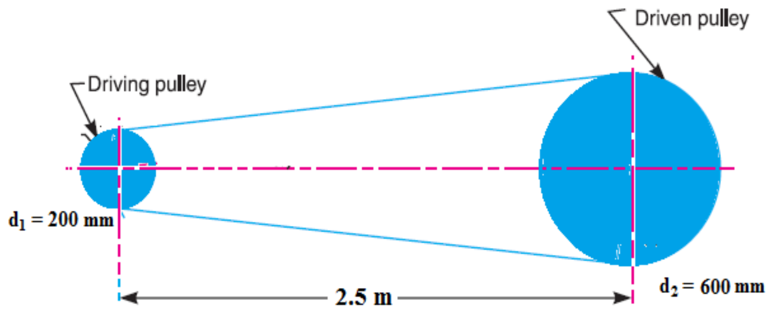
The reaction *R* must, therefore, be equal and opposite to the resultant of *W* and *P* and will be inclined at an angle ϕ to the normal reaction R_N. This angle ϕ is known as the ***limiting angle of friction***. It may be defined as the angle which the resultant reaction *R* makes with the normal reaction R_N.

From Fig. $\tan \phi = F/R_N = \mu R_N / R_N = \mu$

$$\boxed{\tan \phi = \mu}$$

(ii)

Given:



$d_1 = 200$ mm

$d_2 = 600$ mm

x = 2.5 m

P = 7.5kW = 7.5 X 10^3 W

$N_1 = 300$ rpm

$\mu = 0.3$

To Find: T_1 , T_2 & T_0

Solution:

(i) Tension on Tight and Slack Side (T_1 & T_2):

We know that velocity of the belt,

$$v = \frac{\pi \cdot d_1 N_1}{60} = \frac{\pi \times 0.20 \times 300}{60} = \mathbf{3.14} \text{ m/s}$$

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

∴ Power transmitted (P),

$$\mathbf{7500} = (T_1 - T_2)\,v = (T_1 - T_2)\,\mathbf{3.14}$$

$$T_1 - T_2 = \mathbf{7500 / 3.14} = \mathbf{2388.5\ N} \longrightarrow (1)$$

We know that for an open belt drive,

$$\sin\alpha = \frac{r_1 - r_2}{x} = \frac{d_1 - d_2}{2x} = \frac{0.6 - 0.2}{2 \times 2.5} = \mathbf{0.08} \quad \text{or} \quad \boxed{\alpha = \mathbf{4.58}^\circ}$$

and angle of lap on the smaller pulley,

$$\theta = 180^\circ - 2\alpha = 180^\circ - 2 \times \mathbf{4.58}^\circ = \mathbf{170.84}^\circ$$

$$= \mathbf{170.84}^\circ \times \pi / 180 = \mathbf{2.98} \text{ rad}$$

$$\boxed{\theta = \mathbf{2.98} \text{ rad}}$$

$$\left(\frac{T_1}{T_2}\right) = e^{\mu.\theta} = e^{\mathbf{0.3\ x\ 2.98}}$$

$$\left(\frac{T_1}{T_2}\right) = \mathbf{2.44} \longrightarrow (2)$$

From equations **1 & 2**

$$\boxed{T_1 = \mathbf{1658.68}\text{ N, and } T_2 = \mathbf{679.78}\text{ N}}$$

(ii) Initial Tension T_0:

We know that initial belt tension,

$$T_0 = \frac{T_1 + T_2}{2} = \frac{\mathbf{1658.68 + 679.78}}{2} = \mathbf{1169.23}\text{ N} \quad \textbf{Ans.}$$

$$\boxed{T_0 = \mathbf{1169.23}\text{ N}}$$

31. b) i) Neatly sketch a Simple Band Brake and derive the equations for braking torque for both directions of rotation separately and compare them. (8)

ii) The outer and inner radii of a flat collar thrust bearing are 120 mm and 72 mm respectively. The total axial thrust is 60 kN and the intensity of uniform pressure is 0.25 MPa. If the coefficient of friction is 0.05 and the shaft rotates at 600 rpm, determine the power lost in overcoming the friction and the number of collars required to withstand the axial thrust. (5)

[APRIL/MAY-2018]

(i)

Simple Band Brake

A band brake consists of a flexible band of leather, one or more ropes,or a steel lined with friction material, which embraces a part of the circumference of the drum. A band brake, as shown in Fig., is called a ***simple band brake*** in which one end of the band is attached to a fixed pin or fulcrum of the lever while the other end is attached to the lever at a distance b from the fulcrum.

When a force P is applied to the lever at C, the lever turns about the fulcrum pin O and tightens the band on the drum and hence the brakes are applied. The friction between the band and the drum provides the braking force. The force P on the lever at C may be determined as discussed below :

Let T_1 = Tension in the tight side of the band,

T_2 = Tension in the slack side of the band,

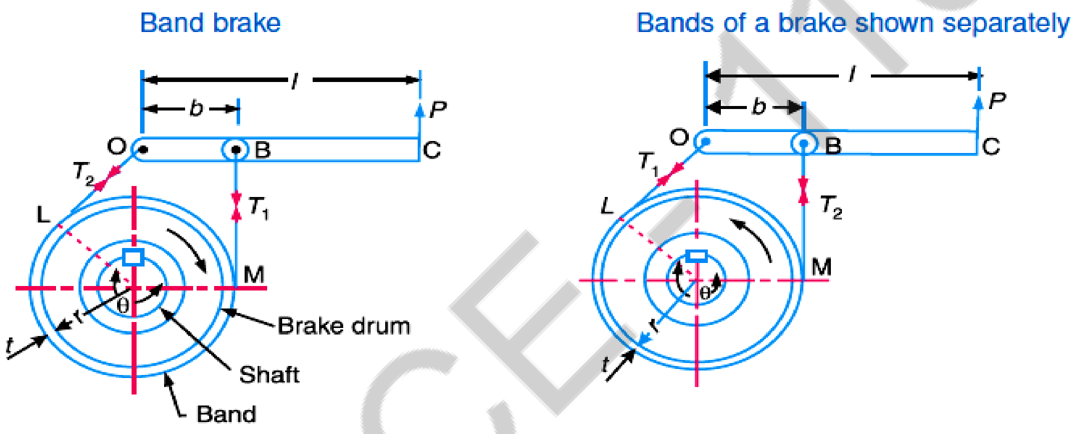
θ = Angle of lap (or embrace) of the band on the drum,

μ = Coefficient of friction between the band and the drum,

r = Radius of the drum,

t = Thickness of the band, and

r_e = Effective radius of the drum = $r + \frac{t}{2}$



(*a*) Clockwise rotation of drum. (*b*) Anticlockwise rotation of drum.

Fig. Simple band brake.

We know that limiting ratio of the tensions is given by the relation,

$$\frac{T_1}{T_2} = e^{\mu\theta} \quad \text{or} \quad 2.3\log\left(\frac{T_1}{T_2}\right) = \mu\theta$$

and braking force on the drum = $T_1 - T_2$

$\therefore$ Braking torque on the drum,

$$T_B = (T_1 - T_2)\, r \qquad \text{. . . (Neglecting thickness of band)}$$

$$= (T_1 - T_2)\, r_e \qquad \text{. . . (Considering thickness of band)}$$

Now considering the equilibrium of the lever *OBC*. It may be noted that when the drum rotates in the clockwise direction, as shown in Fig. 19.11 (*a*), the end of the band attached to the fulcrum *O* will be slack with tension T_2 and end of the band attached to *B* will be tight with tension T_1. On the other hand, when the drum rotates in the anticlockwise direction, as shown in Fig. 19.11 (*b*), the tensions in the band will reverse, *i.e.* the end of the band attached to the fulcrum *O* will be tight with tension T_1 and the end of the band attached to *B* will be slack with tension T_2. Now taking moments about the fulcrum *O*, we have

$$P.l = T_1.b \qquad \text{. . . (For clockwise rotation of the drum)}$$

and

$$P.l = T_2.b \qquad \text{. . . (For anticlockwise rotation of the drum)}$$

where l = Length of the lever from the fulcrum (*OC*), and

b = Perpendicular distance from *O* to the line of action of T_1 or T_2.

1. When the brake band is attached to the lever, as shown in Fig. 19.11 (*a*) and (*b*), then the force (*P*) must act in the upward direction in order to tighten the band on the drum.

2. If the permissible tensile stress (σ) for the material of the band is known, then maximum tension in the band is given by

$$T_1 = \sigma.w.t$$

where w = Width of the band, and

t = thickness of the band.

(ii)

Given:

r_1 = 120 mm

r_2 = 72 mm

W = 60 kN = 60 X 10^3 N

p = 0.25 MPa = 0.25 N/mm^2

N = 600 rpm

μ = 0.05

To Find:

(i) Power lost in Friction (P)

(ii) No. of Collars required (n).

Solution:

(i) Power Lost to Overcoming Friction:

We know that for uniform pressure, total frictional torque transmitted,

$$T = \frac{2}{3} \times \mu.W \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \frac{2}{3} \times 0.05 \times 60 \times 10^3 \left[\frac{(120)^3 - (72)^3}{(120)^2 - (72)^2}\right] \text{N-mm}$$

$$= 2000 \text{ X } 147 = 294 \times 10^3 \text{ N-mm} = 294 \text{ N-m}$$

$\therefore$ **Power Lost:**

$$P = T.\omega$$

$$\omega = 2\pi \times 600/60 = 62.83 \text{ rad/s}$$

$$= 294 \text{ X } 62.83 = 18472.02 \text{ W}$$

$$\mathbf{P = 18.47\ kW}$$

(i) <u>No. of Collars required (n):</u>

Let n = Number of collars required.

We know that the intensity of uniform pressure (p),

$$0.25 = \frac{W}{n.\pi[(r_1)^2 - (r_2)^2]} = \frac{60 \times 10^3}{n.\pi[(120)^2 - (72)^2]} = \frac{2.07}{n}$$

$\therefore$ **n = 2.07/0.25 = 8.28 ≈ 9**

Number of collars required. n = **9**

32. A multi disc clutch has three discs on the driving shaft and two on the driven shaft. The outside diameter of the contact surface is 240 mm and the inside diameter is 120 mm. Assuming uniform wear and coefficient of friction as 0.3. Find the maximum axial intensity of pressure between discs for transmitting 25 KW at 1575 rpm. (13)

[NOV/DEC-2018]

Solution. Given : $n_1 = 3$; $n_2 = 2$; $d_1 = 240$ mm or $r_1 = 120$ mm ; $d_2 = 120$ mm or $r_2 = 60$ mm ; $\mu = 0.3$; $P = 25$ kW $= 25 \times 10^3$ W ; $N = 1575$ r.p.m. or $\omega = 2\,\pi \times 1575/60 = 165$ rad/s

Let T = Torque transmitted in N-m, and

W = Axial force on each friction surface.

We know that the power transmitted (P),

$$25 \times 10^3 = T.\omega = T \times 165 \quad \text{or} \quad T = 25 \times 10^3/165 = 151.5 \text{ N-m}$$

Number of pairs of friction surfaces,

$$n = n_1 + n_2 - 1 = 3 + 2 - 1 = 4$$

and mean radius of friction surfaces for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{120 + 60}{2} = 90 \text{ mm} = 0.09 \text{ m}$$

We know that torque transmitted (T),

$$151.5 = n.\mu.W.R = 4 \times 0.3 \times W \times 0.09 = 0.108\,W$$

$$\therefore \quad W = 151.5/0.108 = 1403 \text{ N}$$

Let p = Maximum axial intensity of pressure.

Since the intensity of pressure (p) is maximum at the inner radius (r_2), therefore for uniform wear

$$p.r_2 = C \quad \text{or} \quad C = p \times 60 = 60\,p \text{ N/mm}$$

We know that the axial force on each friction surface (W),

$$1403 = 2\,\pi.C\,(r_1 - r_2) = 2\,\pi \times 60\,p\,(120 - 60) = 22\,622\,p$$

$$\therefore \quad p = 1403/22\,622 = 0.062 \text{ N/mm}^2$$ **Ans.**

33. Derive an expression to find the length of a belt in an open belt drive. (13)

[NOV/DEC-2018]

Length of an Open Belt Drive

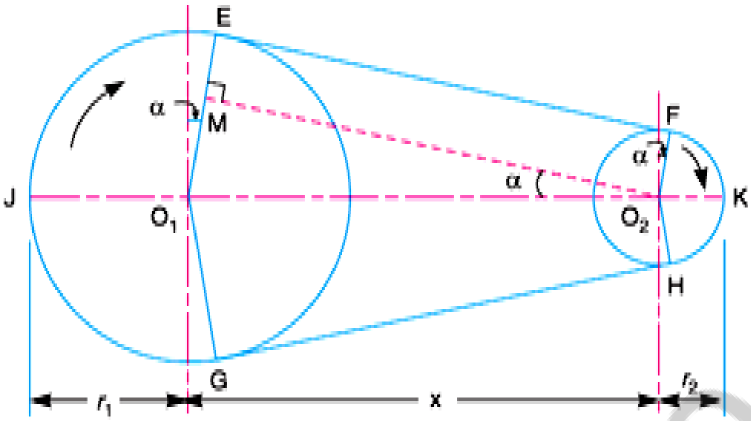


Fig. Length of an open belt drive.

We have already discussed in Art. 11.6 that in an open belt drive, both the pulleys rotate in the *same* direction as shown in Fig.

Let r_1 and r_2 = Radii of the larger and smaller pulleys,

x = Distance between the centres of two pulleys (*i.e.* $O_1 O_2$), and

L = Total length of the belt.

Let the belt leaves the larger pulley at E and G and the smaller pulley at F and H as shown in Fig. Through O_2, draw O_2M parallel to FE.

From the geometry of the figure, we find that O_2M will be perpendicular to O_1E.

Let the angle MO_2O_1 = α radians.

We know that the length of the belt,

$$L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$$

$$= 2\,(\text{Arc } JE + EF + \text{Arc } FK) \qquad \ldots(i)$$

From the geometry of the figure, we find that

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E - EM}{O_1O_2} = \frac{r_1 - r_2}{x}$$

Since α is very small, therefore putting

$$\sin\alpha = \alpha \text{ (in radians)} = \frac{r_1 - r_2}{x} \qquad \ldots(ii)$$

$$\therefore \quad \text{Arc } JE = r_1\left(\frac{\pi}{2} + \alpha\right) \qquad \ldots(iii)$$

Similarly

$$\text{Arc } FK = r_2\left(\frac{\pi}{2} - \alpha\right) \qquad \ldots(iv)$$

and

$$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2} = \sqrt{x^2 - (r_1 - r_2)^2}$$

$$= x\sqrt{1-\left(\frac{r_1-r_2}{x}\right)^2}$$

Expanding this equation by binomial theorem,

$$EF = x\left[1-\frac{1}{2}\left(\frac{r_1-r_2}{x}\right)^2+ ....\right] = x - \frac{(r_1-r_2)^2}{2x} \quad ...(v)$$

Substituting the values of arc *JE* from equation *(iii)*, arc *FK* from equation *(iv)* and *EF* from equation *(v)* in equation *(i)*, we get

$$L = 2\left[r_1\left(\frac{\pi}{2}+\alpha\right)+x-\frac{(r_1-r_2)^2}{2x}+r_2\left(\frac{\pi}{2}-\alpha\right)\right]$$

$$= 2\left[r_1\times\frac{\pi}{2}+r_1.\alpha+x-\frac{(r_1-r_2)^2}{2x}+r_2\times\frac{\pi}{2}-r_2.\alpha\right]$$

$$= 2\left[\frac{\pi}{2}(r_1+r_2)+\alpha(r_1-r_2)+x-\frac{(r_1-r_2)^2}{2x}\right]$$

$$= \pi(r_1+r_2)+2\alpha(r_1-r_2)+2x-\frac{(r_1-r_2)^2}{x}$$

Substituting the value of $\alpha = \frac{r_1-r_2}{x}$ from equation *(ii)*,

$$L = \pi(r_1+r_2)+2\times\frac{(r_1-r_2)}{x}\times(r_1-r_2)+2x-\frac{(r_1-r_2)^2}{x}$$

$$= \pi(r_1+r_2)+\frac{2(r_1-r_2)^2}{x}+2x-\frac{(r_1-r_2)^2}{x}$$

$$= \pi(r_1+r_2)+2x+\frac{(r_1-r_2)^2}{x} \quad \text{...(In terms of pulley radii)}$$

$$= \frac{\pi}{2}(d_1+d_2)+2x+\frac{(d_1-d_2)^2}{4x} \quad \text{...(In terms of pulley diameters)}$$

$$\boxed{L = \frac{\pi}{2}(d_1+d_2)+2x+\frac{(d_1-d_2)^2}{4x}}$$

34. An open belt running over two pulleys 240 mm and 600 mm diameter connects two parallel shafts 3 metres apart and transmits 4 kW from the smaller pulley that rotates at 300 r.p.m. Co-efficient of friction between the belt and the pulley is 0.3 and the safe working tension is 10 N per mm width. Determine

(i) Minimum width of the belt.

(ii) Initial belt tension, and

(iii) Length of the belt required. (13)

[APR/MAY-2019]

Solution. Given : $d_2 = 240$ mm $= 0.24$ m ; $d_1 = 600$ mm $= 0.6$ m ; $x = 3$ m ; $P = 4$ kW $= 4000$ W; $N_2 = 300$ r.p.m. ; $\mu = 0.3$; $T_1 = 10$ N/mm width

1. ***Minimum width of belt***

We know that velocity of the belt,

$$v = \frac{\pi d_2 . N_2}{60} = \frac{\pi \times 0.24 \times 300}{60} = 3.77 \text{ m/s}$$

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

$\therefore$ Power transmitted (P),

$$4000 = (T_1 - T_2)\, v = (T_1 - T_2)\, 3.77$$

or $$T_1 - T_2 = 4000 / 3.77 = 1061 \text{ N} \qquad ...(i)$$

We know that for an open belt drive,

$$\sin \alpha = \frac{r_1 - r_2}{x} = \frac{d_1 - d_2}{2x} = \frac{0.6 - 0.24}{2 \times 3} = 0.06 \quad \text{or} \quad \alpha = 3.44°$$

and angle of lap on the smaller pulley,

$$\theta = 180° - 2\alpha = 180° - 2 \times 3.44° = 173.12°$$

$$= 173.12 \times \pi / 180 = 3.022 \text{ rad}$$

We know that

$$2.3 \log \left(\frac{T_1}{T_2} \right) = \mu . \theta = 0.3 \times 3.022 = 0.9066$$

$$\log \left(\frac{T_1}{T_2} \right) = \frac{0.9066}{2.3} = 0.3942 \quad \text{or} \quad \frac{T_1}{T_2} = 2.478 \qquad ...(ii)$$

...(Taking antilog of 0.3942)

From equations (*i*) and (*ii*),

$$T_1 = 1779 \text{ N}, \text{ and } T_2 = 718 \text{ N}$$

Since the safe working tension is 10 N per mm width, therefore minimum width of the belt,

$$b = \frac{T_1}{10} = \frac{1779}{10} = 177.9 \text{ mm} \quad \textbf{Ans.}$$

2. ***Initial belt tension***

We know that initial belt tension,

$$T_0 = \frac{T_1 + T_2}{2} = \frac{1779 + 718}{2} = 1248.5 \text{ N} \quad \textbf{Ans.}$$

3. ***Length of the belt required***

We know that length of the belt required,

$$L = \frac{\pi}{2}(d_1 - d_2) + 2x + \frac{(d_1 - d_2)^2}{4x}$$

$$= \frac{\pi}{2}(0.6 + 0.24) + 2 \times 3 + \frac{(0.6 - 0.24)^2}{4 \times 3}$$

$$= 1.32 + 6 + 0.01 = 7.33 \text{ m} \quad \textbf{Ans.}$$

35. A dry single plate clutch is to be designed for an automotive vehicle whose engine is rated to give 100 kW at 2400 r.p.m. and maximum torque 500 N-m. The outer radius of friction plate is 25% more than the inner radius. The intensity of pressure between the plates is not to exceed 0.07 N/mm². The coefficient of friction may be assumed equal to 0.3. The helical springs required by this clutch to provide axial force necessary to engage the clutch are eight. If each spring has stiffness equal to 40 N/mm, determine the initial compression in the springs and dimensions of the friction plate. (13)

[APR/MAY-2019]

Solution. Given : $P = 100$ kW $= 100 \times 10^3$ W ; $T = 500$ N-m $= 500 \times 10^3$ N-mm ; $p = 0.07$ N/mm^2 ; $\mu = 0.3$; Number of springs = 8 ; Stiffness = 40 N/mm

Dimensions of the friction plate

Let r_1 and r_2 = Outer and inner radii of the friction plate respectively.

Since the outer radius of the friction plate is 25% more than the inner radius, therefore

$$r_1 = 1.25\, r_2$$

We know that, for uniform wear,

$$p.r_2 = C \quad \text{or} \quad C = 0.07\, r_2 \text{ N/mm}$$

and load transmitted to the friction plate,

$$W = 2\,\pi\, C\,(r_1 - r_2) = 2\,\pi \times 0.07\, r^2\,(1.125\, r_2 - r_2) = 0.11\,(r_2)^2 \text{ N} \qquad \ldots(i)$$

We know that mean radius of the plate for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{1.25\, r_2 + r_2}{2} = 1.125\, r_2$$

$\therefore$ Torque transmitted (T),

$$500 \times 10^3 = n.\mu.W.R = 2 \times 0.3 \times 0.11\,(r_2)^2 \times 1.125\, r_2 = 0.074\,(r_2)^3 \qquad \ldots(\because\; n = 2)$$

$\therefore$ $(r_2)^3 = 500 \times 10^3/0.074 = 6757 \times 10^3$ or $r_2 = 190$ mm **Ans.**

and $r_1 = 1.25\, r_2 = 1.25 \times 190 = 273.5$ mm **Ans.**

Initial compression of the springs

We know that total stiffness of the springs,

$$s = \text{Stiffness per spring} \times \text{No. of springs} = 40 \times 8 = 320 \text{ N/mm}$$

Axial force required to engage the clutch,

$$W = 0.11\,(r_2)^2 = 0.11\,(190)^2 = 3970 \text{ N} \qquad \ldots\text{[From equation } (i)\text{]}$$

$\therefore$ Initial compression in the springs

$$= W/s = 3970/320 = 12.5 \text{ mm} \quad \textbf{Ans.}$$

36. A simple band brake operates on a drum of 600 mm in diameter that is running at 200 rpm. The coefficient of friction is 0.25. The brake band has a contact of 270°, one end is fastened to a fixed pin and the other end to the brake arm 125 mm from the fixed pin. The straight brake arm is 750 mm long and placed perpendicular to the diameter that bisects the angle of contact.

(i) What is the pull necessary on the end of the brake arm to stop the wheel if 35 KW is being absorbed? What is the direction for minimum pull?

(ii) What width of steel band of 2.5 mm thick is required for this brake if the maximum tensile stress is not to exceed 50 N/mm²? (15)

[NOV/DEC-2018]

Solution. Given : d = 600 mm or r = 300 mm ; N = 200 r.p.m. ; $\mu = 0.25$; $\theta = 270° = 270 \times \pi/180 = 4.713$ rad ; Power = 35 kW = 35 × 10^3 W ; t = 2.5 mm ; σ = 50 N/mm²

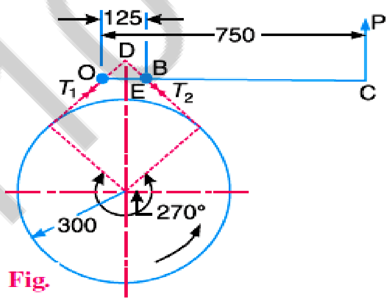


Fig.

All dimensions in mm

1. Pull necessary on the end of the brake arm to stop the wheel

Let P = Pull necessary on the end of the brake arm to stop the wheel.

The simple band brake is shown in Fig. Since one end of the band is attached to the fixed pin O, therefore the pull P on the end of the brake arm will act upward and when the wheel rotates anticlockwise, the end of the band attached to O will be tight with tension T_1 and the end of the band attached to B will be slack with tension T_2.

First of all, let us find the tensions T_1 and T_2. We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times 4.713 = 1.178$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{1.178}{2.3} = 0.5122 \text{ or } \frac{T_1}{T_2} = 3.25 \quad \text{... (Taking antilog of 0.5122) ... } (i)$$

Let T_B = Braking torque.

We know that power absorbed,

$$35 \times 10^3 = \frac{2\pi \times N\, T_B}{60} = \frac{2\pi \times 200 \times T_B}{60} = 21\, T_B$$

$\therefore$ $T_B = 35 \times 10^3 / 21 = 1667$ N-m $= 1667 \times 10^3$ N-mm

We also know that braking torque (T_B),

$1667 \times 10^3 = (T_1 - T_2)\ r = (T_1 - T_2)\ 300$

$\therefore$ $T_1 - T_2 = 1167 \times 10^3/300 = 5556$ N ...(*ii*)

From equations (*i*) and (*ii*), we find that

$T_1 = 8025$ N; and $T_2 = 2469$ N

Now taking moments about O, we have

$P \times 750 = T_2 \times {}^*OD = T_2 \times 62.5\sqrt{2} = 2469 \times 88.4 = 218\ 260$

$\therefore$ $P = 218260 / 750 = 291$ N **Ans.**

2. *Width of steel band*

Let w = Width of steel band in mm.

We know that maximum tension in the band (T_1),

$8025 = \sigma.w.t = 50 \times w \times 2.5 = 125\ w$

$\therefore$ $w = 8025 / 125 = 64.2$ mm **Ans.**

37. The mean diameter of the screw jack having pitch of 10 mm is 50 mm. A load of 20 KN is lifted through a distance of 170 mm. Find the work done in lifting the load and efficiency of screw jack when

(i) The load rotates with the screw and

(ii) The load rests on the loose head which does not rotate with the screw the external and internal diameters of the bearing surface of the loose head are 60 mm and 10 mm respectively. The coefficient of friction for the screw as well as the bearing surface may be taken as 0.08. (15)

[NOV/DEC-2018]

Solution. Given : $p = 10$ mm ; $d = 50$ mm ; $W = 20$ kN $= 20 \times 10^3$ N ; $D_2 = 60$ mm or $R_2 = 30$ mm ; $D_1 = 10$ mm or $R_1 = 5$ mm ; $\mu = \tan\phi = \mu_1 = 0.08$

We know that $\tan\alpha = \dfrac{p}{\pi d} = \dfrac{10}{\pi \times 50} = 0.0637$

∴ Force required at the circumference of the screw to lift the load,

$$P = W \tan(\alpha + \phi) = W \left[\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha . \tan \phi} \right]$$

$$= 20 \times 10^3 \left[\frac{0.0637 + 0.08}{1 - 0.0637 \times 0.08} \right] = 2890 \text{ N}$$

and torque required to overcome friction at the screw,

$$T = P \times d/2 = 2890 \times 50/2 = 72250 \text{ N-mm} = 72.25 \text{ N-m}$$

Since the load is lifted through a vertical distance of 170 mm and the distance moved by the screw in one rotation is 10 mm (equal to pitch), therefore number of rotations made by the screw,

$$N = 170/10 = 17$$

1. *When the load rotates with the screw*

We know that work done in lifting the load

$$= T \times 2\pi N = 72.25 \times 2\pi \times 17 = 7718 \text{ N-m} \quad \textbf{Ans.}$$

and efficiency of the screw jack,

$$\eta = \frac{\tan \alpha}{\tan(\alpha + \phi)} = \frac{\tan \alpha (1 - \tan \alpha . \tan \phi)}{\tan \alpha + \tan \alpha}$$

$$= \frac{0.0637 (1 - 0.0637 \times 0.08)}{0.0637 + 0.08} = 0.441 \quad \text{or} \quad 44.1\% \quad \textbf{Ans.}$$

2. *When the load does not rotate with the screw*

We know that mean radius of the bearing surface,

$$R = \frac{R_1 + R_2}{2} = \frac{30 + 5}{2} = 17.5 \text{ mm}$$

and torque required to overcome friction at the screw and the collar,

$$T = P \times d/2 + \mu_1 . W . R$$

$$= 2890 \times 50/2 + 0.08 \times 20 \times 10^3 \times 17.5 = 100\ 250 \text{ N-mm}$$

$$= 100.25 \text{ N-m}$$

∴ Work done by the torque in lifting the load

$$= T \times 2\pi N = 100.25 \times 2\pi \times 17 = 10\ 710 \text{ N-m} \quad \textbf{Ans.}$$

We know that the torque required to lift the load, neglecting friction,

$$T_0 = P_0 \times d/2 = W \tan \alpha \times d/2 \qquad \text{...}(\because P_0 = W \tan \alpha)$$

$$= 20 \times 10^3 \times 0.0637 \times 50/2 = 31\ 850 \text{ N-mm} = 31.85 \text{ N-m}$$

∴ Efficiency of the screw jack,

$$\eta = T_0 / T = 31.85/100.25 = 0.318 \quad \text{or} \quad 31.8\% \quad \textbf{Ans.}$$

38. A double start square threaded screw with 50 mm major diameter has 8 mm pitch. The coefficient of friction between screw and nut is 0.1. If the nut is held fixed, determine the torque required on the screw to raise and to lower a load of 40 kN assuming the load to rotate with the screw. State giving reasons whether the screw is self locking or over hauling. **(15)**

[APR/MAY-2019]

Given:

$d = 50$ mm

$p = 8$ mm

$W = 40$ kN $= 40 \times 10^3$ N

$\mu = \tan \phi = 0.1$

To Find:

(i) Torque required raising & lowering the Load.

(ii) Check for Self locking?

Solution:

(i) Torque required to raise the Load:

We know that, $\tan \alpha = \frac{p}{\pi d} = \frac{8}{\pi \times 50} = 0.05$

and force required on the screw to raise the load,

$$P = W \tan(\alpha + \phi) = W\left[\frac{\tan \phi - \tan \alpha}{1 + \tan \phi . \tan \alpha}\right]$$

$$= 40 \times 10^3 \left[\frac{0.05 + 0.1}{1 - 0.05 \times 0.1}\right] = 6030 \text{ N}$$

We know that the torque required on the screw to raise the load,

$$T_1 = P \times d/2 = 6030 \times 50/2 = 150750 \text{ N-mm}$$ **Ans.**

Torque required to lower the Load:

We know that the force required on the screw to lower the load,

$$P = W \tan(\phi - \alpha) = W\left[\frac{\tan \phi - \tan \alpha}{1 + \tan \phi . \tan \alpha}\right]$$

$$= 40 \times 10^3 \left[\frac{0.05 - 0.1}{1 + 0.05 \times 0.1}\right] = 1990 \text{ N}$$

and torque required to lower the load

$$T_2 = P \times d/2 = 1990 \times 50/2 = 49750 \text{ N-mm}$$

(ii) *Self locking of the screw*

We know that efficiency of the screw jack,

$$\eta = \frac{\tan \alpha}{\tan(\alpha + \phi)} = \frac{\tan \alpha (1 - \tan \alpha . \tan \phi)}{\tan \alpha + \tan \phi}$$

$$= 0.326 \quad \text{or} \quad 32.6\ \%$$

Since the efficiency of the screw jack is less than 50%, therefore the screw is a self locking screw. **Ans.**