UNIT – I

STATIC ELECTRIC FIELD

PART - A

1. Define Divergence:

The divergence of we vector $\vec{D}$ at any point is defined as the limit of the surface integral per unit volume as the volume enclosed by the surface around the point shrinks to zero.

$$\vec{D} = \lim_{\Delta v \to 0} \frac{\int \vec{D}.\hat{n}.ds}{\Delta V}$$

2. Define curl:

Curl of a vector at any point is defined as the limit of the ratio of two integral of its cross product with the outward drawn normal over a closed surface, to the volume enclosed by the surface as the volume shrinks to zero.

$$\mathrm{Curl}\vec{F} = \lim_{v \to 0} \frac{\oint \vec{F} \times n}{v}$$

3. State Stoke's theorem:

The line integral of the vector to enclosed path is equal to the integral of the normal component of its curl are any surface balanced by the continuous $\oint_s \vec{D}dl = \oint_s (\nabla \times \vec{F})n.c$.

4. State Gauss law:

The surface integral of a normal component of electric flux density vector D over a closed surface is equal to the charge enclosed by that surface.

$$\oint_s D.n.ds = Q$$

5. State Divergence theorem:

The integral of the divergence to the vector field over a volume V is equal the surface integral of the normal component the vector over any surface bounding the role.

$$\iiint_V \nabla.\vec{F}\ dv = \oint_S \vec{f}.n.\ ds$$

6. Define electric field intensity:

The electric field intensity is defined as the electric force her unit charge it is given by

$$E = \frac{F}{Q}$$

According to coulomb's law,

$$E = \frac{Q}{4\pi\omega r^2}$$

7. State Coulomb's law:

Coulomb's law of electrostatic force states that the force of attractive or repulsion is directly proportional to the product of the magnitude to the square of the distance between them

$$F \infty \frac{Q_1 Q_2}{R^2} N \text{ (or) } C^2 / m^2$$

8. Define electric flux:

Electric flux is defined as the lines or force. It denoted by the symbol $\psi(P_{si})$. Its unit is coulomb

$$E = \frac{Q}{4\pi\varepsilon r^2}$$

9. Define electric flux density:

It is defined as the flux per unit area. Its unit is C/m^2.

$$D = \frac{\psi}{4\pi r^2} = E\varepsilon$$

Where D is called electric flux density.

10. Define electric Dipole:

The equal and opposite charge constitute the electric dipole separated by a small distance .

11. Define absolute potential:

It is defined as the work done per coulomb to being to the point index consider.

$$V = \frac{Q}{4\pi\varepsilon_{or}}$$

12. What is the integral form of Gauss law:

$$\oiint_s D.n\ ds = Q$$

13. What is the point form of Gauss law:

$$\nabla.D = \rho$$

This is called the point form (or) differential form of Gauss law.

14. Define electric Dipole moment:

The product of charge and spacing is called electric dipole moment.

15. Define potential:

The potential is a scalar quantity and is found to be moving that test charge against the field from a reference point say from infinity to its final position.

16. What are the co-ordinate systems?

In order to describe a vector accurately, some specific lengths, directions, angles, projections or components must be given. There are three simple methods of doing this and they are

1. The rectangular or Cartesian system of co- ordinates
2. The circular cylindrical system of co –ordinates
3. The circular spherical system of co – ordinates

17. Give relation between electric field & potential:

If two points are separated by an infinitesimal distance dr, the work done in moving at point charge from one point to other is given by

$$dv = -E.\,dr$$

Since scalar potential V is a function of x, y, z.

$$\frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy + \frac{\partial v}{\partial z}dz = -E.\ dr$$

$$\left(\overline{a_x}\frac{\partial v}{\partial x} + \overline{a_y}\frac{\partial v}{\partial y} + \overline{a_z}\frac{\partial v}{\partial z}\right).\left(\overline{a_x}\ dx + \overline{a_y}\ dy\ \ + \overline{a_z}\ dz\right) = -E.\ dr$$

$$\nabla r.\ dr = -E.\ dr$$

$$\therefore E = -\nabla V$$

18. State the principle of superposition?

Principle of superposition:-

Let Q1 be at a distance of r1 from origin and Q2 be at a distance of r2, then E at P.

$$E(r) = \frac{Q_1}{4\pi\varepsilon_0 |r - r_1|^2}\overline{a_1} + \frac{Q_1}{4\pi\varepsilon_0 |r - r_1|^2}\overline{a_2}$$

If we add more charges at other positions, the field due to 'h' point charges is

$$E(r) = \frac{Q_1}{4\pi\varepsilon_0 |r - r_1|^2}\overline{a_1} + \frac{Q_1}{4\pi\varepsilon_0 |r - r_1|^2}\overline{a_2} + \ldots\ldots.\frac{Q_n}{4\pi\varepsilon_0 |r - r_n|^2}\overline{a_n}$$

$$E(r) = \sum_{m=1}^{n} \frac{Q_m}{4\pi\varepsilon_0 |r - r_m|^2} a_m$$

19. Give electric flux density due to line, surface * volume density electric flux density D or electric displacement

$$D = \varepsilon_0\ E\ c/m^2$$

Electric flux $\Psi = \oiint_s \overline{D}.\ ds$

For line charge, $D = \int \frac{\rho_L\ dl}{4\pi R^2}\ \ a_R$

For surface charge, $D = \int \frac{\rho_S\ ds}{4\pi R^2}\ \ a_R$

For volume charge, $D = \int \frac{\rho_V\ dv}{4\pi R^2}\ \ a_R$

20) What do you meant by conservation field?

If the line integral of a vector field around a closed path is zero, such a field is called conservative field.

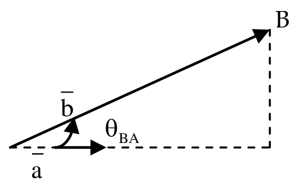
21) Define DOT product.

Given two vector A & B, the dot product or scalar product is defined as the product of the magnitude of A, the magnitude of B & the cosine of the smaller angle between them.

$$A.B = |A||B|\cos\theta_{AB}$$

The dot product obeys communicative law

A .B = B. A

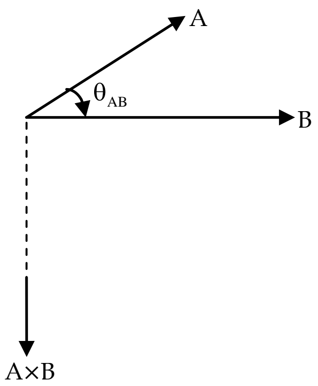


$$a_x.a_x = a_y.a_y = a_z.a_z = 1$$
$$a_x.a_y = a_y.a_z = a_z.a_x = 0$$

22) Define CROSS Product.

The cross product:-

$$A \times B = a_N |A||B| \sin \theta_{AB}$$
$$A \times B = \begin{vmatrix} a_x & a_y & a_{xz} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$



PART-B

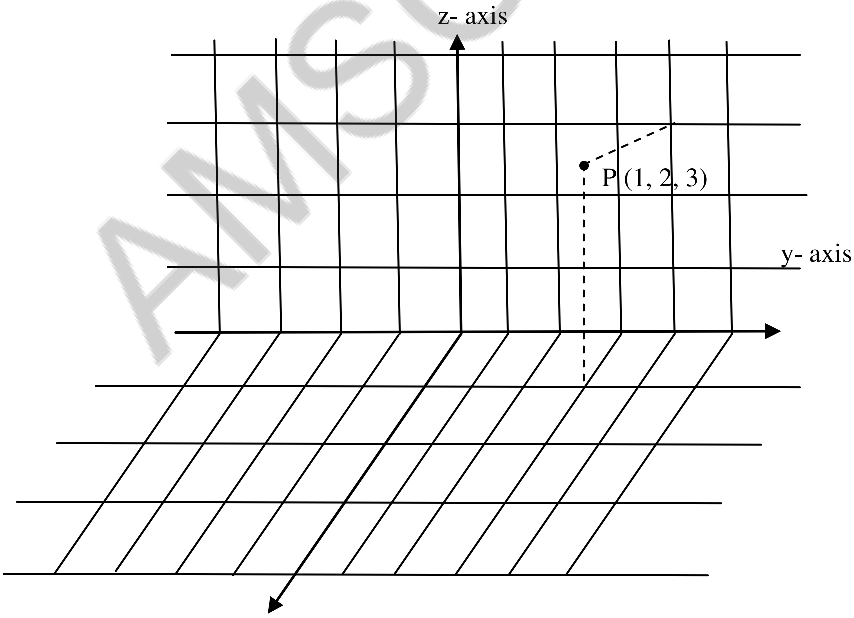
1) Explain the rectangular co ordinate systems?

This system has three co – ordinates axis mutually at right angles to each other and we name them as x, y and z axis. A rotation of x – axis into y – axis would cause a right handled screw to programs in the directions of z – axis.

A point is located by giving its x, y and z co – ordinates. These are, respectively the distance from the origin to the intersection of a perpendicular dropped from the point to the x, y and z axis.

An alternative method of interpreting co –ordinal values is to consider a point at the common intersection of three surface, the planes x = constant, y = constant & z = constant, the constant being the co –ordinates values of the point.

The following figure 1.1 (a) shows the points p(1, 2, 3) & (2, -2, 1) respectively

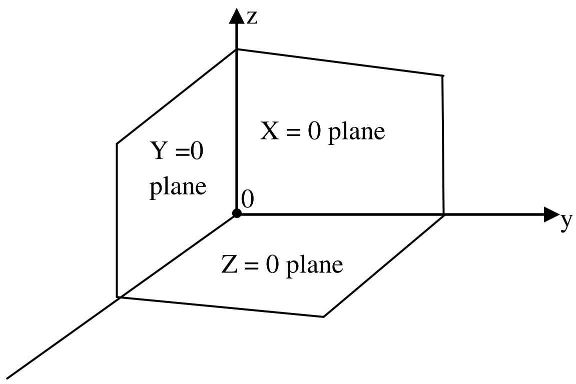


This figure (1.1b) shows a rectangular co ordinates system

If we visualise three planes interesting at the general point P, whose co – ordinates are x, y and z, we may increase each co ordinates value by a differential amount and obtain three slightly displaced planes intersecting at point 'P'. Whose co – ordinates are x+dx, y+dy and z+dz. The six planes define a rectangular parallelopiped whose volume is dv = dx dy dz the surface have differential areas $ds = dx\,dy; ds = dydz; ds = dxdz$

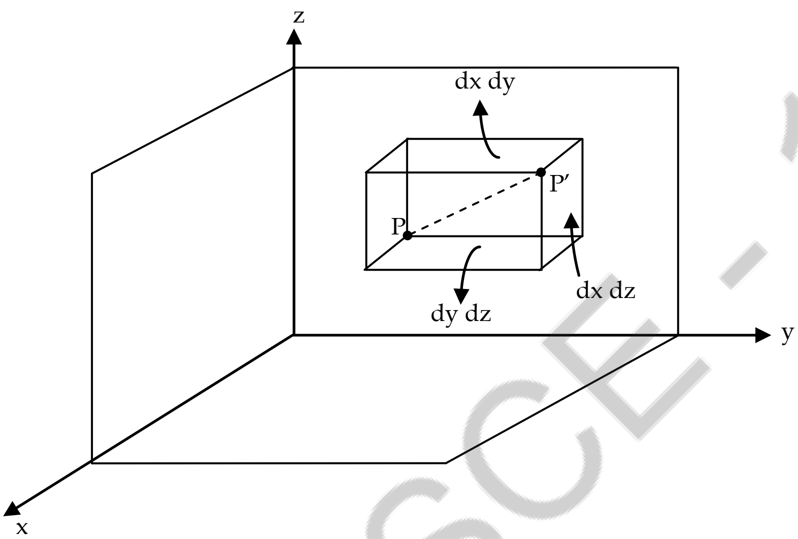
x- axis

Fig (1.1a)



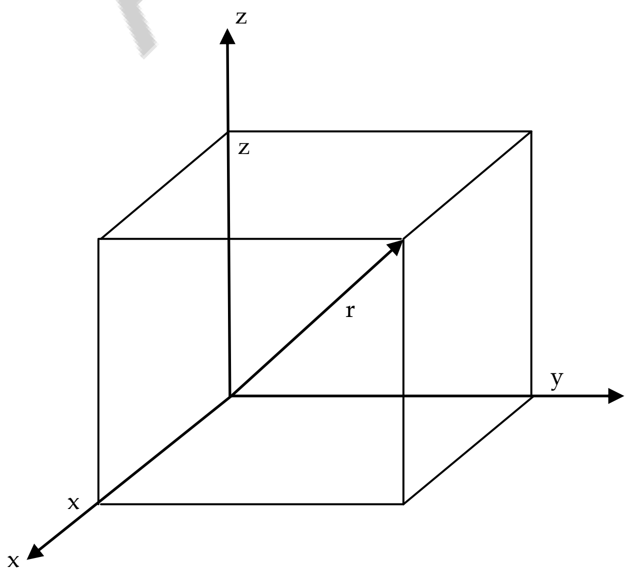
Finally the distance dL from p to p¹ is diagonal of the parallelopiped & has a length of $\sqrt{(dx)^2 + (dy)^2 + (dz)^2}$

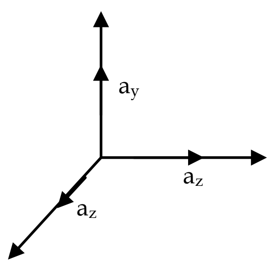
The volume element is given in fig (1.1c)



Vector components and unit vector:-

Let us first consider a vector ‘r’ extending outward from the origin. A local way to identify this vector is by giving the three component vectors, lying along the three co – ordinates axis, whose vector sum must be the given vector. If the component of the vector are x, y and z, then r = x+ y+ z.





Unit vector are those which have unit magnitude and directed along the co –ordinates axis in the direction of increasing co –ordinates values. Any vector 'B' may be described by

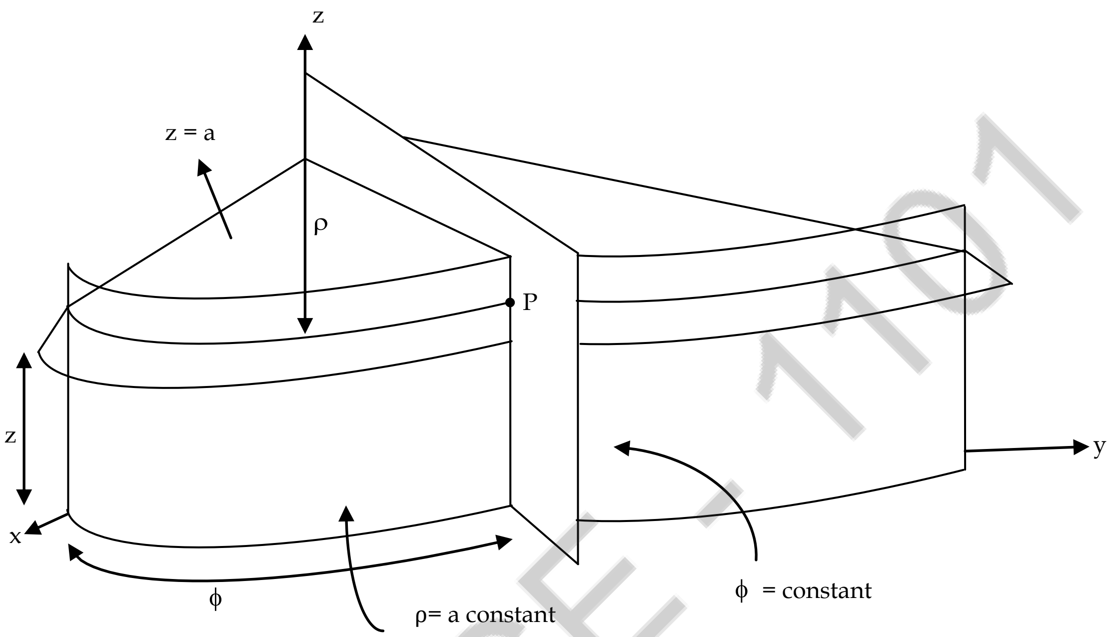
$$\bar{B} = B_x \bar{a}_x + B_y \bar{a}_y + B_z \bar{a}_z$$

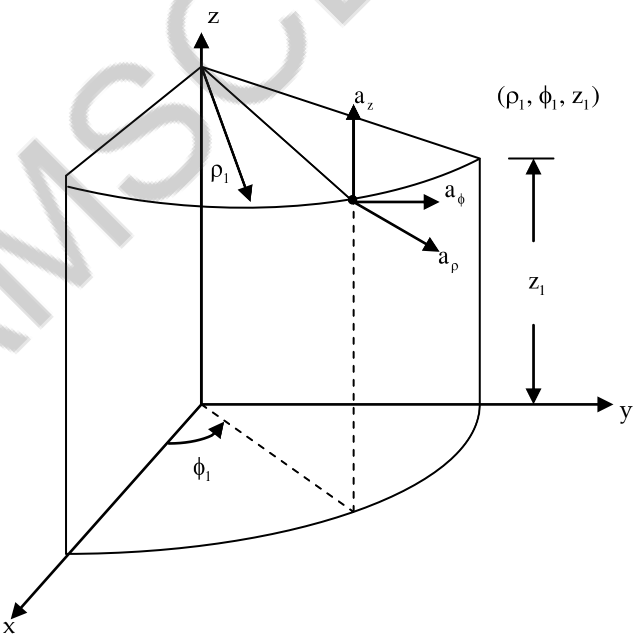
$$|B| = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

$$a_B = \frac{\bar{B}}{|B|}$$

2) Explain Circular cylindrical co –ordinates:-

A point was located in a plane by giving its distance ρ from the origin & the angle ϕ between the line from the point to the origin & an arbitrary radial line taken as $\phi = 0$, a distance Z of the point

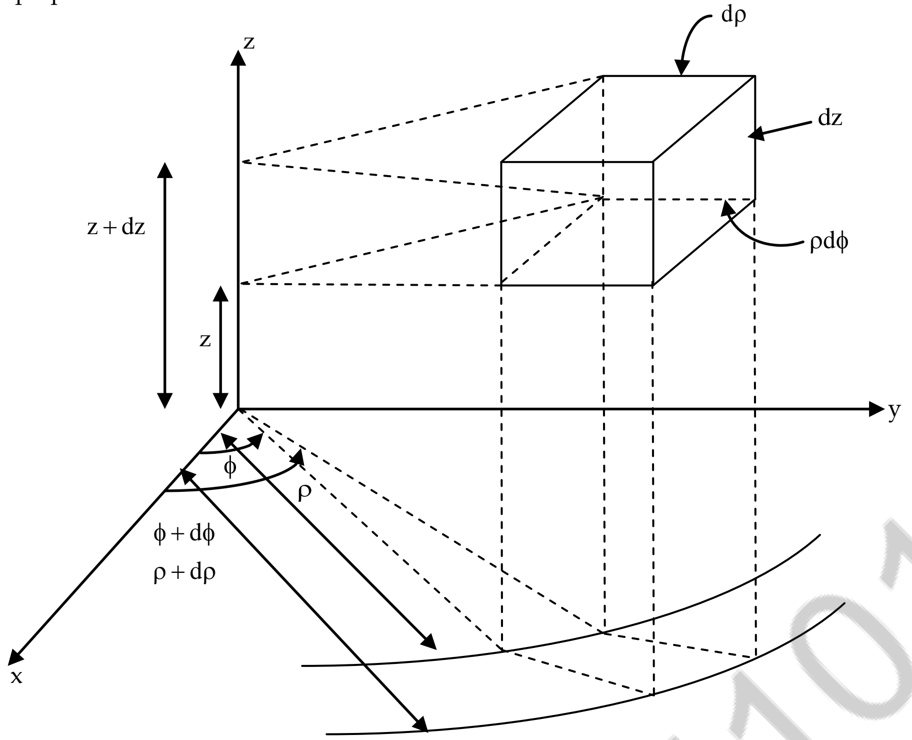




There are three unit vectors designed as follows.

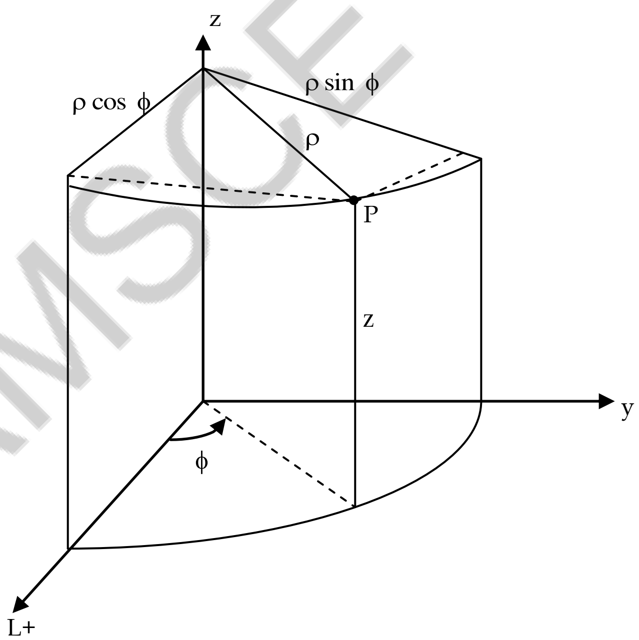
- a_ρ at a point P (ρ, ϕ, z) is directed radially outward, normal to the cylindrical surface $\rho = \rho_1$. It lies in the planes $\phi = \phi_1$ & $z = z_1$
- $a\phi$ is normal to the plane $\phi = \phi_1$, points in the direction of increasing ϕ, lies in the plane $Z = Z_1$, is tangent to the cylindrical surface $\rho = \rho_1$
- The unit vector a_z is same as unit vector a_z of the rectangular system.

- The unit vectors are again mutually perpendicular for each is normal to one of the three mutual perpendicular surfaces.



$$a_\rho \times a_\phi = a_z$$
$$a_\phi \times a_z = a_\rho$$
$$a_z \times a_\rho = a_\phi$$

The surfaces have areas of $\rho d\rho d\phi$, $d\rho dz$, & $\rho d\phi dz$ the volume becomes $\rho d\rho d\phi$



The above figure can give the rectangular of Cartesian & rectangular co –ordinates

$$x = \rho\cos\phi \Rightarrow x^2 = \rho^2\cos^2\phi \ \rightarrow (1)$$

$$y = \rho\sin\phi \Rightarrow y^2 = \rho^2\sin^2\phi \ \rightarrow (2)$$

$z = z$

Equ (1) + (2) $\Rightarrow x^2 + y^2 = \rho^2\left[\cos^2\phi + \sin^2\phi\right]$

$$\rho = \sqrt{x^2 + y^2}$$

$$\cos\phi = x/\rho,\ \sin\phi = y/\rho\ \ \frac{\sin\phi}{\cos\phi} = y/x$$

$$\phi = \tan^{-1} y/x$$

Conversion of Cartesian to cylindrical :-

A vector function is Cartesian co–ordinates is given as follows.

$$A = A_x a_x + A_y a_y + A_z a_z \rightarrow (1)$$

& we need a vector in cylindrical co – ordinates

$$A = A_\rho a_\rho + A_\phi a_\rho + A_z a_z \rightarrow (2)$$

To find Aρ:

$$A_\rho = A.a_\rho$$
$$A_\rho = (A_x a_x + A_y a_y + A_z a_z).a_\rho$$

$$A_\rho = A_x a_x.a_\rho + A_y a_y.\ a_\rho \rightarrow (3)$$

To find $A\phi$:

$$A_\phi = (A_x a_x + A_y a_y + A_z a_z) a_\phi$$
$$A_\phi = A_x a_x.\ a_\phi + A_y a_y.a_\phi$$
$$A_z = A_z$$

$$a_z.a_\rho = a_z.a_\phi = 0$$

	a_ρ	a_ϕ	a_z
a_x	$\cos\phi$	$-\sin\phi$	0
a_y	$-\sin\phi$	$\cos\phi$	0
a_z	0	0	1

Sub in (3) & (4) we get

$$A_\rho = A_x \cos\phi + A_y \sin\phi$$
$$A_\phi = -A_x \sin\phi + A_y \cos\phi$$
$$A = (A_x \cos\phi + A_y \sin\phi)\bar{a}_\rho + (-A_x \sin\phi + A_y \cos\phi) a_\phi + A_z a_z$$

Cylindrical to Cartesian:-

$$A = A_\rho a_\rho + A_\phi a_\phi + A_z a_z$$
$$A_x = A.\ a_x$$
$$= (A_\rho a_\rho + A_\phi a_\phi + A_z a_z).\ a_x$$
$$A_x = A_\rho \cos\phi - A\phi\ \sin\phi$$
$$A_y = A.\ a_y$$
$$= (A_\rho a_\rho\ a_y + A_\phi a_\phi.a_y)$$
$$A_y = A_\rho \sin\phi - A\phi\ \cos\phi$$
$$A = (A_\rho \cos\phi - A\phi\ \sin\phi)\ a_x + (A_\rho \sin\phi - A\phi\ \cos\phi) a_y + A_z a_z$$

3) Explain about spherical co – ordinates system:

Let us draw a spherical co ordinates system on three rectangular axis as shown in fig (a)

We first define the distance from the origin to any point as r. The surface r = constant is a sphere.

The second co – ordinates is angle Q between the z – axis & the line drawn from the origin to the point in question. The surface Q = constant is a core & the two surface, core &sphere are everywhere perpendicular along their intersection which is a circle of radius r sin θ.

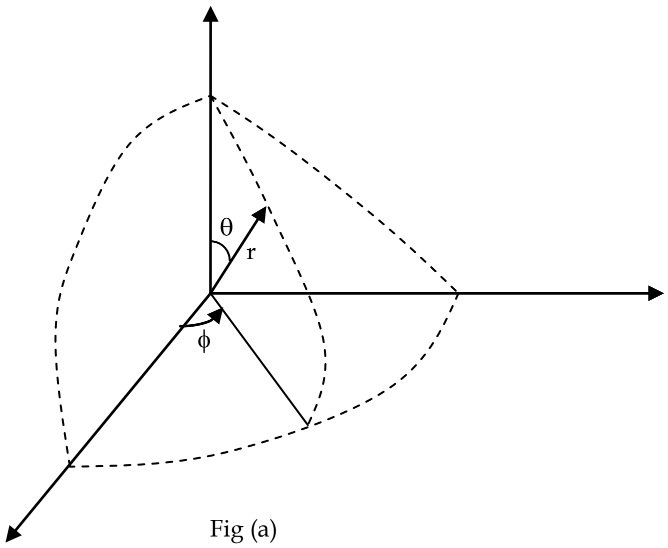


Fig (a)

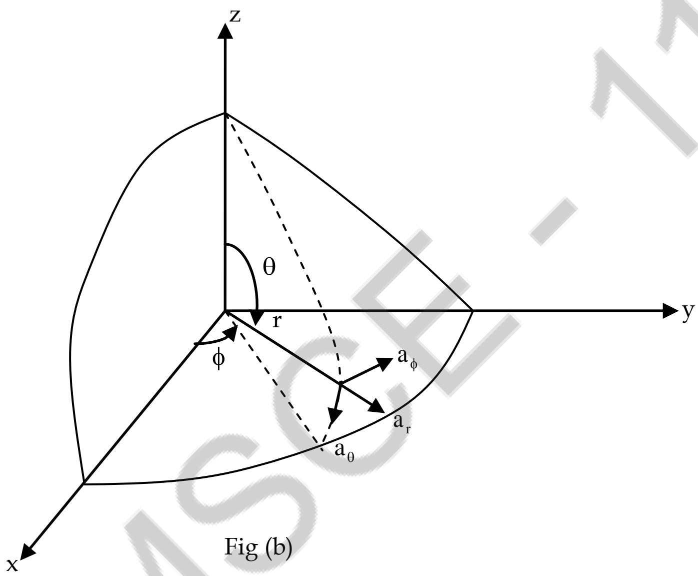


Fig (b)

The third co – ordinates ϕ is also an angle and is exactly the same as the angle ϕ of the cylindrical co – ordinates. It is angle between the x – axis & the propagation in the z = 0 plane of the line drawn from the origin to the point.

Three unit vectors may again be defined at any point. Each unit vector is perpendicular to one of the three mutually perpendicular surfaces & oriented in that direction in which the co – ordinate increases.

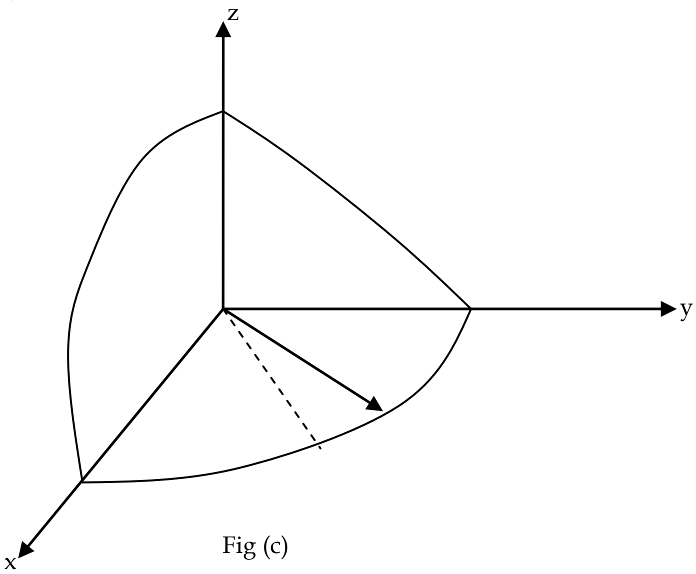


Fig (c)

The unit vector a_r is directed radially outward, normal to the sphere r = constant & lies in the cone θ = constant & the plane ϕ = constant.

The unit vector a_θ is normal to the conical surfaces, lies in the plane & is tangent to the sphere.

$$a_r \times a_Q = a_\phi$$

The surfaces have areas of rd rdQ, $r\sin Q\, dr d\phi$ & $r^2 \sin\theta d\theta\, d\phi$. the volume is $r^2 \sin\theta dr d\theta\, d\phi$

$$x = r\sin\theta\cos\phi \Rightarrow x^2 = r^2\sin^2\theta\cos^2\phi$$
$$y = r\sin\theta\sin\phi \Rightarrow y^2 = r^2\sin^2\theta\sin^2\phi$$
$$z = r\cos\theta \quad \Rightarrow z^2 = r^2\cos^2\theta$$
$$r = \sqrt{x^2 + y^2 + +z^2}$$
$$\theta = \cos^{-1}\frac{z}{\sqrt{x^2+y^2+z^2}}$$
$$\phi = \tan^1\left(y/x\right)$$

	a_r	a_θ	a_ϕ
a_x	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
a_y	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
a_z	$\cos\theta$	$-\sin\theta$	0

Transformation of vector in Cartesian to Spherical:

Let us consider a vector in Cartesian co – ordinate

$$A = A_x a_x + A_y a_y + A_z a_z$$

& to find the components of spherical vector

$$\begin{aligned} A_r &= A.a_r \\ &= (A_x a_x + A_y a_y + A_z a_z).a_r \\ &= A_x a_x a_r + A_y a_y a_r + A_z a_z.a_r \\ &= A_x \sin\theta\cos\phi + A_y \sin\theta\sin\phi + A_z \cos \\ A_\theta &= A.a_\theta \\ &= (A_x a_x + A_y a_y + A_z a_z).a_\theta \\ &= A_x \sin\theta\cos\phi + A_y \cos\theta\sin\phi + A_z \sin\theta \\ A_\phi &= A.a_\phi \\ &= A_x \sin\phi + A_y \cos\phi \end{aligned}$$

$$A = (A_x \sin\phi\cos\phi + A_y \sin\theta\sin\phi + A_z\cos\theta)a_r + (A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta)a_\theta + (-A_x\sin\phi + A_y\cos\phi)A_\phi$$

The line, surface & volume integral let us consider a charge Q whose density along line, surface, & volume is given by ρ_L, ρ_S, ρ_V

Line integral $Q = \int_L \rho_L\, dl \;\; \rho_L = \frac{\Delta Q}{\Delta L}$ as $\Delta L \to 0$

Surface integral $Q = \int_S \rho_S\, ds \;\; \rho_s = \frac{\Delta Q}{\Delta S}$ as $\Delta S \to 0$

Volume integral $Q = \int_V \rho_V\, dv \;\; \rho_v = \frac{\Delta Q}{\Delta V}$ as $\Delta V \to 0$

4) State and Explain Divergence theorem.

Divergence:

Definition: The divergence of the vector $\vec{F}$ at any point is defined as the limit of its surface integral per unit volume as per the volume enclosed by the surface around the point shrinks to zero.

$$\vec{F} = \lim_{\Delta v \to 0} \frac{\int \vec{F}.\, n\ ds}{2}$$

Proof:

Consider an elemental volume $\Delta v = \Delta x\ \Delta y\ \Delta z$ of a parallopiped.

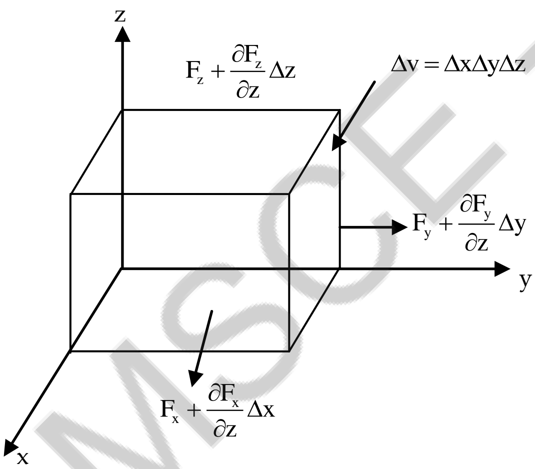
Let F is a vector field. The flux of any vector F through a surface is given by the surface integral of the vector over that surface.

The flux passing out of the volume is taken as positive and that passing inward as negative. Let Fx, y, Fz be the components of F along the co –ordinates axis, so that

$F = F_x + F_y + F_z$.

Consider the volume element , the flux of the vector F in y direction into hand face is and $F_y(\Delta x\ \Delta z)$ and out of right hand face it is $\left(F_y + \frac{\partial F_y}{\partial y}\Delta y\right)\Delta x\Delta z$. Therefore, the net increase of flux along the positive Y direction is

flux along Y axis $= \left(F_y + \frac{\partial F_y}{\partial y}\Delta y\right)\Delta x\Delta z - F_y\ \Delta x\Delta z$



$$= F_y \cancel{\Delta x} \Delta z + \frac{\partial F_y}{\partial y}\Delta x\Delta z\Delta y - F_y\Delta x\Delta z$$

$$= \frac{\partial F_y}{\partial y}\Delta x\Delta z\Delta y$$

Similarly in x direction,

Flux along x axis $= \left(F_x + \frac{\partial F_x}{\partial x}\Delta x\right)\Delta y\Delta z - F_x\ \Delta y\Delta z$

$$= F_x \cancel{\Delta y} \Delta z + \frac{\partial F_x}{\partial x}\Delta x\Delta y\Delta z - F_x \cancel{\Delta y} \Delta z$$

$$= \frac{\partial F_x}{\partial x}\Delta x\Delta y\Delta z$$

Now, along the z – direction, the net influx of the vector is given by.

Flux along z axis $= \left(F_z + \frac{\partial F_z}{\partial z}\Delta z\right)\Delta x\Delta y - F_x\ \Delta x\Delta y$

$$= F_z \cancel{\Delta z}\,\Delta y + \frac{\partial F_z}{\partial z}\Delta x\Delta y\Delta z - F_x \cancel{\Delta x}\,\Delta y$$

$$= \frac{\partial F_z}{\partial z}\Delta x\Delta y\Delta z$$

Now, total increase is given by,

Total flux

$$= \frac{\partial F_x}{\partial x}\Delta x\Delta y\Delta z + = \frac{\partial F_y}{\partial y}\Delta x\Delta y\Delta z + = \frac{\partial F_z}{\partial z}\Delta x\Delta y\Delta z$$

$$\oiint_s \vec{F}.\,n\,ds = \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}\right).\Delta x\Delta y\Delta z$$

$$= \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}\right)\Delta v$$

$$\frac{\oiint_s \vec{F}.n\,ds}{\Delta v} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

By definition of divergence, we get

$$\because \nabla.\vec{F} = \lim_{\Delta v \to 0}\frac{1}{\Delta v}\left[\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}\right]\Delta v$$

$$\nabla.\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\therefore \oiint_s \frac{\vec{F}.\,n\,ds}{\Delta v} = \nabla.\vec{F}$$

Divergence theorem:

The integral of the divergence of the vector field over a volume V is equal to the surface integral of the normal component of the vector over any surface bounding the volume.

Mathematically for any field vector $\vec{F}$

$$\iiint_v \nabla.\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

Proof:

By definition of divergence,

$$\iiint_v \nabla.\,\vec{F} = \iiint_v \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}\right)dx\,dy\,dz$$

When dv = dx dy dz

Now,

$$\iiint_v \nabla.\,\vec{F}\,dv = \int\left(\frac{\partial}{\partial x}a_x + \frac{\partial}{\partial y}a_y + \frac{\partial}{\partial z}a_z\right)(F_x a_x + F_y a_y + F_z a_z)\,dx\,dy\;\;dz$$

$$= \iiint_v \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}\right)dx\;dy\;dz$$

$$\iiint_v \nabla.\,\vec{F}\,dv = \iiint_v \frac{\partial F_x}{\partial x}dx\;dy\;dz + \iiint_v \frac{\partial F_y}{\partial y}dy\;dx\;dz + \iiint_v \frac{\partial F_z}{\partial z}\;dz\;dx\;dy$$

$$= \iint_s F_x\;dydz + \iint_s F_y\;dxdz + \iint_s F_z\;dxdy$$

$$= \iint F_{Sx}dy\;dz + \iint F_{Sy}dx\;dz + \iint F_{Sz}dx\;dy$$

$$\iiint_v \nabla.\,\vec{F}\,dv = \oiint_s \vec{F}.\,n\,ds$$

Hence proved.

5) Explain Stoke's theorem.

The line integral of the vector around a closed path is equal to the integral of the normal component of its well over any surface bounded by the contour.

$$\oint_c \vec{F}.dl = \oint_s (\nabla \times \vec{F}).\, n c / s$$

Where C is closed contour which bounds the surface S.

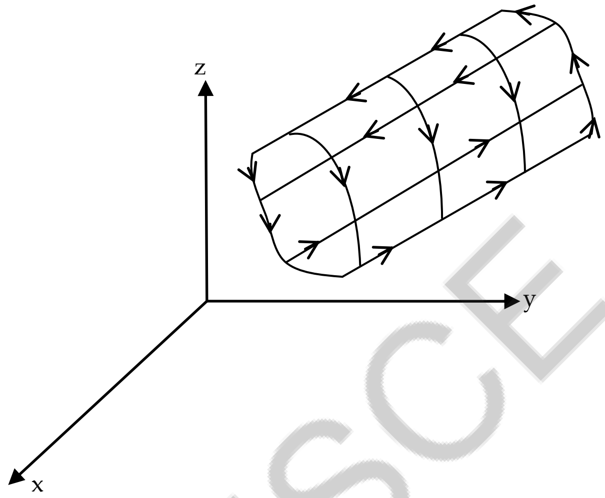
Proof:

Consider the arbitrary surface S as shown below.

If F is the field vector, then by definition of curl, the line integral $\oint F.\,dL$, divided by the surface area gives the curl of F normal to the surface at the point around which the surface shrinks to zero.

Thus $\lim\limits_{s\to 0} \dfrac{\oint F.\,dl}{S} = \text{curl nF}$

Where curl nF is the component of curl of F normal to the surface S.



Now divide up the area S into a large number of still smaller elements 1, 2 etc. Each area element can be represented by a vector directed outwardly normal to the surface.

For each such element, find the line integral in the positive direction, normally anti–clockwise which corresponds to the positive direction of the surface elements.

If all the elements 1, 2 etc are summed up, the contributions of the common boundary of any two adjacent elements neutralize each other, as they are oppositely directed along the common boundary

$$\oint_c F.dl = \int \vec{F}\, dl_1 + \int \vec{F}\, dl_2 + \int \vec{F}\, dl_3$$

Applying the definition of curl F, we have

$$\oint_c F\, .dl = \text{curl Fds}_1 + \text{curl Fds}_2 + \text{curl ds}_1 + \text{curl Fds}_2 = \iint_s \text{curl}\,F n\; ds.$$

(i.e) curl Fds_1 + curl Fds_2 +..... denotes the summation of normal components of curl F over the whole surface S.

Therefore,

$$\oint_c F.dl = \iint_s \text{curl F.n ds}$$

Hence stoke's theorem is proved.

Curl:

The curl of a vector at any point is defined as the limit of the ratio of the integral of its cross product with the outward drawn normal over a closed surface, to the volume enclosed by the surface as the volume shrinks to zero.

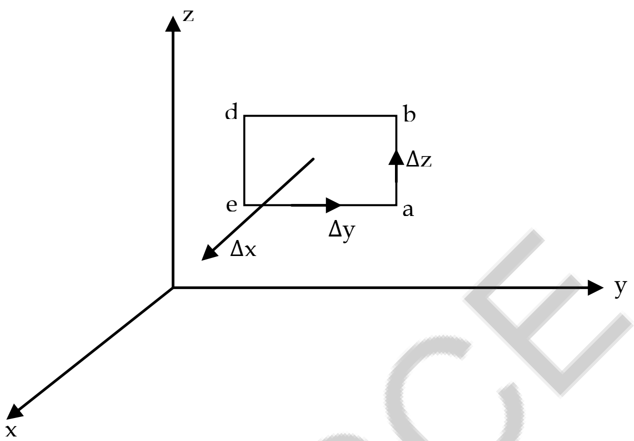
$$|\text{curl}\,F| = \lim_{v\to 0}\frac{1}{v}\oint_s n \times F\, ds$$

The component of curl of vector in the direction of the unit vector n is the ratio the line integral of the vector around a closed contour, to the enclosed area bounded by the contour, as the enclosed area diminishes to zero.

$$n\ \text{curl}F = \lim_{a\to 0}\frac{1}{s}\oint_c f.\,dl$$

Referring to fig, consider an elemental plane surface of area $\Delta Y\ \Delta Z$ bounded by a contour C marked as able.

$$|\text{curl } B|\ x = \text{curl of B in the x – direction}$$



$$\lim_{\substack{\Delta y\to 0\\ \Delta z\to 0}} \frac{\oint_e B\,.dl}{\Delta y\ \Delta z}$$

Where B is a vector, choosing the value of B at centre, we have the closed integral of B around the abde,

$$= \oint B\,.dl$$

Integral along $e_a = \left(By - \frac{\partial By}{\partial z}\frac{\Delta z}{2}\right)\Delta y$

Integral along $ab = \left(Bz - \frac{\partial Bz}{\partial y}\frac{\Delta y}{2}\right)\Delta z$

Integral along $bd = -\left(By + \frac{\partial By}{\partial z}\frac{\Delta z}{2}\right)\Delta y$

Integral along $de = -\left(Bz - \frac{\partial Bz}{\partial y}\frac{\Delta y}{2}\right)\Delta z$

Adding

$$\oint_L B\,.dl = By\ \Delta y - \frac{\partial By}{\partial z}\frac{\Delta y\Delta z}{2} + Bz\ \Delta z + \frac{\partial Bz}{\partial y}\frac{\Delta y\Delta z}{2}$$

$$\text{curl}_x\ B = \left(\frac{\partial Bz}{\partial y} - \frac{\partial By}{\partial z}\right) = \mu_0 J_x$$

Simarlly

$$\text{curl}_y\ B = \left(\frac{\partial Bx}{\partial z} - \frac{\partial Bz}{\partial x}\right) = \mu_0 J_y$$

$$\text{curl}_y\ B = \left(\frac{\partial By}{\partial x} - \frac{\partial Bx}{\partial y}\right) = \mu_0 J_z$$

$$\text{curl } B = |\text{curl}_x\ B| a_x + |\text{curl}_y\ B| a_y + |\text{curl}_z\ B| a_z$$

$$= \mu_0 (J_x\ a_x + J_y\ a_y + J_z\ a_z)$$

$$\text{curl }\ B = \left(\frac{\partial Bz}{\partial y} - \frac{\partial By}{\partial z}\right)\overline{a_x} + \left(\frac{\partial Bx}{\partial z} - \frac{\partial Bz}{\partial x}\right)\overline{a_y} + \left(\frac{\partial By}{\partial x} - \frac{\partial Bx}{\partial y}\right)\overline{a_z}$$

$$\text{curl } B = \begin{vmatrix} a_x & a_y & alz \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ B_x & B_y & B_z \end{vmatrix}$$

The main difference of divergence & curl is that in the former the parameter along x varies only with x but in curl the parameter in x varies along y & z.

6) Define coulomb's law. Derive coulomb's law in vector form

Coulomb's law:-

The law states that "two charges Q_1 and Q_2 separated by a distance 'r' in free space or vacuum, the force of attraction or repulsion is directly proportional to the product of the magnitude of charges and is inversely proportion to the square of the distance between them

$$F \propto \frac{Q_1 Q_2}{r^2}$$

$$F = \frac{KQ_1 Q_2}{r^2}$$

Where, k = proportionality constant $= \frac{1}{4\pi\varepsilon}$

Where

ε = permittivity of the medium

$\varepsilon = \varepsilon_0 \varepsilon_r$

ε_0 = absolute permittivity $= = \frac{1}{36\pi \times 10^9} = 8.854 \times 10^{-12}$

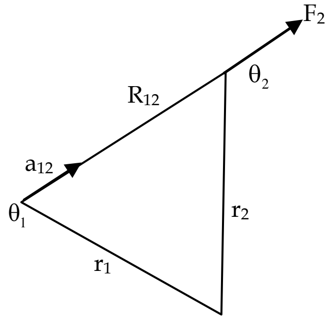
ε_r = Relative permittivity of the medium which is 1 for free space / air.

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon r^2}$$

$$= \frac{Q_1 Q_2}{4\pi\varepsilon_0 \varepsilon_r r^2}$$

$$F = 9 \times 10^9 \frac{Q_1 Q_2}{r^2}\ \text{ newton or }\ C^2 / m^2$$

Coulomb's law in vector form:-



In vector form, we need an additional fact that the force all along the line joining origin the two charges & is repulsive if they are of opposite sign.

Let r_1 locate Q_1 and r_2 locate Q_2. Then the vector $R_{12}=r_2 - r_1$ represent the direct line segment from Q_1 to Q_2.

Let F_2 be the force on Q_2 by Q_1. Note Q_1 and Q_2 have the same sign.

$$\overline{F_2} = \frac{Q_1 Q_2}{4\pi\varepsilon R_{12}^2}\overrightarrow{a_{12}}$$

Let F_2 be the force on Q_2 by Q_1 where $\bar{a}_{12} =$ a unit vector in the direction of the R_{12}.

$$\bar{a}_{12} = \frac{\overline{R_{12}}}{|R_{12}|} = \frac{\overline{R_{12}}}{R_{12}} = \frac{r_2 - r_1}{|r_2 - r_1|}$$

Let Q_1 be located at $x_1\bar{a}_x + y_1\bar{a}_y + z_1\bar{a}_z$ and Q_2 be located at $x_2\bar{a}_x + y_2\bar{a}_y + z_2\bar{a}_z$

Then $r_2 - r_1 = (x_2 - x_1)\bar{a}_x + (y_2 - y_1)\bar{a}_y + (z_2 - z_1)\bar{a}_z$

$$R_{12} = |r_2 - r_1| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Therefore,

$$F_2 = \frac{Q_1 Q_2}{4\pi\varepsilon\left[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\right]} \times \frac{\bar{r}_2 - \bar{r}_1}{|r_2 - r_1|}$$

$$F_2 = \frac{Q_1 Q_2\left[(x_2 - x_1)\bar{a}_x + (y_2 - y_1)\bar{a}_y + (z_2 - z_1)\bar{a}_z\right]}{4\pi\varepsilon\left[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\right]^{3/2}}$$

7) Explain electric field intensity?

Electric field intensity:-

Consider one charge in fixed position say Q1 and none a second change slowly around. The change Qt will experience a force exerted by Q1.

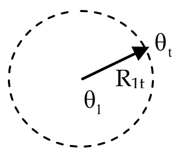
As per coulomb's law, force on Qt by Q1 is given by

$$F_t = \frac{Q_1 Q_t}{4\pi\varepsilon_0 R_{1t}^2}\overline{a_{1t}}$$

The force exerted on the test charge Qt is defined as the electric field intensity

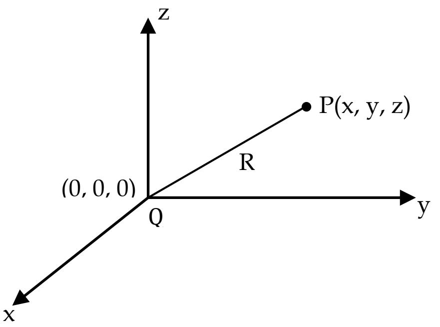
$$E = \frac{F_t}{Q_t} = \frac{Q_1}{4\pi\varepsilon_0 R_{1t}^2}\overline{a_{1t}}$$

Generally, $E = \dfrac{Q}{4\pi\varepsilon_0 R^2}\overline{a_R}$



Case (1):- charge at origin

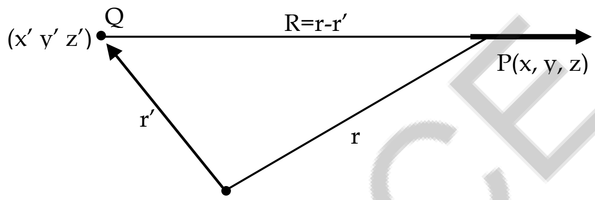
Let us consider a charge at the origin and to find electric field intensity at point whose co – ordinates are (x, y, z)



x

$$R = (x\bar{a}_x + y\bar{a}_y + z\bar{a}_z) - (0\bar{a}_x + 0\bar{a}_y + 0\bar{a}_z)$$
$$\bar{R} = x\bar{a}_x + y\bar{a}_y + z\bar{a}_z = \sqrt{x^2 + y^2 + z^2}$$
$$a_R = \frac{\bar{R}}{|R|} = \frac{x\bar{a}_x + y\bar{a}_y + z\bar{a}_z}{\sqrt{x^2 + y^2 + z^2}}$$
$$E = \frac{Q}{4\pi\varepsilon_0 R^2}\bar{a}_R$$
$$E = \frac{Q\left(x\bar{a}_x + y\bar{a}_y + z\bar{a}_z\right)}{4\pi\varepsilon_0\left(x^2 + y^2 + z^2\right)^{3/2}}$$

Case (ii):- Charge not at origin



If we consider that the charges are not at origin but located at r1 and fixed electric field intensity a P at distance r from the origin.

$$R = r - r^1$$
$$E = \frac{Q}{4\pi\varepsilon_0 (r - r^1)^2} \times \frac{\overline{r - r^1}}{|r - r^1|}$$
$$E = \frac{Q\overline{r - r^1}}{4\pi\varepsilon_0 |r - r^1|^3}$$
$$E = \frac{Q\overline{r - r^1}}{4\pi\varepsilon_0 |r - r^1|^{3/2}}$$
$$r = x\bar{a}_x + y\bar{a}_y + z\bar{a}_z$$
$$r^1 = x^1\bar{a}_x + y^1\bar{a}_y + z^1\bar{a}_z$$
$$r - r^1 = \sqrt{(x - x^1)^2 + (y - y^1)^2 + (z - z^1)^2}$$
$$E = \frac{Q\left[(x - x^1)a_x + (y - y^1)a_y + (z - z^1)a_z\right]}{4\pi\varepsilon_0\left[(x - x^1)^2 + (y - y^1)^2 + (z - z^1)^2\right]^{3/2}}$$

8) Discuss electric field intensity due to continues charger electric field intensity due to continuous charges:-

If charge Q is uniformly distributed throughout a volume V, the charge density ρ is given by

$$\rho = \frac{Q}{V}$$

To define the value of the charge density at point P, let us consider the charge ΔQ in a volume ΔV. The ratio of infinitesimal charge ΔQ clinded by volume ΔV as ΔV = 0

$$\rho_V = \lim_{\Delta V \to 0} \frac{\Delta Q}{\Delta V}$$

Similarly we can write for ρ_S and ρ_ℓ

$$\rho_\ell = \lim_{\Delta \ell \to 0} \frac{\Delta Q}{\Delta \ell}; \rho_S = \lim_{\Delta s \to 0} \frac{\Delta Q}{\Delta S}$$

The charge element dQ and total charge Q due to these charge distribution are,

$$dQ = \rho_\ell \; dl \rightarrow Q = \int_L \rho_\ell \; dl$$
$$dQ = \rho_s \; ds \rightarrow Q = \int_S \rho_s \; ds$$
$$dQ = \rho_v \; dv \rightarrow Q = \int_V \rho_v \; dv$$

$$E = \frac{Q}{4\pi\varepsilon_0 R^2} \overline{a_R}$$

$$E = \int \frac{\rho_L \; dl}{4\pi\varepsilon_0 R^2} \overline{a_R} \quad \text{(line charge)}$$

Here, $$E = \int \frac{\rho_S \; ds}{4\pi\varepsilon_0 R^2} \overline{a_R} \quad \text{(surface charge)}$$

$$E = \int \frac{\rho_V \; dv}{4\pi\varepsilon_0 R^2} \overline{a_R} \quad \text{(volume charge)}$$

9) Explain Electric Potential difference and Potential

<u>**Electric Potential difference and Potential**</u>

$$F = q\,E$$

Since there is a moment of charge in electric field from one point r1 to another point r2, there will be work done against force. $W = -\int_{r_1}^{r_2} qE.\,dr \Rightarrow -\,q\int_{r_1}^{r_2} E.\,dr$

Potential difference is defined as the work done in moving a unit positive charge from one point to another in an electric field.

Work done on unit positive charge per charge is

$$V = \frac{W}{q}$$
$$V = -\int_{r_1}^{r_2} E.\,dr \quad J/C$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$V = -\frac{Q}{4\pi\varepsilon_0} \int_{r_1}^{r_2} \frac{1}{r^2}\,dr$$

But $$= -\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{r_1}^{r_2}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$V = \frac{Q}{4\pi\varepsilon_0 r_2} - \frac{Q}{4\pi\varepsilon_0 r_1}$$

$$V_{AB} = V_B - V_A \text{ or } V_{12} = V_2 - V_1$$

E = Negative of potential gradient at that point.

10) Determine electric filed intensity due to infinite line charge

I) Using cylindrical co – ordinates?

An infinite line of charge having the charge density ρ_L c/m is considered along the Z – axis. i.e, the line extends from - ∞ to +∞ along the z – axis. A point P is considered along the axis perpendicular to z, where 'E' is to be measured the point 'P' located at a distance ρ from the origin.

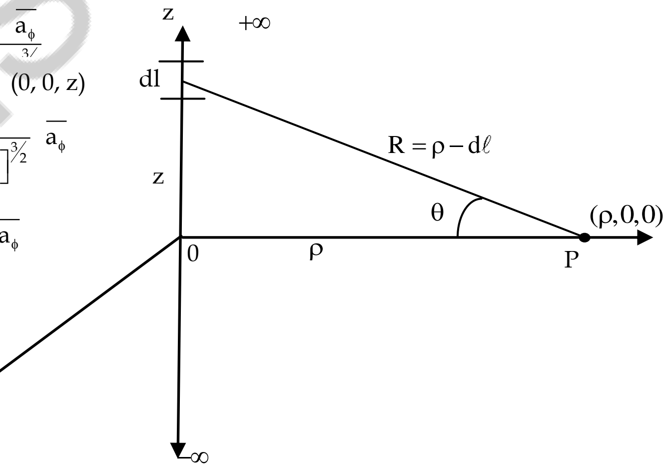
Consider a small differential length dL carrying char dQ along the z – axis.

$$dQ = \rho_L\ dl$$
$$= \rho_L\ dz$$
$$E = \frac{\int \rho_L\ dl}{4\pi\varepsilon_0 R^2}\overline{a_R}$$

$$E = \int_{-\infty}^{+\infty} \frac{\rho_L\ dz\ \ \overline{a_Z}}{4\pi\varepsilon_0(\rho^2+z^2)} \cdot \frac{\rho\overline{a_e} - z\overline{a_Z}}{\sqrt{\rho^2+z^2}}$$
$$= \frac{\rho_L}{4\pi\varepsilon_0(\rho^2+z^2)^{3/2}} \int_{-\infty}^{+\infty} (\rho a_e - z a_z)\ dz\ x\overline{a_z}$$
$$= \frac{\rho_L}{4\pi\varepsilon_0(\rho^2+z^2)^{3/2}} \int_{-\infty}^{+\infty} \rho a_\phi\, dz\ [\overline{a_z}]$$
$$E = \frac{\rho_L}{4\pi\varepsilon_0} \int_{-\infty}^{+\infty} \frac{\rho dz}{(\rho^2+z^2)^{3/2}}\ \overline{a_\phi}$$

$$\tan\theta = z/\rho \Rightarrow z = \rho\tan\theta$$
$$dz = \rho\sec^2\theta$$
$$z = -\infty,\ \theta = \frac{-\pi}{2}$$
$$z \neq +\infty,\ \theta = \frac{\pi}{2}$$

$$\overline{E} = \frac{\rho_L}{4\pi\varepsilon_0} \int_{-\pi/2}^{+\pi/2} \frac{\rho(\rho\sec^2\theta)d\theta\ \ \overline{a_\phi}}{(\rho^2+\rho^2\tan^2{}^{3/}}$$

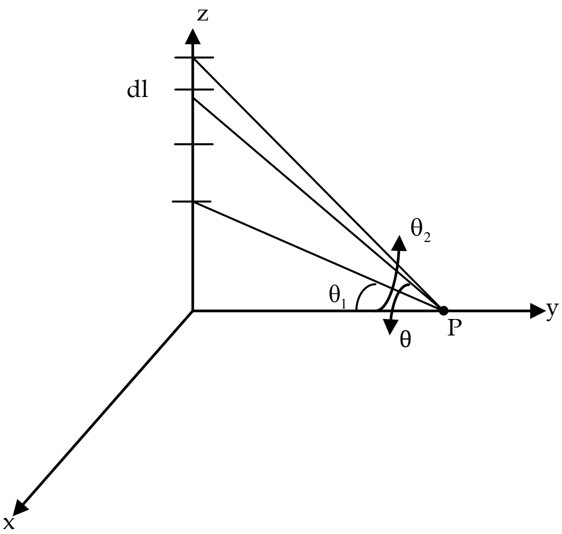


$$= \frac{\rho_L}{4\pi\varepsilon_0} \int_{-\pi/2}^{+\pi/2} \frac{\rho^2\sec^2\theta\ d\theta}{\left[\rho^2(1+\tan^2\theta)\right]^{3/2}}\ \overline{a_\phi}$$
$$= \frac{\rho_L}{4\pi\varepsilon_0} \int_{-\pi/2}^{+\pi/2} \frac{\rho^2\sec^2\theta\ d\theta}{\left[\rho^3\sec^3\theta\right]}\ \overline{a_\phi}$$
$$= \frac{\rho_L}{4\pi\varepsilon_0\rho} \int_{-\pi/2}^{+\pi/2} \cos\theta\ d\theta\ \ \overline{a_\phi}$$
$$= \frac{\rho_L}{4\pi\varepsilon_0\rho} \left[\sin\theta\right]_{-\pi/2}^{+\pi/2} \overline{a_\phi}$$
$$= \frac{\rho_L}{4\pi\varepsilon_0\rho}[2]\ \overline{a_\phi}$$
$$\overline{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\ \overline{a_\phi}$$

11) Derive the electric field intensity due to finite line charge using cylindrical co – ordinates?

Due to finite line charge:-

Let us consider the differential element dl along z – axis let the minimum point be z1 and maximum point z2



$$E = \frac{Q}{4\pi\varepsilon_0 R^2}\,\overline{a_\phi}$$

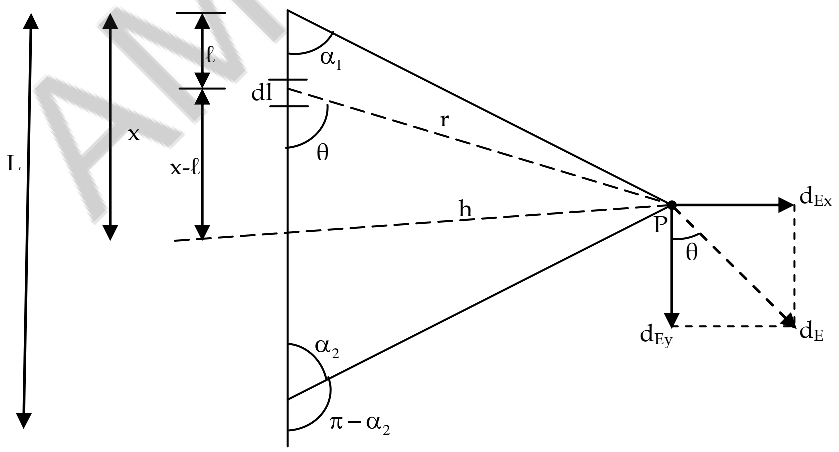
$$= \frac{\int_{z_1}^{z_2} \rho_L\,dz}{4\pi\varepsilon_0(\rho^2+z^2)}\,\frac{(\rho a_e - z a_z)}{\sqrt{\rho^2+z^2}}$$

$$= \frac{\rho_L}{4\pi\varepsilon_0}\,\frac{\int_{z_1}^{z_2} \rho_L\,dz\,a\phi}{(\rho^2+z^2)^{3/2}}$$

$$E = \frac{\rho_L}{4\pi\varepsilon_0}\int_{\theta_1}^{\theta_2}\frac{\rho^2\sec^2\theta\,d\theta}{\rho^3\sec^3\theta}\,\overline{a_\phi}$$

12. Derive the expression for electric field intensity due to finite and infinite line charge using Cartesian co – ordinates

Consider a uniformly charged line of length 'L' whose charge density ρL c/m. Consider a small element dl at a distance 'l' from one end of the charge line. Let be any point a distance 'r' from the element ρ dl.



The electric field at a point P due to the charge element ρ dl is

$$dE = \frac{\rho_e\,dl}{4\pi\varepsilon_0 r^2} \rightarrow (1)$$

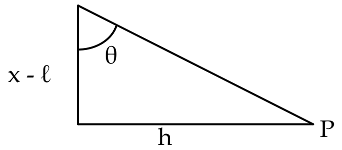
The x and y component of electric field dE is given by,

$$dE_x = dE \sin\theta \rightarrow (2)$$

$$dE_y = dE \cos\theta \rightarrow (3)$$

$$dE_x = \frac{\rho_e \ dl \sin\theta}{4\pi\varepsilon r^2} \rightarrow (4)$$

From fig



$$\tan\theta = \frac{h}{x-\ell}$$

$$x-\ell = \frac{h}{\tan\theta} = h\cot\theta \rightarrow (5)$$

$$-dL = -h \operatorname{cosec}^2\theta \, d\theta \rightarrow (6)$$

$$\sin\theta = \frac{h}{r} \Rightarrow r = \frac{h}{\sin\theta}$$

$$r = h \operatorname{cosec}\theta \rightarrow (7)$$

$$dE_x = \frac{\rho_l \, dl \sin\theta}{4\pi\varepsilon r^2}$$
$$= \frac{\rho_l \, h \operatorname{cosec}^2\theta \sin\theta \, d\theta}{4\pi\varepsilon \, h^2 \operatorname{cosec}^2\theta}$$

$$E_x = \int dE_x = \frac{\rho_l}{4\pi\varepsilon h}\int_{\alpha_1}^{\pi-\alpha_2} \sin\theta \, d\theta$$
$$= \frac{\rho_l}{4\pi\varepsilon h}\left[-\cos\theta\right]_{\alpha_1}^{\pi-\alpha_2}$$
$$= \frac{\rho_l}{4\pi\varepsilon h}\left[\cos\alpha_1 + \cos\alpha_2\right]$$

$$dE_y = \frac{\rho_\ell \, dl \cos\theta}{4\pi\varepsilon r^2} = \frac{\rho_\ell \, h \operatorname{cosec}^2\theta \cos\theta \, d\theta}{4\pi\varepsilon h^2 \operatorname{cosec}^2\theta}$$

$$E_y = \int_{\alpha_2}^{\pi-\alpha_1} \frac{\rho_\ell}{4\pi\varepsilon h}\cos\theta \, d\theta = \frac{\rho_L}{4\pi\varepsilon h}\left[\sin\theta\right]_{\alpha_1}^{\pi-\alpha_2}$$

$$E_y = \frac{\rho_L}{4\pi\varepsilon h}\left[\sin\alpha_2 - \sin\alpha_1\right]$$

Case (i):- If the point 'P' is at bisector of a line, then $\alpha_1 = \alpha_2 = \alpha$

$E_y = 0$, $\therefore$ E becomes E_x

$$E = \frac{\rho_L}{2\pi\varepsilon h}\left[\cos\alpha\right]$$

Case (ii):- If the line is infinity long, then $\alpha=0$, $E_y = 0$

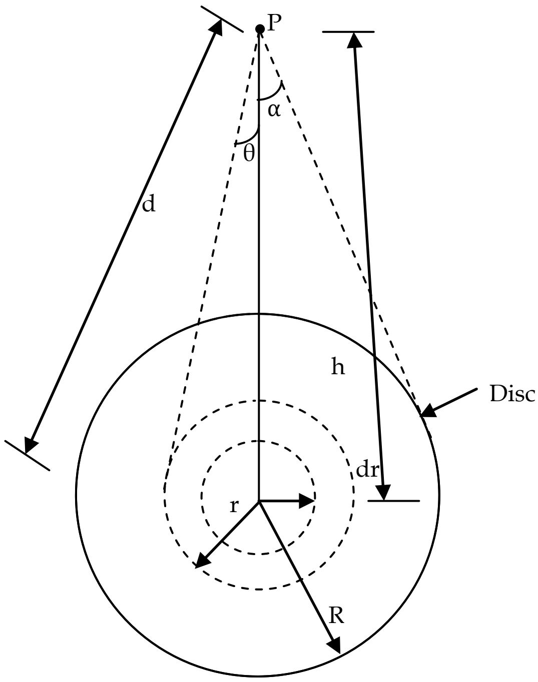
$$E = E_x \frac{\rho_L}{2\pi\varepsilon h}$$

13. Explain the electric field intensity due to a charged circular disc

Consider a circular disc of radius R charged uniformly with charge density of $\rho_s \ C/m^2$. Let be any point on the axis of disc at a distance from the centre.

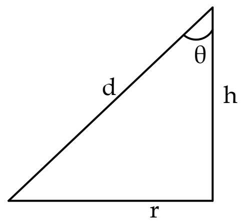
Consider an angular size of radius 'r' & of radial thickness the area of the annular ring ds = $2\pi r$ dr. The field intensity at point p due to the charged annular ring is

$$dE = \frac{\rho_s\ ds}{4\pi\varepsilon d^2}$$



Since the horizontal component of electric field intensity is zero, the vertical component is

$$dE_y = \frac{\rho_s\, ds \cos\theta}{4\pi\varepsilon\ d^2}$$



$$\because dE_y = dE \cos\theta$$
$$\sin\theta = r/d$$
$$\cos\theta = h/d$$
$$\tan\theta = r/h$$

From fig,

$$\tan\theta = r/h \Rightarrow r = h\tan\theta$$
$$r/d \quad dr = h\ \sec^2\ d\theta$$
$$\sin\ \theta = 1 \quad \Rightarrow d = \frac{r}{\sin\theta}$$

$$dE_y = \frac{\rho_s 2\pi r\ dr\ \cos\theta}{4\pi\varepsilon\ d^2}$$
$$= \frac{\rho_s 2\pi r\ h\ \sec^2\ \theta d\theta\cos\theta}{4\pi\varepsilon\ \dfrac{r^2}{\sin^2\theta}}$$

$$dE_y == \frac{\rho_s\ \sec\theta \sin^2\theta\ d\theta}{2\varepsilon\ \tan\theta} = \frac{\rho_s}{2\varepsilon}\sin\theta\ d\theta$$

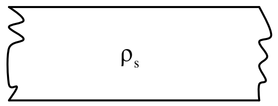
$$E_y = \frac{\rho_s}{2\varepsilon}\int_0^\alpha \sin\theta\ d\theta = \frac{\rho_s}{2\varepsilon}\left[-\cos\theta\right]_0^\alpha$$

$$E = \frac{\rho_s}{2\varepsilon}(1-\cos\alpha)$$

$$E = \frac{\rho_s}{2\varepsilon}\left[1 - \frac{h}{\sqrt{h^2+R^2}}\right]$$

14. Explain the electric field intensity due to infinite plane sheet of charge and two infinitely conducting planes

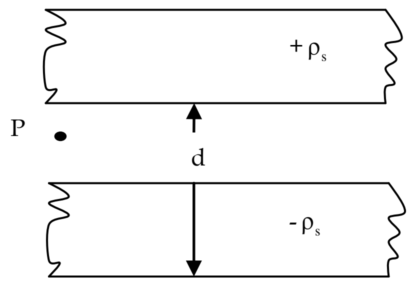
Consider an infinite plane sheet which is uniformly charged with density ρ_s c/m^2 ρ_s.



The field intensity at any point 'p' due infinite plane sheet of charged can be evaluate by applying expression of charged disc.

$$E = \frac{\rho_s}{2\varepsilon}(1 - \cos\alpha)\ \alpha = 90°$$
$$E = \frac{\rho_s}{2\varepsilon}$$

Two infinitely conducting planes



Consider two infinite plane sheet with charge density $+\rho_s$ and $-\rho_s$ c/m^2 separated by distance d.

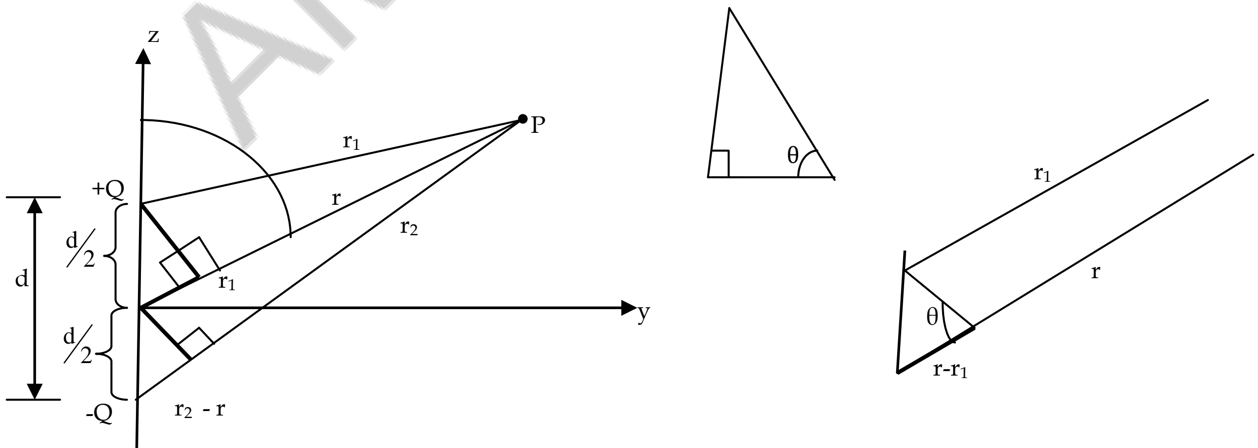
$$E = \frac{\rho_s}{2\varepsilon} + \frac{\rho_s}{2\varepsilon} = \frac{\rho_s}{\varepsilon}$$

15. Derive an expression to determine electric field intensity and at P due to an electric dipole and its torque moment.

Electric dipole:-

An electric dipole or dipole is two equal and opposite charges separated by a very small distance.

The product of charge and spacing is called electric dipole moment.



Electric dipole

Let +Q and –Q be the two charges separated by a small distance 'd'. The product of charge Q and spacing 'd' is called the dipole moment m = Q d

Let P be any point at distance of r_1, r_2 and r from +Q and –Q and midpoint of dipole.

Potential at due to +Q, $V_1 = \frac{Q}{4\pi\varepsilon r_1}$

Potential at due to –Q, $V_2 = \frac{Q}{4\pi\varepsilon r_2}$

Resultant potential at p, $V = V_1 + V_2 = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

If the point 'P' is for away from the dipole the distance r_1 and r_2 are given by

$$r_1 = r - d/2 \cos\theta$$
$$r_2 = r + d/2 \cos\theta$$

Potential at P due to dipole

$$V = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{r - d/2 \cos\theta} - \frac{1}{r + d/2 \cos\theta}\right]$$
$$= \frac{Q}{4\pi\varepsilon}\left[\frac{d\cos\theta}{r^2 - d^2/4 \cos^2\theta}\right]$$
$$V = \frac{Q}{4\pi\varepsilon}\left(\frac{d\cos\theta}{r^2}\right) \qquad \text{since } d/2 << r^2$$
$$V = \frac{Q\,d\cos\theta}{4\pi\varepsilon r^2}$$
$$\therefore m = Qd; \quad V = \frac{m\cos\theta}{4\pi\varepsilon r^2}$$

This shows that the potential is directly proportional to the dipole moment and is inversely proportional to the square of the distance between them.

Electric field produced at P due to the dipole:

As $V = -\int E.\, dr$, this relation can be used for evaluation of field at P due to +Q and –Q separately.

The electric field E has components along radial distance r and angle Q.

$$E = E_r + E_Q$$

$$E = -a_r\frac{\partial v}{\partial r} - a_Q\frac{\partial v}{r\,\partial Q}$$
$$\frac{\partial v}{\partial r} = \frac{-m\cos\theta}{2\pi\varepsilon_0 r^3}; \frac{\partial v}{\partial r} = \frac{-m\sin\theta}{4\pi\varepsilon_0 r^2}$$
$$\therefore E = \overline{a_r}\left(\frac{m\cos\theta}{2\pi\varepsilon_0 r^3}\right) + \overline{a_Q}\frac{-m\sin\theta}{4\pi\varepsilon_0 r^2}$$
$$E_r = \left(\frac{m\cos\theta}{2\pi\varepsilon_0 r^3}\right) \Rightarrow \varepsilon_0 E_r = \frac{m\cos\theta}{2\pi\varepsilon_0 r^3} = D_r$$
$$E_r = \left(\frac{m\sin\theta}{4\pi\varepsilon_0 r^3}\right) \Rightarrow \varepsilon_0 E_Q = \frac{m\sin\theta}{4\pi\varepsilon_0 r^3} = D_Q$$

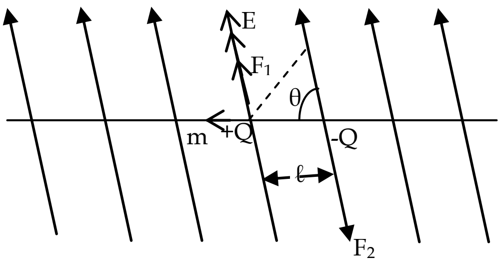
The expression above clearly indicate that the field component (radial as well as angular) vary inversely as the cute of the distance

If Q = 90°, E_r = vanishes, but E_Q persists

If Q = 90°,obviously p is somewhere in alignment with dipole axis.

Torque experimented by a dipole in a uniform:-

Uniform Field E



The dipole moment m = QL is a vector direct from negative to positive charge forming the dipole .

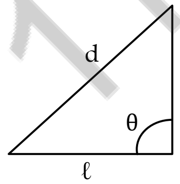
The numerator of the potential is given by m cos θ, which can be written as m . ar

$$V = \frac{m.a_r}{4\pi\varepsilon r^2}$$

There are two charges +Q and –Q placed in a uniform electric field. These charges experience a force QE but opposite in direction

These two forces form a couple whose torque is equal in magnitude to the produced of force and the aim of couple.

$$\begin{aligned} \text{Torque} &= QE\ell \sin\theta \\ &= Q\ell\ E \sin\theta \\ &= mE \sin\theta \\ T &= m \times E \end{aligned}$$



$$\sin\ \theta = d/L$$
$$d = L \sin\theta$$

In conclusion, although a dipole in a uniform field E does not experiences a translation force, it does experiences a torque.

16) State & Prove Gauss law?

Gauss law:-

The electric flux passing through any closed surface is equal to the total charge enclosed by the surface.

$$\Psi = Q_{encl}$$

Proof:

Consider a small element of area ds in a plane surface having charge Q and P be a point in element. At every point of surface, the electric flux density D will have Ds.

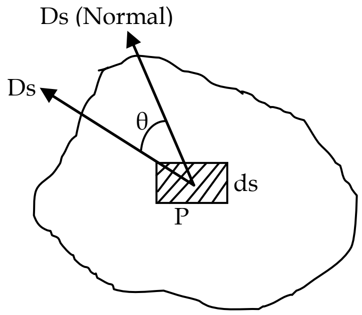


Fig . closed surface having charge

From fig, $\cos\theta = \dfrac{D_S(\text{normal})}{D_S}$

$$D_S(\text{normal}) = D_S \cos\theta$$

Let Ds makes an angle θ with ds the flux crossing ds is the product of normal component of Ds and ds.

$$\begin{aligned} d\phi &= D_S(\text{normal}).\, ds \\ &= D_S \cos\theta.\, ds \\ d\phi &= D_S.ds \end{aligned}$$

Total flux passing through closed surface,

$$\Psi = \int d\Psi = \oiint_s D_s.\, ds$$

$$E = \frac{Q}{4\pi\varepsilon r^2} \Rightarrow D = \varepsilon E \Rightarrow D = \frac{Q}{4\pi r^2}$$

The small element of area ds on surface of sphere is

$$\begin{aligned} ds_r &= r\, d\theta \sin\theta\, d\phi \\ ds &= r^2 \sin\theta\ d\theta\ d\phi \\ \Psi &= \oiint_s \frac{Q}{4\pi r^2}.\, r^2 \sin\theta\, d\theta\ d\phi \\ &= \frac{Q}{4\pi}\int_{\phi=0}^{2\pi}\int_{Q=0}^{\pi} \sin\ \theta\, d\theta\, d\phi \\ &= \frac{Q}{4\pi}[-\cos]_0^{\pi}\,[\phi]_0^{2\pi} \\ &= \frac{Q}{4\pi}[1+1][2\pi] = Q \\ \Psi &= Q \end{aligned}$$

Note:-

Total charge enclosed $Q = \int_Y \rho_v dv \ \rightarrow (1)$

$$Q = \oiint_s D.\, ds = \int_v \rho_v dv \ \rightarrow (2)$$

Applying divergence to (2),

$$\oiint_s D.\, ds = \int_x \nabla.\, D\, dv \ \rightarrow (3)$$

Comparing 2 & 3 we get

$$\nabla.D = \rho_v$$

Differential from Gauss Law.

17) Explain the application of Gauss law?

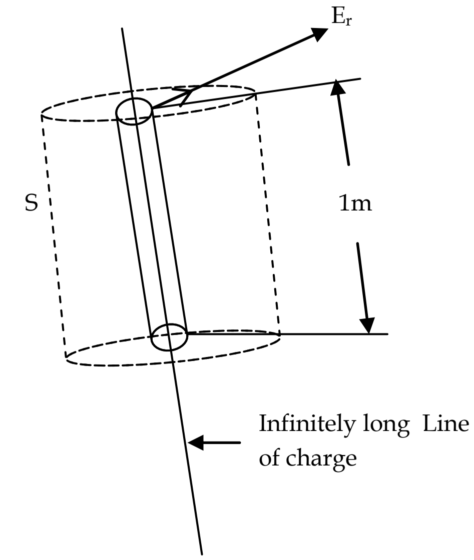
Application of Gauss law:-

Application 1: To determine the field at a distance r from an line charge of strength λ c/m.

The figure shows a theoretically infinite charged line with length alone particular signified. Imagine that a coaxial cylindrical surface surrounds the charged over a meter length this is a Gaussian surface.

The electric field at any point is radial an independent of both positions thro along and angular position around the wire.

As the electric field is in the same plate as the circular ends at top and bottom of the cylindrical, no flux passes through the end surfaces.



Applying Gauss's law for a metre length the charged line

$$\oiint E.n\,ds = 2\pi r\,Er = \lambda/\varepsilon_0$$

$$E = E_r = \frac{\lambda}{2\pi\varepsilon_0 r}\overline{a}_r$$

Where ar is unit vector at any point P on the cylindrical surface and is disc radially outward, being perpendicular to the axis of the charged line.

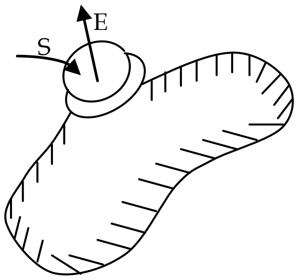
Application2:

Consider a closed pill box shaped surface S resting on the surface of a charged on the conductor be given by the surface charge density function ρ_s. Consider an elementary surface area Δs; the applying gauss law to the small pill box shaped surface S, we have the surface. Integral of the normal component of $E = E.n\ \Delta s$ Thus,

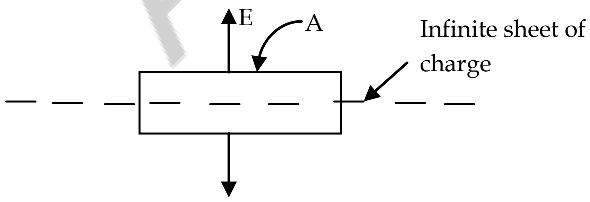
$$\varepsilon_0 E.n\,\Delta s = Q = \rho_S \Delta s$$

$$E.n\,\Delta s = \rho_S \frac{\Delta s}{\varepsilon_0}$$

$$E = \frac{\rho_S}{\varepsilon_0}.\ n$$



Application 3:



By summery it is obvious that the field can only be perpendicular to the surface of the infinite plane sheet of charge.

$$\oiint_s D.\ n\ ds = Q\ \rho_s\ A\ \left[\because\ D = \varepsilon_0 E\right]$$

$$\oiint_s E.n\,ds = \frac{\rho_S\ A}{\varepsilon_0}$$

$$E(2A) = \frac{\rho_S\ A}{\varepsilon_0}$$

$$E = \frac{\rho_S\ A}{2\varepsilon_0}.\ n$$

Application 4:

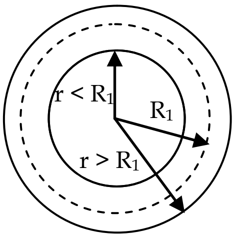
To determine the variation of field the point to point due to

(i) A single spherical shell of charge with radius R_1.

(ii) Two concentric spherical shells of charge of radii R_1 (inner) & R_2 (outer)

(iii) Spherical volume distribution of charge of radius R density L

(i) Single shell of charge



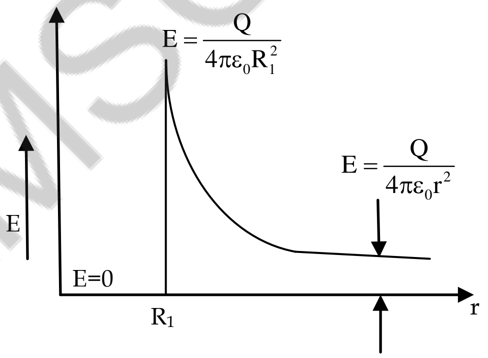
Let us suppose a total charge Q to be uniformly distributed over an imaginary shell of radius R_1 in a medium of a free space. At any radius $r < R_1$. Inside the shell of charge. Integral of $\overline{D}$ over a spherical surface.

$$\oiint_s \overline{D}.\ n\ ds = \varepsilon_0 \oiint_s E.\ n\ ds = 0$$
$$E = 0\ (\text{for}\ r < R_1)$$

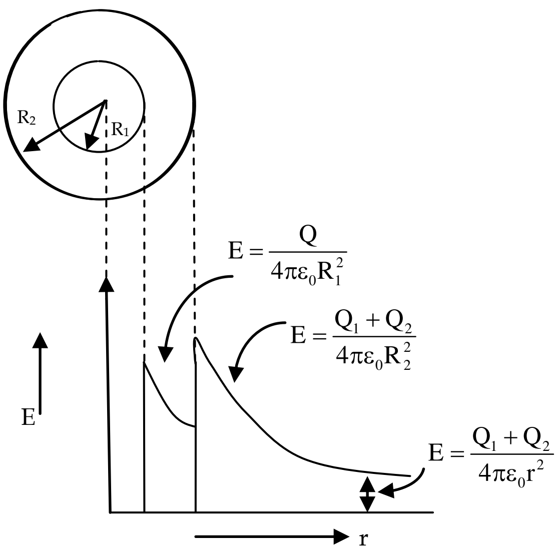
On the other hand, at any radius $r \geq R_1$, integral of $\overline{D}$ over s spherical surface is equal to the charge by itself.

Thus,

$$\varepsilon_0 \oiint E.\ n\ ds = \varepsilon_0\ \ E(4\pi R_1^2) = Q$$
$$E = \frac{Q}{4\pi\varepsilon_0 R_1^2}$$



(ii) Two concentric shells of charge



Consider the two spherical shell charges Q_1 & Q_2 at radius R_1 & R_2 respectively

$R < R_1$, $E = 0$ (in the charge free region)

$$R_1 \le r \le R_2,\ E = \frac{Q}{4\pi\varepsilon_0 r^2}\overline{a_r}$$

Just outside the shells of charges Q_1 & Q_2 respective

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}\overline{a_r}(\text{i.e; at } r = R_1 + dr)$$

$$E = \frac{Q_1 + Q_2}{4\pi\varepsilon_0 R_2^2}\,a_r(\text{i.e; at } r = R_2 + dr)$$

At any point outside both the shells of charge (i.e) $r \ge R_2$

$$E = \frac{Q_1 + Q_2}{4\pi\varepsilon_0 r^2}\,\overline{a_r}$$

(iii) Spherical volume distribution of charge:

Consider any point with in the spherical volume of charge (ie) for $r < R$. As we have uniformly distributed by a concentric sphere of radius $r < R$ is proportional to cube radius $< R$ to the total volume charge

$$\frac{Q_r}{Q_t} = \left(\frac{r}{R}\right)^3$$

$$Q_r = Q_t\left(\frac{r}{R}\right)^3$$

$$E = \frac{Q_r}{4\pi\varepsilon_0 r^2}a_r(r \le R)$$

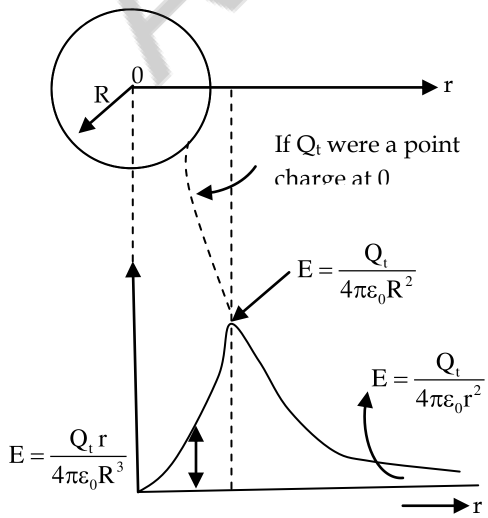
$$E = \frac{Q_t r}{4\pi\varepsilon_0 R^3}a_r\,(r \le R)$$

The above relation indicates that the field is zero at the centre of the sphere and increases uniformly to a maximum.

At $r \ge R$

$$E = \frac{Q_t}{4\pi\varepsilon_0 r^2}$$

Uniformly distributed volume charge of density ρv,



UNIT – I

STATIC ELECTRIC FIELD

INTRODUCTION TO CO – ORDINATE SYSTEM:

In order to describe a vector accurately, some specific lengths, directions, angles, projections or components must be given. There are three simple methods of doing this and they are

1. The rectangular or Cartesian system of co- ordinates
2. The circular cylindrical system of co –ordinates
3. The circular spherical system of co – ordinates

Explain the rectangular co ordinate systems?

This system has three co – ordinates axis mutually at right angles to each other and we name them as x, y and z axis. A rotation of x – axis into y – axis would cause a right handled screw to programs in the directions of z – axis.

A point is located by giving its x, y and z co – ordinates. These are, respectively the distance from the origin to the intersection of a perpendicular dropped from the point to the x, y and z axis.

An alternative method of interpreting co –ordinal values is to consider a point at the common intersection of three surface, the planes x = constant, y = constant & z = constant, the constant being the co –ordinates values of the point.

The following figure 1.1 (a) shows the points p(1, 2, 3) & (2, -2, 1) respectively

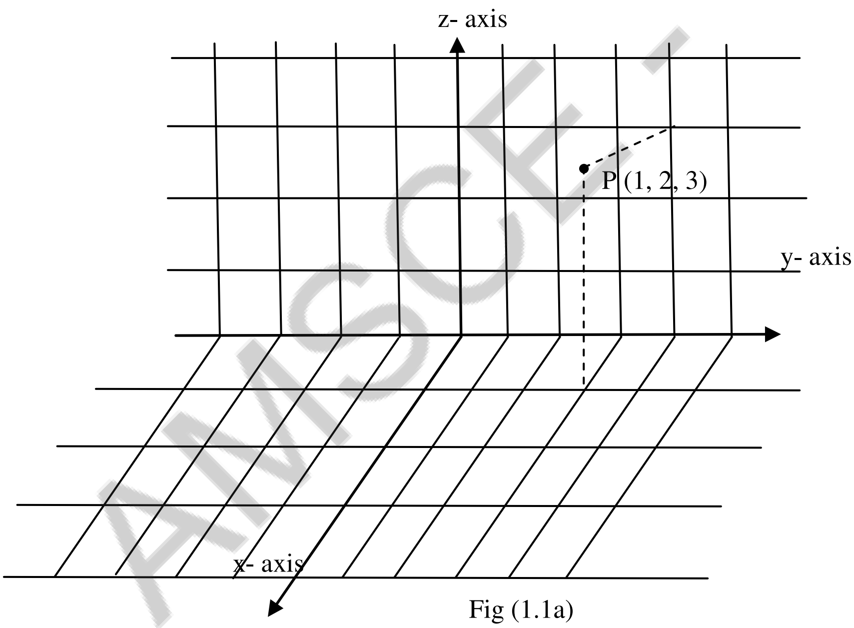
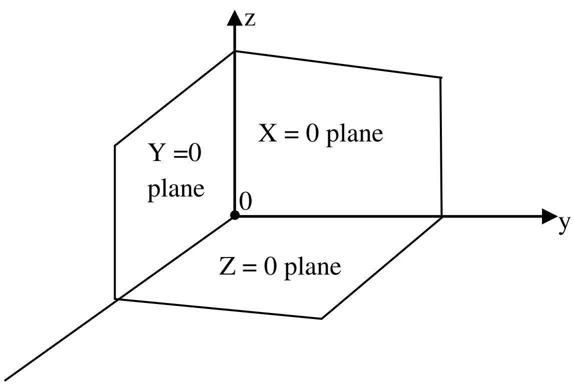


Fig (1.1a)

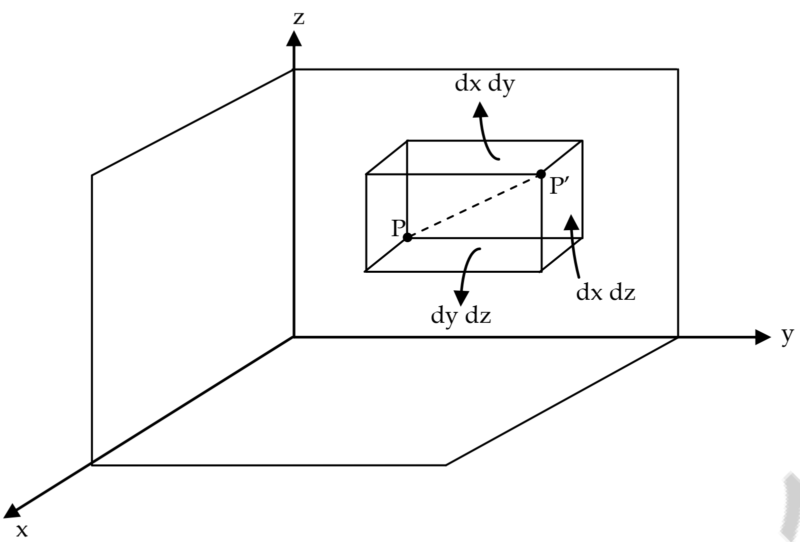


This figure (1.1b) shows a rectangular co ordinates system

If we visualise three planes interesting at the general point P, whose co – ordinates are x, y and z, we may increase each co ordinates value by a differential amount and obtain three slightly displaced planes intersecting at point 'P'. Whose co – ordinates are x+dx, y+dy and z+dz. The six planes define a rectangular parallelopiped whose volume is dv = dx dy dz the surface have differential areas $ds = dx\,dy; ds = dydz; ds = dxdz$

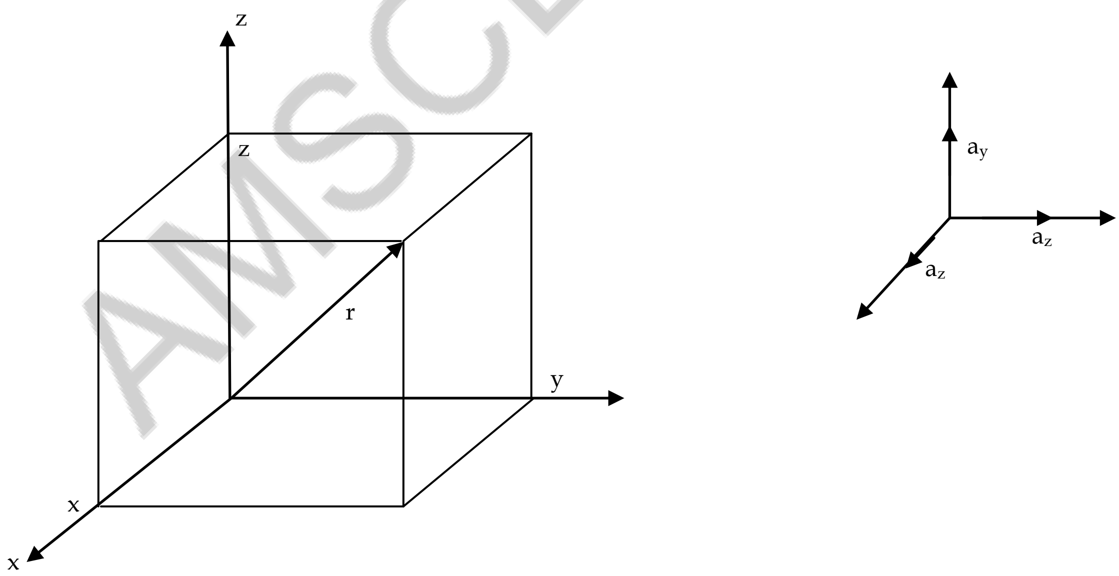
Finally the distance dL from p to p[1] is diagonal of the parallelopiped & has a length of $\sqrt{(dx)^2 + (dy)^2 + (dz)^2}$

The volume element is given in fig (1.1c)



Vector components and unit vector:-

Let us first consider a vector 'r' extending outward from the origin. A local way to identify this vector is by giving the three component vectors, lying along the three co – ordinates axis, whose vector sum must be the given vector. If the component of the vector are x, y and z, then r = x+ y+ z.



Unit vector are those which have unit magnitude and directed along the co –ordinates axis in the direction of increasing co –ordinates values. Any vector 'B' may be described by

$$\bar{B} = B_x \bar{a}_x + B_y \bar{a}_y + B_z \bar{a}_z$$

$$|B| = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

$$a_B = \frac{\bar{B}}{|B|}$$

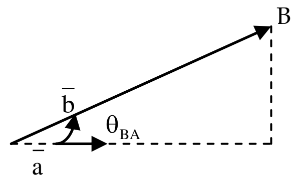
DOT product:-

Given two vector A & B, the dot product or scalar product is defined as the product of the magnitude of A, the magnitude of B & the cosine of the smaller angle between them.

$$A.B = |A||B|\cos\theta_{AB}$$

The dot product obeys communicative law

$$A.B = B.A$$

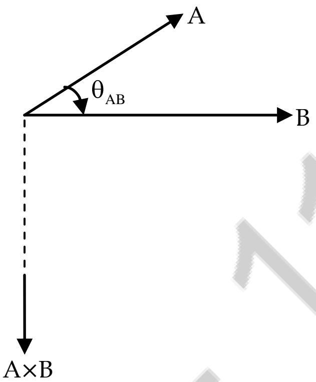


$$a_x.a_x = a_y.a_y = a_z.a_z = 1$$
$$a_x.a_y = a_y.a_z = a_z.a_x = 0$$

The cross product:-

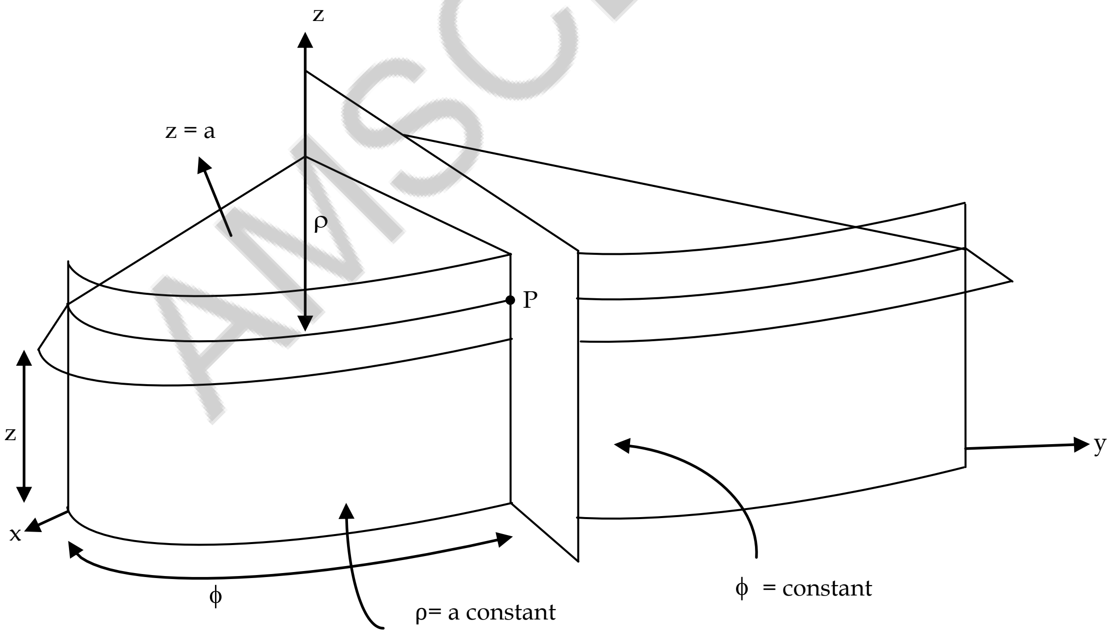
$$A \times B = a_N |A||B| \sin\theta_{AB}$$

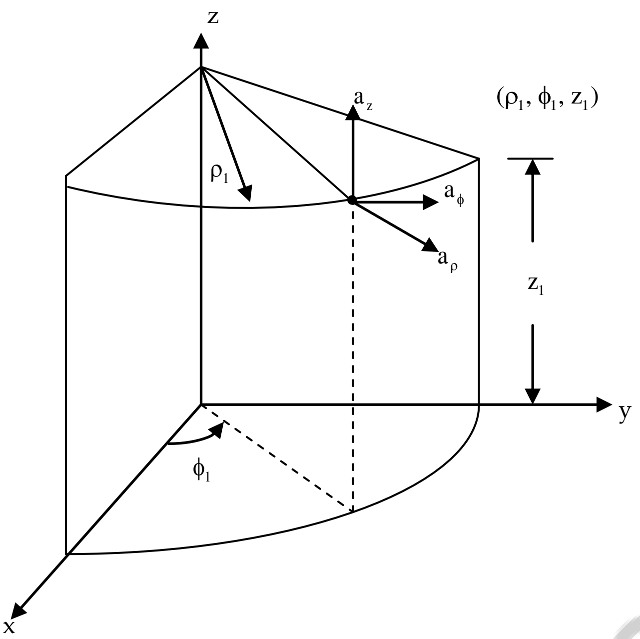
$$A \times B = \begin{vmatrix} a_x & a_y & a_{xz} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

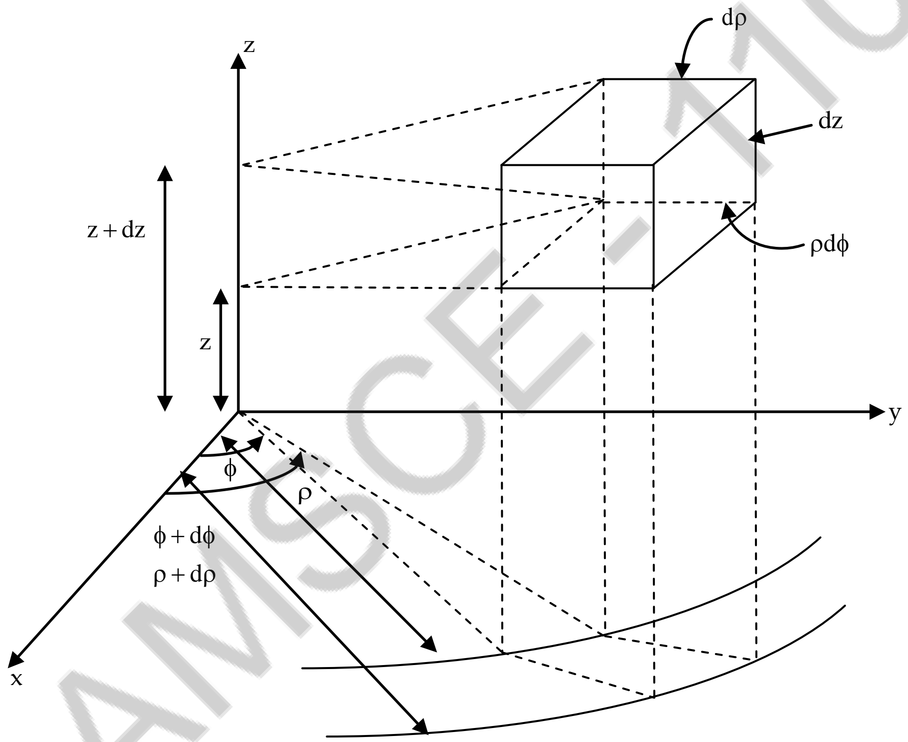


Circular cylindrical co –ordinates:-

A point was located in a plane by giving its distance ρ from the origin & the angle ϕ between the line from the point to the origin & an arbitrary radial line taken as $\phi = 0$, a distance Z of the point





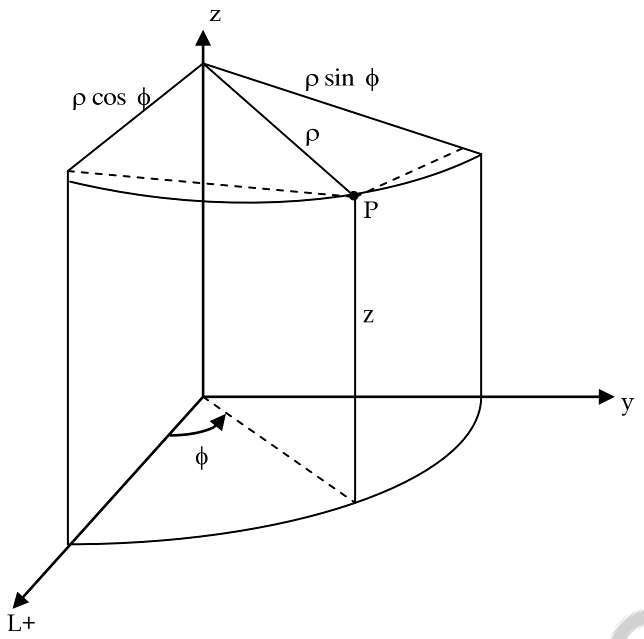


There are three unit vectors designed as follows.

- a_ρ at a point $P(\rho, \phi, z)$ is directed radially outward, normal to the cylindrical surface $\rho = \rho_1$. It lies in the planes $\phi = \phi_1$ & $z = z_1$
- $a\phi$ is normal to the plane $\phi = \phi_1$, points in the direction of increasing ϕ, lies in the plane $Z = Z_1$, is tangent to the cylindrical surface $\rho = \rho_1$
- The unit vector a_z is same as unit vector a_z of the rectangular system.
- The unit vectors are again mutually perpendicular for each is normal to one of the three mutual perpendicular surfaces.

$$a_\rho \times a_\phi = a_z$$
$$a_\phi \times a_z = a_\rho$$
$$a_z \times a_\rho = a_\phi$$

The surfaces have areas of $\rho d\rho d\phi$, $d\rho dz$, & $\rho d\phi dz$ the volume becomes $\rho d\rho d\phi$



The above figure can give the rectangular of Cartesian & rectangular co –ordinates

$$x = \rho\cos\phi \Rightarrow x^2 = \rho^2\cos^2\phi \rightarrow (1)$$

$$y = \rho\sin\phi \Rightarrow y^2 = \rho^2\sin^2\phi \rightarrow (2)$$

$$z = z$$

Equ (1) + (2) $\Rightarrow x^2 + y^2 = \rho^2\left[\cos^2\phi + \sin^2\phi\right]$

$$\rho = \sqrt{x^2 + y^2}$$

$$\cos\phi = x/\rho, \ \sin\phi = y/\rho \ \frac{\sin\phi}{\cos\phi} = y/x$$

$$\phi = \tan^{-1} y/x$$

Conversion of Cartesian to cylindrical :-

A vector function is Cartesian co–ordinates is given as follows.

$$A = A_x a_x + A_y a_y + A_z a_z \rightarrow (1)$$

& we need a vector in cylindrical co – ordinates

$$A = A_\rho a_\rho + A_\phi a_\rho + A_z a_z \rightarrow (2)$$

To find Aρ:

$$A_\rho = A.a_\rho$$
$$A_\rho = (A_x a_x + A_y a_y + A_z a_z).a_\rho$$

$$A_\rho = A_x a_x.a_\rho + A_y a_y . a_\rho \rightarrow (3)$$

To find A ϕ :

$$A_\phi = (A_x a_x + A_y a_y + A_z a_z)a_\phi$$
$$A_\phi = A_x a_x. \ a_\phi + A_y a_y.a_\phi$$
$$A_z = A_z$$

$a_z.a_\rho = a_z.a_\phi = 0$

	a_ρ	a_ϕ	a_z
a_x	$\cos\phi$	$-\sin\phi$	0
a_y	$-\sin\phi$	$\cos\phi$	0
a_z	0	0	1

Sub in (3) & (4) we get

$$A_\rho = A_x \cos\phi + A_y \sin\phi$$
$$A_\phi = -A_x \sin\phi + A_y \cos\phi$$
$$A = (A_x \cos\phi + A_y \sin\phi)\bar{a}_\rho + (-A_x \sin\phi + A_y \cos\phi)a_\phi + A_z\, a_z$$

Cylindrical to Cartesian:-

$$A = A_\rho a_\rho + A_\phi a_\phi + A_z a_z$$
$$A_x = A.\, a_x$$
$$= (A_\rho a_\rho + A_\phi a_\phi + A_z a_z).\ a_x$$
$$A_x = A_\rho \cos\phi - A\phi\ \sin\phi$$
$$A_y = A.\, a_y$$
$$= (A_\rho a_\rho\ a_y + A_\phi a_\phi.a_y)$$
$$A_y = A_\rho \sin\phi - A\phi\ \cos\phi$$
$$A = (A_\rho \cos\phi - A\phi\ \sin\phi)\ a_x + (A_\rho \sin\phi - A\phi\ \cos\phi)a_y + A_z a_z$$

The spherical co – ordinates system:

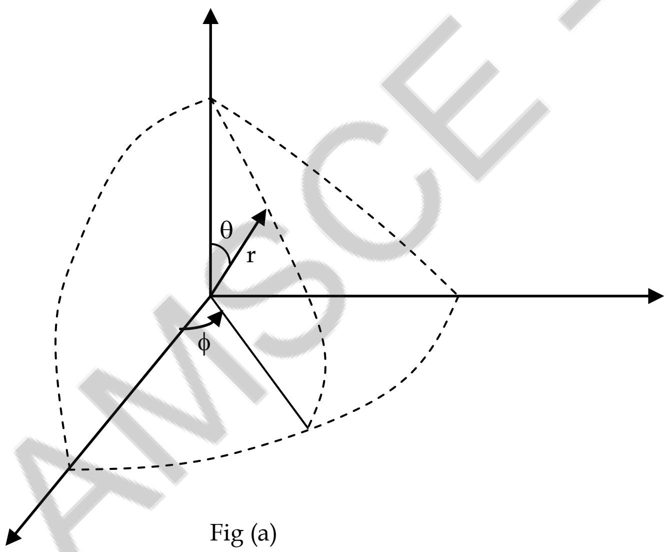


Fig (a)

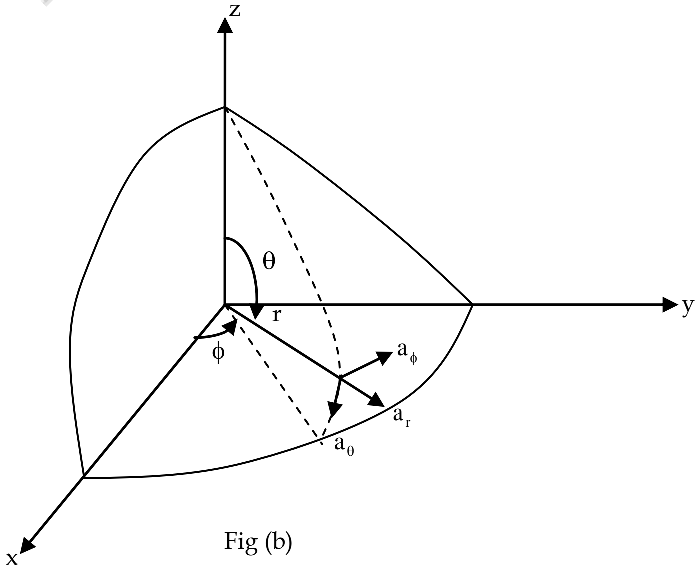


Fig (b)

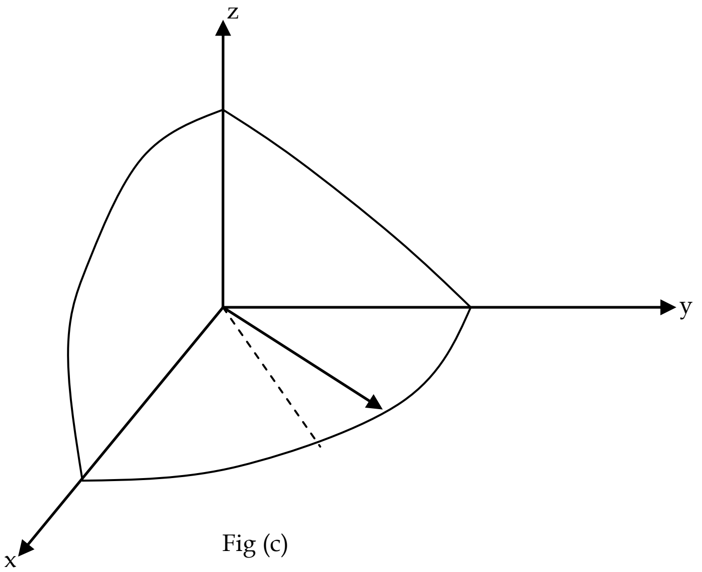


Fig (c)

Let us draw a spherical co ordinates system on three rectangular axis as shown in fig (a)

We first define the distance from the origin to any point as r. The surface r = constant is a sphere.

The second co – ordinates is angle Q between the z – axis & the line drawn from the origin to the point in question. The surface Q = constant is a core & the two surface, core &sphere are everywhere perpendicular along their intersection which is a circle of radius r sin θ.

The third co – ordinates ϕ is also an angle and is exactly the same as the angle ϕ of the cylindrical co – ordinates. It is angle between the x – axis & the propagation in the z = 0 plane of the line drawn from the origin to the point.

Three unit vectors may again be defined at any point. Each unit vector is perpendicular to one of the three mutually perpendicular surfaces & oriented in that direction in which the co – ordinate increases.

The unit vector a_r is directed radially outward, normal to the sphere r = constant & lies in the cone θ = constant & the plane ϕ = constant.

The unit vector a_θ is normal to the conical surfaces, lies in the plane & is tangent to the sphere.

$$a_r \times a_Q = a_\phi$$

The surfaces have areas of rd rdQ, $r \sin Q \, dr d\phi$ & $r^2 \sin\theta d\theta \, d\phi$. the volume is $r^2 \sin\theta dr d\theta \, d\phi$

$$x = r\sin\theta\cos\phi \Rightarrow x^2 = r^2\sin^2\theta\cos^2\phi$$
$$y = r\sin\theta\sin\phi \Rightarrow y^2 = r^2\sin^2\theta\sin^2\phi$$
$$z = r\cos\theta \qquad \Rightarrow z^2 = r^2\cos^2\theta$$
$$r = \sqrt{x^2 + y^2 + +z^2}$$
$$\theta = \cos^{-1}\frac{z}{\sqrt{x^2 + y^2 + z^2}}$$
$$\phi = \tan^{1}\left(y/x\right)$$

	a_r	a_θ	a_ϕ
a_x	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
a_y	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
a_z	$\cos\theta$	$-\sin\theta$	0

Transformation of vector in Cartesian to Spherical:

Let us consider a vector in Cartesian co – ordinate

$$A = A_x a_x + A_y a_y + A_z a_z$$

& to find the components of spherical vector

$$\begin{aligned} A_r &= A.a_r \\ &= (A_x a_x + A_y a_y + A_z a_z).a_r \\ &= A_x a_x a_r + A_y a_y a_r + A_z a_z.a_r \\ &= A_x \sin\theta\cos\phi + A_y \sin\theta\sin\phi + A_z \cos \\ A_\theta &= A.a_\theta \\ &= (A_x a_x + A_y a_y + A_z a_z).a_\theta \\ &= A_x \sin\theta\cos\phi + A_y \cos\theta\sin\phi + A_z \sin\theta \\ A_\phi &= A.a_\phi \\ &= A_x \sin\phi + A_y \cos\phi \end{aligned}$$

$$A = (A_x \sin\phi\cos\phi + A_y \sin\theta\sin\phi + A_z \cos\theta)a_r + (A_x \cos\theta\cos\phi + A_y \cos\theta\sin\phi - A_z \sin\theta)a_\theta + (-A_x \sin\phi + A_y \cos\phi)A_\phi$$

The line, surface & volume integral let us consider a charge Q whose density along line, surface, & volume is given by ρ_L, ρ_S, ρ_V

Line integral $Q = \int_L \rho_L \; dl \;\; \rho_L = \dfrac{\Delta Q}{\Delta L}$ as $\Delta L \to 0$

Surface integral $Q = \int_S \rho_S \; ds \;\; \rho_s = \dfrac{\Delta Q}{\Delta S}$ as $\Delta S \to 0$

Volume integral $Q = \int_V \rho_V \; dv \;\; \rho_v = \dfrac{\Delta Q}{\Delta V}$ as $\Delta V \to 0$

State and Explain Divergence theorem and Stoke's theorem

Divergence:

Definition: The divergence of the vector $\vec{F}$ at any point is defined as the limit of its surface integral per unit volume as per the volume enclosed by the surface around the point shrinks to zero.

$$\vec{F} = \lim_{\Delta v \to 0} \frac{\int \vec{F}.\, n \;\, ds}{2}$$

Proof:

Consider an elemental volume $\Delta v = \Delta x \;\, \Delta y \;\, \Delta z$ of a parallopiped.

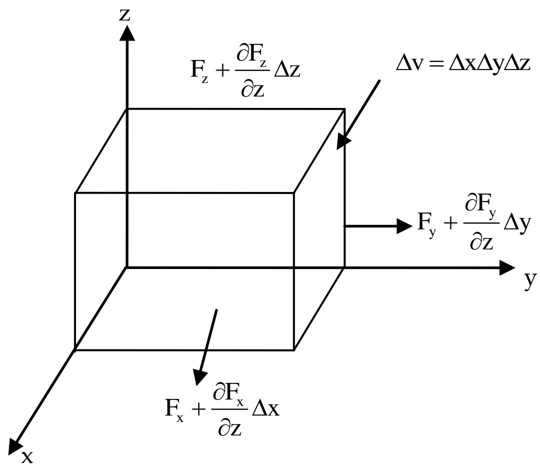
Let F is a vector field. The flux of any vector F through a surface is given by the surface integral of the vector over that surface.

The flux passing out of the volume is taken as positive and that passing inward as negative. Let Fx, y, Fz be the components of F along the co –ordinates axis, so that

$$F = F_x + F_y + F_z.$$

Consider the volume element , the flux of the vector F in y direction into hand face is and $F_y(\Delta x \;\Delta z)$ and out of right hand face it is $\left(F_y + \dfrac{\partial F_y}{\partial y}\Delta y\right)\Delta x\Delta z$. Therefore, the net increase of flux along the positive Y direction is

flux along Y axis $= \left(F_y + \dfrac{\partial F_y}{\partial y}\Delta y\right)\Delta x\Delta z - F_y \;\Delta x\Delta z$



$$= F_y \cancel{\Delta x}\Delta z + \frac{\partial F_y}{\partial y}\Delta x \Delta z \Delta y - F_y \Delta x \Delta z$$
$$= \frac{\partial F_y}{\partial y}\Delta x \Delta z \Delta y$$

Similarly in x direction,

$$\text{Flux along x axis } = \left(F_x + \frac{\partial F_x}{\partial x}\Delta x\right)\Delta y \Delta z - F_x\ \Delta y \Delta z$$

$$= F_x \cancel{\Delta y}\Delta z + \frac{\partial F_x}{\partial x}\Delta x \Delta y \Delta z - F_x \cancel{\Delta y}\Delta z$$
$$= \frac{\partial F_x}{\partial x}\Delta x \Delta y \Delta z$$

Now, along the z – direction, the net influx of the vector is given by.

$$\text{Flux along z axis } = \left(F_z + \frac{\partial F_z}{\partial z}\Delta z\right)\Delta x \Delta y - F_x\ \Delta x \Delta y$$

$$= F_z \cancel{\Delta z}\Delta y + \frac{\partial F_z}{\partial z}\Delta x \Delta y \Delta z - F_x \cancel{\Delta x}\Delta y$$
$$= \frac{\partial F_z}{\partial z}\Delta x \Delta y \Delta z$$

Now, total increase is given by,

Total flux

$$= \frac{\partial F_x}{\partial x}\Delta x \Delta y \Delta z + = \frac{\partial F_y}{\partial y}\Delta x \Delta y \Delta z + = \frac{\partial F_z}{\partial z}\Delta x \Delta y \Delta z$$
$$\oint\!\!\oint_s \vec{F}.\,n\,ds = \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}\right).\Delta x \Delta y \Delta z$$
$$= \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}\right)\Delta v$$
$$\frac{\oint\!\!\oint_s \vec{F}\,.n\,ds}{\Delta v} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

By definition of divergence, we get

$$\because \nabla.\vec{F} = \lim_{\Delta v \to 0} \frac{1}{\Delta v}\left[\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}\right]\Delta v$$

$$\nabla.\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\therefore \oint_s \frac{\vec{F}.\,n\,ds}{\Delta v} = \nabla.\vec{F}$$

Divergence theorem:

The integral of the divergence of the vector field over a volume V is equal to the surface integral of the normal component of the vector over any surface bounding the volume.

Mathematically for any field vector $\vec{F}$

$$\iiint_v \nabla.\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

Proof:

By definition of divergence,

$$\iiint_v \nabla.\,\vec{F} = \iiint_v \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}\right) dx\,dy\,dz$$

When dv = dx dy dz

Now,

$$\iiint_v \nabla.\,\vec{F}\,dv = \int\left(\frac{\partial}{\partial x}a_x + \frac{\partial}{\partial y}a_y + \frac{\partial}{\partial z}a_z\right)(F_x a_x + F_y a_y + F_z a_z)\,dx\,dy\;\;dz$$

$$= \iiint_v \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}\right) dx\;dy\;dz$$

$$\iiint_v \nabla.\,\vec{F}\,dv = \iiint_v \frac{\partial F_x}{\partial x} dx\;dy\;dz + \iiint_v \frac{\partial F_y}{\partial y} dy\;dx\;dz + \iiint_v \frac{\partial F_z}{\partial z}\;dz\;dx\;dy$$

$$= \iint_s F_x\;dydz + \iint_s F_y\;dxdz + \iint_s F_z\;dxdy$$

$$= \iint F_{Sx}dy\;dz + \iint F_{Sy}dx\;dz + \iint F_{Sz}dx\;dy$$

$$\iiint_v \nabla.\,\vec{F}\,dv = \oiint_s \vec{F}.\,\hat{n}\,ds$$

Hence proved.

Stoke's theorem:

The line integral of the vector around a closed path is equal to the integral of the normal component of its well over any surface bounded by the contour.

$$\oint_c \vec{F}.dl = \oint_s (\nabla \times \vec{F}).\,n c / s$$

Where C is closed contour which bounds the surface S.

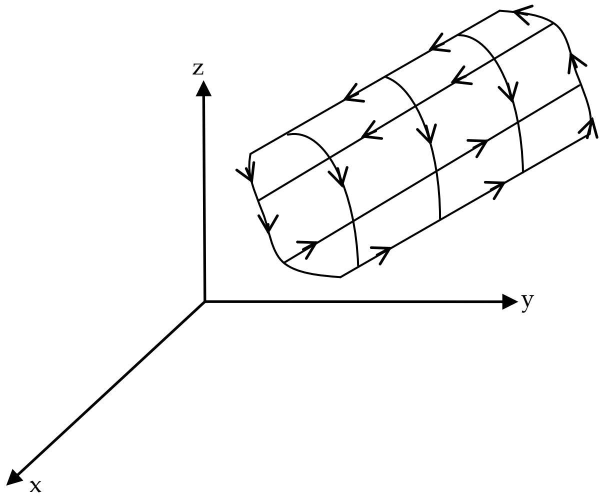
Proof:

Consider the arbitrary surface S as shown below.

If F is the field vector, then by definition of curl, the line integral $\oint F.\,dL$, divided by the surface area gives the curl of F normal to the surface at the point around which the surface shrinks to zero.

Thus $\lim_{s \to 0} \frac{\oint F.\,dl}{S} = \text{curl } nF$

Where curl nF is the component of curl of F normal to the surface S.



Now divide up the area S into a large number of still smaller elements 1, 2 etc. Each area element can be represented by a vector directed outwardly normal to the surface.

For each such element, find the line integral in the positive direction, normally anti–clockwise which corresponds to the positive direction of the surface elements.

If all the elements 1, 2 etc are summed up, the contributions of the common boundary of any two adjacent elements neutralize each other, as they are oppositely directed along the common boundary

$$\oint_c F.dl = \int \vec{F}\, dl_1 + \int \vec{F}\, dl_2 + \int \vec{F}\, dl_3$$

Applying the definition of curl F, we have

$$\oint_c F\,.dl = \text{curl } Fds_1 + \text{curl } Fds_2 + \text{curl } ds_1 + \text{curl } Fds_2 = \iint_s \text{curl}\, F\, n\ ds.$$

(i.e) curl Fds_1 + curl Fds_2 +..... denotes the summation of normal components of curl F over the whole surface S.

Therefore,

$$\oint_c F.dl = \iint_s \text{curl } F.n\ ds$$

Hence stoke's theorem is proved.

Curl:

The curl of a vector at any point is defined as the limit of the ratio of the integral of its cross product with the outward drawn normal over a closed surface, to the volume enclosed by the surface as the volume shrinks to zero.

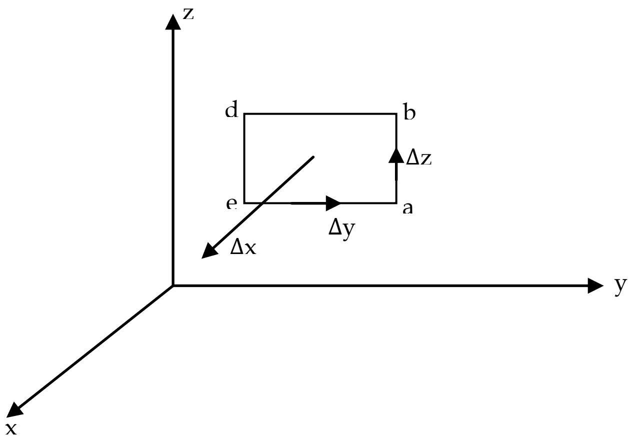
$$|\text{curl}\, F| = \lim_{v \to 0} \frac{1}{v} \oint_s n \times F\, ds$$

The component of curl of vector in the direction of the unit vector n is the ratio the line integral of the vector around a closed contour, to the enclosed area bounded by the contour, as the enclosed area diminishes to zero.

$$n \text{ curl} F = \lim_{a \to 0} \frac{1}{s} \oint_c f.\ dl$$

Referring to fig, consider an elemental plane surface of area $\Delta Y\ \Delta Z$ bounded by a contour C marked as able.

$|\text{curl } B|\ x$ = curl of B in the x – direction



$$\lim_{\substack{\Delta y \to 0 \\ \Delta z \to 0}} \frac{\oint_e B\,.dl}{\Delta y\ \Delta z}$$

Where B is a vector, choosing the value of B at centre, we have the closed integral of B around the abde,

$$= \oint B\,.dl$$

$$\text{Integral along } e_a = \left(By - \frac{\partial By}{\partial z}\frac{\Delta z}{2} \right)\Delta y$$

$$\text{Integral along } ab = \left(Bz - \frac{\partial Bz}{\partial y}\frac{\Delta y}{2} \right)\Delta z$$

$$\text{Integral along } bd = -\left(By + \frac{\partial By}{\partial z}\frac{\Delta z}{2} \right)\Delta y$$

$$\text{Integral along } de = -\left(Bz - \frac{\partial Bz}{\partial y}\frac{\Delta y}{2} \right)\Delta z$$

Adding

$$\oint_L B\,.dl = By\ \Delta y - \frac{\partial By}{\partial z}\frac{\Delta y \Delta z}{2} + Bz\ \Delta z + \frac{\partial Bz}{\partial y}\frac{\Delta y \Delta z}{2}$$

$$\text{curl}_x\ B = \left(\frac{\partial Bz}{\partial y} - \frac{\partial By}{\partial z} \right) = \mu_0 J_x$$

Simarlly

$$\text{curl}_y\ B = \left(\frac{\partial Bx}{\partial z} - \frac{\partial Bz}{\partial x} \right) = \mu_0 J_y$$

$$\text{curl}_y\ B = \left(\frac{\partial By}{\partial x} - \frac{\partial Bx}{\partial y} \right) = \mu_0 J_z$$

$$\text{curl } B = |\text{curl}_x\ B| a_x + |\text{curl}_y\ B| a_y + |\text{curl}_z\ B| a_z$$

$$= \mu_0 (J_x\ a_x + J_y\ a_y + J_z\ a_z)$$

$$\text{curl } B = \left(\frac{\partial Bz}{\partial y} - \frac{\partial By}{\partial z} \right)\overline{a_x} + \left(\frac{\partial Bx}{\partial z} - \frac{\partial Bz}{\partial x} \right)\overline{a_y} + \left(\frac{\partial By}{\partial x} - \frac{\partial Bx}{\partial y} \right)\overline{a_z}$$

$$\text{curl } B = \begin{vmatrix} a_x & a_y & alz \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ B_x & B_y & B_z \end{vmatrix}$$

The main difference of divergence & curl is that in the former the parameter along x varies only with x but in curl the parameter in x varies along y & z.

1. Define coulomb's law

Coulomb's law:-

The law states that "two charges Q_1 and Q_2 separated by a distance 'r' in free space or vacuum, the force of attraction or repulsion is directly proportional to the product of the magnitude of charges and is inversely proportion to the square of the distance between them

$$F \propto \frac{Q_1 Q_2}{r^2}$$
$$F = \frac{K Q_1 Q_2}{r^2}$$

Where, k = proportionality constant $= \frac{1}{4\pi\varepsilon}$

Where

ε = permittivity of the medium

$\varepsilon = \varepsilon_0 \varepsilon_r$

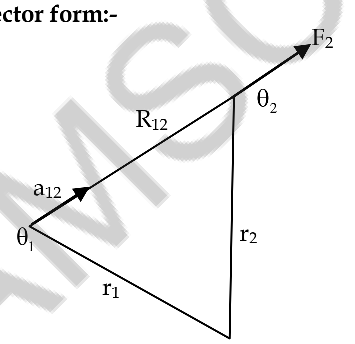
ε_0 = absolute permittivity $= = \frac{1}{36\pi \times 10^9} = 8.854 \times 10^{-12}$

ε_r = Relative permittivity of the medium which is 1 for free space / air.

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon r^2}$$
$$= \frac{Q_1 Q_2}{4\pi\varepsilon_0 \varepsilon_r r^2}$$
$$F = 9 \times 10^9 \frac{Q_1 Q_2}{r^2} \text{ newton or } C^2/m^2$$

2. Derive coulomb's law in vector form?

Coulomb's law in vector form:-



In vector form, we need an additional fact that the force all along the line joining origin the two charges & is repulsive if they are of opposite sign.

Let r_1 locate Q_1 and r_2 locate Q_2. Then the vector $R_{12}=r_2 - r_1$ represent the direct line segment from Q_1 to Q_2.

Let F_2 be the force on Q_2 by Q_1. Note Q_1 and Q_2 have the same sign.

$$\overline{F_2} = \frac{Q_1 Q_2}{4\pi\varepsilon R_{12}^2} \overrightarrow{a_{12}}$$

Let F_2 be the force on Q_2 by Q_1 where $\bar{a}_{12} = $ a unit vector in the direction of the R_{12}.

$$\bar{a}_{12} = \frac{\overline{R_{12}}}{|R_{12}|} = \frac{\overline{R_{12}}}{R_{12}} = \frac{r_2 - r_1}{|r_2 - r_1|}$$

Let Q_1 be located at $x_1\bar{a}_x + y_1\bar{a}_y + z_1\bar{a}_z$ and Q_2 be located at $x_2\bar{a}_x + y_2\bar{a}_y + z_2\bar{a}_z$

Then $r_2 - r_1 = (x_2 - x_1)\bar{a}_x + (y_2 - y_1)\bar{a}_y + (z_2 - z_1)\bar{a}_z$

$$R_{12} = |r_2 - r_1| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Therefore,

$$F_2 = \frac{Q_1 Q_2}{4\pi\varepsilon\left[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\right]} \times \frac{\bar{r}_2 - \bar{r}_1}{|r_2 - r_1|}$$
$$F_2 = \frac{Q_1 Q_2\left[(x_2 - x_1)\bar{a}_x + (y_2 - y_1)\bar{a}_y + (z_2 - z_1)\bar{a}_z\right]}{4\pi\varepsilon\left[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\right]^{3/2}}$$

3. Explain electric field intensity?

Electric field intensity:-

Consider one charge in fixed position say Q1 and none a second change slowly around. The change Qt will experience a force exerted by Q1.

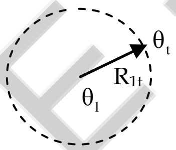
As per coulomb's law, force on Qt by Q1 is given by

$$F_t = \frac{Q_1 Q_t}{4\pi\varepsilon_0 R_{1t}^2}\overline{a_{1t}}$$

The force exerted on the test charge Qt is defined as the electric field intensity

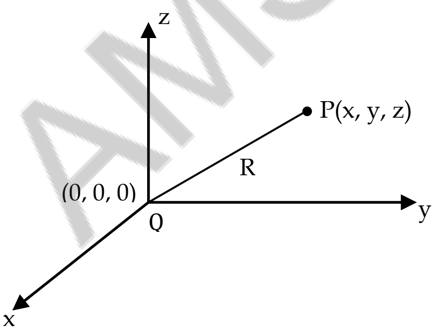
$$E = \frac{F_t}{Q_t} = \frac{Q_1}{4\pi\varepsilon_0 R_{1t}^2}\overline{a_{1t}}$$

Generally, $E = \dfrac{Q}{4\pi\varepsilon_0 R^2}\overline{a_R}$



Case (1):- charge at origin

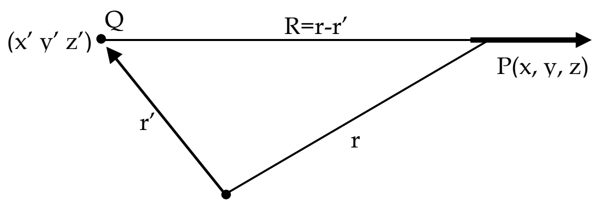
Let us consider a charge at the origin and to find electric field intensity at point whose co – ordinates are (x, y, z)



x

$$R = (x\bar{a}_x + y\bar{a}_y + z\bar{a}_z) - (0\bar{a}_x + 0\bar{a}_y + 0\bar{a}_z)$$
$$\bar{R} = x\bar{a}_x + y\bar{a}_y + z\bar{a}_z = \sqrt{x^2 + y^2 + z^2}$$
$$a_R = \frac{\bar{R}}{|R|} = \frac{x\bar{a}_x + y\bar{a}_y + z\bar{a}_z}{\sqrt{x^2 + y^2 + z^2}}$$
$$E = \frac{Q}{4\pi\varepsilon_0 R^2}\bar{a}_R$$
$$E = \frac{Q\left(x\bar{a}_x + y\bar{a}_y + z\bar{a}_z\right)}{4\pi\varepsilon_0\left(x^2 + y^2 + z^2\right)^{3/2}}$$

Case (ii):- Charge not at origin



If we consider that the charges are not at origin but located at r1 and fixed electric field intensity a P at distance r from the origin.

$$R = r - r^1$$

$$E = \frac{Q}{4\pi\varepsilon_0 (r - r^1)^2} \times \frac{\overline{r - r^1}}{|r - r^1|}$$

$$E = \frac{Q\overline{r - r^1}}{4\pi\varepsilon_0 |r - r^1|^3}$$

$$E = \frac{Q\overline{r - r^1}}{4\pi\varepsilon_0 |r - r^1|^{3/2}}$$

$$r = x\bar{a}_x + y\bar{a}_y + z\bar{a}_z$$

$$r^1 = x^1\bar{a}_x + y^1\bar{a}_y + z^1\bar{a}_z$$

$$r - r^1 = \sqrt{(x - x^1)^2 + (y - y^1)^2 + (z - z^1)^2}$$

$$E = \frac{Q\left[(x - x^1)a_x + (y - y^1)a_y + (z - z^1)a_z\right]}{4\pi\varepsilon_0\left[(x - x^1)^2 + (y - y^1)^2 + (z - z^1)^2\right]^{3/2}}$$

4. State and explain the principle of superposition?

Principle of superposition:-

Let Q1 be at a distance of r1 from origin and Q2 be at a distance of r2, then E at P.

$$E(r) = \frac{Q_1}{4\pi\varepsilon_0 |r - r_1|^2}\bar{a}_1 + \frac{Q_1}{4\pi\varepsilon_0 |r - r_1|^2}\bar{a}_2$$

If we add more charges at other positions, the field due to 'h' point charges is

$$E(r) = \frac{Q_1}{4\pi\varepsilon_0 |r - r_1|^2}\bar{a}_1 + \frac{Q_1}{4\pi\varepsilon_0 |r - r_1|^2}\bar{a}_2 + \ldots\ldots \frac{Q_n}{4\pi\varepsilon_0 |r - r_n|^2}\bar{a}_n$$

$$E(r) = \sum_{m=1}^{n} \frac{Q_m}{4\pi\varepsilon_0 |r - r_m|^2} a_m$$

5. Discuss electric field intensity due to continues charger electric field intensity due to continuous charges:-

If charge Q is uniformly distributed throughout a volume V, the charge density ρ is given by

$$\rho = \frac{Q}{V}$$

To define the value of the charge density at point P, let us consider the charge ΔQ in a volume ΔV. The ratio of infinitesimal charge ΔQ clinded by volume ΔV as ΔV = 0

$$\rho_V = \lim_{\Delta V \to 0} \frac{\Delta Q}{\Delta V}$$

Similarly we can write for ρ_S and ρ_ℓ

$$\rho_\ell = \lim_{\Delta\ell \to 0} \frac{\Delta Q}{\Delta \ell}; \rho_S = \lim_{\Delta s \to 0} \frac{\Delta Q}{\Delta S}$$

The charge element dQ and total charge Q due to these charge distribution are,

$$dQ = \rho_\ell \ dl \to Q = \int_L \rho_\ell \ dl$$
$$dQ = \rho_s \ ds \to Q = \int_S \rho_s \ ds$$
$$dQ = \rho_v \ dv \to Q = \int_V \rho_v \ dv$$

$$E = \frac{Q}{4\pi\varepsilon_0 R^2} \overline{a_R}$$

Here,

$$E = \int \frac{\rho_L \ dl}{4\pi\varepsilon_0 R^2} \overline{a_R} \quad \text{(line charge)}$$
$$E = \int \frac{\rho_S \ ds}{4\pi\varepsilon_0 R^2} \overline{a_R} \quad \text{(surface charge)}$$
$$E = \int \frac{\rho_V \ dv}{4\pi\varepsilon_0 R^2} \overline{a_R} \quad \text{(volume charge)}$$

7. Explain Electric Potential difference and Potential

<u>Electric Potential difference and Potential</u>

$$F = q\,E$$

Since there is a moment of charge in electric field from one point r1 to another point r2, there will be work done against force.

$$W = -\int_{r_1}^{r_2} qE.\,dr \Rightarrow -\ q\int_{r_1}^{r_2} E.\ dr$$

Potential difference is defined as the work done in moving a unit positive charge from one point to another in an electric field.

Work done on unit positive charge per charge is

$$V = \frac{W}{q}$$
$$V = -\int_{r_1}^{r_2} E.\,dr \quad J/C$$

But

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$
$$V = -\frac{Q}{4\pi\varepsilon_0} \int_{r_1}^{r_2} \frac{1}{r^2}\,dr$$
$$= -\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{r_1}^{r_2}$$
$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$
$$V = \frac{Q}{4\pi\varepsilon_0 r_2} - \frac{Q}{4\pi\varepsilon_0 r_1}$$
$$V_{AB} = V_B - V_A \text{ or } V_{12} = V_2 - V_1$$

8. Give relation between electric field & potential:

Relation between electric field and potential

If 2 point are separated by an infinitesimal distance dr, the work done in moving at point charge from one point to other is given by

$$dv = -E.\ dr$$

Since scalar potential V is a function of x, y, z.

$$\frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy + \frac{\partial v}{\partial z}dz = -E.\ dr$$

$$\left(\overline{a}_x \frac{\partial v}{\partial x} + \overline{a}_y \frac{\partial v}{\partial y} + \overline{a}_z \frac{\partial v}{\partial z}\right).\left(\overline{a}_x\ dx + \overline{a}_y\ dy\ \ + \overline{a}_z\ dz\right) = -E.\ dr$$

$$\nabla r.\ dr = -E.\ dr$$

$$\therefore E = -\nabla V$$

E = Negative of potential gradient at that point.

12. Determine electric filed intensity due to infinite line charge

I) Using cylindrical co – ordinates?

An infinite line of charge having the charge density ρ_L c/m is considered along the Z – axis. i.e, the line extends from - ∞ to +∞ along the z – axis. A point P is considered along the axis perpendicular to z, where 'E' is to be measured the point 'P' located at a distance ρ from the origin.

Consider a small differential length dL carrying char dQ along the z – axis.

$$dQ = \rho_L\ dl$$

$$= \rho_L\ dz$$

$$E = \frac{\int \rho_L\ dl}{4\pi\varepsilon_0 R^2}\overline{a}_R$$

$$E = \int_{-\infty}^{+\infty} \frac{\rho_L\ dz\ \ \overline{a}_Z}{4\pi\varepsilon_0(\rho^2 + z^2)} . \frac{\rho\overline{a}_e - z\overline{a}_Z}{\sqrt{\rho^2 + z^2}}$$

$$= \frac{\rho_L}{4\pi\varepsilon_0(\rho^2 + z^2)^{3/2}} \int_{-\infty}^{+\infty} (\rho a_e - za_z)\ dz\ x\overline{a_z}$$

$$= \frac{\rho_L}{4\pi\varepsilon_0(\rho^2 + z^2)^{3/2}} \int_{-\infty}^{+\infty} \rho a_\phi\, dz\ [\overline{a_z}]$$

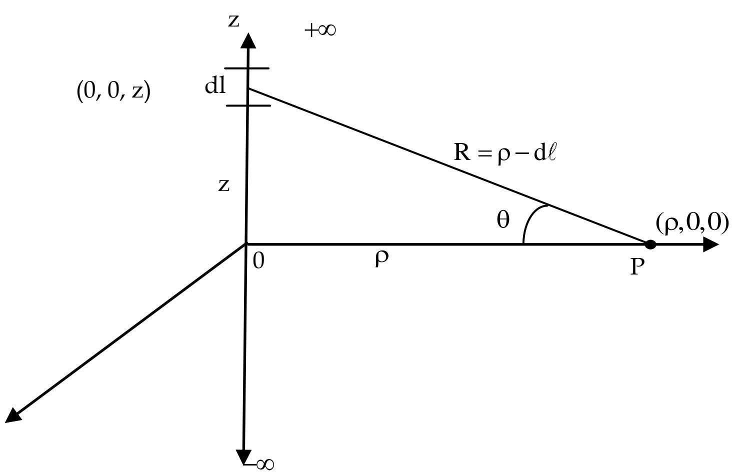
$$E = \frac{\rho_L}{4\pi\varepsilon_0} \int_{-\infty}^{+\infty} \frac{\rho dz}{(\rho^2 + z^2)^{3/2}}\ \overline{a}_\phi$$

$$\tan\theta = z/\rho \Rightarrow z = \rho\tan\theta$$

$$dz = \rho\sec^2\theta$$

$$z = -\infty,\ \theta = \frac{-\pi}{2}$$

$$z \neq +\infty,\ \theta = \frac{\pi}{2}$$



$$\overline{E}=\frac{\rho_L}{4\pi\varepsilon_0}\int_{-\pi/2}^{+\pi/2}\frac{\rho(\rho\sec^2\theta)d\theta\ \overline{a_\phi}}{(\rho^2+\rho^2\tan^2\theta)^{3/2}}$$

$$=\frac{\rho_L}{4\pi\varepsilon_0}\int_{-\pi/2}^{+\pi/2}\frac{\rho^2\sec^2\theta\, d\theta}{\left[\rho^2(1+\tan^2\theta)\right]^{3/2}}\ \overline{a_\phi}$$

$$=\frac{\rho_L}{4\pi\varepsilon_0}\int_{-\pi/2}^{+\pi/2}\frac{\rho^2\sec^2\theta\, d\theta}{\left[\rho^3\sec^3\theta)\right]}\ \overline{a_\phi}$$

$$=\frac{\rho_L}{4\pi\varepsilon_0\rho}\int_{-\pi/2}^{+\pi/2}\cos\theta\, d\theta\ \overline{a_\phi}$$

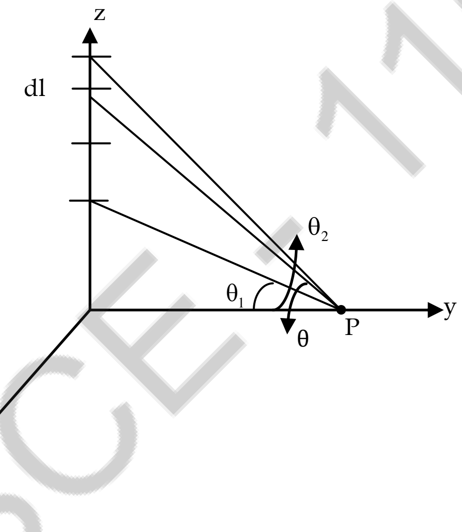
$$=\frac{\rho_L}{4\pi\varepsilon_0\rho}\left[\sin\theta\right]_{-\pi/2}^{+\pi/2}\ \overline{a_\phi}$$

$$=\frac{\rho_L}{4\pi\varepsilon_0\rho}[2]\ \overline{a_\phi}$$

$$\overline{E}=\frac{\rho_L}{2\pi\varepsilon_0\rho}\ \overline{a_\phi}$$

13. Derive the electric field intensity due to finite line charge using cylindrical co – ordinates?

Due to finite line charge:-



Let us consider the differential element dl along z – axis let the minimum point be z1 and maximum point z2.

$$E=\frac{Q}{4\pi\varepsilon_0 R^2}\ \overline{a_\phi}$$

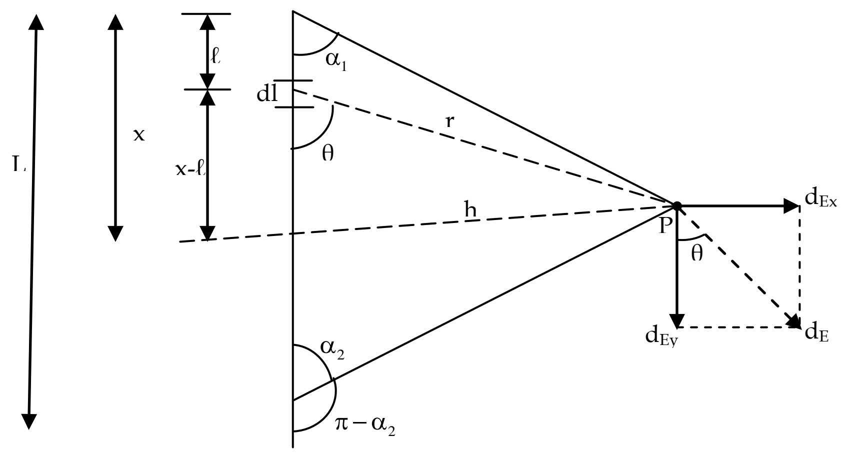
$$=\frac{\int_{z_1}^{z_2}\rho_L\, dz}{4\pi\varepsilon_0(\rho^2+z^2)}\ \frac{(\rho a_e - z a_z)}{\sqrt{\rho^2+z^2}}$$

$$=\frac{\rho_L}{4\pi\varepsilon_0}\ \frac{\int_{z_1}^{z_2}\rho_L\, dz\ a\phi}{(\rho^2+z^2)^{3/2}}$$

$$E=\frac{\rho_L}{4\pi\varepsilon_0}\int_{\theta_1}^{\theta_2}\frac{\rho^2\sec^2\theta\, d\theta}{\rho^3\sec^3\theta}\ \overline{a_\phi}$$

14. Derive the expression for electric field intensity due to finite and infinite line charge using Cartesian co – ordinates

Consider a uniformly charged line of length 'L' whose charge density ρL c/m. Consider a small element dl at a distance 'l' from one end of the charge line. Let be any point a distance 'r' from the element ρ dl.



The electric field at a point P due to the charge element ρ dl is

$$dE = \frac{\rho_e\ dl}{4\pi\varepsilon_0 r^2} \rightarrow (1)$$

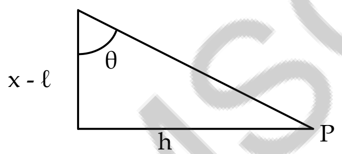
The x and y component of electric field dE is given by,

$$dE_x = dE \sin\theta \rightarrow (2)$$

$$dE_y = dE \cos\theta \rightarrow (3)$$

$$dE_x = \frac{\rho_e\ dl \sin\ \theta}{4\pi\varepsilon r^2} \rightarrow (4)$$

From fig



$$\tan\theta = \frac{h}{x-\ell}$$

$$x-\ell = \frac{h}{\tan\theta} = h\cot\theta \rightarrow (5)$$

$$-dL = -h\ \mathrm{cosec}^2\theta\ d\theta \rightarrow (6)$$

$$\sin\theta = \frac{h}{r} \Rightarrow r = \frac{h}{\sin\ \theta}$$

$$r = h\ \mathrm{cosec}\ \theta \rightarrow (7)$$

$$dE_x = \frac{\rho_l \, dl \sin\theta}{4\pi\varepsilon r^2}$$

$$= \frac{\rho_l \, h \, \text{cosec}^2\theta \sin\theta \, d\theta}{4\pi\varepsilon \, h^2 \, \text{cosec}^2\theta}$$

$$E_x = \int dE_x = \frac{\rho_l}{4\pi\varepsilon h} \int_{\alpha_1}^{\pi-\alpha_2} \sin\theta \, d\theta$$

$$= \frac{\rho_l}{4\pi\varepsilon h} \left[-\cos\theta\right]_{\alpha_1}^{\pi-\alpha_2}$$

$$= \frac{\rho_l}{4\pi\varepsilon h} \left[\cos\alpha_1 + \cos\alpha_2\right]$$

$$dE_y = \frac{\rho_\ell \, dl \cos\theta}{4\pi\varepsilon r^2} = \frac{\rho_\ell \, h \, \text{cosec}^2\theta \cos\theta \, d\theta}{4\pi\varepsilon h^2 \, \text{cosec}^2\theta}$$

$$E_y = \int_{\alpha_2}^{\pi-\alpha_1} \frac{\rho_\ell}{4\pi\varepsilon h} \cos\theta d\theta = \frac{\rho_L}{4\pi\varepsilon h}\left[\sin\theta\right]_{\alpha_1}^{\pi-\alpha_2}$$

$$E_y = \frac{\rho_L}{4\pi\varepsilon h}\left[\sin\alpha_2 - \sin\alpha_1\right]$$

Case (i):- If the point 'P' is at bisector of a line, then $\alpha_1 = \alpha_2 = \alpha$

$E_y = 0$, $\therefore$ E becomes E_x

$$E = \frac{\rho_L}{2\pi\varepsilon h}\left[\cos\alpha\right]$$

Case (ii):- If the line is infinity long, then α=0, $E_y = 0$

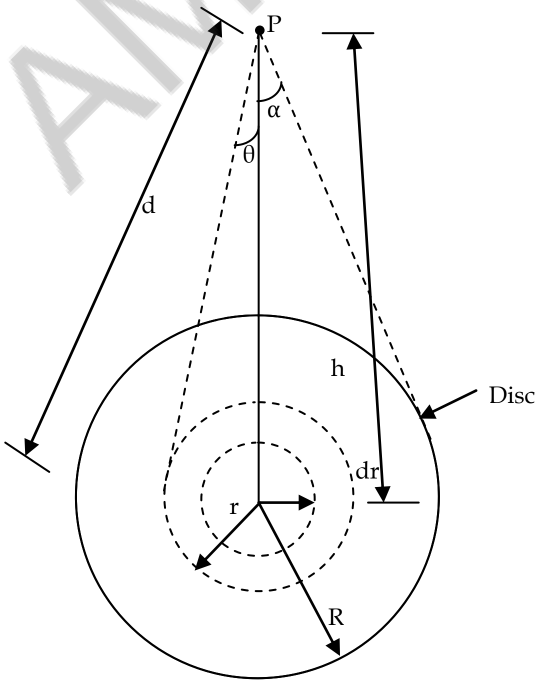
$$E = E_x \frac{\rho_L}{2\pi\varepsilon h}$$

16. Explain the electric field intensity due to a charged circular disc

Consider a circular disc of radius R charged uniformly with charge density of $\rho_s \, C/m^2$. Let be any point on the axis of disc at a distance from the centre.

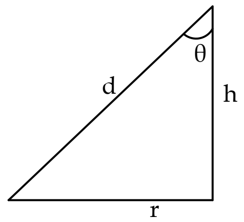
Consider an angular size of radius 'r' & of radial thickness the area of the annular ring ds = 2πr dr. The field intensity at point p due to the charged annular ring is

$$dE = \frac{\rho_s \, ds}{4\pi\varepsilon d^2}$$



Since the horizontal component of electric field intensity is zero, the vertical component is

$$dE_y = \frac{\rho_s \, ds \cos\theta}{4\pi\varepsilon \, d^2}$$



$$\because dE_y = dE \cos\theta$$
$$\sin\theta = r/d$$
$$\cos\theta = h/d$$
$$\tan\theta = r/h$$

From fig,

$$\tan\theta = r/h \Rightarrow r = h \tan\theta$$
$$r/d \quad dr = h \sec^2 d\theta$$
$$\sin\theta = 1 \Rightarrow d = \frac{r}{\sin\theta}$$

$$dE_y = \frac{\rho_s 2\pi r \, dr \cos\theta}{4\pi\varepsilon \, d^2}$$
$$= \frac{\rho_s 2\pi r \, h \sec^2\theta d\theta \cos\theta}{4\pi\varepsilon \, \frac{r^2}{\sin^2\theta}}$$
$$dE_y == \frac{\rho_s \sec\theta \sin^2\theta \, d\theta}{2\varepsilon \tan\theta} = \frac{\rho_s}{2\varepsilon} \sin\theta \, d\theta$$
$$E_y = \frac{\rho_s}{2\varepsilon}\int_0^\alpha \sin\theta \, d\theta = \frac{\rho_s}{2\varepsilon}\left[-\cos\theta\right]_0^\alpha$$
$$E = \frac{\rho_s}{2\varepsilon}(1-\cos\alpha)$$
$$E = \frac{\rho_s}{2\varepsilon}\left[1-\frac{h}{\sqrt{h^2+R^2}}\right]$$

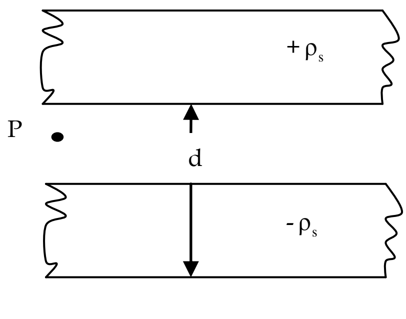
17. Explain the electric field intensity due to infinite plane sheet of charge

Consider an infinite plane sheet which is uniformly charged with density $\rho_s \; c/m^2 \; \rho_s$.

The field intensity at any point 'p' due infinite plane sheet of charged can be evaluate by applying expression of charged disc.

$$E = \frac{\rho_s}{2\varepsilon}(1-\cos\alpha) \; \alpha = 90°$$
$$E = \frac{\rho_s}{2\varepsilon}$$

18. Derive the expression for electric field intensity due to two infinitely conducting planes



Consider two infinite plane sheet with charge density $+\rho_s$ and $-\rho_s$ c / m² separated by distance d.

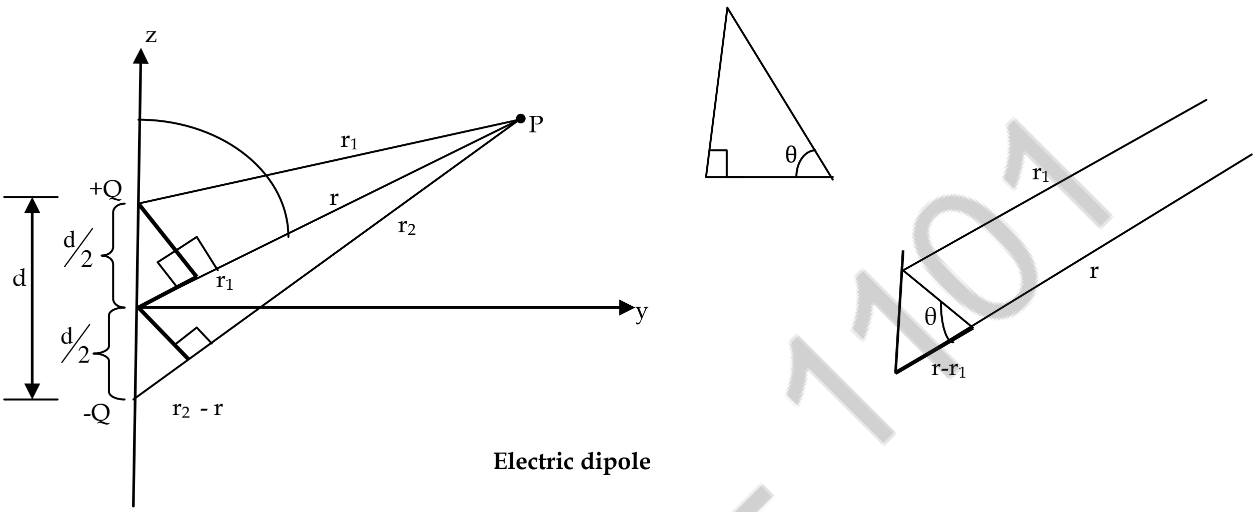
$$E = \frac{\rho_s}{2\varepsilon} + \frac{\rho_s}{2\varepsilon} = \frac{\rho_s}{\varepsilon}$$

19. Derive an expression to determine electric field intensity and at P due to an electric dipole

Electric dipole:-

An electric dipole or dipole is two equal and opposite charges separated by a very small distance.

The product of charge and spacing is called electric dipole moment.



Electric dipole

Let +Q and –Q be the two charges separated by a small distance 'd'. The product of charge Q and spacing 'd' is called the dipole moment $m = Q\,d$

Let P be any point at distance of r_1, r_2 and r from +Q and –Q and midpoint of dipole.

Potential at due to +Q, $V_1 = \frac{Q}{4\pi\varepsilon r_1}$

Potential at due to –Q, $V_2 = \frac{Q}{4\pi\varepsilon r_2}$

Resultant potential at p, $V = V_1 + V_2 = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

If the point 'P' is for away from the dipole the distance r_1 and r_2 are given by

$$r_1 = r - \frac{d}{2}\cos\theta$$
$$r_2 = r + \frac{d}{2}\cos\theta$$

Potential at P due to dipole

$$V = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{r - d/2\cos\theta} - \frac{1}{r + d/2\cos\theta}\right]$$

$$= \frac{Q}{4\pi\varepsilon}\left[\frac{d\cos\theta}{r^2 - d^2/4\cos^2\theta}\right]$$

$$V = \frac{Q}{4\pi\varepsilon}\left(\frac{d\cos\theta}{r^2}\right) \qquad \text{since } d/2 << r^2$$

$$V = \frac{Q\,d\cos\theta}{4\pi\varepsilon r^2}$$

$$\therefore m = Qd;\quad V = \frac{m\cos\theta}{4\pi\varepsilon r^2}$$

This shows that the potential is directly proportional to the dipole moment and is inversely proportional to the square of the distance between them.

Electric field produced at P due to the dipole:

As $V = -\int E.\ dr$, this relation can be used for evaluation of field at P due to +Q and –Q separately.

The electric field E has components along radial distance r and angle Q.

$$E = E_r + E_Q$$

$$E = -a_r\frac{\partial v}{\partial r} - a_Q\frac{\partial v}{r\,\partial Q}$$

$$\frac{\partial v}{\partial r} = \frac{-m\cos\theta}{2\pi\varepsilon_0 r^3}; \frac{\partial v}{\partial r} = \frac{-m\sin\theta}{4\pi\varepsilon_0 r^2}$$

$$\therefore E = \overline{a_r}\left(\frac{m\cos\theta}{2\pi\varepsilon_0 r^3}\right) + \overline{a_Q}\frac{-m\sin\theta}{4\pi\varepsilon_0 r^2}$$

$$E_r = \left(\frac{m\cos\theta}{2\pi\varepsilon_0 r^3}\right) \Rightarrow \varepsilon_0 E_r = \frac{m\cos\theta}{2\pi\varepsilon_0 r^3} = D_r$$

$$E_r = \left(\frac{m\sin\theta}{4\pi\varepsilon_0 r^3}\right) \Rightarrow \varepsilon_0 E_Q = \frac{m\sin\theta}{4\pi\varepsilon_0 r^3} = D_Q$$

The expression above clearly indicate that the field component (radial as well as angular) vary inversely as the cute of the distance

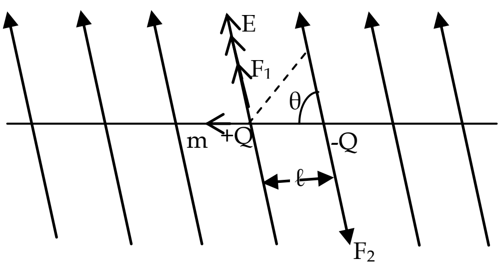
If Q = 90°, E_r = vanishes, but E_Q persists

If Q = 90°,obviously p is somewhere in alignment with dipole axis.

20. Discuss torque experiment by dipole in a uniform field?

Torque experimented by a dipole in a uniform:-

Uniform Field E



The dipole moment m = QL is a vector direct from negative to positive charge forming the dipole .

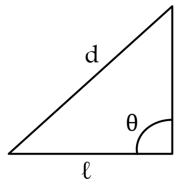
The numerator of the potential is given by m cos θ, which can be written as m . ar

$$V = \frac{m.a_r}{4\pi\varepsilon r^2}$$

There are two charges +Q and –Q placed in a uniform electric field. These charges experience a force QE but opposite in direction

These two forces form a couple whose torque is equal in magnitude to the produced of force and the aim of couple.

$$\begin{aligned} \text{Torque} &= QE\ell \sin\theta \\ &= Q\ell\ E \sin\theta \\ &= mE \sin\theta \\ T &= m \times E \end{aligned}$$



$$\sin\theta = d/L$$
$$d = L \sin\theta$$

In conclusion, although a dipole in a uniform field E does not experiences a translation force, it does experiences a torque.

9. Give electric flux density due to line, surface * volume density electric flux density D or electric displacement

$$D = \varepsilon_0\ E\ c/m^2$$

Electric flux $\Psi = \oiint_s \vec{D}.\ ds$

For line charge, $D = \int \frac{\rho_L\ dl}{4\pi R^2}\ a_R$

For surface charge, $D = \int \frac{\rho_S\ ds}{4\pi R^2}\ a_R$

For volume charge, $D = \int \frac{\rho_V\ dv}{4\pi R^2}\ a_R$

10) State & Prove Gauss law?

Gauss law:-

The electric flux passing through any closed surface is equal to the total charge enclosed by the surface.

$$\Psi = Q_{encl}$$

Proof:

Consider a small element of area ds in a plane surface having charge Q and P be a point in element. At every point of surface, the electric flux density D will have Ds.

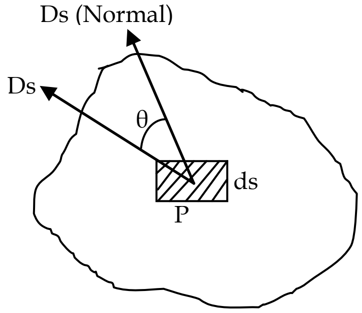


Fig . closed surface having charge

From fig, $\cos\theta = \frac{D_S(\text{normal})}{D_S}$

$$D_S(\text{normal}) = D_S \cos\theta$$

Let Ds makes an angle θ with ds the flux crossing ds is the product of normal component of Ds and ds.

$$\begin{aligned} d\phi &= D_S(\text{normal}).\, ds \\ &= D_S \cos\theta.\, ds \\ d\phi &= D_S.ds \end{aligned}$$

Total flux passing through closed surface,

$$\Psi = \int d\Psi = \oint_s D_s.\, ds$$

$$E = \frac{Q}{4\pi\varepsilon r^2} \Rightarrow D = \varepsilon E \Rightarrow D = \frac{Q}{4\pi r^2}$$

The small element of area ds on surface of sphere is

$$\begin{aligned} ds_r &= r\, d\theta \sin\theta\, d\phi \\ ds &= r^2 \sin\theta\, d\theta\, d\phi \\ \Psi &= \oint_s \frac{Q}{4\pi r^2}.\, r^2 \sin\theta\, d\theta\, d\phi \\ &= \frac{Q}{4\pi} \int_{\phi=0}^{2\pi} \int_{Q=0}^{\pi} \sin\theta\, d\theta\, d\phi \\ &= \frac{Q}{4\pi} [-\cos]_0^{\pi} [\phi]_0^{2\pi} \\ &= \frac{Q}{4\pi} [1+1][2\pi] = Q \\ \Psi &= Q \end{aligned}$$

Note:-

Total charge enclosed $Q = \int_Y \rho_v dv \rightarrow (1)$

$$Q = \oint_s D.\, ds = \int_v \rho_v dv \rightarrow (2)$$

Applying divergence to (2),

$$\oint_s D.\, ds = \int_x \nabla.\, D\, dv \rightarrow (3)$$

Comparing 2 & 3 we get

$$\nabla.D = \rho_v$$

Differential from Gauss Law.

21. Explain the application of Gauss law?

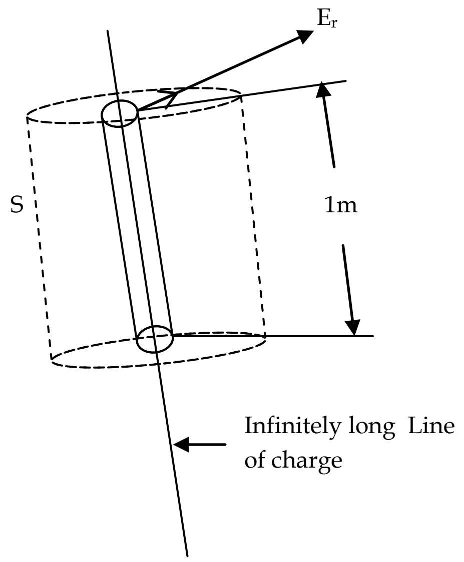
Application of Gauss law:-

Application 1: To determine the field at a distance r from an line charge of strength λ c/m.

The figure shows a theoretically infinite charged line with length alone particular signified. Imagine that a coaxial cylindrical surface surrounds the charged over a meter length this is a Gaussian surface.

The electric field at any point is radial an independent of both positions thro along and angular position around the wire.

As the electric field is in the same plate as the circular ends at top and bottom of the cylindrical, no flux passes through the end surfaces.



Applying Gauss's law for a metre length the charged line

$$\oiint E.n\ ds = 2\pi r\ Er = \lambda/\varepsilon_0$$

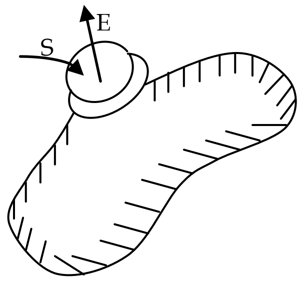
$$E = E_r = \frac{\lambda}{2\pi\varepsilon_0 r}\overline{a}_r$$

Where ar is unit vector at any point P on the cylindrical surface and is disc radially outward, being perpendicular to the axis of the charged line.

Application2:

Consider a closed pill box shaped surface S resting on the surface of a charged on the conductor be given by the surface charge density function ρ_s. Consider an elementary surface area Δs; the applying gauss law to the small pill box shaped surface S, we have the surface. Integral of the normal component of $E = E.n\ \Delta s$

Thus,

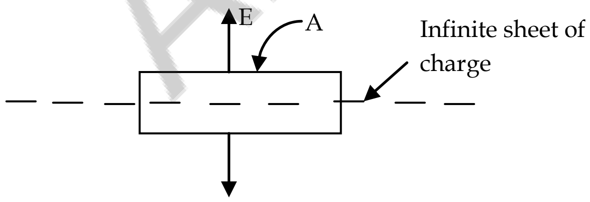


$$\varepsilon_0 E.n\,\Delta s = Q = \rho_S \Delta s$$

$$E.n\,\Delta s = \rho_S \frac{\Delta s}{\varepsilon_0}$$

$$E = \frac{\rho_S}{\varepsilon_0}.\ n$$

Application 3:



By summery it is obvious that the field can only be perpendicular to the surface of the infinite plane sheet of charge.

$$\oiint_s D.\ n\ \ ds = Q\ \ \rho_s\ A\ \ \left[\because\ D = \varepsilon_0 E\right]$$

$$\oint_s E.n\,ds = \frac{\rho_S\ A}{\varepsilon_0}$$

$$E(2A) = \frac{\rho_S\ A}{\varepsilon_0}$$

$$E = \frac{\rho_S\ A}{2\varepsilon_0}.\ n$$

Application 4:

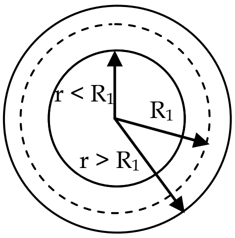
To determine the variation of field the point to point due to

(i) A single spherical shell of charge with radius R_1.

(ii) Two concentric spherical shells of charge of radii R_1 (inner) & R_2 (outer)

(iii) Spherical volume distribution of charge of radius R density L

(i) Single shell of charge



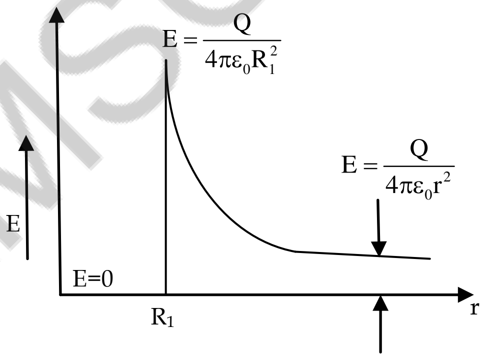
Let us suppose a total charge Q to be uniformly distributed over an imaginary shell of radius R_1 in a medium of a free space. At any radius $r < R_1$. Inside the shell of charge. Integral of $\overline{D}$ over a spherical surface.

$$\oiint_s \overline{D}.\, n\, ds = \varepsilon_0 \oiint_s E.\, n\, ds = 0$$
$$E = 0\ (\text{for } r < R_1)$$

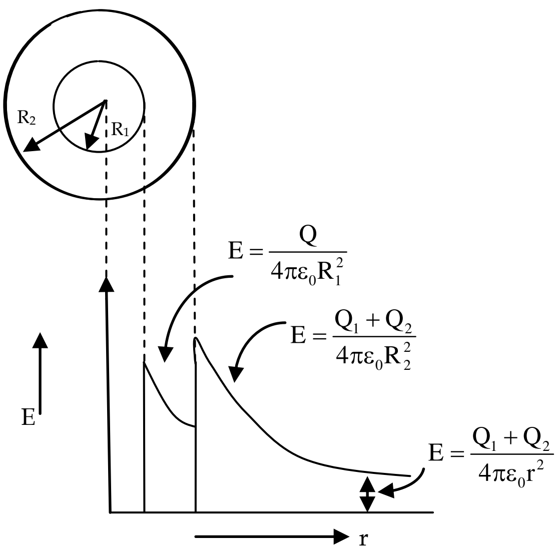
On the other hand, at any radius $r \geq R_1$, integral of $\overline{D}$ over s spherical surface is equal to the charge by itself.

Thus,

$$\varepsilon_0 \oiint E.\ n\, ds = \varepsilon_0\ E(4\pi R_1^2) = Q$$
$$E = \frac{Q}{4\pi\varepsilon_0 R_1^2}$$



(ii) Two concentric shells of charge



Consider the two spherical shell charges Q_1 & Q_2 at radius R_1 & R_2 respectively

$R < R_1$, E =0 (in the charge free region)

$$R_1 \le r \le R_2,\ E = \frac{Q}{4\pi\varepsilon_0 r^2}\overline{a_r}$$

Just outside the shells of charges Q_1 & Q_2 respective

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}\overline{a_r}(\text{i.e; at } r = R_1 + dr)$$
$$E = \frac{Q_1 + Q_2}{4\pi\varepsilon_0 R_2^2}\, a_r(\text{i.e; at } r = R_2 + dr)$$

At any point outside both the shells of charge (i.e) $r \ge R_2$

$$E = \frac{Q_1 + Q_2}{4\pi\varepsilon_0 r^2}\,\overline{a_r}$$

(iii) Spherical volume distribution of charge:

Consider any point with in the spherical volume of charge (ie) for r < R. As we have uniformly distributed by a concentric sphere of radius r < R is proportional to cube radius < R to the total volume charge

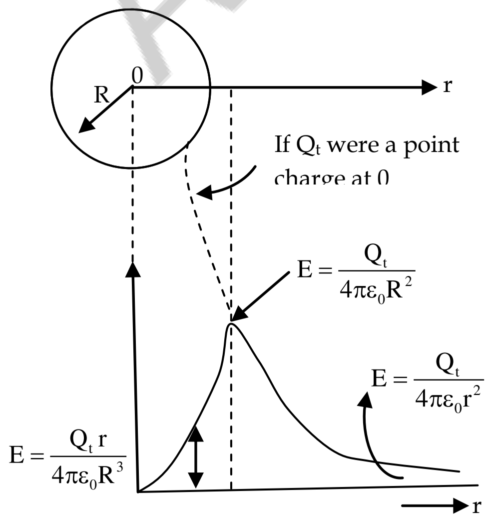
$$\frac{Q_r}{Q_t} = \left(\frac{r}{R}\right)^3$$
$$Q_r = Q_t\left(\frac{r}{R}\right)^3$$
$$E = \frac{Q_r}{4\pi\varepsilon_0 r^2}a_r(r \le R)$$
$$E = \frac{Q_t r}{4\pi\varepsilon_0 R^3}a_r\,(r \le R)$$

The above relation indicates that the field is zero at the centre of the sphere and increases uniformly to a maximum.

At $r \ge R$

$$E = \frac{Q_t}{4\pi\varepsilon_0 r^2}$$

Uniformly distributed volume charge of density ρv,



UNIT II

CONDUCTORS AND DIELECTRICS

PART-A

1. Write the poisson's and Laplace's equation.

Poisson's equation:

$$\nabla^2 v = -\frac{\ell}{E}$$
$$\nabla^2 v = 0.$$

This is the Laplace's equation.

$$\nabla^2 v = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0 \text{ [cartesian]}$$
$$\nabla^2 v = \frac{1}{e}\frac{\partial}{\partial e}\left[e\frac{\partial v}{\partial e}\right] + \frac{1}{e}\left[\frac{\partial^2 v}{\partial \phi^2}\right] + \frac{\partial^2 v}{\partial z^2} = 0$$
$$\nabla^2 v = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial v}{\partial r}\right] + \frac{1}{r^2 \sin\theta}\ \frac{\partial}{\theta}\left[\sin\theta\frac{\partial v}{\partial \theta}\right] + \frac{1}{r^2 \sin\theta^2}\ \frac{\partial^2 v}{\partial \phi^2} = 0$$

2. Define capacitance:

Capacitance between two conductor defined as the ratio of the magnitude of the total charge on eitherconductor to the potential difference between conductors.

$$C = \frac{Q}{v} \text{ farads}$$

Where,

C is capacitance in Farads

Q is charge in coulombs

V is potential difference between the conductor due to equal to opposite charges.

3. Define electric current:-

It is defined as the rate of flow of charges the direction of the current flow is opposite to the flow of charges . the unit of current is Amperes.

$$I = -\frac{dQ}{dt}$$

4. Define electric current density

It is defined as the current per unit area. It is denoted by 'J' and its unit is Ampere /m^2.

$$J = \frac{I}{A}.$$
$$I = J.A$$
$$I = \oiint_s J.\,ds$$

5. Write the equation of continuity.

$$\nabla.\,J = -\frac{\partial \ell v}{\partial t}$$

6. Write the point form of ohm's law.

$$J = \sigma E$$

This is the point form of ohm's law.

Where, $\sigma = |\rho_e \ \mu_e|$. Its unit is Mho/m

7. Define dipole moment

Dipole moment is denoted by P is defined as the product of a charge in the distance of separated by the charge

$$P = qd$$

8. Define polarization:-

Polarization is defined as the total dipole moment unit volume.

$$P = \frac{\text{Pltotal}}{\Delta v} = \frac{1}{\Delta v}\sum_{1=1}^{n\Delta r} P_i$$

9. Define inductance:

The inductance is defined as the rate of total magnetic flux linkage to the current through the coil and it is denoted by symbol

$$L = \frac{d\Lambda}{di} = \frac{Nd\phi}{di} \qquad \therefore \left[\frac{d\phi}{di} = \frac{\phi}{I}\right]$$

$$L = \frac{N\phi}{I}$$

10. Define flux linkage.

Flux is defined as the product of N – turns in the coil, and the total flux linked with the coil. It is defined by the symbol (Λ)

$$\Lambda = N\phi$$

11. Define Mutual inductance:-

Mutual inductance is defined as the flux linked is one coil due to the current following the second coil.

12. What is meant by dielectric breakdown?

When the electric field in a dielectric is sufficiently large, it begins to pull electrons completely out of the molecules & the dielectric becomes conducting.

13. Define dielectric strength of material & give its unit.

The maximum electric field intensity that a dielectric material can with and without break down is the dielectric strength of the material, unit: V/m

14. Find the capacitance of cylindrical (co – axial) capacitor shown in fig. here each dielectric occupies one has the volume with a =3cm & b = 12cm $\varepsilon_{r1} = 2.5$ & $\varepsilon_{r2} = 4$. The voltage difference is 50 v.

Solution:-

$$C_1 = \frac{\pi\varepsilon_0\varepsilon_{r1}}{\ln\left(b/a\right)} \qquad C_2 = \frac{\pi\varepsilon_0\varepsilon_{r2}}{\ln\left(b/a\right)}$$

$$C = C_1 + C_2 = \frac{\pi\varepsilon_0}{\ln\left(b/a\right)}(\varepsilon_{r1} + \varepsilon_{r2})$$

$$C = \frac{\pi \times 8.854 \times 10^{-12}}{\ln\left(12/3\right)}[2.5 + 4]$$

C = 130.6 pF/m

15. Find the resistance a copper wire of length 200 km and uniform cross section area 40mm?. Given that the conductivity of Cu is 5.8 × 10^7 S/m.

Solution:-

$$R = \frac{\ell}{\sigma A} = \frac{200\times10^3}{5.8\times10^7\times40\times10^{-6}}$$
$$R = \frac{2\times10^5}{5.8\times4\times10^8\times10^{-6}} = \frac{2\times10^5}{5.8\times4\times10^2}$$
$$= \frac{2\times10^3}{23.2} = 86.2\Omega$$

R = 86.2 Ω

16. A Cu bar of 30mm × 80mm in cross section and 2m in length has 50mv ends. Find resistance intensity.

For Cu, σ = 5.8 × 10^7 S/m.

Solution:-

$$R = \frac{\ell}{\sigma A} = \frac{2}{5.8\times10^7\times300}$$

R= 14.268× 10^{-6}

$$E = \frac{V}{d} = \frac{50\times10^{-3}}{2} = 25\times10^{-3} = 25\,\text{mV}/\text{m}$$

17. A parallel plate capacitor has an area of 0.8 mm separation of 0.1mm with a dielectric for which $\varepsilon_r = 1000$ **and a field of 10^6 v / m. Determine the capacitance and voltage across the two plates.**

Solution:-

Given A = 0.8m^2 d= 0.1 mm

$\varepsilon_r = 1000$ E = 10^6 V/ m

$$C = \frac{\varepsilon A}{d} = \frac{\varepsilon_0\varepsilon_r A}{d} = \frac{8.854\times10^{-12}\times1000\times0.8}{0.1\times10^{-3}}$$
$$= \frac{8.854\times10^{-12}\times10^3\times0.8\times10^3}{0.1}$$
$$= 8.854\times8\times10^{-6}$$
$$C = 70.732\,\mu F$$
$$V = Ed = 10^6\times0.1\times10^{-3} = 100v$$

18. A capacitor consists of two similar square aluminium plates each of 10cm × 10 cm mounted parallel and opposite to each other. What is the capacitance when the distance between them is 1cm and dielectric is air.

Solution:-

Given A = 10 × 10^{-2} × 10 × 10^{-2}=10^{-2}m^2

D = 1cm = 1 × 10^{-2}m

$$C = \frac{\varepsilon_0\varepsilon_r A}{d} = \frac{8.854\times10^{-2}\times10^{-2}}{1\times10^{-2}}$$

C = 8.854 Pf

19. Determine capacitance of area 1sqcm, separated by 1cm placed in a liquid whose dielectric constant is 6 & $\varepsilon_0 = 8.854\times10^{-2}$

Solution:-

A = 1cm^2 = (1 × 10^{-2}) = 1 × 10^{-4}

d = 1cm = 1 × 10^{-2}

$$C = \frac{\varepsilon_0\varepsilon_r A}{d} = \frac{8.854\times10^{-2}\times6\times10^{-4}}{1\times10^{-2}}$$

C = 0.5312 pF

20. If C = 40 nF, d = 0.1mm & A = 0.15m^2. Determine the relative permittivity of dielectric material used in a parallel plate capacitor?

Solution:-

$$C = \frac{\varepsilon_0\varepsilon_r A}{d}$$

$$40\times10^{-9} = \frac{8.8854\times10^{-2}\times\varepsilon_r\times0.15}{0.1\times10^{-3}}$$

$$\varepsilon_r = \frac{40\times10^{-9}\times0.1\times10^{-3}}{8.854\times10^{-12}\times0.15}$$

$$\varepsilon_r = 3.01$$

21. Find the capacitance / unit length between two plate cylindrical conductor in air of radius 1.5 cm & coil a centre separation of 85 cm.

Solution:-

$$C' = \frac{C}{\ell} = \frac{\pi\varepsilon_0}{\ln\left(d/2\right)} = \frac{\pi\times8.854\times10^{-12}}{\ln\left(8.5/11.5\right)}$$

C' = 6.89 pF/m

22. Calculate the capacitance / km length of two identical parallel wires of diameter 1 cm each and plate 1 m apart. Also find the potential difference between them, which will make the 'E' at the conductor surface just 4×10^6 v/m.

Solution:-

Radius a = 0.5 × 10-2 m, d = 1m, E = 4×10^6

$$C' = \frac{C}{\ell} = \frac{\pi\varepsilon_0}{\ln\left(d/2\right)} = \frac{\pi\times8.854\times10^{-12}}{\ln\left(\frac{1}{0.5\times10^{-2}}\right)}$$

C' = 5.25pF

$$E = V/d \Rightarrow V = E.d$$
$$= 4\times10^6\times1$$

V=4 × =10^6 Volts

23. The conductors of two wire transmission line of length 4 km are spaced 45 cm between centre. If each conductor has a diameter of 1.5 cm, then calculate capacitance of the line.

Solution:-

$$C = \frac{\pi\varepsilon_0\ell}{\ln\left(\frac{d}{a}\right)} = \frac{\pi\times 8.854\times 10^{-12}\times 4\times 10^{3}}{\ln\left(\frac{0.45}{0.75\times 10^{-2}}\right)}$$

C = 27.17 nF

24. Consider that two copper wires of 1.299 mm diameter are parallel with separation d between the axes. Determine 'd', so that the capacitance in air is 30 pF/nF

Solution:-

$$C' = \frac{C}{\ell} = \frac{\pi\varepsilon_0}{\ln\left(\frac{d}{a}\right)}$$

$$30\times 10^{-2} = \frac{\pi\times 8.854\times 10^{-12}}{\ln\left(\frac{d}{a}\right)}$$

$$\ln\left(\frac{d}{a}\right) = \frac{\pi\times 8.854}{30} = 0.927$$

$$\frac{d}{a} = e^{0.927} = 2.53$$

$$d = 2.53a = 2.53\times\frac{1.29\times 10^{-3}}{2}$$

d = 1.63 mm

25. Calculate the capacitance of the co –axial cable with the radius of inner conductor a 10 mm and outer conductor b = 10 mm and has $\varepsilon_r = 3.5$. The inner conductor is at potential 1 kv and the outer shield is grounded. The cable is 8 km long.

Solution:-

Given a = 10mm, b = 15mm $\varepsilon_r = 3.5$ $\ell = 8\text{km}$

$$C = \frac{2\pi\ell}{\ln\left(\frac{b}{a}\right)} = \frac{2\pi\times 3.5\times 8.854\times 10^{-12}\times 8\times 10^{3}}{\ln\left(\frac{15}{10}\right)}$$

C = 3.84 µF

26. Find the capacitance / unit length of a co axial conductor with outer radiys 4 mm and inner radius 0.5 mm if $\varepsilon_r = 2$

Solution:-

$$C' = \frac{C}{\ell} = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{b}{a}\right)} = \frac{2\pi\times 8.854\times 10^{-12}\times 2}{\ln\left(\frac{4}{0.5}\right)}$$

C' = 139. 116 pF/km

27. The radius of inner and outer spheres are 10 cm & 20cm respectively. The space between the two spheres is field with $\varepsilon_r = 3$. Find the capacitance.

Solution:-

$$C = \frac{4\pi\varepsilon}{\frac{1}{a}-\frac{1}{b}} = \frac{4\pi\varepsilon_0\varepsilon_r}{\frac{1}{0.1}-\frac{1}{0.2}} = \frac{4\pi\times 8.854\times 10^{-12}\times 3}{5}$$

C = 66.76 pF

28. Two capacitance $10\mu F \& 25\mu F$ are connected in series & parallel. Find the equivalent values of capacitance.

Solution:-

$$C_1 = 10\mu F,\ C_2 = 25\mu F$$

(i) In series:-

$$C_{eq} = \frac{C_1C_2}{C_1 + C_2} = \frac{25\times 10\times 10^{-12}}{35\times 10^{-6}}$$

C_{eq} = 7.142 μF

(ii) In parallel:-

$C_{eq} = C_1 + C_2 =$ **35μF**

29. The radii of inner and outer sphere are 10 cm & 20cm respectively. The space between the two sphere is field with insulating material of ε_0. Find the capacitance formed by the two conductor sphere.

Solution:-

$$C = \frac{4\pi\varepsilon_0\varepsilon_r}{\frac{1}{a} - \frac{1}{b}}$$
$$= \frac{4\pi\times\frac{1}{36\pi\times 10^9}\times 3}{\frac{1}{0.1} - \frac{1}{0.2}}$$
$$= \frac{\left(0.33\times 10^{-9}\right)3}{5}$$
$$= \frac{0.99\times 10^{-9}}{5}$$

C = 66.76 pF

30. The radii of two sphere fifer by 4 cm with air dielectric & the capacitance of the spherical capacitor is $\frac{160}{3}$ pF. If the outer sphere is grounded, determine the radii. The capacitance of spherical capacitor is

Solution:-

$$C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$$
$$b - a = 4\times 10^{-2}$$
$$C = 4\pi\times 8.854\times 10^{-12}\left(\frac{ab}{4\times 10^{-2}}\right)$$
$$\frac{160}{3}\times 10^{-12} = 4\pi\times 8.854\times 10^{-12}\left(\frac{ab}{4\times 10^{-12}}\right)$$
$$ab = 0.019 m^2$$

Solving we get,

a = 0.12 m & b = 0.16 m

PART- B

1) Explain the significance of Poisson's and Laplace equation:

Poisson's Laplace equation:

Gauss law states that the surface integral of the normal component of electric flux density vector over a closed surface is equal to the charge enclosed by the closed surface.

$$\oint D.\,n.ds = Q = \iiint \rho\, dv \;\rightarrow (1)$$

As per divergence theorem,

$$\oint D.\,n\, ds = \iiint_V (\nabla.V)\, dv \;\rightarrow (2)$$

Equating (1) & (2), we get

$$\iiint (\nabla.D) dv = \iiint \rho\, dv$$

$$\nabla.D = \rho \;\rightarrow (3)$$

This is point form of Gauss law,

Sub D = εE in eqn (3), we get

$$\nabla.\varepsilon E = \rho \Rightarrow \nabla.E = \frac{\rho}{\varepsilon} \;\rightarrow (4)$$

Relating E &V, as $E = -\nabla V \;\rightarrow (5)$

Sub (5) in (4), we get

$$\nabla^2 V = \frac{-\rho}{\varepsilon}$$

This is known as Poisson's equation.

In a non- conducting region, charge density is equal to zero $(\rho = 0)$

$$\nabla^2 V = 0$$

This is known as Laplace's equation

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \qquad \text{(cartesian)}$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right) + \frac{\partial^2 V}{\partial z^2} = 0 \qquad \text{(cylindrical)}$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0 \qquad \text{(spherical)}$$

2) Define the term capacitance? Explain it.

Capacitance:-

Capacitance between two conductors is defined as the ratio of the magnitude of the total charge on either conductor to the potential difference between conductors.

$$C = \frac{Q}{V} \text{ Farads}$$

C = Capacitance in farads

Q = Charge in Coulomb

V = Potential difference between conductors due to equal & opposite charges on them magnitude Q

When the capacitance of a single conductor is referred to, it is assumed that the other conductor is a spherical shell of infinite large radius.

Consider two conductors of arbitrary shape (1) & (2) as shown in below figure

(1) If initially both the conductors are uncharged & if a charge is removed from (2) to (1), the conductor (2) will be left with – Q

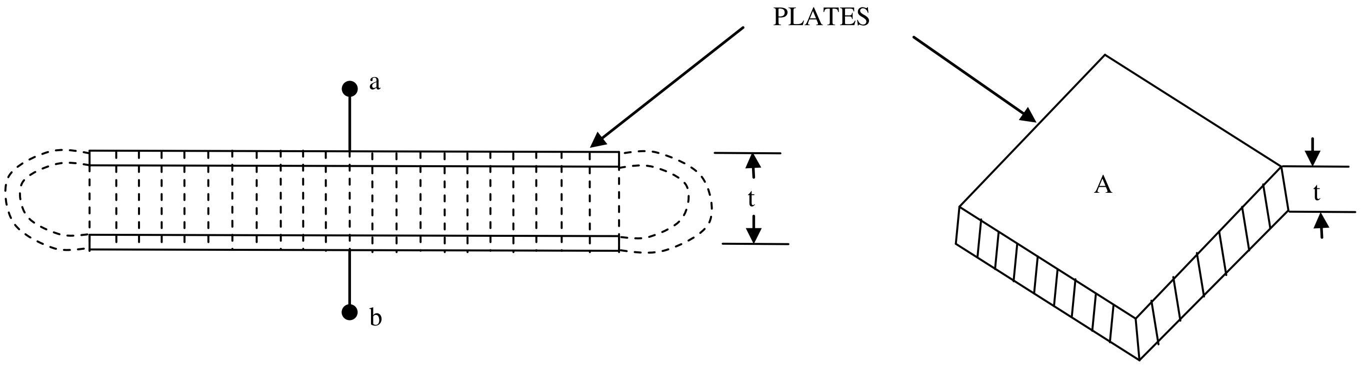
(2) Work is done in moving a charge from (2) to (1) resulting in a potential difference developed between two conductors. There will be an electronic field around them.

(3) Conversely, if a potential difference of V volts is applied. Then a charge of +Q & -Q is developed along the conductor. So, there exist a relationship between Q & V, and the ratio is constant.

If acharge of 1 coulomb is associated with a potential difference of 1 volt, the capacitance between the two conductors is said to be one farad.

3) Derive the expression of capacitance for various geometries :

(1) Parallel plate capacitor:-



A typical parallel plate capacitor which consists of a pair of flat parallel plates with surface area A separated by distance ‘t’ and through a dielectric of permittivity $\varepsilon = \varepsilon_0 \varepsilon_r$.

The capacitor may be charged by connecting the terminals a and b to a source of potential difference. Let us assume that there is uniform charge density over a plate surface, $\rho s C/m^2$ and also across the dielectric.

$$\oiint D.\, ds = Q$$
$$D.A = Q$$
$$D = Q/A = \rho_s$$

So that the filed intensity is

$$E = \frac{D}{\varepsilon} = \frac{\rho_s}{\varepsilon_0 \varepsilon_r}$$

Potential difference between the plates is given by the integral of the field E over the thickness t. As the field is uniform, we can write

$$V = Et = \frac{\rho_s t}{\varepsilon_0 \varepsilon_r}$$

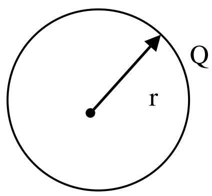
Capacitance may be expressed as,

$$C = \frac{Q}{V} = \frac{\rho_s\, A}{\frac{\rho_s t}{\varepsilon_0 \varepsilon_r}} = \frac{\varepsilon_0 \varepsilon_r A}{t}$$

Capacitance of an isolated sphere

In the case of an isolated conductor, the other conductor forming part of the capacitor is a spherical of infinite radius. Let it be radius r_1 the potential of an isolated sphere is the work done in moving a positive test charge from infinity to the sphere consequently, the absolute potential is given by

$$V = \frac{Q}{4\pi\varepsilon_0 r_1} \quad \text{[For a free space medium } \varepsilon_r = 1 \text{]}$$



So the capacitance is given by,

$$C = \frac{Q}{V} = 4\pi\varepsilon_0 r_1$$

Capacitance between two concentric spherical shells:-

A spherical capacitor is composed of two concentric, spherical, conducting shells separated through a dielectric medium; say free space in the case. Let ‘a’ and ‘b’ be the radii of the inner and outer shells respectively.

If a charge ‘Q’ is distributed uniformly over the outer surface of inner shell of radius ‘a’ then there will be equal and opposite charges induced on the outer shell of radius ‘b’.

The filed at any point between the shells is given by

$$E_r = \frac{Q}{4\pi\varepsilon_0 r^2} \ (a \le r \le b)$$

Capacitance of co- axial cable with two dielectrics

Let us consider a cable with two dielectric with permittivities $\varepsilon_1 \ \& \varepsilon_2$. If E_1 is the field intensity at any radial distance r in the dielectric (1) and E_2 that in the dielectric (2).

$$E_1 = \frac{\rho_L}{2\pi\varepsilon_1 r} \ (r_1 \le r \le r_2)$$

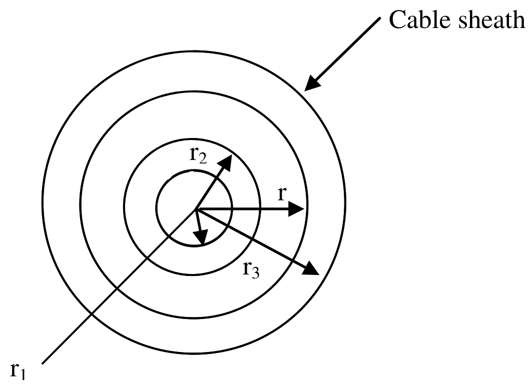
$$E_2 = \frac{\rho_L}{2\pi\varepsilon_2 r} \ (r_2 \le r \le r_3)$$

$$V_1 = -\int_{r_2}^{r_1} E_1 \, dr; \quad V_2 = -\int_{r_2}^{r_1} E_2 \, dr$$

$$V_1 = \frac{\rho_L}{2\pi\varepsilon_0\varepsilon_{r1}} \ln\frac{r_2}{r_1} \quad V_1 = \frac{\rho_L}{2\pi\varepsilon_0\varepsilon_{r2}} \ln\frac{r_3}{r_2}$$

$$V = V_1 + V_2$$

$$V = \frac{\rho_L}{2\pi\varepsilon_0}\left[\frac{1}{\varepsilon_{r1}}\ln\frac{r_2}{r_1} + \frac{1}{\varepsilon_{r2}}\ln\frac{r_3}{r_2}\right]$$



The capacitance / m length is given by

$$C = \frac{\rho_L}{V} = \frac{2\pi\varepsilon_0\varepsilon_{r1}\varepsilon_{r2}}{2.303\left[\varepsilon_{r2}\log_{10}\frac{r_2}{r_1} + \varepsilon_{r1}\log_{10}\frac{r_3}{r_2}\right]} \ p/m$$

$$C = \frac{0.0241\varepsilon_{r1}\varepsilon_{r2}}{\varepsilon_{r2}\log_{10}\frac{r_2}{r_1} + \varepsilon_{r1}\log_{10}\frac{r_3}{r_2}} \ \mu F/km$$

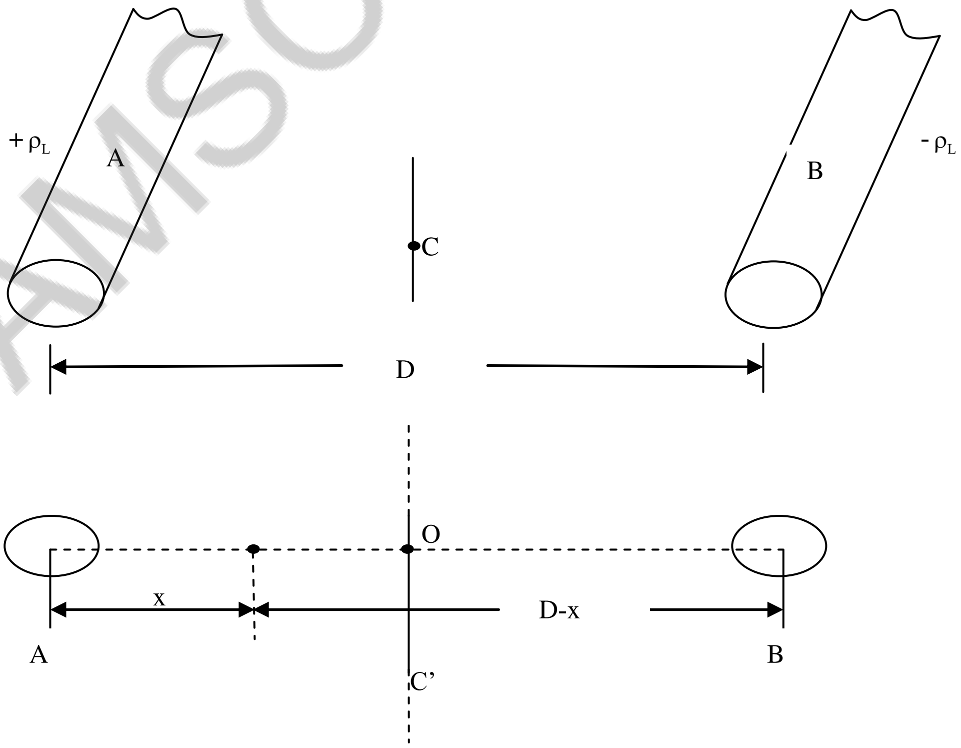
Capacitance between two parallel wires:-

Assume that $+\rho_L$ and $-\rho_L$ are the charge in c/m of the wires A and B, spaced D me ties apart and radius of each wire is r metre, remembering that D >>r.

In order to determine the capacitance between A and B, we need to find the potential difference

$$V_{BA} = -\int_B^A E_x . d_x$$

Let x = r, D –x = -r.



4) Derive the expression of energy and energy density?

Energy in a capacitance:-

Potential is defined as the work done/ unit charge. If the capacitor is connected to a source of potential the capacitor acquires charge. It involves the work to charge a capacitor.

Potential may be expressed as the infinitesimal work per infinitesimal charge.

$$V = \frac{dW}{dQ}$$
$$dW = VdQ$$

If ‘Q’ is the charge corresponding to V,

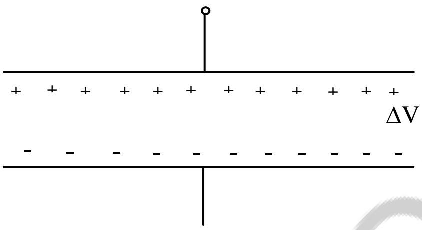
$$V = \frac{Q}{C}$$
$$dW = \frac{1}{C}QdQ$$

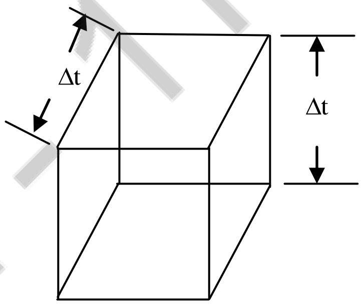
If the capacitor is initially uncharged and the process of charging continued until a charge Q is reached, the total work done is

$$W = \frac{1}{c}\int_0^q Q\,dQ = \frac{Q^2}{2C} \quad \text{Where } Q = CV$$

$$W = \frac{Q^2}{2C} \text{ or } \frac{1}{2}CV^2 \text{or } \frac{1}{2}QV \text{ Joules}$$

Energy density:-





When a capacitor is charged to a V between the plates, the energy stored is given by

$$W = \frac{1}{2}CV^2$$

The potential difference between the parallel faces of the volume element is

$$\Delta V = E(\Delta t)$$
$$\Delta W = \frac{1}{2}\Delta C(\Delta V)^2$$
$$= \frac{1}{2}\varepsilon(\Delta t)E^2(\Delta t)^2$$
$$\Delta W = \frac{1}{2}\varepsilon E^2(\Delta t)^3$$
$$\Delta W = \frac{1}{2}\varepsilon E^2(\Delta \vartheta)$$
$$\omega = \frac{\Delta W}{\Delta \vartheta} = \frac{1}{2}\varepsilon E^2$$

Energy $W = \frac{1}{2}CV^2 / \frac{1}{2}QV / \frac{1}{2}\frac{Q^2}{C}$ Joules

Energy density $\omega = \frac{1}{2}\varepsilon E^2 = \frac{1}{2}DE = \frac{1}{2}\frac{D^2}{\varepsilon}$ Joules

5) Explain current, current density and equation of continuity?

Electric current:-

It is defined as the rate of flow of charges. The directions of the current flow is opposite to the flow of charges. The unit of current is Amperes

$$I = \frac{-dQ}{dt} \rightarrow (1)$$

Let us consider a charge Q in a volume V. Let ρ_L be the volume charge density given by

$$Q = \oiiint_V \rho_v \, dv \rightarrow (2)$$

Sub (2) in (1), we get

$$I = \frac{-d}{dt} \oiiint_v \rho_v \, dv \rightarrow (3)$$

Electric current density:-

It is defined as the current per unit area. It is denoted by 'J' and its unit is Ampere/ m^2.

$$J = \frac{I}{A} \Rightarrow I = J.A = \oiint_s J.ds \rightarrow (4)$$

Equation of continuity or continuity equation of current:-

Let us consider a closed surface S, the current through the closed surface is I, due to outward flow of positive charges.

$$I = \frac{-dQ}{dt}$$

$$\oiint_s J.ds = \frac{-d}{dt} \oiiint_v \rho_v \, dv \rightarrow (5)$$

By applying Divergence theorem,

$$\oiint_s J.ds = \iiint_v (\nabla.J) dV \rightarrow (6)$$

Equating (5) & (6), we get

$$\iiint_v (\nabla.J) dV = \iiint_v \frac{-\partial \rho_v}{\partial t} . dV$$

$$\nabla.J = \frac{-\partial \rho_v}{\partial t}$$

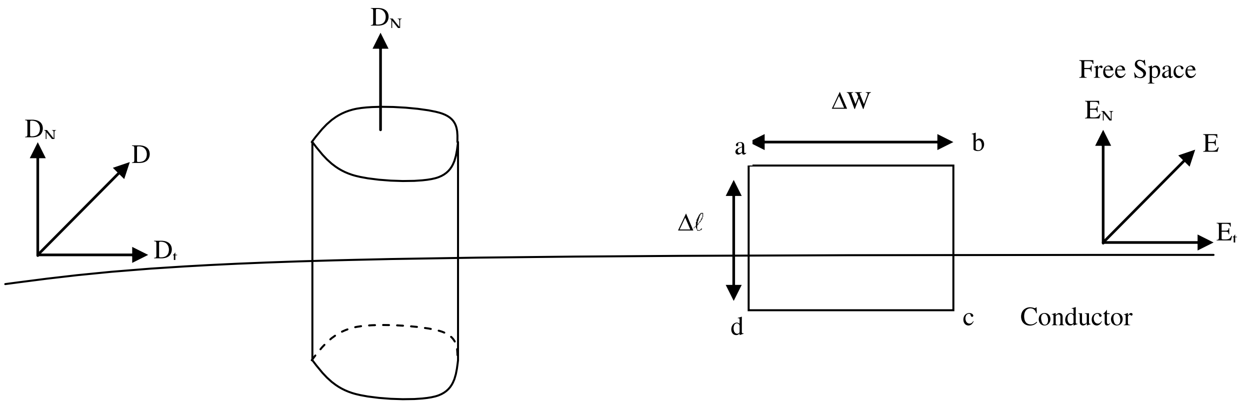
This follows principle of conversation of charges

6) Explain in detail about the boundary conditions of electric field

Boundary conditions of electric field:-

- **Conductor – Free space**
- **Conductor – Dielectric**
- **Dielectric – Dielectric**

Conductor – Free space:



We know that for a conservative field,

$$\int E.\ dl = 0$$
$$\int_{ab} E.dl + \int_{bc} E.dl + \int_{da} E.dl = 0$$
$$\int E_t\ \Delta W + E_N.\frac{\Delta \ell}{2} - E_N.\frac{\Delta L}{2} = 0$$
$$E_t = 0$$
$$\therefore \varepsilon Dt = 0$$

Hence Dt=0

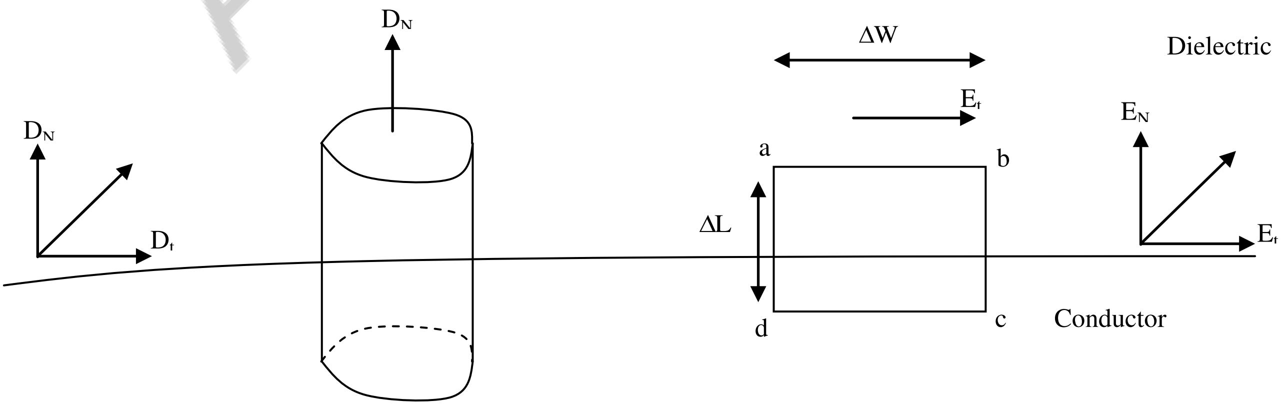
As per gauss's law,

$$\oiint_s D.N\ ds = Q$$
$$D_N.\Delta s = \rho_s\ \Delta s$$
$$D_N = \rho_s$$
$$\varepsilon E_N = \rho_s$$
$$E_N = \frac{\rho_s}{\varepsilon_0 \varepsilon_r}$$
$$E_N = \frac{\rho_s}{\varepsilon_0}$$

1). $E_t = 0;\ D_t = 0$

2). $E_N = \frac{\rho_s}{\varepsilon_0}; D_N = \rho_S$

2. Conductor – Dielectric:-



$$\oint_S D_N.ds = Q$$

$$D_N \Delta s = \rho_s \Delta s$$

$$\varepsilon E_N = \rho_s$$

$$E_N = \frac{\rho}{\varepsilon_0 \varepsilon_r}$$

$$D_N = \rho_s$$

$$E_N = \frac{\rho_s}{\varepsilon_0 \varepsilon_r}$$

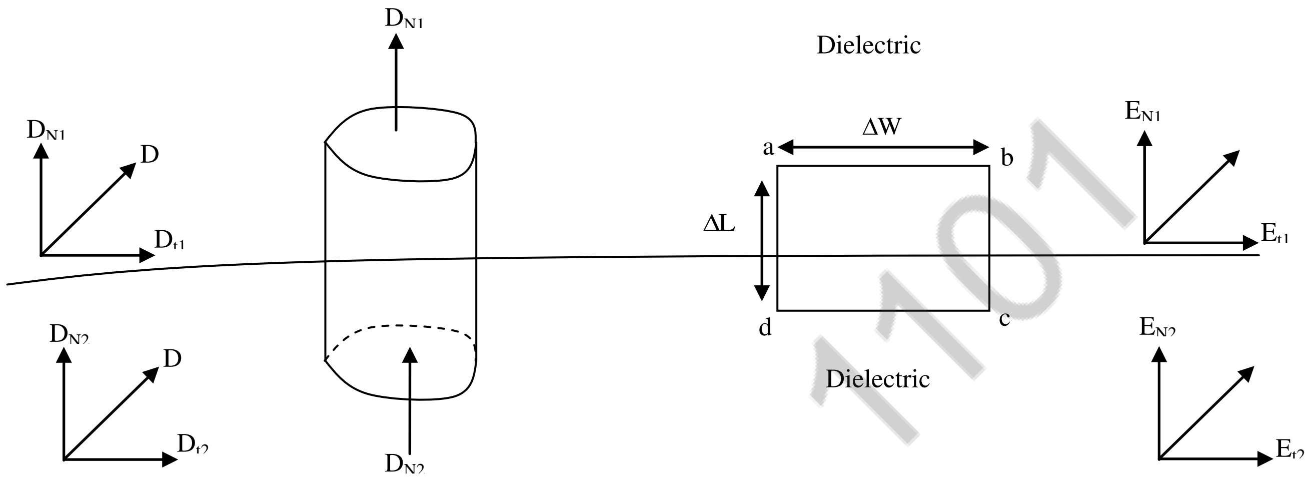
$$\int_{ab} E.dl + \int_{bc} E.dl + \int_{cd} E.dl + \int_{da} E.dl = 0$$

$$\int E_t.\Delta W + \int E_N.\frac{\Delta L}{2} - \int E_N.\frac{\Delta L}{2} = 0$$

$$E_t = 0$$

$$D_t = 0$$

3. Dielectric – Dielectric:-



$$\oint_s (D_{N1} - D_{N2}).n.ds = Q = \rho_s \ \Delta s$$

$$(D_{N1} - D_{N2})\Delta s = \rho_s \ \Delta s$$

$$D_{N1} - D_{N2} = \rho_s$$

$$\varepsilon_1 E_{N1} - \varepsilon_2 E_{N2} = \rho_s$$

Assume $\Delta L \to 0$

$$E_{t1}\,\Delta W - E_{t2}\,\Delta W = 0$$

$$E_{t1} = E_{t2}$$

$$\frac{D_{t1}}{\varepsilon_1} = \frac{D_{t2}}{\varepsilon_2}$$

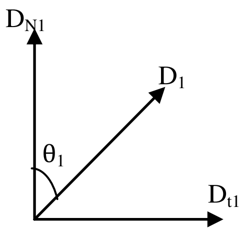
$$\frac{D_{t1}}{D_{t2}} = \frac{\varepsilon_1}{\varepsilon_2}$$

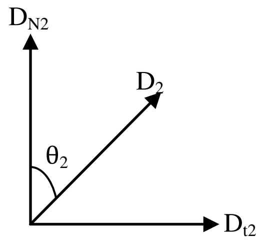
For a perfect Dielectric, $\rho_s = 0$

$$D_{N1} = D_{N2}$$

$$\varepsilon_1 E_{N1} = \varepsilon_2 E_{N2}$$

$$\frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_2}{\varepsilon_1}$$





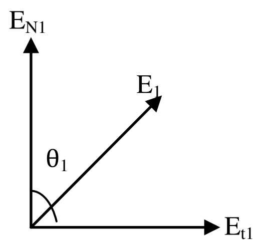
$$\cos\theta_1 = \frac{D_{N1}}{D_1}$$

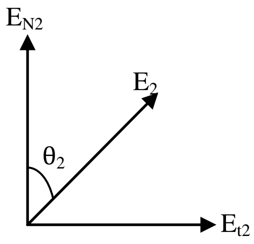
$$D_{N1} = D_1 \cos\theta_1$$

$$\cos\theta_2 = \frac{D_{N2}}{D_2}$$

$$D_{N2} = D_2 \ \cos\theta_2$$

$$\boxed{D_1 \cos\theta_1 = D_2 \cos\theta_2} \rightarrow (1)$$





$$\sin\theta_1 = \frac{E_{t1}}{E_1} \Rightarrow E_{t1} = E_1 \sin\theta_1$$

$$\sin\theta_2 = \frac{E_{t2}}{E_2} \Rightarrow E_{t2} = E_2 \sin\theta_2$$

$$\boxed{E_1 \sin\theta_1 = E_2 \sin\theta_2} \rightarrow (2)$$

(2) ÷ (1)

$$\frac{E_1 \sin\theta_1}{\varepsilon_1 E_1 \cos\theta_1} = \frac{E_2 \sin\theta_2}{\varepsilon_2 E_2 \cos\theta_2}$$

$$\frac{\tan\theta_1}{\varepsilon_1} = \frac{\tan\theta_2}{\varepsilon_2}$$

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2}$$

Summary of boundary condition:

Conductor – Free space:-

$D_t = 0;\ E_t = 0;\ D_N = \rho_s;$

The tangential component of electric field & electric flux density is equal zero.

The normal component of the electric flux is equal to the surface charge density.

The normal component of electric field is ratio of the surface charge density to the absolute permittivity.

Conductor – Dielectric:-

$D_t = E_t = 0; D_N = \rho_s;$

The tangential component of electric field and electric flux density is equal to the zero.

The normal component of electric flux density equal to the surface density.

The normal component of the electric field is equal to the ratio of surface electric density to permittivity.

Dielectric – Dielectric

$$E_{t1} = E_{t2} \qquad D_{N1} = D_{N2}$$

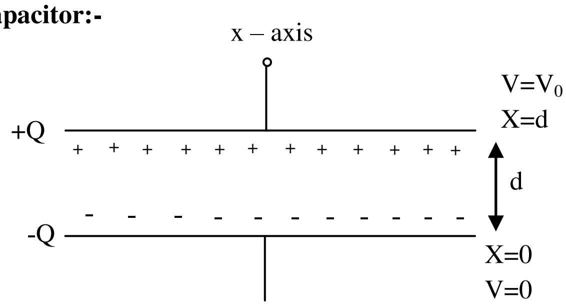
$$\frac{D_{t1}}{D_{t2}} = \frac{\varepsilon_1}{\varepsilon_2} \qquad \frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_1}{\varepsilon_2}$$

$$\frac{\tan_{\theta 1}}{\tan_{\theta 2}} = \frac{\varepsilon_1}{\varepsilon_2}$$

7) Discuss about the capacitor of various geometries using Laplace equation

Capacitance of various Geometries using Laplace equation:-

1) Due to a parallel plate capacitor:-



Let us consider two parallel plates placed along the x- axis

We know that, Laplace equation for Cartesian co – ordinates is

$$\nabla^2 V = 0$$
$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

Since the plates are along x- axis, variation of potential in Y and Z direction is equal to zero.

$$\frac{\partial^2 V}{\partial x^2} = 0$$
$$\int \frac{\partial^2 V}{\partial x^2} = \int 0$$
$$\frac{\partial V}{\partial x} = A$$
$$\int \frac{dV}{dx} = A$$
$$\int dV = \int A\, dx$$

$$V = Ax + B \qquad \rightarrow (1)$$

Boundary conditions are

$$x = 0, v = 0$$
$$x = d,\ v = v_0$$

Applying boundary condition in Equation (1), we get

$$B = 0$$
$$V_0 = A(d)$$
$$A = \frac{V_0}{d}$$

$$V = \frac{V_0}{d} x + 0 \qquad \rightarrow (2)$$

(a) Calculate E from $E = -\nabla V$

$$E = \frac{-\partial V}{\partial x} a_x$$
$$E = \frac{-\partial}{\partial x}\left(\frac{V_0 x}{d}\right) a_x$$

$$E = \frac{-V_0}{d} a_x \qquad \rightarrow (3)$$

(b) Calculate $D = +\varepsilon E$

$$D = \frac{-\varepsilon V_0}{d} a_x$$

Sub Equation (3) in $D = \varepsilon E$

(c) By gauss law,

$$\oiint D.n\,ds = Q$$
$$Q = \int D\,ds = \int \rho_s\,ds = \frac{-\varepsilon V_0}{d}$$
$$Q = \frac{-\varepsilon V_0 A}{d}$$
$$C = \frac{|Q|}{V_0} = \frac{\varepsilon A}{d}\ \text{Farad}$$

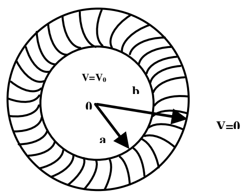
8) Explain the capacitance of co – axial capacitance using Laplace equations

2. Due to co – axial capacitor:-

Let us consider a co – axial cylindrical capacitor with inner radius ‘a’ & outer radius ‘b’.

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial V^2}{\partial\phi^2} + \frac{\partial V^2}{\partial z^2} = 0$$

Since potential varies with respect to ρ, variation with respect to $\phi \,\&\, z = 0$. The above equation reduces to



$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) = 0$$

Multiply by $\rho \int_\rho$ the above equation,

$$\rho\frac{\partial V}{\partial\rho} = A \Rightarrow \frac{\partial V}{\partial\rho} = A/\rho$$

$$V = A\ln\rho + B \qquad \rightarrow (1)$$

When $\rho = a, V = V_0; \rho = b, V = 0;$

$$0 = A\ln b + B \qquad V_0 = A\ln a - A\ln b$$
$$\boxed{B = -A\ln b} \qquad V_0 = A\ln\left(a/b\right)$$
$$B = \frac{-V_0}{\ln\left(a/b\right)}\ln(b) \qquad A = \frac{V_0}{\ln\left(a/b\right)}$$

Sub the values of A & B in eqn (1)

$$V = \frac{V_0}{\ln\left(a/b\right)}\ln\rho - \frac{V_0}{\ln\left(a/b\right)}\ln(b)$$
$$\boxed{V = \frac{V_0}{\ln\left(a/b\right)}\ln\left(\rho/b\right)}$$

$$E = -\nabla V$$
$$= -\left(\frac{\partial V}{\partial \rho}\ a_\rho\right)$$
$$= -\left[\frac{\partial V}{\partial \rho}\left\{\frac{V_0 \ln\left(\rho/b\right)}{\ln\left(a/b\right)}\right\}\right] a_\rho$$
$$= -\frac{V_0}{\ln\left(a/b\right)} \times \frac{1}{\rho/b} \times \frac{1}{b}$$
$$= \frac{V_0}{\ln\left(a/b\right)^{-1}} \times \frac{1}{\rho}$$
$$\boxed{E = \frac{V_0}{\rho \ln\left(b/a\right)} a_\rho}$$

$$D = \varepsilon E$$
$$D = \frac{\varepsilon V_0}{\rho \ln\left(b/a\right)}\ \overline{a_\rho}$$
$$Q = \int \rho_s\ ds$$
$$Q = \int \frac{\varepsilon V_0}{\rho \ln\left(b/a\right)}.\ ds$$

$$Q = \frac{\varepsilon V_0}{\rho \ln\left(b/a\right)}\ 2\pi a \ell \Bigg]_{\rho = a}$$

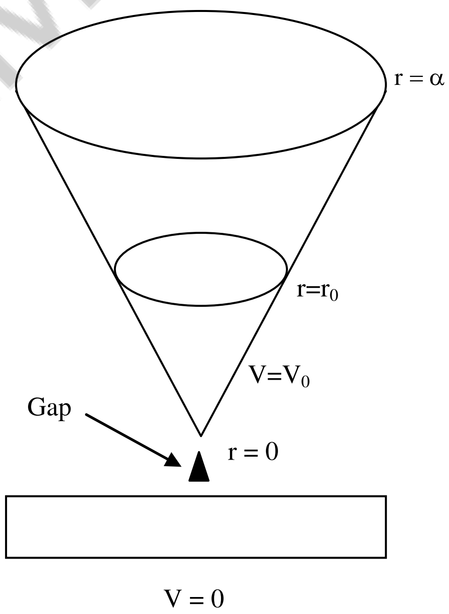
$$C = \frac{Q}{V_0} = \frac{2\pi \varepsilon_0 \varepsilon_r \ell}{\ln\left(b/a\right)}$$

$$Q = \frac{\varepsilon V_0}{a \ln\left(b/a\right)} \times 2\pi a \ell$$
$$Q = \frac{2\pi \varepsilon V_0 \ell}{\ln\left(b/a\right)}$$

(iii) Capacitance due to a cone separate from the conductor along its vertex with air gap as dielectric

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0$$

Since the potential is constant with r and ϕ, the above equation reduces to



$$\frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)=0$$

$$\frac{d}{d\theta}\left(\sin\theta\frac{dV}{d\theta}\right)=0$$

$$\sin\theta\frac{dV}{d\theta}=A$$

$$\frac{dV}{d\theta}=A \quad \mathrm{cosec}\,\theta\, d\theta$$

$$V=A\log\tan {}^{0}\!/\!_{2}+B$$

$$\theta=\frac{\pi}{2};V=0$$

$$\theta=\alpha;V=V_0$$

$$0=A\log_e\tan {}^{\pi}\!/\!_{4}+B$$

$$0=A\log 1+B$$
$$B=0$$

$$V_0=A\log_e\tan {}^{\alpha}\!/\!_{2}+0$$

$$A=\frac{V_0}{\log_e\tan {}^{\alpha}\!/\!_{2}}$$

$$V=\frac{V_0}{\log_e\tan {}^{\alpha}\!/\!_{2}}\log\tan {}^{0}\!/\!_{2}$$

$$E=-\nabla V$$

$$=-\left[\frac{\partial V}{\partial r}\bar{a}_r+\frac{1}{r}\frac{\partial V}{\partial\theta}a_\theta+\frac{1}{r\sin\theta}\right]$$

$$E=\frac{-1}{r}\frac{\partial V}{\partial\theta}a_\theta$$

$$E=\frac{-1}{r}\frac{V_0}{\log_e\tan {}^{\alpha}\!/\!_{2}}\frac{\partial}{\partial\theta}\left[\log\tan {}^{0}\!/\!_{2}\right]$$

$$E=\frac{-1}{r}.\frac{V_0}{\log_e\tan {}^{\alpha}\!/\!_{2}}.\frac{1}{\tan {}^{\theta}\!/\!_{2}}.\frac{d}{d\theta}\left(\tan {}^{\theta}\!/\!_{2}\right)a_\theta$$

$$=\frac{-1}{r}.\frac{V_0}{\log_e\tan {}^{\alpha}\!/\!_{2}}.\frac{\sec^2 {}^{\theta}\!/\!_{2}.\frac{1}{2}}{\tan {}^{\theta}\!/\!_{2}}$$

$$=\frac{-1}{r}.\frac{V_0}{\log_e\tan {}^{\alpha}\!/\!_{2}}.\frac{1}{\frac{2\sin {}^{\theta}\!/\!_{2}.\cos {}^{\theta}\!/\!_{2}}{\cos {}^{\theta}\!/\!_{2}}}a_\theta$$

$$E=\frac{-1}{r}.\frac{V_0}{\log_e\tan {}^{\alpha}\!/\!_{2}}.\frac{1}{\sin\theta}a_\theta$$

$$D=\varepsilon E$$

$$D=\frac{-\varepsilon V_0}{r\sin\theta\log_e\tan {}^{\alpha}\!/\!_{2}}=\rho_s$$

At $\theta=\alpha$

$$Q=\int_S\rho_s\, ds=\int_S\frac{-\varepsilon V_0}{r\sin\theta\log_e\tan {}^{\alpha}\!/\!_{2}}.ds=\rho_S$$

$$=\frac{-\varepsilon V_0}{r\sin\alpha\log_e\tan {}^{\alpha}\!/\!_{2}}\int_0^{\infty}\int_0^{2\pi}\frac{r\sin\alpha\, d\phi}{r}$$

$$Q=\frac{-\varepsilon V_0 2\pi}{\log_e\tan {}^{\alpha}\!/\!_{2}}\left[r\right]_0^{\infty}\Rightarrow Q=\infty$$

So the limit of r is changed from 0 to r_0

$$Q=\frac{-\varepsilon V_0 2\pi}{\log\tan {}^{\alpha}\!/\!_{2}}\times r_0$$

$$C=\frac{Q}{V_0}=\frac{2\pi\varepsilon r_0}{\left(\log\tan {}^{\alpha}\!/\!_{2}\right)^{-1}}=\frac{2\pi\varepsilon_0\varepsilon_r\, r_0}{\log\tan {}^{\alpha}\!/\!_{2}}\text{ Farads}$$

9) Derive the capacitance due to concentric spherical shell

Capacitance due to concentric spherical shell:-

Let us consider two spherical conducting shells separated by a dielectric with permittivity ε .

Let ‘a’ and ‘b’ be the radii of inner and outer shells respectively.

Let the potential V =0 at r = b, V =V_0 at r = a

$$\nabla^2 V = 0 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial V^2}{\partial\phi^2}$$

Since the potential is

Constant with $\phi \& \theta$, the above equation reduces to

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) = 0$$
$$\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) = 0$$
$$r^2\frac{\partial V}{\partial r} = A$$
$$\frac{\partial V}{\partial r} = A/r^2$$
$$V = -A/r + B$$

When V= 0 at r = b;

V = V_0 at r = a;

$$0 = -A/b + B \Rightarrow B = A/b$$
$$V_0 = -A/a + A/b$$
$$V_0 = A\left(\frac{1}{b} - \frac{1}{a}\right)$$

$$A = V_0\left(\frac{1}{b} - \frac{1}{a}\right)$$
$$B = \frac{V_0}{b\left(\frac{1}{b} - \frac{1}{a}\right)}$$
$$V = \frac{-V_0}{r\left(\frac{1}{b} - \frac{1}{a}\right)} + \frac{V_0}{b\left(\frac{1}{b} - \frac{1}{a}\right)} \Rightarrow \frac{V_0}{\frac{1}{b} - \frac{1}{a}}\left[\frac{1}{b} - \frac{1}{r}\right]$$

$$E = -\nabla V$$
$$= \frac{-\partial V}{\partial r}\bar{a}_r$$
$$= \frac{+\partial V}{\partial r}\left[\frac{V_0}{r^2\left(\frac{1}{b} - \frac{1}{a}\right)}\right]$$
$$E = \frac{-V_0}{r^2\left(\frac{1}{b} - \frac{1}{a}\right)}$$

$$D = \varepsilon E = \frac{-\varepsilon V_0}{r^2\left(\frac{1}{b} - \frac{1}{a}\right)}$$
$$D_w = \rho_S$$
$$\int \frac{-\varepsilon V_0}{r^2\left(\frac{1}{b} - \frac{1}{a}\right)}.ds = Q$$
$$\frac{-\varepsilon V_0}{r^2\left(\frac{1}{b} - \frac{1}{a}\right)} \times 4\pi r^2 = Q$$

$$\frac{4\pi\varepsilon V_0}{\frac{1}{b}-\frac{1}{a}} = |Q|$$

$$\boxed{C = \frac{Q}{V_0} = \frac{4\pi\varepsilon}{\frac{1}{b}-\frac{1}{a}}}$$

10) Explain about the nature of dielectric materials

The nature of dielectric materials

Dielectric in an electric field can be named as a free space arrangement of microscopic electric dipoles which are composed of positive and negative changes whose centre do not co inside.

These are not free charges and they cannot contribute to the conduction process. Rather, they are found in place by atomic and molecular forces and can only shift positions slightly in response to external fields. They are called bound charges, in contrast to the free charges that determine conductivity.

The characteristics of the dielectric material are store electric energy. This storage takes place by means of a shift in the relative positions of the internal, found positive and negative charges against the normal molecular and atomic forces.

There are two types of molecular.

1. Polar molecule → a dipole is formed without the application of E
2. Non- polar molecule → a dipole is formed with the application of E.

A dipole moment is defined as the product of the charge and distance of separation between them. It is denoted by P and its unit is coulomb.

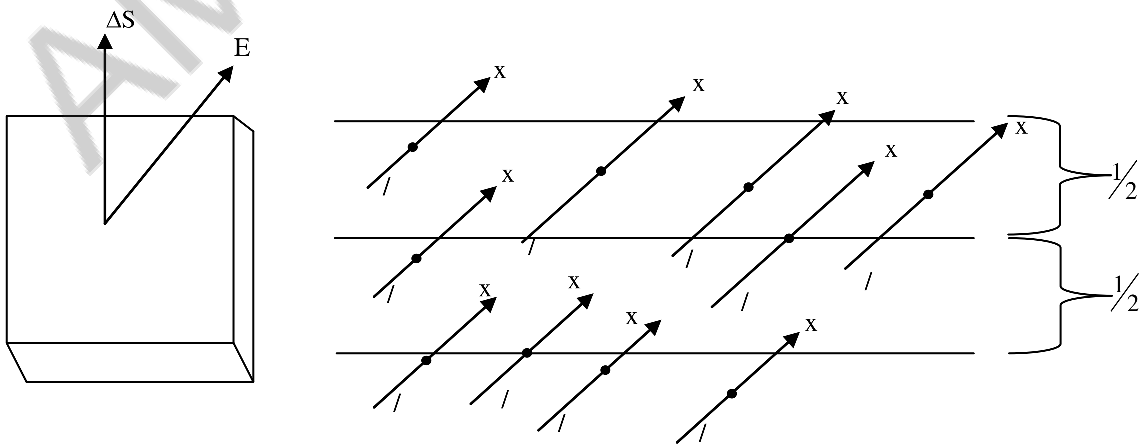
$$P = qd$$

If there are 'n'dipole in a volume ΔV , then the total dipole moment is given by

$$P_{total} = \sum_{i=1}^{n\Delta V} P_i$$

The term polarization is defined as the total dipole moment per unit volume

$$P = \frac{1}{\Delta r}\sum_{i=1}^{n\Delta V} P_i$$



The flow of current is due to bounded charges & free charges

$$Q_T = Q_b + Q$$
$$\Delta Q_b = -P.\Delta s$$
$$Q_b = -\oint\!\!\oint_s P.ds$$
$$Q_T = \oint\!\!\oint_s D.ds$$
$$Q = -Q_b + Q_T$$
$$Q = \oint\!\!\oint_S (D + P).ds$$

The relationship between E and P is given by

$$P = X_e\ \varepsilon_0\ E$$
$$Q = \oint\!\!\oint_s (\varepsilon_0 E + X_e \varepsilon_0 E)\,ds$$
$$= \oint\!\!\oint_s \varepsilon_0 E(1 + X_e)\ ds$$
$$Q = \oint\!\!\oint_s \varepsilon_0 \varepsilon_r\ E\ ds$$
$$\boxed{Q = \oint\!\!\oint_s D.\ ds}$$

Thus $\varepsilon_r = 1 + X_e$

X_e = electrical susceptibility. It is a dimensionless quantity.

Summary

$$C = Q/V \text{ Farads}$$

$$\nabla^2 V = \frac{-\rho}{\varepsilon} \text{ (poisson's equation)}$$
$$\nabla^2 V = 0 \text{ (Laplace equation)}$$
$$W = \frac{1}{2}CV^2 = \frac{1}{2}Q.V = \frac{1}{2}\frac{Q^2}{C}$$
$$\omega = \frac{1}{2}\varepsilon E^2 = \frac{1}{2}D.E = \frac{1}{2}\frac{D^2}{\varepsilon}$$

Boundary conditions:-

$E_t = 0$	$E_t = 0$	$E_{t1} = E_{t2}$
$D_t = 0$	$D_t = 0$	$\frac{D_{t1}}{D_{t2}} = \frac{\varepsilon_1}{\varepsilon_2}\ \frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_2}{\varepsilon_1}$
$D_N = \rho_S$	$D_N = \rho_S$	$D_{N1} = D_{N2}$
$E_N = \frac{\rho_S}{\varepsilon_0}$	$E_N = \frac{\rho_S}{\varepsilon_0 \varepsilon_r}$	$\frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_2}{\varepsilon_1}$

11) Explain in detail about polarization and its types?

Polarization:-

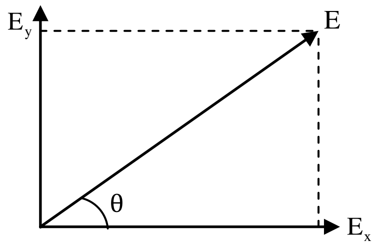
Polarization of a uniform plane wave refers to the time varying behaviour of the electric field strength vector at some fixed point in space.

Consider a uniform plane travelling in z – direction with e and H vectors lying in the x- y plane.

If Ey = 0 and Ex is present, the wave is said to be polarized in the x – direction. If Ex = 0.Ey = is present, the wave is said to be polarization in the y – direction.

(i) Linear polarization:-

If both Ex and Ey are present and are in phase, the resultant electric field has a direction at an angle of $\tan^{-1}\left(E_y / E_x\right)$. If the direction of the resultant vector is constant with time, the wave is said to be linearly polarized.



$$\theta = \tan^{-1}\left(\frac{E_y}{E_x}\right)$$

$$E = \sqrt{E_x^2 + E_y^2}$$

(ii) Circular polarization:-

If Ex and Ey have equal magnitudes and a II phase difference, the locus of the resultant 'E' is a circule and the wave is said to be circularly polarized.

If Ex and Ey have same magnitude Ea and differ in phase by 90°.

The resultant electric field in vector form is

$$\overline{E} = \bar{a}_x E_x + j\bar{a}_y E_y$$

The corresponding time varying field is

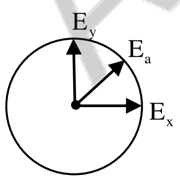
$$E_x = \bar{a}_x E_a \cos\omega t - \bar{a}_y E_a \sin\omega t$$

The component are

$$E_x = E_a \cos\omega t \qquad E_y = -E_a \sin\omega t$$

$$E_x^2 + E_y^2 = E_a^2$$

The equation shows that the locus of the resultant e is circle whose radius is Ea.



(ii) Elliptical polarization:-

If Ex and Ey hare different amplitude and a II phase difference, the locus of electric field is an ellipse and the ware is said to be elliptically polarized.

Let Ex has magnitude A and Ey has magnitude B and differ 90° in phase.

The resultant electric field in vector form

$$E = \bar{a}_x A + j\bar{a}_y B$$

The corresponding time carrying field is

$$E = \bar{a}_x A\cos\omega t + \bar{a}_y B\sin\omega t$$

The components are

$$E_x = A\cos\omega t \;\&\; E_y = -B\sin\omega t$$

$$\frac{E_x}{A} = \cos\omega t \;\&\; \frac{E_y}{B} = -\sin\omega t$$

$$\frac{E^2{}_x}{A^2} + \frac{E^2{}_y}{A^2} = 1$$

The equation shows that the locus of the resultant E is an ellipse

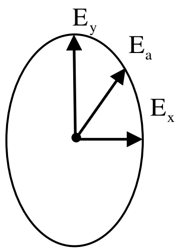


Fig:- Elliptical polarization

UNIT – II PROBLEMS

1. A condenser is composed of two plates separate by a sheet of insulating material 3 mm thick and of ε_{r1} = 4. The distance between the plates us increased so as to allow the insulation of a see and sheet of 5 mm thick and ε_{r2}. If the capacitance of the condenser so former is 1/3 of the original capacitance, find ε_{r2}.

Solution:-

The capacitance of a parallel plate capacitor is $C = \frac{\varepsilon_0 \varepsilon_r A}{d}$

Therefore, $C_1 = \frac{\varepsilon_0 \varepsilon_{r1} A}{d_1} = \frac{\varepsilon_0 \times 4 \times A}{3 \times 10^{-3}}$

$$C_1 = \frac{4000}{3}\varepsilon_0 A$$

$$C_2 = \frac{\varepsilon_0 \varepsilon_{r2} A}{d_2} = \frac{\varepsilon_0 \times \varepsilon_r \times A}{5 \times 10^{-3}}$$

$$C_2 = \frac{10^3 \varepsilon_0 \varepsilon_{r2} A}{5}$$

When $C_2 = \frac{1}{3} C_1$, the above equation can be equated as,

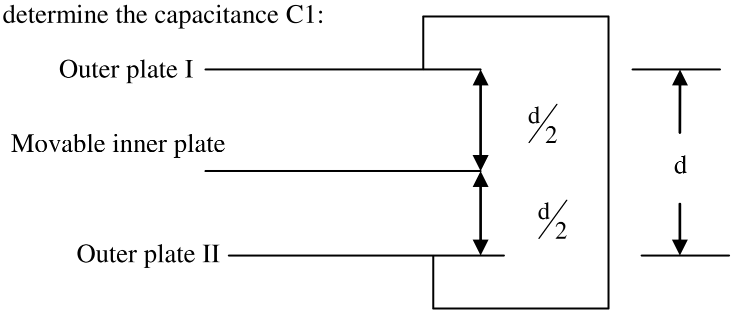
$$\frac{10^3 \varepsilon_0 \varepsilon_{r2} A}{5} = \frac{1}{3} \times \frac{4 \times 10^3 \times A\varepsilon_0}{2}$$

$$\varepsilon_r = \frac{20}{9} = 2.22$$

2. A parallel plate capacitor has three similar plate the outside two being joined together the inner plate is immovable so that it can be used as a variable capacitor. If C1 is the capacitance when the inner plate is exactly midway between the outer plates and C2 is the capacitance when inner plate is 3 times here the plate than outer plate.

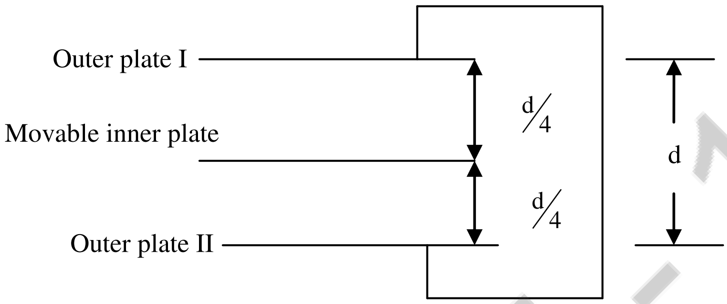
Solution:-

To determine the capacitance C1:



$$C_1 = \frac{\varepsilon A}{d/2} + \frac{\varepsilon A}{d/2}$$
$$C_1 = 4\frac{\varepsilon A}{d}$$

To determine the capacitance C2:-



$$C_2 = \frac{\varepsilon A}{d/4} + \frac{\varepsilon A}{d/4} = \frac{4\varepsilon A}{d} + \frac{4}{3}\frac{\varepsilon A}{d}$$
$$C_2 = \frac{16}{3}\frac{\varepsilon A}{d} = \left(\frac{4\varepsilon A}{d}\right)\frac{4}{3}$$

3. The capacitance of the condenser formed by the two parallel metal sheets, each 100 cm^2 in area separated by dielectric of 2 mm thick is 2 ×10^{-4} μ F. A potential of 20kv is applied into it. Find (i) Electric flux (ii)Potential gradient in Kv/m (iii) relative permittivity (iv) Electro flux density

Solution:-

Given A = 100 cm2, d = 2 mm, C= 2×10^{-4} **μ** F

V = 20 Kv

(i) The capacitance $C = \frac{Q}{V}$

$$2\times10^{-4}\times10^{-6} = \frac{Q}{20\times10^{3}}$$
$$Q = 2\times10^{-4}\times2\times10^{4}\times10^{-6}$$
$$Q = 4\mu C$$

(ii) The Electric flux $\Psi = Q = 4\mu C$

(iii) $E = \frac{V}{d} = \frac{20\times10^{3}}{2\times10^{-3}} = 10\times10^{6}\ v/m$

(iv) Capacitance between parallel plates $C = \frac{\varepsilon_0\varepsilon_r A}{d}$

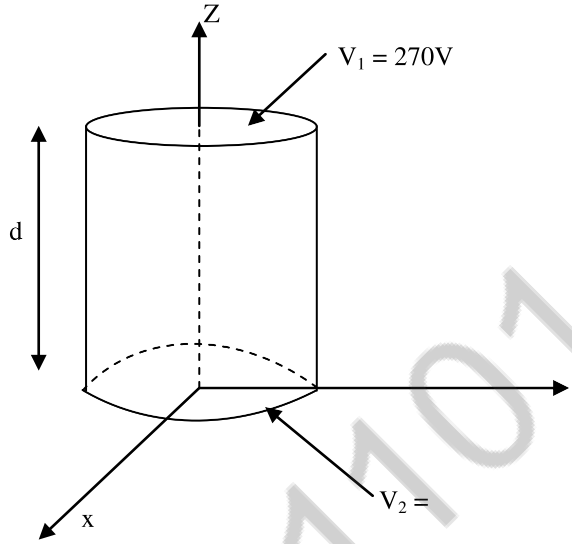
$$2\times10^4\times10^6=\frac{8.854\times10^{-12}\times\varepsilon_r\times100\times10^{-4}}{2\times10^{-3}}$$

$$\times=4.5177$$

$$D_n=\rho_s=\frac{Q}{A}=\frac{4\times10^{-6}}{100\times10^{-4}}$$

$$D_n=4\times10^{-4}\,c/m^2$$

4. The parallel conducting disks are separated by 6 mm and contain a dielectric for ε_r = 4. Determine the charge densities on the disks .



$$E=\frac{\Delta V}{d}=\frac{270-90}{6\times10^{-3}}=3\times10^4\ V/m$$

$$E=-\nabla V=-3\times10^4\,a_z V/ma$$

$$D=\varepsilon E=\varepsilon_0\varepsilon_r E$$

$$=8.854\times10^{-2}\times4\times\left(-3\times10^4\right)$$

$$=-10.62\times10^{-7}\ \bar{a}_z\ c/m^2$$

$$\rho_s=\pm10.62\times10^{-7}$$

$$=\pm1.062\mu C/m^2$$

5. An air condenser consisting of a parallel square plate of 50 cm side is charged to a p.d of 250 v. When the plate are 1mm apart. Find the 100 rt line. Assume perfect insulation.

Solution:-

A = 50 × 10^{-2} × 50 × 10^{-2} = 25×10^{-2}m

When d_1 = 1×10^{-3}, $C_1=\frac{\varepsilon_0 A}{d_1}=\frac{8.854\times10^{-12}\times25\times10^{-2}}{1\times10^{-3}}$

C_1 = 0.22× 10-8 F

When d2 = 3× 10^{-3}, $C_2=\frac{\varepsilon_0 A}{d_2}=\frac{8.854\times10^{-2}\times25\times10^{-2}}{3\times10^{-3}}$

C2 = 0.07×10^{-8} F

The energy stored in C1 is $W_{e1}=\frac{1}{2}C_1V^2$

The energy stored in C2 is $W_{e2}=\frac{1}{2}C_2V^2$

$$W_e=W_{e1}-W_{e2}$$

$$=\frac{1}{2}C_1V^2-\frac{1}{2}C_2V^2$$

$$=\frac{1}{2}V^2\left(C_1-C_2\right)$$

$$=\frac{1}{2}(250)^2\left[0.22-0.07\right]\times10^{-5}$$

We = 4.583 × = 10^{-5} J

6. The radius of two sphere differ by 4cm with air as dielectric and the capacitor of the spherical capacitor is $\frac{160}{3}$ pF. If the outer sphere is grounded, determine the ratio.

Solution:-

$$C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$$
$$b-a = 4\times10^{-2}$$
$$C = 4\pi\times8.854\times10^{-12}\left(\frac{ab}{4\times10^{-2}}\right)$$
$$\frac{160}{3}\times10^{-12} = \frac{4\pi\times8.854\times10^{-12}}{4\times10^{-2}}(ab)$$
$$ab = 0.019\,m^2$$
$$b-a = 4\times10^{-2}$$
$$b = a+4\times10^{-2}$$
$$a\left[a+4\times10^{-2}\right] = 0.019$$
$$a^2+4\times10^{-2}a-0.019 = 0$$

Solving for a, we get

a = 0.12 m

b = 0.16 m.

7. The radius of outer sphere

$$C = \frac{4\pi\varepsilon}{\frac{1}{a}-\frac{1}{b}} = \frac{4\pi\times8.854\times110^{-12}\times2^{-6}}{\frac{1}{0.1\times10^{-2}}\times\frac{1}{0.25\times10^{-2}}}$$

C = 0.46pF

$$V = \frac{Q}{C} = \frac{1}{0.464\times10^{-12}} = 2.15\times10^{12}$$
$$E_{max} = \frac{V}{(b-a)\ln\left(\frac{b}{a}\right)} = \frac{2.15\times10^{12}}{(0.15\times10^{-2})\ln\left(\frac{0.25}{0.1}\right)}$$

E_{max} = 1.56 × 10^5 V/m

8. Determine the voltage across each dielectric in the series plate capacitor conducting two dielectric $\varepsilon_{r1} = 3\ \&\ \varepsilon_{r2} = 1$, when the applied voltage is 200. Here $A = 1m^2$, d_1 = 1mm & d_2 = 4 mm.

Solution:-

Ceq = C1.C2/C1+C2

C = 2.043nF

$$D_n = \rho_s = \frac{Q}{A} = \frac{\rho V}{A} = \frac{2.043\times10^{-7}\times20}{1}$$

D_n = 4.086 × 10^{-7}c/m^2.

$$E_1 = \frac{D}{\varepsilon_0 \varepsilon_{r1}} = \frac{4.086\times10^{-7}}{8.854\times10^{-12}\times3} = 15.4\times10^{-3}\ \text{v/m}$$

$$E_2 = \frac{D}{\varepsilon_0 \varepsilon_{r1}} = \frac{4.086\times10^{-7}}{8.854\times10^{-12}\times1} = 46.15\times10^{-2}\ \text{v/m}$$

$$V_1 = E_1 d_1 = 15.4\times10^{3}\times10^{-3} = 15.48$$

$$V_2 = E_2 d_2 = 46.15\times10^{3}\times4\times10^{-3} = 184.6$$

9. A spherical capacitor with radius a 20 ccm & b = 4 cm has a non homogeneous dielectric of $\varepsilon = \frac{10\varepsilon_0}{r}$. Calculate the capacitance of the capacitor.

Solution:-

$$V = -\int_b^a E.dl = -\int_b^a \left(\frac{Q}{4\pi\varepsilon r^2}\bar{a}_r\right).\left(dr\,\bar{a}_r\right)$$

$$= \frac{-Q}{4\pi}\int_b^a \frac{dr}{\varepsilon r^2}$$

$$= \frac{-Q}{4\pi}\int_b^a \frac{dr}{\frac{10\varepsilon_0 r^2}{r}} = \frac{-Q}{40\pi\varepsilon_0}\int_b^a \frac{dr}{r}$$

$$= \frac{-Q}{40\pi\varepsilon_0}\left[\ln(r)\right]_b^a$$

$$V = \frac{-Q}{40\pi\varepsilon_0}\left[\ln\left(\frac{b}{a}\right)\right]$$

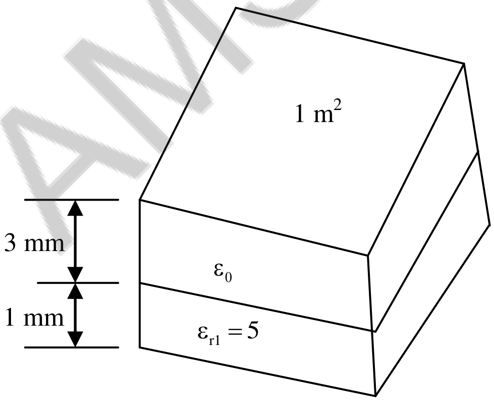
$$\frac{40\pi\varepsilon_0}{\ln\left(\frac{b}{a}\right)} = C = \frac{Q}{V}$$

$$\frac{40\pi\times\frac{1}{36\pi}\times10^{-9}}{\ln\left(\frac{4}{2}\right)} = C$$

C = 1.6 nF

10. Determine the voltage across each dielectric ion the capacitor as shown in the figure, when the applied voltage is 200v.

Solution:-



$$C_1 = \frac{\varepsilon_1 A}{d} = \frac{\varepsilon_0 \varepsilon_{r1} A}{d_1} = \frac{\varepsilon_0\times5\times1}{10^{-3}} = 5000\varepsilon_0$$

$$C_1 = \frac{\varepsilon_2 A}{d} = \frac{\varepsilon_0 \varepsilon_{r2} A}{d_2} = \frac{\varepsilon_0\times1\times1}{3\times10^{-3}} = \frac{1000}{3}\varepsilon_0$$

Since the capacitor are in series,

$$C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{(5000\varepsilon_0)\times\left(\frac{1000\varepsilon_0}{3}\right)}{5000\varepsilon_0 + \frac{1000\varepsilon_0}{3}}$$
$$= \frac{5\times10^6\times{\varepsilon_0}^2}{16\times10^3\varepsilon_0}$$
$$= \frac{5\times10^3\times10^3\times\varepsilon_0}{16\times10^3}$$
$$= \frac{5000}{16}\times8.854\times10^{-12}$$
$$= \frac{44.270}{16}\times10^{-9}$$

C_{eq} = 2.766 n F

$$D_n = \rho_s = \frac{Q}{A} = \frac{CV}{A} = \frac{2.766\times10^{-9}\times200}{1}$$

$D_n = 5.54\times10^{-7}$ C/m²

$$E_1 = \frac{D}{\varepsilon_0\varepsilon_{r1}} = \frac{5.54\times10^{-7}}{8.854\times10^{-12}\times5} = 1.25\times10^4\ V/m$$

$$E_1 = \frac{D}{\varepsilon_0\varepsilon_{r2}} = \frac{5.54\times10^{-7}}{8.854\times10^{-12}\times1} = 6.25\times10^4\ V/m$$

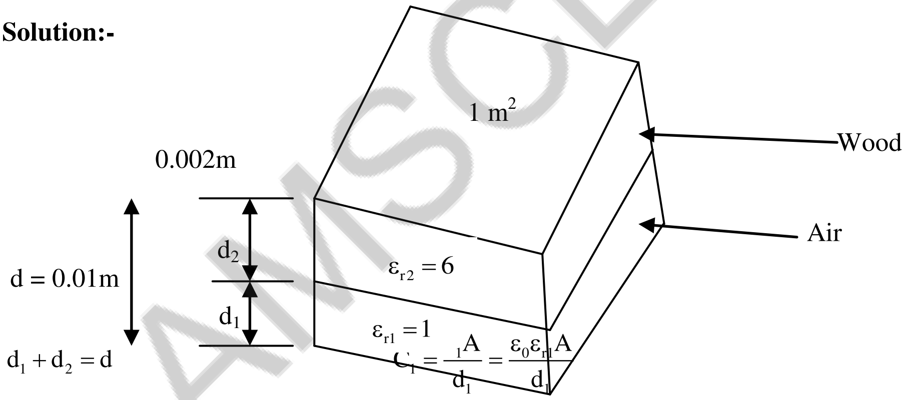
$V_1 = E_1d_1 \quad V_2 = E_2d_2$

$V_1 = 1.25\times10^4\times10^{-3} = 12.5\,v$

$V_2 = 6.25\times10^4\times3\times10^{-3} = ?$

11. A parallel plate capacitance has an area of 1m with the distance b/w the plates 0.01m & thickness of the wood is 0.002 m. The relate dielectric constant of wood is 6 the that of calculate the capacitance.

Solution:-



$d_1 + d_2 = d$

$$C_1 = \frac{\varepsilon_1 A}{d_1} = \frac{\varepsilon_0\varepsilon_{r1}A}{d_1}$$

$d_1 + 0.002 = 0.01$

$$C_1 = \frac{8.854\times10^{-2}\times1\times1}{0.008}$$

$d_1 = 0.01 - 0.02; \quad d_1 = 0.008$

$C_1 = 1.106\text{nF}$

$$C_2 = \frac{\varepsilon_2 A}{d_2} = \frac{\varepsilon_0\varepsilon_{r2}A}{d_2}$$
$$= \frac{8.854\times10^{-12}\times6\times1}{0.002}$$

C_2 = 26.562 n F

Capacitance in series

$$C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{1.106\times26.56\times10^{-18}}{27.668\times10^{-9}} = 1.06\ nF$$

12. Three capacitor of 10μF, 25μF & 50μF are connected in series & parallel. Find the C_{eq} & energy stored in each case, when the combination is connected across 500v supply.

Solution:-

(i) In series

$$\frac{1}{C_s}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_{36}}=\left(\frac{1}{10}+\frac{1}{25}+\frac{1}{50}\right)\frac{1}{10^{-6}}$$
$$=(0.1+0.04+0.02)10^6$$
$$=6.25\mu F$$
$$W_e=\frac{1}{2}\rho_s V^2=\frac{1}{2}\times 6.25\times 10^{-6}\times (500)^3$$
$$W_e=0.781\text{ J}$$

(ii) In parallel

$$C_p=(10+25+50)\times 10^{-6} = \mathbf{85\ \mu F}$$

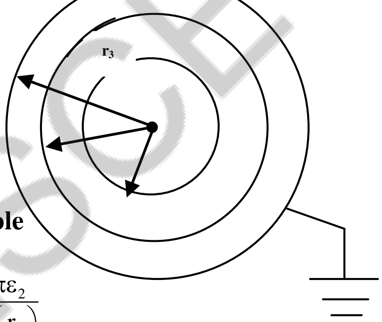
$$W_e=\frac{1}{2}C_p V^2=\frac{1}{2}\times 85\times 10^{6}\times (500)^2$$

$$\mathbf{W_e = 10.625\ J}$$

13. Referring is the figure, determine

(i) Capacitor / unit length of the cable

(ii) Maximum of in each dielectric with the data $V_t=1.2\text{kv},\ \varepsilon_{r1}=4.5\ \&\ \varepsilon_{r2}=??\&\ r_3=2r_2\ 4r_1=40\text{mm}$



r_1

r_2

(i) The capacitance of co axial cable

$$C_1=\frac{2\pi\varepsilon_1}{\ln\left(\frac{r_2}{r_1}\right)} \qquad C_2=\frac{2\pi\varepsilon_2}{\ln\left(\frac{r_3}{r_2}\right)}$$
$$C=\frac{C_1C_2}{C_1+C_2}=\frac{2\pi\varepsilon_1\varepsilon_r}{\varepsilon_2\ln\left(\frac{r_2}{r_1}\right)+\varepsilon_1\ln\left(\frac{r_3}{r_2}\right)}$$

(ii) To find maximum E,

$$C_1=\frac{2\pi\times 8.854\times 10^{-12}\times 4.5}{\ln\left(\frac{20}{10}\right)}=0.36\text{nF/m}$$
$$C_2=\frac{2\pi\times 8.854\times 10^{-12}\times 3}{\ln\left(\frac{40}{20}\right)}=0.24\text{nF/m}$$
$$\frac{V_2}{V_1}=\frac{C_1}{C_2}=\frac{0.36}{0.24}=1.5\ \ \&V_1+V_2=1200V$$
$$V_1=480V \qquad Q=C_1V_1=0.36\times 10^{-9}\times 480=172.8\text{nC/m}$$
$$E_{r(\max)}=\frac{\rho_s}{\varepsilon}=\frac{Q}{2\pi r\varepsilon}=\frac{172.8\times 10^{-9}}{2\pi\times 0.01\times 8.854\times 10^{-12}\times 4.5}$$

$E_{r(max)}$ **= 69.1 kv/m**

At r = r_2= $2r_1$

$$E_{r\,max} = \frac{172.8\times10^{-9}}{2\pi\times0.02\times8.854\times10^{-12}\times3} = 518\text{kv}/\text{m}$$

V_2 = 720V, C_2 = 0.24 n F/m

Q = C_2 V_2 = 172.8 n C/m

14. If two parallel plate, of area $4m^2$ are separate by a distance of 6mm, field the capacitor between these 2 plate. If a rubber sheet of 4mm thick with $\varepsilon_r = 2.4$ is introduced in between the plates leaving a gap of 1 mm on both sides, dielectric this capacitance.

Solution:-

Given, $A = 4m^2 \ \& d = 6\times10^{-3}$

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.854\times10^{-12}\times4}{6\times10^{-3}} = 5.90\times10^{9}F$$

Capacitance of two introducing rubber is

$$\begin{aligned}
C &= \frac{\varepsilon_0 A}{d_1} + \frac{\varepsilon_0\varepsilon_r A}{d_2} + \frac{\varepsilon_0 A}{d_3} \\
&= 8.854\times10^{-12}\times4\left[\frac{1}{1\times10^{-3}} + \frac{2.4}{4\times10^{-3}} + \frac{1}{1\times10^{-3}}\right] \\
C &= 8.854\times10^{-12}\times4\left[10^3 + \frac{1000+4\times1000}{4}\right] \\
&= 8.854\times10^{-12}\times4\left[4\times10^3 + \frac{1000+4\times1000}{4}\right] \\
&= 8.854\times11.4\times10^{-12}\times10^3 \\
&= 92.082\text{nF}
\end{aligned}$$

C = 92 n F

Problems in Laplace's equation

15. Show that the expression for the potential due to electric dipole satisfies the Laplace's equation

Solution:-

$$V = \frac{Qd\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{k\cos\theta}{r^2}$$

where $k = \dfrac{Qd}{4\pi\varepsilon_0}$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial V}{\partial\theta}\right] + \frac{1}{r^2\sin\theta}\frac{\partial^2 V}{\partial\phi^2}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 k\cos\theta\frac{\partial}{\partial r}\left(\frac{1}{r^2}\right)\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial}{\partial\theta}\left(\frac{k\cos\theta}{r}\right)\right]$$

$$+ \frac{1}{r^2\sin\theta}\frac{\partial^2}{\partial\phi^2}\left[\frac{k\cos\theta}{r}\right]$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 k\cos\theta\left(\frac{-2}{r^3}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{k(-\sin\theta)}{r^2}\right]\right]$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left[\frac{2k\cos\theta}{r^2}\right] + \frac{1}{r^2\sin\theta}\left(\frac{-2k\sin\theta\cos\theta}{r^2}\right)$$

$$= \frac{2k\cos\theta}{r^4} - \frac{2k\cos\theta}{r^4}$$

$$\nabla^2 V = 0$$

Thus the potential due to dipole satisfies Laplace's equation.

16. Determine whether or not the following potential fields satisfy the Laplace's equation.

(i) $V = x^2 - y^2 + z^2$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$= \frac{\partial^2}{\partial x^2}\left[x^2 - y^2 + z^2\right] + \frac{\partial^2}{\partial y^2}\left[x^2 - y^2 + z^2\right] + \frac{\partial^2}{\partial z^2}\left[x^2 - y^2 + z^2\right]$$

$$= \frac{\partial}{\partial x}(\partial x) + \frac{\partial}{\partial y}(-\partial y) + \frac{\partial}{\partial z}(\partial z)$$

$= 2 - 2 + 2$

=2.

This equation does not satisfy the Laplace's equation.

(ii) $V = \rho\cos\phi + z$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{1}{\rho}\left(\frac{\partial^2 V}{\partial\phi^2}\right) + \frac{\partial^2 V}{\partial z^2}$$

$$\frac{\partial V}{\partial\rho} = \frac{\partial}{\partial\rho}(\rho\cos\phi + z).\cos\phi$$

$$\frac{\partial V}{\partial\phi} = \frac{\partial}{\partial\phi}(\rho\cos\phi + z) = -\rho\sin\phi$$

$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}(\rho\cos\phi + z) = 1$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\cos\phi) + \frac{1}{\rho^2}\frac{\partial}{\partial\phi}(-\rho\sin\phi) + \frac{\partial}{\partial z} = 1$$

$$= \frac{\cos\phi}{\rho} - \frac{\cos\phi}{\rho} + 0$$

$$\nabla^2 V = 0$$

Satisfies Laplace equation

(iii) $V = r\cos\theta + \phi$ In spherical co – ordinates

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial\phi^2} = 0$$

$$= r^2 \frac{\partial V}{\partial r} + r^2 \frac{\partial}{\partial r}(r\cos\theta + \phi) - r^2\cos\theta\sin\theta\frac{\partial V}{\partial\theta} + \sin\theta\frac{\partial}{\partial\theta}(r\cos\theta + \phi) - r\sin^2\theta$$

$$+ \frac{1}{r^2\sin\theta}\frac{\partial^2 V}{\partial\phi^2} + \frac{1}{r^2\sin\theta}\frac{\partial^2}{\partial\phi^2}(r\cos\theta + \phi) = 0$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\cos\theta) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(-r\sin\theta)$$

$$= \frac{2\cos\theta}{r} - \frac{2\cos\theta}{r} = 0$$

$$\nabla^2 V = 0$$

This field satisfies Laplace equations.

UNIT II

CONDUCTORS AND DIELECTRICS

PART-A

1. Write the poisson's and Laplace's equation.

Poisson's equation:

$$\nabla^2 v = -\frac{\ell}{E}$$
$$\nabla^2 v = 0.$$

This is the Laplace's equation.

$$\nabla^2 v = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0 \text{ [cartesian]}$$
$$\nabla^2 v = \frac{1}{e}\frac{\partial}{\partial e}\left[e\frac{\partial v}{\partial e}\right] + \frac{1}{e}\left[\frac{\partial^2 v}{\partial \phi^2}\right] + \frac{\partial^2 v}{\partial z^2} = 0$$
$$\nabla^2 v = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial v}{\partial r}\right] + \frac{1}{r^2 \sin\theta}\ \frac{\partial}{\theta}\left[\sin\theta\frac{\partial v}{\partial \theta}\right] + \frac{1}{r^2 \sin\theta^2}\ \frac{\partial^2 v}{\partial \phi^2} = 0$$

2. Define capacitance:

Capacitance between two conductor defined as the ratio of the magnitude of the total charge on eitherconductor to the potential difference between conductors.

$$C = \frac{Q}{v} \text{ farads}$$

Where,

C is capacitance in Farads

Q is charge in coulombs

V is potential difference between the conductor due to equal to opposite charges.

3. Define electric current:-

It is defined as the rate of flow of charges the direction of the current flow is opposite to the flow of charges . the unit of current is Amperes.

$$I = -\frac{dQ}{dt}$$

4. Define electric current density

It is defined as the current per unit area. It is denoted by 'J' and its unit is Ampere /m^2.

$$J = \frac{I}{A}.$$
$$I = J.A$$
$$I = \iint_s J.\, ds$$

5. Write the equation of continuity.

$$\nabla.\, J = -\frac{\partial \ell v}{\partial t}$$

6. Write the point form of ohm's law.

$$J = \sigma E$$

This is the point form of ohm's law.

Where, $\sigma = |\rho_e \ \mu_e|$. Its unit is Mho/m

7. Define dipole moment

Dipole moment is denoted by P is defined as the product of a charge in the distance of separated by the charge

$$P = qd$$

8. Define polarization:-

Polarization is defined as the total dipole moment unit volume.

$$P = \frac{\text{Pltotal}}{\Delta v} = \frac{1}{\Delta v}\sum_{1=1}^{n\Delta r} P_i$$

9. Define inductance:

The inductance is defined as the rate of total magnetic flux linkage to the current through the coil and it is denoted by symbol

$$L = \frac{d\Lambda}{di} = \frac{Nd\phi}{di} \qquad \therefore \left[\frac{d\phi}{di} = \frac{\phi}{I}\right]$$

$$L = \frac{N\phi}{I}$$

10. Define flux linkage.

Flux is defined as the product of N – turns in the coil, and the total flux linked with the coil. It is defined by the symbol (Λ)

$$\Lambda = N\phi$$

11. Define Mutual inductance:-

Mutual inductance is defined as the flux linked is one coil due to the current following the second coil.

12. What is meant by dielectric breakdown?

When the electric field in a dielectric is sufficiently large, it begins to pull electrons completely out of the molecules & the dielectric becomes conducting.

13. Define dielectric strength of material & give its unit.

The maximum electric field intensity that a dielectric material can with and without break down is the dielectric strength of the material, unit: V/m

14. Find the capacitance of cylindrical (co – axial) capacitor shown in fig. here each dielectric occupies one has the volume with a =3cm & b = 12cm $\varepsilon_{r1} = 2.5 \,\&\, \varepsilon_{r2} = 4$. The voltage difference is 50 v.

Solution:-

$$C_1 = \frac{\pi\varepsilon_0\varepsilon_{r1}}{\ln\left(b/a\right)} \qquad C_2 = \frac{\pi\varepsilon_0\varepsilon_{r2}}{\ln\left(b/a\right)}$$

$$C = C_1 + C_2 = \frac{\pi\varepsilon_0}{\ln\left(b/a\right)}\left(\varepsilon_{r1} + \varepsilon_{r2}\right)$$

$$C = \frac{\pi \times 8.854 \times 10^{-12}}{\ln\left(12/3\right)}\left[2.5 + 4\right]$$

C = 130.6 pF/m

15. Find the resistance a copper wire of length 200 km and uniform cross section area 40mm?. Given that the conductivity of Cu is 5.8 × 10^7 S/m.

Solution:-

$$R = \frac{\ell}{\sigma A} = \frac{200\times10^3}{5.8\times10^7\times40\times10^{-6}}$$
$$R = \frac{2\times10^5}{5.8\times4\times10^8\times10^{-6}} = \frac{2\times10^5}{5.8\times4\times10^2}$$
$$= \frac{2\times10^3}{23.2} = 86.2\Omega$$

R = 86.2 Ω

16. A Cu bar of 30mm × 80mm in cross section and 2m in length has 50mv ends. Find resistance intensity.

For Cu, σ = 5.8 × 10^7 S/m.

Solution:-

$$R = \frac{\ell}{\sigma A} = \frac{2}{5.8\times10^7\times300}$$

R= 14.268× 10^{-6}

$$E = \frac{V}{d} = \frac{50\times10^{-3}}{2} = 25\times10^{-3} = 25\,\text{mV / m}$$

17. A parallel plate capacitor has an area of 0.8 mm separation of 0.1mm with a dielectric for which $\varepsilon_r = 1000$ and a field of 10^6 v / m. Determine the capacitance and voltage across the two plates.

Solution:-

Given A = 0.8m^2 d= 0.1 mm

$\varepsilon_r = 1000$ E = 10^6 V/ m

$$C = \frac{\varepsilon A}{d} = \frac{\varepsilon_0\varepsilon_r A}{d} = \frac{8.854\times10^{-12}\times1000\times0.8}{0.1\times10^{-3}}$$
$$= \frac{8.854\times10^{-12}\times10^3\times0.8\times10^3}{0.1}$$
$$= 8.854\times8\times10^{-6}$$
$$C = 70.732\,\mu F$$
$$V = Ed = 10^6\times0.1\times10^{-3} = 100v$$

18. A capacitor consists of two similar square aluminium plates each of 10cm × 10 cm mounted parallel and opposite to each other. What is the capacitance when the distance between them is 1cm and dielectric is air.

Solution:-

Given A = 10 × 10^{-2} × 10 × 10^{-2}=$10^{-2}$$m^2$

D = 1cm = 1 × 10^{-2}m

$$C = \frac{\varepsilon_0\varepsilon_r A}{d} = \frac{8.854\times10^{-2}\times10^{-2}}{1\times10^{-2}}$$

C = 8.854 Pf

19. Determine capacitance of area 1sqcm, separated by 1cm placed in a liquid whose dielectric constant is 6 & $\varepsilon_0 = 8.854 \times 10^{-2}$

Solution:-

A = 1cm^2 = (1 × 10^{-2}) = 1 × 10^{-4}

d = 1cm = 1 × 10^{-2}

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{8.854 \times 10^{-2} \times 6 \times 10^{-4}}{1 \times 10^{-2}}$$

C = 0.5312 pF

20. If C = 40 nF, d = 0.1mm & A = 0.15m^2. Determine the relative permittivity of dielectric material used in a parallel plate capacitor?

Solution:-

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

$$40 \times 10^{-9} = \frac{8.8854 \times 10^{-2} \times \varepsilon_r \times 0.15}{0.1 \times 10^{-3}}$$

$$\varepsilon_r = \frac{40 \times 10^{-9} \times 0.1 \times 10^{-3}}{8.854 \times 10^{-12} \times 0.15}$$

$$\varepsilon_r = 3.01$$

21. Find the capacitance / unit length between two plate cylindrical conductor in air of radius 1.5 cm & coil a centre separation of 85 cm.

Solution:-

$$C' = \frac{C}{\ell} = \frac{\pi \varepsilon_0}{\ln\left(d/2\right)} = \frac{\pi \times 8.854 \times 10^{-12}}{\ln\left(8.5/11.5\right)}$$

C' = 6.89 pF/m

22. Calculate the capacitance / km length of two identical parallel wires of diameter 1 cm each and plate 1 m apart. Also find the potential difference between them, which will make the 'E' at the conductor surface just 4×10^6 v/m.

Solution:-

Radius a = 0.5 × 10-2 m, d = 1m, E = 4×10^6

$$C' = \frac{C}{\ell} = \frac{\pi \varepsilon_0}{\ln\left(d/2\right)} = \frac{\pi \times 8.854 \times 10^{-12}}{\ln\left(\frac{1}{0.5 \times 10^{-2}}\right)}$$

C' = 5.25pF

$$E = V/d \Rightarrow V = E.d$$
$$= 4 \times 10^6 \times 1$$

V=4 × =10^6 Volts

23. The conductors of two wire transmission line of length 4 km are spaced 45 cm between centre. If each conductor has a diameter of 1.5 cm, then calculate capacitance of the line.

Solution:-

$$C = \frac{\pi\varepsilon_0\ell}{\ln\left(d/a\right)} = \frac{\pi\times 8.854\times 10^{-12}\times 4\times 10^{3}}{\ln\left(\frac{0.45}{0.75\times 10^{-2}}\right)}$$

C = 27.17 nF

24. Consider that two copper wires of 1.299 mm diameter are parallel with separation d between the axes. Determine 'd', so that the capacitance in air is 30 pF/nF

Solution:-

$$C' = \frac{C}{\ell} = \frac{\pi\varepsilon_0}{\ln\left(d/a\right)}$$

$$30\times 10^{-2} = \frac{\pi\times 8.854\times 10^{-12}}{\ln\left(d/a\right)}$$

$$\ln\left(d/a\right) = \frac{\pi\times 8.854}{30} = 0.927$$

$$\frac{d}{a} = e^{0.927} = 2.53$$

$$d = 2.53a = 2.53\times\frac{1.29\times 10^{-3}}{2}$$

d = 1.63 mm

25. Calculate the capacitance of the co –axial cable with the radius of inner conductor a 10 mm and outer conductor b = 10 mm and has $\varepsilon_r = 3.5$. The inner conductor is at potential 1 kv and the outer shield is grounded. The cable is 8 km long.

Solution:-

Given a = 10mm, b = 15mm $\varepsilon_r = 3.5$ $\ell = 8\text{km}$

$$C = \frac{2\pi\ell}{\ln\left(b/a\right)} = \frac{2\pi\times 3.5\times 8.854\times 10^{-12}\times 8\times 10^{3}}{\ln\left(\frac{15}{10}\right)}$$

C = 3.84 μF

26. Find the capacitance / unit length of a co axial conductor with outer radiys 4 mm and inner radius 0.5 mm if $\varepsilon_r = 2$

Solution:-

$$C' = \frac{C}{\ell} = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(b/a\right)} = \frac{2\pi\times 8.854\times 10^{-12}\times 2}{\ln\left(\frac{4}{0.5}\right)}$$

C' = 139. 116 pF/km

27. The radius of inner and outer spheres are 10 cm & 20cm respectively. The space between the two spheres is field with $\varepsilon_r = 3$. Find the capacitance.

Solution:-

$$C = \frac{4\pi\varepsilon}{1/a - 1/b} = \frac{4\pi\varepsilon_0\varepsilon_r}{\frac{1}{0.1}-\frac{1}{0.2}} = \frac{4\pi\times 8.854\times 10^{-12}\times 3}{5}$$

C = 66.76 pF

28. Two capacitance $10\mu F \& 25\mu F$ are connected in series & parallel. Find the equivalent values of capacitance.

Solution:-

$$C_1 = 10\mu F, C_2 = 25\mu F$$

(i) In series:-

$$C_{eq} = \frac{C_1C_2}{C_1 + C_2} = \frac{25\times10\times10^{-12}}{35\times10^{-6}}$$

C_{eq} = 7.142 μF

(ii) In parallel:-

$C_{eq} = C_1+C_2 =$ **35μF**

29. The radii of inner and outer sphere are 10 cm & 20cm respectively. The space between the two sphere is field with insulating material of ε_0. Find the capacitance formed by the two conductor sphere.

Solution:-

$$C = \frac{4\pi\varepsilon_0\varepsilon_r}{\frac{1}{a}-\frac{1}{b}}$$

$$= \frac{4\pi\times\frac{1}{36\pi\times10^9}\times3}{\frac{1}{0.1}-\frac{1}{0.2}}$$

$$= \frac{\left(0.33\times10^{-9}\right)3}{5}$$

$$= \frac{0.99\times10^{-9}}{5}$$

C = 66.76 pF

30. The radii of two sphere fifer by 4 cm with air dielectric & the capacitance of the spherical capacitor is $\frac{160}{3}$ pF. If the outer sphere is grounded, determine the radii. The capacitance of spherical capacitor is

Solution:-

$$C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$$

$$b-a = 4\times10^{-2}$$

$$C = 4\pi\times8.854\times10^{-12}\left(\frac{ab}{4\times10^{-2}}\right)$$

$$\frac{160}{3}\times10^{-12} = 4\pi\times8.854\times10^{-12}\left(\frac{ab}{4\times10^{-12}}\right)$$

$$ab = 0.019m^2$$

Solving we get,

a = 0.12 m & b = 0.16 m

PART- B

1) Explain the significance of Poisson's and Laplace equation:

Poisson's Laplace equation:

Gauss law states that the surface integral of the normal component of electric flux density vector over a closed surface is equal to the charge enclosed by the closed surface.

$$\oiint D.\, n.ds = Q = \iiint \rho \, dv \;\rightarrow (1)$$

As per divergence theorem,

$$\oiint D.\, n\, ds = \iiint_V (\nabla.V)\, dv \;\rightarrow (2)$$

Equating (1) & (2), we get

$$\iiint (\nabla.D) dv = \iiint \rho \, dv$$

$$\nabla.D = \rho \;\rightarrow (3)$$

This is point form of Gauss law,

Sub D = εE in eqn (3), we get

$$\nabla.\varepsilon E = \rho \Rightarrow \nabla.E = \frac{\rho}{\varepsilon} \;\rightarrow (4)$$

Relating E &V, as $E = -\nabla V \;\rightarrow (5)$

Sub (5) in (4), we get

$$\nabla^2 V = \frac{-\rho}{\varepsilon}$$

This is known as Poisson's equation.

In a non- conducting region, charge density is equal to zero $(\rho = 0)$

$$\nabla^2 V = 0$$

This is known as Laplace's equation

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \qquad \text{(cartesian)}$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right) + \frac{\partial^2 V}{\partial z^2} = 0 \qquad \text{(cylindrical)}$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0 \qquad \text{(spherical)}$$

2) Define the term capacitance? Explain it.

Capacitance:-

Capacitance between two conductors is defined as the ratio of the magnitude of the total charge on either conductor to the potential difference between conductors.

$$C = \frac{Q}{V} \text{ Farads}$$

C = Capacitance in farads

Q = Charge in Coulomb

V = Potential difference between conductors due to equal & opposite charges on them magnitude Q

When the capacitance of a single conductor is referred to, it is assumed that the other conductor is a spherical shell of infinite large radius.

Consider two conductors of arbitrary shape (1) & (2) as shown in below figure

(1) If initially both the conductors are uncharged & if a charge is removed from (2) to (1), the conductor (2) will be left with – Q

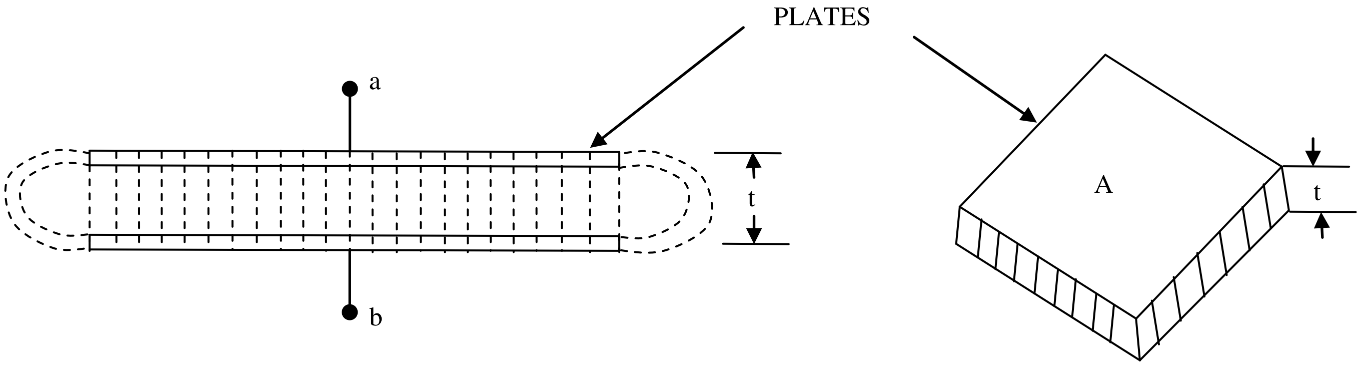
(2) Work is done in moving a charge from (2) to (1) resulting in a potential difference developed between two conductors. There will be an electronic field around them.

(3) Conversely, if a potential difference of V volts is applied. Then a charge of +Q & -Q is developed along the conductor. So, there exist a relationship between Q & V, and the ratio is constant.

If acharge of 1 coulomb is associated with a potential difference of 1 volt, the capacitance between the two conductors is said to be one farad.

3) Derive the expression of capacitance for various geometries :

(1) Parallel plate capacitor:-



A typical parallel plate capacitor which consists of a pair of flat parallel plates with surface area A separated by distance 't' and through a dielectric of permittivity $\varepsilon = \varepsilon_0 \varepsilon_r$.

The capacitor may be charged by connecting the terminals a and b to a source of potential difference. Let us assume that there is uniform charge density over a plate surface, $\rho s C/m^2$ and also across the dielectric.

$$\oint D.\,ds = Q$$
$$D.A = Q$$
$$D = Q/A = \rho_s$$

So that the filed intensity is

$$E = \frac{D}{\varepsilon} = \frac{\rho_s}{\varepsilon_0 \varepsilon_r}$$

Potential difference between the plates is given by the integral of the field E over the thickness t. As the field is uniform, we can write

$$V = Et = \frac{\rho_s t}{\varepsilon_0 \varepsilon_r}$$

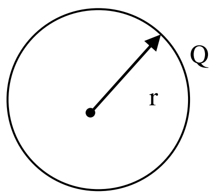
Capacitance may be expressed as,

$$C = \frac{Q}{V} = \frac{\rho_s A}{\frac{\rho_s t}{\varepsilon_0 \varepsilon_r}} = \frac{\varepsilon_0 \varepsilon_r A}{t}$$

Capacitance of an isolated sphere

In the case of an isolated conductor, the other conductor forming part of the capacitor is a spherical of infinite radius. Let it be radius r_1 the potential of an isolated sphere is the work done in moving a positive test charge from infinity to the sphere consequently, the absolute potential is given by

$$V = \frac{Q}{4\pi\varepsilon_0 r_1} \quad \text{[For a free space medium } \varepsilon_r = 1 \text{]}$$



So the capacitance is given by,

$$C = \frac{Q}{V} = 4\pi\varepsilon_0 r_1$$

Capacitance between two concentric spherical shells:-

A spherical capacitor is composed of two concentric, spherical, conducting shells separated through a dielectric medium; say free space in the case. Let 'a' and 'b' be the radii of the inner and outer shells respectively.

If a charge 'Q' is distributed uniformly over the outer surface of inner shell of radius 'a' then there will be equal and opposite charges induced on the outer shell of radius 'b'.

The filed at any point between the shells is given by

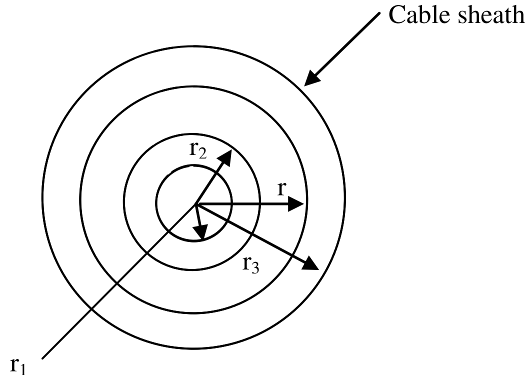
$$E_r = \frac{Q}{4\pi\varepsilon_0 r^2} \ (a \le r \le b)$$

Capacitance of co- axial cable with two dielectrics

Let us consider a cable with two dielectric with permittivities $\varepsilon_1 \,\&\, \varepsilon_2$. If E_1 is the field intensity at any radial distance r in the dielectric (1) and E_2 that in the dielectric (2).

$$E_1 = \frac{\rho_L}{2\pi\varepsilon_1 r}\ (r_1 \le r \le r_2)$$

$$E_2 = \frac{\rho_L}{2\pi\varepsilon_2 r}\ (r_2 \le r \le r_3)$$



$$V_1 = -\int_{r_2}^{r_1} E_1\, dr; \quad V_2 = -\int_{r_2}^{r_1} E_2\, dr$$

$$V_1 = \frac{\rho_L}{2\pi\varepsilon_0\varepsilon_{r1}} \ln\frac{r_2}{r_1} \quad V_1 = \frac{\rho_L}{2\pi\varepsilon_0\varepsilon_{r2}} \ln\frac{r_3}{r_2}$$

$$V = V_1 + V_2$$

$$V = \frac{\rho_L}{2\pi\varepsilon_0}\left[\frac{1}{\varepsilon_{r1}}\ln\frac{r_2}{r_1} + \frac{1}{\varepsilon_{r2}}\ln\frac{r_3}{r_2}\right]$$

The capacitance / m length is given by

$$C = \frac{\rho_L}{V} = \frac{2\pi\varepsilon_0\varepsilon_{r1}\varepsilon_{r2}}{2.303\left[\varepsilon_{r2}\log_{10}\frac{r_2}{r_1} + \varepsilon_{r1}\log_{10}\frac{r_3}{r_2}\right]}\ p/m$$

$$C = \frac{0.0241\,\varepsilon_{r1}\varepsilon_{r2}}{\varepsilon_{r2}\log_{10}\frac{r_2}{r_1} + \varepsilon_{r1}\log_{10}\frac{r_3}{r_2}}\ \mu F/km$$

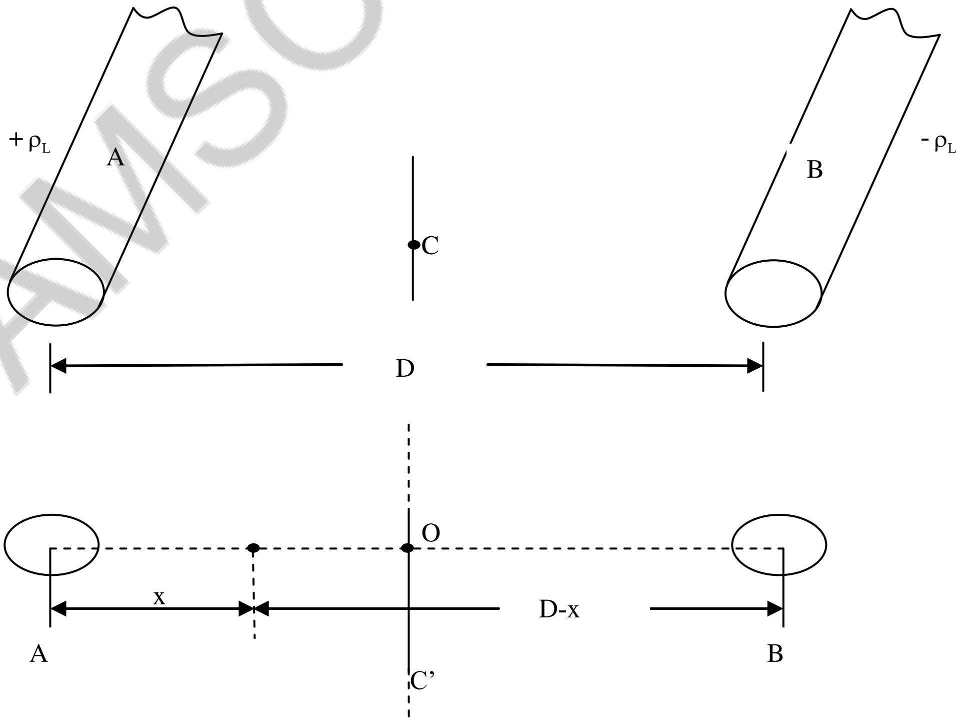
Capacitance between two parallel wires:-

Assume that $+\rho_L$ and $-\rho_L$ are the charge in c/m of the wires A and B, spaced D me ties apart and radius of each wire is r metre, remembering that D >>r.

In order to determine the capacitance between A and B, we need to find the potential difference

$$V_{BA} = -\int_B^A E_x . d_x$$

Let x = r, D –x = -r.



4) Derive the expression of energy and energy density?

Energy in a capacitance:-

Potential is defined as the work done/ unit charge. If the capacitor is connected to a source of potential the capacitor acquires charge. It involves the work to charge a capacitor.

Potential may be expressed as the infinitesimal work per infinitesimal charge.

$$V = \frac{dW}{dQ}$$
$$dW = VdQ$$

If ‘Q’ is the charge corresponding to V,

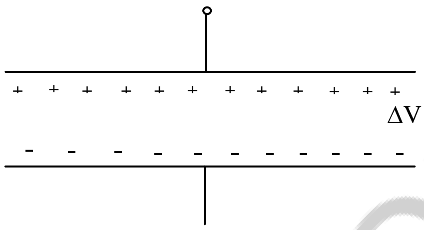
$$V = \frac{Q}{C}$$
$$dW = \frac{1}{C}QdQ$$

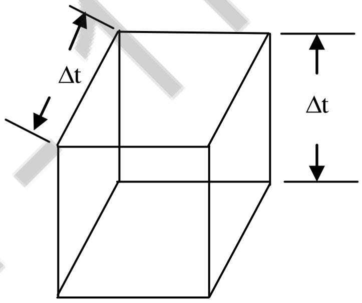
If the capacitor is initially uncharged and the process of charging continued until a charge Q is reached, the total work done is

$$W = \frac{1}{c}\int_0^q Q\,dQ = \frac{Q^2}{2C} \quad \text{Where } Q = CV$$

$$W = \frac{Q^2}{2C} \text{ or } \frac{1}{2}CV^2 \text{or } \frac{1}{2}QV \text{ Joules}$$

Energy density:-





When a capacitor is charged to a V between the plates, the energy stored is given by

$$W = \frac{1}{2}CV^2$$

The potential difference between the parallel faces of the volume element is

$$\Delta V = E(\Delta t)$$
$$\Delta W = \frac{1}{2}\Delta C(\Delta V)^2$$
$$= \frac{1}{2}\varepsilon(\Delta t)E^2(\Delta t)^2$$
$$\Delta W = \frac{1}{2}\varepsilon E^2(\Delta t)^3$$
$$\Delta W = \frac{1}{2}\varepsilon E^2(\Delta\vartheta)$$
$$\omega = \frac{\Delta W}{\Delta\vartheta} = \frac{1}{2}\varepsilon E^2$$

Energy $W = \frac{1}{2}CV^2 / \frac{1}{2}QV / \frac{1}{2}\frac{Q^2}{C}$ Joules

Energy density $\omega = \frac{1}{2}\varepsilon E^2 = \frac{1}{2}DE = \frac{1}{2}\frac{D^2}{\varepsilon}$ Joules

5) Explain current, current density and equation of continuity?

Electric current:-

It is defined as the rate of flow of charges. The directions of the current flow is opposite to the flow of charges. The unit of current is Amperes

$$I = \frac{-dQ}{dt} \rightarrow (1)$$

Let us consider a charge Q in a volume V. Let ρ_L be the volume charge density given by

$$Q = \oiiint_V \rho_v \, dv \rightarrow (2)$$

Sub (2) in (1), we get

$$I = \frac{-d}{dt} \oiiint_v \rho_v \, dv \rightarrow (3)$$

Electric current density:-

It is defined as the current per unit area. It is denoted by 'J' and its unit is Ampere/ m^2.

$$J = \frac{I}{A} \Rightarrow I = J.A = \oiint_s J.ds \rightarrow (4)$$

Equation of continuity or continuity equation of current:-

Let us consider a closed surface S, the current through the closed surface is I, due to outward flow of positive charges.

$$I = \frac{-dQ}{dt}$$

$$\oiint_s J.ds = \frac{-d}{dt} \oiiint_v \rho_v \, dv \rightarrow (5)$$

By applying Divergence theorem,

$$\oiint_s J.ds = \iiint_v (\nabla.J) dV \rightarrow (6)$$

Equating (5) & (6), we get

$$\iiint_v (\nabla.J) dV = \iiint_v \frac{-\partial \rho_v}{\partial t}. dV$$

$$\nabla.J = \frac{-\partial \rho_v}{\partial t}$$

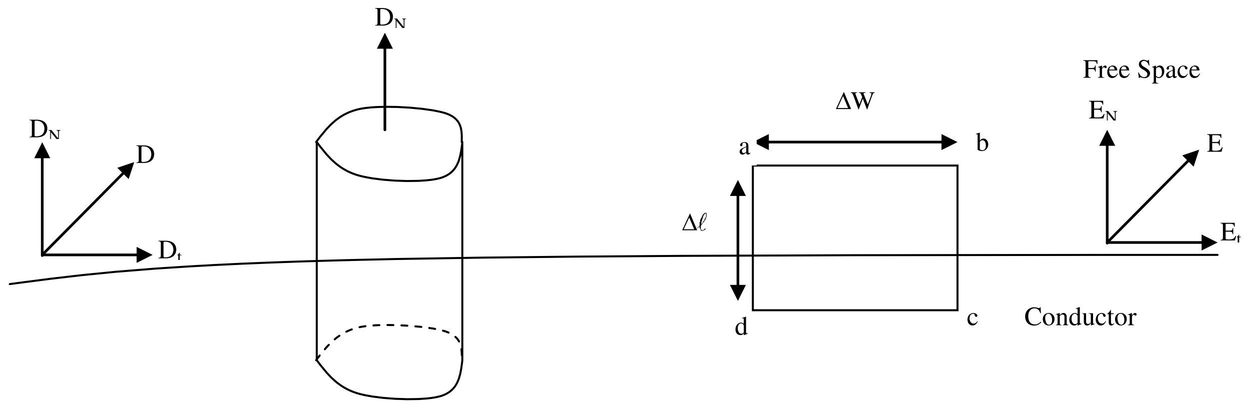
This follows principle of conversation of charges

6) Explain in detail about the boundary conditions of electric field

Boundary conditions of electric field:-

- **Conductor – Free space**
- **Conductor – Dielectric**
- **Dielectric – Dielectric**

Conductor – Free space:



We know that for a conservative field,

$$\oint E.\ dl = 0$$
$$\int_{ab} E.dl + \int_{bc} E.dl + \int_{da} E.dl = 0$$
$$\int E_t \ \Delta W + E_N.\frac{\Delta\ell}{2} - E_N.\frac{\Delta L}{2} = 0$$
$$E_t = 0$$
$$\therefore \varepsilon Dt = 0$$

Hence Dt=0

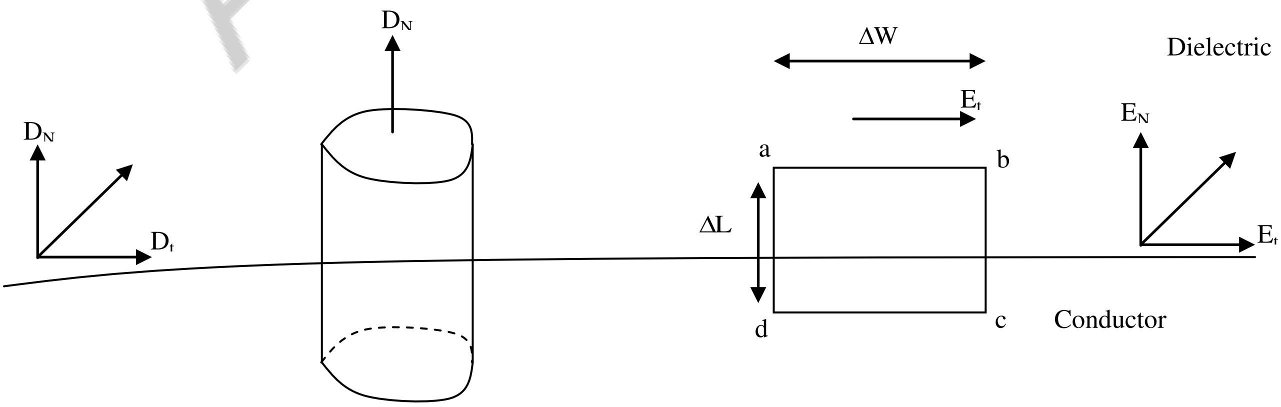
As per gauss's law,

$$\oiint_s D.N\ ds = Q$$
$$D_N.\Delta s = \rho_s\ \Delta s$$
$$D_N = \rho_s$$
$$\varepsilon E_N = \rho_s$$
$$E_N = \frac{\rho_s}{\varepsilon_0 \varepsilon_r}$$
$$E_N = \frac{\rho_s}{\varepsilon_0}$$

1). $E_t = 0;\ D_t = 0$

2). $E_N = \frac{\rho_s}{\varepsilon_0}; D_N = \rho_S$

2. Conductor – Dielectric:-



$$\oiint_S D_N . ds = Q$$

$$D_N \Delta s = \rho_s \Delta s$$

$$\varepsilon E_N = \rho_s$$

$$E_N = \frac{\rho}{\varepsilon_0 \varepsilon_r}$$

$$D_N = \rho_s$$

$$E_N = \frac{\rho_s}{\varepsilon_0 \varepsilon_r}$$

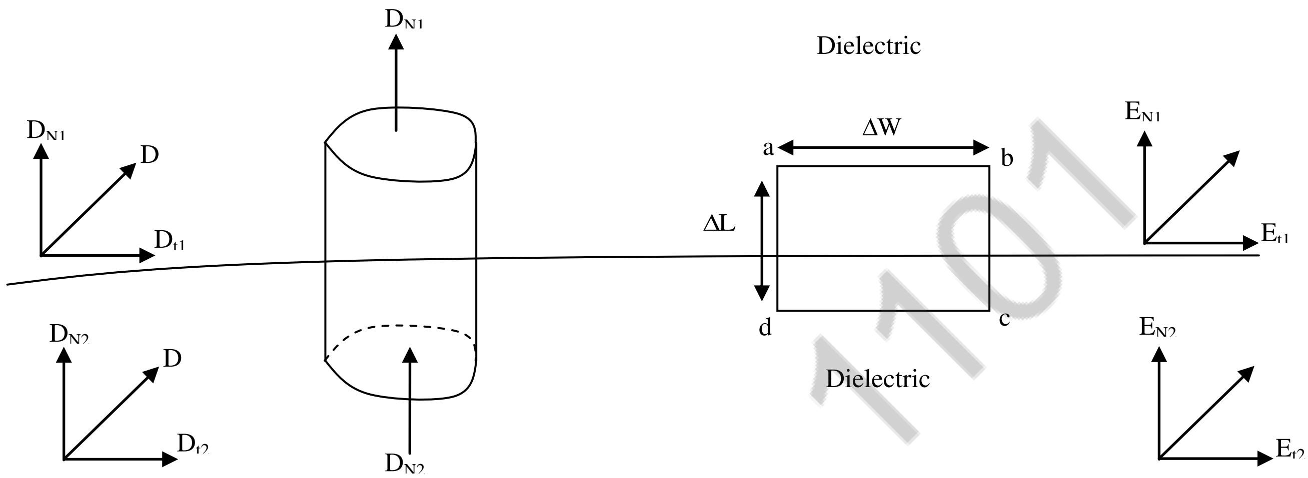
$$\int_{ab} E.dl + \int_{bc} E.dl + \int_{cd} E.dl + \int_{da} E.dl = 0$$

$$\int E_t . \Delta W + \int E_N . \frac{\Delta L}{2} - \int E_N . \frac{\Delta L}{2} = 0$$

$$E_t = 0$$

$$D_t = 0$$

3. Dielectric – Dielectric:-



$$\oiint_s (D_{N1} - D_{N2}).n.ds = Q = \rho_s \ \Delta s$$

$$(D_{N1} - D_{N2}) \Delta s = \rho_s \ \Delta s$$

$$D_{N1} - D_{N2} = \rho_s$$

$$\varepsilon_1 E_{N1} - \varepsilon_2 E_{N2} = \rho_s$$

$$\text{Assume } \Delta L \to 0$$

$$E_{t1} \Delta W - E_{t2} \Delta W = 0$$

$$E_{t1} = E_{t2}$$

$$\frac{D_{t1}}{\varepsilon_1} = \frac{D_{t2}}{\varepsilon_2}$$

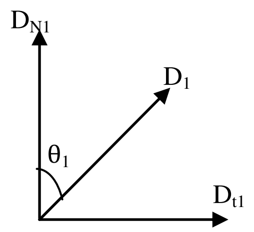
$$\frac{D_{t1}}{D_{t2}} = \frac{\varepsilon_1}{\varepsilon_2}$$

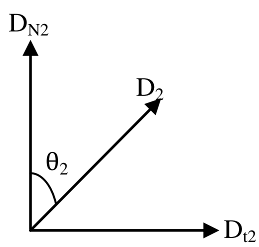
For a perfect Dielectric, $\rho_s = 0$

$$D_{N1} = D_{N2}$$

$$\varepsilon_1 E_{N1} = \varepsilon_2 E_{N2}$$

$$\frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_2}{\varepsilon_1}$$





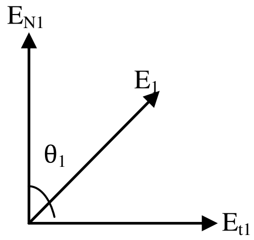
$$\cos\theta_1 = \frac{D_{N1}}{D_1}$$

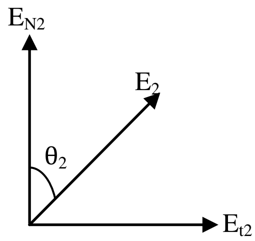
$$D_{N1} = D_1 \cos\theta_1$$

$$\cos\theta_2 = \frac{D_{N2}}{D_2}$$

$$D_{N2} = D_2 \ \cos\theta_2$$

$$\boxed{D_1 \cos\theta_1 = D_2 \cos\theta_2} \rightarrow (1)$$





$$\sin\theta_1 = \frac{E_{t1}}{E_1} \Rightarrow E_{t1} = E_1 \sin\theta_1$$

$$\sin\theta_2 = \frac{E_{t2}}{E_2} \Rightarrow E_{t2} = E_2 \sin\theta_2$$

$$\boxed{E_1 \sin\theta_1 = E_2 \sin\theta_2} \rightarrow (2)$$

(2) ÷ (1)

$$\frac{E_1 \sin\theta_1}{\varepsilon_1 E_1 \cos\theta_1} = \frac{E_2 \sin\theta_2}{\varepsilon_2 E_2 \cos\theta_2}$$

$$\frac{\tan\theta_1}{\varepsilon_1} = \frac{\tan\theta_2}{\varepsilon_2}$$

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2}$$

Summary of boundary condition:

Conductor – Free space:-

$D_t = 0;\ E_t = 0;\ D_N = \rho_s;$

The tangential component of electric field & electric flux density is equal zero.

The normal component of the electric flux is equal to the surface charge density.

The normal component of electric field is ratio of the surface charge density to the absolute permittivity.

Conductor – Dielectric:-

$D_t = E_t = 0; D_N = \rho_s;$

The tangential component of electric field and electric flux density is equal to the zero.

The normal component of electric flux density equal to the surface density.

The normal component of the electric field is equal to the ratio of surface electric density to permittivity.

Dielectric – Dielectric

$$E_{t1} = E_{t2} \qquad D_{N1} = D_{N2}$$

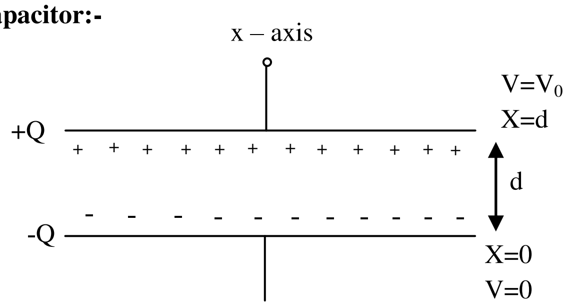
$$\frac{D_{t1}}{D_{t2}} = \frac{\varepsilon_1}{\varepsilon_2} \qquad \frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_1}{\varepsilon_2}$$

$$\frac{\tan_{\theta 1}}{\tan_{\theta 2}} = \frac{\varepsilon_1}{\varepsilon_2}$$

7) Discuss about the capacitor of various geometries using Laplace equation

Capacitance of various Geometries using Laplace equation:-

1) Due to a parallel plate capacitor:-



Let us consider two parallel plates placed along the x- axis

We know that, Laplace equation for Cartesian co – ordinates is

$$\nabla^2 V = 0$$
$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

Since the plates are along x- axis, variation of potential in Y and Z direction is equal to zero.

$$\frac{\partial^2 V}{\partial x^2} = 0$$
$$\int \frac{\partial^2 V}{\partial x^2} = \int 0$$
$$\frac{\partial V}{\partial x} = A$$
$$\int \frac{dV}{dx} = A$$
$$\int dV = \int A\ dx$$

$$V = Ax + B \qquad \rightarrow (1)$$

Boundary conditions are

$$x = 0, v = 0$$
$$x = d,\ v = v_0$$

Applying boundary condition in Equation (1), we get

$$B = 0$$
$$V_0 = A(d)$$
$$A = \frac{V_0}{d}$$

$$V = \frac{V_0}{d}x + 0 \qquad \rightarrow (2)$$

(a) Calculate E from $E = -\nabla V$

$$E = \frac{-\partial V}{\partial x} a_x$$
$$E = \frac{-\partial}{\partial x}\left(\frac{V_0 x}{d}\right) a_x$$

$$E = \frac{-V_0}{d} a_x \qquad \rightarrow (3)$$

(b) Calculate $D = +\varepsilon E$

$$D = \frac{-\varepsilon V_0}{d} a_x$$

Sub Equation (3) in $D = \varepsilon E$

(c) By gauss law,

$$\oint D .n\, ds = Q$$

$$Q = \int D\, ds = \int \rho_s\, ds = \frac{-\varepsilon V_0}{d}$$

$$Q = \frac{-\varepsilon V_0 A}{d}$$

$$C = \frac{|Q|}{V_0} = \frac{\varepsilon A}{d} \text{ Farad}$$

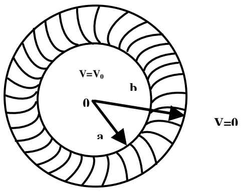
8) Explain the capacitance of co – axial capacitance using Laplace equations

2. Due to co – axial capacitor:-

Let us consider a co – axial cylindrical capacitor with inner radius 'a' & outer radius 'b'.

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial V^2}{\partial \phi^2} + \frac{\partial V^2}{\partial z^2} = 0$$

Since potential varies with respect to ρ , variation with respect to $\phi \,\&\, z = 0$. The above equation reduces to



$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) = 0$$

Multiply by $\rho \int_{\rho}$ the above equation,

$$\rho \frac{\partial V}{\partial \rho} = A \Rightarrow \frac{\partial V}{\partial \rho} = A/\rho$$

$$V = A \ln \rho + B \qquad \rightarrow (1)$$

When $\rho = a, V = V_0; \rho = b, V = 0;$

$$0 = A \ln b + B \qquad V_0 = A \ln a - A \ln b$$

$$\boxed{B = -A \ln b} \qquad V_0 = A \ln\left(a/b\right)$$

$$B = \frac{-V_0}{\ln\left(a/b\right)} \ln(b) \qquad A = \frac{V_0}{\ln\left(a/b\right)}$$

Sub the values of A & B in eqn (1)

$$V = \frac{V_0}{\ln\left(a/b\right)} \ln \rho - \frac{V_0}{\ln\left(a/b\right)} \ln(b)$$

$$\boxed{V = \frac{V_0}{\ln\left(a/b\right)} \ln\left(\rho/b\right)}$$

$$E = -\nabla V$$

$$= -\left(\frac{\partial V}{\partial \rho}\ a_\rho\right)$$

$$= -\left[\frac{\partial V}{\partial \rho}\left\{\frac{V_0 \ln\left(\rho/b\right)}{\ln\left(a/b\right)}\right\}\right] a_\rho$$

$$= -\frac{V_0}{\ln\left(a/b\right)} \times \frac{1}{\rho/b} \times \frac{1}{b}$$

$$= \frac{V_0}{\ln\left(a/b\right)^{-1}} \times \frac{1}{\rho}$$

$$\boxed{E = \frac{V_0}{\rho \ln\left(b/a\right)} a_\rho}$$

$$D = \varepsilon E$$

$$D = \frac{\varepsilon V_0}{\rho \ln\left(b/a\right)}\ \overline{a_\rho}$$

$$Q = \int \rho_s\ ds$$

$$Q = \int \frac{\varepsilon V_0}{\rho \ln\left(b/a\right)} .\ ds$$

$$Q = \left.\frac{\varepsilon V_0}{\rho \ln\left(b/a\right)}\ 2\pi a \ell\right]_{\rho = a} \qquad C = \frac{Q}{V_0} = \frac{2\pi \varepsilon_0 \varepsilon_r \ell}{\ln\left(b/a\right)}$$

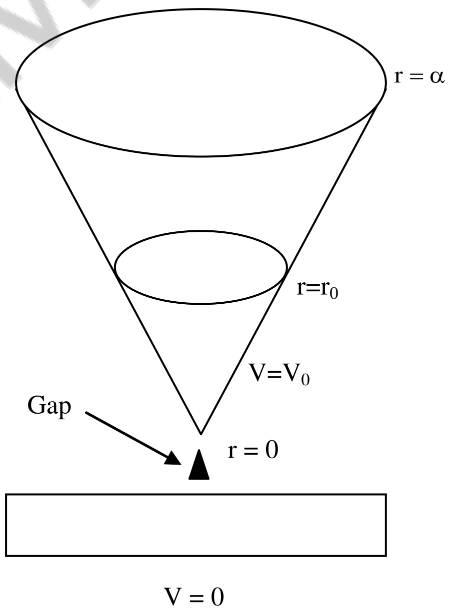
$$Q = \frac{\varepsilon V_0}{a \ln\left(b/a\right)} \times 2\pi a \ell$$

$$Q = \frac{2\pi \varepsilon V_0 \ell}{\ln\left(b/a\right)}$$

(iii) Capacitance due to a cone separate from the conductor along its vertex with air gap as dielectric

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0$$

Since the potential is constant with r and ϕ, the above equation reduces to



$$\frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)=0$$

$$\frac{d}{d\theta}\left(\sin\theta\frac{dV}{d\theta}\right)=0$$

$$\sin\theta\frac{dV}{d\theta}=A$$

$$\frac{dV}{d\theta}=A \quad \text{cosec}\,\theta\, d\theta$$

$$V=A\log\tan{}^{0}\!/_{2}+B$$

$$\theta=\frac{\pi}{2}; V=0$$

$$\theta=\alpha; V=V_0$$

$$0=A\log_e\tan{}^{\pi}\!/_{4}+B$$

$$0=A\log 1+B$$

$$B=0$$

$$V_0=A\log_e\tan{}^{\alpha}\!/_{2}+0$$

$$A=\frac{V_0}{\log_e\tan{}^{\alpha}\!/_{2}}$$

$$V=\frac{V_0}{\log_e\tan{}^{\alpha}\!/_{2}}\log\tan{}^{0}\!/_{2}$$

$$E=-\nabla V$$

$$=-\left[\frac{\partial V}{\partial r}\bar{a}_r+\frac{1}{r}\frac{\partial V}{\partial\theta}a_\theta+\frac{1}{r\sin\theta}\right]$$

$$E=\frac{-1}{r}\frac{\partial V}{\partial\theta}a_\theta$$

$$E=\frac{-1}{r}\frac{V_0}{\log_e\tan{}^{\alpha}\!/_{2}}\frac{\partial}{\partial\theta}\left[\log\tan{}^{0}\!/_{2}\right]$$

$$E=\frac{-1}{r}.\frac{V_0}{\log_e\tan{}^{\alpha}\!/_{2}}.\frac{1}{\tan{}^{\theta}\!/_{2}}.\frac{d}{d\theta}\left(\tan{}^{\theta}\!/_{2}\right)a_\theta$$

$$=\frac{-1}{r}.\frac{V_0}{\log_e\tan{}^{\alpha}\!/_{2}}.\frac{\sec^2{}^{\theta}\!/_{2}.\frac{1}{2}}{\tan{}^{\theta}\!/_{2}}$$

$$=\frac{-1}{r}.\frac{V_0}{\log_e\tan{}^{\alpha}\!/_{2}}.\frac{1}{\frac{2\sin{}^{\theta}\!/_{2}.\cos{}^{\theta}\!/_{2}}{\cos{}^{\theta}\!/_{2}}}a_\theta$$

$$E=\frac{-1}{r}.\frac{V_0}{\log_e\tan{}^{\alpha}\!/_{2}}.\frac{1}{\sin\theta}a_\theta$$

$$D=\varepsilon E$$

$$D=\frac{-\varepsilon V_0}{r\sin\theta\log_e\tan{}^{\alpha}\!/_{2}}=\rho_s$$

At $\theta=\alpha$

$$Q=\int_S\rho_s\,ds=\int_S\frac{-\varepsilon V_0}{r\sin\theta\log_e\tan{}^{\alpha}\!/_{2}}.ds=\rho_S$$

$$=\frac{-\varepsilon V_0}{r\sin\alpha\log_e\tan{}^{\alpha}\!/_{2}}\int_0^\infty\int_0^{2\pi}\frac{r\sin\alpha\, d\phi}{r}$$

$$Q=\frac{-\varepsilon V_0 2\pi}{\log_e\tan{}^{\alpha}\!/_{2}}[r]_0^\infty\Rightarrow Q=\infty$$

So the limit of r is changed from 0 to r_0

$$Q=\frac{-\varepsilon V_0 2\pi}{\log\tan{}^{\alpha}\!/_{2}}\times r_0$$

$$C=\frac{Q}{V_0}=\frac{2\pi\varepsilon r_0}{\left(\log\tan{}^{\alpha}\!/_{2}\right)^{-1}}=\frac{2\pi\varepsilon_0\varepsilon_r r_0}{\log\tan{}^{\alpha}\!/_{2}}\text{ Farads}$$

9) Derive the capacitance due to concentric spherical shell

Capacitance due to concentric spherical shell:-

Let us consider two spherical conducting shells separated by a dielectric with permittivity ε .

Let ‘a’ and ‘b’ be the radii of inner and outer shells respectively.

Let the potential V =0 at r = b, V =V_0 at r = a

$$\nabla^2 V = 0 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial V^2}{\partial\phi^2}$$

Since the potential is

Constant with $\phi \& \theta$, the above equation reduces to

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) = 0$$
$$\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) = 0$$
$$r^2\frac{\partial V}{\partial r} = A$$
$$\frac{\partial V}{\partial r} = A/r^2$$
$$V = -A/r + B$$

When V= 0 at r = b;

V = V_0 at r = a;

$$0 = -A/b + B \Rightarrow B = A/b$$
$$V_0 = -A/a + A/b$$
$$V_0 = A\left(\frac{1}{b} - \frac{1}{a}\right)$$

$$A = V_0\left(\frac{1}{b} - \frac{1}{a}\right)$$
$$B = \frac{V_0}{b\left(\frac{1}{b} - \frac{1}{a}\right)}$$
$$V = \frac{-V_0}{r\left(\frac{1}{b} - \frac{1}{a}\right)} + \frac{V_0}{b\left(\frac{1}{b} - \frac{1}{a}\right)} \Rightarrow \frac{V_0}{\frac{1}{b} - \frac{1}{a}}\left[\frac{1}{b} - \frac{1}{r}\right]$$

$$E = -\nabla V$$
$$= \frac{-\partial V}{\partial r}\bar{a}_r$$
$$= \frac{+\partial V}{\partial r}\left[\frac{V_0}{r^2\left(\frac{1}{b} - \frac{1}{a}\right)}\right]$$
$$E = \frac{-V_0}{r^2\left(\frac{1}{b} - \frac{1}{a}\right)}$$

$$D = \varepsilon E = \frac{-\varepsilon V_0}{r^2\left(\frac{1}{b} - \frac{1}{a}\right)}$$
$$D_w = \rho_S$$
$$\int \frac{-\varepsilon V_0}{r^2\left(\frac{1}{b} - \frac{1}{a}\right)}.ds = Q$$
$$\frac{-\varepsilon V_0}{r^2\left(\frac{1}{b} - \frac{1}{a}\right)} \times 4\pi r^2 = Q$$

$$\frac{4\pi\varepsilon V_0}{\frac{1}{b}-\frac{1}{a}} = |Q|$$

$$\boxed{C = \frac{Q}{V_0} = \frac{4\pi\varepsilon}{\frac{1}{b}-\frac{1}{a}}}$$

10) Explain about the nature of dielectric materials

The nature of dielectric materials

Dielectric in an electric field can be named as a free space arrangement of microscopic electric dipoles which are composed of positive and negative changes whose centre do not co inside.

These are not free charges and they cannot contribute to the conduction process. Rather, they are found in place by atomic and molecular forces and can only shift positions slightly in response to external fields. They are called bound charges, in contrast to the free charges that determine conductivity.

The characteristics of the dielectric material are store electric energy. This storage takes place by means of a shift in the relative positions of the internal, found positive and negative charges against the normal molecular and atomic forces.

There are two types of molecular.

1. Polar molecule → a dipole is formed without the application of E
2. Non- polar molecule → a dipole is formed with the application of E.

A dipole moment is defined as the product of the charge and distance of separation between them. It is denoted by P and its unit is coulomb.

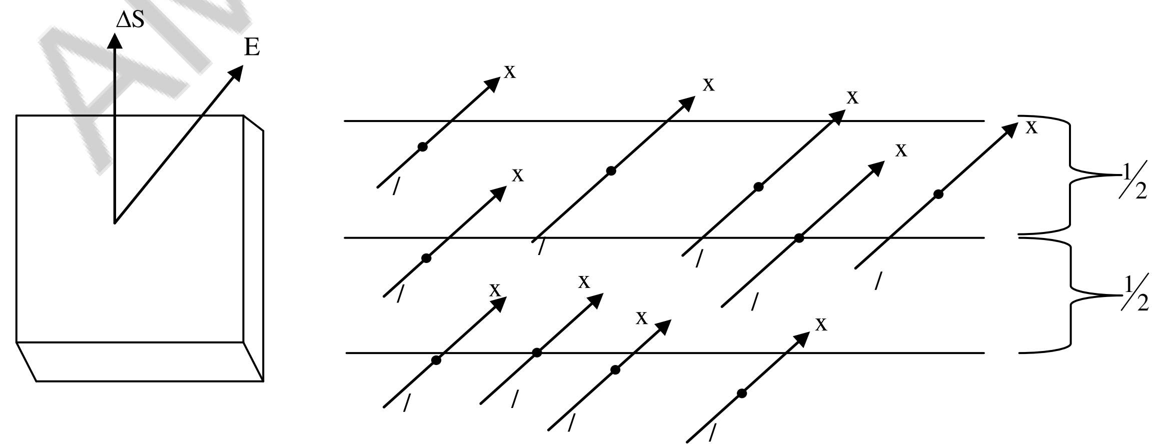
$$P = qd$$

If there are ‘n’dipole in a volume ΔV , then the total dipole moment is given by

$$P_{total} = \sum_{i=1}^{n\Delta V} P_i$$

The term polarization is defined as the total dipole moment per unit volume

$$P = \frac{1}{\Delta r}\sum_{i=1}^{n\Delta V} P_i$$



The flow of current is due to bounded charges & free charges

$$Q_T = Q_b + Q$$
$$\Delta Q_b = -P.\Delta s$$
$$Q_b = -\oiint_s P.ds$$
$$Q_T = \oiint_s D.ds$$
$$Q = -Q_b + Q_T$$
$$Q = \oiint_S (D+P).ds$$

The relationship between E and P is given by

$$P = X_e\ \varepsilon_0\ E$$
$$Q = \oiint_s (\varepsilon_0 E + X_e \varepsilon_0 E)\,ds$$
$$= \oiint_s \varepsilon_0 E(1+X_e)\ ds$$
$$Q = \oiint_s \varepsilon_0 \varepsilon_r\ E\ ds$$
$$\boxed{Q = \oiint_s D.\ ds}$$

Thus $\varepsilon_r = 1 + X_e$

X_e = electrical susceptibility. It is a dimensionless quantity.

Summary

$C = Q/V$ Farads

$$\nabla^2 V = \frac{-\rho}{\varepsilon} \text{ (poisson's equation)}$$
$$\nabla^2 V = 0 \text{ (Laplace equation)}$$
$$W = \frac{1}{2}CV^2 = \frac{1}{2}Q.V = \frac{1}{2}\frac{Q^2}{C}$$
$$\omega = \frac{1}{2}\varepsilon E^2 = \frac{1}{2}D.E = \frac{1}{2}\frac{D^2}{\varepsilon}$$

Boundary conditions:-

$E_t = 0$	$E_t = 0$	$E_{t1} = E_{t2}$
$D_t = 0$	$D_t = 0$	$\frac{D_{t1}}{D_{t2}} = \frac{\varepsilon_1}{\varepsilon_2}\ \frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_2}{\varepsilon_1}$
$D_N = \rho_S$	$D_N = \rho_S$	$D_{N1} = D_{N2}$
$E_N = \frac{\rho_S}{\varepsilon_0}$	$E_N = \frac{\rho_S}{\varepsilon_0 \varepsilon_r}$	$\frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_2}{\varepsilon_1}$

UNIT – II PROBLEMS

1. A condenser is composed of two plates separate by a sheet of insulating material 3 mm thick and of ε_{r1} = 4. The distance between the plates us increased so as to allow the insulation of a see and sheet of 5 mm thick and ε_{r2}. If the capacitance of the condenser so former is 1/3 of the original capacitance, find ε_{r2}.

Solution:-

The capacitance of a parallel plate capacitor is $C = \frac{\varepsilon_0 \varepsilon_r A}{d}$

Therefore, $C_1 = \frac{\varepsilon_0 \varepsilon_{r1} A}{d_1} = \frac{\varepsilon_0 \times 4 \times A}{3 \times 10^{-3}}$

$$C_1 = \frac{4000}{3}\varepsilon_0 A$$

$$C_2 = \frac{\varepsilon_0\varepsilon_{r2}A}{d_2} = \frac{\varepsilon_0 \times \varepsilon_r \times A}{5\times10^{-3}}$$

$$C_2 = \frac{10^3\varepsilon_0\varepsilon_{r2}A}{5}$$

When $C_2 = \frac{1}{3}C_1$, the above equation can be equated as,

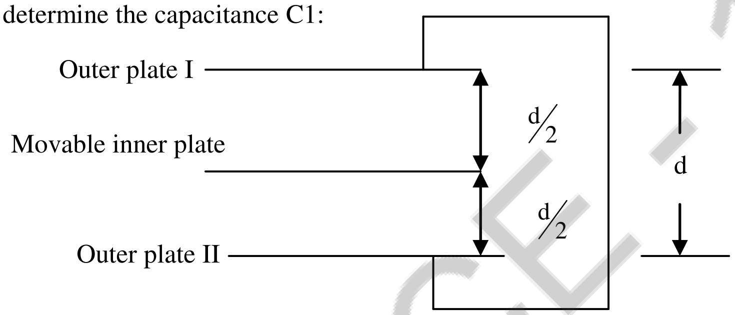
$$\frac{10^3\varepsilon_0\varepsilon_{r2}A}{5} = \frac{1}{3}\times\frac{4\times10^3\times A\varepsilon_0}{2}$$

$$\varepsilon_r = \frac{20}{9} = 2.22$$

2. A parallel plate capacitor has three similar plate the outside two being joined together the inner plate is immovable so that it can be used as a variable capacitor. If C1 is the capacitance when the inner plate is exactly midway between the outer plates and C2 is the capacitance when inner plate is 3 times here the plate than outer plate.

Solution:-

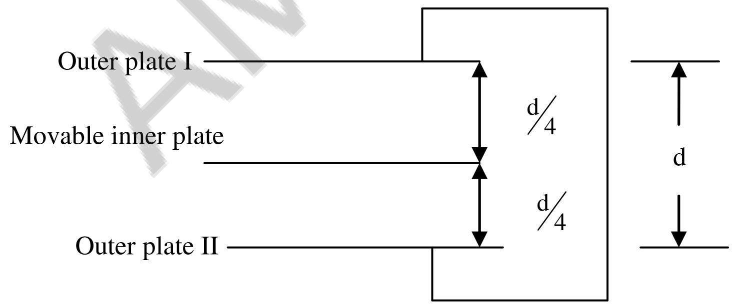
To determine the capacitance C1:



$$C_1 = \frac{\varepsilon A}{d/2} + \frac{\varepsilon A}{d/2}$$

$$C_1 = 4\frac{\varepsilon A}{d}$$

To determine the capacitance C2:-



$$C_2 = \frac{\varepsilon A}{d/4} + \frac{\varepsilon A}{d/4} = \frac{4\varepsilon A}{d} + \frac{4}{3}\frac{\varepsilon A}{d}$$

$$C_2 = \frac{16}{3}\frac{\varepsilon A}{d} = \left(\frac{4\varepsilon A}{d}\right)\frac{4}{3}$$

3. The capacitance of the condenser formed by the two parallel metal sheets, each 100 cm^2 in area separated by dielectric of 2 mm thick is 2 ×10^{-4} µ F. A potential of 20kv is applied into it. Find (i) Electric flux (ii)Potential gradient in Kv/m (iii) relative permittivity (iv) Electro flux density

Solution:-

Given A = 100 cm2, d = 2 mm, C= 2×10^{-4} **μ** F

V = 20 Kv

(i) The capacitance $C = \frac{Q}{V}$

$$2\times10^{-4}\times10^{-6} = \frac{Q}{20\times10^{3}}$$
$$Q = 2\times10^{-4}\times2\times10^{4}\times10^{-6}$$
$$Q = 4\mu C$$

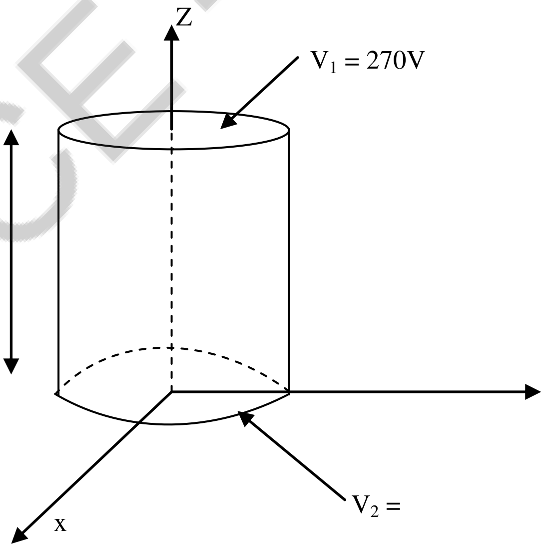
(ii) The Electric flux $\Psi = Q = 4\mu C$

(iii) $E = \frac{V}{d} = \frac{20\times10^{3}}{2\times10^{-3}} = 10\times10^{6}\ v/m$

(iv) Capacitance between parallel plates $C = \frac{\varepsilon_0\varepsilon_r A}{d}$

$$2\times10^{4}\times10^{6} = \frac{8.854\times10^{-12}\times\varepsilon_r\times100\times10^{-4}}{2\times10^{-3}}$$
$$\times = 4.5177$$
$$D_n = \rho_s = \frac{Q}{A} = \frac{4\times10^{-6}}{100\times10^{-4}}$$
$$D_n = 4\times10^{-4}\,c/m^2$$

4. The parallel conducting disks are separated by 6 mm and contain a dielectric for ε_r = 4. Determine the charge densities on the disks .



$$E = \frac{\Delta V}{d} = \frac{270-90}{6\times10^{-3}} = 3\times10^{4}\ V/m$$

$$E = -\nabla V = -3\times10^{4}\,a_z V/ma$$

$$D = \varepsilon E = \varepsilon_0\varepsilon_r E$$
$$= 8.854\times10^{-2}\times4\times\left(-3\times10^{4}\right)$$
$$= -10.62\times10^{-7}\ \bar{a}_z\ c/m^2$$
$$\rho_s = \pm10.62\times10^{-7}$$
$$= \pm1.062\mu C/m^2$$

5. An air condenser consisting of a parallel square plate of 50 cm side is charged to a p.d of 250 v. When the plate are 1mm apart. Find the 100 rt line. Assume perfect insulation.

Solution:-

A = 50 × 10^{-2} × 50 × 10^{-2} = 25×10^{-2}m

When $d_1 = 1\times10^{-3}$, $C_1 = \frac{\varepsilon_0 A}{d_1} = \frac{8.854\times10^{-12}\times25\times10^{-2}}{1\times10^{-3}}$

C_1 = 0.22× 10-8 F

When d2 = 3× 10^{-3}, $C_2 = \frac{\varepsilon_0 A}{d_2} = \frac{8.854\times10^{-2}\times25\times10^{-2}}{3\times10^{-3}}$

C2 = 0.07×10^{-8} F

The energy stored in C1 is $W_{e1} = \frac{1}{2}C_1V^2$

The energy stored in C2 is $W_{e2} = \frac{1}{2}C_2V^2$

$$
\begin{aligned}
W_e &= W_{e1} - W_{e2} \\
&= \frac{1}{2}C_1V^2 - \frac{1}{2}C_2V^2 \\
&= \frac{1}{2}V^2(C_1 - C_2) \\
&= \frac{1}{2}(250)^2[0.22 - 0.07]\times10^{-5}
\end{aligned}
$$

We = 4.583 $\times$ = 10^{-5} J

6. The radius of two sphere differ by 4cm with air as dielectric and the capacitor of the spherical capacitor is $\frac{160}{3}$ pF. If the outer sphere is grounded, determine the ratio.

Solution:-

$$
\begin{aligned}
&C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right) \\
&b - a = 4\times10^{-2} \\
&C = 4\pi\times8.854\times10^{-12}\left(\frac{ab}{4\times10^{-2}}\right) \\
&\frac{160}{3}\times10^{-12} = \frac{4\pi\times8.854\times10^{-12}}{4\times10^{-2}}(ab) \\
&ab = 0.019\,m^2 \\
&b - a = 4\times10^{-2} \\
&b = a + 4\times10^{-2} \\
&a\left[a + 4\times10^{-2}\right] = 0.019 \\
&a^2 + 4\times10^{-2}a - 0.019 = 0
\end{aligned}
$$

Solving for a, we get

a = 0.12 m

b = 0.16 m.

7. The radius of outer sphere

$$C = \frac{4\pi\varepsilon}{1/a - 1/b} = \frac{4\pi\times8.854\times110^{-12}\times2^{-6}}{\frac{1}{0.1\times10^{-2}}\times\frac{1}{0.25\times10^{-2}}}$$

C = 0.46pF

$$V = \frac{Q}{C} = \frac{1}{0.464\times10^{-12}} = 2.15\times10^{12}$$

$$E_{max} = \frac{V}{(b-a)\ln\left(b/a\right)} = \frac{2.15\times10^{12}}{(0.15\times10^{-2})\ln\left(\frac{0.25}{0.1}\right)}$$

$\mathbf{E_{max} = 1.56 \times 10^5}$ V/m

8. Determine the voltage across each dielectric in the series plate capacitor conducting two dielectric $\varepsilon_{r1} = 3 \,\&\, \varepsilon_{r2} = 1$, when the applied voltage is 200. Here $A = 1m^2$, d_1 = 1mm & d_2 = 4 mm.

Solution:-

Ceq = C1.C2/C1+C2

C = 2.043nF

$$D_n = \rho_s = \frac{Q}{A} = \frac{\rho V}{A} = \frac{2.043\times10^{-7}\times20}{1}$$

$\mathbf{D_n = 4.086 \times 10^{-7}}$ c/m^2.

$$E_1 = \frac{D}{\varepsilon_0\varepsilon_{r1}} = \frac{4.086\times10^{-7}}{8.854\times10^{-12}\times3} = 15.4\times10^{-3} \text{ v/m}$$

$$E_2 = \frac{D}{\varepsilon_0\varepsilon_{r1}} = \frac{4.086\times10^{-7}}{8.854\times10^{-12}\times1} = 46.15\times10^{-2} \text{ v/m}$$

$$V_1 = E_1 d_1 = 15.4\times10^3\times10^{-3} = 15.48$$

$$V_2 = E_2 d_2 = 46.15\times10^3\times4\times10^{-3} = 184.6$$

9. A spherical capacitor with radius a 20 ccm & b = 4 cm has a non homogeneous dielectric of $\varepsilon = \frac{10\varepsilon_0}{r}$. Calculate the capacitance of the capacitor.

Solution:-

$$V = -\int_b^a E.dl = -\int_b^a \left(\frac{Q}{4\pi\varepsilon r^2}\bar{a}_r\right).\left(dr\,\bar{a}_r\right)$$

$$= \frac{-Q}{4\pi}\int_b^a \frac{dr}{\varepsilon r^2}$$

$$= \frac{-Q}{4\pi}\int_b^a \frac{dr}{\frac{10\varepsilon_0 r^2}{r}} = \frac{-Q}{40\pi\varepsilon_0}\int_b^a \frac{dr}{r}$$

$$= \frac{-Q}{40\pi\varepsilon_0}\left[\ln(r)\right]_b^a$$

$$V = \frac{-Q}{40\pi\varepsilon_0}\left[\ln\left(b/a\right)\right]$$

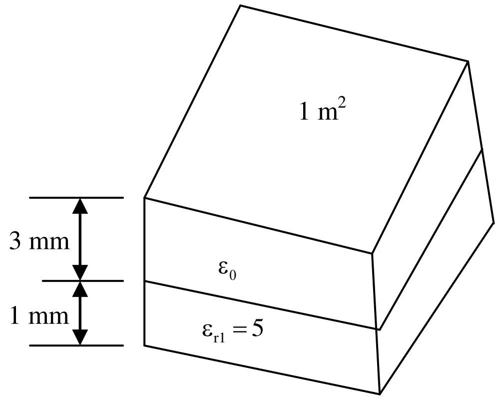
$$\frac{40\pi\varepsilon_0}{\ln\left(b/a\right)} = C = \frac{Q}{V}$$

$$\frac{40\pi\times\frac{1}{36\pi}\times10^{-9}}{\ln\left(4/2\right)} = C$$

C = 1.6 nF

10. Determine the voltage across each dielectric ion the capacitor as shown in the figure, when the applied voltage is 200v.

Solution:-



$$C_1 = \frac{\varepsilon_1 A}{d} = \frac{\varepsilon_0 \varepsilon_{r1} A}{d_1} = \frac{\varepsilon_0 \times 5 \times 1}{10^{-3}} = 5000\varepsilon_0$$

$$C_1 = \frac{\varepsilon_2 A}{d} = \frac{\varepsilon_0 \varepsilon_{r2} A}{d_2} = \frac{\varepsilon_0 \times 1 \times 1}{3 \times 10^{-3}} = \frac{1000}{3}\varepsilon_0$$

Since the capacitor are in series,

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(5000\varepsilon_0) \times \left(\frac{1000\varepsilon_0}{3}\right)}{5000\varepsilon_0 + \frac{1000\varepsilon_0}{3}}$$

$$= \frac{5 \times 10^6 \times \varepsilon_0{}^2}{16 \times 10^3 \varepsilon_0}$$

$$= \frac{5 \times 10^3 \times 10^3 \times \varepsilon_0}{16 \times 10^3}$$

$$= \frac{5000}{16} \times 8.854 \times 10^{-12}$$

$$= \frac{44.270}{16} \times 10^{-9}$$

C_{eq} = 2.766 n F

$$D_n = \rho_s = \frac{Q}{A} = \frac{CV}{A} = \frac{2.766 \times 10^{-9} \times 200}{1}$$

$D_n = 5.54 \times 10^{-7}$ C/m^2

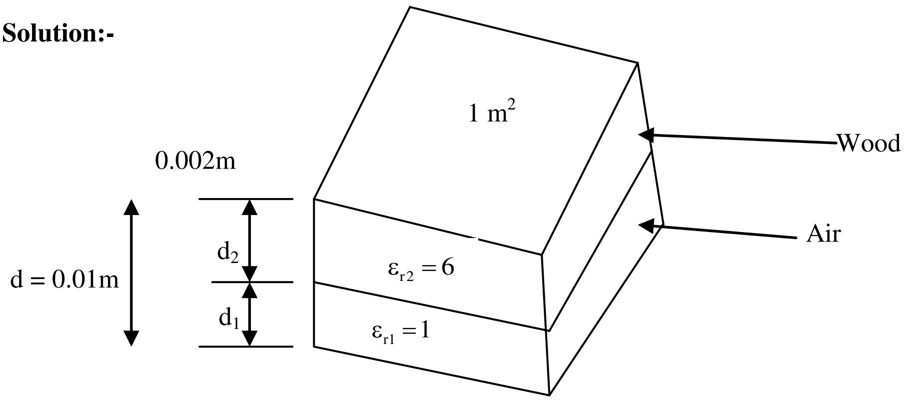
$$E_1 = \frac{D}{\varepsilon_0 \varepsilon_{r1}} = \frac{5.54 \times 10^{-7}}{8.854 \times 10^{-12} \times 5} = 1.25 \times 10^4 \text{ V/m}$$

$$E_1 = \frac{D}{\varepsilon_0 \varepsilon_{r2}} = \frac{5.54 \times 10^{-7}}{8.854 \times 10^{-12} \times 1} = 6.25 \times 10^4 \text{ V/m}$$

$$V_1 = E_1 d_1 \;\; V_2 = E_2 d_2$$
$$V_1 = 1.25 \times 10^4 \times 10^{-3} = 12.5\text{v}$$
$$V_2 = 6.25 \times 10^4 \times 3 \times 10^{-3} = ?$$

11. A parallel plate capacitance has an area of 1m with the distance b/w the plates 0.01m & thickness of the wood is 0.002 m. The relate dielectric constant of wood is 6 the that of calculate the capacitance.

Solution:-



$d_1 + d_2 = d$

$d_1 + 0.002 = 0.01$

$d_1 = 0.01 - 0.02; \quad d_1 = 0.008$

$C_1 = \dfrac{\varepsilon_1 A}{d_1} = \dfrac{\varepsilon_0 \varepsilon_{r1} A}{d_1}$

$C_1 = \dfrac{8.854 \times 10^{-2} \times 1 \times 1}{0.008}$

$C_1 = 1.106\text{nF}$

$$C_2 = \frac{\varepsilon_2 A}{d_2} = \frac{\varepsilon_0 \varepsilon_{r2} A}{d_2}$$
$$= \frac{8.854 \times 10^{-12} \times 6 \times 1}{0.002}$$

C_2 = 26.562 n F

Capacitance in series

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{1.106 \times 26.56 \times 10^{-18}}{27.668 \times 10^{-9}} = 1.06 \text{ nF}$$

12. Three capacitor of 10μF, 25μF & 50μF are connected in series & parallel. Find the C_{eq} & energy stored in each case, when the combination is connected across 500v supply.

Solution:-

(i) In series

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_{36}} = \left(\frac{1}{10} + \frac{1}{25} + \frac{1}{50}\right)\frac{1}{10^{-6}}$$
$$= (0.1 + 0.04 + 0.02)10^6$$
$$= 6.25\,\mu\text{F}$$
$$W_e = \frac{1}{2}\rho_s V^2 = \frac{1}{2} \times 6.25 \times 10^{-6} \times (500)^3$$
$$W_e = 0.781 \text{ J}$$

(ii) In parallel

$$C_p = (10 + 25 + 50) \times 10^{-6} = \mathbf{85\ \mu F}$$

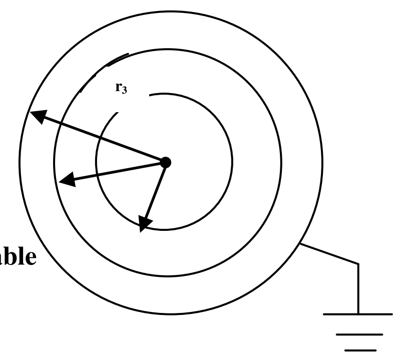
$$W_e = \frac{1}{2} C_p V^2 = \frac{1}{2} \times 85 \times 10^6 \times (500)^2$$

W_e = 10.625 J

13. Referring is the figure, determine

(i) **Capacitor / unit length of the cable**

(ii) **Maximum of in each dielectric with the data** $V_t = 1.2\text{kv},\ \varepsilon_{r1} = 4.5\ \&\ \varepsilon_{r2} = ??\&\ r_3 = 2r_2\ 4r_1 = 40\text{mm}$



r_1

r_2

(i) The capacitance of co axial cable

$$C_1 = \frac{2\pi\varepsilon_1}{\ln\left(\frac{r_2}{r_1}\right)} \qquad C_2 = \frac{2\pi\varepsilon_2}{\ln\left(\frac{r_3}{r_2}\right)}$$

$$C = \frac{C_1C_2}{C_1 + C_2} = \frac{2\pi\varepsilon_1\varepsilon_r}{\varepsilon_2 \ln\left(\frac{r_2}{r_1}\right) + \varepsilon_1 \ln\left(\frac{r_3}{r_2}\right)}$$

(ii) To find maximum E,

$$C_1 = \frac{2\pi \times 8.854 \times 10^{-12} \times 4.5}{\ln\left(\frac{20}{10}\right)} = 0.36\text{nF/m}$$

$$C_2 = \frac{2\pi \times 8.854 \times 10^{-12} \times 3}{\ln\left(\frac{40}{20}\right)} = 0.24\text{nF/m}$$

$$\frac{V_2}{V_1} = \frac{C_1}{C_2} = \frac{0.36}{0.24} = 1.5 \ \& V_1 + V_2 = 1200\text{V}$$

$$V_1 = 480\text{V} \qquad Q = C_1V_1 = 0.36 \times 10^{-9} \times 480 = 172.8\text{nC/m}$$

$$E_{r(\max)} = \frac{\rho_s}{\varepsilon} = \frac{Q}{2\pi r\varepsilon} = \frac{172.8 \times 10^{-9}}{2\pi \times 0.01 \times 8.854 \times 10^{-12} \times 4.5}$$

$E_{r(\max)}$ = 69.1 kv/m

At r = r_2= $2r_1$

$$E_{r\max} = \frac{172.8 \times 10^{-9}}{2\pi \times 0.02 \times 8.854 \times 10^{-12} \times 3} = 518\text{kv/m}$$

V_2 = 720V, C_2 = 0.24 n F/m

Q = C_2 V_2 = 172.8 n C/m

14. If two parallel plate, of area 4m^2 are separate by a distance of 6mm, field the capacitor between these 2 plate. If a rubber sheet of 4mm thick with $\varepsilon_r = 2.4$ is introduced in between the plates leaving a gap of 1 mm on both sides, dielectric this capacitance.

Solution:-

Given, $A = 4\text{m}^2 \ \& d = 6 \times 10^{-3}$

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.854 \times 10^{-12} \times 4}{6 \times 10^{-3}} = 5.90 \times 10^9\text{F}$$

Capacitance of two introducing rubber is

$$C = \frac{\varepsilon_0 A}{d_1} + \frac{\varepsilon_0\varepsilon_r A}{d_2} + \frac{\varepsilon_0 A}{d_3}$$

$$= 8.854 \times 10^{-12} \times 4\left[\frac{1}{1 \times 10^{-3}} + \frac{2.4}{4 \times 10^{-3}} + \frac{1}{1 \times 10^{-3}}\right]$$

$$C = 8.854 \times 10^{-12} \times 4\left[10^3 + \frac{1000 + 4 \times 1000}{4}\right]$$

$$= 8.854 \times 10^{-12} \times 4\left[4 \times 10^3 + \frac{1000 + 4 \times 1000}{4}\right]$$

$$= 8.854 \times 11.4 \times 10^{-12} \times 10^3$$

$$= 92.082\text{nF}$$

C = 92 n F

Problems in Laplace's equation

15. Show that the expression for the potential due to electric dipole satisfies the Laplace's equation

Solution:-

$$V = \frac{Qd\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{k\cos\theta}{r^2}$$

where $k = \dfrac{Qd}{4\pi\varepsilon_0}$

$$\begin{aligned}
\nabla^2 V &= \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial V}{\partial\theta}\right] + \frac{1}{r^2\sin\theta}\frac{\partial^2 V}{\partial\phi^2} \\
&= \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 k\cos\theta\frac{\partial}{\partial r}\left(\frac{1}{r^2}\right)\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial}{\partial\theta}\left(\frac{k\cos\theta}{r}\right)\right] \\
&\qquad + \frac{1}{r^2\sin\theta}\frac{\partial^2}{\partial\phi^2}\left[\frac{k\cos\theta}{r}\right] \\
&= \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 k\cos\theta\left(\frac{-2}{r^3}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{k(-\sin\theta)}{r^2}\right]\right] \\
&= \frac{1}{r^2}\frac{\partial}{\partial r}\left[\frac{2k\cos\theta}{r^2}\right] + \frac{1}{r^2\sin\theta}\left(\frac{-2k\sin\theta\cos\theta}{r^2}\right) \\
&= \frac{2k\cos\theta}{r^4} - \frac{2k\cos\theta}{r^4} \\
\nabla^2 V &= 0
\end{aligned}$$

Thus the potential due to dipole satisfies Laplace's equation.

16. Determine whether or not the following potential fields satisfy the Laplace's equation.

(i) $V = x^2 - y^2 + z^2$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$= \frac{\partial^2}{\partial x^2}\left[x^2 - y^2 + z^2\right] + \frac{\partial^2}{\partial y^2}\left[x^2 - y^2 + z^2\right] + \frac{\partial^2}{\partial z^2}\left[x^2 - y^2 + z^2\right]$$

$$= \frac{\partial}{\partial x}(\partial x) + \frac{\partial}{\partial y}(-\partial y) + \frac{\partial}{\partial z}(\partial z)$$

$= 2 - 2 + 2$

=2.

This equation does not satisfy the Laplace's equation.

(ii) $V = \rho\cos\phi + z$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{1}{\rho}\left(\frac{\partial^2 V}{\partial\phi^2}\right) + \frac{\partial^2 V}{\partial z^2}$$

$$\frac{\partial V}{\partial\rho} = \frac{\partial}{\partial\rho}(\rho\cos\phi + z).\cos\phi$$

$$\frac{\partial V}{\partial\phi} = \frac{\partial}{\partial\phi}(\rho\cos\phi + z) = -\rho\sin\phi$$

$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}(\rho\cos\phi + z) = 1$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho \cos\phi) + \frac{1}{\rho^2}\frac{\partial}{\partial \phi}(-\rho \sin\phi) + \frac{\partial}{\partial z} = 1$$

$$= \frac{\cos\phi}{\rho} - \frac{\cos\phi}{\rho} + 0$$

$$\nabla^2 V = 0$$

Satisfies Laplace equation

(iii) $V = r\cos\theta + \phi$ In spherical co – ordinates

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0$$

$$= r^2 \frac{\partial V}{\partial r} + r^2 \frac{\partial}{\partial r}(r\cos\theta + \phi) - r^2 \cos\theta \sin\theta \frac{\partial V}{\partial \theta} + \sin\theta \frac{\partial}{\partial \theta}(r\cos\theta + \phi) - r\sin^2\theta$$

$$+ \frac{1}{r^2 \sin\theta}\frac{\partial^2 V}{\partial \phi^2} + \frac{1}{r^2 \sin\theta}\frac{\partial^2}{\partial \phi^2}(r\cos\theta + \phi) = 0$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \cos\theta) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}(-r\sin\theta)$$

$$= \frac{2\cos\theta}{r} - \frac{2\cos\theta}{r} = 0$$

$$\nabla^2 V = 0$$

This field satisfies Laplace equations.

UNIT III

STATIC MAGNETIC FIELD

1. Define magnetisation:

Magnetic dipole moment per unit volume is defined as the magnetization (M)

$$M = {}^{m}\!/_{v} \; {}^{A}\!/_{m}$$

2. State Biot savart's law:

Biot savart's law states that the magnetic field intensity at any point P directly proportional to

(i) Current flowing through the conductor
(ii) Infinite smally length of the conductor
(iii) Fine of the and '0' between the conductor and the line joining the conductor add a point P where the magnitude field neet to be calculated.
(iv) And it is inversely proportional to the squares of the distance between them.

$$\vec{H} \propto \frac{I dl \sin\theta}{R^2}$$

3. Define Ampere's circuital law:

The line integral of the magnetic field intensity due to closed control is equal to the current enclosed by **the** path.

$$\oint_{\ell} \vec{H}.\, dl = I$$

This is the ACL in the integral for.

4. Write the differential form of ACL?

$$\nabla \times H = J$$

This is the differential form of ACL.

5. Define magnetic flux:

Magnetic flux is defined as the magnetic lines of force. It is denoted by the symbol φ. It unit is weber (wb).

6. Magnetic flux density:

Magnetic flux density is defined as the magnetic flux per unit area. It is denoted by B. Its unit is wb/m2 (or) Jesla.

7. What is the Gauss law for magnetic field?

The surface integral of the normal component of the magnetic flux density around a closed path is equal to Zero.

$$\oint_{s} B.n\; ds = 0$$

8. Define magnetic moment

The product of current and the area of the loop is defined as the magnitude moment. It is denoted by 'm'.

$$M = I\, A$$

Its unit is Ampere m^2 (or) Am^2.

9. Define torque:

When a current loop is placed parallel in the magnetic field forces act on the loop that tent to rotate it. The tangential force multiplied by the radial distance at which it acts is called torque (or) mechanical mono on the loop.

10. What is Magnetic susceptibility:-

Magnetic susceptibility is defined as the ratio of magnetization to the magnetic field intensity. It is denoted by Xm.

$$X_m = \frac{\mu}{H}$$

It is dimension quantity.

11. Define magnetic scalar potential?

It is defined as dead quantity whose negative gradient given the magnetic intensity if there is no current source present.

$$H = -\nabla V_m$$

Where Vm is the magnetic scalar potential

$$V_m = -\int H.\ dl$$

12. Define magnetic vector potential.

It is defined as the quantity whose curl gives the magnetic flux density.

$$B = \nabla \times A$$

Where A is the magnetic vector potential

$$A = \frac{\mu}{4\pi}\iiint_v \frac{J}{r}\ dr \quad W/m$$

13. Define hysteresis:

The phenomenon which causes magnetic flux density (B) to lag behind magnetic field intensity (H). So that the magnetization curve to incoming and decreasing the hysteresis.

14. What are the major classification of magnetic material.

There are three groups of magnetic material they are

a) Diamagnetic
b) Paramagnetic
c) Ferromagnetic

15. A steady element of $10^{-3}\bar{a}_z$ A. m is located at origin in free space (i) What is the magnetic field intensity due to the current element at (1, 0, 0) & at (0, 0, 1).

d) **Solution:-**

e)
$$d\bar{H} = \frac{Idl \times \bar{a}_R}{4\pi R^2}$$

$$= \frac{10^{-3}\bar{a}_z \times \bar{a}_x}{4\pi(1)^2(1)} = \frac{10^{-3}}{4\pi}\bar{a}_y A/m$$

f)
$$d\bar{H} = \frac{Idl \times \bar{a}_R}{4\pi R^2} = \frac{10^{-3}\bar{a}_z \times \bar{a}_z}{4\pi(1)^2(1)} = 0$$

16. A circular loop located on $x^2+y^2=4,\ z=0$ carries a direct current of 7A along $\bar{a}_\phi$. Determine $\bar{H}$ at (0, 0, 5) & (0, 0, -5).

Solution:-

Equation of circle is $x^2+y^2=a^2 \Rightarrow x^2+y^2=2^2$

From the above equation, $\rho=2, I=7A, h=5$

$$\bar{H}=\frac{I\rho^2}{2\left(\rho^2+h^2\right)^{3/2}}\bar{a}_z=\frac{7\times 4}{2(4+25)^{3/2}}\bar{a}_z$$

$$H=90\bar{a}_z mA/m$$

17. A thin of radius 5 cm is placed on a place z = 1cm so that the centre is at (0, 0, -1) cm. If the ring carries 50 ma along $\bar{a}_\phi$, determine $\bar{H}$ at (0, 0, -1)cm (ii) (0, 0, 10)cm.

Solution:-

$$\bar{H}=\frac{I\rho^2}{2(\rho^2+h^2)^{3/2}}a_z$$

$$=\frac{50\times 10^{-3}(5\times 10^{-2})^2}{2\left[(5\times 10^{-2})+(2\times 10^{-2})^2\right]^{3/2}}\bar{a}_z$$

$$=\frac{25\times 10^{-4}\times 5\times 10^{-3}}{2\left[25\times 10^{-4}+4\times 10^{-4}\right]^{3/2}}=400.23\bar{a}_z\ mA/m$$

(ii) At (0, 0, 10)c,, h = 9cm

$$\bar{H}=\frac{50\times 10^{-3}\times 25\times 10^{-4}}{2(5^2+9^2)^{3/2}\times 10^{-6}}\bar{a}_z$$

$$\bar{H}=57.26\bar{a}_z mA/m.$$

18. Determine the current density for $\bar{H}=28\sin x\,\bar{a}_y A/m$

Solution:-

$$\nabla\times H=\bar{J}$$

$$\begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial y} \\ 0 & 28\sin x & 0 \end{vmatrix}=a_x(0-0)+a_y(0)+a_z\left[\frac{\partial}{\partial x}(28\sin x)\right]$$

$$=28\cos x\bar{a}_z A/m^2$$

19.A wire carrying a current of 8A is formed in the circular loop. If 'H' at the centre of the loop is 40A /m. What is the radius of the loop if the loop has (i) Only one turn (ii) 10 turns

Solution:-

$$\bar{H}=\frac{I}{2\rho}\bar{a}_z$$

If there are 'N' turns,

$$\bar{H}=\frac{NI}{2\rho}$$

$$\rho=\frac{NI}{2H}=\frac{1\times 8}{2\times 40}=0.1m$$

(ii) If N = 10

$$\rho = \frac{10\times 8}{2\times 40} = 1\text{m}$$

20. The portion of the sphere is specified by r = 4,
$0 \le \theta \le 0.1\pi, 0 \le \phi \le 0.3\pi.\ \overline{H} = 6r\sin\phi\,\overline{a}_{r-1}, 18r\sin\theta\cos\phi\,\overline{a}_{\phi}\ \text{A/m}.$ **Determine the current.**

Solution:-

$$\nabla\times H = \frac{1}{r^2\sin\theta}\begin{bmatrix} a_r & ra_\theta & r\sin\theta a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ H_r & rH_\theta & r\sin\theta H_\phi \end{bmatrix}$$

21. A radius = 2cm B = 10Wb/m^2. If the plane of the coil is perpendicular to the field, determine ϕ .

Solution:-

$$\begin{aligned}\phi &= BA \\ &= 10\times\pi r^2 = 10\times\pi(2\times10^{-2})^2 \\ \phi &= 12.56\,\text{mWb}\end{aligned}$$

PART-B

1) Explain BIOT – SAVART'S LAW VECTOR FORM:-

State Biot – savart's law in vector form?

The magnetic field intensity at any point at a distance r from a current carrying conductor is directly proportional to

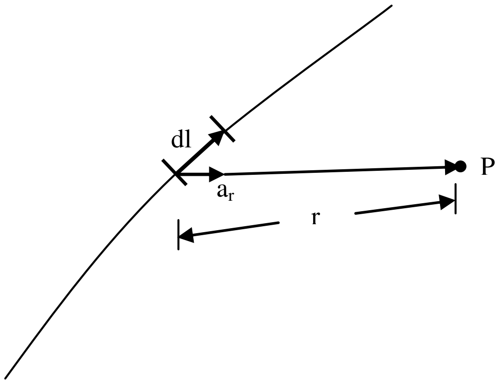
(i) The current flowing through the conductor
(ii) The infinitesimal length of the conductor
(iii) The sine of the angle between the conductor and line joining the conductor with point 'P' where the magnitude field intensity is to be calculated.
(iv) And is inversely proportional to the square of the distance between them

$$\begin{aligned}&\overline{H}\propto I \\ &\quad dl \\ &\quad \sin\theta \\ &\quad \frac{1}{r^2} \\ &\overline{H}\propto\frac{I\,dl\,\sin\theta}{r^2} \\ &\overline{H}=\frac{kI\,dl\,\sin\theta}{r^2} \\ &K=\frac{1}{4\pi}=\text{constant of proportional} \\ &\overline{H}=\frac{I\,dl\,\sin\theta}{4\pi r^2}\end{aligned}$$

The direction of the magnitude field intensity is perpendicular to the plane containing the conductor carrying current the line joining the conducts to the point P where magnitude field intensity to be calculated.

$$\overline{H}=\frac{I\,dl\,\sin\theta}{4\pi r^2}\overline{a}_r$$

BIOT SAVARTS law in vector form $\overline{H}=I\frac{\overline{dl}\times\overline{a}_r}{4\pi r^2} \rightarrow (1)$



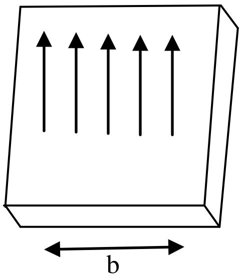
$\bar{a}_r = \frac{\bar{r}}{|r|}$

We know that

$|r| = r$

We know that $\bar{H} = I\frac{\overline{dl} \times \bar{r}}{4\pi r^3}$

Let us a rectangular sheet of width 'v' and sheet current density k, then the total current I is kb.



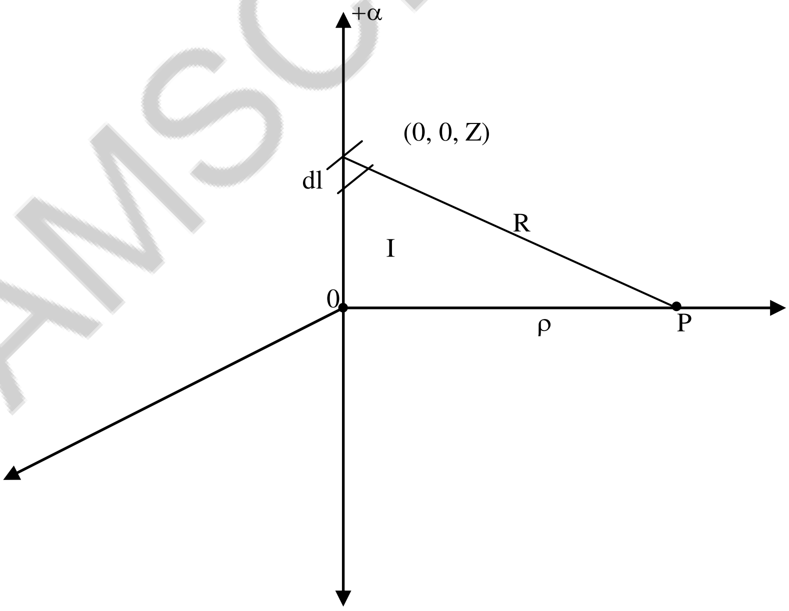
$I\,dl = K\,ds = J\,dv$

$\bar{H} = \oint_S \frac{K \times \bar{a}_r}{4\pi r^2}\,ds$

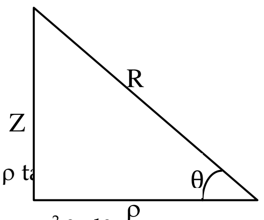
$\bar{H} = \oint_V \frac{J \times \bar{a}_R}{4\pi R^2}\,dv$

2) Determine that magnetic field intensity due to finite and infinite wire carrying a current I:

Due to infinite wire:-



$$R = P - dl$$
$$= \rho\bar{a}_\rho - z\bar{a}_z$$
$$\overline{dH} = \frac{I\overline{dl} \times \bar{a}_R}{4\pi R^2}$$
$$\overline{dl} = dz\ \bar{a}_z$$
$$\bar{a}_R = \frac{\bar{R}}{|R|} = \frac{\rho\bar{a}_\rho - z\bar{a}_z}{\sqrt{\rho^2 + z^2}}$$
$$\overline{dH} = \frac{I\ dz\ \bar{a}_z \times (\rho\bar{a}_\rho - z\bar{a}_z)}{4\pi(\rho^2 + z^2)\ \sqrt{\rho^2 + z^2}}$$
$$\overline{dH} = \frac{I\ \rho dz\ \bar{a}_z \times \bar{a}_\rho - Iz\ \ dz\ \ \bar{a}_z\ \times \bar{a}_z}{4\pi(\rho^2 + z^2)^{3/2}}$$
$$\overline{dH} = \frac{I\rho dz \bar{a}_\phi}{4\pi(\rho^2 + z^2)^{3/2}}$$



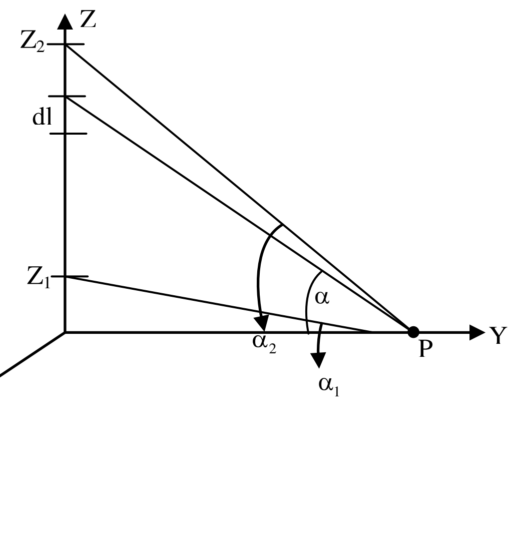
$$\tan\theta = \frac{z}{\rho} \Rightarrow z = \rho \tan\theta$$
$$dz = \rho \sec^2\theta\ \ d\theta$$

When

$$z = +\infty,\ \theta = +\frac{\pi}{2}$$
$$z = -\infty,\ \theta = -\frac{\pi}{2}$$

$$\overline{dH} = \frac{I\rho(\rho\ \sec^2\theta)d\theta\ \bar{a}_\phi}{4\pi(\rho^2 + \rho^2\ \tan^2\theta)^{3/2}}$$
$$\overline{dH} = \frac{I\rho^2\ \sec^2\theta d\theta}{4\pi\left[\rho^2(1 + \tan^2\theta)\right]^{3/2}}\ \ \bar{a}_\phi$$
$$\int_{-\infty}^{+\infty} dH = \frac{I\rho^2\ \sec^2\theta d\theta}{4\pi\rho^3\ \sec^3\theta} \bar{a}_\phi$$
$$d\bar{H} = \frac{I}{4\pi\rho}\cos\theta\ d\theta\ \bar{a}_\phi$$
$$\bar{H} = \int_{-\pi/2}^{+\pi/2} \frac{I}{4\pi\rho}\cos\theta\ d\theta\ \bar{a}_\phi$$
$$= \frac{I}{4\pi\rho}\left[\sin\theta\right]_{-\pi/2}^{+\pi/2}\ \ \bar{a}_\phi$$
$$\bar{H} = \frac{I}{4\pi\rho} \times 2\ \ \bar{a}_\phi$$
$$\bar{H} = \frac{I}{2\pi\rho}\ \ \bar{a}_\phi$$

Due to finite wire:-



Let us consider a current carrying conductor of length z. Let us consider z1 and z2 inclined at α_1 and α_2

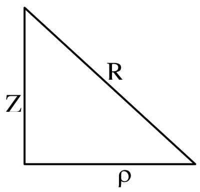
$$\overline{H} = \int_{z_1}^{z_2} \frac{I\overline{dl} \times \overline{a}_R}{4\pi R^2}$$

$$= \int_{z_1}^{z_2} \frac{I dz\ \overline{a}_z \times (\rho \overline{a}_\rho - z\overline{a}_z)}{4\pi(\rho^2 + z^2)\sqrt{\rho^2 + z^2}}$$

$$= \int_{z_1}^{z_2} \frac{I\ \rho dz\ \overline{a}_z \times \overline{a}_\rho - Iz\ dz\ \overline{a}_z \times \overline{a}_z}{4\pi(\rho^2 + z^2)^{3/2}}$$

$$= \int_{z_1}^{z_2} \frac{I\rho dz \overline{a}_\phi}{4\pi(\rho^2 + z^2)^{3/2}}$$

$$= \int_{\infty 1}^{\infty 2} \frac{I\rho^2 \sec^2\alpha\ d\alpha\ \overline{a}_\phi}{4\pi(\rho^2 + \rho^2 \tan^2\alpha)^{3/2}}$$



$$z = \rho \tan\alpha$$
$$dz = \rho \sec^2\alpha\ d\alpha$$

$$H = \int_{\infty 1}^{\infty 2} \frac{I\rho^2 \sec^2\alpha\ d\alpha}{4\pi(\rho^2 + \rho^2\tan^2\alpha)^{3/2}} \overline{a}_\phi$$

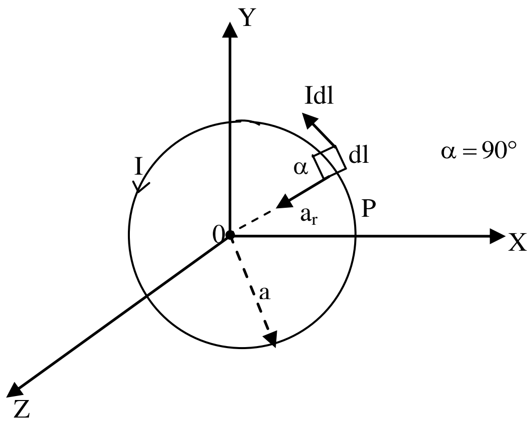
$$= \int_{\infty 1}^{\infty 2} \frac{I\rho^2 \sec^2\alpha\ d\alpha}{4\pi(\rho^3 \sec^3\alpha)} \overline{a}_\phi$$

$$= \frac{I}{4\pi\rho} \int_{\infty_1}^{\infty_2} \cos\alpha\ d\alpha\ \overline{a}_\phi$$

$$\overline{H} = \frac{I}{4\pi\rho}\left[\sin\alpha_1 - \sin\alpha_2\right]\ \overline{a}_\phi$$

3) Determine the magnetic field at the centre of a circular wire carrying a current i in the anti – clockwise direction.

The radius of the circle is ‘a’ & the wire is in XY plane:-



The field intensity at O is given by $H = \oint dH$, where dH is the field intensity at 0 due to any current element Idl. The direction of dl at any point P on the circular wire is given by the tangent at ‘P’ the direction of current flow.

The unit vector at P directed towards the centre ‘0’ is obviously along the radius p0, so that $\alpha = 90°$

$$|dH| = \frac{I\,dl}{4\pi a^2}\sin 90°$$
$$= \frac{I\,dl}{4\pi a^2}$$

The direction of the vector dH is given by dl × ar, along z – axis in the positive direct

Hence, $|dH| = \frac{I\,dl}{4\pi a^2}\bar{a}_z$

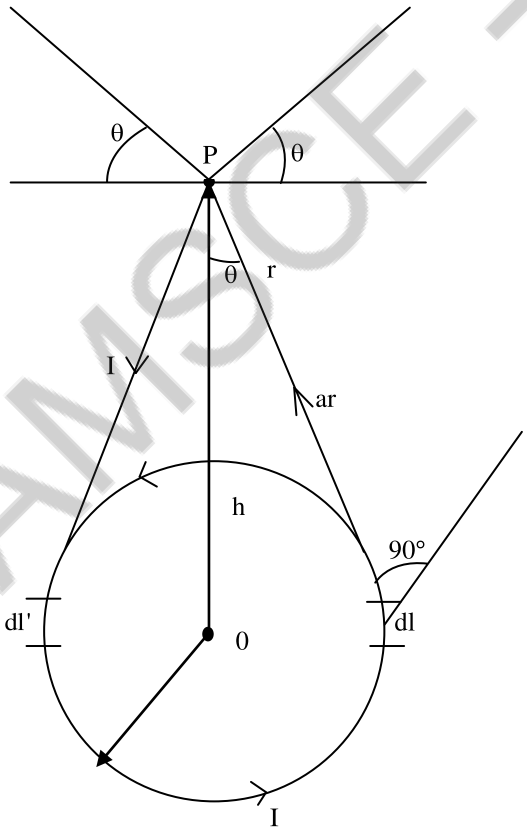
$$H = \bar{a}_z \int \frac{I\,dl}{4\pi a^2}$$

$$\bar{H} = \bar{a}_z \frac{I}{4\pi a^2}\int dl$$

Consequently, $\bar{H} = \bar{a}_z \frac{I}{4\pi a^2}(2\pi a)$

$$\bar{H} = \frac{I}{2a}\bar{a}_z$$

4) Determine the magnetic field at any point on the line through the centre at distance ‘h’ from the centre and perpendicular to the plane of a circular loop of radius of radius ‘a’ & current I.



Consider P as the point distant h from the plane of the loop.

To find the field intensity at P:-

Consider two diametrally opposite elements of the wire loop dl & dl^1. The field intensity at P at distant at r from the current element Idl is given by

$$d\bar{H} = \frac{I\,dl \times \bar{a}_r}{4\pi r^2}$$

As the vectors dl and $\bar{a}_r$ are perpendicular the value of dH is given by

$$dH = \frac{I\,dl}{4\pi r^2}\bar{a}_r .\sin 90°$$

$$dH = \frac{I\,dl}{4\pi r^2}$$

This field is oriented at an angle Q to the plane of loop. The diametrally opposite element Idl will also produce a field of magnitude equal to dH.

$$dH = \frac{I\,dl\sin\theta}{4\pi r^2}$$

Where $\sin\theta = \frac{a}{(a^2+h^2)^{1/2}}$ and $r^2 = a^2 + h^2$

The resultant field intensity at that point is given by integrating the z – component of the field contributions of all the current elements

$$H_p = \oint dHz = \oint \frac{I\,dl}{4\pi(a^2+h^2)} \cdot \frac{a}{(a^2+h^2)^{1/2}}$$

$$= \frac{Ia}{4\pi(a^2+h^2)^{3/2}}\oint dl$$

$$H_p = \frac{Ia^2}{4\pi(a^2+h^2)^{3/2}} \times 2\pi a$$

$$H_p = \frac{Ia^2}{2(a^2+h^2)^{3/2}}$$

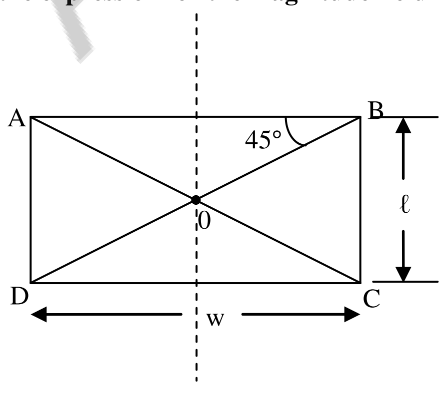
As the field is directed along z – axis

$$H_p = \frac{Ia^2}{2(a^2+h^2)^{3/2}}\,\bar{a}_z$$

If h = 0, P continues with 0,the centre of the wire – loop.

$$\bar{H} = \frac{Ia^2}{2a^3} = \frac{I}{2a}\bar{a}_z$$

5) Derive the expression for the magnitude field intensity due to a rectangular loop:-



Consider a rectangular loop ABCD carrying a current I through the loop. Let 'L' be the breadth of the loop.

$$\left.\begin{array}{l}\text{Magnitude field intensity H due}\\ \text{To and AB at '0'}\end{array}\right\} = \frac{I}{4\pi\rho}(\sin\alpha_2 - \alpha_1)$$

Substitute $\rho = \frac{L}{2};\ \alpha_1 = \alpha_2 = 45°$

$$H = \frac{I}{4\pi \ell/2}\left[\sin(45°) - \sin(-45°)\right]$$
$$= \frac{I}{2\pi\ell}\left[\frac{1}{\sqrt{2}} - \left(\frac{-1}{\sqrt{2}}\right)\right]$$
$$= \frac{I}{2\pi\ell}\left[\frac{2}{\sqrt{2}}\right]$$
$$H = \frac{I}{\sqrt{2}\pi\ell}$$

Similarly 'H' to arm CD at 'o'$\} = \frac{I}{\sqrt{2}\pi\ell}$

$$\left.\begin{array}{r}\text{Magnitude field intensity due to}\\ \text{Arm Ad \& BC}\end{array}\right\} = \frac{2I}{\sqrt{2}\pi\ell} + \frac{2I}{\sqrt{2}\pi b}$$
$$= \frac{2I}{\sqrt{2}\pi}\left[\frac{b+\ell}{\ell b}\right]$$

$$H = \frac{2\sqrt{2}\,I}{2\pi}\left[\frac{\ell+b}{\ell b}\right]$$
$$H = \frac{\sqrt{2}\,I}{\pi}\left[\frac{\ell+b}{\ell b}\right]$$

The above expression can be deduced to a square by substituting $\ell = b = a$

$$H = \frac{\sqrt{2}\,I}{\pi}\left[\frac{2a}{a^2}\right]$$
$$H = \frac{2\sqrt{2}\,I}{\pi a}$$

Magnetic field intensity due to a rectangular loop:-

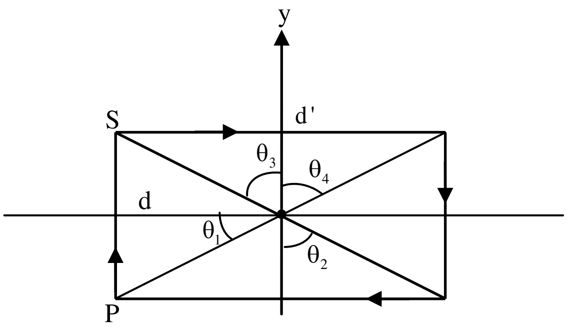
Consider a rectangular loop (PQRS) located in XY plane which carries a current I. 'H' is found at on 0.

Let 'L' = length b rectangular 'b; = breadth of rectangular.

Each side of rectangular is treated as finite length current element.

Consider PS, $\overline{H}$ due to finite length wire,

$$\overline{H} = \frac{I}{4\pi d}\left[\sin\theta_1 + \sin\theta_2\right]P$$
$$d = \frac{L}{2},\ \overline{H}_1 = \frac{I}{2\pi L}\left[\sin\theta_1 + \sin\theta_2\right]\overline{a}_\phi$$



From symmetry of rectangular, for QR

$$\overline{H}_2 = \frac{I}{2\pi L}\left[\sin\theta_1 + \sin\theta_2\right]\overline{a}_\phi$$

Similarly $\overline{H}$ due to finite length RS & PQ

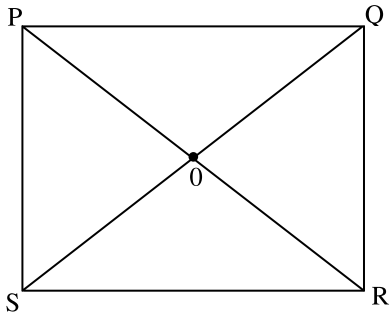
$$H_3 = H_4 = \frac{I}{2\pi d}\left[\sin\theta_3 + \sin\theta_4\right]$$

$$\overline{H}_3 = \overline{H}_4 = \frac{I}{2\pi B}\left[\sin\theta_3 + \sin\theta_4\right]$$

$$\overline{H} = \overline{H}_1 + \overline{H}_2 + \overline{H}_3 + \overline{H}_4$$

$$= \frac{I}{\pi}\left\{\frac{1}{L}(\sin\theta_1 + \sin\theta_2) + \frac{1}{B}(\sin\theta_3 + \sin\theta_4)\right\}$$

6) Determine the expression for the magnitude field intensity due to a square loop:-



Consider a square loop PQRS of side ‘a’. A current “I’ flows through the loop

$$H = \frac{I}{4\pi\rho}\left[\sin\alpha_2 - \sin\alpha_1\right]$$

$$\rho = \frac{a}{2};\ \alpha_2 = 45°\ \ \alpha_1 = -\,45°$$

$$H = \frac{I}{2\pi a}\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]$$

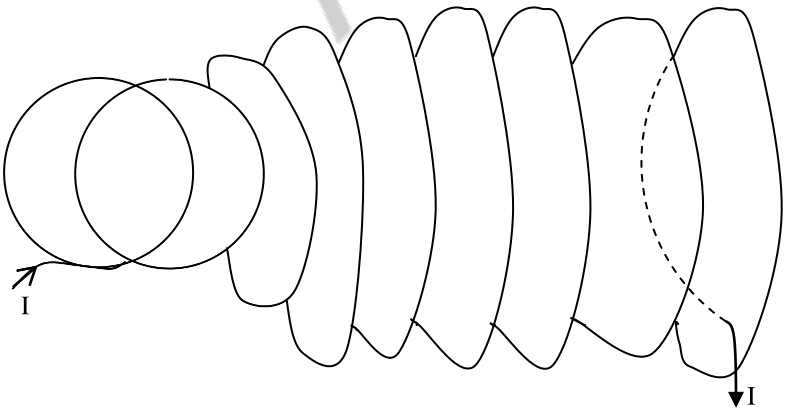
$$H = \frac{I}{2\pi a}\left[\frac{2}{\sqrt{2}}\right] = \frac{I}{\sqrt{2}\pi a}$$

H due to all the four sides $= \frac{4\times I}{\sqrt{2}\pi a} = \frac{4\times\sqrt{2}I}{2\pi a} = \frac{2\sqrt{2}I}{\pi a}$

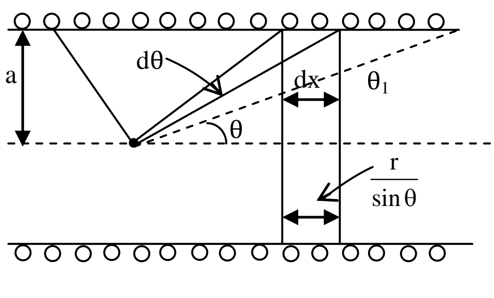
7) Derive the expression for the magnitude field intensity due to a solenoid:-

A helical coil or solenoid is usually used to produce a magnetic field. Let us calculate the magnetic field of such a coil.

A coil of wire wound its the form of a cylindrical as shown in fig. Let us assume there the turns in the windings are closely and ever spaced so that the number of turns per unit to of the solenoid is constant N. The path of the current through the coil is helical



(a) Solenoid



(b) Cut view of solenoid

$$\frac{rd\theta}{dx} = \sin\theta$$
$$\frac{rd\theta}{\sin\theta} = dx$$

If 'N' is the number of turns, $Ndx = \frac{Nrd\theta}{\sin\theta}$ and the total current element in the solenoid

$$Idx = \frac{INrd\theta}{\sin\theta}$$

$$DBx = \frac{\mu Ia^2}{2r^3} = \frac{\mu a^2}{2r^3} \cdot \frac{INrd\theta}{\sin\theta}$$
$$\sin\theta = \frac{a}{r}; \;\; r^2 = \frac{a^2}{\sin^2\theta}$$
$$dBx = \frac{\mu a^2\, I\, N\, d\theta}{2r^3 \sin\theta} = \frac{\mu a^2\, I\, N\, d\theta \sin^2\theta}{2\, a^2 \sin\theta} = \frac{\mu I\, N}{2}\sin\theta\, d\theta$$
$$Bx = \int dBx = \int_{Q_2}^{Q_1} \frac{\mu I\, N}{2}\sin\theta\, d\theta$$
$$Bx = \frac{\mu I\, N}{2}\left[\cos\theta_1 - \cos\theta_2\right]\bar{a}_x$$

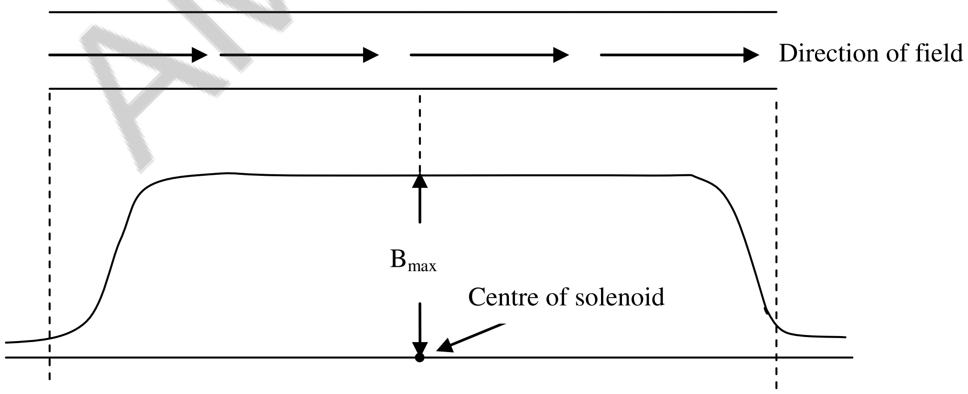
Case 1: In case the solenoid considered to be long and the point P lies in the muddle.

$$Q_1 = 0 \;\; \& Q_2 = \pi$$
$$B = \frac{\mu\, I\, N}{2}\left[\cos 0 - \cos\pi\right]$$
$$B = \frac{\mu\, I\, N}{2}\left[1-(-1)\right] = \mu\, I\, N$$

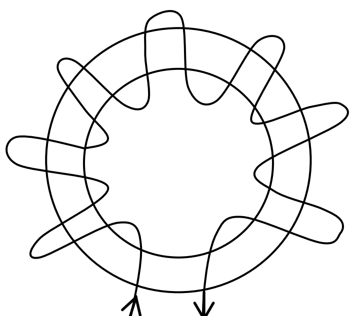
Case ii:- If the point lies at one end of the solenoid. $Q_1 = 0 \; \& Q_2 = \frac{\pi}{2}$

Then
$$B = \frac{\mu\, I\, N}{2}\left[\cos 0 - \cos \pi/2\right]$$
$$B = \frac{\mu\, I\, N}{2} \text{Jesla}$$

From the alone equations, it is clean that the magnitude field is one half at one end than at centre.



8) Derive the expression for the magnetic field intensity due to a toroid:-



Assuming diameter of the core is small and compared to the diameter of ring the circular paths through the core will be approximate of same length 2π r.

Ampere’s circuital law,

$$\int_{\ell} H.dl = \quad NIH$$

$$H\int_{\dagger} dl = NI \Rightarrow 2\pi r\ (H) = NI$$

$$\Rightarrow \overline{H} = \frac{NI}{2\pi r} = \frac{NI}{\ell}$$

Where L – mean circumference b the Toroid.

$$\overline{B} = \mu H$$

$$\overline{B} = \frac{\mu NI}{\ell}$$

9) State and proof Ampere’s circuital law?

Ampere’s circuital law:-

The law states “ the line integral of the magnetic field intensity (H) around a closed path is the ssame as the net current Ienc enclosed by the path

$\oint \overline{H}.dl = I_{enc}$ i.e, $\oint \overline{H}.dl = I \rightarrow$ general form

Ampere’s law is easily applied to determine H when the current distribution is symmetrical

By applying stoke’s theorem

$$I_{enc} = \oint_{L} \overline{H}.dl = \iint_{s} (\nabla \times H).\ ds \rightarrow (1)$$

But $I_{enc} = \iint_{s} J\ .\ ds \rightarrow (2)$

Comparing (1) & (2), $\nabla \times H = J \rightarrow$ point form or differential form

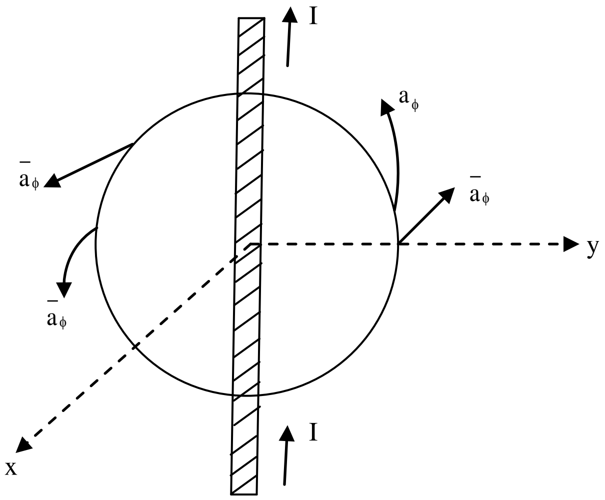
$\oint \overline{H}.dl = \iint_{s} (\nabla \times H).\ ds = \iint_{s} J\ .ds$ integral form of ACL

So, $\nabla \times H\ = J \neq 0$

So magnetic field is not conservative.

PROOF:-

Consider an infinitely long straight conductor carrying current I placed along z – axis consider a closed circular path of radius r. The point P is at a perpendicular distance ‘r’ for the conductor. Consider $\overline{dl}$ at point P which is in $a\phi$ direction.



$\overline{dl} = rd\phi \ \overline{a\phi}$

$\overline{H}$ obtained at point P from BIOT savart's law due infinitely long conductor is

$$\overline{H} = \frac{I}{2\pi r}\bar{a}_\phi$$
$$\overline{H}.\,dl = \frac{I}{2\pi r} rd\phi = \frac{I}{2\pi} d\phi$$
$$\oint \overline{H}.\overline{dl} = \int_{\phi=0}^{2\pi} \frac{I}{2\pi} d\phi \Rightarrow \frac{I}{2\pi}[\phi]_0^{2\pi} = I$$
$$\oint \overline{H}.\,dl = I$$

10) Explain in detail about the Applications of Ampere's circuit law:-

Amperes law is used to find H for symmetrical current distributions. For symmetrical current distribution $\overline{H}$ is either parallel or perpendicular to $\overline{dl}$.

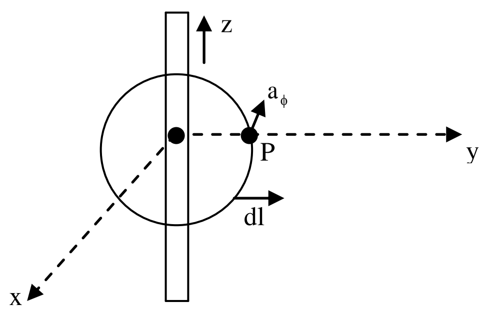
When $\overline{H}$ is parallel to $\overline{dl}$, $|H|$ = constant.

Applications of Ampere's circuit law:-

Case i:- Infinite line current (or) $\overline{H}$ due to infinitely long conduction. Consider an infinitely long straight conductor placed along Z – axis carrying current I. considers a point P on the closed path at which $\overline{H}$ is to be obtained. The radius of path is 'r' and hence P is a perpendicular distance 'r' from the conductor. Consider $\overline{dl}$ at point 'P' in $\vec{a}_\phi$ direction i.e, H_ϕ

$$\vec{H} = H_\phi \, \vec{a}_\phi .r\, d\phi\, \overline{a_\phi}$$

According to Amperes circuital law,

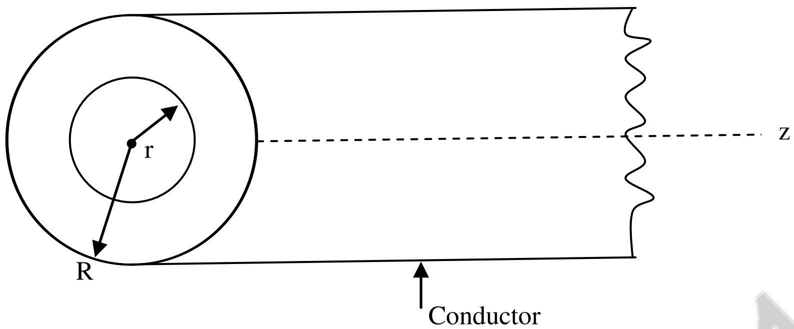


$$\oint H.\overline{dl} = I$$
$$\oint H_\phi \vec{a}_\phi.\, r\, d\phi\, \bar{a}_\phi = I$$
$$\int_0^{2\pi} H_\phi r\, d\phi = I$$
$$H_\phi\, r[\phi]_0^{2\pi} = I \Rightarrow H_\phi = \frac{I}{2\pi r}$$

$\overline{H}$ at point P is given by $\overline{H} = H_\phi a_\phi$

$$\vec{H} = \frac{I}{2\pi r}.\bar{a}_\phi \ \ A/m$$

Case ii:- $\overline{H}$ **due to cylindrical conductor (or) co – axial cable**



Consider a cylindrical conductor of radius R carries an uniform current of I amperes. It is placed along z – axis and has finite length.

$\overline{H}$ is to be obtained considering two regions.

Region 1:- With the conductor, r < R.

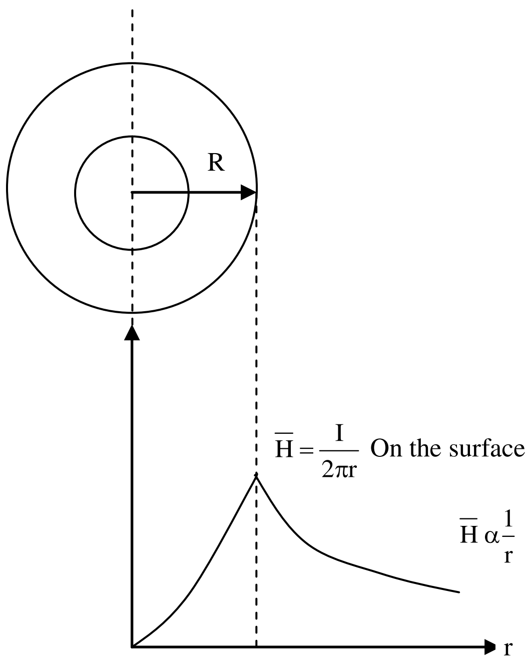
According to ampere's law, $\oint \overline{H}.dl = I_{enc}$

As current I flows uniformly, it flows across the cross sectional area πR^2, while the closed path enclosed only part of current which passed across the cross sectional area πR^2.

Hence current enclosed by path , $I_{enc} = I.\frac{\pi r^2}{\pi R^2}$

$$I_{enc} = I.\frac{r^2}{R^2}$$

$$\vec{H} = H_\phi \, \bar{a}_\phi, \ \overline{dl} = r\, d\phi\, a\phi$$
$$\vec{H}.\vec{dl} = H_\phi \, \bar{a}_\phi .\, r\, d\phi\, a\phi = H_\phi \, r\, d\phi$$
$$\int_0^{2\pi} H_\phi \, r\, d\phi = I.\frac{r^2}{R^2} \quad H_\phi = \frac{Ir}{2\pi R^2}$$
$$\vec{H} = \frac{Ir}{2\pi R^2}\vec{a\phi} \ \ A/m.$$



Region 2:- Outside the conductor, r > R

The conductor is infinite length along z – axis carrying a current I,

$$\vec{H} = \frac{I}{2\pi r}\bar{a}_\phi \ \ r > R$$

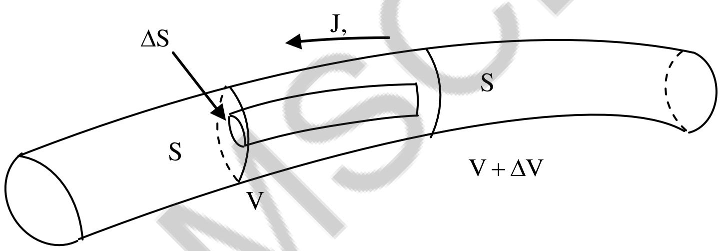
Outside the conductor, $\overline{H} \propto \frac{1}{r}$

Formulae:- Curl, distance & gradient in all the three system of co – ordinates.

Co – ordinate system	Cartesian	Cylindrical	Spherical
Curl $\nabla \times A$	$\nabla \times A = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$	$\nabla \times A = \begin{vmatrix} a_\rho & a_\phi & a_z \\ \frac{\partial}{\partial \rho} & \frac{1}{\rho}\frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & A_\phi & A_z \end{vmatrix}$	$\nabla \times A = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(H_\phi \sin\theta) - \frac{\partial H_\theta}{\partial_\phi}\right]\bar{a}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial H_r}{\partial\phi} - \frac{\partial}{\partial_r}(rH_\phi)\right]\bar{a}_\theta + \frac{1}{r}\left[\frac{\partial}{\partial_r}(rH_\phi) - \frac{\partial H_r}{\partial\theta}\right]\bar{a}_\phi$
Divergence	$\nabla.d = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$	$\nabla.D = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial\phi} + \frac{\partial D_z}{\partial z}$	$\nabla.D = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(D_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi}$
Gradient	$\nabla.d = \frac{\partial v}{\partial x}\bar{a}_x + \frac{\partial v}{\partial y}\bar{a}_y + \frac{\partial v}{\partial z}\bar{a}_z$	$\nabla.v = \frac{\partial v}{\partial\rho}\bar{a}_\rho + \frac{1}{\rho}\frac{\partial v}{\partial\phi}\bar{a}_\phi + \frac{\partial v}{\partial z}\bar{a}_z$	$\nabla.v = \frac{\partial v}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial v}{\partial\theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial v}{\partial\phi}\bar{a}_\phi$

11) State and prove point from of ohm's law.

Point form of ohm's law:-



Ohm's law is given by V = IR where 'R' is the resistance of the given medium. Let S and S' be two surface with potential V and $V + \Delta V$. The plate are separated through a distance $\Delta\ell$.

$$I = \iint_s J.\, N\, ds$$

If there is a charge –q, it will experience a force.

$$F = -\, q\, E$$

In free space, the electrons would get accelerated and its velocity would continuously increase.

In the crystalline material, the progress of the electron is impeded by its continual collisions with the thermally excited attained is drift velocity.

$$V_d = -\mu_e E \rightarrow (1)$$

The electron velocity is in a direction opposite to that of the electric filed.

We know that $J = \rho_e V_d \rightarrow (2)$

Sub (1) in (2), we get

$$J = -\rho_e \mu_e E$$
$$J = \sigma E$$

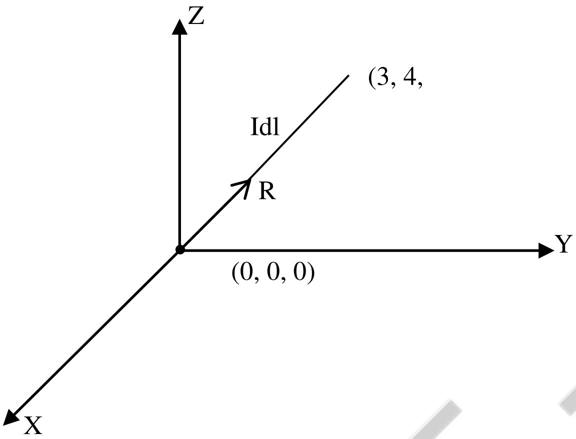
Point form of ohm's law.

PROBLEMS UNIT – III

Problems in BIOT – SAVARTS LAW:-

1. Find the magnetic field intensity at the origin due to a current element $\overline{Idl} = 3\pi(\overline{a}_x + 2\overline{a}_y + 3\overline{a}_z)\mu Am$ **at (3, 4, 5) in free space.**

Solution:-



$$\overline{dH} = \frac{\overline{Idl} \times \overline{a_R}}{4\pi R^2}$$

$$\overline{a_R} = \frac{\overline{R}}{|R|} = \frac{-(3\overline{a}_x + 4\overline{a}_y + 5\overline{a}_z)}{\sqrt{9+16+25}}$$

$$\overline{a_R} = -0.424 a_x - 0.565 a_y - 0.707 a_z$$

$$Idl = 3\pi\left(\overline{a}_x + 2\overline{a}_y + 3\overline{a}_z\right) \times 10^{-6}$$

$$\overline{Idl} \times \overline{a}_R = \begin{vmatrix} a_x & a_y & a_z \\ 3\pi & 6\pi & 9\pi \\ -0.424 & -0.565 & -0.707 \end{vmatrix} \times 10^{-6}$$

$$= (2.661\overline{a}_x - 5.339\overline{a}_y + 2.67\overline{a}_z) \times 10^{-6}$$

$$\overline{dH} = \frac{(2.661\overline{a}_x - 5.339\overline{a}_y + 2.67\overline{a}_z) \times 10^{-6}}{4\pi\left(\sqrt{50}\right)^2}$$

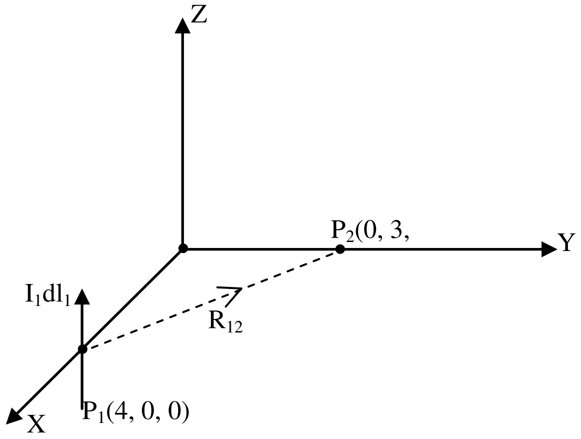
$$\overline{dH} = 4.24\overline{a}_x - 8.51\overline{a}_y + 4.25\overline{a}_z nA$$

2. Find the infinitesimal strength at P2 due to the current element $2\pi\overline{a}_z\mu A.m$ **at P1. The co – ordinates of P1 and P2 are (4, 0, 0) & (0, 3, 0).**

Solution:-

According to B.S.L, the infinitesimal field strength at P2 due to the current element $2\pi\overline{a}_z\mu A.m$ at P1is

$$d\overline{H}_2 = \frac{I_1\overline{dl}_1 \times \overline{a}_{R12}}{4\pi R_1^2} = \overline{R_{12}} = (0-4)\overline{a}_x + (3-0)\overline{a}_y + 0\overline{a}_z = -4\overline{a}_x + 3\overline{a}_y$$



$$\bar{a}_{R12} = \frac{\overline{R_{12}}}{|R_{12}|} = \frac{-4\bar{a}_x + 3\bar{a}_y}{\sqrt{16+9}} = \frac{-4\bar{a}_x + 3\bar{a}_y}{5}$$

$$I_1 dl_1 = 2\pi\bar{a}_z\,\mu Am$$

$$I_1 dl_1 \times \bar{a}_{R12} = \begin{vmatrix} a_x & a_y & a_z \\ 0 & 0 & 2\pi \\ -4/5 & 3/5 & 0 \end{vmatrix}$$

$$= \left(-3/5 \times 2\pi\right)\overline{a_x} - \left(4/5 \times 2\pi\right)\overline{a_y}$$

$$= \frac{-2\pi}{5}\left[3a_x + 4a_y\right]$$

$$dH_2 = \frac{\frac{-2\pi}{5}\left[3\overline{a_x} + 4\overline{a_y}\right]}{4\pi(5)^2}$$

$$dH_2 = -12\overline{a_x} - 16\overline{a_y}\,nA/m$$

3. A filamentary current of 10a is denoted inward from infinity to the origin on the positive x – axis & then outward to infinity along positive y – axis. Find ‘H’ at p(0, 0, 1).

Solution:-

$$\overline{dH} = \frac{\overline{Idl} \times \overline{a_R}}{4\pi R^2} = \frac{\overline{Idl} \times \overline{R}}{4\pi R^3} \qquad \overline{R} = -x\bar{a}_x + \bar{a}_z;\ Idl = 10dx\bar{a}_x$$

$$H_1 = \frac{10}{4\pi}\int_{\infty}^{0} \frac{dx\bar{a}_x \times (-x\bar{a}_x + \bar{a}_z)}{\left[\sqrt{1+x^2}\right]^3} = \frac{-5}{2\pi}\int_{\infty}^{0} \frac{dx a_y}{\left(1+x^2\right)^{3/2}}$$

$$= \frac{-5}{2\pi}\frac{x}{\sqrt{1+x^2}}\Bigg|_{\infty}^{0} \bar{a}_y = \frac{5}{2\pi}\bar{a}_y A/m$$

$$\overline{H}_2 = \frac{5}{2\pi}\int_{0}^{\infty} \frac{(dy\bar{a}_y) \times (-y\bar{a}_y + \bar{a}_z)}{\left[\sqrt{1+y^2}\right]^3}$$

$$= \frac{5}{2\pi}\int_{0}^{\infty} \frac{dy\bar{a}_x}{(Hy^2)^{3/2}}$$

$$= \frac{5}{2\pi}\bar{a}_x$$

$$H = H_1 + H_2 = 0.796\left[\bar{a}_x + \bar{a}_y\right]A/m$$

4. In cylindrical co- ordinates, $\bar{H} = (2\rho - \rho^2)\bar{a}_\phi \, A/m$ for 0 < ρ <1. (i) Determine the current density as a function of ρ within this cylindrical (ii) What is the total current passing through surface z =0, 0 < ρ in $\bar{a}_z$ direction? (iii) Verify the same using stoke's theorem.

Solution:- $\bar{H} = (2\rho - \rho^2)\bar{a}_\phi A/m \; 0 < \rho < 1$

$$J = \nabla \times H = \frac{1}{\rho}\begin{vmatrix} a_\rho & \rho\bar{a}_\phi & \bar{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & \rho(2\rho - \rho^2) & 0 \end{vmatrix}$$
$$= \frac{1}{\rho}\left[a_\rho(0 - \frac{\partial}{\partial z}(2\rho^2 - \rho^3)\right] + 0a_\phi + \frac{\partial}{\partial \rho}(2\rho^2 - \rho^3)\bar{a}_z$$
$$= \frac{1}{\rho}\left[4\rho - 3\rho^2\right]\bar{a}_z$$
$$J = (4 - 3\rho)\bar{a}_z$$

(ii) $$I = \int_s \bar{J} \; \bar{ds} = \int_0^{2\pi} J_2 \; \rho d\rho \; d\phi$$

$$= \int_0^{2\pi}\int_0^1 (4 - 3\rho)\rho d\rho \; d\phi$$
$$= \int_0^{2\pi} d\phi \int_0^1 (4 - 3\rho)\rho \; d\rho$$
$$= (\phi)_0^{2\pi}\left(\frac{4\rho^2}{2} - \frac{3\rho^3}{3}\right)_0^1$$
$$= 2\pi(2 - 1) = 2\pi$$

I = 6.28 A

(ii) $$\int_\ell \bar{H}.\,\bar{dl} = \int_0^{2\pi} (2\rho - \rho^2)\bar{a}_\phi \; \rho d\phi \; a_\phi$$

$$= 2\pi(2 - 1) = 2\pi = 6.28A$$

5. The $\bar{H} = \left[y\cos(\alpha x)\right]\bar{a}_x + (y + e^x)\bar{a}_z A/m$. Determine the current density over the yz plane.

Solution:-

$$J = \nabla \times H = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial y} \\ y\cos(\alpha x) & 0 & y + e^x \end{vmatrix}$$
$$= a_x\left[\frac{\partial}{\partial y}(y + e^x) - 0\right] + \left[\frac{\partial}{\partial z}(y\cos(\alpha x) - \frac{\partial}{\partial x}(y + e^x)\right] + \left[0 - \frac{\partial}{\partial y}(y\cos(\alpha x)\right]\bar{a}_z$$
$$\bar{J} = a_x + (-e^x)\bar{a}_y - \cos\alpha x\,\bar{a}_z$$

Since x = 0, on yz plane ,

$$\bar{J} = \bar{a}_x - e^0\bar{a}_y - \cos(0)\bar{a}_z$$
$$J = \bar{a}_x - \bar{a}_y - \bar{a}_z A/m^2$$

6. A flat perfectly conducting surface in xy plane is siluated in a magnetic field,

$$\overline{H} = \begin{cases} 3\cos x\,\overline{a}_x + z\cos x\,\overline{a}_y, & z \geq 0 \\ 0, & z < 0 \end{cases}$$ **, find 'J' on the conductor surface.**

Solution:-

$$J = \nabla \times \overline{H} = \begin{vmatrix} \overline{a}_x & \overline{a}_y & \overline{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 3\cos x & z\cos x & 0 \end{vmatrix} = a_x\left[0 - \frac{\partial}{\partial z}(z\cos x)\right] + a_y\left[0 - \frac{\partial}{\partial y}(3\cos x)\right] + a_z\left[\frac{\partial}{\partial x}(z\cos x) \frac{-\partial}{\partial y}(3\cos x)\right]$$

$$= a_x(-\cos x) + a_y(0) + a_z(-z\sin x)$$

$$J = \begin{cases} -\cos x\,\overline{a}_x - z\sin x\,\overline{a}_z, & z \geq 0 \\ 0, & z < 0 \end{cases}$$

7. If $\overline{H} = y\overline{a}_x - x\overline{a}_y\, a/m$ on plane z = 0, (i) Determine J (ii) Verify Ampere's law by taking the circulation of $\overline{H}$ around the rectangular path with z = 0, 0 < x < 5 & -2 < y < 6.

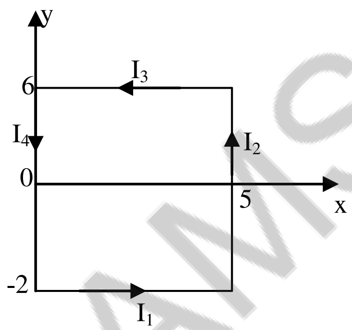
Solution:-

$$J = \nabla \times H = \begin{vmatrix} a_x & a_y & a_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ y & -x & 0 \end{vmatrix}$$

(i) $$= a_x[0-0] + a_y[0-0] + a_z\left[\frac{\partial}{\partial x}(-x) \frac{-\partial}{\partial y}\right]$$

$$= -2\overline{a}_z A/m^2$$

(ii) $$\int_{\ell} \overline{H}.\,\overline{dl} = \left(\int_{\ell 1} + \int_{\ell 2} + \int_{\ell 3} + \int_{\ell 4}\right)\overline{H}\ \overline{dl}$$



$$\int_{\ell 1} \overline{H}.dl = \int_0^5 (y\overline{a}_x - x\overline{a}_y)(dx\overline{a}_x)$$

$$= \int_0^5 ydx = (xy)_0^5\Big|_{y=0} = -10$$

$$\int_{\ell 2} \overline{H}.dl = \int_{-2}^{6} (y\overline{a}_x - x\overline{a}_y)(dx\overline{a}_y)$$

$$= \int_{-2}^{6} xdy = -[xy]_{-2}^{6}\Big|_{x=5} = -(5y)_{-2}^{6}$$

$$= -[5\times 6 - (5\times -2] = -40$$

$$\int_{\ell 3} \overline{H}.dl = \int_5^0 (y\overline{a}_x - x\overline{a}_y)(dx\overline{a}_x)$$

$$= \int_5^0 ydx = [yx]_5^0\Big|_{y=0} = (6x)_5^0 = -30$$

$$\int_{\ell 4} \overline{H}.dl = \int_6^{-2} (y\overline{a}_x - x\overline{a}_y)(dx\overline{a}_y)$$

$$= \int_6^{-2} -xdy\Big|_{x=0} = 0$$

$$\int_{\ell} H.dl = -10 - 40 - 30 = -80A$$

$$I = \int \overline{J}.\ \overline{ds} = \int_{y=-2}^{6}\int_0^5 (-2a_z)(dxdydz)$$

$$= -2\int_{-2}^{6}\int_0^5 dxdy$$

$$= -2(x)_0^5(y)_{-2}^6$$

$$= -2(5)(6+2) = -80A$$

Thus verified.

8. If an in rotational field $\overline{F} = (x+2y+\ell z)\overline{a}_x + (mx-3y-z)\overline{a}_y + (4x+hy+2z)\overline{a}_z$, Determine the constants ℓ, m & n for the above field.

Solution:-

$$\nabla\times F = 0$$

$$\nabla\times F = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

$$a_x\left[\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right] = a_x\left[\frac{\partial}{\partial y}(4x+hy+2z) - \frac{\partial}{\partial z}(mx-3y-z)\right] = a_x(n+1)$$

$$a_y\left[\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z}\right] = a_y\left[\frac{\partial}{\partial x}(4x+hy+2z) - \frac{\partial}{\partial z}(x+2y-\ell z)\right] = a_x(4-\ell)$$

$$a_z\left[\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right] = a_z\left[\frac{\partial}{\partial x}(mx-3y-z) - \frac{\partial}{\partial z}(x+2y-\ell z)\right] = a_z(m+2)$$

h = -1, ℓ=4, m = +2

9. $\overline{H} = \frac{x+2y}{z^2}\overline{a}_y + \frac{2}{z}\overline{a}_z$, Find $\nabla\times H, \overline{J}$ & 'I' passing through the surface z = 4, 1 < x < 2 & 3 < y < 5 in the $\overline{a}_z$ direction using $\overline{J}$

Solution:-

(i) $\nabla \times H = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & \frac{x+2y}{z^2} & \frac{2}{z} \end{vmatrix}$

$$= a_x\left[\frac{\partial}{\partial y}(2z^{-1}) - \frac{\partial}{\partial z}\left[(x+2y)z^{-2}\right]\right] + a_y\left[\frac{\partial}{\partial x}(2z^{-1}) - 0\right] + a_z\left[\frac{\partial}{\partial x}\left[(x+2y)z^{-2}\right] - 0\right]$$

$$\nabla \times \bar{H} = \frac{2(x+2y)}{z^3}\bar{a}_x + \frac{1}{z^2}\bar{a}_z$$

(ii) $\nabla \times H = \bar{J}$

$$J = 2\left(\frac{x+2y}{z^3}\right)\bar{a}_x + \frac{1}{z^2}\bar{a}_z = A/m^2$$

$$I = \int \bar{J}.ds = \left(\frac{2(x+2y)}{z^3}\bar{a}_x + \frac{1}{z^2}\bar{a}_z\right)dxdydz$$

$$= \int_3^5\int_1^2 \frac{1}{z^2}dxdy\Big|_{z=4}$$

(iii) $I = \int_3^5\int_1^2 \frac{1}{16}dxdy$

$$= \frac{1}{16}(x)_1^2(y)_3^5 = \frac{1}{16}[2-1][5-3]$$

$$I = \frac{1}{8}A$$

10. $\bar{H} = x^2\bar{a}_x + 2yz\,\bar{a}_y + (-x^2)\bar{a}_z A/m$ **. Find the 'J' at point (i) (2, 3, 4) (ii)** $\rho = 6, \phi = 45°, z = 3$ **(iii)** $r = 3.6, \theta = 60°, \phi = 90°$

Solution:-

Given $\bar{H} = x^2\bar{a}_x + 2yz\,\bar{a}_y + (-x^2)\bar{a}_z$

$$\nabla \times H = \bar{J} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & 2yz & -x^2 \end{vmatrix}$$

$$= a_x\left[\frac{\partial}{\partial y}(-x^2) - \frac{\partial}{\partial x}(2yz)\right] + a_y\left[\frac{\partial}{\partial x}(-x^2) - \frac{\partial}{\partial z}(x^2)\right] + a_z\left[\frac{\partial}{\partial x}(2yz) - \frac{\partial}{\partial y}(x^2)\right]$$

$$J = -2y\bar{a}_x + 2x\bar{a}_y$$

At P(2, 3, 4) $= -2(3)\bar{a}_x + 2(2)\bar{a}_y = -6\bar{a}_x + 4\bar{a}_y A/m^2$

At (6, 45°, 3)

$x = \rho\cos\phi,\qquad y = \rho\sin\phi$

$x = 6\cos 45°,\qquad y = 6\sin 45°$

$x = 4.242\qquad y = 4.242$

$$\nabla \times H = -2y\bar{a}_x + 2x\bar{a}_y$$
$$= -2(4.242)\bar{a}_x + 2(4.242)\bar{a}_y$$
$$\nabla \times H = -8.484\bar{a}_x + 8.484\bar{a}_y$$

(iii) At (3.6, 60°,90°)

$$x = r\sin\theta\cos\phi = 3.6\sin 60^\circ \cos 90^\circ = 0$$
$$y = r\sin\theta\cos\phi = 3.6\sin 60^\circ \sin 90^\circ = 1.8R3 = 3.117$$
$$z = r\cos\theta \quad = 3.6\cos 60^\circ \quad = 1.8$$

$$\nabla \times H = -2(3.117)\bar{a}_x + 2(0)\bar{a}_y$$
$$= -6.234\bar{a}_x \ A/m^2$$

11. $\bar{H} = -y(x^2+y^2)\bar{a}_x + x(x^2+y^2)\bar{a}_y \ A/m$ **in the z = 0 plane for -5 < x, y < 5. Calculate the current passing thro z = 0 plane & through the region -1 < x < 1 & -2 < y < 2.**

Solution:-

$$J = \nabla \times H = \begin{vmatrix} a_x & a_Y & a_Z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ -y(x^2+y^2) & x(x^2+y^2) & 0 \end{vmatrix}$$

$$= a_x(0-0) - a_y(0-0) + a_z\left[\frac{\partial}{\partial x}(x^3+xy^2) - \frac{\partial}{\partial y}(-x^2y+y^3)\right]$$
$$= (3x^2+y^2) + x^2 + 3y^2 = 4(x^2+y^2)\bar{a}_z A/m^2$$

$$I = \int_s (\nabla \times H).\, ds = \int \bar{J}\, ds$$

$$= \int_{-2}^{2}\int_{-1}^{1} 4(x^2+y^2)\,dx\,dy$$
$$= \int_{-2}^{2}\left[\frac{4x^3}{3} + 4y^2x\right]_{-1}^{1} dy$$
$$= \int_{-2}^{2}\left[\left(\frac{4}{3}+4y^2\right) + \frac{4}{3} + 4y^2\right]dy$$
$$= \int_{-2}^{2}\left(\frac{4}{3}+8y^2\right)dy$$
$$= 2\int_{0}^{2}\left(\frac{4}{3}+8y^2\right)dy = 2\left[\frac{4}{3}+\frac{8y^3}{3}\right]_0^2$$
$$= 2\left[\frac{8}{3}+\frac{64}{3}\right] = \frac{144}{3} = 48A.$$

Problems on magnetic flux and magnetic flux density

1. If $H = r\sin\phi\bar{a}_r + 2.5r\sin\theta\cos\phi\bar{a}_\phi A/m$ **exists in a medium** $\mu_r = 3$, **determine** $\bar{B}$.

Solution:-

$$\bar{H} = r\sin\phi\bar{a}_r + 2.5r\sin\theta\cos\phi\bar{a}_\phi A/m \ \& \mu_r = 3$$
$$B = \mu H = \mu_0\mu_r H = 4\pi\times 10^{-7}\times 3\left[r\sin\phi\bar{a}_r + 2.5r\sin\theta\cos\phi\bar{a}_\phi\right]$$
$$B = 3.77r\sin\phi\bar{a}_r + 9.42r\sin\theta\cos\phi\bar{a}_\phi \ Wb/m^2$$

2. IF $\bar{B} = 2.5\sin\dfrac{\pi x}{2}e^{-2y}\bar{a}_z \ Wb/m^2$ **, find the flux for z = 0,** $y \ge 0 \ \& 0 \le x \le 2m.$

Solution:-

$$
\begin{aligned}
\phi &= \int_s \overline{B}.\ \overline{ds} \\
&= \int_s \left(2.5\sin\frac{\pi x}{2} e^{-2y}\overline{a}_z\right).(dxdy\overline{a}_z) \\
&= \int_{x=0}^{2}\int_{y=0}^{\infty} 2.5\sin\frac{\pi x}{2} e^{-2y} dx\, dy \\
&= 2.5\left[-\cos\frac{\pi x}{2}\right]_{x=0}^{2}\left[\frac{e^{-2y}}{-2}\right]_{y=0}^{\infty} \\
&= 2.5\left[\frac{-\cos\pi+\cos 0}{\pi/2}\right]\left[\frac{e^{-\infty}}{-2}\frac{-e^{0}}{-2}\right] \\
&= \frac{2.5\times 2}{\pi}\times 2\times\frac{1}{2} \\
&= 1.592\,\text{Wb.}
\end{aligned}
$$

3. If $\overline{H} = 2.39\times 10^{-6}\cos\phi\overline{a}_\rho\ A/m.$ **Determine the flux defined by** $0\le\phi\le\pi/4\ \&\ 0\le z\le 2m.$

Solution:-

$$
\begin{aligned}
\phi &= \int_s \overline{B}.\ ds = \int_s \mu_0\mu_r\overline{H} = 4\pi\times 10^{-7}\int_0^2\int_{\phi=0}^{\pi/4}\left(\frac{2.39\times 10^6}{\rho}\cos\phi\overline{a}_\rho\right)\left(\rho d\phi dz\overline{a}_\rho\right) \\
\phi &= 2.39\times 10^6\times 4\pi\times 10^{-7}\left[\sin\phi\right]_0^{\pi/4}\left[z\right]_0^2 \\
&= 2.39\times 4\pi\times 10^{-1}\left[\sin\pi/4 - \sin 0\right]\left[2\right] \\
\phi &= 4.24\,\text{Wb}
\end{aligned}
$$

4. Given $\overline{B} = \rho\sin\phi\overline{a}_\phi\ Wb/m^2$ **. Determine total flux crossing the surface defined by** $1\le\rho\le 2, \phi = \pi/4\ \&\ 0\le z\le 5.$

Solution:-

$$
\begin{aligned}
\overline{B} &= \rho\sin\phi\overline{a}_\phi \\
\phi &= \int\overline{B}.ds = \int_{z=0}^{5}\int_{\rho=1}^{2}\left(\rho\sin\phi\overline{a}_\phi\right)(d\rho dz\overline{a}_\phi) \\
\phi &== \left[z\right]_0^5\left[\frac{\rho^2}{2}\right]_1^2\sin\phi
\end{aligned}
$$

At $\phi = \pi/4, \phi = \left[5\right]\left[\frac{4}{2}-\frac{1}{2}\right]\sin 45° = 5\times 3/2\times\frac{1}{\sqrt{2}} = 5.303\text{Wb}$

5. Given $\overline{B} = \left(2/\rho\right)\overline{a}_\phi T.$ **Determine the magnetic flux** ϕ **crossing the plane defined by** $0.5\le\rho\le 2.5m\ \&\ 0\le z\le 3.$

Solution:-

$$
\begin{aligned}
\phi &= \int_0^3\int_{0.5}^{2.5}\left(\frac{2}{\rho}\overline{a}_\phi\right).\ (d\rho dz\overline{a}_\phi) \\
&= 2\left[\ln\rho\right]_{0.5}^{2.5}\left[z\right]_0^3 \\
&= 6\ln\left[2.5\right] = 9.66k/b
\end{aligned}
$$

6. Given mean circumference $\ell = 1.2m$ **& area of 8cm^2. N = 480turns**. I = 2A, ϕ **=1 Wb. What is the permeability**

Solution:-

$$\ell = 1.2\text{m},\ A = 8\times\left(10^{-2}\right),\ I = 2A$$
$$N = 480\,\text{turn} \quad \phi = 1\,\text{Wb}$$

$$B = \frac{\phi}{a} = \frac{1}{8\times 10^{-4}} = \frac{100\times 10^{2}}{8} = 1250\,\text{Wb}/\text{m}^2$$
$$B = \mu H = \mu_0\mu_r H$$
$$H = \frac{NI}{\ell} = \frac{480\times 2}{1.2} = 800\text{A}-\text{T}/\text{m}.$$
$$\mu_r = \frac{B}{\mu_0 H} = \frac{1250}{4\pi\times 10^{-7}\times 800} = 0.124\times 10^{7}$$

7. Calculate b, due to a coil N = 1000 A- T, area = 100 cm^2 & h = 10m.

Solution:-

NI = 1000Ampere turns h = 10m, area = 100×10^{-4}

$$\pi a^2 = 100\times 10^{-4} \Rightarrow a = 5.64\,\text{cm}.$$

$$H = \frac{1000\times(5.64\times 10^{-2})^2}{2\left[100+(5.61\times 10^{-2})^2\right]^{3/2}} = 1.59\times 10^{-3}$$
$$B = \mu H = 1.59\times 10^{-3}\times 4\pi\times 10^{-7}\times 1$$

UNIT III

STATIC MAGNETIC FIELD

BIOT – SAVART'S LAW VECTOR FORM:-

State Biot – savart's law in vector form?

The magnetic field intensity at any point at a distance r from a current carrying conductor is directly proportional to

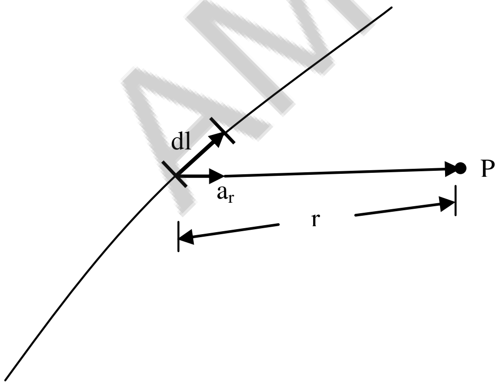
(i) The current flowing through the conductor
(ii) The infinitesimal length of the conductor
(iii) The sine of the angle between the conductor and line joining the conductor with point 'P' where the magnitude field intensity is to be calculated.
(iv) And is inversely proportional to the square of the distance between them

$$\bar{H} \propto I$$
$$dl$$
$$\sin\theta$$
$$\frac{1}{r^2}$$
$$\bar{H} \propto \frac{I\,dl\,\sin\theta}{r^2}$$
$$\bar{H} = \frac{kI\,dl\,\sin\theta}{r^2}$$
$$K = \frac{1}{4\pi} = \text{constant of proportional}$$
$$\bar{H} = \frac{I\,dl\,\sin\theta}{4\pi r^2}$$

The direction of the magnitude field intensity is perpendicular to the plane containing the conductor carrying current the line joining the conducts to the point P where magnitude field intensity to be calculated.

$$\bar{H} = \frac{I\,dl\,\sin\theta}{4\pi r^2}\bar{a}_r$$

BIOT SAVARTS law in vector form $\bar{H} = I\frac{\overline{dl} \times \bar{a}_r}{4\pi r^2} \rightarrow (1)$



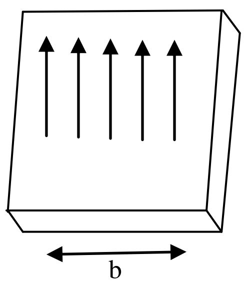
$$\bar{a}_r = \frac{\bar{r}}{|r|}$$

We know that

$$|r| = r$$

We know that $\bar{H} = I\frac{\overline{dl} \times \bar{r}}{4\pi r^3}$

Let us a rectangular sheet of width 'v' and sheet current density k, then the total current I is kb.



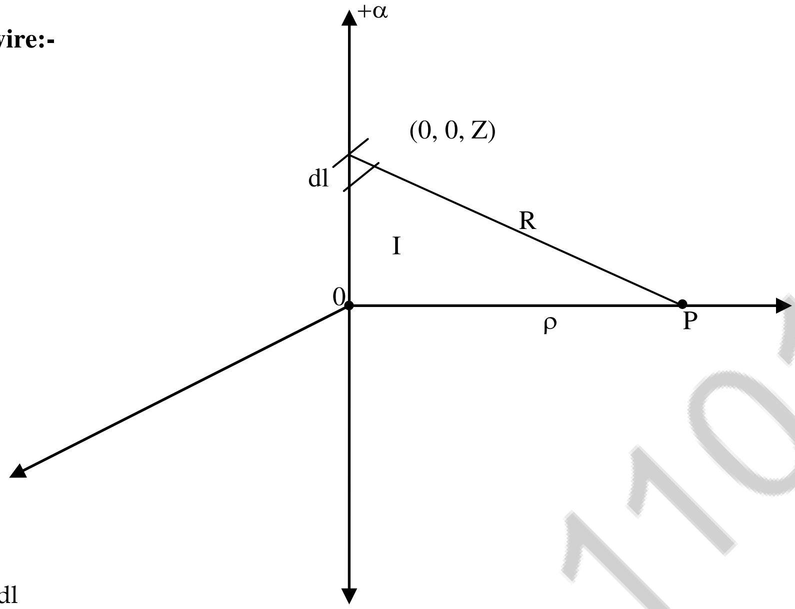
$I\,dl = K\,ds = J\,dv$

$$\overline{H} = \oint_S \frac{K \times \overline{a}_r}{4\pi r^2}\,ds$$

$$\overline{H} = \oint_V \frac{J \times \overline{a}_R}{4\pi R^2}\,dv$$

Determine that magnetic field intensity due to finite and infinite wire carrying a current I:

Due to infinite wire:-



$$R = P - dl$$
$$= \rho\overline{a}_\rho - z\overline{a}_z$$

$$\overline{dH} = \frac{I\overline{dl} \times \overline{a}_R}{4\pi R^2}$$

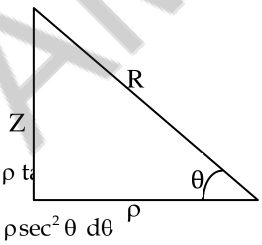
$$\overline{dl} = dz\,\overline{a}_z$$

$$\overline{a}_R = \frac{\overline{R}}{|R|} = \frac{\rho\overline{a}_\rho - z\overline{a}_z}{\sqrt{\rho^2 + z^2}}$$

$$\overline{dH} = \frac{I\,dz\,\overline{a}_z \times (\rho\overline{a}_\rho - z\overline{a}_z)}{4\pi(\rho^2 + z^2)\sqrt{\rho^2 + z^2}}$$

$$\overline{dH} = \frac{I\,\rho dz\,\overline{a}_z \times \overline{a}_\rho - I z\,dz\,\overline{a}_z \times \overline{a}_z}{4\pi(\rho^2 + z^2)^{3/2}}$$

$$\overline{dH} = \frac{I\rho dz\overline{a}_\phi}{4\pi(\rho^2 + z^2)^{3/2}}$$



$$\tan\theta = \frac{z}{\rho} \Rightarrow z = \rho\tan\theta$$

$$dz = \rho\sec^2\theta\,d\theta$$

When

$$z = +\infty,\ \theta = +\frac{\pi}{2}$$

$$z = -\infty,\ \theta = -\frac{\pi}{2}$$

$$\overline{dH} = \frac{I\rho(\rho \sec^2\theta)d\theta\,\bar{a}_\phi}{4\pi(\rho^2+\rho^2\tan^2\theta)^{3/2}}$$

$$\overline{dH} = \frac{I\rho^2 \sec^2\theta d\theta}{4\pi\left[\rho^2(1+\tan^2\theta)\right]^{3/2}}\ \bar{a}_\phi$$

$$\int_{-\infty}^{+\infty} dH = \frac{I\rho^2 \sec^2\theta d\theta}{4\pi\rho^3 \sec^3\theta}\bar{a}_\phi$$

$$d\overline{H} = \frac{I}{4\pi\rho}\cos\theta\, d\theta\, \bar{a}_\phi$$

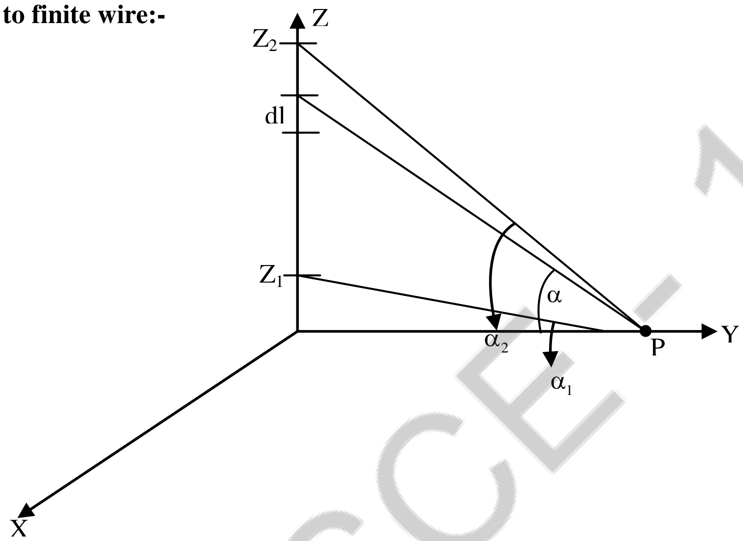
$$\overline{H} = \int_{-\pi/2}^{+\pi/2} \frac{I}{4\pi\rho}\cos\theta\, d\theta\, \bar{a}_\phi$$

$$= \frac{I}{4\pi\rho}\left[\sin\theta\right]_{-\pi/2}^{+\pi/2}\ \bar{a}_\phi$$

$$\overline{H} = \frac{I}{4\pi\rho}\times 2\ \bar{a}_\phi$$

$$\overline{H} = \frac{I}{2\pi\rho}\ \bar{a}_\phi$$

Due to finite wire:-



Let us consider a current carrying conductor of length z. Let us consider z1 and z2 inclined at α_1 and α_2

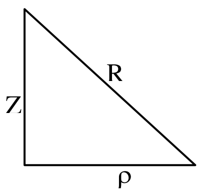
$$\overline{H} = \int_{z_1}^{z_2} \frac{I\overline{dl}\times\bar{a}_R}{4\pi R^2}$$

$$= \int_{z_1}^{z_2} \frac{I dz\ \bar{a}_z \times(\rho\bar{a}_\rho - z\bar{a}_z)}{4\pi(\rho^2+z^2)\sqrt{\rho^2+z^2}}$$

$$= \int_{z_1}^{z_2} \frac{I\,\rho dz\,\bar{a}_z\times\bar{a}_\rho - Iz\ dz\ \bar{a}_z\ \times\bar{a}_z}{4\pi(\rho^2+z^2)^{3/2}}$$

$$= \int_{z_1}^{z_2} \frac{I\rho dz\bar{a}_\phi}{4\pi(\rho^2+z^2)^{3/2}}$$

$$= \int_{\infty_1}^{\infty_2} \frac{I\rho^2\sec^2\alpha\ d\alpha\ \bar{a}_\phi}{4\pi(\rho^2+\rho^2\tan^2\alpha)^{3/2}}$$



$z = \rho \tan \alpha$

$dz = \rho \sec^2 \alpha \, d\alpha$

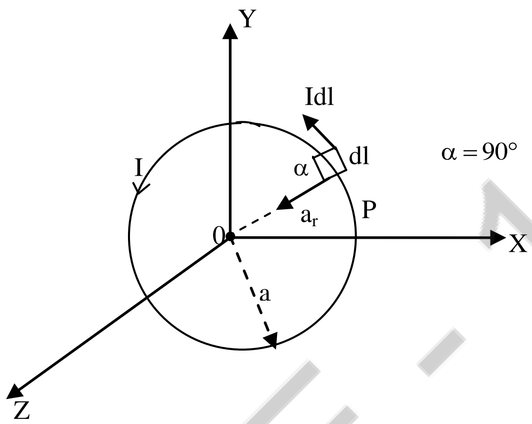
$$H = \int_{\infty_1}^{\infty_2} \frac{I\rho^2 \sec^2 \alpha \; d\alpha}{4\pi(\rho^2 + \rho^2 \tan^2 \alpha)^{3/2}} \bar{a}_\phi$$

$$= \int_{\infty_1}^{\infty_2} \frac{I\rho^2 \sec^2 \alpha \; d\alpha}{4\pi(\rho^3 \sec^3 \alpha)} \bar{a}_\phi$$

$$= \frac{I}{4\pi\rho} \int_{\infty_1}^{\infty_2} \cos \alpha \, d\alpha \; \bar{a}_\phi$$

$$\bar{H} = \frac{I}{4\pi\rho}\left[\sin \alpha_1 - \sin \alpha_2\right] \; \bar{a}_\phi$$

Determine the magnetic field at the centre of a circular wire carrying a current i in the anti – clockwise direction. The radius of the circle is 'a' & the wire is in XY plane:-



The field intensity at O is given by $H = \oint dH$, where dH is the field intensity at 0 due to any current element Idl. The direction of dl at any point P on the circular wire is given by the tangent at 'P' the direction of current flow.

The unit vector at P directed towards the centre '0' is obviously along the radius p0, so that α = 90°

$$|dH| = \frac{I\,dl}{4\pi a^2} \sin 90°$$

$$= \frac{I\,dl}{4\pi a^2}$$

The direction of the vector dH is given by dl × ar, along z – axis in the positive direct

Hence, $|dH| = \frac{I\,dl}{4\pi a^2} \bar{a}_z$

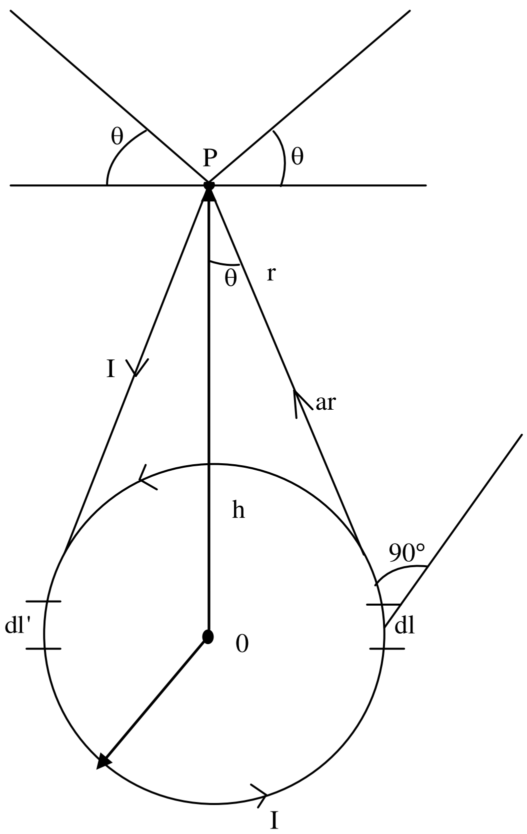
$$H = \bar{a}_z \int \frac{I\,dl}{4\pi a^2}$$

$$\bar{H} = \bar{a}_z \frac{I}{4\pi a^2} \int dl$$

Consequently, $\bar{H} = \bar{a}_z \frac{I}{4\pi a^2}(2\pi a)$

$$\bar{H} = \frac{I}{2a} \bar{a}_z$$

Determine the magnetic field at any point on the line through the centre at distance 'h' from the centre and perpendicular to the plane of a circular loop of radius of radius 'a' & current I.



Consider P as the point distant h from the plane of the loop.

To find the field intensity at P:-

Consider two diametrally opposite elements of the wire loop dl & dl^1. The field intensity at P at distant at r from the current element Idl is given by

$$d\overline{H} = \frac{I\,dl \times \bar{a}_r}{4\pi r^2}$$

As the vectors dl and $\bar{a}_r$ are perpendicular the value of dH is given by

$$dH = \frac{I\,dl}{4\pi r^2}\bar{a}_r .\sin 90°$$

$$dH = \frac{I\,dl}{4\pi r^2}$$

This field is oriented at an angle Q to the plane of loop. The diametrally opposite element Idl will also produce a field of magnitude equal to dH.

$$dH = \frac{I\,dl\,\sin\theta}{4\pi r^2}$$

Where $\sin\theta = \frac{a}{(a^2 + h^2)^{1/2}}$ and $r^2 = a^2 + h^2$

The resultant field intensity at that point is given by integrating the z – component of the field contributions of all the current elements

$$H_p = \oint dHz = \oint \frac{I\,dl}{4\pi(a^2+h^2)} \cdot \frac{a}{(a^2+h^2)^{1/2}}$$

$$= \frac{Ia}{4\pi(a^2+h^2)^{3/2}} \oint dl$$

$$H_p = \frac{Ia^2}{4\pi(a^2+h^2)^{3/2}} \times 2\pi a$$

$$H_p = \frac{Ia^2}{2(a^2+h^2)^{3/2}}$$

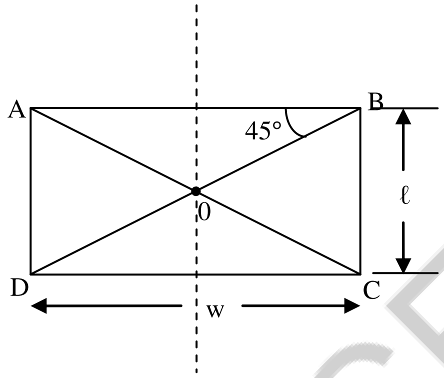
As the field is directed along z – axis

$$H_p = \frac{Ia^2}{2(a^2+h^2)^{3/2}}\ \bar{a}_z$$

If h = 0, P continues with 0,the centre of the wire – loop.

$$\bar{H} = \frac{Ia^2}{2a^3} = \frac{I}{2a}\ \bar{a}_z$$

Derive the expression for the magnitude field intensity due to a rectangular loop:-



Consider a rectangular loop ABCD carrying a current I through the loop. Let 'L' be the breadth of the loop.

$$\left.\begin{array}{l}\text{Magnitude field intensity H due}\\ \text{To and AB at '0'}\end{array}\right\} = \frac{I}{4\pi\rho}(\sin\alpha_2 - \alpha_1)$$

Substitute $\rho = \frac{L}{2};\ \alpha_1 = \alpha_2 = 45°$

$$H = \frac{I}{4\pi \ell/2}\left[\sin(45°) - \sin(-45°)\right]$$

$$= \frac{I}{2\pi\ell}\left[\frac{1}{\sqrt{2}} - \left(\frac{-1}{\sqrt{2}}\right)\right]$$

$$= \frac{I}{2\pi\ell}\left[\frac{2}{\sqrt{2}}\right]$$

$$H = \frac{I}{\sqrt{2}\pi\ell}$$

$$\text{Similarly 'H' to arm CD at 'o'}\} = \frac{I}{\sqrt{2}\pi\ell}$$

$$\left.\begin{array}{r}\text{Magnitude field intensity due to}\\ \text{Arm Ad \& BC}\end{array}\right\} = \frac{2I}{\sqrt{2}\pi\ell} + \frac{2I}{\sqrt{2}\pi b}$$

$$= \frac{2I}{\sqrt{2}\pi}\left[\frac{b+\ell}{\ell b}\right]$$

$$H = \frac{2\sqrt{2}\,I}{2\pi}\left[\frac{\ell + b}{\ell b}\right]$$
$$H = \frac{\sqrt{2}\,I}{\pi}\left[\frac{\ell + b}{\ell b}\right]$$

The above expression can be deduced to a square by substituting $\ell = b = a$

$$H = \frac{\sqrt{2}\;I}{\pi}\left[\frac{2a}{a^2}\right]$$
$$H = \frac{2\sqrt{2}\;I}{\pi a}$$

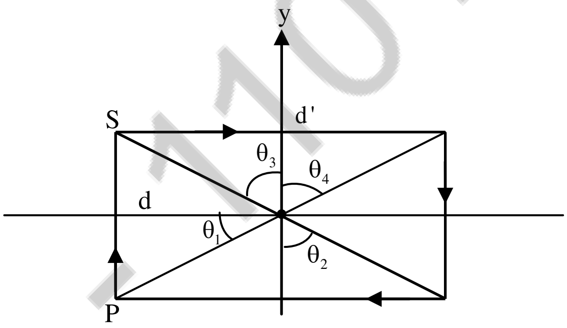
Magnetic field intensity due to a rectangular loop:-

Consider a rectangular loop (PQRS) located in XY plane which carries a current I. 'H' is found at on 0.

Let 'L' = length b rectangular 'b; = breadth of rectangular.

Each side of rectangular is treated as finite length current element.

Consider PS, $\overline{H}$ due to finite length wire,



$$\overline{H} = \frac{I}{4\pi d}[\sin\theta_1 + \sin\theta_2]P$$
$$d = \frac{L}{2},\ \overline{H}_1 = \frac{I}{2\pi L}[\sin\theta_1 + \sin\theta_2]\overline{a}_\phi$$

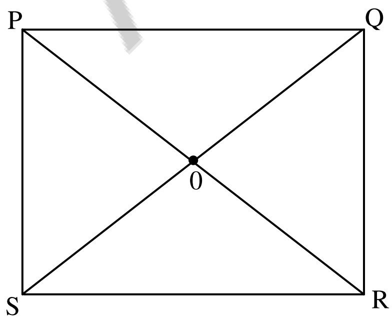
From symmetry of rectangular, for QR

$$\overline{H}_2 = \frac{I}{2\pi L}[\sin\theta_1 + \sin\theta_2]\overline{a}_\phi$$

Similarly $\overline{H}$ due to finite length RS & PQ

$$H_3 = H_4 = \frac{I}{2\pi d}[\sin\theta_3 + \sin\theta_4]$$
$$\overline{H}_3 = \overline{H}_4 = \frac{I}{2\pi B}[\sin\theta_3 + \sin\theta_4]$$
$$\overline{H} = \overline{H}_1 + \overline{H}_2 + \overline{H}_3 + \overline{H}_4$$
$$= \frac{I}{\pi}\left\{\frac{1}{L}(\sin\theta_1 + \sin\theta_2) + \frac{1}{B}(\sin\theta_3 + \sin\theta_4)\right\}$$

Determine the expression for the magnitude field intensity due to a square loop:-



Consider a square loop PQRS of side 'a'. A current "I" flows through the loop

$$H = \frac{I}{4\pi\rho}[\sin\alpha_2 - \sin\alpha_1]$$

$$\rho = \frac{a}{2};\ \alpha_2 = 45°\ \ \alpha_1 = -\,45°$$

$$H = \frac{I}{2\pi a}\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]$$

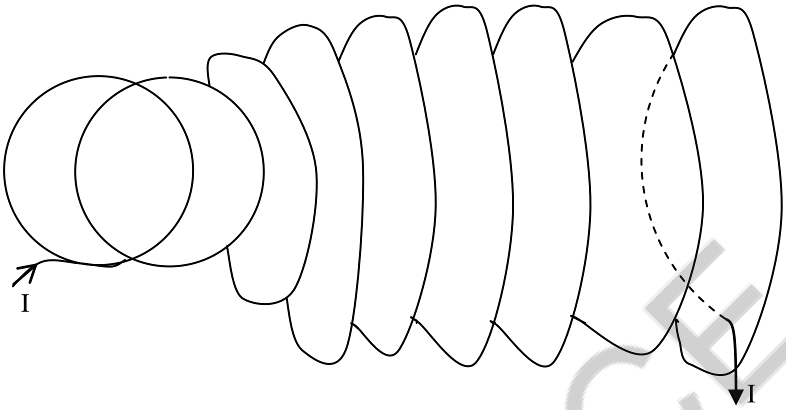
$$H = \frac{I}{2\pi a}\left[\frac{2}{\sqrt{2}}\right] = \frac{I}{\sqrt{2}\pi a}$$

H due to all the four sides $= \frac{4 \times I}{\sqrt{2}\pi a} = \frac{4 \times \sqrt{2} I}{2\pi a} = \frac{2\sqrt{2} I}{\pi a}$

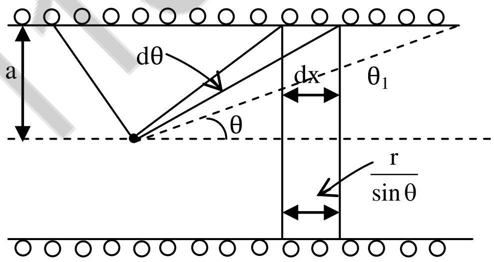
Derive the expression for the magnitude field intensity due to a solenoid:-

A helical coil or solenoid is usually used to produce a magnetic field. Let us calculate the magnetic field of such a coil.

A coil of wire wound its the form of a cylindrical as shown in fig. Let us assume there the turns in the windings are closely and ever spaced so that the number of turns per unit to of the solenoid is constant N. The path of the current through the coil is helical



(a) Solenoid



(b) Cut view of solenoid

$$\frac{r d\theta}{dx} = \sin\theta$$

$$\frac{r d\theta}{\sin\theta} = dx$$

If 'N' is the number of turns, $N dx = \frac{N r d\theta}{\sin\theta}$ and the total current element in the solenoid

$$I dx = \frac{I N r d\theta}{\sin\theta}$$

$$DBx = \frac{\mu I a^2}{2r^3} = \frac{\mu a^2}{2r^3} \cdot \frac{I N r d\theta}{\sin\theta}$$

$$\sin\theta = \frac{a}{r};\ \ r^2 = \frac{a^2}{\sin^2\theta}$$

$$dBx = \frac{\mu a^2\ I\ N\ d\theta}{2r^3 \sin\theta} = \frac{\mu a^2\ I\ N\ d\theta \sin^2\theta}{2\ a^2 \sin\theta} = \frac{\mu I\ N}{2}\sin\theta\ d\theta$$

$$Bx = \int dBx = \int_{Q_2}^{Q_1} \frac{\mu I\ N}{2}\sin\theta\ d\theta$$

$$Bx = \frac{\mu I\ N}{2}[\cos\theta_1 - \cos\theta_2]\bar{a}_x$$

Case 1: In case the solenoid considered to be long and the point P lies in the muddle.

$$Q_1 = 0 \;\; \& Q_2 = \pi$$

$$B = \frac{\mu \, I \, N}{2}\left[\cos 0 - \cos \pi\right]$$

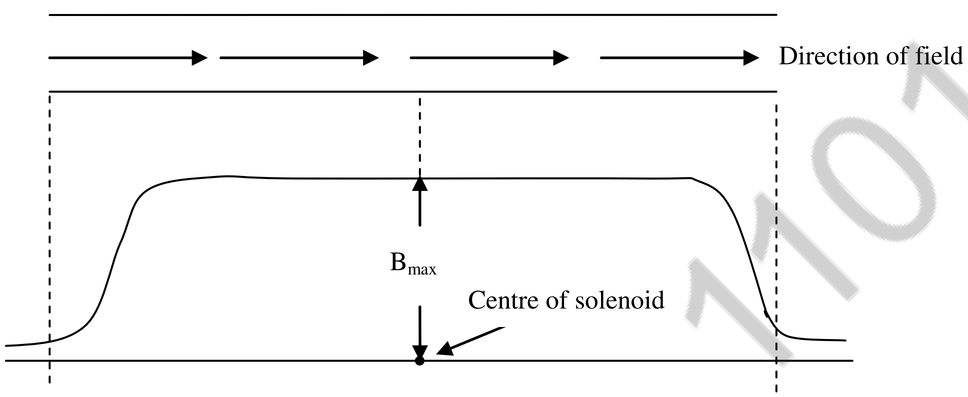
$$B = \frac{\mu \, I \, N}{2}\left[1 - (-1)\right] = \mu \, I \, N$$

Case ii:- If the point lies at one end of the solenoid. $Q_1 = 0 \;\& Q_2 = \frac{\pi}{2}$

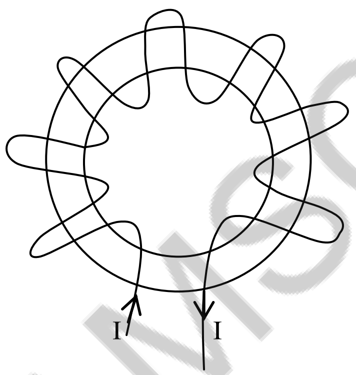
Then
$$B = \frac{\mu \, I \, N}{2}\left[\cos 0 - \cos \pi/2\right]$$

$$B = \frac{\mu \, I \, N}{2} \text{Jesla}$$

From the alone equations, it is clean that the magnitude field is one half at one end than at centre.



Derive the expression for the magnetic field intensity due to a toroid:-



Assuming diameter of the core is small and compared to the diameter of ring the circular paths through the core will be approximate of same length 2π r.

Ampere's circuital law,

$$\int_{\ell} H.dl = \quad NIH$$

$$H\int_{\dagger} dl = NI \Rightarrow 2\pi r \,(H) = NI$$

$$\Rightarrow \overline{H} = \frac{NI}{2\pi r} = \frac{NI}{\ell}$$

Where L – mean circumference b the Toroid.

$$\overline{B} = \mu H$$

$$\overline{B} = \frac{\mu NI}{\ell}$$

State and proof Ampere's circuital law?

Ampere's circuital law:-

The law states " the line integral of the magnetic field intensity (H) around a closed path is the ssame as the net current Ienc enclosed by the path

$\oint \overline{H}.dl = I_{enc}$ i.e, $\oint \overline{H}.dl = I \rightarrow$ general form

Ampere's law is easily applied to determine H when the current distribution is symmetrical

By applying stoke's theorem

$$I_{enc} = \oint_L \overline{H}.dl = \oint_s (\nabla \times H).\,ds \rightarrow (1)$$

But $I_{enc} = \oint_s J.\,ds \rightarrow (2)$

Comparing (1) & (2), $\nabla \times H = J \rightarrow$ point form or differential form

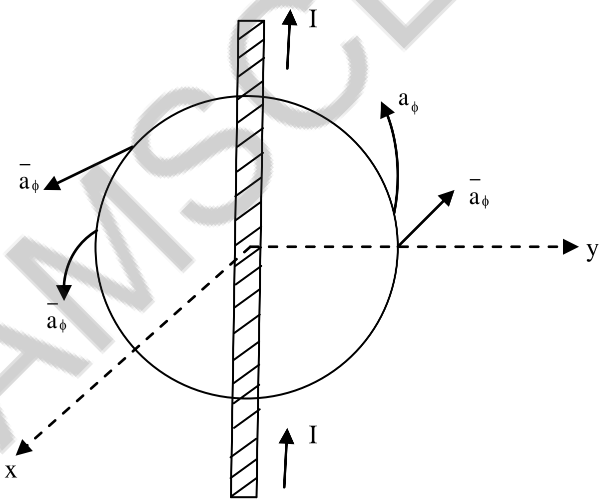
$\oint \overline{H}.dl = \oint_s (\nabla \times H).\,ds = \oint_s J.ds$ integral form of ACL

So, $\nabla \times H = J \neq 0$

So magnetic field is not conservative.

PROOF:-

Consider an infinitely long straight conductor carrying current I placed along z – axis consider a closed circular path of radius r. The point P is at a perpendicular distance 'r' for the conductor. Consider $\overline{dl}$ at point P which is in $a\phi$ direction.



$\overline{dl} = rd\phi\ \overline{a\phi}$

$\overline{H}$ obtained at point P from BIOT savart's law due infinitely long conductor is

$$\overline{H} = \frac{I}{2\pi r}\bar{a}_\phi$$

$$\overline{H}.\,dl = \frac{I}{2\pi r} rd\phi = \frac{I}{2\pi}d\phi$$

$$\oint \overline{H}.\overline{dl} = \int_{\phi=0}^{2\pi} \frac{I}{2\pi}d\phi \Rightarrow \frac{I}{2\pi}[\phi]_0^{2\pi} = I$$

$$\oint \overline{H}.\,dl = I$$

Explain in detail about the Applications of Ampere's circuit law:-

Amperes law is used to find H for symmetrical current distributions. For symmetrical current distribution $\overline{H}$ is either parallel or perpendicular to $\overline{dl}$.

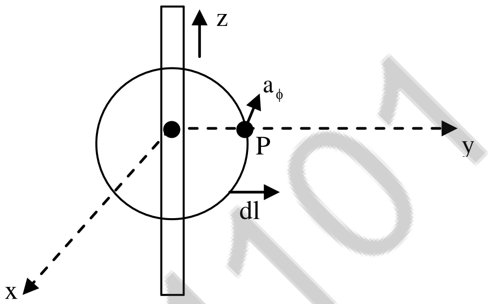
When $\overline{H}$ is parallel to $\overline{dl}$, $|H|$ = constant.

Applications of Ampere's circuit law:-

Case i:- Infinite line current (or) $\overline{H}$ due to infinitely long conduction. Consider an infinitely long straight conductor placed along Z – axis carrying current I. considers a point P on the closed path at which $\overline{H}$ is to be obtained. The radius of path is 'r' and hence P is a perpendicular distance 'r' from the conductor. Consider $\overline{dl}$ at point 'P' in $\vec{a}_\phi$ direction i.e, H_ϕ

$$\vec{H} = H_\phi\, \vec{a}_\phi . r\, d\phi\, \overline{a_\phi}$$

According to Amperes circuital law,



$$\oint H . \overline{dl} = I$$

$$\oint H_\phi \vec{a}_\phi .\, r\, d\phi\, \overline{a}_\phi = I$$

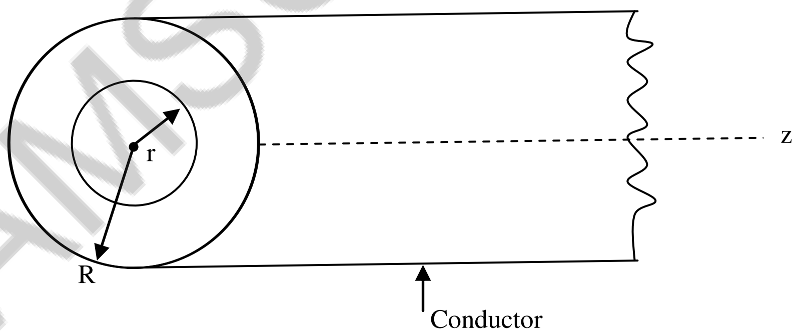
$$\int_0^{2\pi} H_\phi r\, d\phi = I$$

$$H_\phi\, r[\phi]_0^{2\pi} = I \Rightarrow H_\phi = \frac{I}{2\pi r}$$

$\overline{H}$ at point P is given by $\vec{H} = H_\phi a_\phi$

$$\vec{H} = \frac{I}{2\pi r} . \overline{a}_\phi \;\; A/m$$

Case ii:- $\overline{H}$ **due to cylindrical conductor (or) co – axial cable**



Consider a cylindrical conductor of radius R carries an uniform current of I amperes. It is placed along z – axis and has finite length.

$\overline{H}$ is to be obtained considering two regions.

Region 1:- With the conductor, r < R.

According to ampere's law, $\oint \overline{H} . dl = I_{enc}$

As current I flows uniformly, it flows across the cross sectional area πR^2, while the closed path enclosed only part of current which passed across the cross sectional area πR^2.

Hence current enclosed by path , $I_{enc} = I . \frac{\pi r^2}{\pi R^2}$

$$I_{enc} = I.\frac{r^2}{R^2}$$

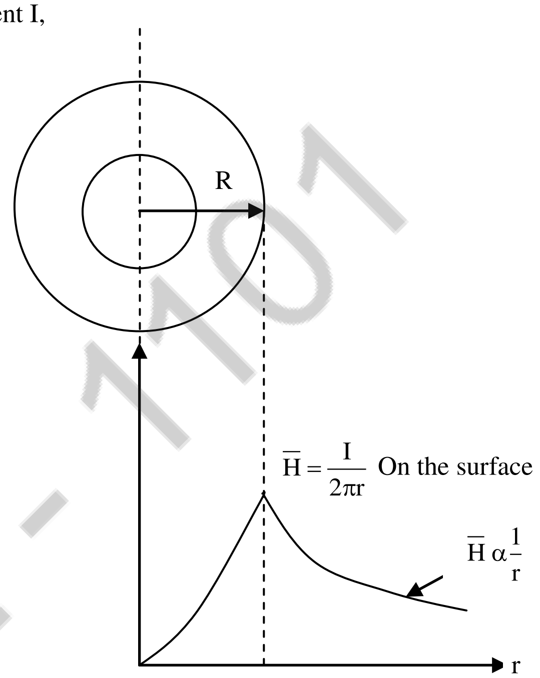
$$\vec{H} = H_\phi\, \bar{a}_\phi,\ \overline{dl} = r\, d\phi\, a\phi$$
$$\vec{H}.\overline{dl} = H_\phi\, \bar{a}_\phi.\, r\,d\phi\, a\phi = H_\phi\, r\,d\phi$$
$$\int_0^{2\pi} H_\phi\, r\,d\phi = I.\frac{r^2}{R^2} \quad H_\phi = \frac{Ir}{2\pi R^2}$$
$$\vec{H} = \frac{Ir}{2\pi R^2}\vec{a\phi}\ \ A/m.$$

Region 2:- Outside the conductor, r > R

The conductor is infinite length along z – axis carrying a current I,

$$\vec{H} = \frac{I}{2\pi r}\,\bar{a}_\phi \quad r > R$$

Outside the conductor, $\overline{H} \propto \frac{1}{r}$



Formulae:- Curl, distance & gradient in all the three system of co – ordinates.

Co – ordinate system	Cartesian	Cylindrical	Spherical
Curl $\nabla\times A$	$\nabla\times A = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$	$\nabla\times A = \begin{vmatrix} a_\rho & a_\phi & a_z \\ \frac{\partial}{\partial \rho} & \frac{1}{\rho}\frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & A_\phi & A_z \end{vmatrix}$	$\nabla\times A = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(H_\phi\sin\theta) - \frac{\partial H_\theta}{\partial_\phi}\right]\bar{a}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial H_r}{\partial\phi} - \frac{\partial}{\partial_r}(rH_\phi)\right]\bar{a}_\theta + \frac{1}{r}\left[\frac{\partial}{\partial_r}(rH_\phi) - \frac{\partial H_r}{\partial\theta}\right]\bar{a}_\phi$
Divergence	$\nabla.d = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$	$\nabla.D = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial\phi} + \frac{\partial D_z}{\partial z}$	$\nabla.D = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(D_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi}$

Gradient	$\nabla.d = \frac{\partial v}{\partial x}\bar{a}_x + \frac{\partial v}{\partial y}\bar{a}_y + \frac{\partial v}{\partial z}\bar{a}_z$	$\nabla.v = \frac{\partial v}{\partial \rho}\bar{a}_\rho + \frac{1}{\rho}\frac{\partial v}{\partial \phi}\bar{a}_\phi + \frac{\partial v}{\partial z}\bar{a}_z$	$\nabla.v = \frac{\partial v}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial v}{\partial \theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial v}{\partial \phi}\bar{a}_\phi$

PROBLEMS UNIT – III

Problems in BIOT – SAVARTS LAW:-

1. A steady element of $10^{-3}\bar{a}_z$ A. m is located at origin in free space (i) What is the magnetic field intensity due to the current element at (1, 0, 0) & at (0, 0, 1).

Solution:-

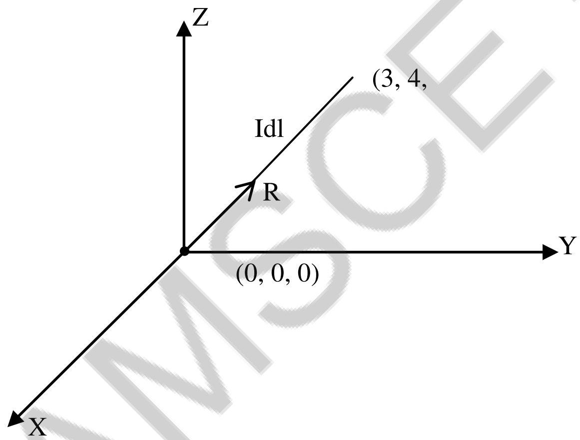
$$d\overline{H} = \frac{Idl \times \bar{a}_R}{4\pi R^2}$$

$$= \frac{10^{-3}\bar{a}_z \times \bar{a}_x}{4\pi(1)^2(1)} = \frac{10^{-3}}{4\pi}\bar{a}_y A/m$$

$$d\overline{H} = \frac{Idl \times \bar{a}_R}{4\pi R^2} = \frac{10^{-3}\bar{a}_z \times \bar{a}_z}{4\pi(1)^2(1)} = 0$$

2. Find the magnetic field intensity at the origin due to a current element $\overline{Idl} = 3\pi(\bar{a}_x + 2\bar{a}_y + 3\bar{a}_z)\mu Am$ at (3, 4, 5) in free space.

Solution:-



$$\overline{dH} = \frac{\overline{Idl} \times \overline{a_R}}{4\pi R^2}$$

$$\overline{a_R} = \frac{\overline{R}}{|R|} = \frac{-(3\bar{a}_x + 4\bar{a}_y + 5\bar{a}_z)}{\sqrt{9+16+25}}$$

$$\overline{a_R} = -0.424a_x - 0.565a_y - 0.707a_z$$

$$Idl = 3\pi\left(\overline{a}_x + 2\overline{a}_y + 3\overline{a}_z\right)\times 10^{-6}$$

$$\overline{Idl}\times\overline{a}_R = \begin{vmatrix} a_x & a_y & a_z \\ 3\pi & 6\pi & 9\pi \\ -0.424 & -0.565 & -0.707 \end{vmatrix}\times 10^{-6}$$

$$= (2.661\overline{a}_x - 5.339\overline{a}_y + 2.67\overline{a}_z)\times 10^{-6}$$

$$\overline{dH} = \frac{(2.661\overline{a}_x - 5.339\overline{a}_y + 2.67\overline{a}_z)\times 10^{-6}}{4\pi\left(\sqrt{50}\right)^2}$$

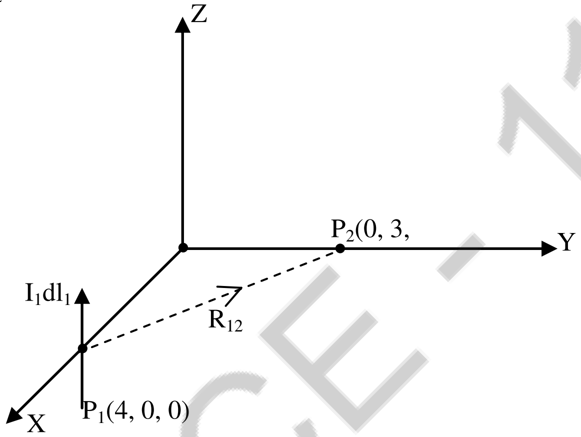
$$\overline{dH} = 4.24\overline{a}_x - 8.51\overline{a}_y + 4.25\overline{a}_z nA$$

3. Find the inenemental strength at P2 due to the current element $2\pi\overline{a}_z\mu A.m$ **at P1. The co – ordinates of P1 and P2 are (4, 0, 0) & (0, 3, 0).**

Solution:-

According to B.S.L, the inenemental field strength at P2 due to the current element $2\pi\overline{a}_z\mu A.m$ at P1is

$$d\overline{H}_2 = \frac{I_1\overline{dl}_1\times\overline{a}_{R12}}{4\pi R_1^2} = \overline{R_{12}} = (0-4)\overline{a}_x + (3-0)\overline{a}_y + 0\overline{a}_z = -4\overline{a}_x + 3\overline{a}_y$$



$$\overline{a}_{R12} = \frac{\overline{R_{12}}}{|R_{12}|} = \frac{-4\overline{a}_x + 3\overline{a}_y}{\sqrt{16+9}} = \frac{-4\overline{a}_x + 3\overline{a}_y}{5}$$

$$I_1dl_1 = 2\pi\overline{a}_z\,\mu Am$$

$$I_1dl_1\times\overline{a}_{R12} = \begin{vmatrix} a_x & a_y & a_z \\ 0 & 0 & 2\pi \\ -4/5 & 3/5 & 0 \end{vmatrix}$$

$$= \left(-3/5\times 2\pi\right)\overline{a_x} - \left(4/5\times 2\pi\right)\overline{a_y}$$

$$= \frac{-2\pi}{5}\left[3a_x + 4a_y\right]$$

$$dH_2 = \frac{\frac{-2\pi}{5}\left[3\overline{a_x} + 4\overline{a_y}\right]}{4\pi(5)^2}$$

$$dH_2 = -12\overline{a_x} - 16\overline{a_y} nA/m$$

4. A filamentary current of 10a is denoted inward from infinity to the origin on the positive x – axis & then outward to infinity along positive y – axis. Find 'H' at p(0, 0, 1).

Solution:-

$$\overline{dH} = \frac{\overline{Idl} \times \overline{a_R}}{4\pi R^2} = \frac{\overline{Idl} \times \overline{R}}{4\pi R^3} \qquad \overline{R} = -x\overline{a}_x + \overline{a}_z;\ Idl = 10dx\overline{a}_x$$

$$H_1 = \frac{10}{4\pi}\int_{\infty}^{0} \frac{dx\overline{a}_x \times (-x\overline{a}_x + \overline{a}_z)}{\left[\sqrt{1+x^2}\right]^3} = \frac{-5}{2\pi}\int_{\infty}^{0} \frac{dxa_y}{\left(1+x^2\right)^{3/2}}$$

$$= \frac{-5}{2\pi}\frac{x}{\sqrt{1+x^2}}\Bigg|_{\infty}^{0} \overline{a}_y = \frac{5}{2\pi}\overline{a}_y A/m$$

$$\overline{H}_2 = \frac{5}{2\pi}\int_{0}^{\infty} \frac{(dy\overline{a}_y) \times (-y\overline{a}_y + \overline{a}_z)}{\left[\sqrt{1+y^2}\right]^3}$$

$$= \frac{5}{2\pi}\int_{0}^{\infty} \frac{dy\overline{a}_x}{(Hy^2)^{3/2}}$$

$$= \frac{5}{2\pi}\overline{a}_x$$

$$H = H_1 + H_2 = 0.796\left[\overline{a}_x + \overline{a}_y\right]A/m$$

5. a wire carrying a current of 8A is formed in the circular loop. If 'H' at the centre of the loop is 40A /m. What is the radius of the loop if the loop has (i) Only one turn (ii) 10 turns

Solution:-

$$\overline{H} = \frac{I}{2\rho}\overline{a}_z$$

If there are 'N' turns,

$$\overline{H} = \frac{NI}{2\rho}$$

$$\rho = \frac{NI}{2H} = \frac{1 \times 8}{2 \times 40} = 0.1m$$

(ii) If N = 10

$$\rho = \frac{10 \times 8}{2 \times 40} = 1m$$

6. a circular loop located on $x^2 + y^2 = 4,\ z = 0$ carries a direct current of 7A along $\overline{a}_\phi$. Determine $\overline{H}$ at (0, 0, 5) & (0, 0, -5).

Solution:-

Equation of circle is $x^2 + y^2 = a^2 \Rightarrow x^2 + y^2 = 2^2$

From the above equation, $\rho = 2, I = 7A, h = 5$

$$\overline{H} = \frac{I\rho^2}{2\left(\rho^2 + h^2\right)^{3/2}}\overline{a}_z = \frac{7 \times 4}{2(4+25)^{3/2}}\overline{a}_z$$

$$H = 90\overline{a}_z mA/m$$

7. A thin of radius 5 cm is placed on a place z = 1cm so that the centre is at (0, 0, -1) cm. If the ring carries 50 ma along $\overline{a}_\phi$, determine $\overline{H}$ at (0, 0, -1)cm (ii) (0, 0, 10)cm.

Solution:-

$$\overline{H} = \frac{I\rho^2}{2(\rho^2 + h^2)^{3/2}} a_z$$

$$= \frac{50\times10^{-3}(5\times10^{-2})^2}{2\left[(5\times10^{-2}) + (2\times10^{-2})^2\right]^{3/2}} \bar{a}_z$$

$$= \frac{25\times10^{-4}\times5\times10^{-3}}{2\left[25\times10^{-4} + 4\times10^{-4}\right]^{3/2}} = 400.23\bar{a}_z \text{ mA / m}$$

(ii) At (0, 0, 10)c,, h = 9cm

$$\overline{H} = \frac{50\times10^{-3}\times25\times10^{-4}}{2(5^2 + 9^2)^{3/2}\times10^{-6}} \bar{a}_z$$

$$\overline{H} = 57.26\bar{a}_z \text{mA / m.}$$

8. Determine the current density for $\overline{H} = 28\sin x\, \bar{a}_y A/m$

Solution:-

$$\nabla \times H = \bar{J}$$

$$\begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial y} \\ 0 & 28\sin x & 0 \end{vmatrix} = a_x(0-0) + a_y(0) + a_z\left[\frac{\partial}{\partial x}(28\sin x)\right]$$

$$= 28\cos x\, \bar{a}_z A/m^2$$

9. In cylindrical co- ordinates, $\overline{H} = (2\rho - \rho^2)\bar{a}_\phi A/m$ for 0 < ρ <1. (i) Determine the current density as a function of ρ within this cylindrical (ii) What is the total current passing through surface z =0, 0 < ρ in $\bar{a}_z$ direction? (iii) Verify the same using stoke's theorem.

Solution:- $\overline{H} = (2\rho - \rho^2)\bar{a}_\phi A/m\ 0 < \rho < 1$

$$J = \nabla \times H = \frac{1}{\rho}\begin{vmatrix} a_\rho & \rho\bar{a}_\phi & \bar{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & \rho(2\rho - \rho^2) & 0 \end{vmatrix}$$

$$= \frac{1}{\rho}\left[a_\rho(0 - \frac{\partial}{\partial z}(2\rho^2 - \rho^3)\right] + 0a_\phi + \frac{\partial}{\partial \rho}(2\rho^2 - \rho^3)\bar{a}_z$$

$$= \frac{1}{\rho}\left[4\rho - 3\rho^2\right]\bar{a}_z$$

$$J = (4 - 3\rho)\bar{a}_z$$

(ii) $I = \int_s \bar{J}\ \overline{ds} = \int_0^{2\pi} J_2\ \rho d\rho\ d\phi$

$$= \int_0^{2\pi}\int_0^1 (4 - 3\rho)\rho d\rho\ \ d\phi$$

$$= \int_0^{2\pi} d\phi \int_0^1 (4 - 3\rho)\rho\ d\rho$$

$$= (\phi)_0^{2\pi}\left(\frac{4\rho^2}{2} - \frac{3\rho^3}{3}\right)_0^1$$

$$= 2\pi(2 - 1) = 2\pi$$

I = 6.28 A

(ii) $\int_{\ell} \overline{H}.\,\overline{dl} = \int_0^{2\pi} (2\rho - \rho^2)\overline{a}_\phi\ \rho d\phi\ a_\phi$

$= 2\pi(2-1) = 2\pi = 6.28A$

10. The $\overline{H} = [y\cos(\alpha x)]\overline{a}_x + (y+e^x)\overline{a}_z A/m$. Determine the current density over the yz plane.

Solution:-

$$J = \nabla \times H = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial y} \\ y\cos(\alpha x) & 0 & y+e^x \end{vmatrix}$$

$$= a_x\left[\frac{\partial}{\partial y}(y+e^x) - 0\right] + \left[\frac{\partial}{\partial z}(y\cos(\alpha x) - \frac{\partial}{\partial x}(y+e^x)\right] + \left[0 - \frac{\partial}{\partial y}(y\cos(\alpha x)\right]\overline{a}_z$$

$$\overline{J} = a_x + (-e^x)\overline{a}_y - \cos\alpha x\,\overline{a}_z$$

Since x = 0, on yz plane ,

$$\overline{J} = \overline{a}_x - e^0\overline{a}_y - \cos(0)\overline{a}_z$$
$$J = \overline{a}_x - \overline{a}_y - \overline{a}_z A/m^2$$

11. a flat perfectly conducting surface in xy plane is siluated in a magnetic field,

$$\overline{H} = \begin{cases} 3\cos x\,\overline{a}_x + z\cos x\,\overline{a}_y, & z \ge 0 \\ 0 & , \quad z < 0 \end{cases}$$ **, find 'J' on the conductor surface.**

Solution:-

$$J = \nabla \times \overline{H} = \begin{vmatrix} \overline{a}_x & \overline{a}_y & \overline{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3\cos x & z\cos x & 0 \end{vmatrix} = a_x\left[0 - \frac{\partial}{\partial z}(z\cos x)\right] + a_y\left[0 - \frac{\partial}{\partial y}(3\cos x)\right] + a_z\left[\frac{\partial}{\partial x}(z\cos x)\frac{-\partial}{\partial y}(3\cos x)\right]$$

$$= a_x(-\cos x) + a_y(0) + a_z(-z\sin x)$$

$$J = \begin{cases} -\cos x\,\overline{a}_x - z\sin x\,\overline{a}_z, & z \ge 0 \\ 0 & , \quad z < 0 \end{cases}$$

12. If $\overline{H} = y\overline{a}_x - x\overline{a}_y\, a/m$ on plane z = 0, (i) Determine J (ii) Verify Ampere's law by taking the circulation of $\overline{H}$ around the rectangular path with z = 0, 0 < x < 5 & -2 < y < 6.

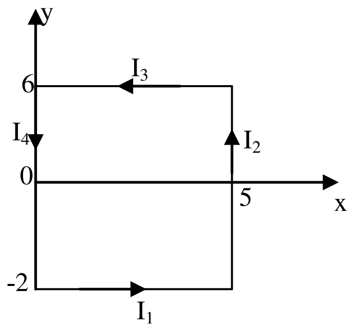
Solution:-

$$J = \nabla \times H = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & -x & 0 \end{vmatrix}$$

(i) $= a_x[0-0] + a_y[0-0] + a_z\left[\frac{\partial}{\partial x}(-x)\frac{-\partial}{\partial y}\right]$

$= -2\overline{a}_z A/m^2$

(ii) $\int_{\ell} \overline{H}.\,\overline{dl} = \left(\int_{\ell 1} + \int_{\ell 2} + \int_{\ell 3} + \int_{\ell 4}\right)\overline{H}\ \overline{dl}$



$$\int_{\ell 1} \overline{H}.dl = \int_0^5 (y\overline{a}_x - x\overline{a}_y)(dx\overline{a}_x)$$

$$= \int_0^5 ydx = (xy)_0^5\Big|_{y=0} = -10$$

$$\int_{\ell 2} \overline{H}.dl = \int_{-2}^6 (y\overline{a}_x - x\overline{a}_y)(dx\overline{a}_y)$$

$$= \int_{-2}^6 xdy = -[xy]_{-2}^6\Big|_{x=5} = -(5y)_{-2}^6$$

$$= -[5\times 6 - (5\times -2] = -40$$

$$\int_{\ell 3} \overline{H}.dl = \int_5^0 (y\overline{a}_x - x\overline{a}_y)(dx\overline{a}_x)$$

$$= \int_5^0 ydx = [yx]_5^0\Big|_{y=0} = (6x)_5^0 = -30$$

$$\int_{\ell 4} \overline{H}.dl = \int_6^{-2} (y\overline{a}_x - x\overline{a}_y)(dx\overline{a}_y)$$

$$= \int_6^{-2} -xdy\Big|_{x=0} = 0$$

$$\int_{\ell} H.dl = -10 - 40 - 30 = -80A$$

$$I = \int \overline{J}.\ \overline{ds} = \int_{y=-2}^{6} \int_0^5 (-2a_z)(dxdydz)$$

$$= -2\int_{-2}^{6}\int_0^5 dxdy$$

$$= -2(x)_0^5 (y)_{-2}^6$$

$$= -2(5)(6+2) = -80A$$

Thus verified.

13. If an in rotational field $\overline{F} = (x + 2y + \ell z)\overline{a}_x + (mx - 3y - z)\overline{a}_y + (4x + hy + 2z)\overline{a}_z$ **, Determine the constants ℓ, m & n for the above field.**

Solution:-

$$\nabla \times F = 0$$

$$\nabla \times F = \begin{vmatrix} a_x & a_y & a_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

$$a_x\left[\frac{\partial F_z}{\partial y}-\frac{\partial F_y}{\partial z}\right]=a_x\left[\frac{\partial}{\partial y}(4x+hy+2z)-\frac{\partial}{\partial z}(mx-3y-z)\right]=a_x(n+1)$$

$$a_y\left[\frac{\partial F_z}{\partial x}-\frac{\partial F_x}{\partial z}\right]=a_y\left[\frac{\partial}{\partial x}(4x+hy+2z)-\frac{\partial}{\partial z}(x+2y-\ell z)\right]=a_x(4-\ell)$$

$$a_z\left[\frac{\partial F_y}{\partial x}-\frac{\partial F_x}{\partial y}\right]=a_z\left[\frac{\partial}{\partial x}(mx-3y-z)-\frac{\partial}{\partial z}(x+2y-\ell z)\right]=a_z(m+2)$$

h = -1, ℓ=4, m = +2

14. $\bar{H}=\frac{x+2y}{z^2}\bar{a}_y+\frac{2}{z}\bar{a}_z$, **Find** $\nabla\times H, \bar{J}$ **& 'I' passing through the surface z = 4, 1 < x < 2 & 3 < y < 5 in the** $\bar{a}_z$ **direction using** $\bar{J}$

Solution:-

(i) $$\nabla\times H=\begin{vmatrix}\bar{a}_x & \bar{a}_y & \bar{a}_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 0 & \frac{x+2y}{z^2} & \frac{2}{z}\end{vmatrix}$$

$$=a_x\left[\frac{\partial}{\partial y}(2z^{-1})-\frac{\partial}{\partial z}\left[(x+2y)z^{-2}\right]\right]+a_y\left[\frac{\partial}{\partial x}(2z^{-1})-0\right]+a_z\left[\frac{\partial}{\partial x}\left[(x+2y)z^{-2}\right]-0\right]$$

$$\nabla\times\bar{H}=\frac{2(x+2y)}{z^3}\bar{a}_x+\frac{1}{z^2}\bar{a}_z$$

(ii) $\nabla\times H=\bar{J}$

$$J=2\left(\frac{x+2y}{z^3}\right)\bar{a}_x+\frac{1}{z^2}\bar{a}_z=A/m^2$$

$$I=\int\bar{J}.ds=\left(\frac{2(x+2y)}{z^3}\bar{a}_x+\frac{1}{z^2}\bar{a}_z\right)dxdydz$$

$$=\int_3^5\int_1^2\frac{1}{z^2}dxdy\bigg|_{z=4}$$

(iii) $$I=\int_3^5\int_1^2\frac{1}{16}dxdy$$

$$=\frac{1}{16}(x)_1^2(y)_3^5=\frac{1}{16}[2-1][5-3]$$

$$I=\frac{1}{8}A$$

15. $\bar{H}=x^2\bar{a}_x+2yz\,\bar{a}_y+(-x^2)\bar{a}_z A/m$. **Find the 'J' at point (i) (2, 3, 4) (ii)** $\rho=6, \phi=45°, z=3$ **(iii)** $r=3.6, \theta=60°, \phi=90°$

Solution:-

Given $\bar{H}=x^2\bar{a}_x+2yz\,\bar{a}_y+(-x^2)\bar{a}_z$

$$\nabla\times H=\bar{J}=\begin{vmatrix}a_x & a_y & a_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ x^2 & 2yz & -x^2\end{vmatrix}$$

$$= a_x\left[\frac{\partial}{\partial y}(-x^2) - \frac{\partial}{\partial x}(2yz)\right] + a_y\left[\frac{\partial}{\partial x}(-x^2) - \frac{\partial}{\partial z}(x^2)\right] + a_z\left[\frac{\partial}{\partial x}(2yz) - \frac{\partial}{\partial y}(x^2)\right]$$

$$J = -2y\bar{a}_x + 2x\bar{a}_y$$

At P(2, 3, 4) $= -2(3)\bar{a}_x + 2(2)\bar{a}_y = -6\bar{a}_x + 4\bar{a}_y\ A/m^2$

At (6, 45°, 3)

$$x = \rho\cos\phi, \qquad y = \rho\sin\phi$$
$$x = 6\cos 45°, \qquad y = 6\sin 45°$$
$$x = 4.242 \qquad\qquad y = 4.242$$

$$\nabla\times H = -2y\bar{a}_x + 2x\bar{a}_y$$
$$= -2(4.242)\bar{a}_x + 2(4.242)\bar{a}_y$$
$$\nabla\times H = -8.484\bar{a}_x + 8.484\bar{a}_y$$

(iii) At (3.6, 60°,90°)

$$x = r\sin\theta\cos\phi = 3.6\sin 60°\cos 90° = 0$$
$$y = r\sin\theta\cos\phi = 3.6\sin 60°\sin 90° = 1.8R3 = 3.117$$
$$z = r\cos\theta \quad = 3.6\cos 60° \quad = 1.8$$

$$\nabla\times H = -2(3.117)\bar{a}_x + 2(0)\bar{a}_y$$
$$= -6.234\bar{a}_x\ A/m^2$$

16. $\bar{H} = -y(x^2+y^2)\bar{a}_x + x(x^2+y^2)\bar{a}_y\ A/m$ **in the z = 0 plane for -5 < x, y < 5. Calculate the current passing thro z = 0 plane & through the region -1 < x < 1 & -2 < y < 2.**

Solution:-

$$J = \nabla\times H = \begin{vmatrix} a_x & a_Y & a_Z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y(x^2+y^2) & x(x^2+y^2) & 0 \end{vmatrix}$$

$$= a_x(0-0) - a_y(0-0) + a_z\left[\frac{\partial}{\partial x}(x^3+xy^2) - \frac{\partial}{\partial y}(-x^2y+y^3)\right]$$
$$= (3x^2+y^2) + x^2 + 3y^2 = 4(x^2+y^2)\bar{a}_z A/m^2$$

$$I = \int_s (\nabla\times H).\,ds = \int \bar{J}\,ds$$

$$= \int_{-2}^{2}\int_{-1}^{1} 4(x^2+y^2)\,dx\,dy$$
$$= \int_{-2}^{2}\left[\frac{4x^3}{3} + 4y^2x\right]_{-1}^{1} dy$$
$$= \int_{-2}^{2}\left[\left(\frac{4}{3}+4y^2\right) + \frac{4}{3} + 4y^2\right]dy$$
$$= \int_{-2}^{2}\left(\frac{4}{3}+8y^2\right)dy$$
$$= 2\int_{0}^{2}\left(\frac{4}{3}+8y^2\right)dy = 2\left[\frac{4}{3}+\frac{8y^3}{3}\right]_0^2$$
$$= 2\left[\frac{8}{3}+\frac{64}{3}\right] = \frac{144}{3} = 48A.$$

17. The portion of the sphere is specified by r = 4,
$0 \le \theta \le 0.1\pi, 0 \le \phi \le 0.3\pi.\ \overline{H} = 6r\sin\phi\,\overline{a}_{r-1}, 18r\sin\theta\cos\phi\,\overline{a}_{\phi}\ A/m.$ **Determine the current.**

Solution:-

$$\nabla \times H = \frac{1}{r^2 \sin\theta}\begin{bmatrix} a_r & ra_\theta & r\sin\theta a_\phi \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\ H_r & rH_\theta & r\sin\theta H_\phi \end{bmatrix}$$

Problems on magnetic flux and magnetic flux density

20. If $H = r\sin\phi\overline{a}_r + 2.5r\sin\theta\cos\phi\overline{a}_\phi A/m$ **exists in a medium** $\mu_r = 3$, **determine** $\overline{B}$.

Solution:-

$$\begin{aligned} &\overline{H} = r\sin\phi\overline{a}_r + 2.5r\sin\theta\cos\phi\overline{a}_\phi A/m\ \&\ \mu_r = 3 \\ &B = \mu H = \mu_0\mu_r H = 4\pi\times10^{-7}\times3\left[r\sin\phi\overline{a}_r + 2.5r\sin\theta\cos\phi\overline{a}_\phi\right] \\ &B = 3.77r\sin\phi\overline{a}_r + 9.42r\sin\theta\cos\phi\overline{a}_\phi\ Wb/m^2 \end{aligned}$$

21. radius = 2cm B = 10Wb/m^2. If the plane of the coil is perpendicular to the field, determine ϕ **.**

Solution:-

$$\begin{aligned} \phi &= BA \\ &= 10\times\pi r^2 = 10\times\pi(2\times10^{-2})^2 \\ \phi &= 12.56\,mWb \end{aligned}$$

22. $\overline{B} = 2.5\sin\frac{\pi x}{2}e^{-2y}\overline{a}_z\ Wb/m^2$ **, find the flux for z = 0,** $y \ge 0\ \&\ 0 \le x \le 2m.$

Solution:-

$$\begin{aligned} \phi &= \int_s \overline{B}.\ \overline{ds} \\ &= \int_s \left(2.5\sin\frac{\pi x}{2}e^{-2y}\overline{a}_z\right).(dxdy\overline{a}_z) \\ &= \int_{x=0}^{2}\int_{y=0}^{\infty} 2.5\sin\frac{\pi x}{2}e^{-2y}dx\,dy \\ &= 2.5\left[-\cos\frac{\pi x}{2}\right]_{x=0}^{2}\left[\frac{e^{-2y}}{-2}\right]_{y=0}^{\infty} \\ &= 2.5\left[\frac{-\cos\pi + \cos 0}{\pi/2}\right]\left[\frac{e^{-\infty}}{-2} \frac{-e^0}{-2}\right] \\ &= \frac{2.5\times2}{\pi}\times2\times\frac{1}{2} \\ &= 1.592\,Wb. \end{aligned}$$

23. $\overline{H} = 2.39\times10^{-6}\cos\phi\overline{a}_\rho\ A/m.$ **Determine the flux defined by** $0 \le \phi \le \pi/4\ \&\ 0 \le z \le 2m.$

Solution:-

$$\phi = \int_s \overline{B}.\ ds = \int_s \mu_0\mu_r \overline{H} = 4\pi\times10^{-7}\int_0^2\int_{\phi=0}^{\pi/4}\left(\frac{2.39\times10^6}{\rho}\cos\phi\,\overline{a}_\rho\right)\left(\rho d\phi dz\overline{a}_\rho\right)$$

$$\phi = 2.39\times10^6\times4\pi\times10^{-7}\left[\sin\phi\right]_0^{\pi/4}\left[z\right]_0^2$$

$$= 2.39\times4\pi\times10^{-1}\left[\sin\pi/4-\sin0\right]\left[2\right]$$

$$\phi = 4.24\,\text{Wb}$$

24. Given $\overline{B}=\rho\sin\phi\overline{a}_\phi\ \text{Wb}/\text{m}^2$. Determine total flux crossing the surface defined by $1\le\rho\le2, \phi=\pi/4\ \&\ 0\le z\le5.$

Solution:-

$$\overline{B}=\rho\sin\phi\overline{a}_\phi$$

$$\phi=\int\overline{B}.ds=\int_{z=0}^{5}\int_{\rho=1}^{2}\left(\rho\sin\phi\overline{a}_\phi\right)(d\rho dz\overline{a}_\phi)$$

$$\phi==\left[z\right]_0^5\left[\frac{\rho^2}{2}\right]_1^2\sin\phi$$

At $\phi=\pi/4, \phi=\left[5\right]\left[\frac{4}{2}-\frac{1}{2}\right]\sin45^\circ=5\times3/2\times\frac{1}{\sqrt{2}}=5.303\text{Wb}$

25. Given $\overline{B}=\left(2/\rho\right)\overline{a}_\phi T.$ Determine the magnetic flux ϕ crossing the plane defined by $0.5\le\rho\le2.5\text{m}\ \&\ 0\le z\le3.$

Solution:-

$$\phi=\int_0^3\int_{0.5}^{2.5}\left(\frac{2}{\rho}\overline{a}_\phi\right).\ (d\rho dz\overline{a}_\phi)$$

$$=2\left[\ln\rho\right]_{0.5}^{2.5}\left[z\right]_0^3$$

$$=6\ln\left[2.5\right]=9.66k/b$$

26. Given mean circumference $\ell=1.2\text{m}$ & area of 8cm^2. N = 480turns. I = 2A, ϕ **=1 Wb. What is the permeability**

Solution:-

$$\ell=1.2\text{m},\ A=8\times\left(10^{-2}\right),\ I=2A$$

$$N=480\,\text{turn}\quad \phi=1\text{Wb}$$

$$B=\frac{\phi}{a}=\frac{1}{8\times10^{-4}}=\frac{100\times10^2}{8}=1250\,\text{Wb}/\text{m}^2$$

$$B=\mu H=\mu_0\mu_r H$$

$$H=\frac{NI}{\ell}=\frac{480\times2}{1.2}=800\text{A}-\text{T}/\text{m}.$$

$$\mu_r=\frac{B}{\mu_0 H}=\frac{1250}{4\pi\times10^{-7}\times800}=0.124\times10^7$$

27. Calculate b, due to a coil N = 1000 A- T, area = 100 cm^2 & h = 10m.

Solution:-

NI = 1000Ampere turns h = 10m, area = 100×10^{-4}

$\pi a^2=100\times10^{-4}\Rightarrow a=5.64\text{cm}.$

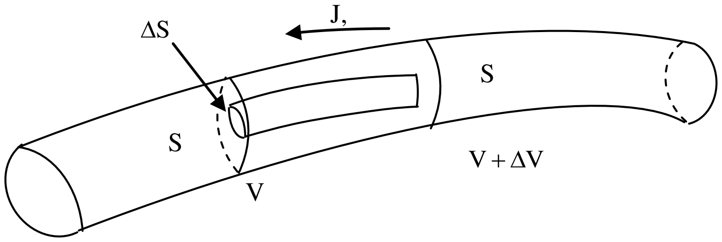
$$H = \frac{1000 \times (5.64 \times 10^{-2})^2}{2\left[100 + (5.61 \times 10^{-2})^2\right]^{3/2}} = 1.59 \times 10^{-3}$$

$$B = \mu H = 1.59 \times 10^{-3} \times 4\pi \times 10^{-7} \times 1$$

UNIT III

State and prove point from of ohm's law.

Point form of ohm's law:-



Ohm's law is given by V = IR where 'R' is the resistance of the given medium. Let S and S' be two surface with potential V and $V + \Delta V$. The plate are separated through a distance $\Delta \ell$.

$$I = \iint_s J.\ N\ ds$$

If there is a charge –q, it will experience a force.

$$F = -\ q\ E$$

In free space, the electrons would get accelerated and its velocity would continuously increase.

In the crystalline material, the progress of the electron is impeded by its continual collisions with the thermally excited attained is drift velocity.

$$V_d = -\mu_e E \ \rightarrow (1)$$

The electron velocity is in a direction opposite to that of the electric filed.

We know that $J = \rho_e V_d \ \rightarrow (2)$

Sub (1) in (2), we get

$$J = -\rho_e \mu_e E$$
$$J = \sigma E$$

Point form of ohm's law.

UNIT – IV

MAGNETIC FORCES AND MATERIALS

PART-A

1. State Farad's law of electromagnetic Induction

The total electromagnetic force in a circuit is equal to the time rate of decrease of total magnetic flux in the circuit.

$$\varepsilon = -\frac{d\phi}{dt}$$

2. State Lenz law:-

It states that the current in the loop is always in the such a direction as to opposite the change of magnetic flux produce it.

3. What is transformer emf?

The emf induced in a stationary conductor due to the change in flux linked with is called transformer emf (or) static induced emf

$$\text{emf} = -\iint \frac{\partial B}{\partial t}.\ ds \ \text{. eg. Transformer.}$$

4. Find the displacement current density for the field

Solution:-

$$J_d = \frac{\partial D}{\partial t} = \frac{\partial}{\partial t}(\varepsilon E) = \frac{\partial}{\partial t}(\varepsilon_0 \varepsilon_r E)$$
$$= \frac{\partial}{\partial t}\left[8.854\times10^{-12}\times1\times300\sin10^9\ t\right]$$
$$= 8.854\times10^{-12}\times300\times10^9\cos10^9 t$$
$$= 8.854\times0.3\cos10^9 t$$
$$J_d = 2.6562\cos10^9\ t\ A/m^2$$

5. The parallel plates in a capacitor have an area of 5cm^2 & separated by 0.5cm. A voltage of 10sin 10^3 t v is applied to the capacitor. Find the displacement current with $\varepsilon_r = 5$.

Solution:-

Given d = 0.5cm

$$A = 5cm = 5\times10^{-4}\ m^2$$
$$\varepsilon_r = 5,\ v = 10\sin10^3 t$$

$$J_d = \frac{\partial D}{\partial t} = \varepsilon\frac{\partial E}{\partial t} = \frac{\varepsilon_0\varepsilon_r}{d}\frac{\partial v}{\partial t}$$

$$I_d = J_d \times A$$

$$I_d = \frac{\varepsilon_0\varepsilon_r A}{d}\ \frac{dV}{dt}$$

$$= \frac{8.854\times10^{-12}\times5\times5\times10^{-4}}{0.5\times10^{-2}}\frac{d}{dt}(10\sin10^{3}t)$$
$$= \frac{8.854\times10^{-12}\times5\times5\times10^{-4}\times10^{3}}{5}\times10\times10^{3}\times\cos10^{3}t$$
$$= 8.854\times5\times10^{-9}\cos10^{3}t$$

$$I_d = 44.27\cos10^{3}t\,nA$$

6. Find displacement current density (Jd) with $\varepsilon_r = 10$ area of the plates 0.01m^2, d = 0.5mm & v = 200 sin 200 t.

Solution:-

$$J_d = \frac{\partial D}{\partial t} = \frac{\partial(\varepsilon E)}{\partial t} = \frac{\varepsilon_0\varepsilon_r}{d}\frac{dv}{dt}$$

$$= \frac{8.854\times10^{-12}\times10}{0.05\times10^{-3}}\frac{d}{dt}(200\sin200t)$$
$$= \frac{8.854\times10^{-12}\times10\times200\times200\cos t}{5\times10^{-5}}$$
$$= 8.854\times8\times10^{-3}\cos200t$$
$$= 70.832\cos200t\,mA/m^{2}$$

7. A Cu wire carries conduction current of 1A. determine the displacement current at 1MHz. For Cu, $\varepsilon = \varepsilon_0$ & $\sigma = 5.8\times10^{7}\ s/m$

Solution:-

$$I_c = J_C\times A$$
$$= \sigma E\times A$$
$$E = \frac{I_c}{\sigma A}$$
$$J_d = \frac{\partial D}{\partial t} = \frac{\partial(\varepsilon E)}{\partial t}$$

Since $\frac{\partial}{\partial t} = j\omega,\ J_d = \omega\varepsilon E = \omega\varepsilon_0\frac{I_c}{\sigma A}$

$$J_d = 2\pi f\ \varepsilon_0\frac{I_c}{\sigma A}$$
$$I_d = J_d\times A$$
$$= 2\pi f\frac{\varepsilon_0 I_c}{\sigma A}\times A$$
$$= \frac{2\times3.14\times1\times10^{6}\times8.854\times10^{-12}}{5.8\times10^{7}}$$
$$I_d = 9.585\times10^{-13}\ A$$

8. Define Magnetic moment:-

Magnetic moment is defined as the maximum torque on loop per uniform magnitude flux density

$$m = \frac{T}{B}$$

In case of magnitude dipole, magnetic moment is given by m= Qm . L

Qm = charge ; L = length

In case of current loop m = IA

I = current; A = area of the loop.

9. Define magnetic dipole moment

Magnetic dipole moment

$$m = Q_m \ell$$

Consider a far magnet of length ℓ and area of cross section A.

Magnetisation is define as the net dipole moment / per unit volume

$$M = \frac{m}{V} = \frac{Q_m \ell}{A\ell} \bar{a}_\ell$$

$$\boxed{M = \frac{Q}{A} \bar{a}_\ell}$$

10) Define magnetic susceptibility

Magnetic susceptibility (Xm)

$$\begin{aligned}
B &= \mu_0 (H + M) \\
&= \mu_0 H \left(1 + \frac{M}{H}\right) \\
B &= \mu_0 H (1 + X_m) \\
X_m &= \frac{M}{H} = \text{Magnetic susceptibility} \\
B &= \mu_0 \mu_r H \\
\text{where } \mu_r &= 1 + X_m
\end{aligned}$$

11. A current sheet, $\bar{k} = 8.5\bar{a}_z\ A/m$, at x = 0 separated region 1, x < 0, where $\bar{H}_1 = 20\bar{a}_y A/m$ & region 2, x > 0 find $\bar{H}_2$ at x = 0

Solution:-

Given $\bar{H}_1 = 20\bar{a}_y A/m$ in region 1

The normal component is in z – direction. Hn1=Hn2 > 0 & Bn1=Bn2>0

$$\begin{aligned}
&(H_{t1} - H_{t2}) \times \bar{a}_{n12} = \bar{k} \\
&(20\bar{a}_y - H_{y2}\bar{a}_y) \times a_x = 8.5a_z \\
&(20 - H_{y2})\bar{a}_y \times \bar{a}_x = 8.5a_z \\
&(20 - H_{y2})(-a_z) = 8.5a_z \\
&-20a_z + H_{y2}a_z = 8.5a_z \\
&H_{y2} = 28.5\bar{a}_y\ A/m
\end{aligned}$$

12. Find the maximum torque on a 75 turn, rectangular coil, 0.5m by 0.6m carrying a current of 4A in a magnetic filed of B = 5T

Solution:-

Given ℓ= 0.5m w = 0.6m I =4A, N = 75 B = 5T

$$T = N\ B\ I\ A \sin\theta$$

Maximum torque is obtained when $\theta = 90°$

$$\begin{aligned}
T_{max} &= NBI(\ell w) \\
&= 75 \times 5 \times 4 \times (0.5 \times 0.6)
\end{aligned}$$

T_{max}=450

13. A 200 turn coil of 30cm $\times$ 15cm with a current of 5A is placed in a uniform field of flux density B = 0.2T. Determine the magnetic moment m & maximum torque.

Solution:-

Given $\ell = 30\text{cm} = 30\times10^{-2}$; $w = 15\text{cm} = 15\times10^{-2}$; $I = 5A,\ N = 200;\ B = 0.2T$

$$m = NIA = 200\times5\times30\times15\times10^{-4}$$
$$= 45 A.m^2$$
$$T_{max} = mB = 45\times0.2$$
$$T_{max} = 9Nm$$

14. A square coil of 200 turns & 0.5 m long sides is in a region of uniform field with density 0.2T. If the maximum torque is 4×10^{-2}N. m , what is the current?

Solution:-

Given $A = a^2 = (0.5)^2 = 0.25m^2$

$$N = 200,\ B = 0.2T\ \ T_{max} = 4\times10^{-2}$$

$$T_{max} = NIAB$$

$$\frac{4\times10^{-2}}{200\times0.25\times0.2} = I$$
$$I = 4\times10^{-2}A = 0.4mA$$

15. Find Ic in a circular conductor of radius 4mm if the current density varies to $\bar{J} = \frac{10^4}{\rho}\bar{a}_z\ A/m^2$

Solution:-

$$I_c = \int \bar{J}\,\bar{d}_s = \int_{\phi=0}^{2\pi}\int_{z=0}^{0.004}\left(\frac{10^4}{\rho}a_z\right)\left(\rho\ d\rho\ d\phi\ \bar{a}_z\right)$$
$$I_c = 10^4\ (0.004)(2\pi) = 80\pi A.$$

16. In a cylindrical conductor of radius 2mm, $J = 10^3\ e^{-400\rho}\ \bar{a}_z A/m$ **find Ic.**

Solution:-

$$I_c = \oint_s \bar{J}.\ \bar{d}_s = \int_0^{2\pi}\int_0^{0.002}(10^3 e^{-400\rho}\bar{a}_z).(\rho\ d\rho\ d\phi\ \bar{a}_z)$$

$$= \int_0^{2\pi} d\phi \int_0^{0.002} 10^3\rho\ e^{-400\rho}\ d\rho$$
$$= 2\pi\times10^3\left[\frac{-\rho}{400}e^{-400\rho}\ \frac{e^{-400\rho}}{(-400)^2}\right]_0^{0.002}$$
$$= \frac{2\pi\times10^3}{16\times10^4}\left[-e^{-400\rho}\ (400\rho+1)\right]_0^{0.002}$$

$$I_c = 7.51mA$$

17. Find Id, where $\varepsilon = 100\varepsilon_0,\ A = 0.01m^2\ \ d = 0.05\times10^{-3}\ \ \&\ v = 100\sin 200\,\pi t$

Solution:-

$$C = \frac{\varepsilon A}{d} = \frac{100\varepsilon_0 \times 0.01}{0.05 \times 10^{-3}} = \frac{8.854 \times 10^{-12} \times 10^2 \times 10^2}{5 \times 10 \times -5} = 1.7708 \times 10^{-7} = 0.1771 \mu F$$

$$I_d = C\frac{dv}{dt} = 0.1771 \times 10^{-6} \frac{d}{dt}(100 \sin 200\pi t)$$
$$= 0.1771 \times 10^6 \times 100 \times 200\pi \times \cos\ 200\pi t$$
$$= 0.3542 \times 10^{-2} \pi \cos 200\pi t$$
$$I_d = (11.13 \cos\ 200\pi t)\ mA$$

18. Find the electric flux density & volume charge density if $\overline{E} = 2x^2\overline{a}_x + 4y^2\overline{a}_y + 2z^2\overline{a}_z\ v/m;\ \varepsilon_r = 4$

Solution:-

$$\overline{E} = 2x^2\overline{a}_x + 4y^2\overline{a}_y + 2z^2\overline{a}_z\ v/m$$

$$D = \varepsilon E = \varepsilon_0 \varepsilon_r E$$
$$= 8.854 \times 10^{-12} \times 4 \times (2x^2\overline{a}_x + 4y^2\overline{a}_y + 2z^2\overline{a}_z)$$
$$D = 70.83x^2\overline{a}_x + 141.66y^2\overline{a}_y + 70.83z^2\overline{a}_z\ \ pc/m^2$$
$$\rho_v = \nabla.D$$
$$= \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$
$$= 141.66x + 283.32y + 141.662pc/m^2$$

19. A rectangular coil carrying a current of 5A is placed in the magnetic field of flux density $\overline{B} = 0.3(\overline{a}_x + \overline{a}_y)Wb/m^2$**. The coil is lying the yz plane & has dimensions 0.8m** $\times$ **0.4m. Find the torque on the coil.**

Solution;-

$$I = 5A\quad \overline{B} = 0.3(a_x + a_y)\,Wb/m^2$$

$$A = (0.8 \times 0.4)a_x = 0.32\overline{a}_x m^2$$
$$t = \overline{m} \times \overline{B}$$
$$T = 5(0.32a_x) \times 0.3(a_x + a_y)$$
$$= 5(0.3)(0.3)a_z$$
$$= 0.48\overline{a}_z\ N.m$$

20. A rod of length 'ℓ' rotates about the z – axis with an angular velocity 10. If $\overline{B} = B_0\,\overline{a}_z Telsa$ **, calculate the voltage induced.**

Solution:-

$$EMF = \int (v \times B).\,dl$$

$$= \int_{\rho=0}^{\ell} \left(\rho\omega a_\phi \times B_0\,\overline{a}_z\right).d\rho a_\rho$$
$$= \int_0^{\ell} B_0\,\rho\omega\overline{a}_\rho\ d\rho\ \overline{a}_\rho$$

$$= B_0\omega\int_0^{\ell} \rho\ d\rho$$

$$= B_0\omega\left[\frac{\rho^2}{2}\right]_0^{\ell}$$

$$\text{EMF} = \frac{1}{2}B_0\omega\ell^2 V$$

PART-B

1) State and explain the Lorentz force equation for a moving charge;-

Consider that a charged particle is moving in a magnitude field of flux density B. It experience a force given by

$$F = Q(\overline{V}\times\overline{B} \rightarrow (1)$$

The force is proportional to the product of the magnitude of the charge Q, its velocity V & flux density B & to the sine of the angle between V and b. The direction of the force is perpendicular to both V and B.

$$F = Qv\,B\sin\theta \rightarrow (2)$$

The electric force on a charged particular in the electric of intensity E is

$$F = Q\overline{E} \rightarrow (3)$$

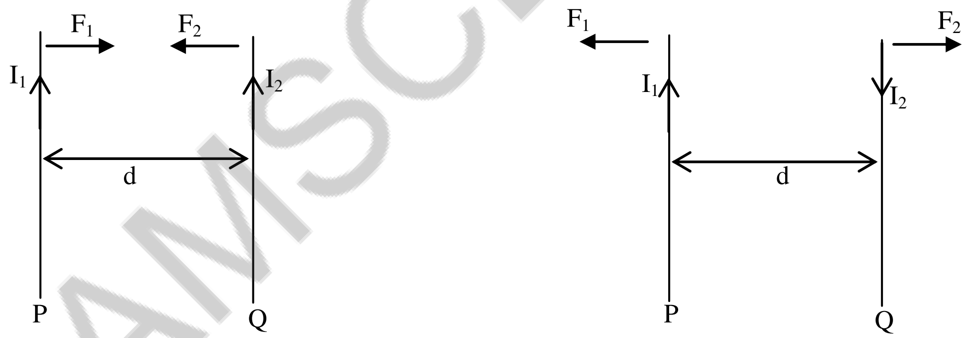
The force on a moving particle due to combined electric field and magnitude field is obtained

$$F = Q\left[\overline{E} + (\vec{v}\times\vec{B})\right]$$

This force is called Lorentz force.

Obtain an expression for the force between two parallel conductors

Consider two straight, long parallel conductor P and Q separated by a distance d. Let I1 and I2 be the currents flowing in conductors P and Q.



Consider a conductor P produces a magnetic field whose flux density is B at conductor

$$B = \frac{\mu I_1}{2\pi d}$$

The force on conductor Q due to P, F1 = B.

Where L = length of the conductor

$$F_1 = \frac{\mu I_1 I_2 \ell}{2\pi d}$$

If the current is flowing in the same direction, then there is force of attraction

$$F_2 = F_1$$

If the currents are flowing in opposite direction, then there is force of repulsion

$$F_2 = -F_1$$
$$F_2 = \frac{-\mu I_1 I_2 \ell}{2\pi d} N$$

If the conductor is infinitely long, the force per unit length is,

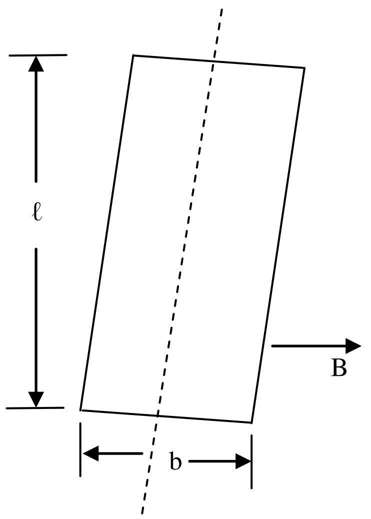
$$F = \frac{\mu I_1 I_2 \ell}{2\pi d} N/m$$

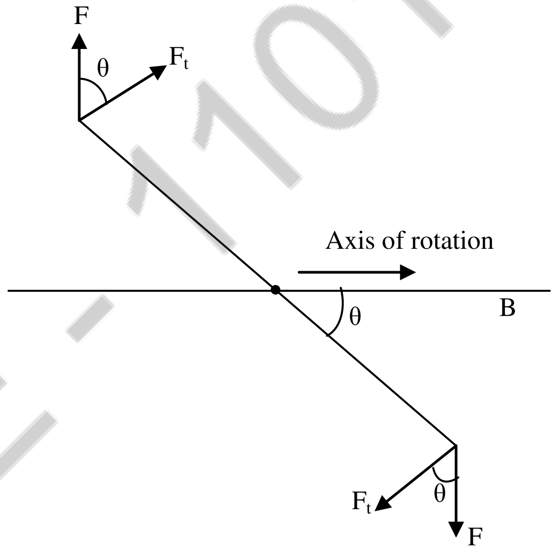
2) Explain the magnitude moment Torque on a loop carrying a current I:-

When a current loop is placed parallel to H, force at on the loop that end to rotate it. The tangential force multiplied by the radial distance at which it at is called torque.

Unit of the Torque → Newton – metre (N – m)

Consider the rectangular loop of length 'L' and breadth 'b' carrying a current I in a uniform magnetic of flux density B.





The force acting on the loop, $F = BIL\sin\theta$

If the loop plane is parallel to the magnetic field total torque on the loop is T.

$T = 2 \times$ Torque on each side

$= 2 \times$ tangential force $\times$ radial distance

$= 2 \times BIL \times b/2$

$T = BIA$

If the loop plane makes an angle Q with respect to the magnitude flux density B, the tangential component of the force is $F_t = F\cos\theta$.

Total torque on the loop, $T = BIA$

Magnetic moment, $m = IA$ A/m^2

$T = BIA\sin\theta$

$= mB\sin\theta$

$\overline{T} = m \times \overline{B}$

$m = \frac{T}{B}$ [If torque is maximum, i.e Q = 90°]

Magnetic moment:-

Magnetic moment is defined as the maximum torque on loop per uniform magnitude flux density

$$m = \frac{T}{B}$$

In case of magnitude dipole, magnetic moment is given by m= Qm . L

Qm = charge ; L = length

In case of current loop m = IA

I = current; A = area of the loop.

3) Detail about the scalar and vector magnetic potential:-

The potential can be of two types

a) Magnetic vector potential (or) vector magnitude potential ($\bar{A}$)
b) Magnetic scalar potential (or) scalar magnitude potential (Vm)

(i) magnetic vector potential:-

Scalar magnetic potential exists if there is no current enclosed (i.e) $\oint H.dl = 0$. If current is enclosed the potential depends upon current element (vector quantity) is no more scalar but it is a vector quantity.

Since the divergence of vector is a scalar the vector potential is expressed in curl.

Let 'A' be any magnetite vector potential and vector potential is expressed in curl.

$$\nabla \times A = B$$

Talking curl on the both the sides

$$\nabla \times \nabla \times A = \nabla \times B \rightarrow (1)$$

By Identity,

$$\nabla \times \nabla \times A = \nabla(\nabla.A) - \nabla^2 A \rightarrow (2)$$

$$\begin{aligned}
&\nabla \times B = \mu J \\
&\nabla \times B = \nabla(\nabla.A) - \nabla^2 A \\
&\mu J = \nabla(\nabla.A) - \nabla^2 A \\
&\text{for steady d.c}, \nabla.A = 0 \\
&-\mu J = \nabla^2 A \\
&\nabla^2 A_x \bar{a}_x + \nabla^2 A_y \bar{a}_y + \nabla^2 A_z \bar{a}_z = -\mu(\bar{a}_x J_x + \bar{a}_y J_y + \bar{a}_z J_z) \\
&\text{Equating,} \\
&\qquad \nabla^2 A_x = -\mu J_x \\
&\qquad \nabla^2 A_y = -\mu J_y \\
&\qquad \nabla^2 A_z = -\mu J_z
\end{aligned}$$

This is in the form of poisson's equation.

In general, magnitude vector potential is expressed as,

$$A = \frac{\mu}{4\pi} \iiint_v \frac{J}{r}\, dv$$

$$A_x = \frac{\mu}{4\pi}\int_v \left(\frac{J_x}{r}\right) dv;\ A_y = \frac{\mu}{4\pi}\int_v \left(\frac{J_y}{r}\right) dv;\ A_z = \frac{\mu}{4\pi}\int_v \left(\frac{J_z}{r}\right) dv$$

r = distance between current element and the point at which $\overline{A}$ is the to be calculated.

Scalar magnetic potential:-

Ampere's law states that the line integral of the field H around a closed path is equal to the current enclosed

$$\oint_{\lambda} H.\,dl = I$$

If no current is enclosed (i.e) J = 0

$$\oint_{\lambda} H.\,dl = 0$$

Magnetic field intensity can be expressed as the negative gradient of a scalar function

$$H = -\nabla V_m$$

Where V_m = scalar magnitude potential.

$$V_m = -\int H\,.dl$$

This scalar potential also satisfied Laplace equation

In free space $\quad \nabla.B = 0 \Rightarrow \mu_0\ \nabla.H = 0$

$$\text{But } H = -\nabla Vm \Rightarrow \mu_0\ \nabla.(-\nabla V_m) = 0$$
$$\mu_0\ \nabla^2 V_m = 0$$
$$\nabla^2 V_m = 0$$

4) Explain the Force on a differential current element (or) Force on a wire carrying current I placed in a magnetic field.

The force exerted on a differential element of charge dQ moving in a steady magnetic field is given by

$$\overline{dF} = dQ(\vec{V} \times \vec{B}) \quad \rightarrow (1)$$

The current density $\bar{J}$ can be expressed as

$$\bar{J} = \rho_v\ \vec{V} \qquad \rightarrow (2)$$

The differential element of charge can be expressed as

$$dQ = \rho_v\ dv \qquad \rightarrow (3)$$

Sub (3) in (1),

$$\overline{dF} = \rho_v\ dv\ (\overline{V} \times \overline{B})$$
$$\overline{dF} = (\rho_v \overline{V} \times \overline{B})dv \quad \rightarrow (4)$$
$$\overline{dF} == (\vec{J} \times \overline{B})\ dv$$

Relationship between current elements

$$\vec{J}\ dv = \overline{k}\ ds = I\ \overline{dl}$$
$$\overline{dF} = \vec{J}\ dv \times \vec{B} \Rightarrow \overline{dF} = I\overline{dl} \times \vec{B}$$

Integrating the above $\overline{F} = I\vec{\ell} \times \overline{B}$

$$F = B\ I\ L \sin\theta$$

5) Define inductance field in magnitude materials

Inductance:-

Any conductor carrying a current produces a field around it. The lines of magnetic flux produced by a current in a solenoid coil form closed loops. If the current in the coil is alternating with respect to times, the flux linked with the coil also varies.

The values of the flux depend on flux density which in turn depends on the current flowing through it.

If there are N – turns in the coil, the total flux linked with the coil is called the flux linkage (Λ)

$$\Lambda = N\phi$$

Due to variation of current, there will be variation of flux linked with the coil which in turn induces an emf.

$$\varepsilon = \frac{-d\Lambda}{di}.\frac{di}{dt}$$
$$\varepsilon = -\left(N\frac{d\phi}{di}\right)\frac{di}{dt} \quad [\because \Lambda = N\phi]$$

Where i = instantaneous value of the current $\frac{d\Lambda}{di}$ or $\frac{N\ d\phi}{di}$ represents the of charge of flux linkage with respect to current. This quantity depends upon the geometrical configuration of the given device and is referred to an inductor and the device which possesses this properly is called Inductor.

Derive the expression of inductance:-

The inductance is defined as the ratio of total magnitude flux linkage to the current through the coil.

Thus, $L = \frac{d\Lambda}{di} = \frac{N\ d\phi}{di}$

If the flux ϕ varies linearly with i,

$$\frac{d\phi}{di} = \frac{\phi}{i}$$
$$L = \frac{\Lambda}{\ell} = \frac{N\phi}{i}$$
$$\varepsilon = -L\frac{di}{dt}$$

The negative sign indicates that the emf is set up in such direction so as opposite the changes in current

$$\varepsilon = -V$$
$$V = +L\frac{di}{dt}$$

The unit of an inductance is Henry. The inductance of coil is said to be 1Henry, if when current in the coil changes at a rate of 1 ampere / second, an emf of 1 volt is induced in it; or if the flux linkage with the coil changes at the rate of 1 wb turn ampere.

6) Describe the equation of solenoids

Solenoids:-

Let b is the flux density and A, the area of cross section of the solenoid, then the flux through the solenoid is $\phi = BA$ and the flux linkage $\Lambda = N\phi = NBA$

Inductance is, therefore, given by

$$L = \frac{\Lambda}{I} = \frac{NBA}{I} \qquad \rightarrow (1)$$

We know that, for long solenoid, $B = \mu_0 \frac{NI}{\ell} \rightarrow (2)$

Sub (2) in (1), we get

$$L = N\left(\mu_0 \frac{NI}{\ell}\right)\frac{A}{I}$$

$$\boxed{L = \frac{\mu_0 N^2 A}{\ell}} \text{ Henry}$$

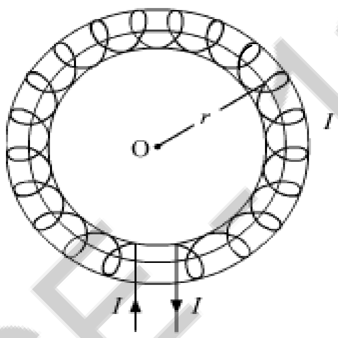
7) Derive the expression of inductance in a Toroid

Inductance in a toroid:-

When a long solenoid is bent into a circular closed on itself, a toroidal coil is obtained. If the toroid has uniform windings, the flux is confined almost interior & the flux at the external of the coil is zero.

Let I be the current flowing in the toroid, B the magnetic field produced at every point of circular path of radius R. $B = \frac{\mu I}{2\pi R}$

Since it has 'N' turns



$$B = \frac{\mu NI}{2\pi R}$$

By Amperes circuital law,

$$\oint_L B.dl = \mu NI$$

$$B\int dl = \mu NI$$

$$B.2\pi R = \mu NI$$

$$B = \frac{\mu NI}{2\pi R}a$$

Flux linkage, $\Lambda = N\phi = NBA = \frac{N\mu NI}{2\pi R}.A = \frac{\mu N^2 IA}{2\pi R}$

Since toroid is a circular section, $A = \pi r^2$

$$\Lambda = \frac{\mu N^2 I \pi r^2}{2\pi R}$$

$$l = \frac{\Lambda}{I} = \frac{\mu N^2 I \pi r^2}{2\pi RI}$$

$$\boxed{L = \frac{\mu N^2 r^2}{2R} \text{ Henry}}$$

8) Derive the expression of Toroid of rectangular

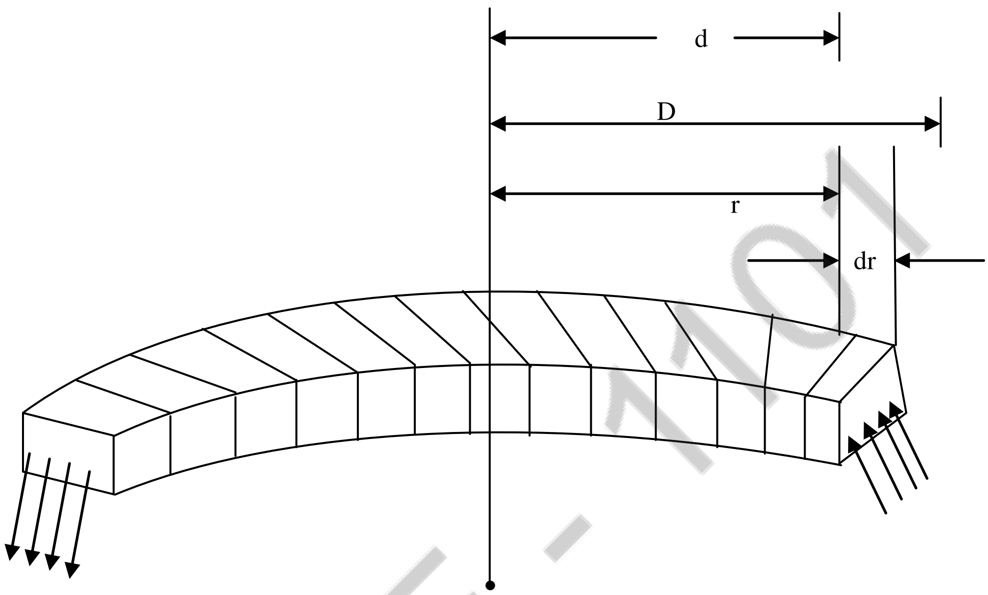
Toroid of rectangular cross section:-

Consider a toroid of rectangular cross section of inner radii ‘d’ & outer radii ‘D’.

Let N be the number of times

$$\oint B.\,dl = \mu NI$$
$$B.\,2\pi r = \mu NI$$
$$B = \frac{\mu NI}{2\pi r}$$

Now consider a rectangular strip of width dr and height h at a distance r from the centre.



$$d\Lambda = Nd\phi = N.BdA$$
$$d\Lambda = N\frac{\mu NI}{2\pi r}\ h.dr$$
$$\Lambda = \frac{\mu N^2 Ih}{2\pi r}\int_d^D \frac{dr}{r} = \frac{\mu N^2 Ih}{2\pi}\log D/d$$
$$\boxed{L = \frac{\Lambda}{I} = \frac{\mu N^2 h}{2\pi}\log D/d}\ \text{Henry}$$

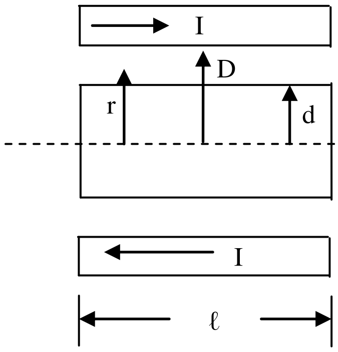
9) Derive the expression of inductance of co – axial cable

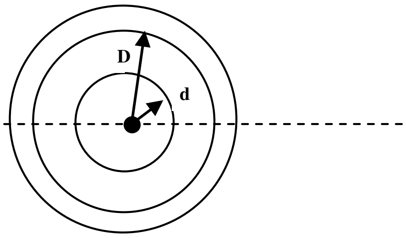
Inductance of a co- axial cable:-

The magnetic flux density B at any radius ‘r’

$$B = \frac{\mu I}{2\pi r}$$

$$d\Lambda = Bxdr$$
$$d\Lambda = \frac{\mu I}{2\pi r}.\ dr$$
$$\int d\Lambda = \Lambda = \int_d^D \frac{\mu I}{2\pi r}.dr$$
$$= \frac{\mu I}{2\pi}\left[\log_e r\right]_d^D$$
$$\Lambda = \frac{\mu I}{2\pi}\log D/d \qquad \text{D = outer diametre; d = inner diametre.}$$
$$\boxed{L = \frac{\Lambda}{I} = \frac{\mu}{2\pi}\log_e D/d\ H/m}$$





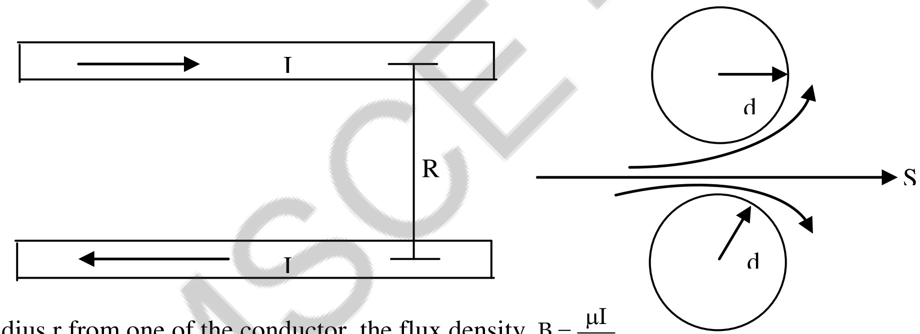
Co – axial cable

$$L = \frac{\mu_0}{2\pi}\log_e D/d$$
$$= \frac{4\pi\times10^{-7}}{2\pi}\times 2.303\,\log_{10} D/d$$
$$= 2\times 2.303\times10^{-7}\log_{10} D/d$$
$$= 4.606\times10^{-7}\log_{10} D/d$$
$$= \frac{4606}{1000}\times10^{-7}\log_{10} D/d$$
$$= 4606\times10^{-7}\log_{10} D/d$$
$$\boxed{L = 0.4606\,\log_{10} D/d\ \text{mH/km}}$$

10) Derive the expression of inductance of a two – wire tx line

Inductance of a two wire transmission line:-

A two wire transmission line is as shown in figure whose conductor radius is d & the spacing between centre iss R.



At any radius r from one of the conductor, the flux density $B = \frac{\mu I}{2\pi r}$

$$\Lambda = \int d\Lambda = 2\int_d^R B.dr = 2\int_d^R \frac{\mu I}{2\pi r}.dr = \frac{\mu I}{2\pi r}\log_e\left(R/d\right)$$
$$L = \frac{\Lambda}{I} = \frac{\mu}{2\pi}\times 2.303\times\log_{10}\left(R/d\right)$$
$$= \frac{4.606}{2}\times10^{-7}\log_{10} R/d = 0.9219\,\log_{10} R/d\ \mu H$$

Derive Energy stored in a magnetic field:- An inductor stores energy when a current through a inductance coil is gradually changed from 0 to I. the energy change of it is opposite by the self induced emf produced due to this change.

$$W = \int_0^I P.dt\ \text{Joules}$$
$$P = VI$$
$$V = L\frac{dI}{dt}$$
$$W = \int_0^I LI\frac{dI}{dt}.dt$$
$$\boxed{W = \frac{1}{2}LI^2}\ \text{Joules} = \frac{1}{2}\Lambda I = \frac{1}{2}$$

Energy density:-

Let us consider the inductance of a solenoid

$$L = \frac{\mu N^2 A}{\ell}$$

$$W = \frac{1}{2} L I^2$$

$$W = \frac{1}{2} \frac{\mu N^2 A}{\ell} I^2$$

$$= \frac{1}{2} \mu \left(\frac{NI}{\ell} \right)^2 \vartheta$$

$$W = \frac{1}{2} \mu H^2 \vartheta$$

$$\omega = \frac{W}{\vartheta} = \frac{1}{2} \mu H^2$$

$$= \frac{1}{2} \beta H$$

$$\omega = \frac{1}{2} \frac{B^2}{\mu} \text{ Joules}$$

11) Derive the mutual inductance of a coil

Mutual inductance:-

In the case of an isolated circuit, the flux produces by the current links only with that circuit. The corresponding inductance representing the flux linkage per unit current is sometimes referred to as self inductance.

Mutual inductance is the flux linked in one coil due to the current in the second coil.

Let us consider the flux linking one of the n circuits 1,2,......n say R^{th} circuit.

$$\Lambda_k = \Lambda_{k1} + \Lambda_k + ...\Lambda_{kk} + ...\Lambda_{kn}$$

$$= \sum_{j=1}^{n} \Lambda_{kj}$$

= total flux linkage with the R^{th} circuit due to all the n circuits composing the system.

The e. m. f induced in the R^{th} circuit may be written as

$$\varepsilon_k = \frac{-d\Lambda_k}{dt} = \frac{-d}{dt} \sum_{j=1}^{n} \Lambda Rj$$

$$= -\sum_{j=1}^{n} \frac{d\Lambda_{kj}}{dt}$$

$$\frac{d\Lambda_{kj}}{dt} = \frac{d\Lambda_{kj}}{dI_j} = \frac{dI_j}{dt}$$

Where $\frac{d\Lambda_{kj}}{dI_j}$ denotes the ratio of flux linkage with R^{th} circuit with respect to the current in j^{th} circuit, has the dimension of inductance is therefore, referred to as mutual – inductance

$$M_{kj} = \left(\frac{1}{2} \frac{d\Lambda_{kj}}{dI_j} \right) k \neq j$$

E. M. F induced in the R^{th} circuit

$$\varepsilon_k = -M_{kj} \frac{dI_j}{dt}$$

If k =j

$$\frac{d\Lambda_{kk}}{dt} = \frac{d\Lambda_{kk}}{dI_k}\frac{dI_k}{dt}$$

$$\frac{d\Lambda_{kk}}{dI_k} = L_{kk}. \quad \text{(self inductance of the circuit)}$$

Consider a toroid with two windings P and S the winding P has N_1 turns & is called primary winding and s with N_2 turns is referred to a secondary winding.

Flux linkage with winding P is

$$\Lambda_H = N_1\,(BA) = \mu_0 N_2 N_1 \frac{I_1 A}{\ell m}$$

Self inductance of coil 'P' is

$$L_H = \frac{\Lambda_H}{I_1} = \frac{\mu_0 N_1^2 A}{\ell m} \qquad [\ \ell m \text{ =mean length of magnetic path = } 2\pi R_m\ ]$$

Mutual inductance of switch P

$$M_{21} = \frac{\Lambda_{21}}{I_1} = \frac{\mu_0 N_2 N_1 A}{\ell m}$$

$$\varepsilon_2 = -M_{21}\frac{dI_1}{dt}$$

Self inductance of coil 'S' is

$$L_{22} = \frac{\mu_0 N_2^2 A}{\ell m}$$

Mutual inductance of P with S

$$M_{12} = \frac{\mu_0 N_1 N_2 A}{\ell m}$$

$$M_{12} = M_{21}$$

$$M_{12} = M_{21} = L_{11}L_{22}$$

$$L_{11} = L_1,\, L_{22} = L_2;\, M_{12} = M_{21} = M$$

$$M^2 = L_1 L_2$$

$$M = \sqrt{L_1 L_2}$$

Suppose R_1 times the flux produced by I_1 links with secondary, then R_2 times the flux produced by I_2 with the secondary

$$\frac{M_{21}}{L_{11}} = k_1\frac{N_2}{N_1};\frac{M_{12}}{M_{22}} = k_2\frac{N_1}{N_2}$$

$$\frac{M_{21}\,M_{21}}{L_{11}L_{22}} = k_1 k_2;\frac{M^2}{L_1 L_2} = k^2$$

12) Derive a Boundary condition of magnetic field

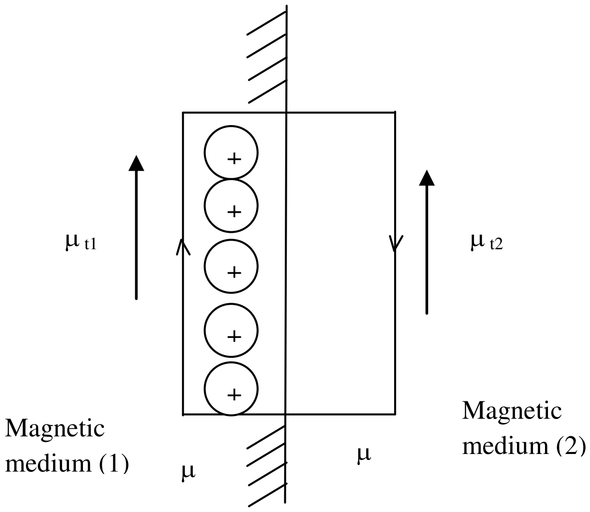
Magnetic Boundary condition:-

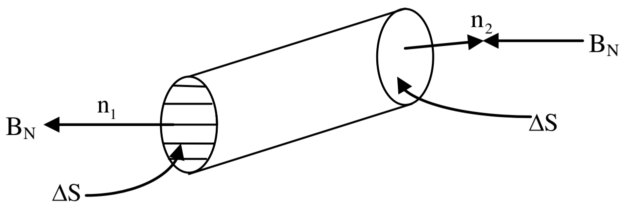
Figure shows an interface between two magnetic media with permeability's μ_1 and μ_2. Consider a Gaussian surface and a closed path to the boundary between the media (1) and (2).

$$\oint_s B.\,n\,ds = 0$$

If B_1 and B_2 are the magnetic flux densities in media (1) and (2),

$$B_1.n_1\Delta s + B_2.n_2\Delta s = 0$$





Where Δs is the pill box surface and $\left(n_1 \text{ and } n_2\right)$ are unit outward normal.

$$(B_1 - B_2).n_1 = 0$$
$$B_{n1} - B_{n2} = 0$$
$$B_{n1} = B_{n2}$$
$$\mu_1 H_{n1} = \mu_2 H_{n2}$$
$$\boxed{\frac{H_{n1}}{H_{n2}} = \frac{\mu_2}{\mu_1}}$$

Applying Ampere's circuital law

$$\oint_{\ell} H.dl = I$$
$$H_{t1}\Delta\ell - H_{t2}\Delta\ell = k.\Delta\ell$$
$$H_{t1} - H_{t2} = K$$

If sheet current density is zero,

$$H_{t1} = H_{t2}$$
$$\frac{B_{t1}}{\mu_1} = \frac{B_{t2}}{\mu_2}$$
$$\boxed{\frac{B_{t1}}{B_{t2}} = \frac{\mu_1}{\mu_2}}$$

13) Discuss about the nature of the magnetic material nature of magnetic material:-

Magnetic material are classified as

(i) Dia magnetic materials
(ii) Para magnetic materials
(iii) Ferro magnetic materials
(iv) Anti Ferro magnetic materials
(v) Ferri magnetic material

Dia magnetic material:-

These are materials which do not hare dipole moment in the absence of an external applied magnetic field. In these materials, magnetisation is opposite is opposed to the applied field, $\mu_r < \mu_0$

e.g:- Silver, Lead, copper, water, Gold, Silicon.

Para magnetic materials:-

1. Permanent magnetic dipole moment
2. In these magnetisation is same dr to the applied field
3. $\mu_r \geq 1$

e.g:- air, aluminium, potassium, oxygen.

Ferro magnetic materials:-

1. In these materials, the dipoles interact strongly and all tend to line up parallel with the applied field.

2. $\mu_r >> 1$

e.g:- Iron, Nicolet, Cobalt

Anti Ferro magnetic materials:-

1. In these materials, the adjacent dipole aligns in anti parallel fashion to the applied field.
2. Magnetic moment is zero
3. Present in only temperatures

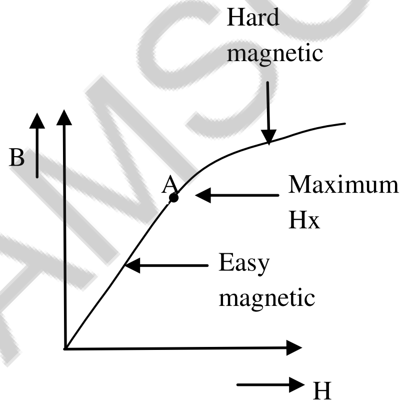
Ferri magnetic materials:-

1. Show an anti – parallel alignment adjacent atomic moments
2. Large increase in flux density

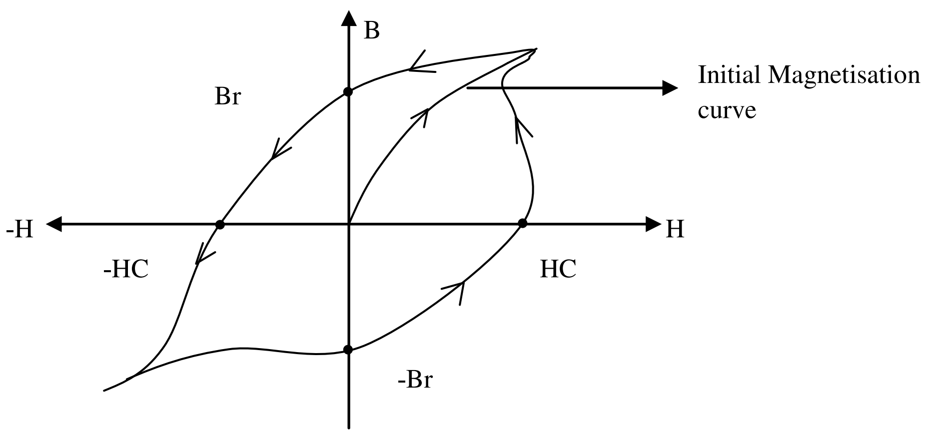
14) Explain the magnetisation curve of B – H curve

Magnetisation curve or B – H curve:-

B increase linearly with h. Till point A, it is called as easy magnetisation region, & Beyond a, it is hard magnetisation.



Hysteresis:-



1) On increasing the value of H to saturation & then decreasing, B decrease less rapidly.

2) When H = 0, B $\neq$ 0 called ass residual flux.

3) To bring B = 0, it is necessary to apply a field H in the negative direction H = -Hc is called the coerctive force.

4) Then B = -Bc at H = +Hc.

This curve is called the hysteresis curve.

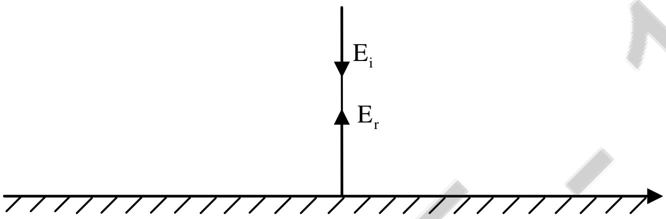
15) Derive the reflection by a perfect conductors

Reflection by a perfect conductor:-

When the electromagnetic wave travelling in one medium strikes upon a second medium,, the wave will be particularly transmitted and partially reflected. Its elepends upon types of wave incidence. The types of incidence are normal and oblique.

(i) Wave incidence normally on a perfect conductor:-

When the plane wave is incident normally upon the surface of a perfect conductor the wave is entirely reflected. Since there can be no loss within a perfect conductor, none of the energy is absorbed.



As a result, the amplitude of E and H are the same as in the reflected wave and differ by π . i.e, $E_i = -E_r$

Fig (1) → Normal incidence

Let the electric field of inci

$$E_i e^{-\gamma x}$$

Since attenuation constant $\alpha = 0$ the propagation constant $\gamma = j\beta$ $\quad \because \gamma = \alpha + j\beta.$ & $\alpha = 0 \quad \therefore \gamma = j\beta$

Incidence wave is $E_i e^{-j\beta x}$

Reflected wave is $E_r e^{+j\beta x}$

$$E_T(x) = E_i e^{-j\beta x} + E_r e^{+j\beta x}$$

But $E_i = -E_r \Rightarrow E_T(x) = E_i e^{-j\beta x} - E_i e^{+j\beta x}$

$$\begin{aligned} E_T(x) &= E_i\left[e^{-j\beta x} - e^{+j\beta x}\right] \\ &= -E_i\left[e^{+j\beta x} - e^{-j\beta x}\right] \\ E_T(x) &= -2jE_i \sin\beta x \end{aligned}$$

Expressing in time variation,

$$\begin{aligned} E_T(x,t) &= -2jE_i \sin\beta x \, e^{j\omega t} \\ &= R_{\in}\left[-2j\sin\beta x\left[\cos\omega t + j\sin\omega t\right]\right] \\ E_T(x,t) &= 2E_i \sin\beta x \sin\omega t \end{aligned}$$

The above equations show that the incident and the reflected waves consider to produce a standing wave does not progress.

To maintain the reversal of direction of energy propagation, H must be reflected without reversal of phase. So incident Hi & reflected Hr are of same phase

$$H_T(x) = H_i\, e^{-j\beta x} + H_r e^{+j\beta x}$$
$$H_T(x) = H_i\left(e^{+j\beta x} + e^{-j\beta x}\right)$$
$$= 2H_i\, \cos\beta x$$

If Hi is real,

$$H_T(x,t) = 2H_i \cos\beta x\left(e^{j\omega t}\right)$$
$$H_T(x,t) = 2H_i \cos\beta x \cos\omega t$$

The equation of E and H shows that E and H differ by $\pi/2$ in phase.

(iii) Wave incident obliquely on a perfect conductor:-

When a wave is incident obliquely on a perfect conductor, it is necessary to consider two spherical easer.

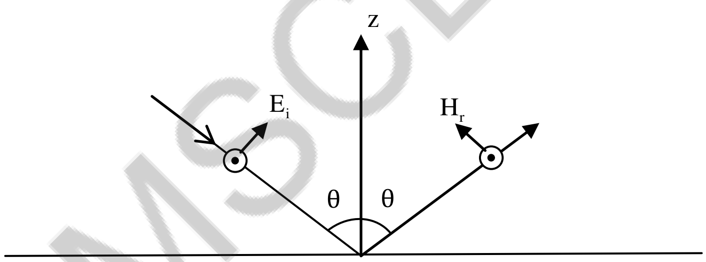
Case(i):- The electric field vector is parallel to boundary surface (or) perpendicular to the plane of incidence. This is called horizontal polarization.

Case(ii):- The electric vector is parallel to the plane of incidence. This is called vertical polarization.

Horizontal polarization:-

E is perpendicular to the plane of incidence.

Let the incident and reflected waves make angle $\theta_i = \theta_r = \theta$ with z – axis.



The incident wave is expressed as

$$E_{in} = E_i e^{-j\beta\bar{n}.r}$$

For the normal of the incident wave

$$\bar{n}.r = x\ \cos\pi/2 + y\ \cos\left(\pi/2 - \theta\right) + z\cos(\pi - \theta)$$
$$= y\sin\theta - z\ \cos\theta$$
$$E_{in} = E_i e^{-j\beta(y\sin\theta - z\operatorname{cso}\theta)}$$

The reflected wave is expressed as

$$E_{ref} = E_r e^{-j\beta(\bar{n}.r)}$$
$$\bar{n}.r = x\ \cos\pi/2 + y\cos\left(\pi/2 - \theta\right) + z\cos\theta$$
$$= y\ \sin\theta + z\cos\theta$$

But $E_{ref} = -E_i$

The total electric field

$$\begin{aligned} E_T &= E_{in} + E_{ref} \\ &= E_i\left[e^{-j\beta(y\sin\theta - z\cos\theta)} - e^{-j\beta(y\sin\theta + z\cos\theta)}\right] \\ &= E_i\left[e^{-j\beta y\sin\theta}e^{+j\beta z\cos\theta} - e^{-j\beta y\sin\theta}e^{-j\beta z\cos\theta}\right] \\ &= E_i\left[e^{j\beta z\cos\theta} - e^{-j\beta z\cos\theta}\right]e^{-j\beta y\sin\theta} \\ &= 2jE_i\sin(\beta z\cos\theta)e^{-j}\text{ By y} \end{aligned}$$

Where $\beta = \dfrac{\omega}{\nu} = \dfrac{2\pi f}{\nu} = \dfrac{2\pi}{\lambda}$

$$\beta_z = \beta\cos\theta;\ \beta_y = \beta\ \sin\theta.$$

The velocity in 'z' direction $\lambda_z = \dfrac{2\pi}{\beta_z} = \dfrac{2\pi}{\beta\cos\theta} = \dfrac{\lambda}{\cos\theta}$

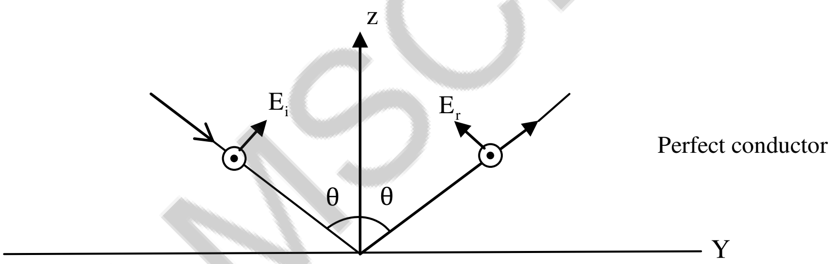
The velocity in y direction $\nu_y = \dfrac{\omega}{\beta_y} = \dfrac{\omega}{\beta\sin\theta}$

$$\nu_y = \frac{\nu}{\sin\theta} \qquad \left[\nu = \frac{\omega}{\beta}\right]$$

$$\lambda_y = \frac{\lambda}{\sin\theta}$$

Vertical polarization:-

The electric field E is parallel to the plane of incidence



The incident wave is expressed as

$$\begin{aligned} H_{in} &= H_i e^{-j\beta(y\sin\theta - z\cos\theta)} \\ H_{ref} &= H_r e^{-j\beta(y\sin\theta + z\cos\theta)} \end{aligned}$$

Since $H_{in} = H_{ref}$

$$\begin{aligned} H_T &= H_{in} + H_{ref} \\ &= H_i\left[e^{-j\beta(y\sin\theta - z\cos\theta)} + e^{-j\beta(y\sin\theta + z\cos\theta)}\right] \\ &= H_i\left[e^{-j\beta y\sin\theta}\left(e^{-j\beta z\cos\theta} + e^{-j\beta z\cos\theta)}\right)\right] \\ &= 2H_i\cos\beta z\cos\theta e^{-j\beta y\sin\theta} \\ H_T &= 2H_i\cos\beta_z z e^{-j\beta_y y} \end{aligned}$$

Where $\beta_z = \beta\cos\theta;\ \ \beta_y = \beta\sin\theta$

16) Derive the reflection by a perfect dielectric

Reflection by a perfect dielectric:-

When plane electromagnetic wave is incident on the surface of a perfect dielectric, part of the energy is transmitted and part of it is reflected. A perfect dielectric is one with zero conductivity, so that there is no less or absorption of power in propagation through the dielectric. Consider two cases.

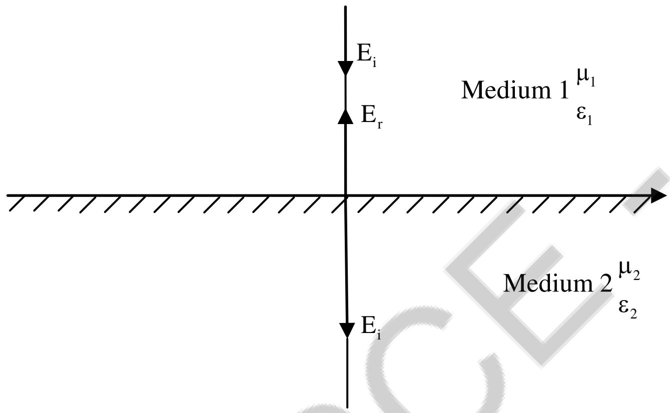
(i) Wave incident normally
(ii) Wave incident obliquely

(i) Wave incident normally perfect dielectric:-

Consider two perfect dielectric media separated by a boundary as shown in fig. Let ε_1 and μ_1 are permittivity ad permittivity of the medium 1 respectively. Let ε_2 and μ_2 are the permittivity and permittivity of medium 2 respectively.

Let E1 be the electric field of incident wave, Er be the electric field of reflected wave and Et be the electric field of transmitted wave

$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} \ \& \ \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$$



$$E_i = \eta_1 H_i$$
$$E_r = -\eta_1 H_r$$
$$E_t = \eta_2 H_t$$

$$H_i + H_r = H_t \quad \& \ E_i + E_r = E_r$$
$$H_i = \frac{E_i}{\eta_1}, H_r = \frac{-E_r}{\eta_1} \ \& \ H_t = \frac{E_t}{\eta_2}$$
$$H_t = H_i + H_r = \frac{1}{\eta_1}(E_i - E_r)$$
$$\frac{E_t}{\eta_2} = \frac{1}{\eta_1}(E_i - E_r)$$
$$\frac{E_i + E_r}{\eta_2} = \frac{1}{\eta_1}(E_i - E_r)$$
$$\eta_1 E_i + \eta_1 E_r = \eta_2 E_i - \eta_2 E_r$$
$$(\eta_1 + \eta_2) E_r = (\eta_2 - \eta_1) E_i$$

$$\boxed{\frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}} \text{ Reflection co – efficient}$$

Also, $\dfrac{E_t}{E_i} = \dfrac{E_i + E_r}{E_i} = 1 + \dfrac{E_r}{E_i}$

$$= 1 + \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\boxed{\frac{E_t}{E_i} = \frac{2\eta_2}{\eta_1 + \eta_2}} \text{ Transmission co – efficient}$$

Similarly for magnetic field,

$$\frac{H_r}{H_t} = -\frac{E_r}{E_i}$$

$$= -\left[\frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}\right]$$

$$\boxed{\frac{H_r}{H_t} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}} \text{ Reflection co – efficient}$$

Also

$$\frac{H_t}{H_i} = \frac{\eta_1}{\eta_2}\frac{E_t}{E_i}$$

$$= \frac{\eta_1}{\eta_2}\left[\frac{2\eta_2}{\eta_1 + \eta_2}\right]$$

$$\boxed{\frac{H_t}{H_i} = \frac{2\eta_1}{\eta_1 - \eta_2}} \text{ Transmission co – efficient}$$

$$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_1}} \text{ and } \eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_2}} \qquad \mu_1 = \mu_2 = \mu_0$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\frac{\mu_0}{\varepsilon_2}} - \sqrt{\frac{\mu_0}{\varepsilon_1}}}{\sqrt{\frac{\mu_0}{\varepsilon_1}} - \sqrt{\frac{\mu_0}{\varepsilon_2}}}$$

$$\frac{E_r}{E_i} = \frac{\frac{1}{\sqrt{\varepsilon_2}} - \frac{1}{\sqrt{\varepsilon_1}}}{\frac{1}{\sqrt{\varepsilon_1}} + \frac{1}{\sqrt{\varepsilon_2}}} = \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$$

$$\boxed{\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}}$$

Similarly, $\frac{E_t}{E_i} = \frac{2\eta_2}{\eta_1 + \eta_2}$

$$= \frac{2\sqrt{\frac{\mu_0}{\varepsilon_2}}}{\sqrt{\frac{\mu_0}{\varepsilon_1}} + \sqrt{\frac{\mu_0}{\varepsilon_2}}} = \frac{2\frac{1}{\sqrt{\varepsilon_2}}\sqrt{\varepsilon_1}\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_2} + \sqrt{\varepsilon_1}}$$

$$\boxed{\frac{E_t}{E_i} = \frac{2\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}}$$

$$\frac{H_r}{H_t} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}$$

$$= \frac{\sqrt{\frac{\mu_0}{\varepsilon_1}} - \sqrt{\frac{\mu_0}{\varepsilon_2}}}{\sqrt{\frac{\mu_0}{\varepsilon_1}} + \sqrt{\frac{\mu_0}{\varepsilon_2}}}$$

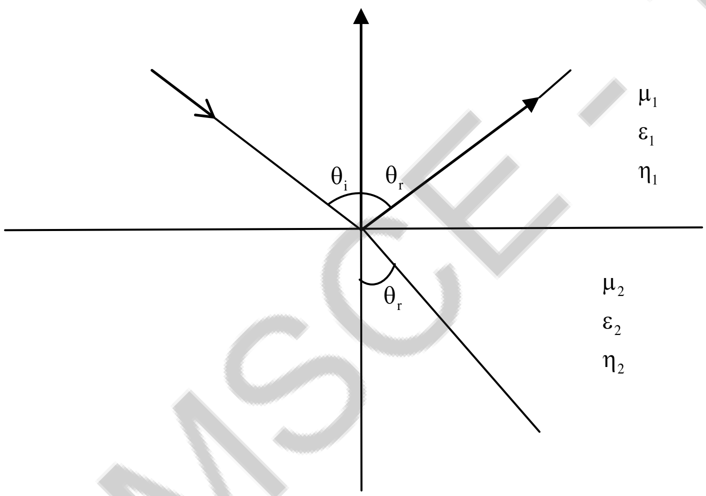
$$\frac{H_r}{H_t} = \frac{\sqrt{\varepsilon_2} - \sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$$

$$\frac{H_t}{H_t} = \frac{2\eta_1}{\eta_1 + \eta_2} = \frac{2\sqrt{\frac{\mu_0}{\varepsilon_1}}}{\sqrt{\frac{\mu_0}{\varepsilon_1}} + \sqrt{\frac{\mu_0}{\varepsilon_2}}}$$

$$\frac{H_r}{H_t} = \frac{2\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$$

(ii) Wave incident obliquely on a perfect dielectric:-

When a plane electromagnetic wave is incident obliquely on the boundary, a part of the wave is transmitted and a part of it reflected, but in this case, the transmitted wave will be refracted. i.e, direction of propagation will be changed.



When the wave is incident obliquely at an angle of θ_i with normal part of the wave reflected at an angle of θ_r in the same of θ_t in second medium.

By snell' law,

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{v_1}{v_2}$$

$$v_1 = \frac{1}{\sqrt{\mu_1\varepsilon_1}} \quad v_2 = \frac{1}{\sqrt{\mu_2\varepsilon_2}}$$

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{\sqrt{\mu_2\varepsilon_2}}{\sqrt{\mu_1\varepsilon_1}}$$

V1= Velocity of wave in medium 1

V2 = Velocity of wave in medium 2

Since $\mu_1 = \mu_2 = \mu_0$

$$\frac{\sin\theta_i}{\sin\theta_r} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$P_i = P_r + P_t$$

The power / unit area = $E \times H = EH \sin \pi/2 = E.H = \frac{E^2}{\eta}$

$$p_i = E_i H_i \cos\theta_i = \frac{E_i^{\,2}}{\eta_1} \cos\,\theta_i$$

$$p_r = E_r H_r \cos\theta_r = \frac{E_r^{\,2}}{\eta_1} \cos\,\theta_r$$

$$p_t = E_t H_t \cos\theta_t = \frac{E_t^{\,2}}{\eta_2} \cos\,\theta_t$$

$$\frac{E_i^{\,2}}{\eta_1} \cos\,\theta_i = \frac{E_r^{\,2}}{\eta_1} \cos\,\theta_r = \frac{E_t^{\,2}}{\eta_2} \cos\,\theta_t$$

By law of reflection, the angle of incidence is equal to the angle of reflection

$$\theta_i = \theta_i$$

$$\frac{E_i^{\,2}}{\eta_1} \cos\theta_i = \frac{E_r^{\,2}}{\eta_1} \cos\theta_r + \frac{E_t^{\,2}}{\eta_2} \cos\theta_t$$

$$\frac{\cos\theta_i}{\eta_1}\left[E_i^2 - E_r^2\right] = \frac{E_t^{\,2}}{\eta_2} \cos\theta_t$$

Dividing by E_i^2 on both sides

$$\frac{\cos\theta_i}{\eta_1}\left(1 - \frac{E_r^2}{E_i^2}\right) = \frac{1}{\eta_2} \frac{E_t^2}{E_i^2} \cos\theta_t$$

$$1 - \frac{E_r^2}{E_i^2} = \frac{\eta_1}{\eta_2} . \frac{E_t^2}{E_i^2} \frac{\cos\theta_t}{\cos\theta_i}$$

$$\frac{E_r^2}{E_i^2} = 1 - \frac{\eta_1 E_t^2 \cos\theta_t}{\eta_2 E_i^2 \cos\theta_i}$$

$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} \;\; \& \;\; \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} \qquad \left[\because \mu_1 = \mu_2 = \mu_0\right]$$

$$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_1}} \;\; \& \;\; \eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_2}}$$

$$\frac{E_r^2}{E_i^2} = 1 - \frac{\sqrt{\varepsilon_2} E_t^2 \cos\theta_t}{\sqrt{\varepsilon_1} E_i^2 \cos\theta_i}$$

Horizontal polarization:-

In this case, E is perpendicular to the plane of incident and parallel to the reflecting surface,

$$E_i + E_r = E_t$$

$$1+\frac{E_r}{E_i}=\frac{E_t}{E_i}$$

$$\frac{E_r^2}{E_i^2}=1-\frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\left(\frac{E_t}{E_i}\right)^2\frac{\cos\theta_t}{\cos\theta_i}$$

$$\frac{E_r^2}{E_i^2}=1-\frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\left(1+\frac{E_r}{E_i}\right)^2\frac{\cos\theta_t}{\cos\theta_i}$$

$$1-\frac{E_r^2}{E_i^2}=\frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\left(1+\frac{E_r}{E_i}\right)^2\frac{\cos\theta_t}{\cos\theta_i}$$

$$\left(1-\frac{E_r}{E_i}\right)\left(1+\frac{E_r}{E_i}\right)=\frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\left(1+\frac{E_r}{E_i}\right)^2\frac{\cos\theta_t}{\cos\theta_i}$$

$$1-\frac{E_r}{E_i}=\frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\left(1+\frac{E_r}{E_i}\right)\frac{\cos\theta_t}{\cos\theta_i}$$

$$1-\frac{E_r}{E_i}=\frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\frac{\cos\theta_t}{\cos\theta_i}+\frac{E_r}{E_i}\frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\frac{\cos\theta_t}{\cos\theta_i}$$

$$1-\frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\frac{\cos\theta_t}{\cos\theta_i}=\frac{E_r}{E_i}\left(1+\frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\frac{\cos\theta_t}{\cos\theta_i}\right)$$

$$\frac{E_r}{E_i}=\frac{1-\dfrac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\dfrac{\cos\theta_t}{\cos\theta_i}}{1+\dfrac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\dfrac{\cos\theta_t}{\cos\theta_i}}$$

$$\frac{E_r}{E_i}=\frac{\sqrt{\varepsilon_1}\cos\theta_i-\sqrt{\varepsilon_2}\cos\theta_t}{\sqrt{\varepsilon_1}\cos\theta_i+\sqrt{\varepsilon_2}\cos\theta_t}$$

$$\sqrt{\varepsilon_2}\cos\theta_t=\sqrt{\varepsilon_2}\sqrt{1-\sin^2\theta_t}$$

But $\dfrac{\sin\theta_i}{\sin\theta_t}=\dfrac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}$

$$\sin^2\theta_t=\frac{\varepsilon_1\sin^2\theta_i}{\varepsilon_2}$$

$$\sqrt{\varepsilon_2}\cos\theta_t=\sqrt{\varepsilon_2}\sqrt{1-\frac{\varepsilon_1\sin^2\theta_t}{\varepsilon_2}}$$

$$=\sqrt{\varepsilon_2-\varepsilon_1\sin^2\theta_t}$$

Substituting this value in $\dfrac{E_r}{E_i}$

$$\frac{E_r}{E_i}=\frac{\sqrt{\varepsilon_1}\cos\theta_i-\sqrt{\varepsilon_2-\varepsilon_1\sin^2\theta_i}}{\sqrt{\varepsilon_1}\cos\theta_i+\sqrt{\varepsilon_2-\varepsilon_1\sin^2\theta_i}}$$

Reflection co – efficient is given by

$$\frac{E_r}{E_i}=\frac{\cos\theta_i-\sqrt{\dfrac{\varepsilon_2}{\varepsilon_1}-\sin^2\theta_i}}{\cos\theta_i+\sqrt{\dfrac{\varepsilon_2}{\varepsilon_1}-\sin^2\theta_i}}$$

Vertical polarization:-

In this case, E is parallel to the plane of incidence.

$$\left(E_i - E_r\right)\cos\theta_i = E_t \cos\theta_t$$

$$1 - \frac{E_r}{E_i} = \frac{E_t}{E_i}\frac{\cos\theta_t}{\cos\theta_i}$$

$$\frac{E_t}{E_i} = \left(1 - \frac{E_r}{E_i}\right)\frac{\cos\theta_t}{\cos\theta_i}$$

$$\frac{E_r^2}{E_i^2} = 1 - \frac{\sqrt{\varepsilon_2}E_t^2\cos\theta_t}{\sqrt{\varepsilon_1}E_i^2\cos\theta_i}$$

$$\frac{E_r^2}{E_i^2} = 1 - \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\left(1 - \frac{E_r}{E_i}\right)^2\frac{\cos^2\theta_i}{\cos^2\theta_t}\times\frac{\cos\theta_t}{\cos\theta_i}$$

$$1 - \frac{E_r^2}{E_i^2} = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\left(1 - \frac{E_r}{E_i}\right)^2\frac{\cos\theta_i}{\cos\theta_t}$$

$$\left(1 - \frac{E_r}{E_i}\right)\left(1 + \frac{E_r}{E_i}\right) = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\left(1 - \frac{E_r}{E_i}\right)^2\frac{\cos\theta_i}{\cos\theta_t}$$

$$1 + \frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\left(1 - \frac{E_r}{E_i}\right)\frac{\cos\theta_i}{\cos\theta_t}$$

$$\frac{E_r}{E_i}\left[1 + \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\frac{\cos\theta_i}{\cos\theta_t}\right] = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\frac{\cos\theta_i}{\cos\theta_t} - 1$$

$$\frac{E_r}{E_i}\left[\frac{\sqrt{\varepsilon_1}\cos\theta_t + \sqrt{\varepsilon_2}\cos\theta_i}{\sqrt{\varepsilon_1}\cos\theta_t}\right] = \frac{\sqrt{\varepsilon_2}\cos\theta_i - \sqrt{\varepsilon_1}\cos\theta_t}{\sqrt{\varepsilon_1}\cos\theta_t}$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_2}\cos\theta_i - \sqrt{\varepsilon_1}\cos\theta_t}{\sqrt{\varepsilon_2}\cos\theta_i + \sqrt{\varepsilon_1}\cos\theta_t}$$

$$\cos\theta_i = \sqrt{1 - \sin^2\theta_t}$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_2}\cos\theta_i - \sqrt{\varepsilon_1\left(1 - \sin^2\theta_t\right)}}{\sqrt{\varepsilon_2}\cos\theta_i + \sqrt{\varepsilon_1\left(1 - \sin^2\theta_t\right)}}$$

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}$$

$$\sin\theta_t = \frac{\sqrt{\varepsilon_1}\sin\theta_i}{\sqrt{\varepsilon_2}}$$

$$\sin^2\theta_t = \frac{\varepsilon_1}{\varepsilon_2}\sin^2\theta_i$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_2}\cos\theta_i - \sqrt{\varepsilon_1 - \frac{\varepsilon_1^{\,2}}{\varepsilon_2^{\,2}}\sin^2\theta_i}}{\sqrt{\varepsilon_2}\cos\theta_i + \sqrt{\varepsilon_1 + \frac{\varepsilon_1^{\,2}}{\varepsilon_2^{\,2}}\sin^2\theta_i}}$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_2}\cos\theta_i - \frac{1}{\sqrt{\varepsilon_2}}\sqrt{\varepsilon_1\varepsilon_2 - \varepsilon_1^{\,2}\sin^2\theta_i}}{\sqrt{\varepsilon_2}\cos\theta_i + \frac{1}{\sqrt{\varepsilon_2}}\sqrt{\varepsilon_1\varepsilon_2 - \varepsilon_1^{\,2}\sin^2\theta_i}}$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_2}\cos\theta_i - \frac{\varepsilon_1}{\sqrt{\varepsilon_2}}\sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}}{\sqrt{\varepsilon_2}\cos\theta_i + \frac{\varepsilon_1}{\sqrt{\varepsilon_2}}\sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}}$$

Dividing numerator & denominator by $\frac{\sqrt{\varepsilon_2}}{\varepsilon_1}$

$$\frac{E_r}{E_i} = \frac{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_i - \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}}{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_i - \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}} \text{ Reflection co – efficient}$$

Brewster's Angle:-

Brewster's angle is a particular angle at which no reflection takes place. This occurs when the numerator of the above equation is zero

$$\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_i - \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i} = 0$$

$$\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_i = \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}$$

$$\frac{\varepsilon_2}{\varepsilon_1}\sqrt{1-\sin^2\theta_i} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}$$

$$\frac{\varepsilon_2^2}{\varepsilon_1^2}\left(1-\sin^2\theta_i\right) = \frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i$$

$$\frac{\varepsilon_2^2}{\varepsilon_1^2} - \frac{\varepsilon_2^2}{\varepsilon_1^2}\sin^2\theta_i = \frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i$$

$$\sin^2\theta_i\left(1-\frac{\varepsilon_2^2}{\varepsilon_1^2}\right) = \frac{\varepsilon_2}{\varepsilon_1}\left(1-\frac{\varepsilon_2}{\varepsilon_1}\right)$$

$$\sin^2\theta_i\left(\varepsilon_1^2 - \varepsilon_2^2\right) = \varepsilon_1\varepsilon_2 - \varepsilon_2^2$$

$$\sin^2\theta_i = \frac{\varepsilon_2(\varepsilon_1 - \varepsilon_2)}{\varepsilon_1^2 - \varepsilon_2^2}$$

$$\sin^2\theta_i = \frac{\varepsilon_2(\varepsilon_1 - \varepsilon_2)}{(\varepsilon_1 - \varepsilon_2) - (\varepsilon_1 + \varepsilon_2)}$$

$$\sin^2\theta_i = \frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}$$

$$\cos^2\theta_i = 1 - \sin^2\theta_i$$

$$= \frac{\varepsilon_1 + \varepsilon_2 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} = \frac{\varepsilon_1}{\varepsilon_1 + \varepsilon_2}$$

$$\tan^2\theta_i = \frac{\varepsilon_2}{\varepsilon_1}; \quad \tan^2\theta_i = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$\theta_i = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

This is called Brewster's angle at which there is no reflected ware when the incident wave is parallel polarized.

PROBLEMS ON UNIT – IV

Problems on Torque

1. A rectangular coil of area 10cm^2 surrounded by uniform magnetic flux density of $B = 0.6a_x + 0.4a_y + 0.5a_z$ carrying current of 50A lies on plane 2x +6y – 3z =7 such that the magnetic moment of the coil is directed away from the origin. Determine (i) magnetic moment (ii) Torque (iii) Maximum torque

Solution:-

Given Area $A = 10cm^2, B = 0.6\bar{a}_x + 0.4\bar{a}_y + 0.5a_z Wb/m^2, I = 50A$

(i) Magnetic moment is

$$m = 1A\bar{a}_n = 50\left(10\times10^{-4}\right)\left(\frac{2a_x + 16a_y - 3a_z}{\sqrt{49}}\right)$$

$$3 = \left(14.29\bar{a}_x + 42.86\bar{a}_y - 21.43\bar{a}_z\right)\times10^{-3}\,A.m^2$$

(ii) The torque on the coil is

$$\bar{T} = m\times\bar{B}$$

$$= \left[\frac{(50)\left(10\times10^{-4}\right)}{7}\left(2a_x + 6a_y - 3a_z\right)\right]\times\left[0.6a_x + 0.4a_y + 0.3a_z\right]$$

$$= \frac{(50)\left(10\times10^{-4}\right)}{7\times10}\begin{bmatrix} a_x & a_y & a_z \\ 2 & 6 & -3 \\ 6 & 4 & 5 \end{bmatrix}$$

$$= 7.143\times10^{-4}\left[42\bar{a}_x - 28\bar{a}_y - 4\bar{a}_z\right]$$

$$= 0.03\bar{a}_x - 0.02\bar{a}_y - 0.025\bar{a}_z\,N.m$$

$$T_{max} = BIA$$

$$= (0.6a_x + 0.4a_y + 0.5a_z)(50)(10\times10^{-4})$$

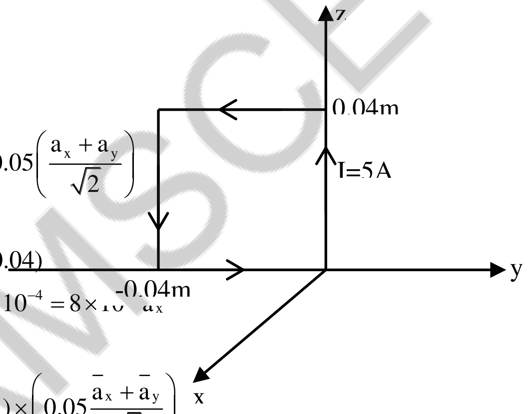
$$= 3a_x + 2a_y + 2.5a_z\times10^{-2}$$

$$= 30a_x + 20a_y + 25a_z\times10^{-3}$$

$$= 43.87\times10^{-3}\,N.m$$

2. A square coil is shown in figure in figure below is placed in the magnetic field of flux density $\bar{B} = 0.05\left(\frac{a_x + a_y}{\sqrt{2}}\right)Wb/m^2$

Solution:-



$$I = 5A\ \&\ B = 0.05\left(\frac{a_x + a_y}{\sqrt{2}}\right)$$

$$m = IA$$

$$= 5(0.04\times0.04)$$

$$= 5\times4\times4\times10^{-4} = 8\times10^{-3}a_x$$

$$\bar{T} = m\times\bar{B}$$

$$= (8\times10^{-3}a_x)\times\left(0.05\frac{\bar{a}_x + \bar{a}_y}{\sqrt{3}}\right)$$

$$\bar{T} = 0.231\bar{a}_z\,mN.m$$

3. A solenoid 25cm long and of 1cm mean diameter of the coil turns has a uniform distribution winding of 2000 turns. If the solenoid is placed in a uniform field of flux density 2Wb/m2 and a current of 5A is passed through the solenoid winding, determine (i) The maximum force on the solenoid (ii) torque on the solenoid

Solution:-

Given $\ell = 25cm = 0.25m$

$d = 1cm = 0.01m$

$I = 5A \quad N = 2000 \quad B = 2Wb/m^2$

Area of solenoid loop = $A = \pi r^2 = \frac{\pi d^2}{2} = 0.25\pi\times10^{-4}\,m^2$

(i) The force on the solenoid is

$$\vec{F} = I\bar{\ell} \times \bar{B}$$
$$F = BI\ell = 2 \times 5 \times 0.25$$
$$= 2.5 \text{newton / unit}$$

For 2000 turns, we have

$$F = 2.5 \times 200 = 5000\text{N}$$

For torque on the solenoid d is

$$T = BIA$$
$$= 2 \times 5 \times 0.2 \times 10^{-4} = 7.85 \times 10^{-4} \text{N.m}$$

For 2000 turns,

$$T = 7.85 \times 10^{-4} \times 2000$$
$$T = 1.57\text{N.m}$$

4. A circular loop conductor of radius 0.1m lies in the z =0 plane & has a resistance of 5L. Given B = 0.2 sin 10^3 t $\bar{a}_z$ T. Find the induced emf & current.

Solution:-

Given $B = 0.2\sin 10^3 t\bar{a}_z \text{Tesla}, \quad \rho = 0.1\text{m}, R = 5\Omega$

Area of circular loop $= \pi\rho^2 = \pi \times (0.1)^2 = 0.01\pi\text{m}^2$

$$\phi = BA$$
$$= 0.2\sin 10^3 t \times 0.01\pi$$
$$= 2\pi \sin 10^3 \text{ mWb}$$

$$\varepsilon = \frac{d\phi}{dt}$$
$$= 2\pi[-\cos 10^3 t \times 10^3 \times 10^{-3}]$$
$$\varepsilon = -2\pi \cos 10^3 t \text{ V.}$$
$$I = \frac{\varepsilon}{k} = \frac{-2\pi \cos 10^3 t}{5}$$
$$I = -0.4\pi \cos 10^3 t$$

Here, the negative sign shows current flow in opposite direction.

5. A 30cm×40cm rectangular loop rotates at 150 rad/s in a magnetic field of 0.06 wb/m^2, normal to the axis of rotation. If the loop has 50 turns, determine the induced voltage in the loop.

Solution:-

Given $A = 0.3 \times 0.4\text{m}, \ N = 50, \ w = 150 \text{rad / s} \ \ B = 0.06\text{Wb / m}^2$

$$\phi = \int_s \vec{B}.\overline{ds} = \int B ds \cos\theta = B\cos\theta \int ds$$
$$\phi = BA\cos\theta$$

$$\varepsilon = -N\frac{d\phi}{dt} = -N\frac{d}{dt}(BA\cos\theta)$$

$$= -N\frac{d}{dt}(BA\cos\omega t)$$

$$= -N\omega\left[BA(-\sin\omega t)\right]$$

$$= \omega NBA\sin\omega t$$

$$= 150\times150\times0.06\times0.12\sin 90^\circ$$

$$= 75\times6\times12\times10^{-4}\times10^{2}$$

$$= 75\times72\times10^{-2}$$

$$= 5400\times10^{-2}$$

$$\varepsilon = 54v$$

6. Determine the emf denoted around a circular path at b = 0 with radius $\rho = 0.5m$ in the plane z =0 if (i)

$\bar{B} = 0.1\sin\left(\frac{377t}{\rho}\right)\bar{a}_\rho T \qquad \bar{B} = 0.1\sin(377t)\bar{a}_\rho T$

Solution:-

Given $\quad \rho = 0.5m$ in $z = 0$

The E. M. F induced in a time varying fiels is

$$EMF = -\iint_s \frac{\partial\bar{B}}{\partial t}.\ \overline{ds}$$

Where $\overline{ds} = \rho d\rho d\phi \rho \bar{a}_z$

$$EMF = -\int \frac{\partial}{\partial t}\left[0.1\sin(377t)\bar{a}_z\right].\left[\rho d\rho d\phi\right]\bar{a}_z$$

$$= -\iint_s 377(0.1)(377t)\ \rho\ d\rho\ d\phi$$

$$= -37.7\cos(377t)\int_0^{0.5}\rho\ d\rho\int_0^{2\pi}d\phi$$

$$= -37.7\cos(377t)\left(\frac{\rho^2}{2}\right)_0^{0.5}(\phi)_0^{2\pi}$$

$$= -37.7\cos(377t)\left[\frac{0.25}{2}\right][2\pi]$$

At t = 0, EMF = -29.59 v

(ii) $\bar{B} = 0.1\sin\left(\frac{377t}{\rho}\right)\bar{a}_\rho$ since $\bar{a}_\rho.\bar{a}_z = 0$ and EMF = 0

7. A magnetic core of uniform cross section 4cm^2 is connected to a120 v, 60Hz generator as shown is figure. Determine the induced EMF V_2 in the secondary coli.

Solution:-

Given $\quad V_1 = 120v, N_1 = 800\ \&\ N_2 = 400$

$$V_1 = -N_1 \frac{d\phi}{dt} \qquad V_2 = -N_2 \frac{d\phi}{dt}$$

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

$$V_2 = \frac{N_2}{N_1} V_1 = \frac{400}{800} \times 120$$

$$V_2 = 60v$$

8. An area of 0.5m2 in the z = 0 plane is enclosed by a filamentary conductor. Find the induced voltage, given that $\bar{B} = 0.65 \cos 10^3 t \left(\frac{\bar{a}_y + \bar{a}_z}{\sqrt{2}} \right) T$

Solution:-

Given $\quad A = 0.5m^2 \qquad \bar{B} = 0.65 \cos 10^3 t \left(\frac{\bar{a}_y + \bar{a}_z}{\sqrt{2}} \right) T$

$$EMF = -\int \frac{\partial B}{\partial t}.ds \qquad \overline{ds} = ds\, \bar{a}_z$$

$$= \int_s 0.65 \times 10^3 \sin 10^3 t \left(\frac{\bar{a}_y + \bar{a}_z}{\sqrt{2}} \right).ds\, \bar{a}_z$$

$$= \frac{650}{\sqrt{2}} \sin 10^3 t \int_s ds = \frac{650}{\sqrt{2}} \sin 10^3 t \times 0.5$$

$$EMF = 229.81 \sin 10^3 t\, V$$

9. Calculate the maximum emf induced in a coil of 4000 turns & radius of 12cm rotating at 30rps in a magnetic field of B = 500 gauss.

Solution:-

Given $\quad \rho = 0.12m,\ B = 500 Gauss; = 500 \times \frac{1}{10000} Wb/m^2 = 0.05 Wb/m^2; \quad N = 4000$

$$\omega = 2\pi \times rps$$

$$= 2\pi \times 30 = 60\pi\, rad/s$$

$$EMF = \oint_{\ell} (v \times B).dl$$

$$= B\ell v \sin\theta$$

For maximum EMF $\quad \theta = 90°$

$$\frac{EMF}{\ell} = Bv \sin 90° = Bv = B\rho\omega$$

$$= 0.05 \times 0.12 \times 60\pi$$

$$= 1.131 V/m$$

For N turns,

$$\frac{EMF}{\ell} = \frac{EMF}{\ell} \times N = 4000 \times 1.131$$

$$\frac{EMF}{\ell} = 4524 V/m$$

10. A conductor 1cm in length is parallel to z – axis and rotates at radius of 25cm at 1200 rpm. Find the induced voltage if the radius field is given by $\bar{B} = 0.5\bar{a}_\rho T$

Solution:-

Given length ℓ=0.01m, radius ρ =0.25m

Velocity = 1200 rpm & $\bar{B} = 0.5\bar{a}_\rho T$

$$\omega = \frac{2\pi \times rpm}{60} = \frac{2\pi \times 1200}{60}$$
$$\omega = 40\pi \text{ rad / s}$$

For a rotating conductor in a stationary magnetic field, the induced emf is

$$EMF = \int (v \times B).dl$$

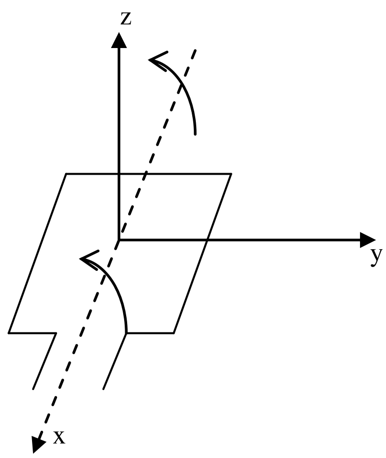
Since the conductor is parallel to z- axis,

$$\bar{dl} = dz\,\bar{a}_z$$

$$EMF = \int_{\rho=0}^{\ell} \left(\rho_w\, a_\phi \times 0.5\bar{a}_\rho\right) dz\ \bar{a}_z$$
$$= \int_{\rho=0}^{\ell} \frac{\rho\omega}{2}(-a_z).dz\,a_z$$
$$= \frac{-40\pi \times 0.25}{2}\int_0^{0.01} dz$$
$$= -5\pi\left[z\right]_0^{0.01} = -0.157$$
$$EMF = -157mV$$

11. A square coil, 0.8m on a side rotates about the x – axis at $\omega = 80\pi$ rad / s in a field $\bar{B} = 0.6\,\bar{a}_z$ as shown in fig. find the induced voltage.

Solution:-



From the diagram, only two sides cut the magnetic field.

$a = 0.8m$ $\bar{B} = 0.6a_z$ $w = 80\pi$

$$EMF = \oint_\ell \left(v \times \bar{B}\right).dl$$
$$= vB\ell\ \sin\theta \quad \theta = \omega t$$

For a square loop with sides a = 0.8m, $v = \frac{a\omega}{2} m / w$

$$EMF = \left(\frac{a\omega}{2}\right)B(2a)\sin\theta$$
$$= \omega Ba^2 \sin\theta$$
$$= \omega Ba^2 \sin\omega t$$
$$= (80\pi)(0.6)(0.8)^2 \sin(80\pi t)$$
$$EMF = 96.46\sin 80\pi t$$

12. A square loop of side 4cm with a resistor of 10Ω on the side is placed in a uniform magnetic field of 50m T in the direction of x- axis. Calculator (i) the induced current at t = 1ms (ii) induced EMF at t= 3ms. It is given that the square loop the axis of solution is perpendicular to the field. The loop lies in yz plane at t =0.

Solution:-

Given

$$a = 4cm = 0.04$$
$$B = 50 \times 10^{-3} T$$
$$\theta = 90°, R = 10\Omega$$
$$f = 10Hz \ \& \ \omega = 2\pi f = 20\pi rad/s$$

$$EMF = \oint_{\ell} (v \times B).dl \quad v = \frac{a\omega}{2}$$

$$= vB\ell \sin\theta$$
$$= \left(\frac{a\omega}{2}\right) B(2a) \sin\theta$$
$$= a^2 \omega B \sin\omega t$$

$$EMF = (0.04)^2 \times 20\pi \times 50 \times 10^{-3} \sin(20\pi t) = 5.03 \sin(20\pi t) mV$$

EMF = 5.52 μV

$$\rho = \frac{EMF}{R} = \frac{5.52 \times 10^{-6}}{10}$$
$$\rho = 0.552 \mu A$$

$$EMF = 5.03 \times 10^{-3} \sin(20\pi \times 10^{-3})$$
$$= 55 \mu V$$

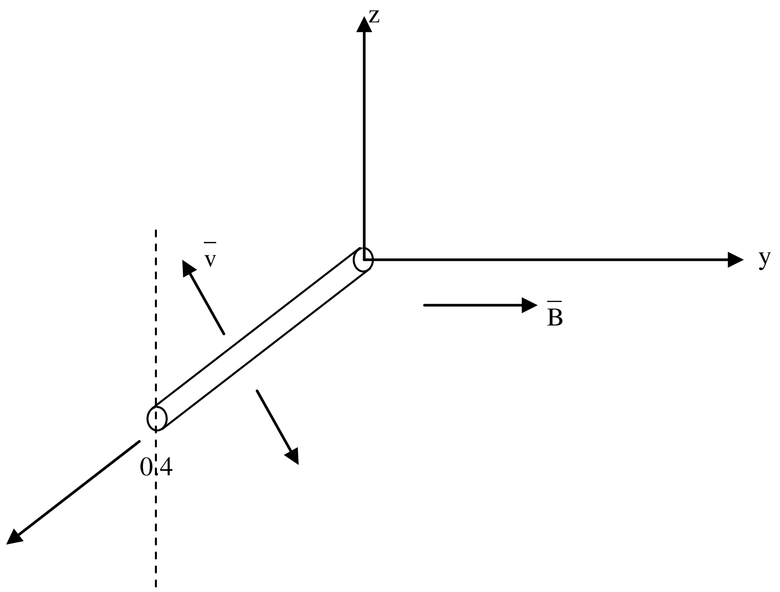
13. If the conductor mores with a velocity $v = 4.5 \sin 10^6 t \ \bar{a}_z$ **. Find the induced voltage is the conductor if (i)** $\bar{B} = 0.08 \bar{a}_y$ **(ii)** $B = 0.08 a_x T$

Solution:-

Given $v = 4.5 \sin 10^6 t \bar{a}_z m/s \qquad \bar{B} = 0.08 \bar{a}_y$

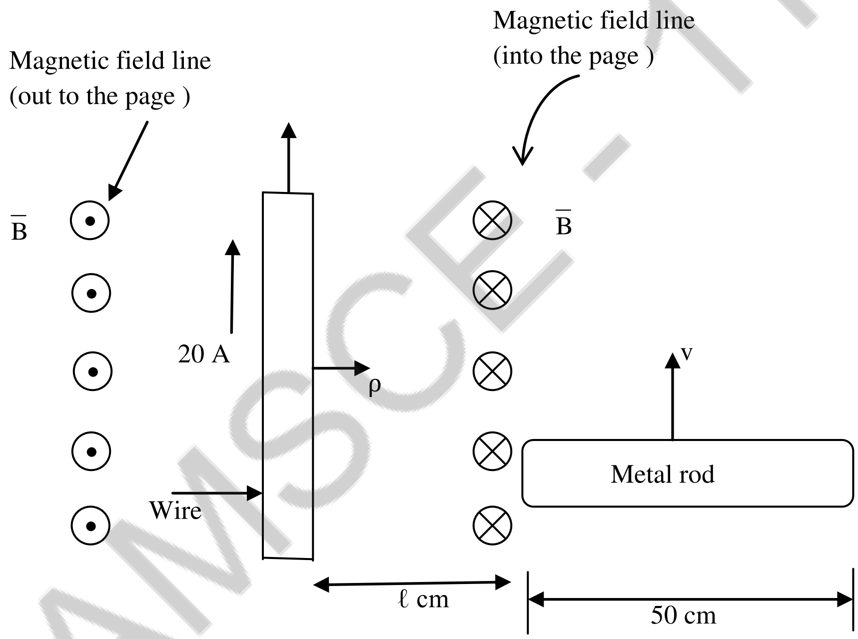
$$EMF = \int (v \times B) dl$$
$$= 4.5 \sin 10^6 t \int (\bar{a}_z \times 0.08 \bar{a}_y).dx \bar{a}_x$$
$$= 0.36 \sin 10^6 t \int_0^{0.4} (-a_x).dx \bar{a}_x$$
$$= -0.36 \sin 10^6 (0.4)$$
$$EMF = -0.144 \sin 10^6 t$$

(ii) Since the conductor is placed parallel the magnetic field, it does not cut any line. Hence the field is zero.



14. The wire shown in the fig is in free space & carnes a current of I = 20 A. A 50cm long metal rod mores at a constant velocity of $\bar{v} = 5\bar{a}_z m/s$ find V_{12}.

Solution:-



Given $v = 5\bar{a}_z m/s \quad I = 20A$

The magnetic field intensity by current carrying wire

$$\bar{H} = \frac{I}{2\pi\rho}\,\bar{a}_\phi \qquad B = \mu_0 H = \frac{\mu_0 I}{2\pi\rho}\,\bar{a}_\phi$$

$$V_{12} = EMF = \int (v \times B).dl$$

$$= \int_{70}^{20}\left(5\bar{a}_z \times \frac{\mu_0 I}{2\pi\rho} a_\phi\right)(d\rho.\bar{a}_\rho)$$
$$= \frac{-5\mu_0 I}{2\pi}\int_{70}^{20}\frac{d\rho}{\rho} = \frac{-5\mu_0 I}{2\pi}\left[\ln(\rho)\right]_{70}^{20}$$
$$= \frac{-5\mu_0 I}{2\pi}\ln\left(\frac{20}{70}\right) = \frac{-5\times 4\pi\times 10^{-7}\times 20}{2\pi}\ln\left(\frac{20}{70}\right)$$

$V_{12} = 25.06\mu v$

15. A faraday's copper (Cu) disc of 0.3m dia is rotated at 60 rotations / sec on a horizontal axis perpendicular to the plane of the disc. The axis is lying in a horizontal fields 20 µT. Determine the emf measured between the bunches.

Solution;-

Given $d = 0.3m, B = 20\times 10^{-6}T,\ \omega = 2\pi f = 2\pi(60) = 120\pi rad/s$

$$EMF = \frac{-\omega B b^2}{2}$$

$$= \frac{-\omega B b^2}{2} \qquad b = radius = \frac{0.3}{2} = 0.15$$
$$= \frac{-120\pi\times 20\times 10^{-6}\times(0.15)^2}{2}$$
$$= -1200\pi\times 10^{-6}\times 0.0225$$
$$= -2700\pi\times 10^{-8}$$
$$= -27\pi\mu V$$

EMF = - 84. 82 μV

16. A Faraday's Cu disc, 0.5m in dia, rotated at 1000rpm on a horizontal axis perpendicular to and thro the centre the disc, the axis lying in a horizontal field 10mT. Determine the emf measured b/w the ????

Solution:-

Given $d = 0.5m,\ \ v = 1000rpm \ \&\ B = 10\times 10^{-3}$

$$\omega = \frac{2\pi\times rpm}{60} = \frac{2\pi\times 1000}{60} = 104.72 rad/s$$

$$EMF = \frac{-\omega B b^2}{2}$$

$$= \frac{-104.72\times 10\times 10^{-3}\times(0.25)^2}{2}$$

EMF = -32.72 m v

17. A conductor bar CD slides freely over two conducting rails as shown in fig. calculate the induced voltage in the bar,

(i) If the bar is stationed at y = 10cm & $\bar{B} = 5\cos 10^6 t\ \ \bar{a}_z m\ Wb/m^2$

(ii) If the bar slides at a velocity $v = 30\bar{a}_y m/s\ \ \&\ \ \bar{B} = 5\bar{a}_z mWb/m^2$

(iii) If the bar slides at a velocity $v = 30\bar{a}_y m/s\ \ \&\ \ \bar{B} = 5\cos(10^6 t - y)\bar{a}_z mWb/m^2$

Solution:-

$$\text{EMF} = -\int \frac{\partial B}{\partial t}.\ \overline{ds}$$

$$= \int_{y=0}^{0.1} \int_{0}^{0.08} 5\times10^{-3}\times10^{6}\sin10^{6}t\ \ dxdy$$

$$= 5\times10^{3}\sin10^{6}t\times0.08\times0.1$$

$$\text{EMF} = 40\ \sin10^{6}\ \ tV$$

(ii) For a sliding bar in a stationary magnetic field, the motional emf is

$$\text{EMF} = \int (v\times B).\ dl$$

$$= \int_{\ell}^{0} \left(v\overline{a}_{y}\times B\overline{a}_{z}\right).\ dx\,\overline{a}_{z}$$

$$= \int_{\ell}^{0} (v\times B).dx$$

$$= vB\left[x\right]_{\ell}^{0}$$

$$= -vB\ell$$

$$= -30\times5\times10^{-3}\times0.08$$

EMF = -12mV

(iii) For a sliding bar in time varying magnetic field, both transformer emf & motional emf an present.

$$\text{EMF} = -\int \frac{\partial B}{\partial t}\ .\overline{ds} + \int (v\times B).\ \overline{dl}$$

$$= \int_{0}^{0.08}\int_{0}^{y} (5\times10^{-3})\times10^{6}\times\sin(10^{6}t-y) + \int_{0.08}^{0}\left[30a_{y}\times5\times10^{-3}\cos(10^{6}t-y)\right]$$

$$= 400\ \cos\ (10^{6}t-y) - 400\ \ \cos10^{6}t\ V$$

$$\overline{E} = 300\sin10^{9}t$$

18. Given $I_c = 4\sin\omega t\ mA,\quad \sigma = 4\times10^{7}\ \ s/m\ \ \&\ \ \varepsilon_r = 1.$ **If w = 10 rad/s. Find Id**

Solution:-

$$I_c = J_C\,A = \sigma EA$$

$$E = \frac{I_C}{\sigma A}$$

$$
\begin{aligned}
I_d &= J_d A \\
&= \frac{\partial D}{\partial t} A \\
&= \frac{\partial}{\partial t}(\varepsilon E) A \\
&= \varepsilon \left(\frac{\partial E}{\partial t} \right) A \\
&= \varepsilon A \frac{\partial E}{\partial t} \\
&= \varepsilon A \frac{\partial}{\partial t} \left(\frac{I_C}{\sigma A} \right) \\
&= \frac{\varepsilon_0 \varepsilon_r A}{\sigma A} \frac{\partial I_C}{\partial t} \\
&= \frac{8.854 \times 10^{-12} \times 1}{4 \times 10^{7}} \frac{\partial}{\partial t} (4 \sin \omega t) \times 10^{-3} \\
&= 8.854 \times 10^{-19} \times 10 \cos \omega t \times 10^{-3} \\
&= 8.854 \times 10^{-19} \times 10^{10} \cos \omega t \times 10^{-3} \\
I_d &= 8.854 \cos \omega t \, nA
\end{aligned}
$$

19. Given $J = 5 \sin(\omega t - 20z) \bar{a}_y + \cos(\omega t - 20z) \bar{a}_z mA/m^2$ **. Find volume charge density** ρ_v

Solution:-

$$\nabla . J = \frac{-\partial \rho_v}{\partial t}$$

$$\frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} = \frac{-\partial \rho_v}{\partial t}$$

$$
\begin{aligned}
0 + 0 + - \sin(\omega t - 20z) x - 20 &= \frac{-\partial \rho_v}{\partial t} \\
20 \sin(\omega t - 20z) &= \frac{-\partial \rho_v}{\partial t} \\
\rho_v &= \int -20 \ \sin \ (\omega t - 20z) dt \\
&= \frac{-20}{\omega} - \cos(\omega t - 20z) \\
&= \frac{20}{\omega} \cos(\omega t - 20z) c/m^3
\end{aligned}
$$

20. If $J = (2y\bar{a}_x + xz\bar{a}_y + z^3 \bar{a}_z) \sin 10^4 t A/m^2$ **. Determine the volume charge density** ρ_v **if** $\rho_v = (x, y, 0, t)$

Solution:-

Given $J = (2y\bar{a}_x + xz\bar{a}_y + z^3 \bar{a}_z) \sin 10^4 t A/m^2$

$$
\begin{aligned}
\nabla . J &= \frac{-\partial \rho_v}{\partial t} \\
\rho_v &= -\int (\nabla . J) . \, dt \\
&= -\int (0 + 0 + 3z^2) \sin 10^4 \ t \\
\rho_v &= \frac{3z^2}{2} \cos 10^4 t + C_0
\end{aligned}
$$

Given ρ_v =0 at z = 0

$$\rho_v = 0 = \frac{3z^2}{10^4}\cos 10^4 t + C_0 \ [z = 0$$

$$\rho_v = 0$$

$$\rho_v = 0.3z^2 \cos 10^4 \, tmC/m^3$$

21. If the $E = 10\cos(\omega t - \beta y)\bar{a}_z v/m,$ **determine . If the same field exist in a medium, whose condutinty is** $5\times10^3 \Omega^{-1}/cm$ **find Jc.**

Solution:-

$$D = \varepsilon E = \varepsilon_0 \varepsilon_r E$$

$$= 8.854\times10^{-2} \times 1\times 10\cos(\omega t - \beta y)a_z$$

$$D = 10\times 8.854\times10^{-12}\cos(\omega t - \beta y)\bar{a}_z$$

$$= 8.854\cos(\omega t - \beta y)\bar{a}_z Pc/m^2$$

$$J_d = \frac{\partial \bar{D}}{\partial t} = \frac{\partial}{\partial t}[8.854\cos(\omega t - \beta y)]\bar{a}_z$$

$$= 8.854\omega \ \sin(\omega t - \beta y)\bar{a}_z \ pA/m^2$$

$$J_c = \sigma E$$

$$= 5\times10^3 \times 10 \cos(\omega t - \beta y)\bar{a}_z A/cm$$

$$= 5\times10^5 \times 10 \cos(\omega t - \beta y)\bar{a}_z A/cm^2$$

$$= 5\times10^6 \times 10 \cos(\omega t - \beta y)\bar{a}_z A/m^2$$

22. A co – axial capacitor has the parameter a = 5mm, b = 30mm, $\ell = 20cm, \ \varepsilon_r = 8 \ \& \ \sigma = 10^{-6} \ s/m$ **. If** $J_C = \left(\frac{2}{\sigma}\right)\sin\ (10^6 t)\bar{a}_\rho A/m$ **determine (i Ic (ii) Jd (iii) Id**

Solution:-

Given $J_C = \left(\frac{2}{\sigma}\right)\sin\ (10^6 t)\bar{a}_\rho A/m \ \ a = 5\times10^{-3} \ \ b = 30\times10^{-3} \ \ \ell = 0.2m \ \ \varepsilon_r = 8 \ \ \sigma = 10^{-6} s/m$

(i) $I_C = \int \bar{J}_c \cdot \bar{d}_s$ $\qquad [ds = \rho d\phi dz \bar{a}_\rho$

$$= \int_s \frac{2}{\rho}\sin\ (10^6 t)\ \bar{a}_\rho \ \rho d\phi dz \bar{a}_\rho$$

$$= \int_0^{2\pi}\int_0^{0.2} 2\sin\ (10^6 t) d\phi dz$$

$$= 2\sin(10^6 t)\,[0.2][2\pi]$$

$$= 0.8\times 3.14\sin(10^6 \ t)$$

$$I_c = 2.512\sin(10^6 \ t)A$$

(ii) $J_d = \frac{\partial D}{\partial t} = \frac{\partial}{\partial t}(\varepsilon E)$

$$= \frac{\partial}{\partial t}\left(\varepsilon \frac{J_c}{\sigma}\right) = \frac{\varepsilon_0 \varepsilon_r}{\sigma}\frac{\partial J_c}{\partial t}$$

$$= \frac{8\varepsilon_0}{\sigma}\frac{\partial}{\partial t}\left(\frac{2}{\rho}\sin\,(10^6)\,t\right)\bar{a}_\rho$$

$$= \frac{8\varepsilon_0}{\sigma\rho} \times 10^6 \times 2\cos(10^6)\,t$$

$$= \frac{16 \times 8.854 \times 10^{-12} \times 10^6}{\rho \times 10^{-6}}\cos\ (10^6)t\bar{a}_\rho$$

$$J_d = \frac{141.664\cos(10^6)\,t}{\rho}\bar{a}_\rho\ A/m^2$$

$$J_{d(max)} = \frac{141.664}{5 \times 10^{-3}} = 28.33 \times 10^3\ A/m^2$$

(iii) $I_d = \int \overline{J_d}\ .\overline{d_s}$

$$= \oint_s \frac{141.664}{\rho}\cos\ (10^6\ \ t)\bar{a}_\rho .\ \rho\ d\phi dz\ \ \bar{a}_\rho$$

$$= 141.664 \int_0^{0.2}\int_0^{2\pi}\cos\ (10^6\ \ t)\ d\phi\ \ dz$$

$$I_d = 178.02\ \cos\ (10^6\ \ t)\ A$$

23. Given $A = 0.05m^2\ \ d = 2 \times 10^{-3} m\ \ \ \varepsilon_r = 5\ \sigma = 5 \times 10^{-4}\ \ s/m\ \ v = 5\sin 10^7\ tV.$ **Find I_{rms}.**

Solution:-

$$E = \frac{v}{d} = \frac{5\sin 10^7\, t}{2 \times 10^{-3}} = 2500\sin 10^7\, t\, v/m$$

$$J_c = \sigma E = 5 \times 10^{-4}(250\sin 10^7\, t) = 125 \times 10^{-2}\sin 10^7\ \ A/m^2$$

$$J_d = \frac{\varepsilon\partial E}{\partial t} = \varepsilon_0\varepsilon_r\frac{\partial}{\partial t}(2500\ \sin 10^7\ \ t)$$

$$= 8.854 \times 10^{-12} \times 5 \times 2500\cos 10^7\, t \times 10^7$$

$$J_d = 1.11\cos 10^7\, t\, A/m^2$$

$$I_c = J_c A = 1.25\sin 10^7\, t \times 0.05 = 0.06\sin 10^7\ t\ A$$

$$I_d = J_d\, A = 1.11\cos 10^7\, t \times 0.05 = 0.056\cos\, 10^7\, t\, A$$

$$I_T = \sqrt{I_c^{\ 2} + I_d^2} = \sqrt{(0.06)^2 + (0.056)^2} = 0.08A$$

$$I_{rms} = \frac{I_T}{\sqrt{2}} = \frac{0.08}{\sqrt{2}} = \frac{0.08\sqrt{2}}{\sqrt{2} \times \sqrt{2}} = 0.04 \times \sqrt{2} = 0.057\,A$$

24. A co – axial capacitor of length ℓ = 6cm with ε_r = 9. The radii are 1cm & 2cm. V= 100 sin (120 πt), what is Id?

Solution:-

Given $\ \ v = 100\sin(120\pi t)\ v,\ \ \ell = 6cm,\ \varepsilon = 9,\ a = 1cm\ \&\ b = 2cm$

$$I_d = \oint_s \frac{\partial D}{\partial t}.\ \bar{d}_s = \oint_s \frac{\partial}{\partial t}(\varepsilon E).\ \bar{d}_s$$
$$= \oint_s \frac{\partial}{\partial t}\left(\frac{\varepsilon V}{\rho \ln\left(b/a\right)}\bar{a}_\rho\right).(\rho\ \ d\phi\ dz\ \bar{a}_\rho)$$
$$= \frac{\varepsilon_0\varepsilon_r}{\ln\left(b/a\right)} \times 100 \times 120\pi \cos\ (120\pi)t \int_0^{0.06} dz \int_0^{2\pi} d\phi$$
$$= \frac{1}{36\pi} \times 10^{-9} \times 9 \times 100 \times 120\pi\ \cos\ (120\pi)t \times 0.06 \times 2\pi$$
$$= 10^{-9} \times 10^2 \times 60\pi \cos\ (120\pi)t \times 6 \times 10^{-2}$$
$$I_d = 360\pi \cos(120\pi)t\ \ n-A$$

Problems on boundary condition for an EM field

1) Region 1 is defined by x < 0 & μ_{r1} =4, when as region 2 is defined by x > 0 & μ_{r2} =8. If $\bar{H}_i = 8\bar{a}_x + 3\bar{a}_y - 6\bar{a}_z\ A/m$ for a source free boundary, determine $\pi 2$ & its magnitude.

Solution:-

Given $\bar{H}_i = 8\bar{a}_x + 3\bar{a}_y - 6\bar{a}_z\ A/m$ & the normal components are

$$H_{t1} = 3\bar{a}_y - 6\bar{a}_z\ \ \&\ H_{n1} = 8\bar{a}_z$$

For a source free interface, $\bar{H}_{t1} = \bar{H}_{t2}$ & $B_{n1} = B_{n2}$ therefore, the tangential & normal components in region 2 (x>0) are

$$\bar{H}_{t2} = 3\bar{a}_y - 6\bar{a}_z \qquad H_{n2} = \frac{\mu_{r1}}{\mu_{r2}}H_{n1} = \frac{4}{8}(8\ a_x)\ \ H_{n2} = 4\bar{a}_x$$
$$\bar{H}_2 = \bar{H}_{t2} + \bar{H}_{n2}$$
$$= 4\bar{a}_x + 3\bar{a}_y - 6\bar{a}_z\ A/m$$
$$|H_z| = \sqrt{4^2 + 3^2 + (-6)^2} = \sqrt{61} = 7.81\ A/m$$

2. A 3d space is divided into region 1 (x<0)& region 2 (x>0) **where** $\sigma_1 = \sigma_2 = 00$ & $\bar{E}_1 = 2\bar{a}_x + 4\bar{a}_y + 6\bar{a}_z\ V/m$**.** **Find $\bar{E}_2$ & $\bar{D}_2$ assume** $\varepsilon_{r1} = 2\ \&\ \varepsilon_{r2} = 4$

Solution:-

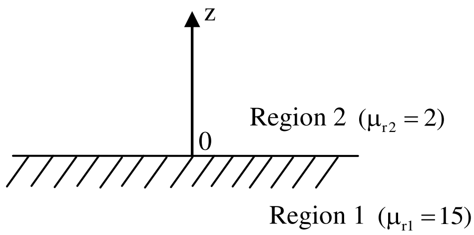
$$\bar{E}_1 = 2\bar{a}_x + 4\bar{a}_y + 6\bar{a}_z\ v/m$$
$$E_{t1} = 4\bar{a}_y + 6\bar{a}_z\ \ \&\ E_{n1} = 2\bar{a}_x$$

$$D_{t1} = \varepsilon_0\varepsilon_r\ E_{t1} = 2\varepsilon_0(44\bar{a}_y + 6\bar{a}_z)$$

$$E_{t1} = E_{t2}\ \ \&\ D_{n1} = D_{n2}$$
$$E_{t2} = 4\bar{a}_y + 6\bar{a}_z\ \ \ E_{n2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}E_{n1};\ E_{n2} = \frac{2}{4}(2a_x);\quad E_{n2} = \bar{a}_x$$
$$\bar{E}_2 = \bar{E}_{n2} + \bar{E}_{t2}$$
$$\bar{E}_2 = \bar{a}_x + 4\bar{a}_y + 6\bar{a}_z$$
$$D_2 = \varepsilon_2 E_2 = \varepsilon_0\varepsilon_r\ \bar{E}_2$$
$$= 4\varepsilon_0(\bar{a}_x + 4\bar{a}_y + 6\bar{a}_z)$$
$$D_2 = \varepsilon_0(4\bar{a}_x + 4\bar{a}_y + 6\bar{a}_z)\ \ c/m^2$$

3. Given $\bar{B}_1 = 1.2\bar{a}_x + 0.8\bar{a}_y + 0.4\bar{a}_y T$ in region 1 as shown in fig. Find $\bar{H}_2$ & the angles between the field vectors & a tangent to the interface

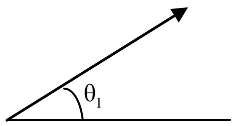
Solution:-



$$\bar{B}_1 = 1.2\bar{a}_x + 0.8\bar{a}_y + 0.4\bar{a}_z \text{ T}$$

$$H_1 = \frac{\beta_1}{\mu_1} = \frac{1.2\bar{a}_x + 0.8\bar{a}_y + 0.4\bar{a}_z}{\mu_0(15)}$$

$$H_1 = \frac{10^{-2}}{\mu_0}(8\bar{a}_x + 5.33\bar{a}_y + 2.67\bar{a}_z)\text{A/ m}$$



$$H_{t1} = \frac{10^{-2}}{\mu_0}(8\bar{a}_x + 5.33\bar{a}_y) H_{n1} = \frac{10^{-2}}{\mu_0}(2.67\bar{a}_z)$$

$$H_{t2} = \frac{10^{-2}}{\mu_0}(8\bar{a}_x + 5.33\bar{a}_y) \quad \bar{H}_{n2} = \frac{\mu_{r1}}{\mu_{r2}}\bar{H}_{n1} = \frac{15}{2} \times \frac{10^{-2}}{\mu_0}(2.67\bar{a}_z)$$

$$H_2 = H_{t2} + H_{n2}$$

$$= \frac{10^{-2}}{\mu_0}(8\bar{a}_x + 5.33\bar{a}_y + 20.03\, a_z) \text{ A / m}$$

$$\theta_1 = 90° - \alpha_1$$

$$\cos\alpha_1 = \frac{B_1 - \bar{a}_z}{|B_1|} = \frac{(1.2a_x + 0.8a_y + 0.4a_z).a_z}{\sqrt{(1.2)^2 + (0.8)^2 + (0.4)^2}}$$

$$\cos\,\alpha_1 = 0.27$$

$$\alpha_1 = \cos^{-1}(0.27) = 74.33$$

$$\theta_1 = 90 - \alpha_1 = 90 - 74.33 = 15.67$$

$$\frac{\tan\,\theta_1}{\tan\,\theta_2} = \frac{\mu_{r2}}{\mu_{r1}}$$

$$\tan\theta_2 = \frac{\mu_{r2}}{\mu_{r1}}\tan\theta_1 = \frac{15}{2}\tan\;(15.67)$$

$$\tan\theta_2 = 2.104$$

$$\theta_2 = \tan^{-1}(2.107) = 64.58$$

4. In region defined by z < 0, $\mu_{r1} = 5 \,\&\, H_1 = \frac{1}{\mu_0}(0.2a_x + 0.5a_y + a_z)\text{A / m}^2$ **find** $\bar{H}_2$ if $\theta = 30°$

Solution:-

Given $\mu_{r1} = 5 \,\&\, H_1 = \frac{1}{\mu_0}(0.2a_x + 0.5a_y + a_z)\text{A / m}^2$

$$\overline{H}_1 = \frac{1}{4\pi\times10^{-7}}(0.2a_x + 0.5a_y + a_z)$$

$$= 10^4(15.92\overline{a}_x + 39.8\overline{a}_y + 79.6\overline{a}_z)A/m$$

$$H_{t1} = 10^4(15.92\overline{a}_x + 39.8\overline{a}_y)\ H_{n1} = 10^4(79.6\overline{a}_z)$$

$$\cos\alpha_1 = \frac{\overline{H}_1 - a_z}{|H_1|} = \frac{10^4(15.92\overline{a}_x + 39.8\overline{a}_y + 79.6\overline{a}_z)}{10^4\sqrt{(15.92)^2 + (39.8)^2 + (79.6)^2}}$$

$$\cos\alpha_1 = 0.88$$

$$\alpha_1 = \cos^{-1}(0.88) = 28.36°$$

$$\theta_1 = 90 - \alpha_1 = 90 - 28.36 = 61.64$$

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_{r2}}{\mu_{r1}} \Rightarrow \mu_{r2} = \mu_{r1}\frac{\tan\theta_1}{\tan\theta_2} = 5\times\frac{\tan 61.64°}{\tan 30°}$$

$$\mu_{r2} = 16$$

$$B_{n1} = B_{n2}\ \text{i.e,}\ \mu_{r1}H_{n1} = \mu_{r2}H_{n2}$$

$$\overline{H}_{n2} = \frac{\mu_{r1}}{\mu_{r2}}H_{n1} = \frac{5}{16}\times10^4(79.6)\overline{a}_z = 10^4(24.87\overline{a}_z)$$

$$H_{t2} = H_{t1} = 10^4(15.92\overline{a}_x + 39.8\overline{a}_y)$$

$$H_2 = 10^4(15.92\overline{a}_x + 39.8\overline{a}_y + 24.87\overline{a}_z)A/m$$

5. Region 1, z < 0 has $\mu_{r1} = 3.5$ & region 2, z > 0 has $\mu_{r2} = 10$. Near the origin $\overline{B}_1 = 2.4\overline{a}_x + 10\overline{a}_y T$ & $B_2 = 25\overline{a}_x + 17\overline{a}_y + 10\overline{a}_z T$. If the interface carries a sheet current, determine its density at the origin.

Solution:-

Given $\mu_{r1} = 3.5\ \overline{B}_1 = 2.4\overline{a}_x + 10\overline{a}_y T$ region 1

$\mu_{r2} = 10 B_2 = 25\overline{a}_x + 17\overline{a}_y + 10\overline{a}_z T$ region 2

$$H_1 = \frac{1}{\mu_0\mu_{r1}}\overline{B}_1 = \frac{1}{3.5\mu_0}(2.4\overline{a}_x + 10\overline{a}_y)$$

$$H_1 = \frac{1}{\mu_0}(0.69\overline{a}_x + 2.86\overline{a}_y)A/m$$

$$H_2 = \frac{1}{\mu_0\mu_{r1}}\overline{B}_2 = \frac{1}{10\mu_0}(25\overline{a}_x - 17\overline{a}_y + 10\overline{a}_z)$$

$$H_2 = \frac{1}{\mu_0}(2.5\overline{a}_x - 1.7\overline{a}_y + \overline{a}_z)A/m$$

$$\overline{k} = (H_1 - H_2)\times\overline{a}\ln 12$$

$$= \frac{1}{\mu_0}(0.69\overline{a}_x + 2.86\overline{a}_y - 2.5\overline{a}_x + 1.7\overline{a}_y - \overline{a}_z)$$

$$= \frac{1}{\mu_0}(-1.51\overline{a}_x + 4.56\overline{a}_y - \overline{a}_z)\times a_z$$

$$= \frac{1}{\mu_0}(-1.51(-a_y) + 4.56(a_x))$$

$$\overline{k} = \frac{(1.51a_y + 4.56a_x)\times10^{-7}}{4\pi}A/m$$

Problems in potential functions

1. If the related scalar electric potential v-(x-Vot) & the vector magnetic potential $\overline{A} = \left(\frac{x}{v_0} - t\right)\overline{a}_x$ where v_0 is the velocity of propagation. Then determine (i)$\nabla.A$, (ii) $\overline{B}, \overline{H}, \overline{E} \& \overline{D}$ (iii) Also S.T $\nabla.\overline{A} = -\mu_0\varepsilon_0\frac{\partial v}{\partial t}$ **in free space**

Solution:-

Given $V = x - \nu_0 t$ & $A = \left(\frac{x}{\nu_0} - t\right)\bar{a}_x$

(i) $\nabla.A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

$$\nabla.A = \frac{\partial A_x}{\partial x} = \frac{\partial}{\partial x}\left(\frac{x}{\nu_0} - t\right) = \frac{1}{\nu_0}$$

(ii) $B = \nabla \times \bar{A} = \begin{pmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{x}{\nu_0} - t & 0 & 0 \end{pmatrix}$

$$= a_x(0) - a_y\left[0 - \frac{\partial}{\partial z}\left(\frac{x}{\nu_0} - t\right)\right] + a_z\left[0 - \frac{\partial}{\partial x}\left(\frac{x}{\nu_0} - t\right)\right]$$

$$B = 0; \quad H = 0$$

$$E = -\nabla V - \frac{\partial A}{\partial t}$$

$$= -\frac{\partial V}{\partial x}a_x - \frac{\partial}{\partial t}\left(\frac{x}{\nu_0} - t\right)a_x$$

$$= -\frac{\partial}{\partial x}(x - \nu_0 t)a_x + \bar{a}_x$$

$$E = -a_x + a_x = 0, \ \therefore D = 0$$

(iii) $V = x - \nu_0 t$

$$\frac{\partial v}{\partial t} = \frac{\partial}{\partial t}(x - \nu_0 t) = -\nu_0$$

$$\mu_0\varepsilon_0 \frac{\partial v}{\partial t} = -\nu_0 \times \mu_0\varepsilon_0$$

$$= -\nu_0 \times \frac{1}{\nu_0^{\ 2}}$$

$$\mu_0\varepsilon_0 \frac{\partial v}{\partial t} = \frac{-1}{\nu_0} = \nabla.A$$

$$\nabla.A = -\mu_0\varepsilon_0 \frac{\partial v}{\partial t}$$

UNIT – IV

MAGNETIC FORCES AND MATERIALS

State and explain the Lorentz force equation for a moving charge;-

Consider that a charged particle is moving in a magnitude field of flux density B. It experience a force given by

$$F = Q(\overline{V} \times \overline{B} \;\rightarrow (1)$$

The force is proportional to the product of the magnitude of the charge Q, its velocity V & flux density B & to the sine of the angle between V and b. The direction of the force is perpendicular to both V and B.

$$F = Qv\,B\sin\theta \;\rightarrow (2)$$

The electric force on a charged particular in the electric of intensity E is

$$F = Q\overline{E} \;\rightarrow (3)$$

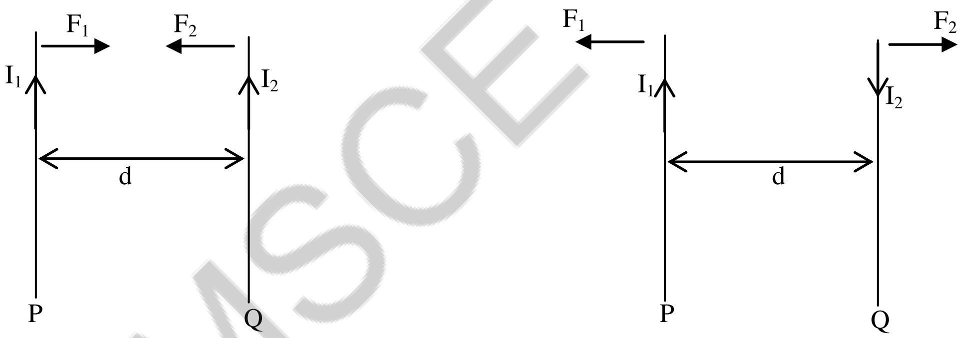
The force on a moving particle due to combined electric field and magnitude field is obtained

$$F = Q\left[\overline{E} + (\vec{v} \times \vec{B})\right]$$

This force is called Lorentz force.

Obtain an expression for the force between two parallel conductors

Consider two straight, long parallel conductor P and Q separated by a distance d. Let I1 and I2 be the currents flowing in conductors P and Q.



Consider a conductor P produces a magnetic field whose flux density is B at conductor

$$B = \frac{\mu I_1}{2\pi d}$$

The force on conductor Q due to P, F1 = B.

Where L = length of the conductor

$$F_1 = \frac{\mu I_1 I_2 \ell}{2\pi d}$$

If the current is flowing in the same direction, then there is force of attraction

$$F_2 = F_1$$

If the currents are flowing in opposite direction, then there is force of repulsion

$$F_2 = -F_1$$
$$F_2 = \frac{-\mu I_1 I_2 \ell}{2\pi d} N$$

If the conductor is infinitely long, the force per unit length is,

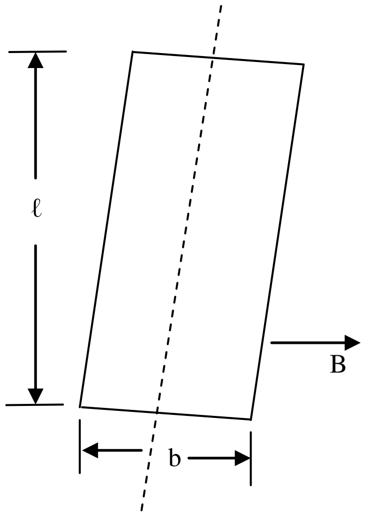
$$F = \frac{\mu I_1 I_2 \ell}{2\pi d} \, N/m$$

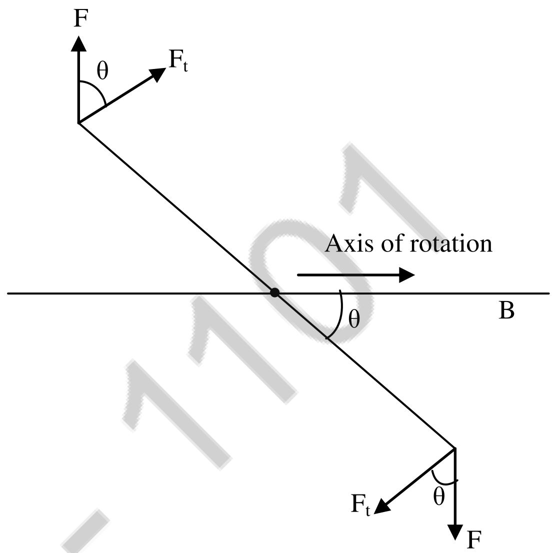
Explain the magnitude moment Torque on a loop carrying a current I:-

When a current loop is placed parallel to H, force at on the loop that end to rotate it. The tangential force multiplied by the radial distance at which it at is called torque.

Unit of the Torque → Newton – metre (N – m)

Consider the rectangular loop of length ‘L’ and breadth ‘b’ carrying a current I in a uniform magnetic of flux density B.





The force acting on the loop, F = BI L sin θ

If the loop plane is parallel to the magnetic field total torque on the loop is T.

T = 2 ×Torque on each side

= 2 ×tangential force × radial distance

= 2 × BIL × b/2

T = BIA

If the loop plane makes an angle Q with respect to the magnitude flux density B, the tangential component of the force is F_t=Fcos θ.

Total torque on the loop, T = BIA

Magnetic moment, m= IA A/m^2

T = BIA sin θ

= m B sin θ

$$\overline{T} = m \times \overline{B}$$

$$m = \frac{T}{B}$$ [If torque is maximum, i.e Q = 90°]

Magnetic moment:-

Magnetic moment is defined as the maximum torque on loop per uniform magnitude flux density

$$m = \frac{T}{B}$$

In case of magnitude dipole, magnetic moment is given by m= Qm . L

Qm = charge ; L = length

In case of current loop m = IA

I = current; A = area of the loop.

Detail about the scalar and vector magnetic potential:-

The potential can be of two types

a) Magnetic vector potential (or) vector magnitude potential ($\overline{A}$)
b) Magnetic scalar potential (or) scalar magnitude potential (Vm)

(i) magnetic vector potential:-

Scalar magnetic potential exists if there is no current enclosed (i.e) $\oint H.dl = 0$. If current is enclosed the potential depends upon current element (vector quantity) is no more scalar but it is a vector quantity.

Since the divergence of vector is a scalar the vector potential is expressed in curl.

Let 'A' be any magnetite vector potential and vector potential is expressed in curl.

$$\nabla \times A = B$$

Talking curl on the both the sides

$$\nabla \times \nabla \times A = \nabla \times B \rightarrow (1)$$

By Identity,

$$\nabla \times \nabla \times A = \nabla(\nabla.A) - \nabla^2 A \rightarrow (2)$$

$$\begin{aligned}
&\nabla \times B = \mu J \\
&\nabla \times B = \nabla(\nabla.A) - \nabla^2 A \\
&\mu J = \nabla(\nabla.A) - \nabla^2 A \\
&\text{for steady d.c}, \nabla.A = 0 \\
&-\mu J = \nabla^2 A \\
&\nabla^2 A_x \overline{a}_x + \nabla^2 A_y \overline{a}_y + \nabla^2 A_z \overline{a}_z = -\mu(\overline{a}_x J_x + \overline{a}_y J_y + \overline{a}_z J_z) \\
&\text{Equating,} \\
&\qquad \nabla^2 A_x = -\mu J_x \\
&\qquad \nabla^2 A_y = -\mu J_y \\
&\qquad \nabla^2 A_z = -\mu J_z
\end{aligned}$$

This is in the form of poisson's equation.

In general, magnitude vector potential is expressed as,

$$A = \frac{\mu}{4\pi} \iiint_v \frac{J}{r}\, dv$$

$$A_x = \frac{\mu}{4\pi}\int_v \left(\frac{J_x}{r}\right) dv;\ A_y = \frac{\mu}{4\pi}\int_v \left(\frac{J_y}{r}\right) dv;\ A_z = \frac{\mu}{4\pi}\int_v \left(\frac{J_z}{r}\right) dv$$

r = distance between current element and the point at which $\overline{A}$ is the to be calculated.

Scalar magnetic potential:-

Ampere's law states that the line integral of the field H around a closed path is equal to the current enclosed

$$\oint_{\lambda} H.\, dl = I$$

If no current is enclosed (i.e) J = 0

$$\oint_{\lambda} H.\, dl = 0$$

Magnetic field intensity can be expressed as the negative gradient of a scalar function

$$H = -\nabla V_m$$

Where V_m = scalar magnitude potential.

$$V_m = -\int H\, .dl$$

This scalar potential also satisfied Laplace equation

In free space $\nabla.B = 0 \Rightarrow \mu_0\ \nabla.H = 0$

$$\text{But } H = -\nabla Vm \Rightarrow \mu_0\ \nabla.(-\nabla V_m) = 0$$
$$\mu_0\ \nabla^2 V_m = 0$$
$$\nabla^2 V_m = 0$$

Force on a differential current element (or) Force on a wire carrying current I placed in a magnetic field.

The force exerted on a differential element of charge dQ moving in a steady magnetic field is given by

$$\overline{dF} = dQ(\vec{V} \times \vec{B}) \quad \rightarrow (1)$$

The current density $\bar{J}$ can be expressed as

$$\bar{J} = \rho_v\ \vec{V} \qquad \rightarrow (2)$$

The differential element of charge can be expressed as

$$dQ = \rho_v\ dv \qquad \rightarrow (3)$$

Sub (3) in (1),

$$\overline{dF} = \rho_v\ dv\ (\vec{V} \times \vec{B})$$
$$\overline{dF} = (\rho_v \vec{V} \times \vec{B}) dv \quad \rightarrow (4)$$
$$\overline{dF} == (\vec{J} \times \vec{B})\ dv$$

Relationship between current elements

$$\vec{J}\ dv = \bar{k}\ ds = I\ \overline{dl}$$
$$\overline{dF} = \vec{J}\ dv \times \vec{B} \Rightarrow \overline{dF} = I \overline{d\vec{l}} \times \vec{B}$$

Integrating the above $\bar{F} = I\vec{\ell} \times \vec{B}$

$$F = B\ I\ L \sin\theta$$

Define inductance field in magnitude materials

Inductance:-

Any conductor carrying a current produces a field around it. The lines of magnetic flux produced by a current in a solenoid coil form closed loops. If the current in the coil is alternating with respect to times, the flux linked with the coil also varies.

The values of the flux depend on flux density which in turn depends on the current flowing through it.

If there are N – turns in the coil, the total flux linked with the coil is called the flux linkage (Λ)

$$\Lambda = N\phi$$

Due to variation of current, there will be variation of flux linked with the coil which in turn induces an emf.

$$\varepsilon = \frac{-d\Lambda}{di}.\frac{di}{dt}$$
$$\varepsilon = -\left(N\frac{d\phi}{di}\right)\frac{di}{dt} \quad [\because \Lambda = N\phi]$$

Where i = instantaneous value of the current $\frac{d\Lambda}{di}$ or $\frac{N\,d\phi}{di}$ represents the of charge of flux linkage with respect to current. This quantity depends upon the geometrical configuration of the given device and is referred to an inductor and the device which possesses this properly is called Inductor.

Derive the expression of inductance:-

The inductance is defined as the ratio of total magnitude flux linkage to the current through the coil.

Thus, $L = \frac{d\Lambda}{di} = \frac{N\,d\phi}{di}$

If the flux ϕ varies linearly with i,

$$\frac{d\phi}{di} = \frac{\phi}{i}$$
$$L = \frac{\Lambda}{\ell} = \frac{N\phi}{i}$$
$$\varepsilon = -L\frac{di}{dt}$$

The negative sign indicates that the emf is set up in such direction so as opposite the changes in current

$$\varepsilon = -V$$
$$V = +L\frac{di}{dt}$$

The unit of an inductance is Henry. The inductance of coil is said to be 1Henry, if when current in the coil changes at a rate of 1 ampere / second, an emf of 1 volt is induced in it; or if the flux linkage with the coil changes at the rate of 1 wb turn ampere.

Describe the equation of solenoids

Solenoids:-

Let b is the flux density and A, the area of cross section of the solenoid, then the flux through the solenoid is $\phi = BA$ and the flux linkage $\Lambda = N\phi = NBA$

Inductance is, therefore, given by

$$L = \frac{\Lambda}{I} = \frac{NBA}{I} \qquad \rightarrow (1)$$

We know that, for long solenoid, $B = \mu_0 \frac{NI}{\ell} \rightarrow (2)$

Sub (2) in (1), we get

$$L = N\left(\mu_0 \frac{NI}{\ell}\right)\frac{A}{I}$$

$$\boxed{L = \frac{\mu_0 N^2 A}{\ell}} \text{ Henry}$$

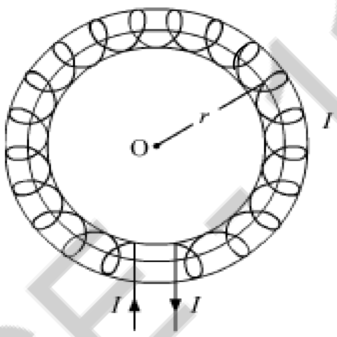
Derive the expression of inductance in a Toroid

Inductance in a toroid:-

When a long solenoid is bent into a circular closed on itself, a toroidal coil is obtained. If the toroid has uniform windings, the flux is confined almost interior & the flux at the external of the coil is zero.

Let I be the current flowing in the toroid, B the magnetic field produced at every point of circular path of radius R. $B = \frac{\mu I}{2\pi R}$

Since it has 'N' turns



$$B = \frac{\mu NI}{2\pi R}$$

By Amperes circuital law,

$$\oint_L B\,.dl = \mu NI$$

$$B\int dl = \mu NI$$

$$B.2\pi R = \mu NI$$

$$B = \frac{\mu NI}{2\pi R} a$$

Flux linkage, $\Lambda = N\phi = NBA = \frac{N\mu NI}{2\pi R}.A = \frac{\mu N^2 IA}{2\pi R}$

Since toroid is a circular section, $A = \pi r^2$

$$\Lambda = \frac{\mu N^2 I \pi r^2}{2\pi R}$$

$$l = \frac{\Lambda}{I} = \frac{\mu N^2 I \pi r^2}{2\pi R I}$$

$$\boxed{L = \frac{\mu N^2 r^2}{2R} \text{ Henry}}$$

Derive the expression of Toroid of rectangular

Toroid of rectangular cross section:-

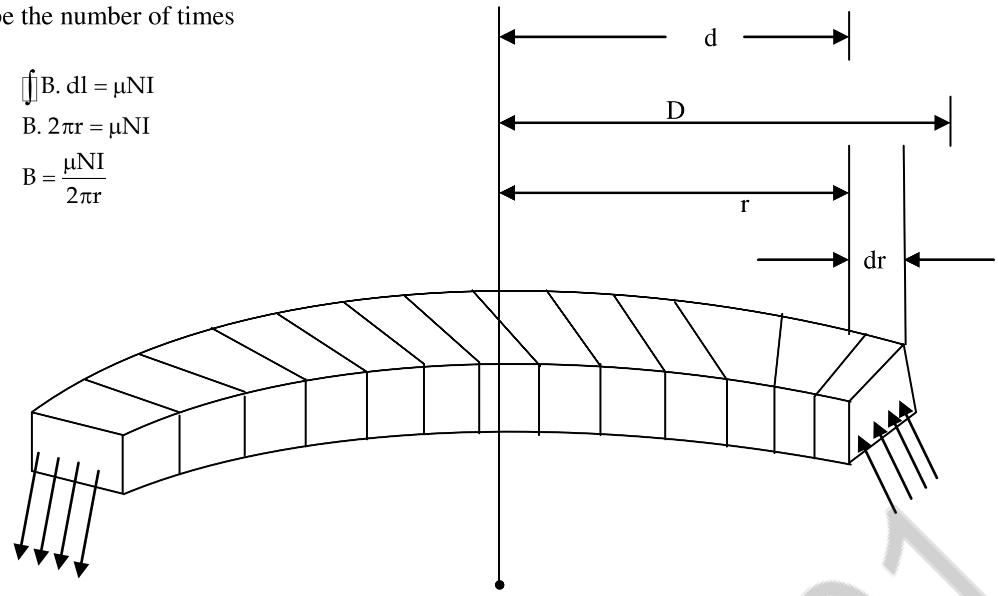
Consider a toroid of rectangular cross section of inner radii 'd' & outer radii 'D'.

Let N be the number of times

$$\oint B.\,dl = \mu NI$$

$$B.\,2\pi r = \mu NI$$

$$B = \frac{\mu NI}{2\pi r}$$



Now consider a rectangular strip of width dr and height h at a distance r from the centre.

$$d\Lambda = Nd\phi = N.BdA$$

$$d\Lambda = N\frac{\mu NI}{2\pi r}\,h.dr$$

$$\Lambda = \frac{\mu N^2 Ih}{2\pi r}\int_d^D \frac{dr}{r} = \frac{\mu N^2 Ih}{2\pi}\log D/d$$

$$\boxed{L = \frac{\Lambda}{I} = \frac{\mu N^2 h}{2\pi}\log D/d}\ \text{Henry}$$

Derive the expression of inductance of co – axial cable

Inductance of a co- axial cable:-

The magnetic flux density B at any radius 'r'

$$B = \frac{\mu I}{2\pi r}$$

$$d\Lambda = Bxdr$$

$$d\Lambda = \frac{\mu I}{2\pi r}.\,dr$$

$$\int d\Lambda = \Lambda = \int_d^D \frac{\mu I}{2\pi r}.dr$$

$$= \frac{\mu I}{2\pi}\left[\log_e r\right]_d^D$$

$$\Lambda = \frac{\mu I}{2\pi}\log D/d$$ D = outer diametre; d = inner diametre.

$$\boxed{L = \frac{\Lambda}{I} = \frac{\mu}{2\pi}\log_e D/d\ H/m}$$

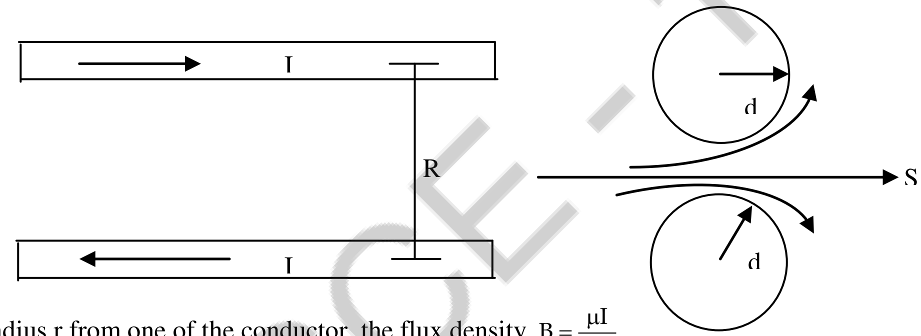
$$
\begin{aligned}
L &= \frac{\mu_0}{2\pi}\log_e {}^{D}\!/\!_{d} \\
&= \frac{4\pi\times10^{-7}}{2\pi}\times 2.303\,\log_{10} {}^{D}\!/\!_{d} \\
&= 2\times 2.303\times10^{-7}\log_{10} {}^{D}\!/\!_{d} \\
&= 4.606\times10^{-7}\log_{10} {}^{D}\!/\!_{d} \\
&= \frac{4606}{1000}\times10^{-7}\log_{10} {}^{D}\!/\!_{d} \\
&= 4606\times10^{-7}\log_{10} {}^{D}\!/\!_{d}
\end{aligned}
$$

$$\boxed{L = 0.4606\,\log_{10} {}^{D}\!/\!_{d}\ \mathrm{mH/km}}$$

Derive the expression of inductance of a two – wire tx line

Inductance of a two wire transmission line:-

A two wire transmission line is as shown in figure whose conductor radius is d & the spacing between centre iss R.



At any radius r from one of the conductor, the flux density $B = \frac{\mu I}{2\pi r}$

$$
\begin{aligned}
\Lambda &= \int d\Lambda = 2\int_d^R B.dr = 2\int_d^R \frac{\mu I}{2\pi r}.dr = \frac{\mu I}{2\pi}\log_e\left({}^{R}\!/\!_{d}\right) \\
L &= \frac{\Lambda}{I} = \frac{\mu}{2\pi}\times 2.303\times\log_{10}\left({}^{R}\!/\!_{d}\right) \\
&= \frac{4.606}{2}\times10^{-7}\log_{10} {}^{R}\!/\!_{d} = 0.9219\,\log_{10} {}^{R}\!/\!_{d}\ \mu H
\end{aligned}
$$

Derive Energy stored in a magnetic field:- An inductor stores energy when a current through a inductance coil is gradually changed from 0 to I. the energy change of it is opposite by the self induced emf produced due to this change.

$$
\begin{aligned}
W &= \int_0^I P.dt\ \text{Joules} \\
P &= VI \\
V &= L\frac{dI}{dt} \\
W &= \int_0^I LI\frac{dI}{dt}.dt
\end{aligned}
$$

$$\boxed{W = \frac{1}{2}LI^2}\ \text{Joules} = \frac{1}{2}\Lambda I = \frac{1}{2}$$

Energy density:-

Let us consider the inductance of a solenoid

$$L = \frac{\mu N^2 A}{\ell}$$

$$W = \frac{1}{2} L I^2$$

$$W = \frac{1}{2} \frac{\mu N^2 A}{\ell} I^2$$

$$= \frac{1}{2} \mu \left(\frac{NI}{\ell}\right)^2 \vartheta$$

$$W = \frac{1}{2} \mu H^2 \vartheta$$

$$\omega = \frac{W}{\vartheta} = \frac{1}{2} \mu H^2$$

$$= \frac{1}{2} \beta H$$

$$\omega = \frac{1}{2} \frac{B^2}{\mu} \text{ Joules}$$

Derive the mutual inductance of a coil

Mutual inductance:-

In the case of an isolated circuit, the flux produces by the current links only with that circuit. The corresponding inductance representing the flux linkage per unit current is sometimes referred to as self inductance.

Mutual inductance is the flux linked in one coil due to the current in the second coil.

Let us consider the flux linking one of the n circuits 1,2,......n say R^{th} circuit.

$$\Lambda_k = \Lambda_{k1} + \Lambda_k + ...\Lambda_{kk} + ...\Lambda_{kn}$$

$$= \sum_{j=1}^{n} \Lambda_{kj}$$

= total flux linkage with the R^{th} circuit due to all the n circuits composing the system.

The e. m. f induced in the R^{th} circuit may be written as

$$\varepsilon_k = \frac{-d\Lambda_k}{dt} = \frac{-d}{dt} \sum_{j=1}^{n} \Lambda Rj$$

$$= -\sum_{j=1}^{n} \frac{d\Lambda_{kj}}{dt}$$

$$\frac{d\Lambda_{kj}}{dt} = \frac{d\Lambda_{kj}}{dI_j} = \frac{dI_j}{dt}$$

Where $\frac{d\Lambda_{kj}}{dI_j}$ denotes the ratio of flux linkage with R^{th} circuit with respect to the current in j^{th} circuit, has the dimension of inductance is therefore, referred to as mutual – inductance

$$M_{kj} = \left(\frac{1}{2} \frac{d\Lambda_{kj}}{dI_j}\right) k \neq j$$

E. M. F induced in the R^{th} circuit

$$\varepsilon_k = -M_{kj} \frac{dI_j}{dt}$$

If k =j

$$\frac{d\Lambda_{kk}}{dt} = \frac{d\Lambda_{kk}}{dI_k}\frac{dI_k}{dt}$$
$$\frac{d\Lambda_{kk}}{dI_k} = L_{kk}. \quad \text{(self inductance of the circuit)}$$

Consider a toroid with two windings P and S the winding P has N_1 turns & is called primary winding and s with N_2 turns is referred to a secondary winding.

Flux linkage with winding P is

$$\Lambda_H = N_1\,(BA) = \mu_0 N_2 N_1 \frac{I_1 A}{\ell m}$$

Self inductance of coil 'P' is

$$L_H = \frac{\Lambda_H}{I_1} = \frac{\mu_0 N_1^2 A}{\ell m} \qquad [\,\ell m = \text{mean length of magnetic path} = 2\pi R_m\,]$$

Mutual inductance of switch P

$$M_{21} = \frac{\Lambda_{21}}{I_1} = \frac{\mu_0 N_2 N_1 A}{\ell m}$$
$$\varepsilon_2 = -M_{21}\frac{dI_1}{dt}$$

Self inductance of coil 'S' is

$$L_{22} = \frac{\mu_0 N_2^2 A}{\ell m}$$

Mutual inductance of P with S

$$M_{12} = \frac{\mu_0 N_1 N_2 A}{\ell m}$$
$$M_{12} = M_{21}$$
$$M_{12} = M_{21} = L_{11}L_{22}$$
$$L_{11} = L_1,\ L_{22} = L_2;\ M_{12} = M_{21} = M$$
$$M^2 = L_1 L_2$$
$$M = \sqrt{L_1 L_2}$$

Suppose R_1 times the flux produced by I_1 links with secondary, then R_2 times the flux produced by I_2 with the secondary

$$\frac{M_{21}}{L_{11}} = k_1\frac{N_2}{N_1};\ \frac{M_{12}}{M_{22}} = k_2\frac{N_1}{N_2}$$
$$\frac{M_{21}\,M_{21}}{L_{11}L_{22}} = k_1 k_2;\ \frac{M^2}{L_1 L_2} = k^2$$

Derive a Boundary condition of magnetic field

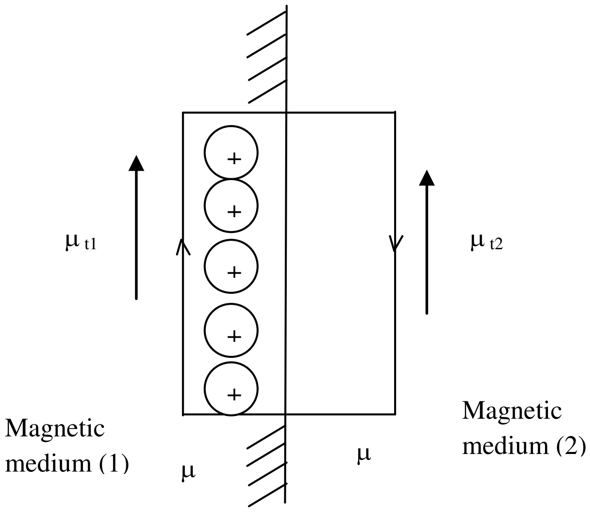
Magnetic Boundary condition:-

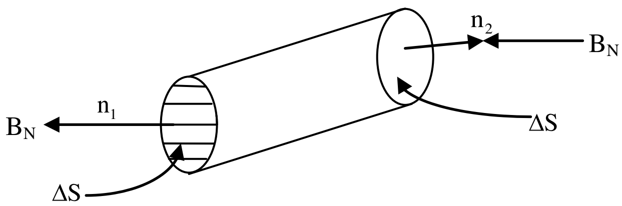
Figure shows an interface between two magnetic media with permeability's μ_1 and μ_2. Consider a Gaussian surface and a closed path to the boundary between the media (1) and (2).

$$\oint_s B.\,n\,ds = 0$$

If B_1 and B_2 are the magnetic flux densities in media (1) and (2),

$$B_1.n_1\Delta s + B_2.n_2\Delta s = 0$$





Where Δs is the pill box surface and $\left(n_1 \text{ and } n_2\right)$ are unit outward normal.

$$(B_1 - B_2).n_1 = 0$$
$$B_{n1} - B_{n2} = 0$$
$$B_{n1} = B_{n2}$$
$$\mu_1 H_{n1} = \mu_2 H_{n2}$$
$$\boxed{\frac{H_{n1}}{H_{n2}} = \frac{\mu_2}{\mu_1}}$$

Applying Ampere's circuital law

$$\oint_{\ell} H.dl = I$$
$$H_{t1}\Delta\ell - H_{t2}\Delta\ell = k.\Delta\ell$$
$$H_{t1} - H_{t2} = K$$

If sheet current density is zero,

$$H_{t1} = H_{t2}$$
$$\frac{B_{t1}}{\mu_1} = \frac{B_{t2}}{\mu_2}$$
$$\boxed{\frac{B_{t1}}{B_{t2}} = \frac{\mu_1}{\mu_2}}$$

Discuss about the nature of the magnetic material nature of magnetic material:-

Magnetic material are classified as

(i) Dia magnetic materials
(ii) Para magnetic materials
(iii) Ferro magnetic materials
(iv) Anti Ferro magnetic materials
(v) Ferri magnetic material

Dia magnetic material:-

These are materials which do not hare dipole moment in the absence of an external applied magnetic field. In these materials, magnetisation is opposite is opposed to the applied field, $\mu_r < \mu_0$

e.g:- Silver, Lead, copper, water, Gold, Silicon.

Para magnetic materials:-

1. Permanent magnetic dipole moment
2. In these magnetisation is same dr to the applied field
3. $\mu_r \geq 1$

e.g:- air, aluminium, potassium, oxygen.

Ferro magnetic materials:-

1. In these materials, the dipoles interact strongly and all tend to line up parallel with the applied field.

2. $\mu_r >> 1$

e.g:- Iron, Nicolet, Cobalt

Anti Ferro magnetic materials:-

1. In these materials, the adjacent dipole aligns in anti parallel fashion to the applied field.
2. Magnetic moment is zero
3. Present in only temperatures

Ferri magnetic materials:-

1. Show an anti – parallel alignment adjacent atomic moments
2. Large increase in flux density

Define magnetic dipole moment, magnetic susceptibility

Magnetic dipole moment

$$m = Q_m \ell$$

Consider a far magnet of length ℓ and area of cross section A.

Magnetisation is define as the net dipole moment / per unit volume

$$M = \frac{m}{V} = \frac{Q_m \ell}{A\ell}\bar{a}_\ell$$

$$\boxed{M = \frac{Q}{A}\bar{a}_\ell}$$

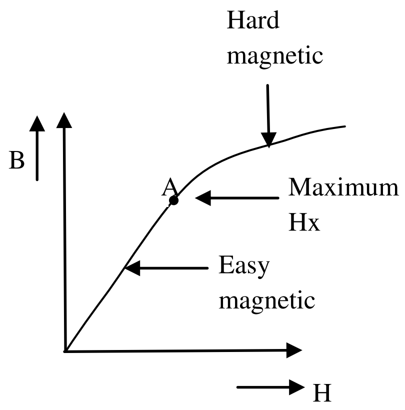
Magnetic susceptibility (Xm)

$$\begin{aligned}
B &= \mu_0 (H + M) \\
&= \mu_0 H\left(1 + \frac{M}{H}\right) \\
B &= \mu_0 H(1 + X_m) \\
X_m &= \frac{M}{H} = \text{Magnetic susceptibility} \\
B &= \mu_0 \mu_r H \\
&\text{where } \mu_r = 1 + X_m
\end{aligned}$$

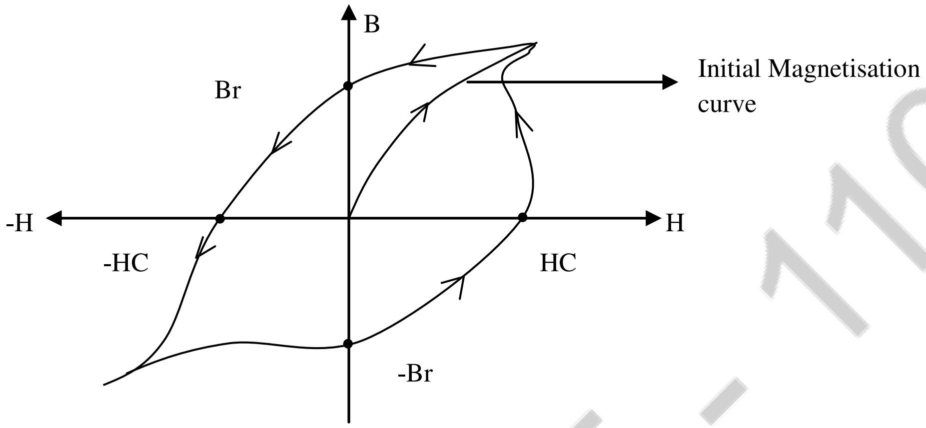
Explain the magnetisation curve of B – H curve

Magnetisation curve or B – H curve:-

B increase linearly with h. Till point A, it is called as easy magnetisation region, & Beyond a, it is hard magnetisation.



Hysteresis:-



1) On increasing the value of H to saturation & then decreasing, B decrease less rapidly.

2) When H = 0, B ≠ 0 called ass residual flux.

3) To bring B = 0, it is necessary to apply a field H in the negative direction H = -Hc is called the coerctive force.

4) Then B = -Bc at H = +Hc.

This curve is called the hysteresis curve.

Explain in detail about polarization and its types?

Polarization:-

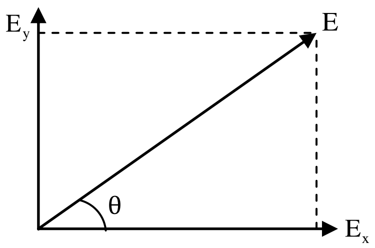
Polarization of a uniform plane wave refers to the time varying behaviour of the electric field strength vector at some fixed point in space.

Consider a uniform plane travelling in z – direction with e and H vectors lying in the x- y plane.

If Ey = 0 and Ex is present, the wave is said to be polarized in the x – direction. If Ex = 0.Ey = is present, the wave is said to be polarization in the y – direction.

(i) Linear polarization:-

If both Ex and Ey are present and are in phase, the resultant electric field has a direction at an angle of $\tan^{-1}\left(E_y / E_x\right)$. If the direction of the resultant vector is constant with time, the wave is said to be linearly polarized.



$$\theta = \tan^{-1}\left(\frac{E_y}{E_x}\right)$$

$$E = \sqrt{E_x^2 + E_y^2}$$

(ii) Circular polarization:-

If Ex and Ey have equal magnitudes and a II phase difference, the locus of the resultant 'E' is a circule and the wave is said to be circularly polarized.

If Ex and Ey have same magnitude Ea and differ in phase by 90°.

The resultant electric field in vector form is

$$\overline{E} = \bar{a}_x E_x + j\bar{a}_y E_y$$

The corresponding time varying field is

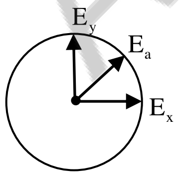
$$E_x = \bar{a}_x E_a \cos\omega t - \bar{a}_y E_a \sin\omega t$$

The component are

$$E_x = E_a \cos\omega t \qquad E_y = -E_a \sin\omega t$$

$$E_x^2 + E_y^2 = E_a^2$$

The equation shows that the locus of the resultant e is circle whose radius is Ea.



(ii) Elliptical polarization:-

If Ex and Ey hare different amplitude and a II phase difference, the locus of electric field is an ellipse and the ware is said to be elliptically polarized.

Let Ex has magnitude A and Ey has magnitude B and differ 90° in phase.

The resultant electric field in vector form

$$E = \bar{a}_x A + j\bar{a}_y B$$

The corresponding time carrying field is

$$E = \bar{a}_x A\cos\omega t + \bar{a}_y B\sin\omega t$$

The components are

$$E_x = A\cos\omega t \;\&\; E_y = -B\sin\omega t$$

$$\frac{E_x}{A} = \cos\omega t \;\&\; \frac{E_y}{B} = -\sin\omega t$$

$$\frac{E^2{}_x}{A^2} + \frac{E^2{}_y}{A^2} = 1$$

The equation shows that the locus of the resultant E is an ellipse

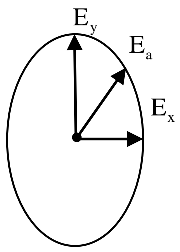


Fig:- Elliptical polarization

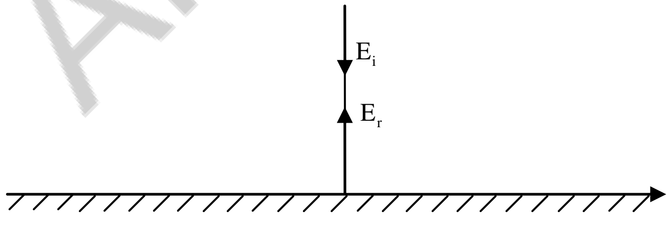
Derive the reflection by a perfect conductors

Reflection by a perfect conductor:-

When the electromagnetic wave travelling in one medium strikes upon a second medium,, the wave will be particularly transmitted and partially reflected. Its elepends upon types of wave incidence. The types of incidence are normal and oblique.

(i) Wave incidence normally on a perfect conductor:-

When the plane wave is incident normally upon the surface of a perfect conductor the wave is entirely reflected. Since there can be no loss within a perfect conductor, none of the energy is absorbed.



As a result, the amplitude of E and H are the same as in the reflected wave and differ by π . i.e, $E_i = -E_r$

Fig (1) → Normal incidence

Let the electric field of inci

$$E_i e^{-\gamma x}$$

Since attenuation constant $\alpha = 0$ the propagation constant $\gamma = j\beta$ $\quad \because \gamma = \alpha + j\beta.\ \&\ \alpha = 0 \quad \therefore \gamma = j\beta$

Incidence wave is $E_i e^{-j\beta x}$

Reflected wave is $E_r e^{+j\beta x}$

$$E_T(x) = E_i e^{-j\beta x} + E_r e^{+j\beta x}$$

But $E_i = -E_r \Rightarrow E_T(x) = E_i e^{-j\beta x} - E_i e^{+j\beta x}$

$$\begin{aligned} E_T(x) &= E_i\left[e^{-j\beta x} - e^{+j\beta x}\right] \\ &= -E_i\left[e^{+j\beta x} - e^{-j\beta x}\right] \\ E_T(x) &= -2jE_i \sin\beta x \end{aligned}$$

Expressing in time variation,

$$\begin{aligned} E_T(x,t) &= -2jE_i \sin\beta x\, e^{j\omega t} \\ &= R_{\in}\left[-2j\sin\beta x\left[\cos\omega t + j\sin\omega t\right]\right] \\ E_T(x,t) &= 2E_i \sin\beta x \sin\omega t \end{aligned}$$

The above equations show that the incident and the reflected waves consider to produce a standing wave does not progress.

To maintain the reversal of direction of energy propagation, H must be reflected without reversal of phase. So incident Hi & reflected Hr are of same phase

$$\begin{aligned} H_T(x) &= H_i e^{-j\beta x} + H_r e^{+j\beta x} \\ H_T(x) &= H_i\left(e^{+j\beta x} + e^{-j\beta x}\right) \\ &= 2H_i \cos\beta x \end{aligned}$$

If Hi is real,

$$\begin{aligned} H_T(x,t) &= 2H_i \cos\beta x\left(e^{j\omega t}\right) \\ H_T(x,t) &= 2H_i \cos\beta x \cos\omega t \end{aligned}$$

The equation of E and H shows that E and H differ by $\pi/2$ in phase.

(iii) Wave incident obliquely on a perfect conductor:-

When a wave is incident obliquely on a perfect conductor, it is necessary to consider two spherical easer.

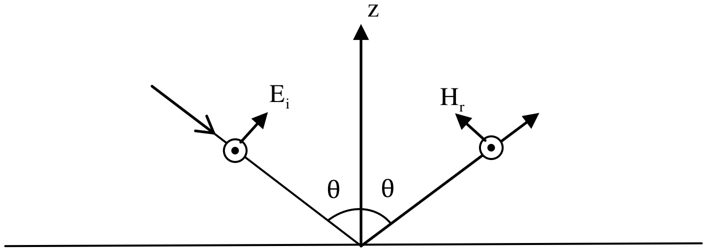
Case(i):- The electric field vector is parallel to boundary surface (or) perpendicular to the plane of incidence. This is called horizontal polarization.

Case(ii):- The electric vector is parallel to the plane of incidence. This is called vertical polarization.

Horizontal polarization:-

E is perpendicular to the plane of incidence.

Let the incident and reflected waves make angle $\theta_i = \theta_r = \theta$ with z – axis.



The incident wave is expressed as

$$E_{in} = E_i e^{-j\beta \bar{n}.r}$$

For the normal of the incident wave

$$\bar{n}.r = x \cos\pi/2 + y\cos\left(\pi/2 - \theta\right) + z\cos(\pi - \theta)$$
$$= y\sin\theta - z\cos\theta$$
$$E_{in} = E_i e^{-j\beta(y\sin\theta - z\csc\theta)}$$

The reflected wave is expressed as

$$E_{ref} = E_r e^{-j\beta(\bar{n}.r)}$$
$$\bar{n}.r = x\cos\pi/2 + y\cos\left(\pi/2 - \theta\right) + z\cos\theta$$
$$= y\sin\theta + z\cos\theta$$

But $E_{ref} = -E_i$

The total electric field

$$E_T = E_{in} + E_{ref}$$
$$= E_i\left[e^{-j\beta(y\sin\theta - z\cos\theta)} - e^{-j\beta(y\sin\theta + z\cos\theta)}\right]$$
$$= E_i\left[e^{-j\beta y\sin\theta}e^{+j\beta z\cos\theta} - e^{-j\beta y\sin\theta}e^{-j\beta z\cos\theta}\right]$$
$$= E_i\left[e^{j\beta z\cos\theta} - e^{-j\beta z\cos\theta}\right]e^{-j\beta y\sin\theta}$$
$$= 2jE_i\sin\left(\beta z\cos\theta\right)e^{-j}\text{ By y}$$

Where $\beta = \dfrac{\omega}{\nu} = \dfrac{2\pi f}{\nu} = \dfrac{2\pi}{\lambda}$

$$\beta_z = \beta\cos\theta;\ \beta_y = \beta\ \sin\theta.$$

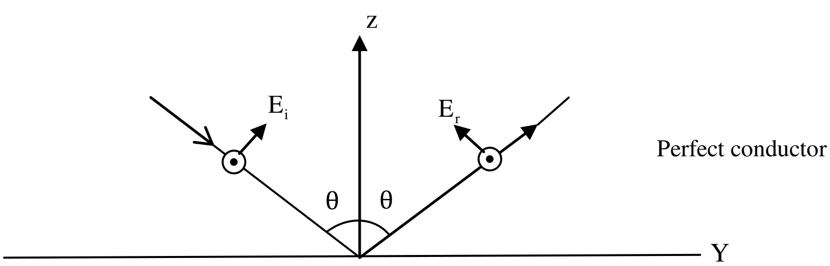
The velocity in 'z' direction $\lambda_z = \dfrac{2\pi}{\beta_z} = \dfrac{2\pi}{\beta\cos\theta} = \dfrac{\lambda}{\cos\theta}$

The velocity in y direction $\nu_y = \dfrac{\omega}{\beta_y} = \dfrac{\omega}{\beta\sin\theta}$

$$\nu_y = \frac{\nu}{\sin\theta} \qquad \left[\nu = \frac{\omega}{\beta}\right]$$
$$\lambda_y = \frac{\lambda}{\sin\theta}$$

Vertical polarization:-

The electric field E is parallel to the plane of incidence



The incident wave is expressed as

$$H_{in} = H_i e^{-j\beta(y\sin\theta - z\cos\theta)}$$
$$H_{ref} = H_r e^{-j\beta(y\sin\theta + z\cos\theta)}$$

Since $H_{in} = H_{ref}$

$$\begin{aligned} H_T &= H_{in} + H_{ref} \\ &= H_i\left[e^{-j\beta(y\sin\theta - z\cos\theta)} + e^{-j\beta(y\sin\theta + z\cos\theta)}\right] \\ &= H_i\left[e^{-j\beta y\sin\theta}\left(e^{-j\beta z\cos\theta} + e^{-j\beta z\cos\theta)}\right)\right] \\ &= 2H_i\cos\beta z\cos\theta e^{-j\beta y\sin\theta} \\ H_T &= 2H_i\cos\beta_z z e^{-j\beta_y y} \end{aligned}$$

Where $\beta_z = \beta\cos\theta;\ \ \beta_y = \beta\sin\theta$

Derive the reflection by a perfect dielectric

Reflection by a perfect dielectric:-

When plane electromagnetic wave is incident on the surface of a perfect dielectric, part of the energy is transmitted and part of it is reflected. A perfect dielectric is one with zero conductivity, so that there is no less or absorption of power in propagation through the dielectric. Consider two cases.

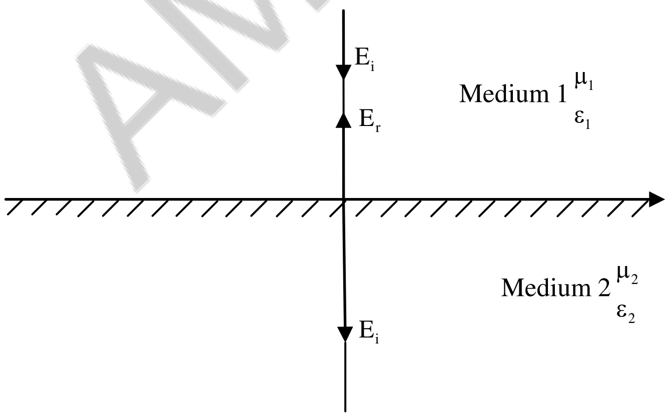
(i) Wave incident normally
(ii) Wave incident obliquely

(i) Wave incident normally perfect dielectric:-

Consider two perfect dielectric media separated by a boundary as shown in fig. Let ε_1 and μ_1 are permittivity ad permittivity of the medium 1 respectively. Let ε_2 and μ_2 are the permittivity and permittivity of medium 2 respectively.

Let E1 be the electric field of incident wave, Er be the electric field of reflected wave and Et be the electric field of transmitted wave

$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}\ \ \&\ \ \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$$



$$E_i = \eta_1 H_i$$
$$E_r = -\eta_1 H_r$$
$$E_t = \eta_2 H_t$$

$$H_i + H_r = H_t \quad \& \; E_i + E_r = E_r$$

$$H_i = \frac{E_i}{\eta_1}, H_r = \frac{-E_r}{\eta_1} \quad \& \; H_t = \frac{E_t}{\eta_2}$$

$$H_t = H_i + H_r = \frac{1}{\eta_1}\left(E_i - E_r\right)$$

$$\frac{E_t}{\eta_2} = \frac{1}{\eta_1}\left(E_i - E_r\right)$$

$$\frac{E_i + E_r}{\eta_2} = \frac{1}{\eta_1}\left(E_i - E_r\right)$$

$$\eta_1 E_i + \eta_1 E_r = \eta_2 E_i - \eta_2 E_r$$

$$\left(\eta_1 + \eta_2\right)E_r = \left(\eta_2 - \eta_1\right)E_i$$

$$\boxed{\frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}} \text{ Reflection co – efficient}$$

Also, $\frac{E_t}{E_i} = \frac{E_i + E_r}{E_i} = 1 + \frac{E_r}{E_i}$

$$= 1 + \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\boxed{\frac{E_t}{E_i} = \frac{2\eta_2}{\eta_1 + \eta_2}} \text{ Transmission co – efficient}$$

Similarly for magnetic field,

$$\frac{H_r}{H_t} = -\frac{E_r}{E_i}$$

$$= -\left[\frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}\right]$$

$$\boxed{\frac{H_r}{H_t} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}} \text{ Reflection co – efficient}$$

Also

$$\frac{H_t}{H_i} = \frac{\eta_1}{\eta_2}\frac{E_t}{E_i}$$

$$= \frac{\eta_1}{\eta_2}\left[\frac{2\eta_2}{\eta_1 + \eta_2}\right]$$

$$\boxed{\frac{H_t}{H_i} = \frac{2\eta_1}{\eta_1 - \eta_2}} \text{ Transmission co – efficient}$$

$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_1}}$ and $\eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_2}}$ $\quad \mu_1 = \mu_2 = \mu_0$

$$\frac{E_r}{E_i} = \frac{\sqrt{\frac{\mu_0}{\varepsilon_2}} - \sqrt{\frac{\mu_0}{\varepsilon_1}}}{\sqrt{\frac{\mu_0}{\varepsilon_1}} - \sqrt{\frac{\mu_0}{\varepsilon_2}}}$$

$$\frac{E_r}{E_i} = \frac{\frac{1}{\sqrt{\varepsilon_2}} - \frac{1}{\sqrt{\varepsilon_1}}}{\frac{1}{\sqrt{\varepsilon_1}} + \frac{1}{\sqrt{\varepsilon_2}}} = \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$$

$$\boxed{\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}}$$

Similarly, $\frac{E_t}{E_i} = \frac{2\eta_2}{\eta_1 + \eta_2}$

$$= \frac{2\sqrt{\frac{\mu_0}{\varepsilon_2}}}{\sqrt{\frac{\mu_0}{\varepsilon_1}} + \sqrt{\frac{\mu_0}{\varepsilon_2}}} = \frac{2\frac{1}{\sqrt{\varepsilon_2}}\sqrt{\varepsilon_1}\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_2} + \sqrt{\varepsilon_1}}$$

$$\boxed{\frac{E_t}{E_i} = \frac{2\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}}$$

$$\frac{H_r}{H_t} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}$$

$$= \frac{\sqrt{\frac{\mu_0}{\varepsilon_1}} - \sqrt{\frac{\mu_0}{\varepsilon_2}}}{\sqrt{\frac{\mu_0}{\varepsilon_1}} + \sqrt{\frac{\mu_0}{\varepsilon_2}}}$$

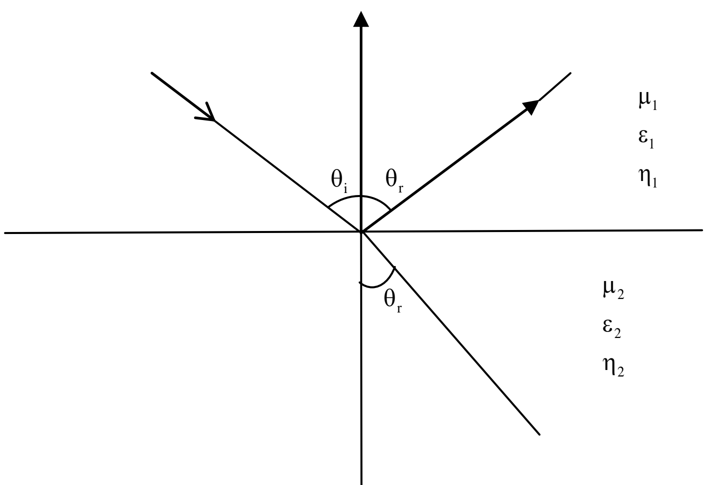
$$\frac{H_r}{H_t} = \frac{\sqrt{\varepsilon_2} - \sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$$

$$\frac{H_t}{H_t} = \frac{2\eta_1}{\eta_1 + \eta_2} = \frac{2\sqrt{\frac{\mu_0}{\varepsilon_1}}}{\sqrt{\frac{\mu_0}{\varepsilon_1}} + \sqrt{\frac{\mu_0}{\varepsilon_2}}}$$

$$\frac{H_r}{H_t} = \frac{2\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$$

(ii) Wave incident obliquely on a perfect dielectric:-

When a plane electromagnetic wave is incident obliquely on the boundary, a part of the wave is transmitted and a part of it reflected, but in this case, the transmitted wave will be refracted. i.e, direction of propagation will be changed.



When the wave is incident obliquely at an angle of θ_i with normal part of the wave reflected at an angle of θ_r in the same of θ_t in second medium.

By snell' law,

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{v_1}{v_2}$$

$$v_1 = \frac{1}{\sqrt{\mu_1\varepsilon_1}} \quad v_2 = \frac{1}{\sqrt{\mu_2\varepsilon_2}}$$

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{\sqrt{\mu_2\varepsilon_2}}{\sqrt{\mu_1\varepsilon_1}}$$

V1= Velocity of wave in medium 1

V2 = Velocity of wave in medium 2

Since $\mu_1 = \mu_2 = \mu_0$

$$\frac{\sin\theta_i}{\sin\theta_r} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$P_i = P_r + P_t$$

The power / unit area = $E \times H = EH\sin \pi/2 = E.H = \frac{E^2}{\eta}$

$$p_i = E_i H_i \cos\theta_i = \frac{E_i^{\,2}}{\eta_1}\cos\theta_i$$

$$p_r = E_r H_r \cos\theta_r = \frac{E_r^{\,2}}{\eta_1}\cos\theta_r$$

$$p_t = E_t H_t \cos\theta_t = \frac{E_t^{\,2}}{\eta_2}\cos\theta_t$$

$$\frac{E_i^{\,2}}{\eta_1}\cos\theta_i = \frac{E_r^{\,2}}{\eta_1}\cos\theta_r = \frac{E_t^{\,2}}{\eta_2}\cos\theta_t$$

By law of reflection, the angle of incidence is equal to the angle of reflection

$$\theta_i = \theta_i$$

$$\frac{E_i^{\,2}}{\eta_1}\cos\theta_i = \frac{E_r^{\,2}}{\eta_1}\cos\theta_r + \frac{E_t^{\,2}}{\eta_2}\cos\theta_t$$

$$\frac{\cos\theta_i}{\eta_1}\left[E_i^2 - E_r^2\right] = \frac{E_t^{\,2}}{\eta_2}\cos\theta_t$$

Dividing by E_i^2 on both sides

$$\frac{\cos\theta_i}{\eta_1}\left(1 - \frac{E_r^2}{E_i^2}\right) = \frac{1}{\eta_2}\frac{E_t^2}{E_i^2}\cos\theta_t$$

$$1 - \frac{E_r^2}{E_i^2} = \frac{\eta_1}{\eta_2}.\frac{E_t^2}{E_i^2}\frac{\cos\theta_t}{\cos\theta_i}$$

$$\frac{E_r^2}{E_i^2} = 1 - \frac{\eta_1 E_t^2\cos\theta_t}{\eta_2 E_i^2\cos\theta_i}$$

$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} \;\; \& \;\; \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} \qquad [\because \mu_1 = \mu_2 = \mu_0]$$

$$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_1}} \;\; \& \;\; \eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_2}}$$

$$\frac{E_r^2}{E_i^2} = 1 - \frac{\sqrt{\varepsilon_2} E_t^2 \cos\theta_t}{\sqrt{\varepsilon_1} E_i^2 \cos\theta_i}$$

Horizontal polarization:-

In this case, E is perpendicular to the plane of incident and parallel to the reflecting surface,

$$E_i + E_r = E_t$$

$$1 + \frac{E_r}{E_i} = \frac{E_t}{E_i}$$

$$\frac{E_r^2}{E_i^2} = 1 - \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}} \left(\frac{E_t}{E_i} \right)^2 \frac{\cos\theta_t}{\cos\theta_i}$$

$$\frac{E_r^2}{E_i^2} = 1 - \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}} \left(1 + \frac{E_r}{E_i} \right)^2 \frac{\cos\theta_t}{\cos\theta_i}$$

$$1 - \frac{E_r^2}{E_i^2} = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}} \left(1 + \frac{E_r}{E_i} \right)^2 \frac{\cos\theta_t}{\cos\theta_i}$$

$$\left(1 - \frac{E_r}{E_i} \right) \left(1 + \frac{E_r}{E_i} \right) = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}} \left(1 + \frac{E_r}{E_i} \right)^2 \frac{\cos\theta_t}{\cos\theta_i}$$

$$1 - \frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}} \left(1 + \frac{E_r}{E_i} \right) \frac{\cos\theta_t}{\cos\theta_i}$$

$$1 - \frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}} \frac{\cos\theta_t}{\cos\theta_i} + \frac{E_r}{E_i} \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}} \frac{\cos\theta_t}{\cos\theta_i}$$

$$1 - \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}} \frac{\cos\theta_t}{\cos\theta_i} = \frac{E_r}{E_i} \left(1 + \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}} \frac{\cos\theta_t}{\cos\theta_i} \right)$$

$$\frac{E_r}{E_i} = \frac{1 - \dfrac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}} \dfrac{\cos\theta_t}{\cos\theta_i}}{1 + \dfrac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}} \dfrac{\cos\theta_t}{\cos\theta_i}}$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_1}\cos\theta_i - \sqrt{\varepsilon_2}\cos\theta_t}{\sqrt{\varepsilon_1}\cos\theta_i + \sqrt{\varepsilon_2}\cos\theta_t}$$

$$\sqrt{\varepsilon_2}\cos\theta_t = \sqrt{\varepsilon_2}\sqrt{1 - \sin^2\theta_t}$$

But $\dfrac{\sin\theta_i}{\sin\theta_t} = \dfrac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}$

$$\sin^2\theta_t = \frac{\varepsilon_1 \sin^2\theta_i}{\varepsilon_2}$$

$$\sqrt{\varepsilon_2}\cos\theta_t = \sqrt{\varepsilon_2}\sqrt{1 - \frac{\varepsilon_1 \sin^2\theta_t}{\varepsilon_2}}$$

$$= \sqrt{\varepsilon_2 - \varepsilon_1 \sin^2\theta_t}$$

Substituting this value in $\dfrac{E_r}{E_i}$

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_1}\cos\theta_i - \sqrt{\varepsilon_2 - \varepsilon_1 \sin^2\theta_i}}{\sqrt{\varepsilon_1}\cos\theta_i + \sqrt{\varepsilon_2 - \varepsilon_1 \sin^2\theta_i}}$$

Reflection co – efficient is given by

$$\frac{E_r}{E_i} = \frac{\cos\theta_i - \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}}{\cos\theta_i + \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}}$$

Vertical polarization:-

In this case, E is parallel to the plane of incidence.

$$(E_i - E_r)\cos\theta_i = E_t \cos\theta_t$$

$$1 - \frac{E_r}{E_i} = \frac{E_t}{E_i}\frac{\cos\theta_t}{\cos\theta_i}$$

$$\frac{E_t}{E_i} = \left(1 - \frac{E_r}{E_i}\right)\frac{\cos\theta_t}{\cos\theta_i}$$

$$\frac{E_r^2}{E_i^2} = 1 - \frac{\sqrt{\varepsilon_2}E_t^2\cos\theta_t}{\sqrt{\varepsilon_1}E_i^2\cos\theta_i}$$

$$\frac{E_r^2}{E_i^2} = 1 - \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\left(1 - \frac{E_r}{E_i}\right)^2 \frac{\cos^2\theta_i}{\cos^2\theta_t} \times \frac{\cos\theta_t}{\cos\theta_i}$$

$$1 - \frac{E_r^2}{E_i^2} = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\left(1 - \frac{E_r}{E_i}\right)^2 \frac{\cos\theta_i}{\cos\theta_t}$$

$$\left(1 - \frac{E_r}{E_i}\right)\left(1 + \frac{E_r}{E_i}\right) = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\left(1 - \frac{E_r}{E_i}\right)^2 \frac{\cos\theta_i}{\cos\theta_t}$$

$$1 + \frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\left(1 - \frac{E_r}{E_i}\right)\frac{\cos\theta_i}{\cos\theta_t}$$

$$\frac{E_r}{E_i}\left[1 + \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\frac{\cos\theta_i}{\cos\theta_t}\right] = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\frac{\cos\theta_i}{\cos\theta_t} - 1$$

$$\frac{E_r}{E_i}\left[\frac{\sqrt{\varepsilon_1}\cos\theta_t + \sqrt{\varepsilon_2}\cos\theta_i}{\sqrt{\varepsilon_1}\cos\theta_t}\right] = \frac{\sqrt{\varepsilon_2}\cos\theta_i - \sqrt{\varepsilon_1}\cos\theta_t}{\sqrt{\varepsilon_1}\cos\theta_t}$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_2}\cos\theta_i - \sqrt{\varepsilon_1}\cos\theta_t}{\sqrt{\varepsilon_2}\cos\theta_i + \sqrt{\varepsilon_1}\cos\theta_t}$$

$$\cos\theta_i = \sqrt{1 - \sin^2\theta_t}$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_2}\cos\theta_i - \sqrt{\varepsilon_1\left(1 - \sin^2\theta_t\right)}}{\sqrt{\varepsilon_2}\cos\theta_i + \sqrt{\varepsilon_1\left(1 - \sin^2\theta_t\right)}}$$

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}$$

$$\sin\theta_t = \frac{\sqrt{\varepsilon_1}\sin\theta_i}{\sqrt{\varepsilon_2}}$$

$$\sin^2\theta_t = \frac{\varepsilon_1}{\varepsilon_2}\sin^2\theta_i$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_2}\cos\theta_i - \sqrt{\varepsilon_1 - \frac{\varepsilon_1^{\,2}}{\varepsilon_2^{\,2}}\sin^2\theta_i}}{\sqrt{\varepsilon_2}\cos\theta_i + \sqrt{\varepsilon_1 + \frac{\varepsilon_1^{\,2}}{\varepsilon_2^{\,2}}\sin^2\theta_i}}$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_2}\cos\theta_i - \frac{1}{\sqrt{\varepsilon_2}}\sqrt{\varepsilon_1\varepsilon_2 - \varepsilon_1{}^2\sin^2\theta_i}}{\sqrt{\varepsilon_2}\cos\theta_i + \frac{1}{\sqrt{\varepsilon_2}}\sqrt{\varepsilon_1\varepsilon_2 - \varepsilon_1{}^2\sin^2\theta_i}}$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_2}\cos\theta_i - \frac{\varepsilon_1}{\sqrt{\varepsilon_2}}\sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}}{\sqrt{\varepsilon_2}\cos\theta_i + \frac{\varepsilon_1}{\sqrt{\varepsilon_2}}\sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}}$$

Dividing numerator & denominator by $\frac{\sqrt{\varepsilon_2}}{\varepsilon_1}$

$$\frac{E_r}{E_i} = \frac{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_i - \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}}{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_i - \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}} \text{ Reflection co – efficient}$$

Brewster's Angle:-

Brewster's angle is a particular angle at which no reflection takes place. This occurs when the numerator of the above equation is zero

$$\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_i - \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i} = 0$$

$$\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_i = \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}$$

$$\frac{\varepsilon_2}{\varepsilon_1}\sqrt{1-\sin^2\theta_i} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}$$

$$\frac{\varepsilon_2^2}{\varepsilon_1^2}\left(1-\sin^2\theta_i\right) = \frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i$$

$$\frac{\varepsilon_2^2}{\varepsilon_1^2} - \frac{\varepsilon_2^2}{\varepsilon_1^2}\sin^2\theta_i = \frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i$$

$$\sin^2\theta_i\left(1-\frac{\varepsilon_2^2}{\varepsilon_1^2}\right) = \frac{\varepsilon_2}{\varepsilon_1}\left(1-\frac{\varepsilon_2}{\varepsilon_1}\right)$$

$$\sin^2\theta_i\left(\varepsilon_1^2-\varepsilon_2^2\right) = \varepsilon_1\varepsilon_2 - \varepsilon_2^2$$

$$\sin^2\theta_i = \frac{\varepsilon_2(\varepsilon_1-\varepsilon_2)}{\varepsilon_1^2-\varepsilon_2^2}$$

$$\sin^2\theta_i = \frac{\varepsilon_2(\varepsilon_1-\varepsilon_2)}{(\varepsilon_1-\varepsilon_2)-(\varepsilon_1+\varepsilon_2)}$$

$$\sin^2\theta_i = \frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2}$$

$$\cos^2\theta_i = 1-\sin^2\theta_i$$

$$= \frac{\varepsilon_1+\varepsilon_2-\varepsilon_2}{\varepsilon_1+\varepsilon_2} = \frac{\varepsilon_1}{\varepsilon_1+\varepsilon_2}$$

$$\tan^2\theta_i = \frac{\varepsilon_2}{\varepsilon_1};\quad \tan^2\theta_i = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$\theta_i = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

This is called Brewster's angle at which there is no reflected ware when the incident wave is parallel polarized.

PROBLEMS ON UNIT – V

Problems in Faraday's law, motioual EMF & transformer EMF

1. A circular loop conductor of radius 0.1m lies in the z =0 plane & has a resistance of 5L. Given B = 0.2 sin 10^3 t $\bar{a}_z$ T. Find the induced emf & current.

Solution:-

Given $B = 0.2\sin 10^3 t\,\bar{a}_z \text{Tesla}, \quad \rho = 0.1\text{m}, R = 5\Omega$

Area of circular loop $= \pi\rho^2 = \pi \times (0.1)^2 = 0.01\pi \text{m}^2$

$$\begin{aligned}\phi &= BA\\ &= 0.2\sin 10^3 t \times 0.01\pi\\ &= 2\pi \sin 10^3 \text{ mWb}\end{aligned}$$

$$\begin{aligned}\varepsilon &= \frac{d\phi}{dt}\\ &= 2\pi[-\cos 10^3 t \times 10^3 \times 10^{-3}]\end{aligned}$$

$$\varepsilon = -2\pi\cos 10^3 t \text{ V.}$$

$$I = \frac{\varepsilon}{k} = \frac{-2\pi\cos 10^3 t}{5}$$

$$I = -0.4\pi\cos 10^3 t$$

Here, the negative sign shows current flow in opposite direction.

2. A 30cm×40cm rectangular loop rotates at 150 rad/s in a magnetic field of 0.06 wb/m^2, normal to the axis of rotation. If the loop has 50 turns, determine the induced voltage in the loop.

Solution:-

Given $A = 0.3 \times 0.4\text{m}, \; N = 50, \; w = 150 \text{rad/s} \; B = 0.06 \text{Wb/m}^2$

$$\phi = \int_s \vec{B}.\overline{ds} = \int B\,ds\,\cos\theta = B\cos\theta \int ds$$

$$\phi = BA\cos\theta$$

$$\begin{aligned}\varepsilon &= -N\frac{d\phi}{dt} = -N\frac{d}{dt}(BA\cos\theta)\\ &= -N\frac{d}{dt}(BA\cos\omega t)\\ &= -N\omega\left[BA(-\sin\omega t)\right]\\ &= \omega NBA\sin\omega t\\ &= 150 \times 150 \times 0.06 \times 0.12 \sin 90°\\ &= 75 \times 6 \times 12 \times 10^{-4} \times 10^2\\ &= 75 \times 72 \times 10^{-2}\\ &= 5400 \times 10^{-2}\end{aligned}$$

$$\varepsilon = 54\text{v}$$

Problems in Transformer EMF:-

3. Determine the emf denoted around a circular path at b = 0 with radius $\rho = 0.5\text{m}$ in the plane z =0 if (i)

$\bar{B} = 0.1\sin\left(\frac{377t}{\rho}\right)\bar{a}_\rho T \qquad \bar{B} = 0.1\sin(377t)\bar{a}_\rho T$

Solution:-

Given $\rho = 0.5\text{m in } z = 0$

The E. M. F induced in a time varying fiels is

$$EMF = -\oint_s \frac{\partial \overline{B}}{\partial t} . \overline{ds}$$

Where $\overline{ds} = \rho d\rho d\phi \rho \overline{a}_z$

$$EMF = -\int \frac{\partial}{\partial t}\left[0.1 \sin(377t)\overline{a}_z\right].\left[\rho d\rho d\phi\right]\overline{a}_z$$

$$= -\oint_s 377(0.1)(377t)\ \rho\ d\rho\ d\phi$$

$$= -37.7 \cos\ (377t) \int_0^{0.5} \rho\ d\rho \int_0^{2\pi} d\phi$$

$$= -37.7 \cos(377t)\left(\frac{\rho^2}{2}\right)_0^{0.5} (\phi)_0^{2\pi}$$

$$= -37.7 \cos(377t)\left[\frac{0.25}{2}\right][2\pi]$$

At t = 0, EMF = -29.59 v

(ii) $\overline{B} = 0.1 \sin\left(\frac{377t}{\rho}\right)\overline{a}_\rho$ since $\overline{a}_\rho.\overline{a}_z = 0$ and $EMF = 0$

4. A magnetic core of uniform cross section 4cm^2 is connected to a120 v, 60Hz generator as shown is figure. Determine the induced EMF V_2 in the secondary coli.

Solution:-

Given $V_1 = 120v, N_1 = 800\ \&\ N_2 = 400$

$$V_1 = -N_1 \frac{d\phi}{dt} \qquad V_2 = -N_2 \frac{d\phi}{dt}$$

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

$$V_2 = \frac{N_2}{N_1} V_1 = \frac{400}{800} \times 120$$

$$V_2 = 60v$$

5. An area of 0.5m2 in the z = 0 plane is enclosed by a filamentary conductor. Find the induced voltage, given that $\overline{B} = 0.65 \cos 10^3 t\left(\frac{\overline{a}_y + \overline{a}_z}{\sqrt{2}}\right)T$

Solution:-

Given $A = 0.5m^2$ $\qquad \overline{B} = 0.65 \cos 10^3 t\left(\frac{\overline{a}_y + \overline{a}_z}{\sqrt{2}}\right)T$

$$EMF = -\int \frac{\partial B}{\partial t}.ds \qquad \overline{ds} = ds\ \overline{a}_z$$

$$= \int_s 0.65\times10^3 \sin10^3 t\left(\frac{\bar{a}_y + \bar{a}_z}{\sqrt{2}}\right).ds\ \bar{a}_z$$

$$= \frac{650}{\sqrt{2}}\sin10^3 t \int_s ds = \frac{650}{\sqrt{2}}\sin10^3 t\times0.5$$

$\text{EMF} = 229.81\sin10^3 t\,\text{V}$

Motional EMF problems

6. A conductor of length 100 cm mores at right angles to a uniform field of strength 10000 lines/ Cm^2 with a velocity of 50m/s. Determine the induced EMF when the conductor mores at an angle of 30° the direction of the field.

Solution:-

Given $\ell = 100\text{cm} = 100\times10^{-2}\text{m},\ \ B = 10000\text{lines}\,C/m^2 = 1\text{Wb}/m^2$

$$v = 50\text{m}/s \,\&\, \theta = 30°$$

$$\text{EMF} = \oint_\ell (v\times B).dl$$

$$= B\ell v\sin\theta = 1\times1\times50\times\sin30°$$

$\text{EMF} = 25\text{V}$

7. Calculate the maximum emf induced in a coil of 4000 turns & radius of 12cm rotating at 30rps in a magnetic field of B = 500 gauss.

Solution:-

Given $\rho = 0.12\text{m},\ B = 500\text{Gauss}; = 500\times\frac{1}{10000}\text{Wb}/m^2 = 0.05\text{Wb}/m^2;\ \ N = 4000$

$$\omega = 2\pi\times\text{rps}$$

$$= 2\pi\times30 = 60\pi\,\text{rad}/s$$

$$\text{EMF} = \oint_\ell (v\times B).dl$$

$$= B\ell v\sin\theta$$

For maximum EMF $\theta = 90°$

$$\frac{\text{EMF}}{\ell} = Bv\sin90° = Bv = B\rho\omega$$

$$= 0.05\times0.12\times60\pi$$

$$= 1.131\text{V}/m$$

For N turns,

$$\frac{\text{EMF}}{\ell} = \frac{\text{EMF}}{\ell}\times N = 4000\times1.131$$

$$\frac{\text{EMF}}{\ell} = 4524\text{V}/m$$

8. A conductor 1cm in length is parallel to z – axis and rotates at radius of 25cm at 1200 rpm. Find the induced voltage if the radius field is given by $\bar{B} = 0.5\bar{a}_\rho T$

Solution:-

Given length ℓ=0.01m, radius ρ =0.25m

Velocity = 1200 rpm & $\overline{B}=0.5\overline{a}_\rho T$

$$\omega=\frac{2\pi\times rpm}{60}=\frac{2\pi\times 1200}{60}$$
$$\omega=40\pi\ rad/s$$

For a rotating conductor in a stationary magnetic field, the induced emf is

$$EMF=\int(v\times B).dl$$

Since the conductor is parallel to z- axis,

$$\overline{dl}=dz\ \overline{a}_z$$

$$EMF=\int_{\rho=0}^{\ell}\left(\rho_w\ a_\phi\times 0.5\overline{a}_\rho\right)dz\ \overline{a}_z$$
$$=\int_{\rho=0}^{\ell}\frac{\rho\omega}{2}(-a_z).dz\,a_z$$
$$=\frac{-40\pi\times 0.25}{2}\int_0^{0.01}dz$$
$$=-5\pi\left[z\right]_0^{0.01}=-0.157$$
$$EMF=-157mV$$

9. A rod of length 'ℓ' rotates about the z – axis with an angular velocity 10. If $\overline{B}=B_0\,\overline{a}_z Telsa$, calculate the voltage induced.

Solution:-

$$EMF=\int(v\times B).\ dl$$

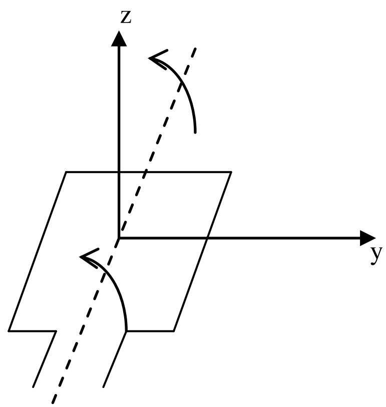
$$=\int_{\rho=0}^{\ell}\left(\rho\omega a_\phi\times B_0\,\overline{a}_z\right).d\rho a_\rho$$
$$=\int_0^{\ell}B_0\,\rho\omega\overline{a}_\rho\ d\rho\ \overline{a}_\rho$$

$$=B_0\omega\int_0^{\ell}\rho\ d\rho$$

$$=B_0\omega\left[\frac{\rho^2}{2}\right]_0^{\ell}$$
$$EMF=\frac{1}{2}B_0\omega\ell^2V$$

10. A square coil, 0.8m on a side rotates about the x – axis at $\omega=80\pi\ rad/s$ in a field $\overline{B}=0.6\,\overline{a}_z$ as shown in fig. find the induced voltage.

Solution:-



From the diagram, only two sides cut the magnetic field.

$$a=0.8m\ \ \overline{B}=0.6a_z\ \ w=80\pi$$

$$EMF=\oint_{\ell}\left(v\times\overline{B}\right).dl$$
$$=vB\ell\ \sin\theta\quad\theta=\omega t$$

For a square loop with sides a = 0.8m, $v = \frac{a\omega}{2} m/w$

$$\begin{aligned} EMF &= \left(\frac{a\omega}{2}\right) B(2a)\sin\theta \\ &= \omega B a^2 \sin\theta \\ &= \omega B a^2 \sin\omega t \\ &= (80\pi)(0.6)(0.8)^2 \sin(80\pi t) \end{aligned}$$

$$EMF = 96.46 \sin 80\pi t$$

11. A square loop of side 4cm with a resistor of 10Ω on the side is placed in a uniform magnetic field of 50m T in the direction of x- axis. Calculator (i) the induced current at t = 1ms (ii) induced EMF at t= 3ms. It is given that the square loop the axis of solution is perpendicular to the field. The loop lies in yz plane at t =0.

Solution:-

Given

$$\begin{aligned} a &= 4cm = 0.04 \\ B &= 50 \times 10^{-3} T \\ \theta &= 90°, R = 10\Omega \\ f &= 10Hz \ \& \ \omega = 2\pi f = 20\pi rad/s \end{aligned}$$

$$EMF = \oint_{\ell} (v \times B).dl \quad v = \frac{a\omega}{2}$$

$$\begin{aligned} &= vB\ell \sin\theta \\ &= \left(\frac{a\omega}{2}\right) B(2a)\sin\theta \\ &= a^2 \omega B \sin\omega t \end{aligned}$$

$$EMF = (0.04)^2 \times 20\pi \times 50 \times 10^{-3} \sin(20\pi t) = 5.03 \sin(20\pi t) mV$$

EMF = 5.52 μV

$$\rho = \frac{EMF}{R} = \frac{5.52 \times 10^{-6}}{10}$$

$$\rho = 0.552 \mu A$$

$$\begin{aligned} EMF &= 5.03 \times 10^{-3} \sin(20\pi \times 10^{-3}) \\ &= 55 \mu V \end{aligned}$$

12. If the conductor mores with a velocity $v = 4.5 \sin 10^6 t \ \bar{a}_z$ **. Find the induced voltage is the conductor if (i)** $\bar{B} = 0.08 \bar{a}_y$ **(ii)** $B = 0.08 a_x T$

Solution:-

Given $v = 4.5 \sin 10^6 t \bar{a}_z m/s$ $\quad \bar{B} = 0.08 \bar{a}_y$

$$\text{EMF} = \int (v \times B)\, dl$$

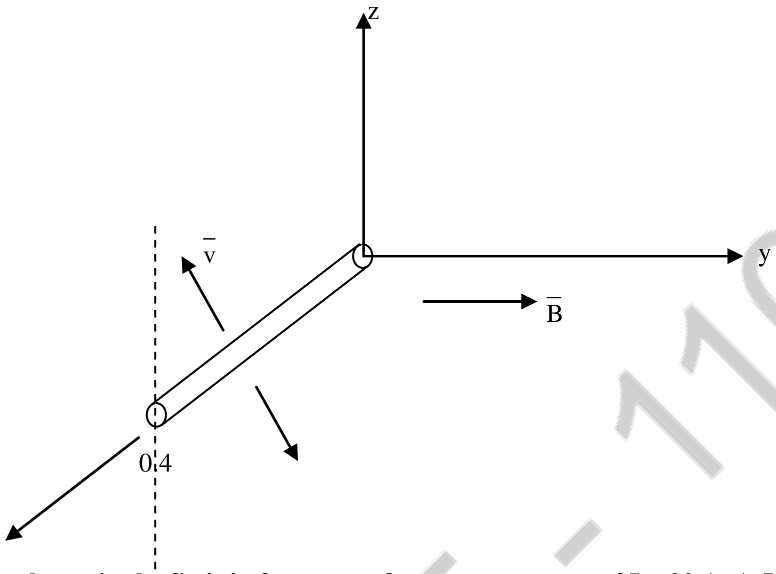
$$= 4.5 \sin 10^6 t \int (\bar{a}_z \times 0.08\bar{a}_y).dx\,\bar{a}_x$$

$$= 0.36 \sin 10^6\ t \int_0^{0.4} (-a_x).dx\,\bar{a}_x$$

$$= -0.36 \sin 10^6 (0.4)$$

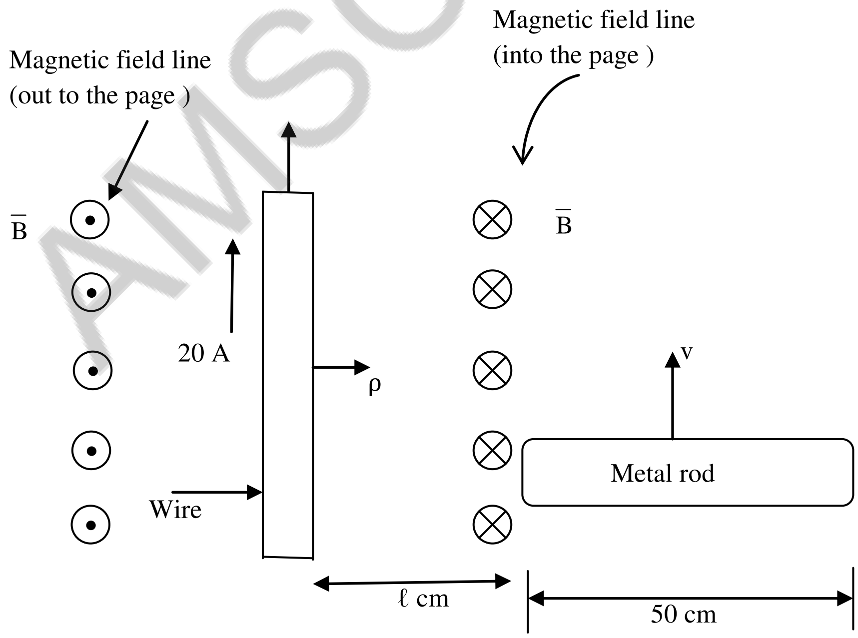
$$\text{EMF} = -0.144 \sin 10^6 t$$

(ii) Since the conductor is placed parallel the magnetic field, it does not cut any line. Hence the field is zero.



13. The wire shown in the fig is in free space & carnes a current of I = 20 A. A 50cm long metal rod mores at a constant velocity of $\bar{v} = 5\bar{a}_z m/s$ find V_{12}.

Solution:-



Given $v = 5\bar{a}_z m/s \quad I = 20A$

The magnetic field intensity by current carrying wire

$$\overline{H} = \frac{I}{2\pi\rho}\,\bar{a}_\phi \qquad B = \mu_0 H = \frac{\mu_0 I}{2\pi\rho}\,\bar{a}_\phi$$

$$V_{12} = EMF = \int (v \times B).dl$$

$$= \int_{70}^{20}\left(5\bar{a}_z \times \frac{\mu_0 I}{2\pi\rho} a_\phi\right)(d\rho.\bar{a}_\rho)$$

$$= \frac{-5\mu_0 I}{2\pi}\int_{70}^{20}\frac{d\rho}{\rho} = \frac{-5\mu_0 I}{2\pi}\left[\ln(\rho)\right]_{70}^{20}$$

$$= \frac{-5\mu_0 I}{2\pi}\ln\left(\frac{20}{70}\right) = \frac{-5\times 4\pi\times 10^{-7}\times 20}{2\pi}\ln\left(\frac{20}{70}\right)$$

$$V_{12} = 25.06\mu v$$

14. A faraday's copper (Cu) disc of 0.3m dia is rotated at 60 rotations / sec on a horizontal axis perpendicular to the plane of the disc. The axis is lying in a horizontal fields 20 µT. Determine the emf measured between the bunches.

Solution;-

Given $d = 0.3m, B = 20\times 10^{-6}T,\ \omega = 2\pi f = 2\pi(60) = 120\pi rad/s$

$$EMF = \frac{-\omega Bb^2}{2}$$

$$= \frac{-\omega Bb^2}{2} \qquad b = radius = \frac{0.3}{2} = 0.15$$

$$= \frac{-120\pi\times 20\times 10^{-6}\times(0.15)^2}{2}$$

$$= -1200\pi\times 10^{-6}\times 0.0225$$

$$= -2700\pi\times 10^{-8}$$

$$= -27\pi\mu V$$

EMF = - 84. 82 μV

15. A Faraday's Cu disc, 0.5m in dia, rotated at 1000rpm on a horizontal axis perpendicular to and thro the centre the disc, the axis lying in a horizontal field 10mT. Determine the emf measured b/w the ????

Solution:-

Given $d = 0.5m,\ \ v = 1000rpm\ \&\ B = 10\times 10^{-3}$

$$\omega = \frac{2\pi\times rpm}{60} = \frac{2\pi\times 1000}{60} = 104.72 rad/s$$

$$EMF = \frac{-\omega Bb^2}{2}$$

$$= \frac{-104.72\times 10\times 10^{-3}\times(0.25)^2}{2}$$

EMF = -32.72 m v

16. A conductor bar CD slides freely over two conducting rails as shown in fig. calculate the induced voltage in the bar,

(i) If the bar is stationed at y = 10cm & $\overline{B} = 5\cos 10^6 t\ \ \bar{a}_z m\ Wb/m^2$

(ii) If the bar slides at a velocity $v = 30\bar{a}_y m/s \ \& \ \bar{B} = 5\bar{a}_z mWb/m^2$

(iii) If the bar slides at a velocity $v = 30\bar{a}_y m/s \ \& \ \bar{B} = 5\cos(10^6 t - y)\bar{a}_z mWb/m^2$

Solution:-

$$EMF = -\int \frac{\partial B}{\partial t}.\ \overline{ds}$$
$$= \int_{y=0}^{0.1}\int_{0}^{0.08} 5\times10^{-3}\times10^{6}\sin10^{6}t\ dxdy$$
$$= 5\times10^{3}\sin10^{6}t\times0.08\times0.1$$
$$EMF = 40\sin10^{6}\ tV$$

(ii) For a sliding bar in a stationary magnetic field, the motional emf is

$$EMF = \int (v\times B).\ dl$$

$$= \int_{\ell}^{0}\left(v\bar{a}_y \times B\bar{a}_z\right).\ dx\,\bar{a}_z$$
$$= \int_{\ell}^{0}(v\times B)dx$$
$$= vB\left[x\right]_{\ell}^{0}$$
$$= -vB\ell$$
$$= -30\times5\times10^{-3}\times0.08$$

EMF = -12mV

(iii) For a sliding bar in time varying magnetic field, both transformer emf & motional emf an present.

$$EMF = -\int\frac{\partial B}{\partial t}.\overline{ds} + \int(v\times B).\ \overline{dl}$$
$$= \int_{0}^{0.08}\int_{0}^{y}(5\times10^{-3})\times10^{6}\times\sin(10^{6}t-y) + \int_{0.08}^{0}\left[30a_y\times5\times10^{-3}\cos(10^{6}t-y)\right]$$
$$= 400\cos(10^{6}t-y) - 400\ \cos10^{6}t\ V$$

Displacement current problems

1. Find the displacement current density for the field $\bar{E} = 300\sin10^{9}t$

Solution:-

$$J_d = \frac{\partial D}{\partial t} = \frac{\partial}{\partial t}(\varepsilon E) = \frac{\partial}{\partial t}(\varepsilon_0\varepsilon_r E)$$
$$= \frac{\partial}{\partial t}\left[8.854\times10^{-12}\times1\times300\sin10^{9}\ t\right]$$
$$= 8.854\times10^{-12}\times300\times10^{9}\cos10^{9}t$$
$$= 8.854\times0.3\cos10^{9}t$$
$$J_d = 2.6562\cos10^{9}\ tA/m^2$$

2. The parallel plates in a capacitor have an area of 5cm^2 & separated by 0.5cm. A voltage of 10sin 10^3 t v is applied to the capacitor. Find the displacement current with $\varepsilon_r = 5$ **.**

Solution:-

Given d = 0.5cm

$$A = 5cm = 5 \times 10^{-4}\ m^2$$
$$\varepsilon_r = 5,\ v = 10\sin 10^3 t$$

$$J_d = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t} = \frac{\varepsilon_0 \varepsilon_r}{d} \frac{\partial v}{\partial t}$$

$$I_d = J_d \times A$$

$$I_d = \frac{\varepsilon_0 \varepsilon_r A}{d} \frac{dV}{dt}$$

$$= \frac{8.854 \times 10^{-12} \times 5 \times 5 \times 10^{-4}}{0.5 \times 10^{-2}} \frac{d}{dt} (10 \sin 10^3 t)$$
$$= \frac{8.854 \times 10^{-12} \times 5 \times 5 \times 10^{-4} \times 10^3}{5} \times 10 \times 10^3 \times \cos 10^3 t$$
$$= 8.854 \times 5 \times 10^{-9} \cos 10^3 t$$

$$I_d = 44.27 \cos 10^3 t\, nA$$

3. Find displacement current density (Jd) with $\varepsilon_r = 10$ area of the plates 0.01m^2, d = 0.5mm & v = 200 sin 200 t.

Solution:-

$$J_d = \frac{\partial D}{\partial t} = \frac{\partial (\varepsilon E)}{\partial t} = \frac{\varepsilon_0 \varepsilon_r}{d} \frac{dv}{dt}$$

$$= \frac{8.854 \times 10^{-12} \times 10}{0.05 \times 10^{-3}} \frac{d}{dt} \left(200 \sin 200t\right)$$
$$= \frac{8.854 \times 10^{-12} \times 10 \times 200 \times 200 \cos t}{5 \times 10^{-5}}$$
$$= 8.854 \times 8 \times 10^{-3} \cos 200t$$
$$= 70.832 \cos 200t\, mA / m^2$$

4. A Cu wire carries conduction current of 1A. determine the displacement current at 1MHz. For Cu, $\varepsilon = \varepsilon_0 \ \ \&\ \ \sigma = 5.8 \times 10^7\ s/m$

Solution:-

$$I_c = J_C \times A$$
$$= \sigma E \times A$$
$$E = \frac{I_c}{\sigma A}$$
$$J_d = \frac{\partial D}{\partial t} = \frac{\partial (\varepsilon E)}{\partial t}$$

Since $\frac{\partial}{\partial t} = j\omega,\ J_d = \omega \varepsilon E = \omega \varepsilon_0 \frac{I_c}{\sigma A}$

$$J_d = 2\pi f\ \varepsilon_0 \frac{I_c}{\sigma A}$$
$$I_d = J_d \times A$$
$$= 2\pi f \frac{\varepsilon_0 I_c}{\sigma A} \times A$$
$$= \frac{2 \times 3.14 \times 1 \times 10^6 \times 8.854 \times 10^{-12}}{5.8 \times 10^7}$$
$$I_d = 9.585 \times 10^{-13}\ A$$

5. Given $I_c = 4\sin\omega t \text{ mA}, \quad \sigma = 4\times10^7 \ \text{s/m} \ \& \quad \varepsilon_r = 1.$ **If w = 10 rad/s. Find Id**

Solution:-

$$I_c = J_C A = \sigma E A$$
$$E = \frac{I_C}{\sigma A}$$

$$I_d = J_d A$$
$$= \frac{\partial D}{\partial t} A$$
$$= \frac{\partial}{\partial t}(\varepsilon E)A$$
$$= \varepsilon\left(\frac{\partial E}{\partial t}\right)A$$
$$= \varepsilon A \frac{\partial E}{\partial t}$$
$$= \varepsilon A \frac{\partial}{\partial t}\left(\frac{I_C}{\sigma A}\right)$$
$$= \frac{\varepsilon_0 \varepsilon_r A}{\sigma A}\frac{\partial I_C}{\partial t}$$
$$= \frac{8.854\times10^{-12}\times1}{4\times10^7}\frac{\partial}{\partial t}(4\sin\omega t)\times10^{-3}$$
$$= 8.854\times10^{-19}\times10\cos\omega t\times10^{-3}$$
$$= 8.854\times10^{-19}\times10^{10}\cos\omega t\times10^{-3}$$
$$I_d = 8.854\cos\omega t \text{ nA}$$

6. Find Id, where $\varepsilon = 100\varepsilon_0, \ A = 0.01\text{m}^2 \ \ d = 0.05\times10^{-3} \quad \& \ v = 100\sin 200\ \pi t$

Solution:-

$$C = \frac{\varepsilon A}{d} = \frac{100\varepsilon_0\times0.01}{0.05\times10^{-3}} = \frac{8.854\times10^{-12}\times10^2\times10^2}{5\times10\times-5} = 1.7708\times10^{-7} = 0.1771\mu F$$

$$I_d = C\frac{dv}{dt} = 0.1771\times10^{-6}\frac{d}{dt}(100\sin 200\pi t)$$
$$= 0.1771\times10^6\times100\times200\pi\times\cos\ \ 200\pi t$$
$$= 0.3542\times10^{-2}\pi\cos 200\pi t$$
$$I_d = (11.13\cos\ 200\pi t)\text{ mA}$$

7. Find the electric flux density & volume charge density if $\overline{E} = 2x^2\overline{a}_x + 4y^2\overline{a}_y + 2z^2\overline{a}_z \ \text{v/m}; \ \varepsilon_r = 4$

Solution:-

$$\overline{E} = 2x^2\overline{a}_x + 4y^2\overline{a}_y + 2z^2\overline{a}_z \ \text{v/m}$$

$$D = \varepsilon E = \varepsilon_0\varepsilon_r E$$
$$= 8.854\times10^{-12}\times4\times(2x^2\overline{a}_x + 4y^2\overline{a}_y + 2z^2\overline{a}_z)$$
$$D = 70.83x^2\overline{a}_x + 141.66y^2\overline{a}_y + 70.83z^2\overline{a}_z \ \ \text{pc/m}^2$$
$$\rho_v = \nabla.D$$
$$= \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$
$$= 141.66x + 283.32y + 141.662\text{pc/m}^2$$

8. Given $J = 5\sin(\omega t - 20z)\overline{a}_y + \cos\ (\omega t - 20z)\overline{a}_z \text{mA/m}^2$ **. Find volume charge density** ρ_v

Solution:-

$$\nabla.J = \frac{-\partial \rho_v}{\partial t}$$

$$\frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} = \frac{-\partial \rho_v}{\partial t}$$

$$0 + 0 + -\sin(\omega t - 20z)x - 20 = \frac{-\partial \rho_v}{\partial t}$$

$$20\sin(\omega t - 20z) = \frac{-\partial \rho_v}{\partial t}$$

$$\rho_v = \int -20 \ \sin \ (\omega t - 20z)dt$$

$$= \frac{-20}{\omega} - \cos(\omega t - 20z)$$

$$= \frac{20}{\omega}\cos(\omega t - 20z)c/m^3$$

9. If $J = (2y\bar{a}_x + xz\bar{a}_y + z^3\bar{a}_z)\sin 10^4 tA/m^2$ **. Determine the volume charge density** ρ_v **if** $\rho_v = (x, y, 0, t)$

Solution:-

Given $J = (2y\bar{a}_x + xz\bar{a}_y + z^3\bar{a}_z)\sin 10^4 tA/m^2$

$$\nabla.J = \frac{-\partial \rho_v}{\partial t}$$

$$\rho_v = -\int (\nabla.J).\, dt$$

$$= -\int (0 + 0 + 3z^2)\sin 10^4 \ t$$

$$\rho_v = \frac{3z^2}{2}\cos 10^4 t + C_0$$

Given ρ_v =0 at z = 0

$$\rho_v = 0 = \frac{3z^2}{10^4}\cos 10^4 t + C_0 \ \ [z = 0$$

$$\rho_v = 0$$

$$\rho_v = 0.3z^2 \cos 10^4 tmC/m^3$$

10. If the $E = 10\cos(\omega t - \beta y)\bar{a}_z v/m,$ **determine . If the same field exist in a medium, whose condutinty is** $5 \times 10^3 \Omega^{-1}/cm$ **find Jc.**

Solution:-

$$D = \varepsilon E = \varepsilon_0 \varepsilon_r E$$

$$= 8.854 \times 10^{-2} \times 1 \times 10\cos(\omega t - \beta y)a_z$$

$$D = 10 \times 8.854 \times 10^{-12} \cos(\omega t - \beta y)\bar{a}_z$$

$$= 8.854\cos(\omega t - \beta y)\bar{a}_z Pc/m^2$$

$$J_d = \frac{\partial \overline{D}}{\partial t} = \frac{\partial}{\partial t}[8.854\cos(\omega t - \beta y)]\overline{a}_z$$
$$= 8.854\omega \ \sin(\omega t - \beta y)\overline{a}_z \ pA/m^2$$
$$J_c = \sigma E$$
$$= 5\times10^3 \times 10 \cos(\omega t - \beta y)\overline{a}_z \ A/cm$$
$$= 5\times10^5 \times 10 \cos(\omega t - \beta y)\overline{a}_z \ A/cm^2$$
$$= 5\times10^6 \times 10 \cos(\omega t - \beta y)\overline{a}_z \ A/m^2$$

11. A co – axial capacitor has the parameter a = 5mm, b = 30mm, $\ell = 20\text{cm},\ \varepsilon_r = 8\ \&\ \sigma = 10^{-6}\ s/m$ **. If** $J_C = \left(\frac{2}{\sigma}\right)\sin\ (10^6 t)\overline{a}_\rho A/m$ **determine (i Ic (ii) Jd (iii) Id**

Solution:-

Given $J_C = \left(\frac{2}{\sigma}\right)\sin\ (10^6 t)\overline{a}_\rho A/m \quad a = 5\times10^{-3} \quad b = 30\times10^{-3} \quad \ell = 0.2m \quad \varepsilon_r = 8 \quad \sigma = 10^{-6} s/m$

(i) $I_C = \int \overline{J}_c \,.\overline{d}_s \qquad [ds = \rho d\phi dz \overline{a}_\rho$

$$= \int_s \frac{2}{\rho} \sin\ (10^6 t)\ \overline{a}_\rho\ \rho d\phi dz \overline{a}_\rho$$
$$= \int_0^{2\pi}\int_0^{0.2} 2\sin\ (10^6 t) d\phi dz$$
$$= 2\sin(10^6 t)\ [0.2][2\pi]$$
$$= 0.8\times3.14\sin(10^6\ t)$$

$$I_c = 2.512\sin(10^6\ t)\,A$$

(ii) $J_d = \frac{\partial D}{\partial t} = \frac{\partial}{\partial t}(\varepsilon E)$

$$= \frac{\partial}{\partial t}\left(\varepsilon \frac{J_c}{\sigma}\right) = \frac{\varepsilon_0\varepsilon_r}{\sigma}\frac{\partial J_c}{\partial t}$$
$$= \frac{8\varepsilon_0}{\sigma}\frac{\partial}{\partial t}\left(\frac{2}{\rho}\sin\ (10^6)t\right)\overline{a}_\rho$$
$$= \frac{8\varepsilon_0}{\sigma\rho}\times10^6\times2\cos(10^6)t$$
$$= \frac{16\times8.854\times10^{-12}\times10^6}{\rho\times10^{-6}}\cos\ (10^6)t\overline{a}_\rho$$

$$J_d = \frac{141.664\cos(10^6)t}{\rho}\overline{a}_\rho\ A/m^2$$
$$J_{d(max)} = \frac{141.664}{5\times10^{-3}} = 28.33\times10^3\ A/m^2$$

(iii) $I_d = \int \overline{J_d}\ .\overline{d_s}$

$$= \oint_s \frac{141.664}{\rho}\cos\ (10^6\ t)\overline{a}_\rho.\ \rho\ d\phi dz\ \overline{a}_\rho$$
$$= 141.664\int_0^{0.2}\int_0^{2\pi}\cos\ (10^6\ t)\ d\phi\ dz$$

$$I_d = 178.02\cos\ (10^6\ t)\ A$$

12. Find Ic in a circular conductor of radius 4mm if the current density varies to $\bar{J} = \frac{10^4}{\rho}\bar{a}_z \ A/m^2$

Solution:-

$$I_c = \int \bar{J}\,\bar{d}_s = \int_{\phi=0}^{2\pi}\int_{z=0}^{0.004}\left(\frac{10^4}{\rho}\,a_z\right)\left(\rho\, d\rho\, d\phi\, \bar{a}_z\right)$$

$$I_c = 10^4\ (0.004)(2\pi) = 80\pi A.$$

13. In a cylindrical conductor of radius 2mm, $J = 10^3\ e^{-400\rho}\ \bar{a}_z A/m$ **find Ic.**

Solution:-

$$I_c = \oiint_s \bar{J}.\ \bar{d}_s = \int_0^{2\pi}\int_0^{0.002}(10^3 e^{-400\rho}\bar{a}_z).(\rho\, d\rho\, d\phi\ \bar{a}_z)$$

$$= \int_0^{2\pi} d\phi \int_0^{0.002} 10^3 \rho\, e^{-400\rho}\ d\rho$$

$$= 2\pi\times 10^3\left[\frac{-\rho}{400}e^{-400\rho}\ \frac{e^{-400\rho}}{(-400)^2}\right]_0^{0.002}$$

$$= \frac{2\pi\times 10^3}{16\times 10^4}\left[-e^{-400\rho}\ (400\rho+1)\right]_0^{0.002}$$

$$I_c = 7.51\text{mA}$$

14. Given $A = 0.05m^2\ \ d = 2\times 10^{-3}m\ \ \varepsilon_r = 5\ \sigma = 5\times 10^{-4}\ s/m\ \ v = 5\sin 10^7\ tV.$ **Find I_{rms}.**

Solution:-

$$E = \frac{v}{d} = \frac{5\sin 10^7 t}{2\times 10^{-3}} = 2500\sin 10^7\, t\, v/m$$

$$J_c = \sigma E = 5\times 10^{-4}(250\sin 10^7 t) = 125\times 10^{-2}\sin 10^7\ A/m^2$$

$$J_d = \frac{\varepsilon\partial E}{\partial t} = \varepsilon_0\varepsilon_r\ \frac{\partial}{\partial t}(2500\ \sin 10^7\ t)$$

$$= 8.854\times 10^{-12}\times 5\times 2500\cos 10^7 t\times 10^7$$

$$J_d = 1.11\cos 10^7\, t\, A/m^2$$

$$I_c = J_c A = 1.25\sin 10^7 t\times 0.05 = 0.06\sin 10^7\ t\ A$$

$$I_d = J_d A = 1.11\cos 10^7 t\times 0.05 = 0.056\cos 10^7\, t\, A$$

$$I_T = \sqrt{I_c^2 + I_d^2} = \sqrt{(0.06)^2 + (0.056)^2} = 0.08A$$

$$I_{rms} = \frac{I_T}{\sqrt{2}} = \frac{0.08}{\sqrt{2}} = \frac{0.08\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = 0.04\times\sqrt{2} = 0.057\,A$$

15. A co – axial capacitor of length ℓ = 6cm with ε_r = 9. The radii are 1cm & 2cm. V= 100 sin (120 πt), what is Id?

Solution:-

Given $v = 100\sin(120\pi t)\ v,\ \ \ell = 6cm,\ \varepsilon = 9,\ a = 1cm\ \&\ b = 2cm$

$$I_d = \oiint_s \frac{\partial D}{\partial t}\cdot \bar{d}_s = \oiint_s \frac{\partial}{\partial t}(\varepsilon E)\cdot \bar{d}_s$$

$$= \oiint_s \frac{\partial}{\partial t}\left(\frac{\varepsilon V}{\rho \ln\left(b/a\right)}\bar{a}_\rho\right)\cdot(\rho\ d\phi\ dz\ \bar{a}_\rho)$$

$$= \frac{\varepsilon_0\varepsilon_r}{\ln\left(b/a\right)}\times 100\times 120\pi\cos\ (120\pi)t\int_0^{0.06} dz\int_0^{2\pi} d\phi$$

$$= \frac{1}{36\pi}\times 10^{-9}\times 9\times 100\times 120\pi\cos\ (120\pi)t\times 0.06\times 2\pi$$

$$= 10^{-9}\times 10^{2}\times 60\pi\cos\ (120\pi)t\times 6\times 10^{-2}$$

$$I_d = 360\pi\cos(120\pi)t\ \ n-A$$

Problems on boundary condition for an EM field

1) Region 1 is defined by x < 0 & μ_{r1} =4, when as region 2 is defined by x > 0 & μ_{r2} =8. If $\bar{H}_i = 8\bar{a}_x + 3\bar{a}_y - 6\bar{a}_z\ A/m$ for a source free boundary, determine $\pi 2$ & its magnitude.

Solution:-

Given $\bar{H}_i = 8\bar{a}_x + 3\bar{a}_y - 6\bar{a}_z\ A/m$ & the normal components are

$$H_{t1} = 3\bar{a}_y - 6\bar{a}_z \ \ \&\ \ H_{n1} = 8\bar{a}_z$$

For a source free interface, $\bar{H}_{t1} = \bar{H}_{t2}$ & $B_{n1} = B_{n2}$ therefore, the tangential & normal components in region 2 (x>0) are

$$\bar{H}_{t2} = 3\bar{a}_y - 6\bar{a}_z \qquad H_{n2} = \frac{\mu_{r1}}{\mu_{r2}}H_{n1} = \frac{4}{8}(8\ a_x)\ \ H_{n2} = 4\bar{a}_x$$

$$\bar{H}_2 = \bar{H}_{t2} + \bar{H}_{n2}$$

$$= 4\bar{a}_x + 3\bar{a}_y - 6\bar{a}_z\ A/m$$

$$|H_z| = \sqrt{4^2+3^2+(-6)^2} = \sqrt{61} = 7.81\ A/m$$

2. A 3d space is divided into region 1 (x<0)& region 2 (x>0) **where** $\sigma_1 = \sigma_2 = 00$ & $\bar{E}_1 = 2\bar{a}_x + 4\bar{a}_y + 6\bar{a}_z\ V/m$**.**
Find $\bar{E}_2$ & $\bar{D}_2$ **assume** $\varepsilon_{r1} = 2$ & $\varepsilon_{r2} = 4$

Solution:-

$$\bar{E}_1 = 2\bar{a}_x + 4\bar{a}_y + 6\bar{a}_z\ v/m$$

$$E_{t1} = 4\bar{a}_y + 6\bar{a}_z\ \ \&\ E_{n1} = 2\bar{a}_x$$

$$D_{t1} = \varepsilon_0\varepsilon_r\ E_{t1} = 2\varepsilon_0(44\bar{a}_y + 6\bar{a}_z)$$

$$E_{t1} = E_{t2}\ \ \&\ D_{n1} = D_{n2}$$

$$E_{t2} = 4\bar{a}_y + 6\bar{a}_z\ \ E_{n2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}E_{n1};\ E_{n2} = \frac{2}{4}(2a_x);\ \ \ E_{n2} = \bar{a}_x$$

$$\bar{E}_2 = \bar{E}_{n2} + \bar{E}_{t2}$$

$$\bar{E}_2 = \bar{a}_x + 4\bar{a}_y + 6\bar{a}_z$$

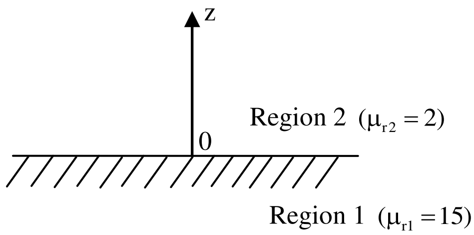
$$D_2 = \varepsilon_2 E_2 = \varepsilon_0\varepsilon_r\ \bar{E}_2$$

$$= 4\varepsilon_0(\bar{a}_x + 4\bar{a}_y + 6\bar{a}_z)$$

$$D_2 = \varepsilon_0(4\bar{a}_x + 4\bar{a}_y + 6\bar{a}_z)\ \ c/m^2$$

3. Given $\bar{B}_1 = 1.2\bar{a}_x + 0.8\bar{a}_y + 0.4\bar{a}_y\,T$ **in region 1 as shown in fig. Find** $\bar{H}_2$ **& the angles between the field vectors & a tangent to the interface**

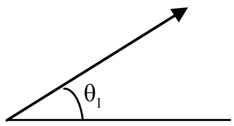
Solution:-



$$\overline{B}_1 = 1.2\overline{a}_x + 0.8\overline{a}_y + 0.4\overline{a}_z \text{ T}$$

$$H_1 = \frac{\beta_1}{\mu_1} = \frac{1.2\overline{a}_x + 0.8\overline{a}_y + 0.4\overline{a}_z}{\mu_0(15)}$$

$$H_1 = \frac{10^{-2}}{\mu_0}(8\overline{a}_x + 5.33\overline{a}_y + 2.67\overline{a}_z) \text{ A/ m}$$



$$H_{t1} = \frac{10^{-2}}{\mu_0}(8\overline{a}_x + 5.33\overline{a}_y) H_{n1} = \frac{10^{-2}}{\mu_0}(2.67\overline{a}_z)$$

$$H_{t2} = \frac{10^{-2}}{\mu_0}(8\overline{a}_x + 5.33\overline{a}_y) \quad \overline{H}_{n2} = \frac{\mu_{r1}}{\mu_{r2}}\overline{H}_{n1} = \frac{15}{2} \times \frac{10^{-2}}{\mu_0}(2.67\overline{a}_z)$$

$$H_2 = H_{t2} + H_{n2}$$

$$= \frac{10^{-2}}{\mu_0}(8\overline{a}_x + 5.33\overline{a}_y + 20.03 \text{ a}_z) \text{ A / m}$$

$$\theta_1 = 90° - \alpha_1$$

$$\cos\alpha_1 = \frac{B_1 - \overline{a}_z}{|B_1|} = \frac{(1.2a_x + 0.8a_y + 0.4a_z).a_z}{\sqrt{(1.2)^2 + (0.8)^2 + (0.4)^2}}$$

$$\cos\alpha_1 = 0.27$$

$$\alpha_1 = \cos^{-1}(0.27) = 74.33$$

$$\theta_1 = 90 - \alpha_1 = 90 - 74.33 = 15.67$$

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_{r2}}{\mu_{r1}}$$

$$\tan\theta_2 = \frac{\mu_{r2}}{\mu_{r1}}\tan\theta_1 = \frac{15}{2}\tan\ (15.67)$$

$$\tan\theta_2 = 2.104$$

$$\theta_2 = \tan^{-1}(2.107) = 64.58$$

4. In region defined by z < 0, $\mu_{r1} = 5 \,\&\, H_1 = \frac{1}{\mu_0}(0.2a_x + 0.5a_y + a_z)\text{A / m}^2$ **find** $\overline{H}_2$ if $\theta = 30°$

Solution:-

Given $\mu_{r1} = 5 \,\&\, H_1 = \frac{1}{\mu_0}(0.2a_x + 0.5a_y + a_z)\text{A / m}^2$

$$\bar{H}_1 = \frac{1}{4\pi\times10^{-7}}(0.2a_x + 0.5a_y + a_z)$$
$$= 10^4(15.92\bar{a}_x + 39.8\bar{a}_y + 79.6\bar{a}_z)A/m$$
$$H_{t1} = 10^4(15.92\bar{a}_x + 39.8\bar{a}_y)\ H_{n1} = 10^4(79.6\bar{a}_z)$$
$$\cos\alpha_1 = \frac{\bar{H}_1 - a_z}{|H_1|} = \frac{10^4(15.92\bar{a}_x + 39.8\bar{a}_y + 79.6\bar{a}_z)}{10^4\sqrt{(15.92)^2 + (39.8)^2 + (79.6)^2}}$$
$$\cos\alpha_1 = 0.88$$
$$\alpha_1 = \cos^{-1}(0.88) = 28.36°$$
$$\theta_1 = 90 - \alpha_1 = 90 - 28.36 = 61.64$$
$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_{r2}}{\mu_{r1}} \Rightarrow \mu_{r2} = \mu_{r1}\frac{\tan\theta_1}{\tan\theta_2} = 5\times\frac{\tan 61.64°}{\tan 30°}$$
$$\mu_{r2} = 16$$
$$B_{n1} = B_{n2} \text{ i.e, } \mu_{r1}H_{n1} = \mu_{r2}H_{n2}$$
$$\bar{H}_{n2} = \frac{\mu_{r1}}{\mu_{r2}}H_{n1} = \frac{5}{16}\times10^4(79.6)\bar{a}_z = 10^4(24.87\bar{a}_z)$$
$$H_{t2} = H_{t1} = 10^4(15.92\bar{a}_x + 39.8\bar{a}_y)$$
$$H_2 = 10^4(15.92\bar{a}_x + 39.8\bar{a}_y + 24.87\bar{a}_z)A/m$$

5. A current sheet, $\bar{k} = 8.5\bar{a}_z\ A/m$, at x = 0 separated region 1, x < 0, where $\bar{H}_1 = 20\bar{a}_y A/m$ & region 2, x > 0 find $\bar{H}_2$ at x = 0

Solution:-

Given $\bar{H}_1 = 20\bar{a}_y A/m$ in region 1

The normal component is in z – direction. Hn1=Hn2 > 0 & Bn1=Bn2>0

$$(H_{t1} - H_{t2})\times\bar{a}_{n12} = \bar{k}$$
$$(20\bar{a}_y - H_{y2}\bar{a}_y)\times a_x = 8.5a_z$$
$$(20 - H_{y2})\bar{a}_y\times\bar{a}_x = 8.5a_z$$
$$(20 - H_{y2})(-a_z) = 8.5a_z$$
$$-20a_z + H_{y2}a_z = 8.5a_z$$
$$H_{y2} = 28.5\bar{a}_y\ A/m$$

6. Region 1, z < 0 has $\mu_{r1} = 3.5$ & region 2, z > 0 has $\mu_{r2} = 10$. Near the origin $\bar{B}_1 = 2.4\bar{a}_x + 10\bar{a}_y T$ & $B_2 = 25\bar{a}_x + 17\bar{a}_y + 10\bar{a}_z T$. If the interface carries a sheet current, determine its density at the origin.

Solution:-

Given $\mu_{r1} = 3.5\ \bar{B}_1 = 2.4\bar{a}_x + 10\bar{a}_y T$ region 1

$\mu_{r2} = 10 B_2 = 25\bar{a}_x + 17\bar{a}_y + 10\bar{a}_z T$ region 2

$$H_1 = \frac{1}{\mu_0\mu_{r1}}\overline{B_1} = \frac{1}{3.5\mu_0}(2.4\bar{a}_x + 10\bar{a}_y)$$

$$H_1 = \frac{1}{\mu_0}(0.69\bar{a}_x + 2.86\bar{a}_y)A/m$$

$$H_2 = \frac{1}{\mu_0\mu_{r1}}\overline{B_2} = \frac{1}{10\mu_0}(25\bar{a}_x - 17\bar{a}_y + 10\bar{a}_z)$$

$$H_2 = \frac{1}{\mu_0}(2.5\bar{a}_x - 1.7\bar{a}_y + \bar{a}_z)A/m$$

$$\bar{k} = (H_1 - H_2)\times \bar{a}\ln 12$$

$$= \frac{1}{\mu_0}(0.69\bar{a}_x + 2.86\bar{a}_y - 2.5\bar{a}_x + 1.7\bar{a}_y - \bar{a}_z)$$

$$= \frac{1}{\mu_0}(-1.51\bar{a}_x + 4.56\bar{a}_y - \bar{a}_z)\times a_z$$

$$= \frac{1}{\mu_0}(-1.51(-a_y) + 4.56(a_x))$$

$$\bar{k} = \frac{(1.51a_y + 4.56a_x)\times 10^{-7}}{4\pi}A/m$$

Problems in potential functions

1. If the related scalar electric potential v-(x-Vot) & the vector magnetic potential $\bar{A} = \left(\frac{x}{v_0} - t\right)\bar{a}_x$ **where** v_0 **is the velocity of propagation. Then determine** (i)$\nabla.A$, (ii) $\bar{B}, \bar{H}, \bar{E} \& \bar{D}$ (iii) Also S.T $\nabla.\bar{A} = -\mu_0\varepsilon_0\frac{\partial v}{\partial t}$ **in free space**

Solution:-

Given $V = x - v_0 t \ \& \ A = \left(\frac{x}{v_0} - t\right)\bar{a}_x$

(i) $\nabla.A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

$$\nabla.A = \frac{\partial A_x}{\partial x} = \frac{\partial}{\partial x}\left(\frac{x}{v_0} - t\right) = \frac{1}{v_0}$$

(ii) $B = \nabla\times\bar{A} = \begin{pmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{x}{v_0} - t & 0 & 0 \end{pmatrix}$

$$= a_x(0) - a_y\left[0 - \frac{\partial}{\partial z}\left(\frac{x}{v_0} - t\right)\right] + a_z\left[0 - \frac{\partial}{\partial x}\left(\frac{x}{v_0} - t\right)\right]$$

$$B = 0; \quad H = 0$$

$$E = -\nabla V - \frac{\partial A}{\partial t}$$

$$= -\frac{\partial V}{\partial x}a_x - \frac{\partial}{\partial t}\left(\frac{x}{v_0} - t\right)a_x$$

$$= -\frac{\partial}{\partial x}(x - v_0 t)a_x + \bar{a}_x$$

$$E = -a_x + a_x = 0, \ \therefore D = 0$$

(iii) $V = x - \nu_0 t$

$$\frac{\partial v}{\partial t} = \frac{\partial}{\partial t}(x - \nu_0 t) = -\nu_0$$
$$\mu_0\varepsilon_0 \frac{\partial v}{\partial t} = -\nu_0 \times \mu_0\varepsilon_0$$
$$= -\nu_0 \times \frac{1}{{\nu_0}^2}$$
$$\mu_0\varepsilon_0 \frac{\partial v}{\partial t} = \frac{-1}{\nu_0} = \nabla.A$$
$$\nabla.A = -\mu_0\varepsilon_0 \frac{\partial v}{\partial t}$$

Problems in pointing theorem & pointing vector

1. In free space $E(z,t) = 60\cos(\omega t - \beta z)\bar{a}_x v/m$**. Find the average power crossing a circular area of radius 4m in the plane z = const**

Solution:-

$$E = 60\cos(\omega t - \beta z)\bar{a}_x = 60 e^{j(\omega t - \beta z)}\bar{a}_x v/m$$
$$\eta_0 = \frac{E}{H} \Rightarrow H = \frac{1}{\eta_0}\bar{a}_k \times E = \frac{60}{120\pi}(a_z) \times e^{j(\omega t - \beta z)}\bar{a}_x$$
$$H = \frac{1}{2\pi} e^{j(\omega t - \beta z)}\bar{a}_y A/m$$

Power density $= \frac{1}{2} R_e \left(\bar{E} \times H^*\right);\ = \frac{1}{2}(60)\left(\frac{1}{2\pi}\right)\bar{a}_z = \frac{15}{\pi} a_z W/m^2$

$$p_{avg} = \left(\frac{15}{\pi}\right) \times \pi(4)^2 = 240W$$

2. When a plane ware trends in free space, it has an average power density of 40W/m2. Calculate $\bar{E}$ & $\bar{B}$

Solution:-

The average power density

$$\left|S_{avg}\right| = \frac{|E|^2}{2\eta_0}$$
$$40 = \frac{|E|^2}{2 \times 120\pi} \Rightarrow E^2 = 80 \times 120\pi$$

$$E^2 = 9600\pi$$

$$E = \sqrt{9600\pi};\quad |E| = 173.62 v/m$$
$$B = \mu_0 H = \mu_0 \frac{|E|}{\eta_0} = \frac{4\pi \times 10^{-7} \times 173.62}{120\pi}$$
$$B = 0.58 \mu Wb/m^2$$

3. A forward travelling plane wave in free space is $E_x = \cos(4\pi \times 10^7 t - \beta z) v/m$ **. Calculate the instantaneous and time average pointing vector**

Solution:-

Given $E_x = \cos(4\pi \times 10^7 t - \beta z) v/m$

$$H_y = \frac{E_x}{\eta_0} = \frac{1}{120\pi}\cos(4\pi\times10^7 t - \beta z)\,A/m$$

Here $\omega = 4\pi\times10^7\,rad/s \qquad \beta = \frac{\omega}{c} = \frac{4\pi\times10^7}{3\times10^8} = 0.42\,rad/m$

$$p = E\times H = E_x\bar{a}_x \times H_y\bar{a}_y = E_x\bar{a}_x \times \frac{E_x}{\eta_0}\bar{a}_y$$
$$= \frac{1}{120\pi}\cos^2(4\pi\times10^7 t - 0.42z)\bar{a}_z$$
$$P = \frac{1}{2}\left[\frac{1}{120\pi}(1+\cos 2(4\pi\times10^7 - 0.42z))\right]$$
$$P_{avg} = \frac{1}{2}\left(\frac{1}{120\pi}\right)\bar{a}_z = 1.326\bar{a}_z\,mW/m^2$$

4. In free space, $\bar{E} = 100\sin(\omega t - \beta z)\bar{a}_x\ v/m$. Calculate the total power passing through a rectangular area of sides 30mm $\times$ 10mm is z = 0 plane. Assume $\eta_0 = \frac{E_m}{H_m}\ \&\ \eta_0 = 120\pi$

Solution:-

$$\bar{E} = 100(\sin\omega t - \beta z)\bar{a}_x v/m$$
$$\bar{H} = \frac{100}{\eta_0}\sin(\omega t - \beta z)\bar{a}_y A/m$$
$$\bar{E} = 100\cos(\omega t - \beta z - \pi/2)\bar{a}_x = 100e^{j(-\beta z - \pi/2)}\bar{a}_x$$
$$\bar{H} = \frac{100}{\eta_0}\cos(\omega t - \beta z - \pi/2)\bar{a}_y = \frac{100}{\eta_0}e^{j(-\beta z - \pi/2)}\bar{a}_y$$
$$H^* = \frac{100}{\eta_0}e^{j(+\beta z + \pi/2)}\bar{a}_y$$
$$P_{avg} = \frac{1}{2}R_e\left(E\times H^*\right)$$
$$= \frac{1}{2}(100)\left(\frac{100}{\eta_0}\right)R_e\left[e^{j(-\beta z - \pi/2)}\bar{a}_x \times e^{j(\beta z + \pi/2)}\bar{a}_y\right]$$
$$= \frac{1}{2}\frac{(100)^2}{120\pi}\bar{a}_z = 13.27a_z\,W/m^2$$

Total power = Average power $\times$ Area

$$= 13.27\bar{a}_z \times 10\times10^{-3}\times 30\times10^{-3}$$

P = 3.98 mW

5. If the field vector of a wave in free space an given by

$$\bar{E} = 50\cos(\omega t + \frac{4\pi}{3}x)\bar{a}_z v/m$$
$$\bar{H} = \cos\ (\omega t + \frac{4\pi}{3}x)\bar{a}_y Nm$$

Determine the pointing vector & calculate power crossing 10m^2 plate of the yz plane.

Solution:-

$$\bar{E} = 50 e^{j\left(\frac{4\pi}{3}\right)x} \bar{a}_z$$

$$\bar{H} = \frac{50}{120\pi} e^{j\left(\frac{4\pi}{3}\right)x} \bar{a}_y \quad \bar{H}^* = \frac{50}{120\pi} e^{j\left(\frac{4\pi}{3}\right)x} \bar{a}_y$$

$$P_{avg} = \frac{1}{2} R_e\left[E \times H^*\right] = \frac{1}{2}\frac{50 \times 50}{120\pi}\left[e^{j\left(\frac{4\pi}{3}\right)x} \bar{a}_z \times e^{j\left(\frac{4\pi}{3}\right)x} \bar{a}_y\right]$$

$$= \frac{(50)^2}{240\pi}(-\bar{a}_x) = -3.31\bar{a}_x W/m^2$$

Total power **=** $\left|P_{avg}\right| \times \text{Area} = 3.31 \times 10 = 33.1W$

6. A uniform plane wave with 10MHz frequency has average pointing vector 4W/m^2. If the medium is perfect dielectric with $\mu_r = 2,\ \varepsilon_r = 3$. Determine (i) Velocity (ii) wavelength (iii) intrinsic impedance (iv) r.m.s value of E

Solution:- $P_{avg} = 4W/m^2 \quad f = 10 \times 10^6 Hz$

For a perfect dielectric $\sigma = 0, \mu_r = 2,\ \varepsilon_r = 3$

(i)Alternation constant & phase constant are

$$\alpha = 0$$

$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}$$

$$= (2\pi \times 10 \times 10^6)\sqrt{4\pi \times 10^{-7} \times 2 \times \frac{1}{36\pi \times 10^9} \times 3}$$

$$= 0.5133 rad/m$$

(ii) $\nu_p = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \frac{2\pi \times 10 \times 10^6}{0.5133} = 122.4 \times 10^6 \ m/s$

(iii) $\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\mu_r}{\varepsilon_r}} = 120\pi\sqrt{\frac{2}{3}} = 307.6\Omega$

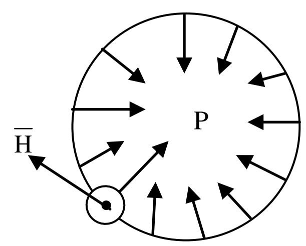
(iv) $P_{avg} = \frac{1}{2}\frac{E_m^2}{\eta}$

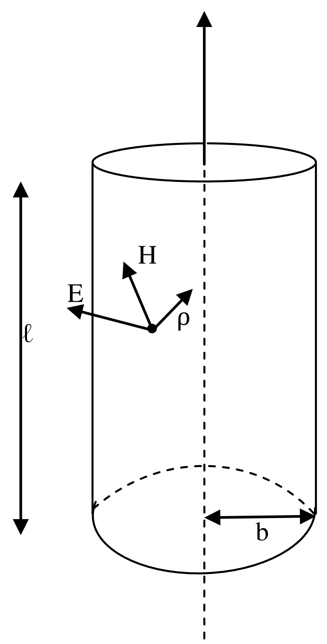
$$E_m = \sqrt{2\eta(p_{avg})} = \sqrt{2 \times 307.6 \times 4} = 49.6 v/m$$

$$E_{rms} = \frac{E_m}{\sqrt{2}} = \frac{49.6}{\sqrt{2}} = 35.07 v/m$$

7. Find the pointing vector on the surface of along straight conducting wire of radius b & conductivity σ that carries a direct current of I verify poynting's theorem.

Solution:-





The axis of the wire coinides with the z – axis

$$J = \frac{I}{A}\bar{a}_z = \frac{I}{\pi b^2}\bar{a}_z$$

$$E = \frac{\bar{J}}{\sigma} = \frac{I}{\sigma \pi b^2}\bar{a}_z$$

$$H = \frac{I}{2\pi\rho}\bar{a}_\phi = \frac{I}{2\pi b}\bar{a}_\phi$$

$$p = \bar{E} \times \bar{H} = \frac{I}{\sigma\pi b^2}\bar{a}_z \times \frac{I}{2\pi b}\bar{a}_\phi = \frac{-I^2}{20\pi^2 b^3}\bar{a}_\rho$$

$$-\oint_s \bar{p}.ds = -\oint_s \left(\bar{p} \,.\, \bar{a}_\rho\right)\left(ds.\ a_\rho\right)$$

$$= -\oint_s \left(\frac{I^2}{20\pi^2 b^3}\right)(2\pi b\ell)$$

$$-\oint_s \bar{p}.ds = I^2\left(\frac{\ell}{\sigma\pi b^2}\right) = I^2\left(\frac{\ell}{\sigma A}\right) = I^2 R$$

The above result shows that the negative surface integral of the pointing vector is exactly equal to I^2 R power loss in the conducting.

UNIT V

TIME VARYING FIELDS AND MAXWELLS EQUATIONS

PART- A

1. Write down the wave equations for E and H is free space (non – dissipative) medium.

$$\nabla^2 E - \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} = 0$$
$$\nabla^2 H - \mu_0 \varepsilon_0 \frac{\partial^2 H}{\partial t^2} = 0$$

2. Write down the wave equations for E and H is a conducting medium.

$$\nabla^2 E - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} - \mu\sigma \frac{\partial E}{\partial t} = 0$$
$$\nabla^2 H - \mu\varepsilon \frac{\partial^2 H}{\partial t^2} - \mu\sigma \frac{\partial H}{\partial t} = 0$$

3. Define a Wave.

If a physical phenomenon that occurs at one place at a given time is reproduced at other places at later times, the time delay being proportional to the space of phenomenon constitutes a wave.

4. Mention the properties of uniform plane wave.

(i) At every point in space, we electric field E and magnetic filed H are perpendicular to each other and to the direction of the travel.
(ii) The fields vary harmonically with time and at the same frequency everywhere in space.
(iii) Each field has the same direction magnitude and phase at every point in any place perpendicular to tthe direction of wave – travel.

5. Define skin depth (or) depth of penetration.

Skip depth (or) depth of penetration (δ) is defined as that of depth in which the wave has been attenuated to $\frac{1}{e}$ or approximately 37% of its original value. $\delta = \frac{1}{\infty} = \sqrt{\frac{2}{j\omega\sigma}}$ for good conductor

6. Define polarization:

Polarization of a uniform plane wave refers to the some fixed point in space

7. Define Linear polarization.

If X and Y component of electric field Ex and Ey are present and are in phase, the resultant electric field has a direction at an angle of $\tan^{-1}\frac{E_y}{E_x}$ and if this angle is constant with time the wave is said to be linearly polarized.

8. Define circular polarization.

If X and Y component of electric field Ex and Ey have equal amplitude and 90°. Phase difference, the low of the resultant electric field E is a circle and the wave is said to be circularly polarized.

9. Define Elliptical polarization.

If X and y component of electric field Ex and Ey have different amplitude and 90° phase difference, the lows of the resultant electric field E is a ellezise and the wave is said to be elliptically polarized.

10. State Snell's law.

When a wave is travelling from one medium 1 to another medium, the angle of incidence is related to angle of reflection.

$$\frac{\sin°\phi_i}{\sin°\phi_t} = \sqrt{\frac{\eta_1}{\eta_2}} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \qquad (\mu_1 = \mu_2 = \mu_0)$$

Where,

ϕ_i is angle of incidence

ε_2 is dielectric constant of medium 2.

ε_1 is dielectric constant of medium 1.

11. What is Brewster angle:

Brewster angle is incident angle at which there is no reflected wave for parallel polarized wave.

$$\theta = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

Where,

ε_1 is dielectric constant of medium 1

ε_2 is dielectric constant of medium 2.

12. State pointing theorem

The theorem states that the vectors product of electric field intensity $\overline{E}$ and the magnetic field intensity $\overline{H}$ at any point is a measure of rate of energy flow per unit area at that point. The direction of the power floe is perpendicular to the direction of the E and direction H and its is given by the vectors $\overline{E} \times \overline{H}$.

13. Write the Maxwell IV eqn integral form & complex form.

Maxwell I eqn:-

Integral form: $\int_\ell H.\ dl = I = \oiint_s \left(J + \frac{\partial D}{\partial t}\right) ds$

Point form: $\nabla \times H = J + \frac{\partial D}{\partial t} = \sigma E + E\frac{\partial D}{\partial t}$

Complex form: $\nabla \times H = \sigma E + j\omega E\varepsilon$

Maxwell II eqn:-

Integral form= $\int_\ell E_e\ dl = -\frac{d\phi}{dt} = -\oiint_s \left(\frac{\partial B}{\partial t}\right) ds$

Point form= $\nabla \times E = -\frac{\partial B}{\partial t} = -\mu \frac{\partial H}{\partial t}$

Complex form= $\nabla \times E = -j\omega\mu H$

Maxwell III eqn:-

Integral form = $\oiint_s \overline{D}.n\ ds = Q = \iiint_v \rho_v dv.$

Point form = $\nabla.D = \rho$

Maxwell IV eqn:-

Integral form = $\oiint_s \overline{B}.n.\ ds = 0$

Point form = $\nabla.B = 0$

14. Find the force on the charged particle of mass 1.7×10^{-27} kg and charge 1.602×10^{-19}c, if it enters a field of flux density B = 10 mw b/m^2 with an initial velocity of 90 km/s.

Solution:-

B = 10 m w b/m^2

Q = 1.602×10^{-19}C

v = 90 km/s

Assume v & B are perpendicular

$$\begin{aligned} F &= QvB \\ &= 1.602\times10^{-19}\times90\times10^{3}\times10\times10^{-3} \\ &= 1.602\times10^{-19}\times9\times10^{4}\times1\times10^{-2} \end{aligned}$$

F = 14.418 $\times$ 10^{-17} N

15. A point charge of 4c moves a velocity of $5\overline{a}_x + 6\overline{a}_y - 7\overline{a}_z$ m/s. Find the force exerted if the flux density is $5a_x + 7a_y + 9a_z Wb/m^2$.

Solution:-

$$Q = 4c \quad \vec{B} = 5\overline{a}_x + 7\overline{a}_y + 9\overline{a}_z \ \&\ v = 5\overline{a}_x + 6\overline{a}_y - 7\overline{a}_z$$

$$\overline{F}_m = Q(v\times B)$$

$$v\times B = \begin{vmatrix} a_x & a_y & a_z \\ 5 & 6 & -7 \\ 5 & 7 & 9 \end{vmatrix}$$

$$= \bar{a}_x[54+49] - \bar{a}_y[45+35] + \bar{a}_z[35-30]$$
$$= 103\bar{a}_x - 80\bar{a}_y + 5\bar{a}_z$$
$$F_m = Q(v \times B) = 4(103\bar{a}_x - 80\bar{a}_y + 5\bar{a}_z)$$
$$F_m = 412\bar{a}_x - 320\bar{a}_y + 20\bar{a}_z \text{ N.}$$
$$|F_m| = \sqrt{(412)^2 + (-320)^2 + (20)^2}$$
$$|F_m| = 522.05\,\text{N}$$

16. If the magnetic field intensity is $\vec{H} = \left[(0.01)/\mu_0 \bar{a}_x\right] A/m$. What is the force on a charge of 1 pc moving with a velocity of $10^6 \vec{a}_y\, m/sv$

Solution:-

Given $\quad \vec{H} = \left(\dfrac{0.01}{\mu_0}\right)\bar{a}_x \; A/m \quad Q = 1pc = 10^{-12}C \;\; \& \; v = 10^6 \bar{a}_y\, m/s$

$$F_m = Qv \times B$$
$$B = \mu_0 H = \mu_0 \left(\frac{0.01}{\mu_0}\right)\bar{a}_x$$
$$\bar{B} = 0.01\bar{a}_x$$

$$v \times B = \begin{vmatrix} a_x & a_y & a_z \\ 0 & 10 & 0 \\ 0.01 & 0 & 0 \end{vmatrix} = \vec{a}_x(10^6 \times 0 - 0 \times 0) - \vec{a}_y(0-0) + \vec{a}_z(0 \times 0 - 10^6(0.01)$$
$$= -0.01 \times 10^6 \bar{a}_z = -1 \times 10^{-2} \times 10^6 = -10^4 \bar{a}_z$$
$$F_m = Q(\vec{v} \times \vec{B}) = 10^{-12} \times (-10^4) = -10^{-8} \bar{a}_z \text{ N}$$

Force on differential current element

17. A magnetic field of flux density $B = 4.5 \times 10^{-2} \bar{a}_z Wb/m^2$ exerts a force on a 0.4m long conductor along x – axis. If a current of 10A flows in $\bar{a}_x$ direction, determine the force that must be applied to hold conductor in position.

Solution:-

Given I = 10A, $\ell = -0.4\bar{a}_x$ & $B = 4.5 \times 10^{-2}\, \bar{a}_z\; Wb/m^2$

The force exerted on a straight conductor is

$$\vec{F} = I\bar{\ell} \times \bar{B}$$
$$= 10(-0.4\bar{a}_x) \times (4.5 \times 10^{-2} \bar{a}_z)$$
$$= -4 \times 4.5 \times (-a_y)$$
$$F = 18\bar{a}_y N$$

18. Calculate the force on a straight conductor of length 30 cm carrying of 5a in a magnetic field along the z – axis. The magnetic flux density is $\bar{B} = 3.5 \times 10^{-3}(\bar{a}_x - \bar{a}_y) Wb/m^2$, where $\bar{a}_x$ and $\bar{a}_y$ are unit vector.

Solution:-

Given $\ell = 30\text{cm} = 30\times10^{-2}\text{m} = 0.3\bar{a}_z, I = 5A, B = 3.5\times10^{-3}(a_x - \bar{a}_y)$

$$F = I\ell \times B = 5\times0.3\bar{a}_z \times 3.5\times10^{-3}(a_x - a_y)$$
$$= 1.5\times3.5\times10^{-3}\times a_z \times (a_x - a_y)$$
$$F = 5.25\bar{a}_x + 5.25\bar{a}_y \text{mN}$$
$$= 5.25\sqrt{1+1}\times10^{-3}$$
$$F = 7.42\text{mN}$$

19. Consider two long parallel wires 2m apart carry current of 50 A and 100 A in the same direction. Determine the magnitude and direction of force between then / unit length.

Solution:-

$$I_1 = 50A \quad I_2 = 100A \qquad d = 2m$$
$$\frac{\vec{F}}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi d} = \frac{4\pi\times10^{-7}\times50\times100}{2\pi\times2}$$
$$= \frac{2\times5\times10^{-4}}{2} = 5\times10^{-4}$$
$$\frac{\vec{F}}{\ell} = 0.5\text{mN/m}\ \bar{a}_n$$

20. Consider two long parallel conductor carry 80A. If they are separated by 3mm, find the force / m of each conductor if the current flowing through them in opposite direction

Solution:-

Given $I_1 = I_2 = 80a \qquad d = 3mm = 3\times10^{-3}$

$$\frac{F}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi d} = \frac{4\pi\times10^{-7}\times80\times80}{2\pi\times3\times10^{-3}} = \frac{2\times\cancel{64}(21.11)\times10^{-5}}{\cancel{3}}\times10^{3}$$
$$= 42.22\times10^{-2} = 0.42\text{N/m}$$

As per currents are in opposite direction, the two conductors will repel with equal force.

21. A conductor of length 100 cm mores at right angles to a uniform field of strength 10000 lines/ Cm² with a velocity of 50m/s. Determine the induced EMF when the conductor mores at an angle of 30° the direction of the field.

Solution:-

Given $\ell = 100\text{cm} = 100\times10^{-2}\text{m},\ B = 10000\text{lines}C/m^2 = 1\text{Wb}/m^2$

$$v = 50\text{m/s} \,\&\, \theta = 30°$$
$$\text{EMF} = \int_\ell (v\times B).dl$$
$$= B\ell v\sin\theta = 1\times1\times50\times\sin30°$$

EMF = 25V

PART- B

1) Sate and prove the Faraday's law of electromagnetic inductor?

Faraday's law:-

The total electromotive force (e. m. f) induced in a circuit is equal to the circuit

$$\varepsilon = \frac{-d\phi}{dt}$$

Where

ε = E. M. F induced in the circuit (volts)

ϕ = total flux (klebers)

t = time in seconds

if the circuit refers to a multiple – turn loop with, say, N turns

$$\varepsilon = -N\frac{d\phi}{dt}$$
$$\varepsilon = \frac{-d\Lambda}{dt}$$

Where $\Lambda = N\phi$

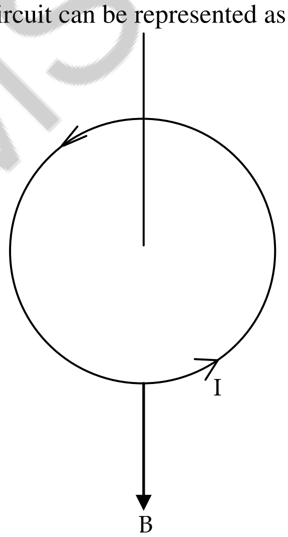
= total flux linkage (Wb – turns)

Consider a closed circuit made of a single turn loop as shown in fig (1). If the field is normal to the plane of the loop & increasing, an e. M. F is induced in the circuit and current flows in the circuit.

The induced current is always so directed as to produce a flux opposing the charge in the magnetic field (B/H)

We know that the e. m. f in a circuit can be represented as the line integral of the electric field around the closed path.

$$\varepsilon = \oint E_e.dl$$



Where E_e is an E.M.F producing electric field caused by the varying magnetic field.

Now the total flux through a circuit is equal to the integral of the normal component of the flux density B over the surface bonded by the circuit. Total flux is, therefore given by

$$\phi = \iint_s B.n\ ds$$

$$\varepsilon = \frac{-d\phi}{dt} = \frac{-d}{dt}\iint_s B.n\ ds$$

$$\varepsilon = \oint_\ell E_e.\ dl = -\iint \frac{\partial B}{\partial t}.\ n\ ds$$

$$\oint_\ell E_e.dl = \iint_s curl\ \ E_e.n\ ds$$

$$\iint_s curl\ E_e.n\ ds = -\iint \frac{\partial B}{\partial t}.n\ ds$$

$$curlE_e = \frac{-\partial B}{\partial t}\ \ for\ \ stationary\ circuits$$

This relation is referred to Maxwell's equation in differential form.

$$\varepsilon = \oint_\ell E_e.dl = -\iint_s \frac{\partial B}{\partial t}.n\ ds$$

This is known as transformer E.M.F.

Let us consider a charge dQ moving with a velocity V in a field with flux density b is given by

$$dF = dQ\vartheta \times B$$

$$E_e = \frac{dF}{dQ} = \vartheta \times B$$

If the charge – element is isolated in a conductor, the EMF induced is a netional EMF & the equation isd given by

$$\varepsilon = \int E_e.dl = \int (\vartheta \times B).\ dl$$

2) Explain in detail the Maxwell's II equation in integral & point form

Maxwell's II equation in integral form from Faraday's law:-

$$\oint_\ell E_e.dl = \iint_s (\nabla \times E).\ ds$$

$$\iint_s (\nabla \times E_e).ds = -\iint_s \frac{\partial B}{\partial t}.n\ ds$$

$$\boxed{\nabla \times E_e = -\frac{\partial B}{\partial t}}$$

Conduction current:-

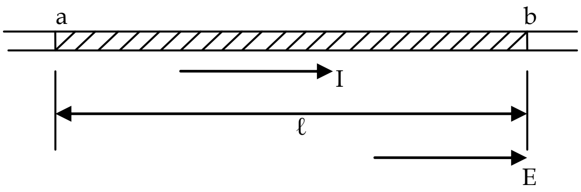
Let I be the current in the conduction of area A. When we say that a current in flowing from a to b, it means that the potential of a is higher than b.

Work is done when charge is carried from the point b which is at a lower potential to the point a which is at a higher potential, against a field E. If Vba is the potential difference between the two points then

$$V_{ba} = E\ell = IR$$

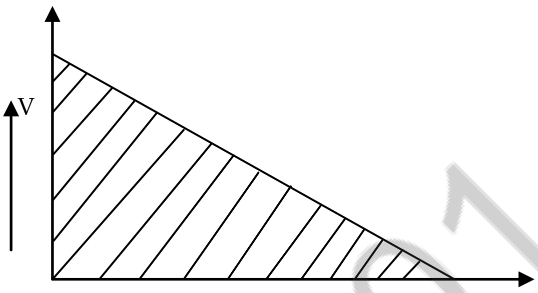
Where R = resistance between a and b

$$= \frac{\ell}{\sigma A}$$



Where σ = conductivity

$$E\ell = I\left(\frac{\ell}{\sigma A}\right)$$
$$= \frac{I}{A} \cdot \frac{1}{\sigma}$$
$$\boxed{E = \frac{J_1}{\sigma}}$$



Where J1 may be referred to as conduction current density which is directly proportional to E.

Displacement current:-

In the case of a capacitor, the current flow can be constant only when the voltage is changing rather than steady.

If Ic is the capacitor current, it is given by

$$I_C = \frac{dQ}{dt} = C\frac{dv}{dt}$$
$$C = \frac{\varepsilon A}{t'}$$
$$I_C = \frac{\varepsilon A}{t'}\frac{dv}{dt}$$
$$= \frac{\varepsilon A t'}{t'}\frac{dE}{dt}$$
$$I_C = \varepsilon A\frac{dE}{dt}$$
$$\frac{I_C}{A} = \text{currrent density} = J_2$$
$$J_2 = \frac{d\vec{D}}{dt}$$
$$\text{Therefore } J = J_1 + J_2 = \sigma E + \frac{d\overline{D}}{dt}$$

3) Discuss about the inconsistency of ACL & derive modes of it

Inconsistency of Ampere's circuital law:-

We know that the equation of continuity,

$$\nabla . J = \frac{-\partial \rho_v}{\partial t}$$

We also know that the Ampere's circuital law,

$$\oint_{\ell} H.dl = I = \iint_{s} J.ds$$

$$\nabla \times H = J \qquad \rightarrow (1)$$

Now talking divergence on both the sides,

$$\nabla.(\nabla \times H) = \nabla.J = 0 \quad \rightarrow (2)$$

But we know $\nabla.J = \frac{-\partial}{\partial t} \qquad \rightarrow (3)$

Equation (2) & (3) is contracting. Thus Ampere's law is inconsistency.

Modified form of Ampere's circuital law:-

$$\oint_{\ell} H.dl = I = \iint_{S} \left(J + \frac{\partial D}{\partial t} \right). \, ds \quad \rightarrow (1)$$

Applying stokes theorem,

$$\oint_{\ell} H.dl = \iint_{s} (\nabla \times H).ds \qquad \rightarrow (2)$$

Equating (1) & (2), we get

$$\iint_{s} (\nabla \times H).ds = \iint_{S} \left(J + \frac{\partial D}{\partial t} \right) ds \quad \text{Integral form of Maxwell I equation}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} \qquad \text{point form of Maxwell's I equation}$$

4) Discuss the Maxwell's Four equation in integral form, point or differentiation form:-

Equation I:- From Ampere's circuital law.

The line integral of the magnetic field intensity around a closed path (contour) is equal to the current enclosed by the path

$$\int_{\ell} H.dl = I = \iint_{s} \left(J + \frac{\partial D}{\partial t} \right). \, ds \quad \text{Integral form}$$

Applying Stoke's theorem,

$$\int_{\ell} H.dl = \iint_{S} (\nabla \times H).ds$$

$$\iint_{S} (\nabla \times H) ds = \iint_{S} \left(J + \frac{\partial D}{\partial t} \right) ds$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} = \sigma E + \frac{\partial(\varepsilon E)}{\partial t}$$

$$\boxed{\nabla \times H = \sigma E + \frac{\varepsilon \partial E}{\partial t}} \quad \text{Point form Maxwell's I equation.}$$

Maxwell's II equation:- (Faraday's law electromagnetic induction)

The total electro motive force induced in a circuit is equal to the time rate of decrease of total magnitude flux linking in the circuit.

$$\varepsilon = \frac{-d\phi}{dt} \quad \phi = \iint_s B.nds$$

$$\varepsilon = \frac{-d}{dt}\iint_s B.n\ ds$$

Integral form of Maxwell's II equation $\quad \varepsilon = \int_\ell E_e.dl = -\iint_s \frac{\partial B}{\partial t}.\ nds \ \rightarrow (1)$

Maxwell's II equation in integral form applying stoke's theorem on the LHS,

$$\int_\ell E_e dl = \oiint_s (\nabla \times E).ds \qquad \rightarrow (2)$$

$$\oiint_s (\nabla \times E).ds = -\iint_S \left(\frac{\partial B}{\partial t}\right).\ nds$$

$\boxed{\nabla \times E = \frac{-\partial B}{\partial t}}$ Point form of Maxwell's II equation

Maxwell's III equation: (From Gauss law) for electric field

The surface integral of the electric flux density vector $\overline{D}$ is equal to the charge enclosed

$\oiint_S \overline{D}.n\ ds = Q \rightarrow$ Integral form of Maxwell's equation

$$\oiint_S \overline{D}.n\ ds = Q = \iiint_v \rho\ dv$$

Applying divergence theorem,

$$\oiint_S \overline{D}.n\ ds = \iiint_V \nabla.D\ dv = \iiint_V \rho\ dv$$

$\boxed{\nabla.D = \rho}$ Point form of Maxwell's equation

Maxwell's IV equation:- (From gauss's law for magnetic field)

The surface integral of the normal component of magnetic flux density vector $\overline{B}$ is equal to the zero.

$$\oiint_S B.\ n\ \ ds = 0$$

$\oiint_S B.\ n\ \ ds = \iiint_V (\nabla.B)$ Integral form Maxwell's equation

$\nabla.B = 0$ Point form of Maxwell's equation IV

Maxwell's Four equations summery

Equation	Law from which the equation is derived	Integral form	Point form	Complex form

number				
I	Ampere's circuital law	$\oint_{\ell} H.dl = I = \iint_{s}\left(J + \frac{\partial D}{\partial t}\right).ds$	$\nabla \times H = J + \frac{\partial D}{\partial t}$	$\nabla \times H = \sigma E + j\omega\mu$
II	Faraday's law of electromagnetic induction	$\oint_{\ell} E_e.dl = \iint_{s} \frac{-\partial B}{\partial t}.ds$	$\nabla \times E = \frac{-\partial B}{\partial t}$	$\nabla \times E = -j\omega\mu$
III	Gauss's law for electric field	$\oiint_{S} \overline{D}.n\ ds = Q$	$\nabla.D = \rho$	$\nabla.D = \rho$
IV	Gauss's law for magnetic field	$\oiint_{S} \overline{B}.n\ ds = 0$	$\nabla.B = 0$	$\nabla.B = 0$

5) Derive the equation of electromagnetic power poynting theorem and poynting vector:-

The theorem states that the vector product of the electric field intensity vector (E) and the magnetic field intensity vector (H) is equal to the measure of the rate of energy flow per unit area at that point. The direction of power flow is perpendicular to e and H in the direction of the vector E× H.

$$P = E \times H VA/m^2 \text{ or } Watts/m^2$$

Let us consider Maxwell's second equation

$$\nabla \times H = J + \frac{\partial D}{\partial t} \rightarrow (1)$$

$$J = \nabla \times H - \frac{\partial D}{\partial t} \rightarrow (2)$$

Multiply the above equation with E,

$$E.J = E.(\nabla \times H) - E.\frac{\partial D}{\partial t}$$

By identity, $\nabla.(E \times H) = H(\nabla \times E) - E(\nabla \times H)$

$$E.J = -\nabla(E \times H) + H(\nabla \times E) - E\frac{\partial D}{\partial t}$$

$$= -\nabla(E \times H) - H\frac{\partial B}{\partial t} - \varepsilon E\frac{\partial E}{\partial t}$$

$$E.J = -\nabla(E \times H) - \mu H\frac{\partial H}{\partial t} - \varepsilon E\frac{\partial E}{\partial t}$$

$$\mu H\frac{\partial H}{\partial t} = \frac{1}{2}\frac{\mu\partial H^2}{\partial t^2} \quad \varepsilon E\frac{\partial E}{\partial t} = \frac{1}{2}\frac{\varepsilon\partial E^2}{\partial t^2}$$

$$E.J = -\nabla(E \times H)\frac{-\partial}{\partial t}\left(\frac{\mu H^2}{2} + \frac{\varepsilon E^2}{2}\right)$$

$$-\nabla(E \times H) = E.J + \frac{-\partial}{\partial t}\left(\frac{\mu H^2}{2} + \frac{\varepsilon E^2}{2}\right)$$

Integrating the above equation throughout the volume V,

$$-\iiint_{V} \nabla.(E \times H)dv = \iiint_{V} E.Jdv + \iiint_{V} \frac{\partial}{\partial t}\left(\frac{\mu H^2}{2} + \frac{\varepsilon E^2}{2}\right)dv$$

Applying divergence theorem to the left hand side,

$$-\oint_S (E\times H).ds = \oint_V (E.J)dv + \oint_V \frac{\partial}{\partial t}\left(\frac{\mu H^2}{2}+\frac{\varepsilon E^2}{2}\right)dv$$

I term II term III term

$\oint_S (E\times H).ds =$ I term = In going power flux in the surface S.

$\oint_V (E.J)dv =$ II term = Total power generated or dissipated within volume v at any instant

$\frac{\partial}{\partial t}\left(\frac{1}{2}\mu H^2+\frac{1}{2}\varepsilon E^2\right)dv$ = III term = The time rate of increase of total electromagnetic energy within the volume

Thus the total power flowing into the volume V is equal to the total power flowing out of the volume V.

$$-\oint_S P.ds = \oint_V (E.J)dv + \int_X \frac{\partial}{\partial t}\left(\frac{\mu H^2}{2}+\frac{\varepsilon E^2}{2}\right)dv$$

6) Explain the instantaneous, Average & complex power instantaneous, Average and complex poynting vector:-

The instantaneous power Pinst is always given by the product of the instantaneous voltage Vinst and instantaneous current Iinst. i.e,

$$\begin{aligned}
P_{inst} &= V_{inst}.I_{inst} \\
V_{inst} &= R_e\left\{Ve^{j\omega t}\right\} \\
&= R_e\left\{|V|e^{j\theta v}.e^{j\omega t}\right\} \\
&= R_e\left\{|V|e^{j(\omega t+\theta v)}\right\} \\
&= R_e\left\{|V|\cos(\omega t+\theta v)+j\sin(\omega t+\theta v)\right\}
\end{aligned}$$

$$\boxed{V_{inst} = V_0\cos(\omega t+\theta v)}$$

And

$$\begin{aligned}
I_{inst} &= R_e\left\{Ie^{j\omega t}\right\} \\
&= R_e\left\{|I_0|.e^{j\theta i}.e^{j\omega t}\right\} \\
&= R_e\left\{|I|e^{j(\omega t+\theta i)}\right\}
\end{aligned}$$

$$\boxed{I_{inst} = I_0\cos(\omega t+\theta i)}$$

$$\begin{aligned}
P_{inst} &= |V_0||I_0|\cos(\omega t+\theta v)\cos(\omega t+\theta i) \\
&= |V_0||I_0|\left\{\frac{1}{2}\left[\cos(\omega t+\theta v-\omega t+\theta i)+\cos(\omega t+\theta v+\omega t+\theta i)\right]\right\} \\
&= \frac{|V_0||I_0|}{2}\left[\cos(\theta v-\theta i)+\cos(2\omega t+\theta v+\theta i)\right] \\
P_{inst} &= \frac{|V_0||I_0|}{2}\left[\cos\theta+\cos(2\omega t+\theta v+\theta i)\right]
\end{aligned}$$

The above equation has an arrange part and an oscillating put

$$P_{av} = \frac{|V_0||I_0|}{2}\cos\theta$$

And also the reactive power

$$P_{react} = \frac{|V_0||I_0|}{2}\sin\theta$$

Since θ is the phase angle, P_{av} and P_{react} are the impulse and out of phase components of the volt ampere product.

Now let us consider the complex power P, defined as one – half the product of v and the complex conjugate of I.

$$\begin{aligned}
P &= \frac{1}{2} VI^* \\
&= \frac{1}{2}|V|e^{j\theta_v}.e^{j\omega t}.\,|I|e^{-j\theta_i}.e^{-j\omega t} \\
&= \frac{1}{2}|V||I|e^{j\theta_v}.e^{-j\theta_i} \\
&= \frac{1}{2}|V||I|e^{j(\theta_v-\theta_i)}
\end{aligned}$$

$$\boxed{P = \frac{1}{2}|V||I|e^{j\theta}}$$

$$P = \frac{1}{2}V_0 I_0 e^{j\theta}$$

$$\boxed{P = P_{av} + jP_{react}}$$

$$P_{inst} = I_{inst} + H_{inst}$$

$$\begin{aligned}
P_{av} &= \frac{1}{T}\int_0^T P\,dt = \frac{1}{2\pi}\int_0^{2\pi} P\,d(\omega t) \\
&= \frac{1}{2}E\times H(\cos\theta) + \frac{1}{2}E\times H\cos(2\omega t + \theta_x - \theta_i) \\
&= R_e\left\{\frac{1}{2}E\times He^{j(\theta_x-\theta_i)}\right\} \\
&= R_e\left\{\frac{1}{2}\left[E\times He^{j(\theta_x-\theta_i)}\right]\right\} \\
&= R_e\left[\frac{1}{2}\left\{E\times He^{j\theta_x}.e^{j\theta_i}\right\}\right] \\
&= R_e\left[\frac{1}{2}Ee^{j\theta_v}\times He^{-j\theta_i}\right] \\
P_{av} &= R_e\left[\frac{1}{2}E_{inst}\times H^*_{inst}\right]
\end{aligned}$$

Thus the complex pointing vector is given by

$$P_{inst} = P_{react} + jP_{react}$$

At time independent complex pointing vector such as its real part is equal to the time – average of the usual pointing vector associated with the electromagnetic field.

$$P = \frac{1}{2} E \times H^*$$

$$P_{av} = \frac{1}{2} R_e \left\{E \times H^*\right\} = \frac{1}{2} R_e \left\{E^* \times H\right\} W / m^2$$

$$P_{react} = \frac{1}{2} I_m \left\{E \times H^*\right\}$$

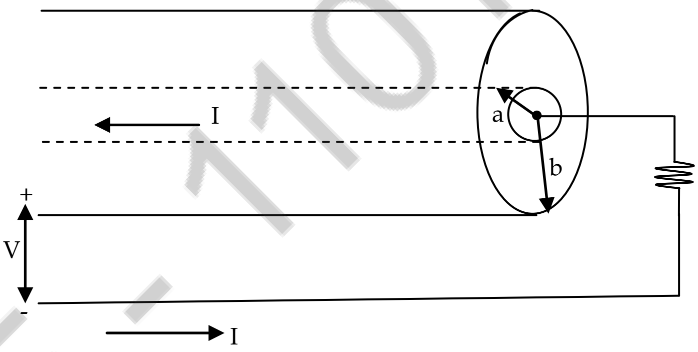
$$P_x = \frac{1}{2}\left(E_y H_z^* - E_z H_y^*\right)$$

$$P_r = \frac{1}{2}\left(E_\theta H_\phi^* - E_\phi H_\theta^*\right)$$

7) Discuss about power flow in a co- axial cable:-

Consider a co- axial cable with a voltage v applied between the conductors.

The radius of inner and outer conductor are ‘a’ & ‘b’



By ACL,

$$\oint H.dl = I$$

$$H.(2\pi r) = I$$

$$H = \frac{I}{2\pi r} \quad a < r < b$$

E due to an infinity long conductor

$$E = \frac{\rho_L}{2\pi\varepsilon_0 t} ; V = \frac{\rho_L}{2\pi\varepsilon} \ln\left(\frac{b}{a}\right)$$

$$E = \frac{V}{r \ln\left(\frac{b}{a}\right)}$$

$$P = E \times H$$

$$P = |E||H| \sin\theta$$

And we know $\theta = 90^\circ$ since e and H are perpendicular

$$P = EH$$

$$= \frac{v}{r \ln\left(\frac{b}{a}\right)} \cdot \frac{I}{2\pi r}$$

Total power $= \int P ds$

$$= \int_a^b \frac{v}{r \ln\left(\frac{b}{a}\right)} \cdot \frac{I}{2\pi r} \cdot (2\pi r)\, dr$$

$$= \frac{vI}{r \ln\left(\frac{b}{a}\right)} \cdot \int_a^b \frac{dr}{r}$$

Total power $= \frac{vI}{r \ln\left(\frac{b}{a}\right)} \ln\left(\frac{b}{a}\right)$

Total power = V I

Summary

1) $\varepsilon = \frac{-d\phi}{dt}$

2) $\int_{\ell} H.dl = I = \oiint_{s} J.ds$

$$\oint_{\ell} E.dl = -\oiint_{s} \frac{\partial B}{\partial t}.ds$$

$$\oiint_{s} \overline{D}.nds = Q = \iiint_{V} \rho_V dv$$

$$\oiint_{s} \overline{B}.n\,ds = 0$$

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$\nabla \times E = \frac{-\partial B}{\partial t}$$

$$\nabla.D = \rho$$

$$\nabla.B = 0$$

3) $P = E \times H \;\; Wwatts / m^2$

8) Discuss about the wave equation of perfect dielectric

Wave equation for a perfect dielectric:-

The Maxwell's equation for a perfect is as follows

$$\nabla \times H = \sigma E + \varepsilon \frac{\partial E}{\partial t}$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

$$\nabla.D = \rho$$

$$\nabla.B = 0$$

For a perfect dielectric, $\sigma = 0 \& \rho = 0$

$$\nabla \times H = \varepsilon \frac{\partial E}{\partial t} \qquad \rightarrow (1)$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t} \qquad \rightarrow (2)$$

$$\nabla.D = 0 \qquad \rightarrow (3)$$

$$\nabla.B = 0 \qquad \rightarrow (4)$$

Talking curl of (1) & differentiating (2), we get

$$\nabla \times \nabla \times H = \varepsilon \frac{\partial}{\partial t}(\nabla \times E) \Rightarrow \varepsilon \nabla \times \frac{\partial E}{\partial t} \qquad \rightarrow (5)$$

$$\nabla \times \frac{\partial E}{\partial t} = -\mu \frac{\partial^2 H}{\partial t^2} \qquad \rightarrow (6)$$

Sub (6) in (5), we get

$$\nabla\times\nabla\times H=-\mu\varepsilon\frac{\partial^2 H}{\partial t^2}$$

$$\nabla(\nabla.H)-\nabla^2 H=-\mu\varepsilon\frac{\partial^2 H}{\partial t^2}$$

$$-\nabla^2 H=-\mu\varepsilon\frac{\partial^2 H}{\partial t^2}$$

$$\nabla^2 H=\mu\varepsilon\frac{\partial^2 H}{\partial t^2} \qquad \rightarrow (7)$$

Similarly, talking curl of (2) & differentiating w. r. t

$$\nabla\times\nabla\times E=-\mu\frac{\partial}{\partial t}(\nabla\times H)$$

$$\nabla\times\nabla\times E=-\mu\left(\nabla\times\frac{\partial H}{\partial t}\right) \qquad \rightarrow (8)$$

$$\nabla\times\frac{\partial H}{\partial t}=\varepsilon\frac{\partial^2 E}{\partial t^2} \qquad \rightarrow (9)$$

Substituting (9) in (8),

$$\nabla\times\nabla\times E=-\mu\varepsilon\frac{\partial^2 E}{\partial t^2}$$

$$\nabla(\nabla.E)-\nabla^2 E=-\mu\varepsilon\frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 E=\mu\varepsilon\frac{\partial^2 E}{\partial t^2} \qquad \rightarrow (10)$$

Wave equation for a conducting medium

$$\nabla\times H=\sigma E+\varepsilon\frac{\partial E}{\partial t} \qquad \rightarrow (1)$$

$$\nabla\times E=-\mu\frac{\partial H}{\partial t} \qquad \rightarrow (2)$$

$$\nabla.D=0 \qquad \rightarrow (3)$$

$$\nabla.B=0 \qquad \rightarrow (4)$$

Talking curl of (1) & differentiating (2), we get

$$\nabla\times\nabla\times H=\sigma(\nabla\times E)+\varepsilon\left(\nabla\times\frac{\partial E}{\partial t}\right) \qquad \rightarrow (5)$$

$$\nabla\times\frac{\partial E}{\partial t}=-\mu\frac{\partial^2 H}{\partial t^2} \qquad \rightarrow (6)$$

Sub (2) & (6) in (5), we get

$$\nabla(\nabla.H) - \nabla^2 H = --\mu\sigma\frac{\partial H}{\partial t} - \mu\varepsilon\frac{\partial^2 H}{\partial t^2}$$
$$-\nabla^2 H = -\mu\sigma\frac{\partial H}{\partial t} - \mu\varepsilon\frac{\partial^2 H}{\partial t^2}$$
$$\boxed{\nabla^2 H = \mu\sigma\frac{\partial H}{\partial t} + \mu\varepsilon\frac{\partial^2 H}{\partial t^2}}$$

Similarly, talking curl of (2) & differentiating (1)

$$\nabla \times \nabla \times E = -\mu\nabla \times \frac{\partial H}{\partial t}$$
$$\nabla \times \frac{\partial H}{\partial t} = \sigma\frac{\partial E}{\partial t} + \varepsilon\frac{\partial^2 E}{\partial t^2}$$
$$\nabla(\nabla.E) - \nabla^2 E^2 = -\mu\sigma\frac{\partial E}{\partial t} - \mu\varepsilon\frac{\partial^2 E}{\partial t^2}$$
$$\boxed{\nabla^2 E = \mu\sigma\frac{\partial E}{\partial t} + \mu\varepsilon\frac{\partial^2 E}{\partial t^2}}$$

Wave equation:-

For perfect Dielectric:- $\nabla^2 E = \mu\varepsilon\frac{\partial^2 E}{\partial t^2}$

$$\nabla^2 H = \mu\varepsilon\frac{\partial^2 H}{\partial t^2}$$

For perfect conducting medium:- $\nabla^2 E = \mu\sigma\frac{\partial E}{\partial t} + \mu\varepsilon\frac{\partial^2 E}{\partial t^2}$

$$\nabla^2 H = \mu\sigma\frac{\partial H}{\partial t} + \mu\varepsilon\frac{\partial^2 H}{\partial t^2}$$

9) Explain about the wave equations in a wave equation in a phasor form:-

$$\nabla^2 E = \mu\varepsilon\frac{\partial^2 E}{\partial t^2}$$
$$\nabla^2 H = \mu\varepsilon\frac{\partial^2 H}{\partial t^2}$$

Let $E = E_0 e^{j\omega t}$

$$\frac{\partial E}{\partial t} = j\omega E_0 e^{j\omega t}$$
$$= j\omega E$$
$$\frac{\partial^2 E}{\partial t^2} = -\omega^2 E$$
$$\nabla^2 E = \mu\varepsilon(-\omega^2 E) \Rightarrow \nabla^2 E + \omega^2\mu\varepsilon E = 0$$
$$\nabla^2 H = \mu\varepsilon(-\omega^2 H) \Rightarrow \nabla^2 H + \omega^2\mu\varepsilon H = 0$$

Wave equation for a perfect dielectric

$$\boxed{\begin{array}{l}\nabla^2 E + \omega^2\mu\varepsilon E = 0\\ \nabla^2 H + \omega^2\mu\varepsilon H = 0\end{array}}$$

$$\nabla^2 E = \mu\sigma\frac{\partial E}{\partial t} + \mu\varepsilon\frac{\partial^2 E}{\partial t^2}$$
$$\nabla^2 H = \mu\sigma\frac{\partial H}{\partial t} + \mu\varepsilon\frac{\partial^2 H}{\partial t^2}$$
$$\nabla^2 E = j\omega\mu\sigma E + j^2\omega^2\mu\varepsilon E$$
$$\nabla^2 E = j\omega\mu E(\sigma + j\omega\varepsilon)$$
$$\nabla^2 H = j\omega\mu H(\sigma + j\omega\varepsilon)$$
$$\boxed{\begin{array}{l}\nabla^2 E - j\omega\mu(\sigma + j\omega\varepsilon)E = 0 \\ \nabla^2 H - j\omega\mu(\sigma + j\omega\varepsilon)H = 0\end{array}}$$

Velocity of the wave:-

$$\nabla^2 E = \mu\varepsilon\frac{\partial^2 E}{\partial t^2}$$

For free space $\mu_r = \varepsilon_r = 1$

$$\nabla^2 E = \mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2}$$

$$= 4\pi \times 10^{-7} \times \frac{1}{36\pi \times 10^9}$$
$$= \frac{1}{9 \times 10^{16}}\frac{\partial^2 E}{\partial t^2}$$
$$\nabla^2 E = \frac{1}{(3 \times 10^8)^2}\frac{\partial^2 E}{\partial t^2}$$

We know that the velocity of light = $= 3 \times 10^8 m/s$

$$\nabla^2 E = \frac{1}{\vartheta^2}\frac{\partial^2 E}{\partial t^2}$$
$$\Rightarrow \nabla^2 E - \frac{1}{\vartheta^2}\frac{\partial^2 E}{\partial t^2} = 0$$

Similarly $\nabla^2 H - \frac{1}{\vartheta^2}\frac{\partial^2 H}{\partial t^2} = 0$

Where

$$\frac{1}{\vartheta^2} = \mu_0\varepsilon_0 \Rightarrow \frac{1}{\mu_0\varepsilon_0} = \vartheta^2$$

$$\boxed{\vartheta = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \quad m/s}$$

10) Discuss about the uniform plane wave uniform plane wave:-

If the phase of a wave is the same for all points on a plane surface, it is called a plane wave.

If the amplitude is also constant, then the wave is called an uniform plane wave.

The following are properties of uniform plane waves.

1) At every point in space electric field (E) * magnetic field H are perpendicular to each other and to the direction of barel.

If the electric field is in x – direction and the magnetic field in y – direction, then the wave is travelling in z – direction.

The wave equation for free space is given by,

$$\nabla^2 E = \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = \mu E \frac{\partial^2 E}{\partial t^2}$$

Consider E varies along 'x' only & independent of y and z, then

$$\frac{\partial^2 E}{\partial x^2} = \mu E \frac{\partial^2 E}{\partial t^2} \qquad \left[\frac{\partial^2 E}{\partial y^2} = \frac{\partial^2 E}{\partial z^2} = 0\right]$$

$$\frac{\partial^2 E_x}{\partial x^2} = \mu E \frac{\partial^2 E_x}{\partial t^2}; \quad \frac{\partial^2 E_y}{\partial x^2} = \mu E \frac{\partial^2 E_y}{\partial t^2}; \quad \frac{\partial^2 E_z}{\partial x^2} = \mu E \frac{\partial^2 E_z}{\partial t^2}$$

$$\nabla.D = \varepsilon \nabla.E = 0 \qquad \nabla.E = 0$$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

For a uniform plane wave, 'E' is independent of y & z.

Then $\frac{\partial E_x}{\partial x} = 0$

Differentiating the above, $\frac{\partial^2 E_x}{\partial x^2} = 0$

If requires that either Ex be zero or constant therefore an uniform wave propagating along x – axis does not have an Ex component.

Similarly,

$$\nabla.B = \mu \nabla.H = 0$$
$$\nabla.H = 0$$
$$\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = 0$$
$$\frac{\partial H_x}{\partial x} = 0 \ \& \ \frac{\partial^2 H_x}{\partial x^2} = 0$$

Since Hx is constant, hx must be zero.

11) Explain the characteristics impedance

Characteristics impedance or intrinsic impedance:-

Consider the plane wave propagating in x – direction the wave equation for free space is

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

The general solution of the differential equation is in the form

$$E = f_1(x - v_0 t) + f_2(x + v_0 t)$$

Where

$$v_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

f_1 and f_2 are any function of $(x - v_0 t) \,\&\, (x + v_0 t)$ respectively.

The solution of wave equation consists of two waves; one travelling in positive direction and other travelling in the negative direction consider the wave travelling in positive direction above

$$E = f(x - v_0 t) \qquad \left[\because f_2(x + v_0 t) = 0\right]$$

$$\nabla \times E = \begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix}$$

Since wave travelling in x direction, E and H are both independent of y and z.

i.e, $\quad E_x = H_x = 0 \,\&\, \quad \frac{\partial E}{\partial y} = \frac{\partial E}{\partial z} = 0$

$$\nabla \times E = \begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & 0 & 0 \\ 0 & E_y & E_z \end{vmatrix}$$

$$\nabla \times E = \frac{-\partial E_z}{\partial x}\bar{a}_y + \frac{\partial E_y}{\partial x}\bar{a}_z \qquad \rightarrow (1)$$

Similarly

$$\nabla \times H = \frac{-\partial H_z}{\partial x}\bar{a}_y + \frac{\partial H_y}{\partial x}\bar{a}_z \qquad \rightarrow (2)$$

But $\quad \nabla \times H = \varepsilon \frac{\partial E}{\partial t} \qquad \rightarrow (3)$

Comparing (2) & (3), we get

$$\frac{-\partial H_z}{\partial x}\bar{a}_y + \frac{\partial H_y}{\partial x}\bar{a}_z = \varepsilon\left[\frac{\partial E_y}{\partial t}\bar{a}_y + \frac{\partial E_z}{\partial t}\bar{a}_z\right]$$

$$\frac{-\partial H_z}{\partial x} = \varepsilon \frac{\partial E_y}{\partial t} \,\&\, \frac{\partial H_y}{\partial x} = \frac{\partial E_z}{\partial t}$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

$$\frac{-\partial E_z}{\partial x}\bar{a}_y + \frac{\partial E_y}{\partial x}\bar{a}_z = -\mu\left[\frac{\partial H_y}{\partial t}\bar{a}_y + \frac{\partial H_z}{\partial t}\bar{a}_z\right]$$

$$\frac{\partial E_z}{\partial x} = -\mu \frac{\partial H_y}{\partial t}; \frac{\partial E_y}{\partial x} = -\mu \frac{\partial H_z}{\partial t}$$

Let the solution of wave equation be,

$$E_y = f(x - v_0 t)$$
$$\frac{\partial E_y}{\partial t} = \frac{\partial f}{\partial (x - v_0 t)} \cdot \frac{\partial (x - v_0 t)}{\partial t}$$
$$= f'(x - v_0 t) \times (-v_0)$$
$$\frac{\partial E_y}{\partial t} = -v_0 F' \qquad [\because f'(x - v_0 t) = F']$$
$$\frac{-\partial H_z}{\partial x} = \varepsilon \frac{\partial E_y}{\partial t} \Rightarrow \frac{-\partial H_z}{\partial x} = \varepsilon(-v_0 F')$$
$$\frac{\partial H_z}{\partial x} = v_0 \varepsilon F'$$
$$= \frac{1}{\sqrt{\mu\varepsilon}} \varepsilon F'$$
$$\frac{\partial H_z}{\partial x} = \sqrt{\varepsilon/\mu}\, F'$$

$$\partial H_z = \sqrt{\varepsilon/\mu} \int F' dx$$
$$= \sqrt{\varepsilon/\mu}\, f$$

$$H_z = \sqrt{\varepsilon/\mu}\, E_y$$
$$\frac{E_y}{H_z} = \sqrt{\mu/\varepsilon}$$

Similarly, it can be shown that,

$$\frac{E_z}{H_y} = -\sqrt{\mu/\varepsilon}$$
$$E = \sqrt{E_y^2 + E_z^2} \ \& \ H = \sqrt{H_y^2 + H_z^2}$$

Therefore $\frac{E}{H} = \sqrt{\mu/\varepsilon}$

This is referred to as characteristics impedance or intrinsic impedance. It is defined as the ratio of permittivity to dielectric constant

$$\eta = \frac{E}{H} = \sqrt{\mu/\varepsilon}$$

For free space $\varepsilon_x = \mu_r = 1,$ then the character impedance for free space is given by

$$\eta_0 = \sqrt{\mu_0/\varepsilon_0}$$
$$= \sqrt{\frac{4\pi \times 10^{-7}}{36\pi \times 10^{9}}}$$
$$\eta_0 = 120\pi / 377\Omega$$

Dot product and cross product of E and H:-

$$E.H = E_yH_y + E_zH_z$$
$$\eta = \frac{E_y}{H_z} = \frac{E_z}{H_y}$$
$$E.H = \eta H_yH_z - \eta H_yH_z$$
$$E.H = 0$$

This proves that E and H both perpendicular to each

$$E \times H = \begin{vmatrix} a_x & a_y & a_z \\ E_x & E_y & E_z \\ H_x & H_z & H_z \end{vmatrix} = \begin{vmatrix} a_x & a_y & a_z \\ 0 & E_y & E_z \\ 0 & H_z & H_z \end{vmatrix}$$
$$= \bar{a}_x\left[E_yH_z - E_zH_z\right]$$
$$= \bar{a}_x\left[\eta H_z^2 + \eta H_y^2\right]$$
$$E \times H = \eta H^2 \bar{a}_x$$

12) Derive the wave propagation in a wave propagation in a lossless medium

Wave propagation in a lossless medium:-

The wave equation for free space lossless medium is,

$$\nabla^2 E = \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

The phasor value of E is

$$E(x,t) = R_e\left[E(x)e^{j\omega t}\right]$$
$$\nabla^2 R_e\left[E(x)e^{j\omega t}\right] = \mu\varepsilon \frac{\partial^2}{\partial t^2} R_e\left[E(x)e^{j\omega t}\right]$$
$$\nabla^2 R_e\left[E(x)e^{j\omega t}\right] = \mu\varepsilon R_e\left[-\omega^2 E e^{j\omega t}\right]$$
$$R_e\left[(\nabla^2 E + \mu\varepsilon\omega^2 E)e^{j\omega t}\right] = 0$$
$$\nabla^2 E + \mu\varepsilon\omega^2 E = 0$$
$$\nabla^2 E + \beta^2 E = 0$$

This is called vector Helmholtz equation

$$\beta^2 = \omega^2\mu\varepsilon$$
$$\beta = \sqrt{\mu\varepsilon}\ \ \omega$$

Where β is called phase shift constant

The velocity of propagation is

$$v = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}}$$

Wave equation in a conducting medium.

The wave equation for conducting medium is

$$\nabla^2 E - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} - \mu\sigma \frac{\partial E}{\partial t} = 0$$

The phasor form of wave equation is,

$$\nabla^2 E - j^2\omega^2\mu\varepsilon E - j\omega\mu\sigma E = 0$$
$$\nabla^2 E - j\omega\mu(\sigma + j\omega\varepsilon)E = 0$$
$$\nabla^2 E - \gamma^2 E = 0$$

Where $\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$

γ is called propagation constant

$$\gamma = \alpha + j\beta$$

Where α = attenuation constant

β =Phase shift

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

Squaring on both sides,

$$\alpha^2 - \beta^2 + 2j\alpha\beta = j\omega\mu(\sigma + j\omega\varepsilon)$$
$$\alpha^2 - \beta^2 = -\omega^2\mu\omega$$
$$\alpha \infty \beta = \omega\mu\sigma$$

We know that,

$$\alpha^2 + \beta^2 = \sqrt{\left(\alpha^2 - \beta^2\right)^2 + 4\alpha^2\beta^2}$$

But $\left(\alpha^2 - \beta^2\right)^2 = \left(-\omega^2\mu\varepsilon\right)^2$

$$\left(\alpha \;\infty\; \beta\right)^2 = \left(\omega\mu\sigma\right)^2$$
$$\alpha^2 + \beta^2 = \sqrt{\omega^4\mu^2\varepsilon^2 + \omega^2\mu^2\sigma^2}$$
$$\alpha^2 - \beta^2 = -\omega^2\mu\varepsilon$$
$$2\alpha^2 = -\omega^2\mu\varepsilon + \sqrt{\omega^4\mu^2\varepsilon^2 + \omega^2\mu^2\sigma^2}$$
$$\alpha^2 = \frac{-\omega^2\mu\varepsilon}{2} + \sqrt{\frac{\omega^4\mu^2\varepsilon^2}{2} + \frac{\omega^2\mu^2\sigma^2}{2}}$$
$$= \frac{-\omega^2\mu\varepsilon}{2} + \frac{\omega^2\mu\varepsilon}{2}\sqrt{1 + \frac{\sigma^2}{\omega^2\varepsilon^2}}$$
$$\alpha = \sqrt{\frac{\omega^2\mu\varepsilon}{2}\left[\sqrt{1 + \frac{\sigma^2}{\omega^2\varepsilon^2}} - 1\right]}$$
$$\boxed{\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[1 + \frac{\sigma^2}{\omega^2\varepsilon^2} - 1\right]}}$$

To find the value of β :-

$$2\beta^2 = \sqrt{\omega^4\mu^2\varepsilon^2 + \omega^2\mu^2\sigma^2} + \omega^2\mu\varepsilon$$

$$\beta^2 = \frac{\omega^2\mu\varepsilon}{2} + \sqrt{1 + \frac{\sigma^2}{\omega^2\varepsilon^2}} + \frac{\omega^2\mu\varepsilon}{2}$$

$$\beta^2 = \frac{\omega^2\mu\varepsilon}{2}\sqrt{\sqrt{1 + \frac{\sigma^2}{\omega^2\varepsilon^2}} - 1}$$

$$\boxed{\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[1 + \frac{\sigma^2}{\omega^2\varepsilon^2} - 1\right]}}$$

Wave propagation in good dielectric:-

$\frac{\sigma}{\omega\varepsilon} >> 1$ For good conductor

$\frac{\sigma}{\omega\varepsilon} << 1$ For good dielectric

For dielectrics, $\frac{\sigma}{\omega\varepsilon} << 1$

$$\sqrt{1 + \frac{\sigma^2}{\omega^2\varepsilon^2}} = \left(1 + \frac{\sigma^2}{\omega^2\varepsilon^2}\right)^{1/2}$$
$$\approx 1 + \frac{\sigma^2}{2\omega^2\varepsilon^2}$$

The attenuation factor is,

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[1 + \frac{\sigma^2}{\omega^2\varepsilon^2} - 1\right]}$$
$$= \omega\sqrt{\frac{\mu\varepsilon}{2}\left[1 + \frac{\sigma^2}{2\omega^2\varepsilon^2} - 1\right]}$$
$$= \omega\sqrt{\frac{\mu\sigma^2\varepsilon}{4\omega^2\varepsilon^2}} = \sqrt{\frac{\mu\sigma^2}{4\varepsilon}} \approx \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

Phase shift, $\beta = \sqrt{\frac{\omega^2\mu\varepsilon}{2}\left[\sqrt{1 + \frac{\sigma^2}{\omega^2\varepsilon^2}} + 1\right]}$

$$\beta = \sqrt{\frac{\omega^2\mu\varepsilon}{2}\left[\sqrt{1 + \frac{\sigma^2}{2\omega^2\varepsilon^2}} + 1\right]}$$
$$= \sqrt{\omega^2\mu\varepsilon\left[\sqrt{1 + \frac{\sigma^2}{4\omega^2\varepsilon^2}}\right]}$$
$$= \omega\sqrt{\mu\varepsilon\left[1 + \frac{\sigma^2}{4\omega^2\varepsilon^2}\right]}$$
$$\beta \approx \omega\sqrt{\mu\varepsilon}\left[1 + \frac{\sigma^2}{4\omega^2\varepsilon^2}\right]^{1/2}$$
$$\beta \approx \omega\sqrt{\mu\varepsilon}\left[1 + \frac{\sigma^2}{8\omega^2\varepsilon^2}\right]$$

Velocity of wave in dielectric is $v = \frac{\omega}{\beta}$

$$\nu = \frac{\omega}{\omega\sqrt{\mu\varepsilon}\left[1+\frac{\sigma^2}{8\omega^2\varepsilon^2}\right]} \approx \frac{1}{\sqrt{\mu\varepsilon}}\left[1+\frac{\sigma^2}{8\omega^2\varepsilon^2}\right]^{-1}$$

$$\nu = \frac{1}{\sqrt{\mu\varepsilon}}\left(1-\frac{\sigma^2}{8\omega^2\varepsilon^2}\right)$$

$$\boxed{\nu \approx \nu_0\left[1-\frac{\sigma^2}{8\omega^2\varepsilon^2}\right]}$$

Intrinsic or characteristics impedance,

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon\left(1+\frac{\sigma}{j\omega\varepsilon}\right)}}$$

$$= \sqrt{\frac{\mu}{\varepsilon}\left(1+\frac{\sigma}{j\omega\varepsilon}\right)^{-1}}$$

$$= \sqrt{\frac{\mu}{\varepsilon}}\left(1+\frac{\sigma}{j\omega\varepsilon}\right)^{1/2}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}\left(1+\frac{j\sigma}{2\omega\varepsilon}\right)$$

$$\boxed{\eta = \eta_0\left[1+\frac{j\sigma}{2\omega\varepsilon}\right]}$$

Wave propagation in good conductors:- or plane waves in good conductors

For good conductor $\frac{\sigma}{\omega\varepsilon} >> 1$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

$$= \sqrt{j\omega\mu\sigma\left[1+\frac{j\omega\varepsilon}{\sigma}\right]}$$

$$\gamma = \sqrt{j\omega\mu\sigma} = \sqrt{\omega\mu\sigma}\underline{|90^\circ}$$

$$\boxed{\gamma = \sqrt{\omega\mu\sigma}\underline{|45^\circ}}$$

W. K.T

$$\alpha = \sqrt{\frac{\omega^2\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}-1\right]}$$

$$\alpha = \sqrt{\frac{\omega^2\mu\varepsilon}{2}\left(\frac{\sigma}{\omega\varepsilon}\right)}$$

$$\boxed{\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}}$$

W>K>T

$$\beta = \sqrt{\frac{\omega^2\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}+1\right]}$$

$$= \sqrt{\frac{\omega^2\mu\varepsilon}{2}\left(\frac{\sigma}{\omega\varepsilon}\right)}$$

$$\boxed{\beta = \sqrt{\frac{\omega\mu\sigma}{2}}}$$

Velocity of wave in conductor, $v = \frac{\omega}{\beta}$

$$v = \frac{\omega}{\sqrt{\frac{\omega\mu\sigma}{2}}} = \sqrt{\frac{2\omega^2}{\omega\mu\sigma}} = \sqrt{\frac{2\omega}{\mu\sigma}}$$

Intrinsic impedance $\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$

$$\eta = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon\left(1+\frac{\sigma}{j\omega\varepsilon}\right)}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon.\frac{\sigma}{j\omega\varepsilon}}} = \sqrt{\frac{j\omega\mu}{\sigma}}$$

$$\boxed{\eta = \sqrt{\frac{\omega\mu}{\sigma}}\underline{|90^\circ}}$$

In good conductor, α and β large. Sine σ is large (i.e) wave is attenuated greatly as it propagation through the conductor.

But velocity and characteristics impedance is considerably reduced.

13) Explain skin depth or penetration?

Depth of penetration:-

In a good conductor, the wave is attenuated as it progress. At radio frequency the ratio of attenuation is very large and the wave may penetration only a very short distance before being reduced to a negligibly small value.

The depth of penetration (δ) or skin depth is defined as the depth in which the wave has been attenuated to $\frac{1}{e}$ or approximate to 37% of its original value.

The amplitude of wave decreases by a factor $e^{-\alpha\delta}$ as it propagates through a distance s.

By definition, $e^{-\alpha S} = e^{-1}$

$$\alpha S = 1$$

$$S = \frac{1}{\alpha}$$

Skin depth is the distance S through which the wave amplitude decrease to factor e^{-1} i.e, about 37% of the original value.

$$S = \frac{1}{\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}-1\right]}}$$

For good conductor, $\frac{\sigma}{\omega\varepsilon} >> 1$

$$S = \frac{1}{\omega\sqrt{\frac{\mu\varepsilon}{2}\left(\frac{\sigma}{\omega\varepsilon}\right)}} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

$$S = \sqrt{\frac{2}{2\pi f\mu\sigma}}$$

$$S = \sqrt{\frac{1}{\pi f\mu\sigma}}$$

$$S = \frac{1}{\sqrt{\pi f\mu\sigma}}$$

PROBLEMS UNIT – V

Problems in pointing theorem & pointing vector

1. Determine the pointing vector & calculate power crossing 10m² plate of the yz plane.

Solution:-

$$\overline{E} = 50e^{j\left(\frac{4\pi}{3}\right)x}\overline{a}_z$$

$$\overline{H} = \frac{50}{120\pi}e^{j\left(\frac{4\pi}{3}\right)x}\overline{a}_y \quad \overline{H}^* = \frac{50}{120\pi}e^{j\left(\frac{4\pi}{3}\right)x}\overline{a}_y$$

$$P_{avg} = \frac{1}{2}R_e\left[E\times H^*\right] = \frac{1}{2}\frac{50\times50}{120\pi}\left[e^{j\left(\frac{4\pi}{3}\right)x}\overline{a}_z\times e^{j\left(\frac{4\pi}{3}\right)x}\overline{a}_y\right]$$

$$= \frac{(50)^2}{240\pi}(-\overline{a}_x) = -3.31\overline{a}_x W/m^2$$

Total power **=** $|P_{avg}|\times \text{Area} = 3.31\times10 = 33.1W$

2. In free space $E(z,t) = 60\cos(\omega t-\beta z)\overline{a}_x v/m$**. Find the average power crossing a circular area of radius 4m in the plane z = const**

Solution:-

$$E = 60\cos(\omega t-\beta z)\overline{a}_x = 60e^{j(\omega t-\beta z)}\overline{a}_x v/m$$

$$\eta_0 = \frac{E}{H} \Rightarrow H = \frac{1}{\eta_0}\overline{a}_k\times E = \frac{60}{120\pi}(a_z)\times e^{j(\omega t-\beta z)}\overline{a}_x$$

$$H = \frac{1}{2\pi}e^{j(\omega t-\beta z)}\overline{a}_y A/m$$

Power density $= \frac{1}{2}R_e\left(\overline{E}\times H^*\right); \ = \frac{1}{2}(60)\left(\frac{1}{2\pi}\right)\overline{a}_z = \frac{15}{\pi}a_z W/m^2$

$$p_{avg} = \left(\frac{15}{\pi}\right) \times \pi(4)^2 = 240W$$

3. When a plane ware trends in free space, it has an average power density of 40W/m2. Calculate $\overline{E}$ & $\overline{B}$

Solution:-

The average power density

$$\left|S_{avg}\right| = \frac{|E|^2}{2\eta_0}$$

$$40 = \frac{|E|^2}{2 \times 120\pi} \Rightarrow E^2 = 80 \times 120\pi$$

$$E^2 = 9600\pi$$

$$E = \sqrt{9600\pi}; \quad |E| = 173.62 v/m$$

$$B = \mu_0 H = \mu_0 \frac{|E|}{\eta_0} = \frac{4\pi \times 10^{-7} \times 173.62}{120\pi}$$

$$B = 0.58 \mu Wb/m^2$$

4. A forward travelling plane wave in free space is $E_x = \cos(4\pi \times 10^7 t - \beta z) v/m$. Calculate the instantaneous and time average pointing vector

Solution:-

Given $E_x = \cos(4\pi \times 10^7 t - \beta z) v/m$

$$H_y = \frac{E_x}{\eta_0} = \frac{1}{120\pi} \cos(4\pi \times 10^7 t - \beta z) A/m$$

Here $\omega = 4\pi \times 10^7 rad/s$ $\quad \beta = \frac{\omega}{c} = \frac{4\pi \times 10^7}{3 \times 10^8} = 0.42 rad/m$

$$p = E \times H = E_x \overline{a}_x \times H_y \overline{a}_y = E_x \overline{a}_x \times \frac{E_x}{\eta_0} \overline{a}_y$$

$$= \frac{1}{120\pi} \cos^2(4\pi \times 10^7 t - 0.42z) \overline{a}_z$$

$$P = \frac{1}{2}\left[\frac{1}{120\pi}(1 + \cos 2(4\pi \times 10^7 - 0.42z))\right]$$

$$P_{avg} = \frac{1}{2}\left(\frac{1}{120\pi}\right)\overline{a}_z = 1.326 \overline{a}_z mW/m^2$$

5. In free space, $\overline{E} = 100 \sin(\omega t - \beta z)\overline{a}_x \ v/m$. Calculate the total power passing through a rectangular area of sides 30mm $\times$ 10mm is z = 0 plane. Assume $\eta_0 = \frac{E_m}{H_m}$ & $\eta_0 = 120\pi$

Solution:-

$$\bar{E} = 100(\sin\omega t - \beta z)\bar{a}_x \, v/m$$
$$\bar{H} = \frac{100}{\eta_0}\sin(\omega t - \beta z)\bar{a}_y A/m$$
$$\bar{E} = 100\cos(\omega t - \beta z - \pi/2)\bar{a}_x = 100 e^{j(-\beta z - \pi/2)}\bar{a}_x$$
$$\bar{H} = \frac{100}{\eta_0}\cos(\omega t - \beta z - \pi/2)\bar{a}_y = \frac{100}{\eta_0}e^{j(-\beta z - \pi/2)}\bar{a}_y$$
$$H^* = \frac{100}{\eta_0}e^{j(+\beta z + \pi/2)}\bar{a}_y$$
$$P_{avg} = \frac{1}{2}R_e\left(E \times H^*\right)$$
$$= \frac{1}{2}(100)\left(\frac{100}{\eta_0}\right)R_e\left[e^{j(-\beta z - \pi/2)}\bar{a}_x \times e^{j(\beta z + \pi/2)}\bar{a}_y\right]$$
$$= \frac{1}{2}\frac{(100)^2}{120\pi}\bar{a}_z = 13.27 a_z W/m^2$$

Total power = Average power $\times$ Area

$$= 13.27\bar{a}_z \times 10 \times 10^{-3} \times 30 \times 10^{-3}$$

P = 3.98 mW

6. If the field vector of a wave in free space an given by

$$\bar{E} = 50\cos(\omega t + \frac{4\pi}{3}x)\bar{a}_z v/m$$
$$\bar{H} = \cos(\omega t + \frac{4\pi}{3}x)\bar{a}_y Nm$$

7. A uniform plane wave with 10MHz frequency has average pointing vector 4W/m². If the medium is perfect dielectric with $\mu_r = 2,\ \varepsilon_r = 3$. Determine (i) Velocity (ii) wavelength (iii) intrinsic impedance (iv) r.m.s value of E

Solution:- $P_{avg} = 4W/m^2 \quad f = 10 \times 10^6 Hz$

For a perfect dielectric $\sigma = 0, \mu_r = 2, \ \varepsilon_r = 3$

(i)Alternation constant & phase constant are

$$\alpha = 0$$
$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}$$
$$= (2\pi \times 10 \times 10^6)\sqrt{4\pi \times 10^{-7} \times 2 \times \frac{1}{36\pi \times 10^9} \times 3}$$
$$= 0.5133 rad/m$$

(ii) $v_p = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \frac{2\pi \times 10 \times 10^6}{0.5133} = 122.4 \times 10^6 \ m/s$

(iii) $\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\mu_r}{\varepsilon_r}} = 120\pi\sqrt{\frac{2}{3}} = 307.6\Omega$

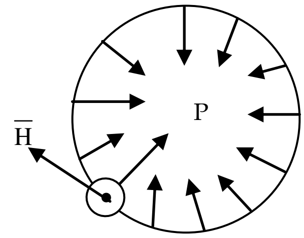
(iv) $P_{avg} = \frac{1}{2}\frac{E_m^2}{\eta}$

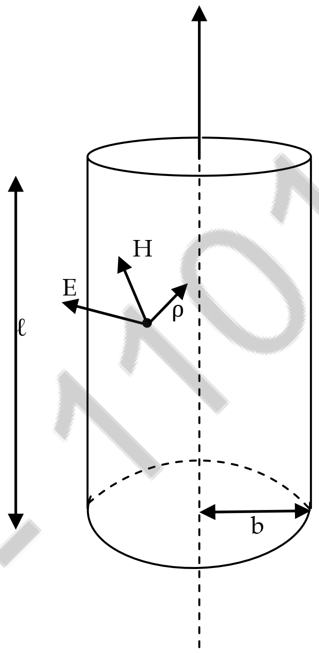
$$E_m = \sqrt{2\eta(p_{avg})} = \sqrt{2\times 307.6\times 4} = 49.6\,v/m$$

$$E_{rms} = \frac{E_m}{\sqrt{2}} = \frac{49.6}{\sqrt{2}} = 35.07\,v/m$$

8. Find the pointing vector on the surface of along straight conducting wire of radius b & conductivity σ that carries a direct current of I verify poynting's theorem.

Solution:-





The axis of the wire coinides with the z – axis

$$J = \frac{I}{A}\bar{a}_z = \frac{I}{\pi b^2}\bar{a}_z$$

$$E = \frac{\bar{J}}{\sigma} = \frac{I}{\sigma\pi b^2}\bar{a}_z$$

$$H = \frac{I}{2\pi\rho}\bar{a}_\phi = \frac{I}{2\pi b}\bar{a}_\phi$$

$$p = \bar{E}\times\bar{H} = \frac{I}{\sigma\pi b^2}\bar{a}_z \times \frac{I}{2\pi b}\bar{a}_\phi = \frac{-I^2}{20\pi^2 b^3}\bar{a}_\rho$$

$$-\oiint_s \bar{p}.ds = -\oiint_s \left(\bar{p}\,.\,\bar{a}_\rho\right)\left(ds.\ a_\rho\right)$$

$$= -\oiint_s \left(\frac{I^2}{20\pi^2 b^3}\right)(2\pi b\ell)$$

$$-\oiint_s \bar{p}.ds = I^2\left(\frac{\ell}{\sigma\pi b^2}\right) = I^2\left(\frac{\ell}{\sigma A}\right) = I^2 R$$

The above result shows that the negative surface integral of the pointing vector is exactly equal to I^2 R power loss in the conducting.

9. A current 4m in length lies along y- axis centred at origin. The current is 10Ain $\bar{a}_y$ direction. If it experiences of force of $15\left(\frac{a_x + a_z}{\sqrt{2}}\right)N$ due to a uniform magnetic filed, determine B & H in free space.

Solution:-

The force exerted on straight current element in uniform magnetic filed is

$$\vec{F} = I\bar{\ell} \times \bar{B}$$

$$\frac{15}{\sqrt{2}}(a_x + a_z) = \left[(10)(4a_y) \times (B_x a_x + B_y a_y + B_z a_z\right]$$

$$10.61(a_x + a_z) = \begin{bmatrix} a_x & a_y & a_z \\ 0 & 40 & 0 \\ B_x & B_y & B_z \end{bmatrix}$$

$$10.61a_x + 10.61a_z = a_x(40B_z) - a_y(0) + a_z(0 - 40B_x)$$

$$10.61a_x + 10.61a_z = 40B_z\bar{a}_x - 40B_z\bar{a}_z$$

$$40B_z = 10.61 \qquad B_z = \frac{10.61}{40} = 0.265$$

$$-40B_x = 10.61 \qquad B_x = -0.265$$

$$\bar{B} = -0.265a_x + 0.265a_z$$

$$= 0.265(-a_x + a_z)T$$

$$\bar{H} = \frac{\bar{B}}{\mu_0} = \frac{0.265(-a_x + a_z)}{4\pi \times 10^{-7}}$$

$$\bar{H} = 0.211 \times 10^6(-a_x + a_z)A/m$$

10. A conductor of length 5m located at z = 0, x = 4m carries a current of 10A in the $-\bar{a}_y$ direction. Find the component of $\bar{B}$ in the region if the force on the conductor is $1.2\times10^{-2}N$ in the direction $(-a_x + \bar{a}_z)/\sqrt{2}$.

Solution:-

$$\bar{F} = (1.2\times10^{-2})\left(\frac{-a_x + \bar{a}_z}{\sqrt{2}}\right) I = 10A \qquad \ell = -5a_y$$

$$\bar{F} = I\bar{\ell} \times \bar{B}$$

$$\frac{1.2\times10^{-2}}{\sqrt{2}}(-a_x + a_z) = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 0 & -50 & 0 \\ B_x & B_y & B_z \end{vmatrix}$$

$$= -50B_z\bar{a}_x + 50B_x\bar{a}_z$$

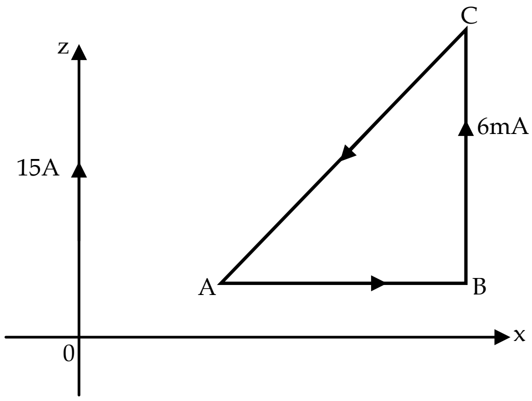
$$\frac{1.2\times10^{-2}}{\sqrt{2}}(-a_x + a_z) = +50(-B_z a_x + B_x a_z)$$

Comparing the co – efficient

$$B_z = B_x = \frac{1.2\times10^{-2}}{50\sqrt{2}} = 1.7\times10^{-4}\,Wb/m^2$$

11. A triangular loop of wire in free space join points A (1, 0, 1) B (3, 0, 1) C (3, 0, 4). The wire carries a current of 6mA flowing in the $\bar{a}_z$ direction from B to C. A filamentary current of 15A flows along the entire z – axis in the $\bar{a}_z$ direction. Find the total force on the loop.

Solution:-



The current flowing through the loop is 6ma & the current along z – axis is I_1= 15A

$$\bar{B} = \mu_0 \bar{H} = \frac{\mu_0 I_1}{2\pi\rho} \bar{a}_\phi$$

In rectangular co – ordinates

$$\bar{B} = \frac{\mu_0 I_1}{2\pi x}(-a_y) = \frac{-4\pi\times10^{-7}\times15\times -a_y}{2\pi x}$$

$$\bar{B} = \frac{-3\times10^{-6}}{x}\bar{a}_y$$

The differential force along AB is

$$d\bar{F}_{AB} = I\bar{dl}_{AB}\times\bar{B}$$

$$\bar{dl}_{AB} = dx\bar{a}_x$$

$$d\bar{F}_{AB} = Idx\bar{a}_x \times \left(\frac{-3\times10^{-6}}{x}\right)\bar{a}_y$$

$$= (6\times10^{-3})\left(\frac{-3\times10^{-6}}{x}\right)dx \;\; a_x\times a_y$$

$$= \frac{-18}{x}10^{-9}dx\,\bar{a}_z$$

$$\bar{F}_{AB} = -18\int_1^3 \frac{dx}{x}\times10^{-9}\bar{a}_z$$

$$= -18\left[\ln(3)-\ln(1)\right]\times10^{-9}a_z$$

$$F_{AB} = -19.77\bar{a}_z nN$$

The differential force along the side BC is

$$d\bar{F}_{BC} = I\bar{dl}\times\bar{B}$$

$$d\bar{F}_{BC} = 6\times10^{-3}dz\; a_z \times \left(\frac{-3\times10^{-6}}{x}\right)a_y$$

$$= 6\times10^{-3}\times\left(\frac{-3\times10^{-6}}{3}\right)dz\times -a_x$$

$$= 6\times10^{-9}dz\,\bar{a}_x$$

$$\bar{F}_{BC} = 6\times10^{-9}\bar{a}_x\int_1^4 dz = 6\times10^{-9}\; a_x[3]$$

$$F_{BC} = 18nN\bar{a}_x$$

The differential force along the side CA is

$$d\bar{F}_{CA} = I\,d\bar{l}_{CA} \times \bar{B}$$

$$d\bar{F}_{CA} = 6\times10^{-3}(-dx\bar{a}_x - dz\bar{a}_z \times \left(\frac{-3\times10^{-6}}{x}\right)a_y$$

$$= 18\times10^{-9}\ \int_3^1 \frac{dx}{x}\bar{a}_z - \int_4^1 \frac{18\times10^{-9}}{3}\bar{a}_x$$

$$F_{CA} = -19.77\times10^{-9}\bar{a}_z + 18\times10^{-9}\bar{a}_x$$

$$\vec{F} = \bar{F}_{AB} + \bar{F}_{BC} + \bar{F}_{CA}$$

$$= -19.77\times10^{-9}\bar{a}_z + 18\times10^{-9}a_x - 19.77\times10^{-9}a_z + 18\times10^{-9}a_x$$

$$\vec{F} = 36a_x - 39.54\bar{a}_z\text{nN}$$

UNIT V

TIME VARYING FIELDS AND MAXWELLS EQUATIONS

Sate and prove the Faraday's law of electromagnetic inductor?

Faraday's law:-

The total electromotive force (e. m. f) induced in a circuit is equal to the circuit

$$\varepsilon = \frac{-d\phi}{dt}$$

Where

ε = E. M. F induced in the circuit (volts)

ϕ = total flux (klebers)

t = time in seconds

if the circuit refers to a multiple – turn loop with, say, N turns

$$\varepsilon = -N\frac{d\phi}{dt}$$

$$\varepsilon = \frac{-d\Lambda}{dt}$$

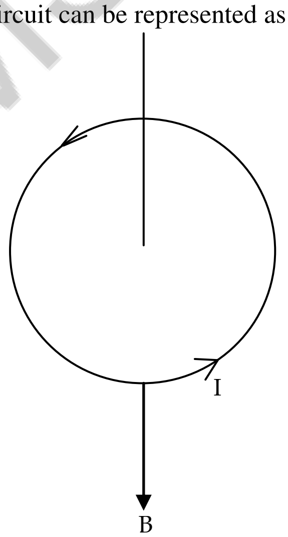
Where $\Lambda = N\phi$

= total flux linkage (Wb – turns)

Consider a closed circuit made of a single turn loop as shown in fig (1). If the field is normal to the plane of the loop & increasing, an e. M. F is induced in the circuit and current flows in the circuit.

The induced current is always so directed as to produce a flux opposing the charge in the magnetic field (B/H)

We know that the e. m. f in a circuit can be represented as the line integral of the electric field around the closed path.



$$\varepsilon = \oint E_e.dl$$

Where E_e is an E.M.F producing electric field caused by the varying magnetic field.

Now the total flux through a circuit is equal to the integral of the normal component of the flux density B over the surface bonded by the circuit. Total flux is, therefore given by

$$\phi = \iint_s B.n\ ds$$
$$\varepsilon = \frac{-d\phi}{dt} = \frac{-d}{dt}\iint_s B.n\ ds$$
$$\varepsilon = \oint_\ell E_e.\ dl = -\iint \frac{\partial B}{\partial t}.\ n\ ds$$
$$\oint_\ell E_e.dl = \iint_s \text{curl}\ \ E_e.n\ ds$$
$$\iint_s \text{curl}\ E_e.n\ ds = -\iint \frac{\partial B}{\partial t}.n\ ds$$
$$\text{curl}E_e = \frac{-\partial B}{\partial t}\ \text{ for stationary circuits}$$

This relation is referred to Maxwell's equation in differential form.

$$\varepsilon = \oint_\ell E_e.dl = -\iint_s \frac{\partial B}{\partial t}.n\ ds$$

This is known as transformer E.M.F.

Let us consider a charge dQ moving with a velocity V in a field with flux density b is given by

$$dF = dQ\vartheta \times B$$
$$E_e = \frac{dF}{dQ} = \vartheta \times B$$

If the charge – element is isolated in a conductor, the EMF induced is a netional EMF & the equation isd given by

$$\varepsilon = \int E_e.dl = \int(\vartheta \times B).\ dl$$

Explain in detail the Maxwell's II equation in integral & point form

Maxwell's II equation in integral form from Faraday's law:-

$$\oint_\ell E_e.dl = \iint_s (\nabla \times E).\ ds$$
$$\iint_s (\nabla \times E_e).ds = -\iint_s \frac{\partial B}{\partial t}.n\ ds$$
$$\boxed{\nabla \times E_e = -\frac{\partial B}{\partial t}}$$

Conduction current:-

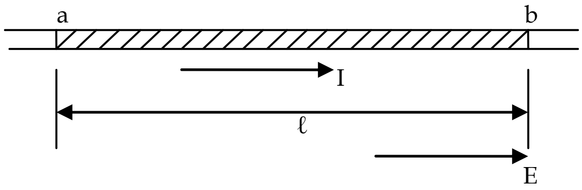
Let I be the current in the conduction of area A. When we say that a current in flowing from a to b, it means that the potential of a is higher than b.

Work is done when charge is carried from the point b which is at a lower potential to the point a which is at a higher potential, against a field E. If Vba is the potential difference between the two points then

$$V_{ba} = E\ell = IR$$

Where R = resistance between a and b

$$= \frac{\ell}{\sigma A}$$

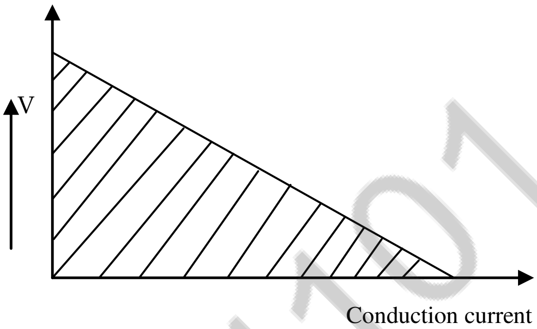


Where σ = conductivity

$$E\ell = I\left(\frac{\ell}{\sigma A}\right)$$

$$= \frac{I}{A}\cdot\frac{1}{\sigma}$$

$$\boxed{E = \frac{J_1}{\sigma}}$$



Where J1 may be referred to as conduction current density which is directly proportional to E.

Displacement current:-

In the case of a capacitor, the current flow can be constant only when the voltage is changing rather than steady.

If Ic is the capacitor current, it is given by

$$I_C = \frac{dQ}{dt} = C\frac{dv}{dt}$$

$$C = \frac{\varepsilon A}{t'}$$

$$I_C = \frac{\varepsilon A}{t'}\frac{dv}{dt}$$

$$= \frac{\varepsilon A t'}{t'}\frac{dE}{dt}$$

$$I_C = \varepsilon A\frac{dE}{dt}$$

$$\frac{I_C}{A} = \text{currrent density} = J_2$$

$$J_2 = \frac{d\vec{D}}{dt}$$

$$\text{Therefore } J = J_1 + J_2 = \sigma E + \frac{d\overline{D}}{dt}$$

Discuss about the inconsistency of ACL & derive mode?????

Inconsistency of Ampere's circuital law:-

We know that the equation of continuity,

$$\nabla.J = \frac{-\partial \rho_v}{\partial t}$$

We also know that the Ampere's circuital law,

$$\oint_{\ell} H.dl = I = \oiint_{s} J.ds$$

$$\nabla \times H = J \qquad \rightarrow (1)$$

Now talking divergence on both the sides,

$$\nabla.(\nabla \times H) = \nabla.J = 0 \ \rightarrow (2)$$

But we know $\nabla.J = \frac{-\partial}{\partial t} \qquad \rightarrow (3)$

Equation (2) & (3) is contracting. Thus Ampere's law is inconsistency.

Modified form of Ampere's circuital law:-

$$\oint_{\ell} H.dl = I = \oiint_{S} \left(J + \frac{\partial D}{\partial t} \right). \ ds \ \rightarrow (1)$$

Applying stokes theorem,

$$\oint_{\ell} H.dl = \oiint_{s} (\nabla \times H).ds \qquad \rightarrow (2)$$

Equating (1) & (2), we get

$$\oiint_{s} (\nabla \times H).ds = \oiint_{S} \left(J + \frac{\partial D}{\partial t} \right) ds \quad \text{Integral form of Maxwell I equation}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} \qquad \text{point form of Maxwell's I equation}$$

Maxwell's Four equation in integral form, point or differentiation form:-

Equation I:- From Ampere's circuital law.

The line integral of the magnetic field intensity around a closed path (contour) is equal to the current enclosed by the path

$$\int_{\ell} H.dl = I = \oiint_{s} \left(J + \frac{\partial D}{\partial t} \right). \ ds \quad \text{Integral form}$$

Applying Stoke's theorem,

$$\int_{\ell} H.dl = \oiint_{S} (\nabla \times H).ds$$

$$\oiint_{S} (\nabla \times H) ds = \oiint_{S} \left(J + \frac{\partial D}{\partial t} \right) ds$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} = \sigma E + \frac{\partial (\varepsilon E)}{\partial t}$$

$$\boxed{\nabla \times H = \sigma E + \frac{\varepsilon \partial E}{\partial t}}$$ Point form Maxwell's I equation.

Maxwell's II equation:- (Faraday's law electromagnetic induction)

The total electro motive force induced in a circuit is equal to the time rate of decrease of total magnitude flux linking in the circuit.

$$\varepsilon = \frac{-d\phi}{dt} \quad \phi = \iint_s B.nds$$

$$\varepsilon = \frac{-d}{dt}\iint_s B.n\ ds$$

Integral form of Maxwell's II equation $\quad \varepsilon = \int_\ell E_e.dl = -\iint_s \frac{\partial B}{\partial t}.\ nds \rightarrow (1)$

Maxwell's II equation in integral form applying stoke's theorem on the LHS,

$$\int_\ell E_e dl = \oint\!\!\!\int_s (\nabla \times E).ds \qquad \rightarrow (2)$$

$$\oint\!\!\!\int_s (\nabla \times E).ds = -\iint_S \left(\frac{\partial B}{\partial t}\right).\ nds$$

$$\boxed{\nabla \times E = \frac{-\partial B}{\partial t}}$$ Point form of Maxwell's II equation

Maxwell's III equation: (From Gauss law) for electric field

The surface integral of the electric flux density vector $\overline{D}$ is equal to the charge enclosed

$$\oint\!\!\!\int_S \overline{D}.n\ ds = Q \rightarrow \text{Integral form of Maxwell's equation}$$

$$\oint\!\!\!\int_S \overline{D}.n\ ds = Q = \iiint_v \rho\ dv$$

Applying divergence theorem,

$$\oint\!\!\!\int_S \overline{D}.n\ ds = \iiint_V \nabla.D\ dv = \iiint_V \rho\ dv$$

$$\boxed{\nabla.D = \rho}$$ Point form of Maxwell's equation

Maxwell's IV equation:- (From gauss's law for magnetic field)

The surface integral of the normal component of magnetic flux density vector $\overline{B}$ is equal to the zero.

$$\oint\!\!\!\int_S B.\ n\ \ ds = 0$$

$$\oint\!\!\!\int_S B.\ n\ \ ds = \iiint_V (\nabla.B) \quad \text{Integral form Maxwell's equation}$$

$\nabla.B = 0$ Point form of Maxwell's equation IV

Maxwell's Four equations summery

Equation number	Law from which the equation is derived	Integral form	Point form	Complex form
I	Ampere's circuital law	$\oint_{\ell} H.dl = I = \oint_{s}\left(J + \frac{\partial D}{\partial t}\right).ds$	$\nabla \times H = J + \frac{\partial D}{\partial t}$	$\nabla \times H = \sigma E + j\omega\mu$
II	Faraday's law of electromagnetic induction	$\oint_{\ell} E_e.dl = \oint_{s} \frac{-\partial B}{\partial t}.ds$	$\nabla \times E = \frac{-\partial B}{\partial t}$	$\nabla \times E = -j\omega\mu$
III	Gauss's law for electric field	$\oint_{S} \overline{D}.n\ ds = Q$	$\nabla.D = \rho$	$\nabla.D = \rho$
IV	Gauss's law for magnetic field	$\oint_{S} \overline{B}.n\ ds = 0$	$\nabla.B = 0$	$\nabla.B = 0$

Derive the equation of electromagnetic power poynting theorem and poynting vector:-

The theorem states that the vector product of the electric field intensity vector (E) and the magnetic field intensity vector (H) is equal to the measure of the rate of energy flow per unit area at that point. The direction of power flow is perpendicular to e and H in the direction of the vector E× H.

$$P = E \times H\,VA/m^2 \text{ or } Watts/m^2$$

Let us consider Maxwell's second equation

$$\nabla \times H = J + \frac{\partial D}{\partial t} \rightarrow (1)$$

$$J = \nabla \times H - \frac{\partial D}{\partial t} \rightarrow (2)$$

Multiply the above equation with E,

$$E.J = E.(\nabla \times H) - E.\frac{\partial D}{\partial t}$$

By identity, $\nabla.(E \times H) = H(\nabla \times E) - E(\nabla \times H)$

$$E.J = -\nabla(E \times H) + H(\nabla \times E) - E\frac{\partial D}{\partial t}$$
$$= -\nabla(E \times H) - H\frac{\partial B}{\partial t} - \varepsilon E\frac{\partial E}{\partial t}$$
$$E.J = -\nabla(E \times H) - \mu H\frac{\partial H}{\partial t} - \varepsilon E\frac{\partial E}{\partial t}$$
$$\mu H\frac{\partial H}{\partial t} = \frac{1}{2}\frac{\mu\partial H^2}{\partial t^2} \quad \varepsilon E\frac{\partial E}{\partial t} = \frac{1}{2}\frac{\varepsilon\partial E^2}{\partial t^2}$$
$$E.J = -\nabla(E \times H)\frac{-\partial}{\partial t}\left(\frac{\mu H^2}{2} + \frac{\varepsilon E^2}{2}\right)$$
$$-\nabla(E \times H) = E.J + \frac{-\partial}{\partial t}\left(\frac{\mu H^2}{2} + \frac{\varepsilon E^2}{2}\right)$$

Integrating the above equation throughout the volume V,

$$-\oint_{V}\nabla.(E \times H)dv = \oint_{V} E.Jdv + \oint_{V}\frac{\partial}{\partial t}\left(\frac{\mu H^2}{2} + \frac{\varepsilon E^2}{2}\right)dv$$

Applying divergence theorem to the left hand side,

$$-\oiint_S (E\times H).ds = \iiint_V (E.J)dv + \iiint_V \frac{\partial}{\partial t}\left(\frac{\mu H^2}{2}+\frac{\varepsilon E^2}{2}\right)dv$$

I term II term III term

$\oiint_S (E\times H).ds =$ I term = In going power flux in the surface S.

$\iiint_V (E.J)dv =$ II term = Total power generated or dissipated within volume v at any instant

$\frac{\partial}{\partial t}\left(\frac{1}{2}\mu H^2+\frac{1}{2}\varepsilon E^2\right)dv$ = III term = The time rate of increase of total electromagnetic energy within the volume

Thus the total power flowing into the volume V is equal to the total power flowing out of the volume V.

$$-\oiint_S P.ds = \iiint_V (E.J)dv + \int_X \frac{\partial}{\partial t}\left(\frac{\mu H^2}{2}+\frac{\varepsilon E^2}{2}\right)dv$$

Explain the instantaneous, Average & complex power instantaneous, Average and complex poynting vector:-

The instantaneous power Pinst is always given by the product of the instantaneous voltage Vinst and instantaneous current Iinst. i.e,

$$\begin{aligned}
P_{inst} &= V_{inst}.I_{inst}\\
V_{inst} &= R_e\left\{Ve^{j\omega t}\right\}\\
&= R_e\left\{|V|e^{j\theta v}.e^{j\omega t}\right\}\\
&= R_e\left\{|V|e^{j(\omega t+\theta v)}\right\}\\
&= R_e\left\{|V|\cos(\omega t+\theta v)+j\sin(\omega t+\theta v)\right\}
\end{aligned}$$

$$\boxed{V_{inst} = V_0\cos(\omega t+\theta v)}$$

And

$$\begin{aligned}
I_{inst} &= R_e\left\{Ie^{j\omega t}\right\}\\
&= R_e\left\{|I_0|.e^{j\theta i}.e^{j\omega t}\right\}\\
&= R_e\left\{|I|e^{j(\omega t+\theta i)}\right\}
\end{aligned}$$

$$\boxed{I_{inst} = I_0\cos(\omega t+\theta i)}$$

$$\begin{aligned}
P_{inst} &= |V_0||I_0|\cos(\omega t+\theta v)\cos(\omega t+\theta i)\\
&= |V_0||I_0|\left\{\frac{1}{2}\left[\cos(\omega t+\theta v-\omega t+\theta i)+\cos(\omega t+\theta v+\omega t+\theta i)\right]\right\}\\
&= \frac{|V_0||I_0|}{2}\left[\cos(\theta v-\theta i)+\cos(2\omega t+\theta v+\theta i)\right]\\
P_{inst} &= \frac{|V_0||I_0|}{2}\left[\cos\theta+\cos(2\omega t+\theta v+\theta i)\right]
\end{aligned}$$

The above equation has an arrange part and an oscillating put

$$P_{av} = \frac{|V_0||I_0|}{2}\cos\theta$$

And also the reactive power

$$P_{react} = \frac{|V_0||I_0|}{2}\sin\theta$$

Since θ is the phase angle, P_{av} and P_{react} are the impulse and out of phase components of the volt ampere product.

Now let us consider the complex power P, defined as one – half the product of v and the complex conjugate of I.

$$\begin{aligned} P &= \frac{1}{2}VI^* \\ &= \frac{1}{2}|V|e^{j\theta_v}.e^{j\omega t}.\ |I|e^{-j\theta_i}.e^{-j\omega t} \\ &= \frac{1}{2}|V||I|e^{j\theta_v}.e^{-j\theta_i} \\ &= \frac{1}{2}|V||I|e^{j(\theta_v-\theta_i)} \end{aligned}$$

$$\boxed{P = \frac{1}{2}|V||I|e^{j\theta}}$$

$$P = \frac{1}{2}V_0 I_0 e^{j\theta}$$

$$\boxed{P = P_{av} + jP_{react}}$$

$$P_{inst} = I_{inst} + H_{inst}$$

$$\begin{aligned} P_{av} &= \frac{1}{T}\int_0^T P\,dt = \frac{1}{2\pi}\int_0^{2\pi} P\,d(\omega t) \\ &= \frac{1}{2}E\times H(\cos\theta) + \frac{1}{2}E\times H\cos(2\omega t + \theta_x - \theta_i) \\ &= R_e\left\{\frac{1}{2}E\times He^{j(\theta_x-\theta_i)}\right\} \\ &= R_e\left\{\frac{1}{2}\left[E\times He^{j(\theta_x-\theta_i)}\right]\right\} \\ &= R_e\left[\frac{1}{2}\left\{E\times He^{j\theta_x}.e^{j\theta_i}\right\}\right] \\ &= R_e\left[\frac{1}{2}Ee^{j\theta_v}\times He^{-j\theta_i}\right] \\ P_{av} &= R_e\left[\frac{1}{2}E_{inst}\times H^*_{inst}\right] \end{aligned}$$

Thus the complex pointing vector is given by

$$P_{inst} = P_{react} + jP_{react}$$

At time independent complex pointing vector such as its real part is equal to the time – average of the usual pointing vector associated with the electromagnetic field.

$$P = \frac{1}{2} E \times H^*$$

$$P_{av} = \frac{1}{2} R_e \left\{E \times H^*\right\} = \frac{1}{2} R_e \left\{E^* \times H\right\} W/m^2$$

$$P_{react} = \frac{1}{2} I_m \left\{E \times H^*\right\}$$

$$P_x = \frac{1}{2}\left(E_y H_z^* - E_z H_y^*\right)$$

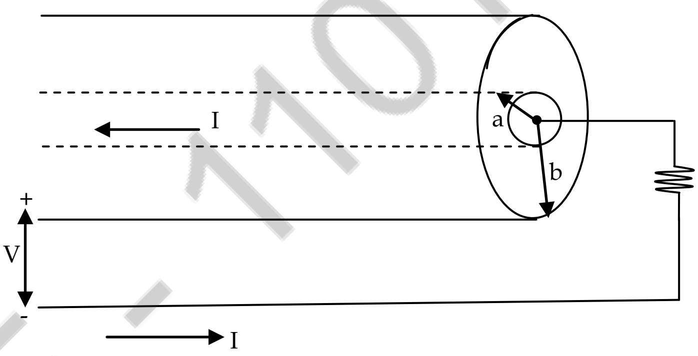
$$P_r = \frac{1}{2}\left(E_\theta H_\phi^* - E_\phi H_\theta^*\right)$$

Discuss about power flow in a co- axial cable:-

Consider a co- axial cable with a voltage v applied between the conductors.

The radius of inner and outer conductor are 'a' & 'b'

By ACL,



$$\oint H.dl = I$$

$$H.(2\pi r) = I$$

$$H = \frac{I}{2\pi r} \quad a < r < b$$

E due to an infinity long conductor

$$E = \frac{\rho_L}{2\pi\varepsilon_0 t}; V = \frac{\rho_L}{2\pi\varepsilon} \ln\left(b/a\right)$$

$$E = \frac{V}{r \ln\left(b/a\right)}$$

$$P = E \times H$$

$$P = |E||H| \sin\theta$$

And we know $\theta = 90°$ since e and H are perpendicular

$$P = EH$$

$$= \frac{v}{r\ln\left(b/a\right)} \cdot \frac{I}{2\pi r}$$

Total power $= \int P ds$

$$= \int_a^b \frac{v}{r\ln\left(b/a\right)} \cdot \frac{I}{2\pi r} . (2\pi r)\, dr$$

$$= \frac{vI}{r\ln\left(b/a\right)} \cdot \int_a^b \frac{dr}{r}$$

Total power $= \frac{vI}{r\ln\left(b/a\right)} \ln\left(b/a\right)$

Total power = V I

Summary

1) $\varepsilon = \frac{-d\phi}{dt}$

2) $\int_{\ell} H.dl = I = \iint_{s} J.ds$

$$\oint_{\ell} E.dl = -\iint_{s} \frac{\partial B}{\partial t}.ds$$
$$\oiint_{s} \overline{D}.nds = Q = \iiint_{V} \rho_V dv$$
$$\oiint_{s} \overline{B}.n\,ds = 0$$

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$
$$\nabla \times E = \frac{-\partial B}{\partial t}$$
$$\nabla.D = \rho$$
$$\nabla.B = 0$$

3) $P = E \times H \;\; Wwatts/m^2$

Discuss about the wave equation of perfect dielectric

Wave equation for a perfect dielectric:-

The Maxwell's equation for a perfect is as follows

$$\nabla \times H = \sigma E + \varepsilon \frac{\partial E}{\partial t}$$
$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$
$$\nabla.D = \rho$$
$$\nabla.B = 0$$

For a perfect dielectric, $\sigma = 0 \& \rho = 0$

$$\nabla \times H = \varepsilon \frac{\partial E}{\partial t} \quad \rightarrow (1)$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t} \quad \rightarrow (2)$$

$$\nabla.D = 0 \quad \rightarrow (3)$$

$$\nabla.B = 0 \quad \rightarrow (4)$$

Talking curl of (1) & differentiating (2), we get

$$\nabla \times \nabla \times H = \varepsilon \frac{\partial}{\partial t}(\nabla \times E) \Rightarrow \varepsilon \nabla \times \frac{\partial E}{\partial t} \quad \rightarrow (5)$$

$$\nabla \times \frac{\partial E}{\partial t} = -\mu \frac{\partial^2 H}{\partial t^2} \quad \rightarrow (6)$$

Sub (6) in (5), we get

$$\nabla\times\nabla\times H=-\mu\varepsilon\frac{\partial^2 H}{\partial t^2}$$

$$\nabla(\nabla.H)-\nabla^2 H=-\mu\varepsilon\frac{\partial^2 H}{\partial t^2}$$

$$-\nabla^2 H=-\mu\varepsilon\frac{\partial^2 H}{\partial t^2}$$

$$\nabla^2 H=\mu\varepsilon\frac{\partial^2 H}{\partial t^2} \quad \rightarrow (7)$$

Similarly, talking curl of (2) & differentiating w. r. t

$$\nabla\times\nabla\times E=-\mu\frac{\partial}{\partial t}(\nabla\times H)$$

$$\nabla\times\nabla\times E=-\mu\left(\nabla\times\frac{\partial H}{\partial t}\right) \quad \rightarrow (8)$$

$$\nabla\times\frac{\partial H}{\partial t}=\varepsilon\frac{\partial^2 E}{\partial t^2} \quad \rightarrow (9)$$

Substituting (9) in (8),

$$\nabla\times\nabla\times E=-\mu\varepsilon\frac{\partial^2 E}{\partial t^2}$$

$$\nabla(\nabla.E)-\nabla^2 E=-\mu\varepsilon\frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 E=\mu\varepsilon\frac{\partial^2 E}{\partial t^2} \quad \rightarrow (10)$$

Wave equation for a conducting medium

$$\nabla\times H=\sigma E+\varepsilon\frac{\partial E}{\partial t} \quad \rightarrow (1)$$

$$\nabla\times E=-\mu\frac{\partial H}{\partial t} \quad \rightarrow (2)$$

$$\nabla.D=0 \quad \rightarrow (3)$$

$$\nabla.B=0 \quad \rightarrow (4)$$

Talking curl of (1) & differentiating (2), we get

$$\nabla\times\nabla\times H=\sigma(\nabla\times E)+\varepsilon\left(\nabla\times\frac{\partial E}{\partial t}\right) \quad \rightarrow (5)$$

$$\nabla\times\frac{\partial E}{\partial t}=-\mu\frac{\partial^2 H}{\partial t^2} \quad \rightarrow (6)$$

Sub (2) & (6) in (5), we get

$$\nabla(\nabla.H) - \nabla^2 H = - - \mu\sigma\frac{\partial H}{\partial t} - \mu\varepsilon\frac{\partial^2 H}{\partial t^2}$$

$$-\nabla^2 H = -\mu\sigma\frac{\partial H}{\partial t} - \mu\varepsilon\frac{\partial^2 H}{\partial t^2}$$

$$\boxed{\nabla^2 H = \mu\sigma\frac{\partial H}{\partial t} + \mu\varepsilon\frac{\partial^2 H}{\partial t^2}}$$

Similarly, talking curl of (2) & differentiating (1)

$$\nabla \times \nabla \times E = -\mu\nabla \times \frac{\partial H}{\partial t}$$

$$\nabla \times \frac{\partial H}{\partial t} = \sigma\frac{\partial E}{\partial t} + \varepsilon\frac{\partial^2 E}{\partial t^2}$$

$$\nabla(\nabla.E) - \nabla^2 E^2 = -\mu\sigma\frac{\partial E}{\partial t} - \mu\varepsilon\frac{\partial^2 E}{\partial t^2}$$

$$\boxed{\nabla^2 E = \mu\sigma\frac{\partial E}{\partial t} + \mu\varepsilon\frac{\partial^2 E}{\partial t^2}}$$

Wave equation:-

For perfect Dielectric:- $\nabla^2 E = \mu\varepsilon\frac{\partial^2 E}{\partial t^2}$

$$\nabla^2 H = \mu\varepsilon\frac{\partial^2 H}{\partial t^2}$$

For perfect conducting medium:- $\nabla^2 E = \mu\sigma\frac{\partial E}{\partial t} + \mu\varepsilon\frac{\partial^2 E}{\partial t^2}$

$$\nabla^2 H = \mu\sigma\frac{\partial H}{\partial t} + \mu\varepsilon\frac{\partial^2 H}{\partial t^2}$$

Explain about the wave equations in a wave equation in a phasor form:-

$$\nabla^2 E = \mu\varepsilon\frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 H = \mu\varepsilon\frac{\partial^2 H}{\partial t^2}$$

Let $E = E_0 e^{j\omega t}$

$$\frac{\partial E}{\partial t} = j\omega E_0 e^{j\omega t}$$

$$= j\omega E$$

$$\frac{\partial^2 E}{\partial t^2} = -\omega^2 E$$

$$\nabla^2 E = \mu\varepsilon(-\omega^2 E) \Rightarrow \nabla^2 E + \omega^2\mu\varepsilon E = 0$$

$$\nabla^2 H = \mu\varepsilon(-\omega^2 H) \Rightarrow \nabla^2 H + \omega^2\mu\varepsilon H = 0$$

Wave equation for a perfect dielectric

$$\boxed{\begin{aligned}\nabla^2 E + \omega^2\mu\varepsilon E = 0\\ \nabla^2 H + \omega^2\mu\varepsilon H = 0\end{aligned}}$$

$$\nabla^2 E = \mu\sigma \frac{\partial E}{\partial t} + \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 H = \mu\sigma \frac{\partial H}{\partial t} + \mu\varepsilon \frac{\partial^2 H}{\partial t^2}$$

$$\nabla^2 E = j\omega\mu\sigma E + j^2\omega^2\mu\varepsilon E$$

$$\nabla^2 E = j\omega\mu E(\sigma + j\omega\varepsilon)$$

$$\nabla^2 H = j\omega\mu H(\sigma + j\omega\varepsilon)$$

$$\boxed{\begin{aligned} &\nabla^2 E - j\omega\mu(\sigma + j\omega\varepsilon)E = 0 \\ &\nabla^2 H - j\omega\mu(\sigma + j\omega\varepsilon)H = 0 \end{aligned}}$$

Velocity of the wave:-

$$\nabla^2 E = \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

For free space $\mu_r = \varepsilon_r = 1$

$$\nabla^2 E = \mu_0\varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$= 4\pi \times 10^{-7} \times \frac{1}{36\pi \times 10^9}$$

$$= \frac{1}{9 \times 10^{16}} \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 E = \frac{1}{(3 \times 10^8)^2} \frac{\partial^2 E}{\partial t^2}$$

We know that the velocity of light = $= 3 \times 10^8 m/s$

$$\nabla^2 E = \frac{1}{\vartheta^2} \frac{\partial^2 E}{\partial t^2}$$

$$\Rightarrow \nabla^2 E - \frac{1}{\vartheta^2} \frac{\partial^2 E}{\partial t^2} = 0$$

Similarly $\nabla^2 H - \frac{1}{\vartheta^2} \frac{\partial^2 H}{\partial t^2} = 0$

Where

$$\frac{1}{\vartheta^2} = \mu_0\varepsilon_0 \Rightarrow \frac{1}{\mu_0\varepsilon_0} = \vartheta^2$$

$$\boxed{\vartheta = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \quad m/s}$$

Discuss about the uniform plane wave uniform plane wave:-

If the phase of a wave is the same for all points on a plane surface, it is called a plane wave.

If the amplitude is also constant, then the wave is called an uniform plane wave.

The following are properties of uniform plane waves.

1) At every point in space electric field (E) * magnetic field H are perpendicular to each other and to the direction of barel.

If the electric field is in x – direction and the magnetic field in y – direction, then the wave is travelling in z – direction.

The wave equation for free space is given by,

$$\nabla^2 E = \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = \mu E \frac{\partial^2 E}{\partial t^2}$$

Consider E varies along 'x' only & independent of y and z, then

$$\frac{\partial^2 E}{\partial x^2} = \mu E \frac{\partial^2 E}{\partial t^2} \qquad \left[\frac{\partial^2 E}{\partial y^2} = \frac{\partial^2 E}{\partial z^2} = 0\right]$$

$$\frac{\partial^2 E_x}{\partial x^2} = \mu E \frac{\partial^2 E_x}{\partial t^2}; \quad \frac{\partial^2 E_y}{\partial x^2} = \mu E \frac{\partial^2 E_y}{\partial t^2}; \quad \frac{\partial^2 E_z}{\partial x^2} = \mu E \frac{\partial^2 E_z}{\partial t^2}$$

$$\nabla . D = \varepsilon \nabla . E = 0 \qquad \nabla . E = 0$$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

For a uniform plane wave, 'E' is independent of y & z.

Then $\frac{\partial E_x}{\partial x} = 0$

Differentiating the above, $\frac{\partial^2 E_x}{\partial x^2} = 0$

If requires that either Ex be zero or constant therefore an uniform wave propagating along x – axis does not have an Ex component.

Similarly,

$$\nabla . B = \mu \nabla . H = 0$$
$$\nabla . H = 0$$
$$\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = 0$$
$$\frac{\partial H_x}{\partial x} = 0 \ \& \ \frac{\partial^2 H_x}{\partial x^2} = 0$$

Since Hx is constant, hx must be zero.

Explain the characteristics impedance

Characteristics impedance or intrinsic impedance:-

Consider the plane wave propagating in x – direction the wave equation for free space is

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

The general solution of the differential equation is in the form

$$E = f_1(x - \nu_0 t) + f_2(x + \nu_0 t)$$

Where

$$\nu_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

f_1 and f_2 are any function of $(x - \nu_0 t) \& (x + \nu_0 t)$ respectively.

The solution of wave equation consists of two waves; one travelling in positive direction and other travelling in the negative direction consider the wave travelling in positive direction above

$$E = f(x - \nu_0 t) \qquad [\because f_2(x + \nu_0 t) = 0]$$

$$\nabla \times E = \begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix}$$

Since wave travelling in x direction, E and H are both independent of y and z.

i.e, $\quad E_x = H_x = 0 \ \& \quad \dfrac{\partial E}{\partial y} = \dfrac{\partial E}{\partial z} = 0$

$$\nabla \times E = \begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & 0 & 0 \\ 0 & E_y & E_z \end{vmatrix}$$

$$\nabla \times E = \frac{-\partial E_z}{\partial x}\bar{a}_y + \frac{\partial E_y}{\partial x}\bar{a}_z \qquad \rightarrow (1)$$

Similarly

$$\nabla \times H = \frac{-\partial H_z}{\partial x}\bar{a}_y + \frac{\partial H_y}{\partial x}\bar{a}_z \qquad \rightarrow (2)$$

But $\quad \nabla \times H = \varepsilon \dfrac{\partial E}{\partial t} \qquad \rightarrow (3)$

Comparing (2) & (3), we get

$$\frac{-\partial H_z}{\partial x}\bar{a}_y + \frac{\partial H_y}{\partial x}\bar{a}_z = \varepsilon\left[\frac{\partial E_y}{\partial t}\bar{a}_y + \frac{\partial E_z}{\partial t}\bar{a}_z\right]$$

$$\frac{-\partial H_z}{\partial x} = \varepsilon\frac{\partial E_y}{\partial t} \ \& \ \frac{\partial H_y}{\partial x} = \frac{\partial E_z}{\partial t}$$

$$\nabla \times E = -\mu\frac{\partial H}{\partial t}$$

$$\frac{-\partial E_z}{\partial x}\bar{a}_y + \frac{\partial E_y}{\partial x}\bar{a}_z = -\mu\left[\frac{\partial H_y}{\partial t}\bar{a}_y + \frac{\partial H_z}{\partial t}\bar{a}_z\right]$$

$$\frac{\partial E_z}{\partial x} = -\mu\frac{\partial H_y}{\partial t}; \ \frac{\partial E_y}{\partial x} = -\mu\frac{\partial H_z}{\partial t}$$

Let the solution of wave equation be,

$$E_y = f(x - \nu_0 t)$$
$$\frac{\partial E_y}{\partial t} = \frac{\partial f}{\partial (x - \nu_0 t)} \cdot \frac{\partial (x - \nu_0 t)}{\partial t}$$
$$= f'(x - \nu_0 t) \times (-\nu_0)$$
$$\frac{\partial E_y}{\partial t} = -\nu_0 F' \qquad [\because f'(x - \nu_0 t) = F']$$
$$\frac{-\partial H_z}{\partial x} = \varepsilon \frac{\partial E_y}{\partial t} \Rightarrow \frac{-\partial H_z}{\partial x} = \varepsilon(-\nu_0 F')$$
$$\frac{\partial H_z}{\partial x} = \nu_0 \varepsilon F'$$
$$= \frac{1}{\sqrt{\mu\varepsilon}} \varepsilon F'$$
$$\frac{\partial H_z}{\partial x} = \sqrt{\varepsilon / \mu}\, F'$$

$$\partial H_z = \sqrt{\varepsilon / \mu} \int F' dx$$
$$= \sqrt{\varepsilon / \mu}\, f$$

$$H_z = \sqrt{\varepsilon / \mu}\, E_y$$
$$\frac{E_y}{H_z} = \sqrt{\mu / \varepsilon}$$

Similarly, it can be shown that,

$$\frac{E_z}{H_y} = -\sqrt{\mu / \varepsilon}$$
$$E = \sqrt{E_y^2 + E_z^2} \;\; \& \;\; H = \sqrt{H_y^2 + H_z^2}$$

Therefore $\frac{E}{H} = \sqrt{\mu / \varepsilon}$

This is referred to as characteristics impedance or intrinsic impedance. It is defined as the ratio of permittivity to dielectric constant

$$\eta = \frac{E}{H} = \sqrt{\mu / \varepsilon}$$

For free space $\varepsilon_x = \mu_r = 1,$ then the character impedance for free space is given by

$$\eta_0 = \sqrt{\mu_0 / \varepsilon_0}$$
$$= \sqrt{\frac{4\pi \times 10^{-7}}{36\pi \times 10^{9}}}$$
$$\eta_0 = 120\pi / 377\Omega$$

Dot product and cross product of E and H:-

$$E.H = E_y H_y + E_z H_z$$

$$\eta = \frac{E_y}{H_z} = \frac{E_z}{H_y}$$

$$E.H = \eta H_y H_z - \eta H_y H_z$$

$$E.H = 0$$

This proves that E and H both perpendicular to each

$$E \times H = \begin{vmatrix} a_x & a_y & a_z \\ E_x & E_y & E_z \\ H_x & H_z & H_z \end{vmatrix} = \begin{vmatrix} a_x & a_y & a_z \\ 0 & E_y & E_z \\ 0 & H_z & H_z \end{vmatrix}$$

$$= \bar{a}_x \left[E_y H_z - E_z H_z \right]$$

$$= \bar{a}_x \left[\eta H_z^2 + \eta H_y^2 \right]$$

$$E \times H = \eta H^2 \bar{a}_x$$

Derive the wave propagation in a wave propagation in a lossless medium

Wave propagation in a lossless medium:-

The wave equation for free space lossless medium is,

$$\nabla^2 E = \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

The phasor value of E is

$$E(x,t) = R_e \left[E(x) e^{j\omega t} \right]$$

$$\nabla^2 R_e \left[E(x) e^{j\omega t} \right] = \mu\varepsilon \frac{\partial^2}{\partial t^2} R_e \left[E(x) e^{j\omega t} \right]$$

$$\nabla^2 R_e \left[E(x) e^{j\omega t} \right] = \mu\varepsilon R_e \left[-\omega^2 E e^{j\omega t} \right]$$

$$R_e \left[(\nabla^2 E + \mu\varepsilon\omega^2 E) e^{j\omega t} \right] = 0$$

$$\nabla^2 E + \mu\varepsilon\omega^2 E = 0$$

$$\nabla^2 E + \beta^2 E = 0$$

This is called vector Helmholtz equation

$$\beta^2 = \omega^2 \mu\varepsilon$$

$$\beta = \sqrt{\mu\varepsilon}\ \ \omega$$

Where β is called phase shift constant

The velocity of propagation is

$$v = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}}$$

Wave equation in a conducting medium.

The wave equation for conducting medium is

$$\nabla^2 E - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} - \mu\sigma \frac{\partial E}{\partial t} = 0$$

The phasor form of wave equation is,

$$\nabla^2 E - j^2\omega^2\mu\varepsilon E - j\omega\mu\sigma E = 0$$
$$\nabla^2 E - j\omega\mu(\sigma + j\omega\varepsilon)E = 0$$
$$\nabla^2 E - \gamma^2 E = 0$$

Where $\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$

γ is called propagation constant

$$\gamma = \alpha + j\beta$$

Where α = attenuation constant

β =Phase shift

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

Squaring on both sides,

$$\alpha^2 - \beta^2 + 2j\alpha\beta = j\omega\mu(\sigma + j\omega\varepsilon)$$
$$\alpha^2 - \beta^2 = -\omega^2\mu\omega$$
$$\alpha \infty \beta = \omega\mu\sigma$$

We know that,

$$\alpha^2 + \beta^2 = \sqrt{\left(\alpha^2 - \beta^2\right)^2 + 4\alpha^2\beta^2}$$

But $\left(\alpha^2 - \beta^2\right)^2 = \left(-\omega^2\mu\varepsilon\right)^2$

$$\left(\alpha \;\infty\; \beta\right)^2 = \left(\omega\mu\sigma\right)^2$$
$$\alpha^2 + \beta^2 = \sqrt{\omega^4\mu^2\varepsilon^2 + \omega^2\mu^2\sigma^2}$$
$$\alpha^2 - \beta^2 = -\omega^2\mu\varepsilon$$
$$2\alpha^2 = -\omega^2\mu\varepsilon + \sqrt{\omega^4\mu^2\varepsilon^2 + \omega^2\mu^2\sigma^2}$$
$$\alpha^2 = \frac{-\omega^2\mu\varepsilon}{2} + \sqrt{\frac{\omega^4\mu^2\varepsilon^2}{2} + \frac{\omega^2\mu^2\sigma^2}{2}}$$
$$= \frac{-\omega^2\mu\varepsilon}{2} + \frac{\omega^2\mu\varepsilon}{2}\sqrt{1 + \frac{\sigma^2}{\omega^2\varepsilon^2}}$$
$$\alpha = \sqrt{\frac{\omega^2\mu\varepsilon}{2}\left[\sqrt{1 + \frac{\sigma^2}{\omega^2\varepsilon^2}} - 1\right]}$$
$$\boxed{\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[1 + \frac{\sigma^2}{\omega^2\varepsilon^2} - 1\right]}}$$

To find the value of β :-

$$2\beta^2 = \sqrt{\omega^4\mu^2\varepsilon^2 + \omega^2\mu^2\sigma^2} + \omega^2\mu\varepsilon$$

$$\beta^2 = \frac{\omega^2\mu\varepsilon}{2} + \sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}} + \frac{\omega^2\mu\varepsilon}{2}$$

$$\beta^2 = \frac{\omega^2\mu\varepsilon}{2}\sqrt{\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}} - 1}$$

$$\boxed{\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[1+\frac{\sigma^2}{\omega^2\varepsilon^2} - 1\right]}}$$

Wave propagation in good dielectric:-

$\frac{\sigma}{\omega\varepsilon} >> 1$ For good conductor

$\frac{\sigma}{\omega\varepsilon} << 1$ For good dielectric

For dielectrics, $\frac{\sigma}{\omega\varepsilon} << 1$

$$\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}} = \left(1+\frac{\sigma^2}{\omega^2\varepsilon^2}\right)^{1/2}$$

$$\approx 1+\frac{\sigma^2}{2\omega^2\varepsilon^2}$$

The attenuation factor is,

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[1+\frac{\sigma^2}{\omega^2\varepsilon^2} - 1\right]}$$

$$= \omega\sqrt{\frac{\mu\varepsilon}{2}\left[1+\frac{\sigma^2}{2\omega^2\varepsilon^2} - 1\right]}$$

$$= \omega\sqrt{\frac{\mu\sigma^2\varepsilon}{4\omega^2\varepsilon^2}} = \sqrt{\frac{\mu\sigma^2}{4\varepsilon}} \approx \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

Phase shift, $\beta = \sqrt{\frac{\omega^2\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}} + 1\right]}$

$$\beta = \sqrt{\frac{\omega^2\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{2\omega^2\varepsilon^2}} + 1\right]}$$

$$= \sqrt{\omega^2\mu\varepsilon\left[\sqrt{1+\frac{\sigma^2}{4\omega^2\varepsilon^2}}\right]}$$

$$= \omega\sqrt{\mu\varepsilon\left[1+\frac{\sigma^2}{4\omega^2\varepsilon^2}\right]}$$

$$\beta \approx \omega\sqrt{\mu\varepsilon}\left[1+\frac{\sigma^2}{4\omega^2\varepsilon^2}\right]^{1/2}$$

$$\beta \approx \omega\sqrt{\mu\varepsilon}\left[1+\frac{\sigma^2}{8\omega^2\varepsilon^2}\right]$$

Velocity of wave in dielectric is $v = \frac{\omega}{\beta}$

$$\nu = \frac{\omega}{\omega\sqrt{\mu\varepsilon}\left[1+\frac{\sigma^2}{8\omega^2\varepsilon^2}\right]} \approx \frac{1}{\sqrt{\mu\varepsilon}}\left[1+\frac{\sigma^2}{8\omega^2\varepsilon^2}\right]^{-1}$$

$$\nu = \frac{1}{\sqrt{\mu\varepsilon}}\left(1-\frac{\sigma^2}{8\omega^2\varepsilon^2}\right)$$

$$\boxed{\nu \approx \nu_0\left[1-\frac{\sigma^2}{8\omega^2\varepsilon^2}\right]}$$

Intrinsic or characteristics impedance,

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon\left(1+\frac{\sigma}{j\omega\varepsilon}\right)}}$$

$$= \sqrt{\frac{\mu}{\varepsilon}\left(1+\frac{\sigma}{j\omega\varepsilon}\right)^{-1}}$$

$$= \sqrt{\frac{\mu}{\varepsilon}}\left(1+\frac{\sigma}{j\omega\varepsilon}\right)^{1/2}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}\left(1+\frac{j\sigma}{2\omega\varepsilon}\right)$$

$$\boxed{\eta = \eta_0\left[1+\frac{j\sigma}{2\omega\varepsilon}\right]}$$

Wave propagation in good conductors:- or plane waves in good conductors

For good conductor $\frac{\sigma}{\omega\varepsilon} >> 1$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

$$= \sqrt{j\omega\mu\sigma\left[1+\frac{j\omega\varepsilon}{\sigma}\right]}$$

$$\gamma = \sqrt{j\omega\mu\sigma} = \sqrt{\omega\mu\sigma\,\underline{|90^\circ}}$$

$$\boxed{\gamma = \sqrt{\omega\mu\sigma}\,\underline{|45^\circ}}$$

W. K.T

$$\alpha = \sqrt{\frac{\omega^2\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}-1\right]}$$

$$\alpha = \sqrt{\frac{\omega^2\mu\varepsilon}{2}\left(\frac{\sigma}{\omega\varepsilon}\right)}$$

$$\boxed{\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}}$$

W>K>T

$$\beta = \sqrt{\frac{\omega^2\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}+1\right]}$$

$$= \sqrt{\frac{\omega^2\mu\varepsilon}{2}\left(\frac{\sigma}{\omega\varepsilon}\right)}$$

$$\boxed{\beta = \sqrt{\frac{\omega\mu\sigma}{2}}}$$

Velocity of wave in conductor, $\nu = \frac{\omega}{\beta}$

$$\nu = \frac{\omega}{\sqrt{\frac{\omega\mu\sigma}{2}}} = \sqrt{\frac{2\omega^2}{\omega\mu\sigma}} = \sqrt{\frac{2\omega}{\mu\sigma}}$$

Intrinsic impedance $\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$

$$\eta = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon\left(1+\frac{\sigma}{j\omega\varepsilon}\right)}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon.\frac{\sigma}{j\omega\varepsilon}}} = \sqrt{\frac{j\omega\mu}{\sigma}}$$

$$\boxed{\eta = \sqrt{\frac{\omega\mu}{\sigma}}\underline{|90^\circ}}$$

In good conductor, α and β large. Sine σ is large (i.e) wave is attenuated greatly as it propagation through the conductor.

But velocity and characteristics impedance is considerably reduced.

Explain skin depth or penetration?

Depth of penetration:-

In a good conductor, the wave is attenuated as it progress. At radio frequency the ratio of attenuation is very large and the wave may penetration only a very short distance before being reduced to a negligibly small value.

The depth of penetration (δ) or skin depth is defined as the depth in which the wave has been attenuated to $\frac{1}{e}$ or approximate to 37% of its original value.

The amplitude of wave decreases by a factor $e^{-\alpha\delta}$ as it propagates through a distance s.

By definition, $e^{-\alpha S} = e^{-1}$

$$\alpha S = 1$$

$$S = \frac{1}{\alpha}$$

Skin depth is the distance S through which the wave amplitude decrease to factor e^{-1} i.e, about 37% of the original value.

$$S = \frac{1}{\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}-1\right]}}$$

For good conductor, $\frac{\sigma}{\omega\varepsilon} >> 1$

$$S = \frac{1}{\omega\sqrt{\frac{\mu\varepsilon}{2}\left(\frac{\sigma}{\omega\varepsilon}\right)}} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

$$S = \sqrt{\frac{2}{2\pi f\mu\sigma}}$$

$$S = \sqrt{\frac{1}{\pi f\mu\sigma}}$$

$$S = \frac{1}{\sqrt{\pi f\mu\sigma}}$$

PROBLEMS UNIT – IV

1. Find the force on the charged particle of mass 1.7×10^{-27} kg and charge 1.602×10^{-19}c, if it enters a field of flux density B = 10 mw b/m^2 with an initial velocity of 90 km/s.

Solution:-

B = 10 m w b/m^2

Q = 1.602×10^{-19}C

v = 90 km/s

Assume v & B are perpendicular

$$\begin{aligned} F &= QvB \\ &= 1.602\times10^{-19}\times90\times10^{3}\times10\times10^{-3} \\ &= 1.602\times10^{-19}\times9\times10^{4}\times1\times10^{-2} \end{aligned}$$

F = 14.418 $\times 10^{-17}$ N

2. A point charge of 4c moves a velocity of $5\bar{a}_x + 6\bar{a}_y - 7\bar{a}_z$ m/s . Find the force exerted if the flux density is $5a_x + 7a_y + 9a_z \text{Wb/m}^2$.

Solution:-

$$Q = 4c \quad \vec{B} = 5\bar{a}_x + 7\bar{a}_y + 9\bar{a}_z \ \& \ v = 5\bar{a}_x + 6\bar{a}_y - 7\bar{a}_z$$

$$\bar{F}_m = Q(v\times B)$$

$$v\times B = \begin{vmatrix} a_x & a_y & a_z \\ 5 & 6 & -7 \\ 5 & 7 & 9 \end{vmatrix}$$

$$= \bar{a}_x[54+49] - \bar{a}_y[45+35] + \bar{a}_z[35-30]$$

$$= 103\bar{a}_x - 80\bar{a}_y + 5\bar{a}_z$$

$$F_m = Q(v \times B) = 4(103\bar{a}_x - 80\bar{a}_y + 5\bar{a}_z)$$

$$F_m = 412\bar{a}_x - 320\bar{a}_y + 20\bar{a}_z \text{ N.}$$

$$|F_m| = \sqrt{(412)^2 + (-320)^2 + (20)^2}$$

$$|F_m| = 522.05\,\text{N}$$

3. If the magnetic field intensity is $\vec{H} = \left[(0.01)/\mu_0 \bar{a}_x\right] A/m$. What is the force on a charge of 1 pc moving with a velocity of $10^6 \vec{a}_y \, m/sv$

Solution:-

Given $\vec{H} = \left(\dfrac{0.01}{\mu_0}\right)\bar{a}_x \ A/m \quad Q = 1pc = 10^{-12}C \ \& \ v = 10^6 \bar{a}_y \, m/s$

$$F_m = Qv \times B$$

$$B = \mu_0 H = \mu_0 \left(\frac{0.01}{\mu_0}\right)\bar{a}_x$$

$$\bar{B} = 0.01\bar{a}_x$$

$$v \times B = \begin{vmatrix} a_x & a_y & a_z \\ 0 & 10 & 0 \\ 0.01 & 0 & 0 \end{vmatrix} = \vec{a}_x(10^6 \times 0 - 0 \times 0) - \vec{a}_y(0-0) + \vec{a}_z(0 \times 0 - 10^6(0.01)$$

$$= -0.01 \times 10^6 \bar{a}_z = -1 \times 10^{-2} \times 10^6 = -10^4 \bar{a}_z$$

$$F_m = Q(\vec{v} \times \vec{B}) = 10^{-12} \times (-10^4) = -10^{-8}\bar{a}_z \, N$$

Force on differential current element

4. A magnetic field of flux density $B = 4.5 \times 10^{-2}\,\bar{a}_z Wb/m^2$ exerts a force on a 0.4m long conductor along x – axis. If a current of 10A flows in $\bar{a}_x$ direction, determine the force that must be applied to hold conductor in position.

Solution:-

Given I = 10A, $\ell = -0.4\bar{a}_x$ & $B = 4.5 \times 10^{-2}\ \bar{a}_z\ Wb/m^2$

The force exerted on a straight conductor is

$$\vec{F} = I\bar{\ell} \times \bar{B}$$

$$= 10(-0.4\bar{a}_x) \times (4.5 \times 10^{-2}\bar{a}_z)$$

$$= -4 \times 4.5 \times (-a_y)$$

$$F = 18\bar{a}_y N$$

5. Calculate the force on a straight conductor of length 30 cm carrying of 5a in a magnetic field along the z – axis. The magnetic flux density is $\vec{B} = 3.5 \times 10^{-3}(\bar{a}_x - \bar{a}_y) Wb/m^2$, where $\bar{a}_x$ and $\bar{a}_y$ are unit vector.

Solution:-

Given $\ell = 30\text{cm} = 30\times10^{-2}\text{m} = 0.3\bar{a}_z, I = 5\text{A}, B = 3.5\times10^{-3}(a_x - \bar{a}_y)$

$$F = I\ell\times B = 5\times0.3\bar{a}_z\times3.5\times10^{-3}(a_x - a_y)$$
$$= 1.5\times3.5\times10^{-3}\times a_z\times(a_x - a_y)$$
$$F = 5.25\bar{a}_x + 5.25\bar{a}_y\text{mN}$$
$$= 5.25\sqrt{1+1}\times10^{-3}$$
$$F = 7.42\text{mN}$$

6. A current 4m in length lies along y- axis centred at origin. The current is 10Ain $\bar{a}_y$ direction. If it experiences of force of $15\left(\frac{a_x + a_z}{\sqrt{2}}\right)\text{N}$ due to a uniform magnetic filed, determine B & H in free space.

Solution:-

The force exerted on straight current element in uniform magnetic filed is

$$\vec{F} = I\bar{\ell}\times\bar{B}$$

$$\frac{15}{\sqrt{2}}(a_x + a_z) = \left[(10)(4a_y)\times(B_x a_x + B_y a_y + B_z a_z\right]$$

$$10.61(a_x + a_z) = \begin{bmatrix} a_x & a_y & a_z \\ 0 & 40 & 0 \\ B_x & B_y & B_z \end{bmatrix}$$

$$10.61a_x + 10.61a_z = a_x(40B_z) - a_y(0) + a_z(0 - 40B_x)$$
$$10.61a_x + 10.61a_z = 40B_z\bar{a}_x - 40B_z\bar{a}_z$$
$$40B_z = 10.61 \qquad B_z = \frac{10.61}{40} = 0.265$$
$$-40B_x = 10.61 \qquad B_x = -0.265$$
$$\bar{B} = -0.265a_x + 0.265a_z$$
$$= 0.265(-a_x + a_z)\text{T}$$
$$\bar{H} = \frac{\bar{B}}{\mu_0} = \frac{0.265(-a_x + a_z)}{4\pi\times10^{-7}}$$
$$\bar{H} = 0.211\times10^6(-a_x + a_z)\text{A/m}$$

7. A conductor of length 5m located at z = 0, x = 4m carries a current of 10A in the $-\bar{a}_y$ direction. Find the component of $\bar{B}$ in the region if the force on the conductor is $1.2\times10^{-2}\text{N}$ in the direction $(-a_x + \bar{a}_z)/\sqrt{2}$.

Solution:-

$$\bar{F} = (1.2\times10^{-2})\left(\frac{-a_x + \bar{a}_z}{\sqrt{2}}\right) I = 10\text{A} \qquad \ell = -5a_y$$

$$\bar{F} = I\ell\times\bar{B}$$

$$\frac{1.2\times10^{-2}}{\sqrt{2}}(-a_x + a_z) = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 0 & -50 & 0 \\ B_x & B_y & B_z \end{vmatrix}$$

$$= -50B_z\bar{a}_x + 50B_x\bar{a}_z$$

$$\frac{1.2\times10^{-2}}{\sqrt{2}}\left(-a_x + a_z\right) = +50(-B_z a_x + B_x a_z)$$

Comparing the co – efficient

$$B_z = B_x = \frac{1.2\times10^{-2}}{50\sqrt{2}} = 1.7\times10^{-4}\ \text{Wb/m}^2$$

8. Consider two long parallel wires 2m apart carry current of 50 A and 100 A in the same direction. Determine the magnitude and direction of force between then / unit length.

Solution:-

$$I_1 = 50\text{A} \quad I_2 = 100\text{A} \qquad d = 2\text{m}$$

$$\frac{\vec{F}}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi d} = \frac{4\pi\times10^{-7}\times50\times100}{2\pi\times2}$$

$$= \frac{2\times5\times10^{-4}}{2} = 5\times10^{-4}$$

$$\frac{\vec{F}}{\ell} = 0.5\text{mN/m}\ \vec{a}_n$$

9. Consider two long parallel conductor carry 80A. If they are separated by 3mm, find the force / m of each conductor if the current flowing through them in opposite direction

Solution:-

Given $I_1 = I_2 = 80a \qquad d = 3\text{mm} = 3\times10^{-3}$

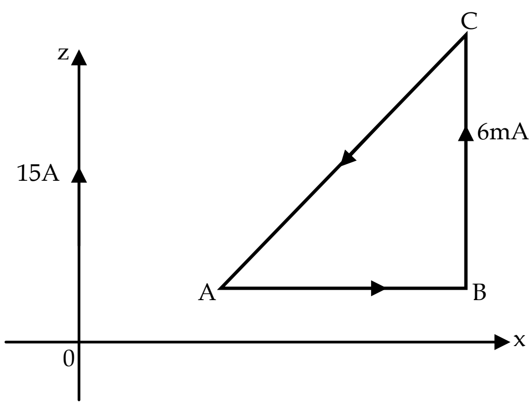
$$\frac{F}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi d} = \frac{4\pi\times10^{-7}\times80\times80}{2\pi\times3\times10^{-3}} = \frac{2\times\cancel{64}(21.11)\times10^{-5}}{\cancel{3}}\times10^{3}$$

$$= 42.22\times10^{-2} = 0.42\text{N/m}$$

As per currents are in opposite direction, the two conductors will repel with equal force.

10. A triangular loop of wire in free space join points A (1, 0, 1) B (3, 0, 1) C (3, 0, 4). The wire carries a current of 6mA flowing in the $\bar{a}_z$ direction from B to C. A filamentary current of 15A flows along the entire z – axis in the $\bar{a}_z$ direction. Find the total force on the loop.

Solution:-



The current flowing through the loop is 6ma & the current along z – axis is I_1= 15A

$$\bar{B} = \mu_0 \bar{H} = \frac{\mu_0 I_1}{2\pi\rho}\bar{a}_\phi$$

In rectangular co – ordinates

$$\bar{B} = \frac{\mu_0 I_1}{2\pi x}(-a_y) = \frac{-4\pi\times 10^{-7}\times 15\times -a_y}{2\pi x}$$

$$\bar{B} = \frac{-3\times 10^{-6}}{x}\bar{a}_y$$

The differential force along AB is

$$d\bar{F}_{AB} = I\bar{dl}_{AB}\times \bar{B}$$

$$\bar{dl}_{AB} = dx\bar{a}_x$$

$$d\bar{F}_{AB} = Idx\bar{a}_x \times \left(\frac{-3\times 10^{-6}}{x}\right)\bar{a}_y$$

$$= (6\times 10^{-3})\left(\frac{-3\times 10^{-6}}{x}\right)dx\ \ a_x \times a_y$$

$$= \frac{-18}{x}10^{-9}dx\,\bar{a}_z$$

$$\bar{F}_{AB} = -18\int_1^3 \frac{dx}{x}\times 10^{-9}\bar{a}_z$$

$$= -18\left[\ln(3)-\ln(1)\right]\times 10^{-9}a_z$$

$$F_{AB} = -19.77\bar{a}_z nN$$

The differential force along the side BC is

$$d\bar{F}_{BC} = I\bar{dl}\times \bar{B}$$

$$d\bar{F}_{BC} = 6\times 10^{-3}dz\ a_z \times \left(\frac{-3\times 10^{-6}}{x}\right)a_y$$

$$= 6\times 10^{-3}\times \left(\frac{-3\times 10^{-6}}{3}\right)dz\times -a_x$$

$$= 6\times 10^{-9}dz\,\bar{a}_x$$

$$\bar{F}_{BC} = 6\times 10^{-9}\bar{a}_x \int_1^4 dz = 6\times 10^{-9}\ a_x\left[3\right]$$

$$F_{BC} = 18nN\,\bar{a}_x$$

The differential force along the side CA is

$$d\bar{F}_{CA} = I\bar{dl}_{CA}\times \bar{B}$$

$$d\bar{F}_{CA} = 6\times 10^{-3}(-dx\bar{a}_x - dz\bar{a}_z \times \left(\frac{-3\times 10^{-6}}{x}\right)a_y$$

$$= 18\times 10^{-9}\int_3^1 \frac{dx}{x}\bar{a}_z - \int_4^1 \frac{18\times 10^{-9}}{3}\bar{a}_x$$

$$F_{CA} = -19.77\times 10^{-9}\bar{a}_z + 18\times 10^{-9}\bar{a}_x$$

$\vec{F} = \bar{F}_{AB} + \bar{F}_{BC} + \bar{F}_{CA}$

$= -19.77 \times 10^{-9} \bar{a}_z + 18 \times 10^{-9} a_x - 19.77 \times 10^{-9} a_z + 18 \times 10^{-9} a_x$

$\vec{F} = 36 a_x - 39.54 \bar{a}_z nN$

11. Find the maximum torque on a 75 turn, rectangular coil, 0.5m by 0.6m carrying a current of 4A in a magnetic filed of B = 5T

Solution:-

Given $\ell = 0.5m$ $w = 0.6m$ $I = 4A$, $N = 75$ $B = 5T$

$$T = N B I A \sin\theta$$

Maximum torque is obtained when $\theta = 90°$

$$T_{max} = NBI(\ell w)$$
$$= 75 \times 5 \times 4 \times (0.5 \times 0.6)$$

$$T_{max} = 450$$

12. A 200 turn coil of 30cm $\times$ 15cm with a current of 5A is placed in a uniform field of flux density B = 0.2T. Determine the magnetic moment m & maximum torque.

Solution:-

Given $\ell = 30cm = 30 \times 10^{-2}$; $w = 15cm = 15 \times 10^{-2}$; $I = 5A$, $N = 200$; $B = 0.2T$

$$m = NIA = 200 \times 5 \times 30 \times 15 \times 10^{-4}$$
$$= 45 A.m^2$$
$$T_{max} = mB = 45 \times 0.2$$
$$T_{max} = 9Nm$$

13. A square coil of 200 turns & 0.5 m long sides is in a region of uniform field with density 0.2T. If the maximum torque is 4×10^{-2}N. m , what is the current?

Solution:-

Given $A = a^2 = (0.5)^2 = 0.25m^2$

$N = 200$, $B = 0.2T$ $T_{max} = 4 \times 10^{-2}$

$$T_{max} = NIAB$$

$$\frac{4 \times 10^{-2}}{200 \times 0.25 \times 0.2} = I$$
$$I = 4 \times 10^{-2} A = 0.4mA$$

Problems on Torque

14. A rectangular coil of area 10cm^2 surrounded by uniform magnetic flux density of $B = 0.6a_x + 0.4a_y + 0.5a_z$ carrying current of 50A lies on plane 2x +6y – 3z =7 such that the magnetic moment of the coil is directed away from the origin. Determine (i) magnetic moment (ii) Torque (iii) Maximum torque

Solution:-

Given Area $A = 10cm^2, B = 0.6\bar{a}_x + 0.4\bar{a}_y + 0.5a_z Wb/m^2, I = 50A$

(i) Magnetic moment is

$$m = 1A\bar{a}_n = 50\left(10\times10^{-4}\right)\left(\frac{2a_x + 16a_y - 3a_z}{\sqrt{49}}\right)$$
$$3 = \left(14.29\bar{a}_x + 42.86\bar{a}_y - 21.43\bar{a}_z\right)\times10^{-3} A.m^2$$

(ii) The torque on the coil is

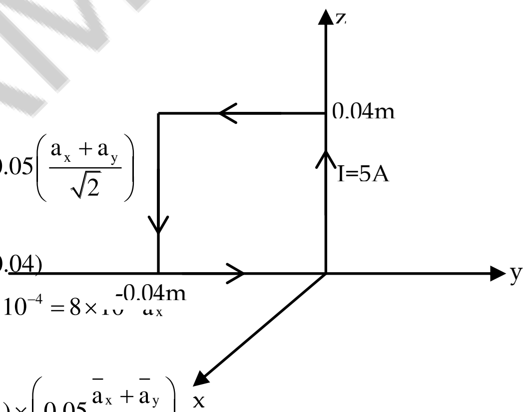
$$\vec{T} = m \times \bar{B}$$
$$= \left[\frac{(50)\left(10\times10^{-4}\right)}{7}\left(2a_x + 6a_y - 3a_z\right)\right]\times\left[0.6a_x + 0.4a_y + 0.3a_z\right]$$
$$= \frac{(50)\left(10\times10^{-4}\right)}{7\times10}\begin{bmatrix} a_x & a_y & a_z \\ 2 & 6 & -3 \\ 6 & 4 & 5 \end{bmatrix}$$
$$= 7.143\times10^{-4}\left[42\bar{a}_x - 28\bar{a}_y - 4\bar{a}_z\right]$$
$$= 0.03\bar{a}_x - 0.02\bar{a}_y - 0.025\bar{a}_z N.m$$

$$T_{max} = BIA$$

$$= (0.6a_x + 0.4a_y + 0.5a_z)(50)(10\times10^{-4})$$
$$= 3a_x + 2a_y + 2.5a_z \times 10^{-2}$$
$$= 30a_x + 20a_y + 25a_z \times 10^{-3}$$
$$= 43.87\times10^{-3} N.m$$

15. A square coil is shown in figure in figure below is placed in the magnetic field of flux density $\bar{B} = 0.05\left(\frac{a_x + a_y}{\sqrt{2}}\right) Wb/m^2$

Solution:-



$$I = 5A \;\&\; B = 0.05\left(\frac{a_x + a_y}{\sqrt{2}}\right)$$
$$m = IA$$
$$= 5(0.04\times0.04)$$
$$= 5\times4\times4\times10^{-4} = 8\times10^{-3} a_x$$
$$\vec{T} = m\times\bar{B}$$
$$= (8\times10^{-3}a_x)\times\left(0.05\frac{\bar{a}_x + \bar{a}_y}{\sqrt{3}}\right)$$
$$\vec{T} = 0.231\bar{a}_z mN.m$$

16. A solenoid 25cm long and of 1cm mean diameter of the coil turns has a uniform distribution winding of 2000 turns. If the solenoid is placed in a uniform field of flux density 2Wb/m2 and a current of 5A is

passed through the solenoid winding, determine (i) The maximum force on the solenoid (ii) torque on the solenoid

Solution:-

Given $\ell = 25\text{cm} = 0.25\text{m}$

$$d = 1\text{cm} = 0.01\text{m}$$
$$I = 5\text{A} \quad N = 2000 \;\; B = 2\text{Wb}/\text{m}^2$$

Area of solenoid loop = $A = \pi r^2 = \dfrac{\pi d^2}{2} = 0.25\pi \times 10^{-4}\text{m}^2$

(i) The force on the solenoid is

$$\vec{F} = I\bar{\ell} \times \bar{B}$$
$$F = BI\ell = 2 \times 5 \times 0.25$$
$$= 2.5\,\text{newton}/\text{unit}$$

For 2000 turns, we have

$$F = 2.5 \times 200 = 5000\text{N}$$

For torque on the solenoid d is

$$T = BIA$$
$$= 2 \times 5 \times 0.2 \times 10^{-4} = 7.85 \times 10^{-4}\,\text{N.m}$$

For 2000 turns,

$$T = 7.85 \times 10^{-4} \times 2000$$
$$T = 1.57\text{N.m}$$

17. A rectangular coil carrying a current of 5A is placed in the magnetic field of flux density $\bar{B} = 0.3(\bar{a}_x + \bar{a}_y)\text{Wb}/\text{m}^2$. The coil is lying the yz plane & has dimensions 0.8m $\times$ 0.4m. Find the torque on the coil.

Solution;-

$$I = 5\text{A} \quad \bar{B} = 0.3(a_x + a_y)\,\text{Wb}/\text{m}^2$$

$$A = (0.8 \times 0.4)a_x = 0.32\bar{a}_x\text{m}^2$$
$$t = \bar{m} \times \bar{B}$$
$$T = 5(0.32a_x) \times 0.3(a_x + a_y)$$
$$= 5(0.3)(0.3)a_z$$
$$= 0.48\bar{a}_z\ \text{N.m}$$