ME-6601 DESIGN OF TRANSMISSION SYSTEMS

UNIT-I DESIGN OF TRANSMISSION SYSTEMS FOR FLEXIBLE ELEMENTS

(PART-A)

1. **What do you understand by 6×19 construction in wire ropes?**

   A 6 × 19 wire rope means a rope is made from 6 strands with 19 wires in each strand.

2. **Mention the losses in belt drives?**
   - The losses in a belt drive are due to:
   - Slip and creep of the belt on the pulleys
   - Air resistance to the movement of belt and pulleys
   - Bending of the belt over the pulleys
   - Friction in the bearings of the pulley

3. **In what ways the timing belts are superior to ordinary V – belts?**

   Flat belt and V – belt drives cannot provide a precise speed ratio, because slippage occurs at the sheaves. But certain applications required an exact output and input speed ratio. In such situations, timing belts are used.

4. **What is meant by 'Chordal action of chain'? Also name a company that produces driving chains?**
   - When chain passes over a sprocket, it moves as a series of chords instead of a continuous arc as in the case of a belt drive. It results in varying speed of the chain drive. This phenomenon is known as chordal action.
   - Roto mechanical equipment, Chennai; Monal Chains Limited; Innotech Engineers Limited., New Delhi.

5. **What is centrifugal effect on belts?**

   In operation, as the belt passes over the pulley the centrifugal effect due to its Self Weight tends to lift the belt from the pulley surface. This reduces the normal reaction and hence the frictional resistance.

   The centrifugal force produces an additional tension in the belt.

6. **What is Chordal action in chain drives?**

   When chain passes over a sprocket, it moves as a series of chords instead of a continuous are as in the case of a belt drive. It results in varying speed of the chain drive. This phenomenon is known as chordal action.

7. **Name the few material for belt drives?**
   - Leather
   - Fabric and cotton
   - Rubber
   - Balata
   - Nylon

8. **Under what circumstances chain drives are preferred over V belt drives?**
   - To transmit more power

9. Define the term 'crowning of pulley'?

- The pulley rims are tapped slightly towards the edges. This slight convexity is known as crowning.

10. What factors will affect the working conditions of the chain drive?

- Lubrication
- Wear
- Strength

11. What are the types of belts?

- (a) Flat Belts
- (b) V Belts.

  (ii) Multiple V belt. (iii) Ribbed Belt.
- (c) Toothed or Timing
- (d), Round, Belts.

12. Indicate some merits and demerits of belt-drive;

Merits

- Belt drives are used for long distance power transmission.
- Their operations are smooth and flexible.
- Simple in design and their manufacturing cost is lower.

Demerits

- They need large space.
- Loss of power due to friction is more.

13. What is meant by the ply of belt?

Flat belts are made of thin strips and laminated one over the other in order to get thick belt. These thin strips or sheets are called as plies of belt. Usually flat belts are made of 11 ply, 4 ply, 5 ply, 6 ply and 8 ply belt etc And 4 ply belt is thicker than 3 ply belt and so-on.

14. Specify the application of round belt.

Round-belt is applied, in sewing machine.

15. Specify the purpose of crowning of bets.

To prevent slipping from pulley due to centrifugal force

16. What factors should be considered during the selection of a belt drive?

a) Amount of power to be transmitted, b) Peripheral and angular speeds. c) Speed ratio. d) Efficiency. e) Centre distance between shafts f) Space available. g) Working environment

17. What are the advantages of chain drives?

Advantages of chain drives

- ❖ Are having more power transmitting capacity.
- ❖ Have higher efficiency and compact size.
- ❖ 3- Exert -less load on shafts since no initial tension is applied on the sprocket shafts.
- ❖ Require easy maintenance

18. **Specify some drawbacks of chain drives**.

- ❖ The design of chain drive is more complicated.
- ❖ The operation is noisy and production cost is high.
- ❖ They require more accurate assembly bf shafts than for belts.

19. **What are the types of ropes?**

They are two type namely

a) Fibre ropes

b) Wire ropes.

20. In what ways wire ropes are superior to fibre ropes?

a) Wire ropes are stronger, more durable than fibre ropes.

b) Wire ropes can withstand' shock loads.

c) Their 'efficiency in high.

d) They can be operated for Very long centre distance even up to 1000 m.

Hence wire-ropes are superior in most of occasions.

21. A longer belt will last more than a shorter belt. Why? (April/May 2017)

The life of a belt is a function of the center distance between the driver and driven shafts. The shorter belt is more often it will be subjected to additional bending stresses while running around the pulleys at a given speed, and quicker it will be destroyed due to fatigue. Hence a longer belt will last more than a shorter belt.

22. List the advantages of wire ropes compared to chains. (April/May 2017)

- ❖ Lighter weight and high strength to weight ratio
- ❖ More reliable in operation
- ❖ Silent operation even at high working speeds

23. Write the advantages of V belts over the Flat belts? (Nov/Dec 2017)

- ❖ Power transmitted is more due to wedging action in the grooved pulley
- ❖ Higher velocity ratio (upto 10) can be obtained
- ❖ V belt is more compact , quiet and shock absorbing
- ❖ The drive positive because the slip is negligible

24. List the chain drive failures? (Nov/Dec 2017)

The four basic modes of chain failures are

- Near
- Fatigue
- Impact
- Galling

25. Define Coefficient of friction? (April/May 2018)

The Coefficient of friction is the ratio of the frictional force to the force acting perpendicular to the two surfaces in contact. This coefficient is a measure of the difficulty with which the surface of one material will slide over another material.

26. What are the advantages of Chain Drives? (April/May 2018)

i. Chain Drive can be used for long as well as short centre distances
ii. They are more compact than belt or gear drives
iii. There is no slip between chain and sprocket, so they provide positive drive
iv. Higher efficiency (upto 98%) of the drive.

27. Name the four types of belts used for transmission of power (Nov/Dec 2018)

i. Flat belts
ii. V belts
iii. Ribbed belts
iv. Toothed or timing belts

28. When do use stepped pulley drive? (Nov/Dec 2018)

A stepped or cone pulley drive is used when the driven or machine shaft is to be started or stopped whenever desired without interfacing with the driving shaft.

29. Which side of the belt should be on the bottom side of the pulley and why? (April/May 2019)

i. The tight side of the belt should be on the bottom side of the pulley
ii. Because the driving pulley pulls the belt from the bottom side and delivers it to the upper side. So it is obvious that the bottom side of the belt is tight

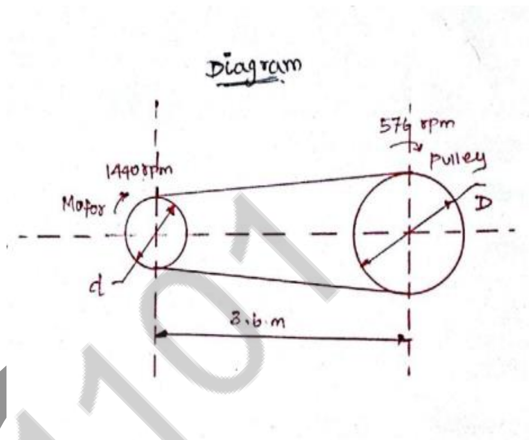
30. What are the various stresses induced in wire ropes? (April/May 2019)

i. Direct stress due to the weight of the load to be lifted and weight of the rope
ii. Bending stress when the rope passes over the sheave
iii. Stress due to acceleration
iv. Stress during starting and stopping

PART-B)

1. A compressor is to run by a motor pulley running at 1440rpm, Speed ratio 2.5. Choose a flat belt crossed drive. Centre distance between pulleys is 3.6m. Take belt speed as 16 m/s. Load factor is 1.3. Take a 5-ply, flat Dunlop belt. Power to be transmitted is 12 KW. High speed load rating is 0.0118 KW/ply/mm, width at v = 5 m/s. Determine the width and length of the belt.

Given data:



$N_1 = 1440\text{rpm}$

$\phi = 2.5$

$c = 3.6\text{m}$

$\gamma = 16\, m/s$

$K_s = 1.3$

Belt = 5 Ply, flat dunlop belt.

$P = 12\text{KW}$

Load rating at $5\, m/s = 0.0118$ KW/Ply/mm

Step 1: **Calculation of Pulley diameters:**

Assume the driven pulley diameter D = 1000 mm.

$$\text{W.K.T } \phi = \frac{D}{d} = \frac{N_1}{N_2}$$

Case (i): To find the driven pulley speed. (N_2).

Case (ii): To find the driver pulley diameter (d):

$$N_2 = \frac{N_1}{\phi}$$

$$= \frac{1440}{2.5}$$

$$N_2 = 576\text{rpm}$$

Case (iii): To find the driver pulley diameter (d):

$$d = \frac{D}{\phi}$$

$$= \frac{1000}{2.5}$$

$$d = 400\text{mm}$$

From PSGDB 7.54, from recommended series of pulley diameters and tolerances.

The standard diameter for the driver pulley $\Big\}\, d = 400mm$

Step 2: Calculation of design power in KW.

$$\text{Design power} = \frac{\text{Rated power}(K_w) \times \text{Load correction factor}(K_s)}{\text{Arc of contact factor}(K_\alpha) \times \text{Small pulley factor}(K_d)}$$

Case (i): To find the arc of contact factor ($K\alpha$)

From PSGDB 7.54

$$\text{Arc of contact} = 180° - \left(\frac{D-d}{c}\right) \times 60°$$

$$= 180° - \left(\frac{1000-400}{3600}\right) \times 60°$$

$$= 170°$$

From PSGDB 7.54, take the value of $K_\alpha = 1.04$. Corresponding to the arc of contact 170°

$$K_\alpha = 1.04$$

Case (ii): To find the small pulley factor (K_d)

Table: Small pulley factor 'K_d'

Small Pulley diameter	K_d
Upto 100mm	0.5
100 – 200mm	0.6
200 – 300mm	0.7
300 – 400mm	0.8
400 – 750mm	0.9
Over 750mm	1.0

From the above table. We take the K_d value 0.8

$$\therefore K_d = 0.8$$

Case (iii):

To find the design power, KW:

W. K. T. $$\text{Design power} = \frac{P \times K_s}{K_a \times K_d}$$

$$= \frac{12 \times 1.3}{1.04 \times 0.8}$$

$$= 18.75\text{KW}$$

Step 3: Selection of belt:

Given: 5 Ply, flat Dunlop belt. Its capacity is given by 0.0118 KW/ply/mm.

Step 4: Load rating correction:

From PSGDB 7.54.

$$\text{Load rating at 'V' m/s} = \text{Load rating at 10 m/s} \times \frac{v}{10}$$

$$\text{Load rating at 16 m/s} = (0.0118 \times 2) \times \frac{16}{10}$$

$$= 0.03776\text{KW/Ply/mm}$$

Step 5: Determination of belt width:

$$\text{Width of the belt} = \frac{\text{Design Power}}{\text{Load rating} \times \text{No. of plies.}}$$

$$= \frac{18.75}{0.03776 \times 5}$$

$$= 99.31\text{mm}$$

From PSGDB 7.52 Specification of transmission belting standard widths.

The standard belt width for 5 Ply belt = 100mm.

Step 6: Determination of Pulley width:

From PSGDB 7.54, Pulley width is given by

Pulley width = Belt width + 18 mm

= 100 + 13 mm

= 113 mm

From PSGDB 7.54, recommended series of width of flat pulleys, mm.

The standard pulley width = 125 mm.

Step 7: Calculation of length of the belt (L):

From PSGDB 7.61,

$$L = 2C + \frac{\pi}{2}(D+d) + \frac{(D-d)^2}{4C}$$

$$= 2\times3600 + \pi/2(1000+400) + \frac{(1000-400)^2}{4\times3600}$$

$$= 7200 + 2199.11 + 25$$

$$L = 9424.11\text{mm}$$

2. **At the construction site, 1 tonne of steel is to be lifted upto a height of 20m with the help of 2 wire ropes of 6 × 19 size, nominal diameter 12 mm and breaking load 78 KN. Determine the factor of safety if the sheave diameter is 56 d and if wire rope is suddenly stopped is 1 second when travelling at a speed of 1.2 m/s. What is the factor of safety if bending load is neglected?**

Given data:

$h- = 20\text{m}$

$W = 1 \text{ tonne} = 1000\text{Kg} = 9810\text{N}$

$n = 2$

Wire rope size $= 6\times19$

$d = 12\text{mm}$

Breaking load $W_{break} = 78\text{KN}$

$D = 56d$

$t = 1\text{ sec}$

$v = 1.2\text{ m/s} = 72\text{ m/min}$

Step 1: Selection of suitable Wire rope:

Given: 6 × 19 size wire rope.

Step 2: Calculation of design load:

Assuming a larger factor of safety of 15, the design load is calculated.

Design load = Load to be lifted × Assumed FOS

= 9810 × 15

= 147150 N

= 147.15 KN

Step 3: Selection of Wire rope diameter (d):

From PSGDB 9.5. For the breaking strength (W_{break}) 78 KN (7.8 tonnes). take the diameter of the rope is 12mm.

$$d = 12mm$$
$$\sigma_u = 1600 \text{ to } 1750\, N/mm^2$$

Step 4: Calculation of sheave diameter (D):

Given:

Sheave diameter D = 56 d

= 56 × 12

= 672 mm

Step 5: Selection of the area of useful cross section of the rope (A):

From PSGDB 9.1

$$A = 0.4 \times \pi/4 \times d^2$$

$$= 0.4 \times \pi/4 \times 12^2$$

$$A = 45.24mm^2$$

Step 6: Calculation of Wire diameter (d_w):

$$d_w = \frac{d}{1.5\sqrt{i}}$$

i = Number of strands × Number of wires in each strand

= 6 × 19

i = 114

$$d_w = \frac{12}{1.5\sqrt{114}}$$

$$\therefore\ d_w = 0.75mm$$

Step 7: Selection of Weight of rope (W_r):

From PSGDB 9.5. Corresponding to the diameter of the rope 12mm, take

Approximate weight $= 0.54\,\text{Kgf/m}$

$= 5.3\,\text{N/m}$

$\therefore$ Weight of rope W_r = Approximate Weight $\times$ h

$= 5.3 \times 20$

$W_r = 106\text{N}$

Step 8: Calculation of various loads:

Case (i): To find the direct load (W_d):

$W_d = W + W_r$

$= 9810\text{N} + 106\text{N}$

$W_d = 9916\text{N}$

Case (ii): To find the acceleration load (W_a):

$$W_a = \left(\frac{W + W_r}{g}\right) a$$

a = acceleration of the load

$$= \frac{V_2 - V_1}{t_1}$$

$$= \frac{1.2 - 0}{1}$$

$a = 1.2\,\text{m/s}^2$

$$\therefore\ W_a = \left(\frac{9810 + 106}{9.81}\right) 1.2$$

$W_a = 1212.97\text{N}$

Step 9: Calculation of effective loads on the rope:

Effective load during acceleration of the load

$$W_{ea} = W_d + W_b + W_a$$

$$= 9916 + 0 + 1212.97$$

$$\begin{bmatrix} \because W_b = 0, \text{From the Question} \\ \text{Bending load is neglected} \end{bmatrix}$$

$$= 11128.97 \text{N}$$

Step 10: Calculation of working factor of safety (FS_w):

$$\left.\begin{matrix}\text{Working factor} \\ \text{of Safety } (F_{sw})\end{matrix}\right\} = \frac{\text{Breaking load}}{\text{Effective load during acceleration } (W_{ea})}$$

$$= \frac{78 \times 10^3}{11128.97}$$

$$F_{sw} = 7$$

Step 11: Check for design:

From PSGDB 9.1, for hoists and class 2, the recommended factor of safety = 5.

Since the working factor of safety is greater than the recommended factor of safety. Therefore the design is safe.

3. Design a V belt drive and calculate the actual belt tensions and average stress for the following data. Power to be transmitted = 7.5 KW, speed of driving wheel = 1000 rpm, speed of driven wheel = 300 rpm, diameter of the driven pulley = 500 mm, diameter of the driver pulley = 150 mm and centre distance = 925 mm

Given data:

P = 7.5 KW

N_1 = 1000 rpm

N_2 = 300 rpm

D = 500 mm

d = 150 mm

C = 925 mm

Step 1: Selection of belt

From PSGDB 7.58,

For 7.5 KW, B section is selected

Step 2: Selection of pulley diameters. d & D:

d = 150 mm, D = 500 mm given.

Step 3: Selection of centre distance (c) :

C= 925 mm given.

Step 4: Calculation of nominal pitch length (L).

From PSGDB 7.61,

$$L = 2C + \frac{\pi}{2}(D+d) + \frac{(D-d)^2}{4C}$$

$$= 2\times 925 + \frac{\pi}{2}(500+150) + \frac{(500-150)^2}{4\times 925}$$

$$= 2904.12mm.$$

From PSGDB 7.60, For B section.

The next standard length L = 3091 mm.

Step 5: Selection of various modification factors.

Case 1: Length correction factor (F_c)

From PSGDB 7.60 for B section corresponding to 'L'

$$F_c = 1.07$$

Case 2: Correction factor for arc of contact (F_d)

From PSGDB 7.68

Arc of contact angle $= 180° - (\frac{D-d}{C}) \times 60°$

$$= 180° - (\frac{500-150}{925}) \times 60°$$

$$= 157.29°$$

Corresponding to the angle 1579.29° □ 160°

F_d = 0.95.

Case 3: Service factor (F_a).

From PSGDB 7.69

$$F_a = 1.3$$

Step 6: Calculation of Maximum power capacity (KW).

From PSGDB 7.62, For B section.

$$KW = (0.79S^{-0.09} - \frac{50.8}{d_e} - 1.32\times10^{-4}S^2)S$$

Where, S = Belt speed $= \frac{\pi d N_1}{60}$

$$= \frac{\pi\times0.150\times1000}{60}$$

$$= 7.854 m/s$$

d $_e$ = equivalent pitch diameter; From PSGDB 7.62 $\frac{D}{d} = \frac{500}{150} = 3.33$ Take F_b=1.14

$$= d_p \times F_b$$

$$= 150\times1.14$$

$$= 171 \text{ mm.}$$

$$\therefore \quad KW = (0.79\times7.854^{-0.09} - \frac{50.8}{171} - 1.32\times10^{-4}\times7.84^2)7.84$$

$$= 2.757 KW$$

Step 7: Calculation of number of belts (n_b)

From PSGDB 7.70

$$n_b = \frac{P\times F_a}{K_w\times F_c\times F_d}$$

$$= \frac{7.5\times1.3}{2.757\times1.07\times0.95}$$

$$= 3.48$$

$$n_b = 4 \text{ belts.}$$

Step 8: Calculation of actual centre distance. (C_{actual}).

From PSGDB 7.61

$$C_{actual} = A + \sqrt{A^2 - B}$$

$$A = \frac{L}{4} - \pi\left[\frac{D+d}{8}\right]$$

$$= \frac{3091}{4} - \pi\left[\frac{500+150}{8}\right]$$

$$A = 517.5mm$$

$$B = \frac{(D-d)^2}{8} = \frac{(500-150)^2}{8}$$

$$= 15312.5 \text{ mm}^2$$

$$\therefore \quad C_{actual} = 517.5 + \sqrt{517.5^2 - 15312.5}$$

$$= 1020 \text{ mm.}$$

Step 9: Calculation of belt tensions (T_1 and T_2).

Power transmitted per belt $= (T_1 - T_2)\,v$

$$\frac{7.5\times10^3}{4} = (T_1 - T_2)7.854$$

$$T_1 - T_2 = 238.73 \quad \text{------1}$$

From PSGDB 7.58 $\Rightarrow$ m = 0.189 Kg/m.

7.70 $\Rightarrow$ 2B =34°

From step 5: $\Rightarrow \alpha = 157.29° \times \frac{\pi}{180°}$

$= 2.745$ rad.

Tension ratio $\Rightarrow \frac{T_1 - mv^2}{T_2 - mv^2} = e^{\mu\alpha\,cosec\beta}$

$$\frac{T_1 - 0.189(7.854)^2}{T_2 - 0.189(7.854)^2} = e^{0.3\times2.745\times cosec17°}$$

$$T_1 - 16.72T_2 = -184.3 \quad \text{------2}$$

Solving equation 1 and 2

$T_2 = 26.9$ N , $T_1 = 265.64$ N

Step 10: Calculation of Stress induced.

$$\text{Stress induced} = \frac{\text{Maximum tension}}{\text{Cross sectional area}}$$

From PSGDB 7.58 Area of B section = 140 mm^2

$$\therefore \text{Stress induced} = \frac{265.64}{140}$$

$$= 1.897 \text{ N/mm}^2$$

4. **A 7.5 KW electric motor running at 1400rpm is used to drive the input shaft of the gear box of a machine. Design a suitable roller chain to connect the motor shaft to the gearbox shaft to give an exact speed ration of 10:1. The center to center distance of the shaft is to be approximately 600mm.**

   Given data:

   N = P = 7.5 KW

   N_1 = 1400 rpm

   i = 10

   a_0=600 mm

Step 1: Selection of transmission ratio. (i)

$$i = \frac{N_1}{N_2} = 10 \quad \text{given.}$$

Then,

$$\frac{N_1}{10} = N_2$$

$$N_2 = \frac{1400}{10}$$

$$N_2 = 140\text{rpm}$$

Step 2: Selection of no. of teeth on the driver sprocket (z_1).

From PSGDB 7.74

$Z_1 = 7$

Step 3: Calculation of no. of teeth on the driven sprocket (Z_2).

From PSGDB 7.74

$$Z_2 = i \times Z_1$$

$$= 10 \times 7$$

$$Z_2 = 70$$

$$Z_{2max} = 100 \text{ to } 120$$

Recommended value of Z_2 should be less than the above value or else the chain may run off the sprocket for a small pull.

$Z_2 = 70$ is satisfactory.

Step 4: Selection of standard pitch (P).

From PSGDB 7.74

$$\text{Centre distance } a = (30 \text{ to } 50)\ P$$

$$\text{Maximum Pitch, } P_{max} = \frac{a}{30} = \frac{600}{30} = 20 \text{ mm}$$

$$\text{Minimum Pitch, } P_{min} = \frac{a}{50} = \frac{600}{50} = 12 \text{ mm}$$

Any standard pitch between 12 mm and 20 mm can be chosen. But to get a quicker solution, it is always preferred to take the standard pitch closer to P_{max}.

From PSGDB 7.72, Standard Pitch P = 15.875 mm.

Step 5: Selection of the chain:

From PSGDB 7.72, assume the chain to be duplex.

$\therefore$ 10 A - 2 / DR50 Chain number is selected.

Step 6: Calculation of total load on the driving side of the chain (P_T):

From PSGDB 7.78,

$$P_T = P_t + P_c + P_a$$

Case 1: To find the tangential force (P_t)

From PSGDB 7.78

$$P_t = \frac{1020N}{V}$$

Where, v = chain velocity $= \frac{Z_1 \times P \times N_1}{60 \times 1000}$

$$= \frac{7 \times 15.875 \times 1400}{60 \times 1000}$$

$$= 2.59\ m/s$$

$$\therefore\ P_t = \frac{1020 \times 7.5}{2.59}$$

$$P_t = 2950.35\ N$$

Case 2: To find the centrifugal tension (P_c).

From PSGDB 7.78. $P_c = \frac{Wv^2}{g} = mv^2$

Where, m= mass of the chain

From PSGDB 7.72, For the selected chain,

m = 1.78 Kg/m [1Kg m/s^2=1N]

$$\therefore\ P_c = 1.78\ (2.59)^2$$

$$P_c = 11.94\ N$$

Case 3: To find the tension due to sagging (P_s).

From PSGDB 7.78,

P_s = K. W. a

Where, K = 6 (for horizontal) From PSGDB 7.78

W = m x g = 1.78 x 9.81 = 17.46 N

A = 600 mm = 0.6 m.

∴ P_s = 6 x 17.46 x 0.6

= 62.82 N

∴ P_T = 2950.35 + 11.94 + 6282

P_T = 3025.11 N

Step 7: Calculation of Service factor (K_s).

From PSGDB 7.76

$$Ks = K_1 \cdot K_2 \cdot K_3 \cdot K_4 \cdot K_5 \cdot K_6$$

From PSGDB 7.76 and 7.77.

- $K_1 = 1.25$ for load with mild shocks
- $K_2 = 1$ for adjustable supports.
- $K_3 = 1$ $\because$ we have used $a_p = (30 \text{ to } 50)P$
- $K_4 = 1$ for horizontal drive.
- $K_5 = 1$ for drop lubrication
- $K_6 = 1.25$ for 16 hrs/day running

$\therefore$ $K_s = 1.25 \times 1 \times 1 \times 1 \times 1 \times 1.25$
$= 1.5625$

Step 8: Calculation of design load.

Design load $= P_T \times K_s$

$= 3025.11 \times 1.5625$

$= 4726.73$ N

Step 9: Calculation of working factor of safety (FS_w)

$$FS_w = \frac{Q}{\text{Design load}}$$

Where, Q = Breaking load = 44400 N. From PGSDB 7.72 for the selected chain

$$\therefore \quad FS_W = \frac{44400}{4726.73}$$

$$FS_W = 9.4$$

Step 10: Check for factor of safety.

From PSGDB 7.77, Recommended factor of safety = 12.45

We find $FS_W < 12.45$, the design is not safe.

In order to overcome this issue we have to increase the pitch = 19.05 mm.

$\therefore$ The chain number 12 A -2 / DR 60 is selected.

For this chain, M= 2.90 Kg/m, Q = 63600 N

By the recalculation of step 6 and step 8, step 9.

$P_T = 2590.28$ N.

Design load = 4047.31 N

FS_w = 15.71

We find FS_w > 12.45 , the design is safe.

Step 11: Check for the bearing stress in the roller.

$$\sigma_{roller} = \frac{P_t \times K_s}{A}$$

Where, $A = 210\ mm^2$ From PSGDB 7.72 for selected chain

$$\therefore \quad \sigma_{roller} = \frac{2459.81*1.5625}{210}$$

$$= 18.30\ N/mm^2$$

From PSGDB 7.77, the allowable bearing stress for the given speed 1400rpm, is 19.75 N/mm^2.

Induced stress is less than the allowable stress i.e 18.30 < 19.75 N/mm^2.

$\therefore$ The design is safe.

Step 12: Calculation of length of chain (L).

From PSGDB 7.75

$$L = l_p \times P$$

Where no.of links $l_p = 2a_p + \left(\frac{Z_1 + Z_2}{2}\right) + \frac{[(Z_2 - Z_1)/2\pi]^2}{a_p}$

Approximate center distance in multiples of pitches $a_p = \frac{a_0}{P} = \frac{600}{19.05} = 31.50$

$$\therefore \quad l_p = 2 \times 31.50 + \left(\frac{7+70}{2}\right) + \frac{[(70-7)/2\pi]^2}{31.50}$$

$$= 63 + 38.5 + 3.19$$

$$l_p = 104.69$$

$$l_p = 106 \text{ links}$$

$$\therefore \left.\begin{array}{l}\text{Actual length} \\ \text{of chain}\end{array}\right\} L = 106 \times 19.05$$

$$L = 2019.3\ mm$$

Step 13: Calculation of exact centre distance (a):

From PSGDB 7.75.

$$m = 100.54 \qquad a = \frac{e + \sqrt{e^2 - 8m}}{4} \times P$$

Case 1: To find e:

$$* \quad e = l_p - \left(\frac{Z_1 + Z_2}{2}\right)$$

$$= 106 - \left(\frac{7 + 70}{2}\right)$$

$$e = 67.5$$

Case 2: To find m:

$$* \quad m = \left(\frac{Z_2 - Z_1}{2\pi}\right)^2$$

$$= \left(\frac{70 - 7}{2\pi}\right)^2$$

$$m = 100.54$$

$$\therefore \quad a = \frac{67.5 + \sqrt{67.5^2 - 8 \times 100.54}}{4} \times 19.05$$

$$a = 613.18 \text{ mm}$$

From PSGDB 7.75, Decrement in centre distance for an initial sag $= 0.01a$

$$= 6.132 \text{ mm}$$

$$\therefore \quad \text{Exact centre distance} = 613.18 - 6.132$$

$$= 607.05 \text{ mm.}$$

Step 14: Calculation of sprocket diameters.

Case 1: Smaller sprocket.

PCD of smaller sprocket $d_1 = \dfrac{P}{\sin\left(\dfrac{180}{Z_1}\right)}$ From PSGDB 7.78

$$= \frac{19.05}{\sin\left(\frac{180}{7}\right)}$$

$$d_1 = 43.91 \text{ mm.}$$

Sprocket outside diameter $d_{01} = d_1 + 0.8d_r$

d_r = diameter of roller $= 11.90$ mm. From PSGDB 7.72 for selected chain.

$$\therefore \quad d_{01} = 43.91 + 0.8 \times 11.90$$

$$d_{01} = 53.43 \text{ mm}$$

Case 2: Larger sprocket:

$$d_2 = \frac{P}{\sin\left(\frac{180}{Z_2}\right)} \quad \text{From PSGDB 7.78}$$

$$= \frac{19.05}{\sin\left(\frac{180}{70}\right)}$$

$$d_2 = 424.61 \text{ mm}$$

$$\left.\begin{matrix}\text{Sprocket outside}\\ \text{diameter}\end{matrix}\right\} d_{02} = d_2 + 0.8d_r$$

$$= 424.61 + 0.8 + 11.90$$

$$d_{02} = 434.13 \text{ mm}$$

5. **Select a suitable v belt and design the drive for a wet grinder. Power is available from a 0.5KN motor running at 750rpm. Drum speed is to be about 100rpm. Drive is to be compact.**

Given data:

$P = 0.5KW$
$N_1 = 750rpm$
$N_2 = 100rpm$

Step 1: Selection of belt:

From PSGDB 7.58, for power 0.5KW.

As per data book the usual load of drive starts from, 0.75KW only. So choose A section, and P=0.75KW.

Step 2: Selection of Pulley diameters (d and D).

$$\text{Speed ratio} = \frac{D}{d} = \frac{N_1}{N_2} = \frac{750}{100} = 7.5$$

Smaller pulley diameter $d = 75mm$.

From PSGDB 7.54,

The standard diameter $d = 80mm$.

Larger Pulley diameter $D = 7.5d$

$= 7.5 \times 80$

$D = 600mm$

From PSGDB 7.54,

The standard $D = 630mm$.

Step 3: Selection of centre distance. (C)

From PSGDB 7.61, For $i = 7.5$, take $C/D = 0.85$

$\therefore C = 0.85 \times D$

$= 0.85 \times 630$

$C = 535.5mm$

Step 4: Calculation of nominal pitch length: (L)

From PSGDB 7.61,

$$L = 2C + \left(\pi/2\right)(D+d) + \frac{(D-d)^2}{4C}$$

$$= 2(535.5) + \left(\pi/2\right)(630+80) + \frac{(630-80)^2}{4 \times 535.5}$$

$$= 1071 + 1115.27 + 141.22$$

$L = 2327.49mm$

From PSGDB 7.59, From A section.

The next standard nominal pitch length

$L = 2474mm.$

Step 5: Selection of various modification factors.

Case 1: Length correction factor. (F_c)

For A section, F_c = 1.08 From PSGDB 7.59

Case 2: Correction factor for arc of contact (F_d)

Arc of contact $= 180° - \left(\frac{D-d}{C}\right) \times 60°$ From PSGDB 7.68

$$= 180° - \left(\frac{630-80}{535.5}\right) \times 60°$$

$$= 118.38°$$

$\therefore$ $F_d = 0.82$ From PSGDB 7.68

Case 3: Service factor (F_a)

For light duty 16 hours continuous service , for driving machines of type II , service factor is selected as $F_a = 1.3$ From PSGDB 7.69

Step 6: Calculation of maximum power capacity.

$KW = \left(0.45S^{-0.09} - \frac{19.62}{d_e} - 0.765 \times 10^{-4} S^2\right) S$ From PSGDB 7.62 for A section.

$$S = \frac{\pi d N_1}{60}$$

$$= \frac{\pi \times 80 \times 750}{60 \times 1000}$$

$$= 3.14 \, m/s$$

$$d_e = d_p \times F_b$$

$$d_p = d = 80mm$$

$F_b = 1.14$ From PSGDB 7.62

$$\therefore d_e = 80 \times 1.14$$

$$= 91.2mm.$$

$$\therefore KW = \left(0.45 \times (3.14)^{-0.09} - \frac{19.62}{91.2} - 0.765 \times 10^{-4} (3.14)^2\right) 3.14$$

$$= 0.6KW.$$

Step 7: Determination of number of belts (n_b)

$$n_b = \frac{P \times F_a}{K_w \times F_c \times F_d}$$

$$= \frac{0.5 \times 1.3}{0.6 \times 1.08 \times 0.82}$$

$$n_b = 1.223$$

$$n_b \approx 2 \text{ belts.}$$

Step 8: Calculation of actual centre distance:

$$C_{Actual} = A + \sqrt{A^2 - B}$$ From PSGDB 7.61

$$A = \frac{L}{4} - \pi\left(\frac{D+d}{8}\right)$$

$$= \frac{2474}{4} - \pi\left(\frac{630+80}{8}\right)$$

$$A = 339.68\text{mm}$$

$$B = \frac{(D-d)^2}{8} = \frac{(630-80)^2}{8} = 37812.5\text{mm}^2$$

$$\therefore \quad C_{actual} = 339.69 + \sqrt{(339.69)^2 - 37812.5}$$

$$= 618.22\text{mm}$$

6. **Select a wire rope for a vertical mine hoist to lift a load of 20KN from a depth of 60 metres. A rope speed of 4 m/sec is to attained on 10 seconds.**

Given data:

Weight to be lifted $= 20$ KN

Depth $= 60\text{m}$

$v_2 = v = 4\,\text{m/sec} = 240\,\text{m/min}$

$t = 10\,\text{sec}$

Step 1: Selection of suitable wire rope.

For hoisting purpose, 6×19 rope is selected. From PSGDB 9.1

Step 2: Calculation of Design load.

Assuming the factor of safety of 15, the design load is calculated.

Design load $= 20 \times 15$

$= 300 KN$

Step 3: To find wire rope diameter (d).

From PSGDB 9.5 For design load 300KN, The next standard value.

$d = 25 mm$

$m = 2.41 Kg/m$

$\sigma_u = 1600 \text{ to } 1750 N/mm^2$

Breaking strength $= 340 KN$

Step 4: Sheave diameter (D)

From PSGDB 9.1. We find $D_{min}/d = 27$ for class 4, for velocity upto $50 m/min$. But the actual speed is $240 m/min \left(i.e \frac{240}{50} \approx 5 \text{ times } 50 m/min \right)$. Therefore $\frac{D_{min}}{d}$ has to be modified.

$$\frac{D_{min}}{d} = 27 \times (1.08)^{5-1} = 36.73 \approx 37 mm.$$

Sheave diameter $D = 37 \times d$

$= 37 \times 25$

$D = 925 mm$

Step 5:Calculation of Area of cross section of the rope (A).

From PSGDB 9.1

$A = 0.4 \times \pi/4 \times d^2$

$= 0.4 \times \pi/4 \times 25^2$

$A = 196.35 mm^2$

Step 6: To find Wire diameter. (d_w).

$$d_w = \frac{d}{1.5\sqrt{i}}$$

$$= \frac{25}{1.5\sqrt{6 \times 19}}$$

$$= 1.56$$

$$d_w = 2\text{mm}$$

Step 7: Weight of the rope. (W_r).

$$W_r \text{ per meter} = 2.41 \times 9.81 = 23.64 \text{ N/m.}$$

$$W_r = 23.64 \times 60 = 1418.53\text{N}$$

$$= 1418.53\text{N}$$

Step 8: Load calculations

Case 1: Direct load (W_d)

$$W_d = W + W_r = 20 + 1418.53 \times 10^{-3} = 21.42\text{KN}$$

Case 2: Bending load (W_b)

$$W_b = \sigma_b \times A = \frac{E_r \times d_w}{D} \times A$$

$$= \frac{0.84 \times 10^5 \times 2}{925} \times 196.35 \qquad \left[\because E_r = 0.84 \times 10^5 \text{ N/mm}^2\right]$$

$$= 35661.41\text{N}$$

$$= 35.66\text{KN.}$$

Case 3: Acceleration load (W_a)

$$W_a = \left(\frac{W + W_r}{g}\right) a \qquad a = \frac{v_2 - v_1}{t}$$

$$= \left(\frac{20 + 1418.53 \times 10^{-3}}{9.81}\right) \times 0.4 \qquad = \frac{4 - 0}{10}$$

$$= 0.87 \text{ KN} \qquad = 0.4 \text{ m/s}^2$$

$$\therefore \left.\begin{array}{c}\text{Effective load on the rope} \\ \text{during acceleration}\end{array}\right\} W_{ea} = W_d + W_b + W_a$$

$$= 21.42 + 35.66 + 0.87$$

$$= 57.95$$

$$W_{ea} = 58KN$$

Step 9:Working factor of Safety (FS_w).

$$FS_w = \frac{\text{Breaking load}}{W_{ea}}$$

$$= \frac{340}{58}$$

$$FS_w = 5.86$$

Step 10: Check for Safe design

* We find $F_{sw} < n'(6)$. $\therefore$ The design is not safe.
* The safe design can be achieved either by selecting the rope with greater breaking strength.

From PSGDB 9.5, for d=25, take breaking strength = 376 KN and $\sigma_u = 1750 \text{ to } 1900 N/mm^2$

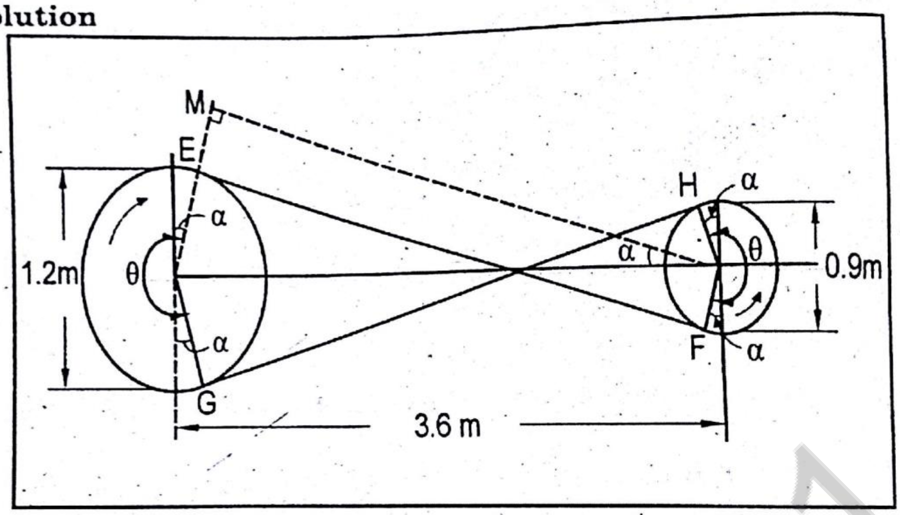
$$\therefore F_{sw} = \frac{376}{58}$$

$$= 6.48$$

Now we find $F_{sw} > n'(6)$. $\therefore$ The design is safe.

7. **Design a flat belt drive to transmit 110kW for a system consisting of two pulleys of diameter 0.9m and 1.2m respectively, for a center distance of 3.6m. Belt speed of 20m/s and coefficient of friction=0.3. There is a slip of 1.2% at each pulley and 5% friction loss at each shaft with 20% over load.**

Solution



Given: $P = 110kW = 150\ HP,\ d_1 = 0.9m = 90cm,$

$\therefore\ r1 = 0.45\ m,\ d2 = 1.2\ m = 120\ cm,\ \therefore r2 = 0.6\ m\ ;$

$x = 3.6m;\ v = 20m/s;\ \mu = 0.3;\ S_1 = S_2 = 1.2\%$

Let N_1 = Speed of the smaller or driving pulley in rpm

N_2 = Speed of the larger or driven pulley in rpm

We know that speed of the belt (v)

$$v = \frac{\pi d_1 N_1}{60}\left(1 - \frac{S_1}{100}\right)$$

$$20 = \frac{\pi \times 0.9 N_1}{60}\left(1 - \frac{1.2}{100}\right)$$

$$\therefore\ N_1 = 430\ rpm$$

And peripheral velocity of driven pulley

$$\frac{\pi d_2 N_2}{60} = v\left(1 - \frac{S_2}{100}\right)$$

$$\frac{\pi \times 1.2 N_2}{60} = 20\left(1 - \frac{1.2}{100}\right)$$

$$\therefore\ N_2 = 315\ rpm$$

We know that the torque acting on the driven shaft

$$= \frac{\text{Power transmitted} \times 4500}{2\pi N_2}$$

$$= \frac{150 \times 4500}{2\pi \times 315}$$

$$= 341 \text{ Kgf} - \text{m}$$

Since there is a 5% friction loss at each shaft, therefore the torque acting on the belt

$$= 1.05 \times 341$$

$$= 358 \text{ Kgf} - \text{m}$$

Since belt is to be designed for 20% overload, therefore the design torque,

$$= 1.2 \times 358$$

$$= 430 \text{ Kgf} - \text{m}$$

Let T_1 = Tension on the tight side of the belt

T_2 = Tension on the slack side of the belt

We know that the torque exerted on the driven pulley.

$$= (T_1 - T_2) r_2 = (T_1 - T_2) 0.6$$

$$= 0.6 (T_1 - T_2) \text{ Kgf} - \text{m}$$

Equating this to the design torque, we have

$$= 0.6 (T_1 - T_2) = 430$$

$$\therefore \quad (T_1 - T_2) = \frac{430}{0.6} = 717 \text{ Kgf}$$

$$\therefore \quad T_1 - T_2 = 717 \text{ Kgf}$$

Now let us find out the angle of contact of the belt on the smaller or driving pulley. From the geometry of the figure, we find that

$$\sin\theta = \frac{O_2M}{O_1O_2} = \frac{r_2 - r_1}{x} = \frac{60 - 45}{360} = 0.0417$$

$$\therefore \quad \theta = 2.4^\circ$$

$$\therefore \quad \theta = 180^\circ - 2\alpha = 180 - 2 \times 2.4 = 175.2^\circ$$

$$=175.2\times\frac{\pi}{180}=3.06\text{rad}$$

We know that

$$2.3\log\left(\frac{T_1}{T_2}\right)=\mu\theta$$

$$=0.3\times3.06$$

$$=0.918$$

$$\therefore \qquad \log\left(\frac{T_1}{T_2}\right)=\frac{0.918}{2.3}=0.3991$$

Or $\frac{T_1}{T_2}=2.51$ 2

From equation (1) & (2) , we have

$T_1=1192\text{Kgf}$ and $T_2=475\text{Kgf}$

Assuming f =safe stress for the belt = 25 Kgf/cm^2

t = thickness of the belt = 1.5 cm

b = Width of the belt.

Since the belt speed is more than 10 m/s, therefore centrifugal tension must be taken into consideration.

Assuming a leather belt for which the density may be taken as 1 gm/cm^2

∴ Weight of the belt per meter length

$$w=\text{Area}\times\text{length}\times\text{density}$$

$$=b\times1.5\times1000\times1$$

$$=0.15b\text{Kg}/\text{m}$$

and centrifugal tension

$$T_c=\frac{w}{g}\times v^2$$

$$=\frac{0.15b}{9.81}(20)^2$$

$$= 6.12bKgf$$

We know that maximum tension in the belt,

$$T = T_1 + T_c = f.b.t$$

$$1192 \div 6.12b = 25 \times b \times 1.5 = 37.5b$$

$$\therefore 37.5b - 6.12b = 1192$$

$$b = 37.98cm.$$

From design data book, the standard width of the belt (b) is 40 cm.

From design data book, Pg.No. 7.53 for open drive

$$L = 2x \div \frac{\pi}{2}(d_2 - d_1) \div \frac{(d_2 - d_1)^2}{4x}$$

$$= 2 \times 360 \div \frac{\pi}{2}(120 - 90) \div \frac{(120 - 90)^2}{4 \times 360}$$

$$= 1050.6cm$$

$$L = 10.506m$$

8. A 7.5 KW electric motor running at 1400rpm is used to drive the input shaft of the gear box of a special purpose machine. Design a suitable roller chain to connect the motor shaft to the gear box shaft to give an exact speed ratio of 10 to 1. Assume the minimum centre distance between driver and driven shaft as 600 rpm.

Given data:

N = P = 7.5 KW

N_1 = 1400 rpm

i = 10

a_0=600 mm

Step 1: Selection of transmission ratio. (i)

$$i = \frac{N_1}{N_2} = 10 \quad \text{given.}$$

Then,

$$\frac{N_1}{10} = N_2$$

$$N_2 = \frac{1400}{10}$$

$$N_2 = 140\text{rpm}$$

Step 2: Selection of no. of teeth on the driver sprocket (z_1).

From PSGDB 7.74

$Z_1 = 7$

Step 3: Calculation of no. of teeth on the driven sprocket (Z_2).

From PSGDB 7.74

$$Z_2 = i \times Z_1$$

$$= 10 \times 7$$

$$Z_2 = 70$$

$$Z_{2max} = 100 \text{ to } 120$$

Recommended value of Z_2 should be less than the above value or else the chain may run off the sprocket for a small pull.

$Z_2 = 70$ is satisfactory.

Step 4: Selection of standard pitch (P).

From PSGDB 7.74

$$\text{Centre distance } a = (30 \text{ to } 50)\ P$$

$$\text{Maximum Pitch, } P_{max} = \frac{a}{30} = \frac{600}{30} = 20 \text{ mm}$$

$$\text{Minimum Pitch, } P_{min} = \frac{a}{50} = \frac{600}{50} = 12 \text{ mm}$$

Any standard pitch between 12 mm and 20 mm can be chosen. But to get a quicker solution, it is always preferred to take the standard pitch closer to P_{max}.

From PSGDB 7.72, Standard Pitch P = 15.875 mm.

Step 5: Selection of the chain:

From PSGDB 7.72, Assume the chain to be duplex.

$\therefore$ 10 A - 2 / DR50 chain number is selected.

Step 6: Calculation of total load on the driving side of the chain (P_T):

From PSGDB 7.78,

$P_T = P_t + P_c + P_a$

Case 1: To find the tangential force (P_t)

From PSGDB 7.78

$$P_t = \frac{1020N}{V}$$

$$\text{Where, v = chain velocity } = \frac{Z_1 \times P \times N_1}{60 \times 1000}$$

$$= \frac{7 \times 15.875 \times 1400}{60 \times 1000}$$

$$= 2.59 \text{ m/s}$$

$$\therefore \quad P_t = \frac{1020 \times 7.5}{2.59}$$

$$P_t = 2950.35 \text{ N}$$

Case 2: To find the centrifugal tension (P_c).

From PSGDB 7.78. $P_c = \frac{Wv^2}{g} = mv^2$

Where, m= mass of the chain

From PSGDB 7.72, For the selected chain,

m = 1.78 Kg/m [1Kg m/s²=1N]

$$\therefore \quad P_c = 1.78\,(2.59)^2$$

$$P_c = 11.94 \text{ N}$$

Case 3: To find the tension due to sagging (P_s).

From PSGDB 7.78,

P_s = K. W. a

Where, K = 6 (for horizontal) From PSGDB 7.78

W = m x g = 1.78 x 9.81 = 17.46 N

A = 600 mm = 0.6 m.

∴ P_s = 6 x 17.46 x 0.6

= 62.82 N

∴ P_T = 2950.35 + 11.94 + 6282

P_T = 3025.11 N

Step 7: Calculation of Service factor (K_s).

From PSGDB 7.76

$K_s = K_1 \cdot K_2 \cdot K_3 \cdot K_4 \cdot K_5 \cdot K_6$

From PSGDB 7.76 and 7.77.

- K_1 = 1.25 for load with mild shocks
- K_2 = 1 for adjustable supports.
- K_3 = 1 ∵ we have used $a_p = (30 \text{ to } 50)P$
- K_4 = 1 for horizontal drive.
- K_5 = 1 for drop lubrication
- K_6 = 1.25 for 16 hrs/day running

∴ K_s = 1.25 × 1 × 1× 1× 1× 1.25
= 1.5625

Step 8: Calculation of design load.

Design load = $P_T \times K_s$

= 3025.11 × 1.5625

= 4726.73 N

Step 9: Calculation of working factor of safety (FS_w)

$$FS_w = \frac{Q}{\text{Design load}}$$

Where, Q = Breaking load = 44400 N. From PGSDB 7.72 for the selected chain

$$\therefore \quad FS_W = \frac{44400}{4726.73}$$

$$FS_W = 9.4$$

Step 10: Check for factor of safety.

From PSGDB 7.77, Recommended factor of safety = 12.45

We find $FS_W < 12.45$, the design is not safe.

In order to overcome this issue we have to increase the pitch = 19.05 mm.

$\therefore$ The chain number 12 A -2 / DR 60 is selected.

For this chain, M= 2.90 Kg/m, Q = 63600 N

By the recalculation of step 6 and step 8, step 9.

P_T = 2590.28 N.

Design load = 4047.31 N

FS_w = 15.71

We find $FS_w > 12.45$, the design is safe.

Step 11: Check for the bearing stress in the roller.

$$\sigma_{roller} = \frac{P_t \times K_s}{A}$$

Where, $A = 210\ mm^2$ From PSGDB 7.72 for selected chain

$$\therefore \quad \sigma_{roller} = \frac{2459.81*1.5625}{210}$$

= 18.30 N/mm^2

From PSGDB 7.77, the allowable bearing stress for the given speed 1400rpm , is 19.75 N/mm^2.

Induced stress is less than the allowable stress i.e 18.30 < 19.75 N/mm^2. $\therefore$ the design is safe.

Step 12: Calculation of length of chain (L).

From PSGDB 7.75

$$L = l_p \times P$$

$$l_p = 2a_p + \left(\frac{Z_1 + Z_2}{2}\right) + \frac{[(Z_2 - Z_1)/2\pi]^2}{a_p}$$

$$a_p = \frac{a_0}{P} = \frac{600}{19.05} = 31.50$$

$$\therefore \quad l_p = 2 \times 31.50 + \left(\frac{7 + 70}{2}\right) + \frac{[(70 - 7)/2\pi]^2}{31.50}$$

$$= 63 + 38.5 + 3.19$$

$$l_p = 104.69$$

$$l_p \approx 106 \text{ links}$$

$$\therefore \left.\begin{array}{l}\text{Actual length}\\ \text{of chain}\end{array}\right\} L = 106 \times 19.05$$

$$L = 2019.3 \text{ mm}$$

Step 13:

Calculation of exact centre distance (a): From PSGDB 7.75.

$$a = \frac{e + \sqrt{e^2 - 8m}}{4} \times P$$

$$* \quad e = l_p - \left(\frac{Z_1 + Z_2}{2}\right)$$

$$= 106 - \left(\frac{7 + 70}{2}\right)$$

$$e = 67.5$$

$$* \quad m = \left(\frac{Z_2 - Z_1}{2\pi}\right)^2$$

$$= \left(\frac{70 - 7}{2\pi}\right)^2$$

$$m = 100.54$$

$$\therefore \quad a = \frac{67.5 + \sqrt{67.5^2 - 8 \times 100.54}}{4} \times 19.05$$

$$a = 613.18 \text{ mm}$$

Decrement in centre distance for an initial sag $= 0.01a$

$$= 6.132 \text{ mm}$$

$$\therefore \quad \text{Exact centre distance } = 613.18 - 6.132$$

$$= 607.05 \text{ mm.}$$

Step 14: Calculation of sprocket diameters.

Case 1: Smaller sprocket.

PCD of smaller sprocket $d_1 = \frac{P}{\sin\left(\frac{180}{Z_1}\right)}$ From PSGDB 7.78

$$= \frac{19.05}{\sin\left(\frac{180}{7}\right)}$$

$$d_1 = 43.91 \text{ mm.}$$

Sprocket outside diameter $d_{01} = d_1 + 0.8d_r$

$d_r =$ diameter of roller $= 11.90$ mm. From PSGDB 7.72 for selected chain.

$$\therefore \quad d_{01} = 43.91 + 0.8 \times 11.90$$

$$d_{01} = 53.43 \text{ mm}$$

Case 2:

Larger sprocket:

$d_2 = \frac{P}{\sin\left(\frac{180}{Z_2}\right)}$ From PSGDB 7.78

$$= \frac{19.05}{\sin\left(\frac{180}{70}\right)}$$

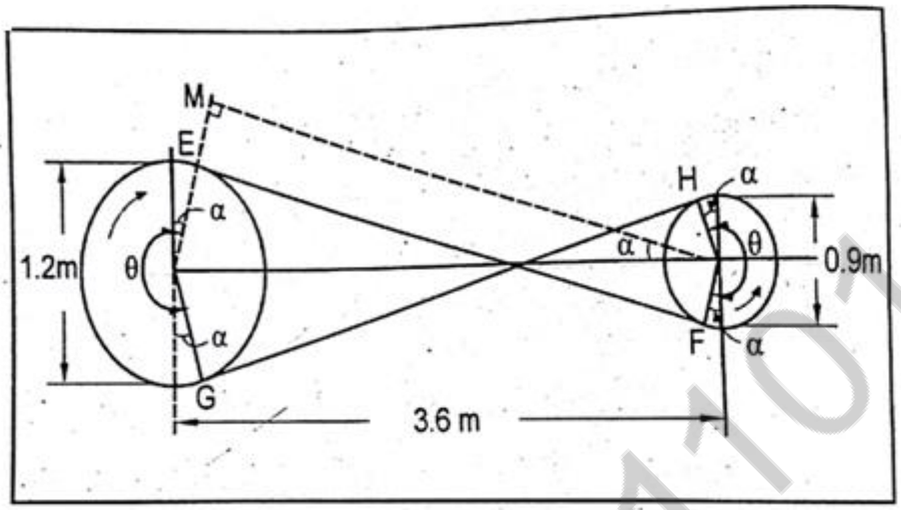
$$d_2 = 424.61 \text{ mm}$$

$$\left.\begin{matrix}\text{Sprocket outside}\\ \text{diameter}\end{matrix}\right\} d_{02} = d_2 + 0.8d_r$$

$$= 424.61 + 0.8 + 11.90$$

$$d_{02} = 434.13 \text{ mm}$$

9. Design a flat belt drive to transmit 110kn for a system. Consisting of two pulleys of diameter 0.9m and 1.2m for a cuter distance of 3.6m, belt speed of 20 m/s and co – efficient of friction is O.S. There is a slip of 1.2 % at each pulley and 5% friction loss at each shaft with 20%, overload.



Given: $P = 110kW = 150\ HP,\ d_1 = 0.9m = 90cm,$

$\therefore\ r1 = 0.45\ m,\ d2 = 1.2\ m = 120\ cm,\ \therefore r2 = 0.6\ m\ ;$

$$x = 3.6m;\ v = 20m/s;\ \mu = 0.3;\ S_1 = S_2 = 1.2\%$$

Let N_1 = Speed of the smaller or driving pulley in rpm

N_2 = Speed of the larger or driven pulley in rpm

We know that speed of the belt (v)

$$v = \frac{\pi d_1 N_1}{60}\left(1 - \frac{S_1}{100}\right)$$

$$20 = \frac{\pi \times 0.9 N_1}{60}\left(1 - \frac{1.2}{100}\right)$$

$$\therefore\ N_1 = 430\ rpm$$

and peripheral velocity of driven pulley

$$\frac{\pi d_2 N_2}{60} = v\left(1 - \frac{S_2}{100}\right)$$

$$\frac{\pi \times 1.2 N_2}{60} = 20\left(1 - \frac{1.2}{100}\right)$$

$\therefore\ N_2 = 315$ rpm

We know that the torque acting on the driven shaft

$$= \frac{\text{Power transmitted} \times 4500}{2\pi N_2}$$

$$= \frac{150 \times 4500}{2\pi \times 315}$$

$$= 341 \text{ Kgf} - \text{m}$$

Since there is a 5% friction loss at each shaft, therefore the torque acting on the belt

$$= 1.05 \times 341$$

$$= 358 \text{ Kgf} - \text{m}$$

Since belt is to be designed for 20% overload, therefore the design torque,

$$= 1.2 \times 358$$

$$= 430 \text{ Kgf} - \text{m}$$

Let T_1 = Tension on the tight side of the belt

T_2 = Tension on the slack side of the belt

We know that the torque exerted on the driven pulley.

$$= (T_1 - T_2) r_2 = (T_1 - T_2) 0.6$$

$$= 0.6 (T_1 - T_2) \text{ Kgf} - \text{m}$$

Equating this to the design torque, we have

$$= 0.6 (T_1 - T_2) = 430$$

$$\therefore\ (T_1 - T_2) = \frac{430}{0.6} = 717 \text{ Kgf}$$

$$\therefore\ T_1 - T_2 = 717 \text{ Kgf}$$

Now let us find out the angle of contact of the belt on the smaller or driving pulley. From the geometry of the figure, we find that

$$\sin\theta = \frac{O_2M}{O_1O_2} = \frac{r_2 - r_1}{x} = \frac{60 - 45}{360} = 0.0417$$

$\therefore \quad \theta = 2.4°$

$\therefore \quad \theta = 180° - 2\alpha = 180 - 2 \times 2.4 = 175.2°$

$= 175.2 \times \frac{\pi}{180} = 3.06 rad$

We know that

$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu\theta$

$= 0.3 \times 3.06$

$= 0.918$

$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{0.918}{2.3} = 0.3991$

Or $\quad \frac{T_1}{T_2} = 2.51 \quad\quad 2$

From equation (1) & (2) , we have

$T_1 = 1192 Kgf$ and $T_2 = 475 Kgf$

Assuming f =safe stress for the belt = 25 Kgf/cm^2

t = thickness of the belt = 1.5 cm

b = Width of the belt.

Since the belt speed is more than 10 m/s, therefore centrifugal tension must be taken into consideration.

Assuming a leather belt for which the density may be taken as 1 gm/cm^2

Weight of the belt permetre length

$w = Area \times length \times density$

$= b \times 1.5 \times 1000 \times 1$

$= 0.15 b Kg / m$

and centrifugal tension

$T_c = \frac{w}{g} \times v^2$

$$= \frac{0.15b}{9.81}(20)^2$$

$$= 6.12b\text{Kgf}$$

We know that maximum tension in the belt,

$$T = T_1 + T_c = f.b.t$$

$$1192 \div 6.12b = 25 \times b \times 1.5 = 37.5b$$

$$\therefore 37.5b - 6.12b = 1192$$

$$b = 37.98\text{cm.}$$

From design data book, the standard width of the belt (b) is 40 cm.

From design data book, Pg.No. 7.53 for open drive

$$L = 2x \div \frac{\pi}{2}(d_2 - d_1) \div \frac{(d_2 - d_1)^2}{4x}$$

$$= 2 \times 360 \div \frac{\pi}{2}(120 - 90) \div \frac{(120 - 90)^2}{4 \times 360}$$

$$= 1050.6\text{cm}$$

$$L = 10.506\text{m}$$

10. **A bucket elevator is to be driven by geared motor and a roller chain drive with the information given below.**

Motor out – put – 3KW, speed of motor shaft – 100 rpm, elevator drive shaft speed – 42rpm, load – even. Distance between centres of sprockets approximately = 1.2 m, period of operation 16 hour / day. Geared motor is mounted on an auxiliary bed for centre distance adjustments. Design the chain drive.

Given data:-

$$N = \rho = 3\text{kw}$$
$$N_1 = 100\,\text{rpm}$$
$$N_2 = 42\text{rpm}$$
$$a_0 = a = 1.2\text{m}$$

Step 1:- Selection of transmission ratio (i)

$$i = \frac{N_1}{N_2} = \frac{100}{42} = 2.38 \approx 3$$

Step 2:- Selection of no. of teeth on driver sprocket (z_1)

$Z_1=25$

Step 3:- Calculation of no. of teeth on driven sprocket (z_2)

$$z_2 = ixz_1$$
$$= 3 \times 25$$
$$z_2 = 75$$
$$z_{max} = 100\,to\,120$$

Recommended value of z_2 should be less than the above value or else the chain may run off the sprocket for a small pull, z_2 = 75 is satisfactory.

Step 4:- Selection of standard pitch (P)

Centre distance $q = (30\ \text{to}\ 50)P$

Maximum pitch, $P_{max} = \frac{a}{30} = \frac{1200}{30} = 40mm$

Minimum pitch $P_{min} = \frac{a}{50} = \frac{1200}{50} = 24mm$

Any standard pitch between 24 mm and 40 mm can be chosen, but to get a quicker solution, it is always preferred to take the standard pitch closer to Pmax.

Standard pitch P = 38.10 mm

Step 5:- Selection of the chain:

Assume the chain to be duplex,

∴ 24 B2 / DR 3825 chain number is selected.

Step 6: Calculation of total load on the diving side of the chain (P_T)

$$P_t = P_t + P_c + P_a$$

Case 1:- To find the tangential force (P_t)

$$P_t = \frac{1020N}{v}$$

Where v = chain velocity = $\frac{z_1 \times P \times N_1}{60 \times 1000} = \frac{25 \times 10 \times 100}{60 \times 1000} = 1.59\,m/2$

$$\therefore P_t = \frac{1020 \times 3}{1.59} = 19.24.53N$$

Case 2:- To find the centrifugal tension (P_c)

$$p_c = \frac{Wv^2}{2} = mv^2$$
$$m = 14.50 kg / m$$
$$\therefore p_c = 14.50 \times (1.59)^2$$
$$= 36.66N$$

Case 3:- To find the tension due to sagging (P_s)

$$p_s = K.W.a$$
$$K = 6 \text{ (for horizontal)}$$
$$W = m \times g = 14.50 \times 9.81 = 142.25N$$
$$a = 1200 mm = 1.2m$$
$$\therefore p_s = 6 \times 142.25 \times 1.2$$
$$= 1024.16N$$
$$P_T = 1924.53 + 36.66 + 1024.16$$
$$= 2985.35N$$

Step 7:- Calculation of service factor (Ks)

$$K_s = K_1 . K_2 . K_3 . K_4 . K_5 . K_6$$

Where, K_1 = 1.25 for load with mid shocks

K_2 = 1 for adjustable supports

K_3 = 1 We have used a_p = (30 to 50) P

K_4 = 1 for horizontal drive

K_5 = 1 for drop lubrication

K_6 = 1.25 for 16/ hurs / day running

$$K_s = 1.25 \times 1 \times 1 \times 1 \times 1 \times 1.25$$
$$= 1.5625$$

Step 8:- Calculation of design load

Design load

$$DL = P_T \times K_s$$
$$= 2985.35 \times 1.5625$$
$$= 4664.62N$$

Step 9:- Calculation of working factor of safety (F_{sw})

$$F_{sw} = \frac{Q}{\text{Design Load}}$$
$$Q = 199600N$$
$$F_{sw} = \frac{199600}{4664.62}$$
$$= 42.8$$

Step 10:- Check for factor of safety.

We find Fsw > 7.4, the design is safe.

Step 11:- Check for the bearing stress in the safe.

$$\sigma_{rouer} = \frac{P_t \times K_s}{A}$$
$$A = 11.09 cm^2 = 1109 mm^2$$
$$\sigma_{rouer} = \frac{1924.53 \times 1.5625}{1109}$$
$$= 2.71 / mm^2$$

The allowable bearing stress for the given speed 100 rpm, is 33.3 N/mm².

Induced stress is less than the allowable stress i.e, 2.71 < 33.33 Nmm². The design is safe.

Step 12:- Calculation of length of chain (L)

$$L = \ell_p \times P$$
$$\ell_p = 2a_p \times \left(\frac{z_1 + z_2}{6}\right) + \frac{[(z_2 - z_1)/2\pi]^2}{a_p}$$
$$a_p = \frac{a_0}{p} = \frac{1200}{38.10} = 31.50$$
$$\ell_p = (2 \times 31.50) \times \left(\frac{25 + 75}{2}\right) + \frac{[(75 - 25)/2\pi]^2}{31.50}$$
$$\ell_p = 3152 \text{ links}$$
$$\left.\begin{array}{l}\text{Actual length}\\ \text{of chain}\end{array}\right\} L = 3152 \times 38.10$$
$$= 120 m$$

Step 13:- Calculation of exact centre distance (a)

$$a = \frac{e + \sqrt{e^2 - 8m}}{4} \times P$$
$$e = \ell_p - \left(\frac{z_1 + z_2}{2}\right)$$
$$= 3152 - \left(\frac{25 + 75}{2}\right)$$
$$e = 3102$$
$$m = \left(\frac{z_2 - z_1}{2\pi}\right)$$
$$= \left(\frac{75 - 25}{2\pi}\right)^2$$
$$= 63.33$$
$$a = \frac{3102 + \sqrt{3102^2 - 8 \times 63.33}}{4} \times 38.10$$
$$a = 59092.32 \text{ mm}$$

Decrement in centre distance for an initial sag = 0.01 a ; = 590.92 mm

Exact centre distance = $= 59092.32 - 59092 = 58501.4$ mm

Step 14:- Calculation of sprocket diameters

Case 1: smaller sprocket

PCD of smaller sprocket

$$d_1 = \frac{P}{\sin\left(180/z_1\right)}$$
$$= \frac{38.10}{\sin\left(180/25\right)} = 304 \text{ mm}$$

Sprocket outside diameter $d_{01} = d_1 + 0.8d_r$

Where,

dr = diameter of rouer = 25.40 mm

$$\therefore d_{01} = 304 + 0.8 \times 25.40$$
$$= 324.32 \text{ mm}$$

Case 2:- Layer sprocket:-

$$d_2 = \frac{P}{\sin\left(\frac{180}{z_2}\right)}$$
$$= \frac{38.10}{\sin\left(\frac{180}{75}\right)}$$
$$d_2 = 909.84 \text{ mm}$$

Sprocket outside diameter $= d_{02} = 909.87 + 0.8(25.40) = 930.16$ mm

11. **Two shafts whose center distance are 1m apart are connected by a V belt drive. The driving pulley is supplied with 100kW and has an effective diameter of 300mm. it runs at 1000rpm, while the driven pulley runs at 375rpm. The angle of groove on the pulleys is 40°. The permissible tension in 400mm² cross sectional area of belt is 2.1MPa. The density of the belt is 1100 kg/m³. Taking μ=0.28. Estimate the number of belts required. Also calculate the length of each belt. (April/May 2017)**

    Given data:

    C=1m=1000mm
    P=100kW
    d=300mm
    N_1=1000rpm

N_2=375rpm

2ß=40°

A=400 mm^2

$\rho = 1100$ kg/m3

σ =2.1MPa

μ=0.28.

Step 1: To find the velocity of the belt 'v':

$$V = \frac{\pi d N1}{60}$$

$$= \frac{\pi x 0.3 x 1000}{60} = 15.71 m/s$$

Step 2: To find the larger pulley diameter 'D'

$$\frac{N_2}{N_1} = \frac{d}{D}$$

$$\frac{375}{1000} = \frac{0.3}{D}$$

D=0.8m

Step 3: To find the number of belts required

For an open belt drive

Case i: To find α:

$$\sin \alpha = \frac{D-d}{2C}$$

$$\sin \alpha = \frac{0.8 - 0.3}{2x1}$$

α=14.48°

Case ii: To find θ:

$$\theta = (180 - 2\alpha) x \frac{\pi}{180}$$

$$= (180 - 2x14.48) x \frac{\pi}{180}$$

$$\theta = 2.636\ rad$$

Case iii: To find T_1& T_2:

$$\frac{T_1}{T_2} = e^{\mu\,\theta Cosec\,\beta}$$

$$\frac{T_1}{T_2} = e^{0.28x\ 2.636xCosec\ 20°}$$

$$\frac{T_1}{T_2} = 2.158----1$$

Mass of the belt per meter length

m=density x area x Length

=1100x400x10^{-6}x1

=0.44kg/m

Centrifugal tension

$$T_c = mv^2$$

$$= 0.44x15.71^2$$

$$T_c = 108.59N$$

Maximum tension in the belt

$$T = \sigma xa$$

$$= 2.1x10^6x400x10^{-6}$$

T=840N

We know that the tension in the tight side of the belt

$$T = T_1 + T_c$$

$$840 = T_1 + 108.59$$

$$T_1 = 731.49N$$

From equation 1

$$\frac{731.49}{T_2} = 2.158$$

$$T_2 = 338.93N$$

Case iv: To find the power transmitted

$$P = (T_1 - T_2)xV$$

$$= (731.41 - 338.93)x15.71$$

$$P = 6165.86\ W$$

Case v: To find the number of belts

$$Number\ of\ belts = \frac{Total\ power\ transmitted}{Power\ transmitted\ per\ belt}$$

$$= \frac{100x10^3}{6165.86}$$

= 16.22

=17 belts

Step 4: To find the length of the each belt:

$$L = 2C + \left(\pi/2\right)(D+d) + \frac{(D-d)^2}{4C}$$

= 3.852m

12. **A 7.5 KW electric motor running at 1400rpm is used to drive the input shaft of the gear box of a machine. Design a suitable roller chain to connect the motor shaft to the gearbox shaft to give an exact speed ration of 10:1. The center to center distance of the shaft is to be approximately 600mm. (April/May 2017)**

Given data:

N = P = 7.5 KW

N_1 = 1400 rpm

i = 10

a_0=600 mm

******similar to this problem, Change the power to be 7.5 kW and the speeds 900 and 400 rpm.***

Step 1: Selection of transmission ratio. (i)

$$i = \frac{N_1}{N_2} = 10 \quad \text{given.}$$

Then,

$$\frac{N_1}{10} = N_2$$

$$N_2 = \frac{1400}{10}$$

$$N_2 = 140 \text{rpm}$$

Step 2: Selection of no. of teeth on the driver sprocket (z_1).

From PSGDB 7.74

$$Z_1 = 7$$

Step 3: Calculation of no. of teeth on the driven sprocket (Z_2).

From PSGDB 7.74

$$Z_2 = i \times Z_1$$

$$= 10 \times 7$$

$$Z_2 = 70$$

$$Z_{2max} = 100 \text{ to } 120$$

Recommended value of Z_2 should be less than the above value or else the chain may run off the sprocket for a small pull.

$Z_2 = 70$ is satisfactory.

Step 4: Selection of standard pitch (P).

From PSGDB 7.74

$$\text{Centre distance } a = (30 \text{ to } 50)\ P$$

$$\text{Maximum Pitch, } P_{max} = \frac{a}{30} = \frac{600}{30} = 20 \text{ mm}$$

$$\text{Minimum Pitch, } P_{min} = \frac{a}{50} = \frac{600}{50} = 12 \text{ mm}$$

Any standard pitch between 12 mm and 20 mm can be chosen. But to get a quicker solution, it is always preferred to take the standard pitch closer to P_{max}.

From PSGDB 7.72, Standard Pitch P = 15.875 mm.

Step 5: Selection of the chain:

From PSGDB 7.72, assume the chain to be duplex.

$\therefore$ 10 A - 2 / DR50 Chain number is selected.

Step 6: Calculation of total load on the driving side of the chain (P_T):

From PSGDB 7.78,

$P_T = P_t + P_c + P_a$

Case 1: To find the tangential force (P_t)

From PSGDB 7.78

$$P_t = \frac{1020N}{V}$$

Where, v = chain velocity $= \frac{Z_1 \times P \times N_1}{60 \times 1000}$

$$= \frac{7 \times 15.875 \times 1400}{60 \times 1000}$$

$$= 2.59 \text{ m/s}$$

$$\therefore \quad P_t = \frac{1020 \times 7.5}{2.59}$$

$$P_t = 2950.35 \text{ N}$$

Case 2: To find the centrifugal tension (P_c).

From PSGDB 7.78. $P_c = \frac{Wv^2}{g} = mv^2$

Where, m= mass of the chain

From PSGDB 7.72, For the selected chain,

m = 1.78 Kg/m [1Kg m/s^2=1N]

$$\therefore \quad P_c = 1.78\ (2.59)^2$$

$$P_c = 11.94 \text{ N}$$

Case 3: To find the tension due to sagging (P_s).

From PSGDB 7.78,

P_s = K. W. a

Where, K = 6 (for horizontal) From PSGDB 7.78

W = m x g = 1.78 x 9.81 = 17.46 N

A = 600 mm = 0.6 m.

∴ P_s = 6 x 17.46 x 0.6

= 62.82 N

$\therefore \quad P_T = 2950.35 + 11.94 + 6282$

$P_T = 3025.11$ N

Step 7: Calculation of Service factor (K_s).

From PSGDB 7.76

$$Ks = K_1 \cdot K_2 \cdot K_3 \cdot K_4 \cdot K_5 \cdot K_6$$

From PSGDB 7.76 and 7.77.

- $K_1 = 1.25$ for load with mild shocks
- $K_2 = 1$ for adjustable supports.
- $K_3 = 1$ $\because$ we have used $a_p = (30 \text{ to } 50)P$
- $K_4 = 1$ for horizontal drive.
- $K_5 = 1$ for drop lubrication
- $K_6 = 1.25$ for 16 hrs/day running

$\therefore \quad K_s = 1.25 \times 1 \times 1 \times 1 \times 1 \times 1.25$

$= 1.5625$

Step 8: Calculation of design load.

Design load $= P_T \times K_s$

$= 3025.11 \times 1.5625$

$= 4726.73$ N

Step 9: Calculation of working factor of safety (FS_w)

$$FS_w = \frac{Q}{\text{Design load}}$$

Where, Q = Breaking load = 44400 N. From PGSDB 7.72 for the selected chain

$$\therefore \quad FS_W = \frac{44400}{4726.73}$$

$$FS_W = 9.4$$

Step 10: Check for factor of safety.

From PSGDB 7.77, Recommended factor of safety = 12.45

We find $FS_W < 12.45$, the design is not safe.

In order to overcome this issue we have to increase the pitch = 19.05 mm.

$\therefore$ The chain number 12 A -2 / DR 60 is selected.

For this chain, M= 2.90 Kg/m, Q = 63600 N

By the recalculation of step 6 and step 8, step 9.

P_T = 2590.28 N.

Design load = 4047.31 N

FS_w = 15.71

We find FS_w > 12.45 , the design is safe.

Step 11: Check for the bearing stress in the roller.

$$\sigma_{roller} = \frac{P_t \times K_s}{A}$$

Where, $A = 210 \text{ mm}^2$ From PSGDB 7.72 for selected chain

$$\therefore \quad \sigma_{roller} = \frac{2459.81*1.5625}{210}$$

= 18.30 N/mm^2

From PSGDB 7.77, the allowable bearing stress for the given speed 1400rpm , is 19.75 N/mm^2.

Induced stress is less than the allowable stress i.e 18.30 < 19.75 N/mm^2. ∴ the design is safe.

Step 12: Calculation of length of chain (L).

From PSGDB 7.75

$$L = l_p \times P$$

$$l_p = 2a_p + \left(\frac{Z_1 + Z_2}{2}\right) + \frac{[(Z_2 - Z_1)/2\pi]^2}{a_p}$$

$$a_p = \frac{a_0}{P} = \frac{600}{19.05} = 31.50$$

$$\therefore \quad l_p = 2 \times 31.50 + \left(\frac{7+70}{2}\right) + \frac{[(70-7)/2\pi]^2}{31.50}$$

$$= 63 + 38.5 + 3.19$$

$$l_p = 104.69$$

$$l_p \approx 106 \text{ links}$$

$$\therefore \left.\begin{array}{l}\text{Actual length} \\ \text{of chain}\end{array}\right\} L = 106 \times 19.05$$

$$L = 2019.3 \text{ mm}$$

Step 13: Calculation of exact centre distance (a): From PSGDB 7.75.

$$a = \frac{e + \sqrt{e^2 - 8m}}{4} \times P$$

$$* \quad e = l_p - \left(\frac{Z_1 + Z_2}{2}\right)$$

$$= 106 - \left(\frac{7 + 70}{2}\right)$$

$$e = 67.5$$

$$* \quad m = \left(\frac{Z_2 - Z_1}{2\pi}\right)^2$$

$$= \left(\frac{70 - 7}{2\pi}\right)^2$$

$$m = 100.54$$

$$\therefore \quad a = \frac{67.5 + \sqrt{67.5^2 - 8 \times 100.54}}{4} \times 19.05$$

$$a = 613.18 \text{ mm}$$

Decrement in centre distance for an initial sag $= 0.01a$

$$= 6.132 \text{ mm}$$

$$\therefore \quad \text{Exact centre distance} = 613.18 - 6.132$$

$$= 607.05 \text{ mm.}$$

Step 14: Calculation of sprocket diameters.

Case 1: Smaller sprocket.

PCD of smaller sprocket $d_1 = \dfrac{P}{\sin\left(\dfrac{180}{Z_1}\right)}$ From PSGDB 7.78

$$= \frac{19.05}{\sin\left(\frac{180}{7}\right)}$$

$$d_1 = 43.91 \text{ mm.}$$

Sprocket outside diameter $d_{01} = d_1 + 0.8d_r$

$d_r =$ diameter of roller $= 11.90$ mm. From PSGDB 7.72 for selected chain.

$$\therefore \quad d_{01} = 43.91 + 0.8 \times 11.90$$

$$d_{01} = 53.43 \text{ mm}$$

Case 2:Larger sprocket:

$$d_2 = \frac{P}{\sin\left(\frac{180}{Z_2}\right)} \quad \text{From PSGDB 7.78}$$

$$= \frac{19.05}{\sin\left(\frac{180}{70}\right)}$$

$$d_2 = 424.61 \text{ mm}$$

$$\left.\begin{array}{r}\text{Sprocket outside}\\ \text{diameter}\end{array}\right\} d_{02} = d_2 + 0.8d_r$$

$$= 424.61 + 0.8 + 11.90$$

$$d_{02} = 434.13 \text{ mm}$$

13. Design a V belt drive and calculate the actual belt tensions and average stress for the following data. Power to be transmitted = 7.5 KW, speed of driving wheel = 1000 rpm, speed of driven wheel = 300 rpm, diameter of the driven pulley = 500 mm, diameter of the driver pulley = 150 mm and centre distance = 925 mm

Given data:

P = 7.5 KW

N_1 = 1000 rpm

N_2 = 300 rpm

D = 500 mm

d = 150 mm

C = 925 mm

******Similar to this problem change the center distance as C=2500mm and slight changes in speeds***

Step 1: Selection of belt

From PSGDB 7.58,

For 7.5 KW, B section is selected

Step 2: Selection of pulley diameters. d & D:

d = 150 mm, D = 500 mm given.

Step 3: Selection of centre distance (c) :

C= 925 mm given.

Step 4: Calculation of nominal pitch length (L).

From PSGDB 7.61,

$$L = 2C + \frac{\pi}{2}(D+d) + \frac{(D-d)^2}{4C}$$

$$= 2\times925 + \frac{\pi}{2}(500+150) + \frac{(500-150)^2}{4\times925}$$

$$= 2904.12mm.$$

From PSGDB 7.60, For B section.

The next standard length L = 3091 mm.

Step 5: Selection of various modification factors.

Case 1: Length correction factor (F_c)

From PSGDB 7.60 for B section corresponding to 'L'

$$F_c = 1.07$$

Case 2: Correction factor for arc of contact (F_α)

From PSGDB 7.68

Arc of contact angle $= 180° - (\frac{D-d}{C}) \times 60°$

$$= 180° - (\frac{500 - 150}{925}) \times 60°$$

$$= 157.29°$$

Corresponding to the angle 1579.29° ≈ 160°

F_d = 0.95.

Case 3: Service factor (F_a).

From PSGDB 7.69 F_a = 1.3

Step 6: Calculation of Maximum power capacity (KW).

From PSGDB 7.62, For B section.

$$KW = (0.79S^{-0.09} - \frac{50.8}{d_e} - 1.32 \times 10^{-4} S^2)S$$

Where, S = Belt speed $= \frac{\pi d N_1}{60}$

$$= \frac{\pi \times 0.150 \times 1000}{60}$$

$$= 7.854 m/s$$

d_e = equivalent pitch diameter; From PSGDB 7.62 $\frac{D}{d} = \frac{500}{150} = 3.33$ Take F_b=1.14

$$= d_p \times F_b$$

$$= 150 \times 1.14$$

$$= 171 mm.$$

$$\therefore \ KW = (0.79 \times 7.854^{-0.09} - \frac{50.8}{171} - 1.32 \times 10^{-4} \times 7.84^2)7.84$$

$$= 2.757 KW$$

Step 7: Calculation of number of belts (n_b)

From PSGDB 7.70

$$n_b = \frac{P \times F_a}{K_w \times F_c \times F_d}$$

$$= \frac{7.5 \times 1.3}{2.757 \times 1.07 \times 0.95}$$

$$= 3.48$$

$$n_b = 4 \text{belts.}$$

Step 8: Calculation of actual centre distance. (C_{actual}).

From PSGDB 7.61

$$C_{actual} = A + \sqrt{A^2 - B}$$

$$A = \frac{L}{4} - \pi\left[\frac{D+d}{8}\right]$$

$$= \frac{3091}{4} - \pi\left[\frac{500+150}{8}\right]$$

$$A = 517.5\text{mm}$$

$$B = \frac{(D-d)^2}{8} = \frac{(500-150)^2}{8}$$

$$= 15312.5 \text{ mm}^2$$

$$\therefore \quad C_{actual} = 517.5 + \sqrt{517.5^2 - 15312.5}$$

$$= 1020 \text{ mm.}$$

Step 9: Calculation of belt tensions (T_1 and T_2).

Power transmitted per belt $= (T_1 - T_2)\,v$

$$\frac{7.5 \times 10^3}{4} = (T_1 - T_2)7.854$$

$$T_1 - T_2 = 238.73 \text{ ------1}$$

From PSGDB 7.58 ⇒ m = 0.189 Kg/m.

7.70 ⇒ 2B =34°

From step 5: ⇒ $\alpha = 157.29° \times \frac{\pi}{180°}$

$$= 2.745 \text{ rad.}$$

Tension ratio $\Rightarrow \quad \frac{T_1 - mv^2}{T_2 - mv^2} = e^{\mu\alpha \operatorname{cosec}\beta}$

$$\frac{T_1 - 0.189(7.854)^2}{T_2 - 0.189(7.854)^2} = e^{0.3\times 2.745\times \operatorname{cosec}17^\circ}$$

$$T_1 - 16.72T_2 = -184.3 \quad \text{------2}$$

Solving equation 1 and 2

T_2 = 26.9 N , T_1 = 265.64 N

Step 10: Calculation of Stress induced.

$$\text{Stress induced} = \frac{\text{Maximum tension}}{\text{Cross sectional area}}$$

From PSGDB 7.58 Area of B section = 140 mm² $\therefore$ Stress induced = $\frac{265.64}{140}$

$$= 1.897\ N/mm^2$$

14. **Select a wire rope for a vertical mine hoist to lift a load of 20KN from a depth of 60 metres. A rope speed of 4 m/sec is to attained on 10 seconds.**

    Given data:

    Weight to be lifted $= 20$ KN

    Depth $= 60$m

    $v_2 = v = 4\,m/sec = 240\,m/min$

    $t = 10\,sec$

Step 1: Selection of suitable wire rope.

For hoisting purpose, 6×19 rope is selected. From PSGDB 9.1

Step 2: Calculation of Design load.

Assuming the factor of safety of 15, the design load is calculated.

Design load $= 20\times 15$

$= 300$KN

Step 3: To find wire rope diameter (d).

From PSGDB 9.5 For design load 300KN, The next standard value.

$$d = 25\text{mm}$$
$$m = 2.41\,\text{Kg/m}$$
$$\sigma_u = 1600 \text{ to } 1750\,\text{N/mm}^2$$

$$\text{Breaking strength} = 340\text{KN}$$

Step 4: Sheave diameter (D)

From PSGDB 9.1. We find $D_{min}/d = 27$ for class 4, for velocity upto 50m/min . But the actual speed is 240m/min $\left(\text{i.e}\,\frac{240}{50} \approx 5 \text{ times } 50\,\text{m/min}\right)$. Therefore $\frac{D_{min}}{d}$ has to be modified.

$$\frac{D_{min}}{d} = 27 \times (1.08)^{5-1} = 36.73 \approx 37\text{mm.}$$

Sheave diameter $D = 37 \times d$

$$= 37 \times 25$$

$$D = 925\,\text{mm}$$

Step 5:Calculation of Area of cross section of the rope (A).

From PSGDB 9.1

$$A = 0.4 \times \pi/4 \times d^2$$

$$= 0.4 \times \pi/4 \times 25^2$$

$$A = 196.35\text{mm}^2$$

Step 6: To find Wire diameter. (d_w).

$$d_w = \frac{d}{1.5\sqrt{i}}$$

$$= \frac{25}{1.5\sqrt{6 \times 19}}$$

$$= 1.56$$

$$d_w = 2\text{mm}$$

Step 7: Weight of the rope. (W_r).

$$W_r \text{ per meter} = 2.41 \times 9.81 = 23.64\,\text{N/m}.$$

$$W_r = 23.64 \times 60 = 1418.53\text{N}$$

$$= 1418.53\text{N}$$

Step 8: Load calculations

Case 1: Direct load (W_d)

$$W_d = W + W_r = 20 + 1418.53 \times 10^{-3} = 21.42\text{KN}$$

Case 2: Bending load (W_b)

$$W_b = \sigma_b \times A = \frac{E_r \times d_w}{D} \times A$$

$$= \frac{0.84 \times 10^5 \times 2}{925} \times 196.35 \qquad \left[\because E_r = 0.84 \times 10^5\,\text{N/mm}^2\right]$$

$$= 35661.41\text{N}$$

$$= 35.66\text{KN.}$$

Case 3: Acceleration load (W_a)

$$W_a = \left(\frac{W + W_r}{g}\right) a \qquad a = \frac{v_2 - v_1}{t}$$

$$= \left(\frac{20 + 1418.53 \times 10^{-3}}{9.81}\right) \times 0.4 \qquad = \frac{4 - 0}{10}$$

$$= 0.87\text{ KN} \qquad = 0.4\,\text{m/s}^2$$

$$\left.\begin{matrix}\therefore \text{Effective load on the rope} \\ \text{during acceleration}\end{matrix}\right\} W_{ea} = W_d + W_b + W_a$$

$$= 21.42 + 35.66 + 0.87$$

$$= 57.95$$

$$W_{ea} = 58\text{KN}$$

Step 9:Working factor of Safety (FS_w).

$$FS_w = \frac{\text{Breaking load}}{W_{ea}}$$

$$= \frac{340}{58}$$

$$FS_w = 5.86$$

Step 10: Check for Safe design

- We find $F_{sw} < n'(6)$. $\therefore$ The design is not safe.
- The safe design can be achieved either by selecting the rope with greater breaking strength.

From PSGDB 9.5, for d=25, take breaking strength = 376 KN and $\sigma_u = 1750 \text{ to } 1900\,\text{N/mm}^2$

$$\therefore\ F_{sw} = \frac{376}{58}$$

$$= 6.48$$

Now we find $F_{sw} > n'(6)$. $\therefore$ The design is safe.

15. **Select a flat belt to drive a mill at 250 rpm from a 10 kW, 730 rpm motor. Centre distance is to be around 2000 mm. The mill shaft pulley is of 1000 mm diameter. (April/May 2018)**

Given data:

N_1=730 rpm

N_2= 360 rpm

P= 10 kW

C= 2000 mm

D=1000 mm

******similar to this problem, Change the power to be 10 kW and the speeds 730 and 360 rpm.***

Step 1: **Calculation of Pulley diameters:**

The driven pulley diameter D = 1000 mm.

$$\text{W.K.T } \phi = \frac{D}{d} = \frac{N_1}{N_2}$$

Case (i): To find the driven pulley speed. (N_2).

Case (ii): To find the driver pulley diameter (d):

$$N_2 = \frac{N_1}{\phi}$$

$$= \frac{1440}{2.5}$$

$$N_2 = 576 rpm$$

Case (iii): To find the driver pulley diameter (d):

$$d = \frac{D}{\phi}$$

$$= \frac{1000}{2.5}$$

$$d = 400 mm$$

From PSGDB 7.54, from recommended series of pulley diameters and tolerances.

$$\left.\begin{array}{l}\text{The standard diameter for}\\ \text{the driver pulley}\end{array}\right\} d = 400 mm$$

Step 2: Calculation of design power in KW.

$$\text{Design power} = \frac{\text{Rated power}(K_w) \times \text{Load correction factor}(K_s)}{\text{Arc of contact factor}(K_\alpha) \times \text{Small pulley factor}(K_d)}$$

Case (i): To find the arc of contact factor ($K\alpha$)

From PSGDB 7.54

$$\text{Arc of contact} = 180° - \left(\frac{D-d}{c}\right) \times 60°$$

$$= 180° - \left(\frac{1000-400}{3600}\right) \times 60°$$

$$= 170°$$

From PSGDB 7.54, take the value of $K_\alpha = 1.04$. Corresponding to the arc of contact 170°

$$K_\alpha = 1.04$$

Case (ii): To find the small pulley factor (K_d)

Table: Small pulley factor 'K_d'

Small Pulley diameter	K_d
Upto 100mm	0.5
100 – 200mm	0.6
200 – 300mm	0.7
300 – 400mm	0.8
400 – 750mm	0.9
Over 750mm	1.0

From the above table. We take the K_d value 0.8

$$\therefore K_d = 0.8$$

Case (iii):

To find the design power, KW:

W. K. T. $$\text{Design power} = \frac{P \times K_s}{K_a \times K_d}$$

$$= \frac{12 \times 1.3}{1.04 \times 0.8}$$

$$= 18.75 \text{KW}$$

Step 3: Selection of belt:

Given: 5 Ply, flat Dunlop belt. Its capacity is given by 0.0118 KW/ply/mm.

Step 4: Load rating correction:

From PSGDB 7.54.

$$\text{Load rating at 'V' m/s} = \text{Load rating at 10 m/s} \times \frac{v}{10}$$

$$\text{Load rating at 16 m/s} = (0.0118 \times 2) \times \frac{16}{10}$$

$$= 0.03776 \text{KW/Ply/mm}$$

Step 5: Determination of belt width:

$$\text{Width of the belt} = \frac{\text{Design Power}}{\text{Load rating} \times \text{No. of plies.}}$$

$$= \frac{18.75}{0.03776 \times 5}$$

$$= 99.31\text{mm}$$

From PSGDB 7.52 Specification of transmission belting standard widths.

The standard belt width for 5 Ply belt = 100mm.

Step 6: Determination of Pulley width:

From PSGDB 7.54, Pulley width is given by

Pulley width = Belt width + 18 mm

= 100 + 13 mm

= 113 mm

From PSGDB 7.54, recommended series of width of flat pulleys, mm.

The standard pulley width = 125 mm.

Step 7: Calculation of length of the belt (L):

From PSGDB 7.61,

$$L = 2C + \frac{\pi}{2}(D + d) + \frac{(D - d)^2}{4C}$$

$$= 2 \times 3600 + \pi/2 (1000 + 400) + \frac{(1000 - 400)^2}{4 \times 3600}$$

$$= 7200 + 2199.11 + 25$$

$$L = 9424.11\text{mm}$$

16. Design a chain drive to accurate a compressor from a 10 Kw electric motor at 960 rpm. The compressor speed is to be 350 rpm. Minimum centre to center should be 500 mm. Motor is mounted on an auxiliary bed. Compressor is to work for 8 hours/day. (April/May 2018)

Given data:

N_1=960rpm

N_2= 350 rpm

P= 10 kW

a= 500 mm

******similar to this problem, Change the power to be 10 kW and the speeds 960 and 350 rpm.***

Step 1: Selection of transmission ratio. (i)

$$i = \frac{N_1}{N_2} = 10 \quad \text{given.}$$

Then,

$$\frac{N_1}{10} = N_2$$

$$N_2 = \frac{1400}{10}$$

$$N_2 = 140 rpm$$

Step 2: Selection of no. of teeth on the driver sprocket (z_1).

From PSGDB 7.74

$Z_1 = 7$

Step 3: Calculation of no. of teeth on the driven sprocket (Z_2).

From PSGDB 7.74

$$Z_2 = i \times Z_1$$

$$= 10 \times 7$$

$$Z_2 = 70$$

$$Z_{2max} = 100 \text{ to } 120$$

Recommended value of Z_2 should be less than the above value or else the chain may run off the sprocket for a small pull.

$Z_2 = 70$ is satisfactory.

Step 4: Selection of standard pitch (P).

From PSGDB 7.74

$$\text{Centre distance } a = (30 \text{ to } 50)\ P$$

$$\text{Maximum Pitch, } P_{max} = \frac{a}{30} = \frac{600}{30} = 20 \text{ mm}$$

$$\text{Minimum Pitch, } P_{min} = \frac{a}{50} = \frac{600}{50} = 12 \text{ mm}$$

Any standard pitch between 12 mm and 20 mm can be chosen. But to get a quicker solution, it is always preferred to take the standard pitch closer to P_{max}.

From PSGDB 7.72, Standard Pitch P = 15.875 mm.

Step 5: Selection of the chain:

From PSGDB 7.72, assume the chain to be duplex.

$\therefore$ 10 A - 2 / DR50 Chain number is selected.

Step 6: Calculation of total load on the driving side of the chain (P_T):

From PSGDB 7.78,

$$P_T = P_t + P_c + P_a$$

Case 1: To find the tangential force (P_t)

From PSGDB 7.78

$$P_t = \frac{1020N}{V}$$

$$\text{Where, v = chain velocity } = \frac{Z_1 \times P \times N_1}{60 \times 1000}$$

$$= \frac{7 \times 15.875 \times 1400}{60 \times 1000}$$

$$= 2.59 \text{ m/s}$$

$$\therefore \quad P_t = \frac{1020 \times 7.5}{2.59}$$

$$P_t = 2950.35 \text{ N}$$

Case 2: To find the centrifugal tension (P_c).

$$\text{From PSGDB 7.78. } P_c = \frac{Wv^2}{g} = mv^2$$

Where, m= mass of the chain

From PSGDB 7.72, For the selected chain,

$m = 1.78$ Kg/m [1Kg m/s^2=1N]

$$\therefore \quad P_c = 1.78\,(2.59)^2$$

$$P_c = 11.94 \text{ N}$$

Case 3: To find the tension due to sagging (P_s).

From PSGDB 7.78,

$$P_s = K.\ W.\ a$$

Where, K = 6 (for horizontal) From PSGDB 7.78

W = m x g = 1.78 x 9.81 = 17.46 N

A = 600 mm = 0.6 m.

$\therefore$ P_s = 6 x 17.46 x 0.6

= 62.82 N

$\therefore$ P_T = 2950.35 + 11.94 + 6282

P_T = 3025.11 N

Step 7: Calculation of Service factor (K_s).

From PSGDB 7.76

$$Ks = K_1 \cdot K_2 \cdot K_3 \cdot K_4 \cdot K_5 \cdot K_6$$

From PSGDB 7.76 and 7.77.

- K_1 = 1.25 for load with mild shocks
- K_2 = 1 for adjustable supports.
- K_3 = 1 $\because$ we have used $a_p = (30 \text{ to } 50)P$
- K_4 = 1 for horizontal drive.
- K_5 = 1 for drop lubrication
- K_6 = 1.25 for 16 hrs/day running

$\therefore$ K_s = 1.25 × 1 × 1× 1× 1× 1.25
= 1.5625

Step 8: Calculation of design load.

Design load = $P_T \times K_s$

= 3025.11 × 1.5625

= 4726.73 N

Step 9: Calculation of working factor of safety (FS_w)

$$FS_w = \frac{Q}{\text{Design load}}$$

Where, Q = Breaking load = 44400 N. From PGSDB 7.72 for the selected chain

$$\therefore \quad FS_W = \frac{44400}{4726.73}$$

$$FS_W = 9.4$$

Step 10: Check for factor of safety.

From PSGDB 7.77, Recommended factor of safety = 12.45

We find $FS_W < 12.45$, the design is not safe.

In order to overcome this issue we have to increase the pitch = 19.05 mm.

∴ The chain number 12 A -2 / DR 60 is selected.

For this chain, M= 2.90 Kg/m, Q = 63600 N

By the recalculation of step 6 and step 8, step 9.

P_T = 2590.28 N.

Design load = 4047.31 N

FS_w = 15.71

We find FS_w > 12.45 , the design is safe.

Step 11: Check for the bearing stress in the roller.

$$\sigma_{roller} = \frac{P_t \times K_s}{A}$$

Where, $A = 210\ mm^2$ From PSGDB 7.72 for selected chain

$$\therefore \quad \sigma_{roller} = \frac{2459.81*1.5625}{210}$$

= 18.30 N/mm^2

From PSGDB 7.77, the allowable bearing stress for the given speed 1400rpm , is 19.75 N/mm^2.

Induced stress is less than the allowable stress i.e 18.30 < 19.75 N/mm^2. ∴ the design is safe.

Step 12: Calculation of length of chain (L).

From PSGDB 7.75

$$L = l_p \times P$$

$$l_p = 2a_p + \left(\frac{Z_1 + Z_2}{2}\right) + \frac{[(Z_2 - Z_1)/2\pi]^2}{a_p}$$

$$a_p = \frac{a_0}{P} = \frac{600}{19.05} = 31.50$$

$$\therefore \quad l_p = 2 \times 31.50 + \left(\frac{7+70}{2}\right) + \frac{[(70-7)/2\pi]^2}{31.50}$$

$$= 63 + 38.5 + 3.19$$

$$l_p = 104.69$$

$$l_p \approx 106 \text{ links}$$

$$\therefore \left.\begin{array}{l}\text{Actual length}\\ \text{of chain}\end{array}\right\} L = 106 \times 19.05$$

$$L = 2019.3 \text{ mm}$$

Step 13: Calculation of exact centre distance (a): From PSGDB 7.75.

$$a = \frac{e + \sqrt{e^2 - 8m}}{4} \times P$$

$$* \quad e = l_p - \left(\frac{Z_1 + Z_2}{2}\right)$$

$$= 106 - \left(\frac{7+70}{2}\right)$$

$$e = 67.5$$

$$* \quad m = \left(\frac{Z_2 - Z_1}{2\pi}\right)^2$$

$$= \left(\frac{70-7}{2\pi}\right)^2$$

$$m = 100.54$$

$$\therefore \quad a = \frac{67.5 + \sqrt{67.5^2 - 8 \times 100.54}}{4} \times 19.05$$

$$a = 613.18 \text{ mm}$$

Decrement in centre distance for an initial sag $= 0.01a$

$$= 6.132 \text{ mm}$$

$$\therefore \quad \text{Exact centre distance } = 613.18 - 6.132$$

$$= 607.05 \text{ mm.}$$

Step 14: Calculation of sprocket diameters.

Case 1: Smaller sprocket.

PCD of smaller sprocket $d_1 = \dfrac{P}{\sin\left(\dfrac{180}{Z_1}\right)}$ From PSGDB 7.78

$$= \frac{19.05}{\sin\left(\frac{180}{7}\right)}$$

$$d_1 = 43.91 \text{ mm.}$$

Sprocket outside diameter $d_{01} = d_1 + 0.8 d_r$

$d_r =$ diameter of roller $= 11.90$ mm. From PSGDB 7.72 for selected chain.

$$\therefore \quad d_{01} = 43.91 + 0.8 \times 11.90$$

$$d_{01} = 53.43 \text{ mm}$$

Case 2:Larger sprocket:

$d_2 = \dfrac{P}{\sin\left(\dfrac{180}{Z_2}\right)}$ From PSGDB 7.78

$$= \frac{19.05}{\sin\left(\frac{180}{70}\right)}$$

$$d_2 = 424.61 \text{ mm}$$

$$\left.\begin{matrix}\text{Sprocket outside}\\ \text{diameter}\end{matrix}\right\} d_{02} = d_2 + 0.8d_r$$

$$= 424.61 + 0.8 + 11.90$$

$$d_{02} = 434.13 \text{ mm}$$

17. Select a V belt drive for 15kW, 1440 rpm motor, which drives a centrifugal pump running at a speed of 576 rpm for a service of 8-10 hours per day. The distance between the driver and the driven shaft is approximately 1.2 m. Service factor Ks=1.1, design factor Na=1.0 , V_R=2.5 (April/May 2018)

Given data:

P = 7.5 KW

N_1 = 1000 rpm

N_2 = 300 rpm

D = 500 mm

d = 150 mm

C = 925 mm

******similar to this problem, Change Given data as per the given question and solve the problem with same procedure.***

Step 1: Selection of belt

From PSGDB 7.58,

For 7.5 KW, B section is selected

Step 2: Selection of pulley diameters. d & D:

d = 150 mm, D = 500 mm given.

Step 3: Selection of centre distance (c) :

C= 925 mm given.

Step 4: Calculation of nominal pitch length (L).

From PSGDB 7.61,

$$L = 2C + \frac{\pi}{2}(D + d) + \frac{(D-d)^2}{4C}$$

$$= 2\times 925 + \frac{\pi}{2}(500 + 150) + \frac{(500-150)^2}{4\times 925}$$

$$= 2904.12 mm.$$

From PSGDB 7.60, For B section.

The next standard length L = 3091 mm.

Step 5: Selection of various modification factors.

Case 1: Length correction factor (F_c)

From PSGDB 7.60 for B section corresponding to 'L'

$$F_c = 1.07$$

Case 2: Correction factor for arc of contact (F_α)

From PSGDB 7.68

Arc of contact angle $= 180° - (\frac{D-d}{C}) \times 60°$

$$= 180° - (\frac{500-150}{925}) \times 60°$$

$$= 157.29°$$

Corresponding to the angle 1579.29° $\approx$ 160°

F_d = 0.95.

Case 3: Service factor (F_a).

From PSGDB 7.69

F_a = 1.3

Step 6: Calculation of Maximum power capacity (KW).

From PSGDB 7.62, For B section.

$$KW = (0.79S^{-0.09} - \frac{50.8}{d_e} - 1.32 \times 10^{-4} S^2)S$$

Where, S = Belt speed $= \frac{\pi d N_1}{60}$

$$= \frac{\pi \times 0.150 \times 1000}{60}$$

$$= 7.854 m/s$$

d_e = equivalent pitch diameter; From PSGDB 7.62 $\frac{D}{d}=\frac{500}{150}=3.33$ Take F_b=1.14

$= d_p \times F_b$

$= 150 \times 1.14$

$= 171\ \text{mm.}$

$$\therefore \quad KW = (0.79 \times 7.854^{-0.09} - \frac{50.8}{171} - 1.32 \times 10^{-4} \times 7.84^2)7.84$$

$= 2.757KW$

Step 7: Calculation of number of belts (n_b)

From PSGDB 7.70

$$n_b = \frac{P \times F_a}{K_w \times F_c \times F_d}$$

$$= \frac{7.5 \times 1.3}{2.757 \times 1.07 \times 0.95}$$

$= 3.48$

$n_b = 4\text{belts.}$

Step 8: Calculation of actual centre distance. (C_{actual}).

From PSGDB 7.61

$$C_{actual} = A + \sqrt{A^2 - B}$$

$$A = \frac{L}{4} - \pi\left[\frac{D+d}{8}\right]$$

$$= \frac{3091}{4} - \pi\left[\frac{500+150}{8}\right]$$

$A = 517.5\text{mm}$

$$B = \frac{(D-d)^2}{8} = \frac{(500-150)^2}{8}$$

$$= 15312.5\ \text{mm}^2$$

$$\therefore \quad C_{actual} = 517.5 + \sqrt{517.5^2 - 15312.5}$$

$$= 1020 \text{ mm.}$$

Step 9: Calculation of belt tensions (T_1 and T_2).

Power transmitted per belt $= (T_1 - T_2)\,v$

$$\frac{7.5\times10^3}{4} = (T_1 - T_2)7.854$$

$$T_1 - T_2 = 238.73 \text{ ------1}$$

From PSGDB 7.58 $\Rightarrow$ m = 0.189 Kg/m.

7.70 $\Rightarrow$ 2B =34°

From step 5: $\Rightarrow \quad \alpha = 157.29° \times \frac{\pi}{180°}$

$$= 2.745 \text{ rad.}$$

Tension ratio $\Rightarrow \quad \frac{T_1 - mv^2}{T_2 - mv^2} = e^{\mu\alpha \operatorname{cosec}\beta}$

$$\frac{T_1 - 0.189(7.854)^2}{T_2 - 0.189(7.854)^2} = e^{0.3\times2.745\times \operatorname{cosec}17°}$$

$$T_1 - 16.72T_2 = -184.3 \quad \text{-------2}$$

Solving equation 1 and 2

T_2 = 26.9 N , T_1 = 265.64 N

Step 10: Calculation of Stress induced.

$$\text{Stress induced} = \frac{\text{Maximum tension}}{\text{Cross sectional area}}$$

From PSGDB 7.58 Area of B section = 140 mm²

$$\therefore \text{Stress induced} = \frac{265.64}{140}$$

$$= 1.897 \text{ N / mm}^2$$

18. A temporary elevator is assembled at the construction site to raise building materials, such a cement, to a height of 20m. It is estimated that the

maximum weight of the material to be raised is 5 kN. It is observed that the acceleration in such applications is 1 m/s², 10mm diameter, 6x19 construction wire ropes with fibre core are used for this application. The tensile designation of the wire is 1570 and the factor of safety should be 10 for the preliminary calculations. Determine the number of wire ropes required for this application. Neglect bending stress. (April/May 2018)

Given data:

$$h- = 20\text{m}$$
$$W = 1 \text{ tonne} = 1000\text{Kg} = 9810\text{N}$$
$$n = 2$$
$$\text{Wire rope size} = 6\times 19$$
$$d = 12\text{mm}$$
$$\text{Breaking load } W_{break} = 78\text{KN}$$
$$D = 56d$$
$$t = 1\,\text{sec}$$
$$v = 1.2\,\text{m/s} = 72\,\text{m/min}$$

******similar to this problem, Change Given data as per the given question and need to find the number of wire ropes***

Step 1: Selection of suitable Wire rope:

Given: 6 × 19 size wire rope.

Step 2: Calculation of design load:

Assuming a larger factor of safety of 15, the design load is calculated.

Design load = Load to be lifted × Assumed FOS

= 9810 × 15

= 147150 N

= 147.15 KN

Step 3: Selection of Wire rope diameter (d):

From PSGDB 9.5. For the breaking strength (W_{break}) 78 KN (7.8 tonnes). take the diameter of the rope is 12mm.

$$d = 12\text{mm}$$
$$\sigma_u = 1600 \text{ to } 1750\,\text{N/mm}^2$$

Step 4: Calculation of sheave diameter (D):

Given:

Sheave diameter $D = 56\ d$

$= 56 \times 12$

$= 672\ mm$

Step 5: Selection of the area of useful cross section of the rope (A):

From PSGDB 9.1

$$A = 0.4 \times \pi/4 \times d^2$$

$$= 0.4 \times \pi/4 \times 12^2$$

$$A = 45.24 mm^2$$

Step 6: Calculation of Wire diameter (d_w):

$$d_w = \frac{d}{1.5\sqrt{i}}$$

i = Number of strands × Number of wires in each strand

= 6 × 19

i = 114

$$d_w = \frac{12}{1.5\sqrt{114}}$$

$$\therefore\ d_w = 0.75 mm$$

Step 7: Selection of Weight of rope (W_r):

From PSGDB 9.5. Corresponding to the diameter of the rope 12mm, take

Approximate weight $= 0.54\ Kgf/m$

$= 5.3\ N/m$

$\therefore$ Weight of rope W_r = Approximate Weight × h

$= 5.3 \times 20$

$W_r = 106 N$

Step 8: Calculation of various loads:

Case (i): To find the direct load (W_d):

$$W_d = W + W_r$$

$$= 9810N + 106N$$

$$W_d = 9916N$$

Case (ii): To find the acceleration load (W_a):

$$W_a = \left(\frac{W + W_r}{g}\right) a$$

$$a = \text{acceleration of the load}$$

$$= \frac{V_2 - V_1}{t_1}$$

$$= \frac{1.2 - 0}{1}$$

$$a = 1.2\, m/s^2$$

$$\therefore\ W_a = \left(\frac{9810 + 106}{9.81}\right) 1.2$$

$$W_a = 1212.97N$$

Step 9: Calculation of effective loads on the rope:

Effective load during acceleration of the load

$$W_{ea} = W_d + W_b + W_a$$

$$= 9916 + 0 + 1212.97$$

$$\left[\begin{array}{l} \because W_b = 0, \text{From the Question} \\ \quad \text{Bending load is neglected} \end{array}\right]$$

$$= 11128.97N$$

Step 10: Calculation of working factor of safety (FS_w):

$$\left.\begin{array}{l}\text{Working factor} \\ \text{of Safety } (F_{sw})\end{array}\right\} = \frac{\text{Breaking load}}{\text{Effective load during acceleration } (W_{ea})}$$

$$= \frac{78 \times 10^3}{11128.97}$$

$$F_{sw} = 7$$

Step 11: Check for design:

From PSGDB 9.1, for hoists and class 2, the recommended factor of safety = 5.

Since the working factor of safety is greater than the recommended factor of safety. Therefore the design is safe.

19. Select a high-speed duck flat belt drive for a fan running at 360 rpm which is driven by 10kW, 1440rpm motor. The belt drive is open and space available for a center distance of 2m approximately. The diameter of the driven pulley 1000 rpm. (Nov/Dec 2018)

Given data:

$N_1 = 1440\text{rpm}$
$\phi = 2.5$
$c = 3.6\text{m}$
$\gamma = 16 \, {}^{m}\!/_{s}$
$K_s = 1.3$
Belt = 5 Ply, flat dunlop belt.
$P = 12\text{KW}$
Load rating at $5 \, {}^{m}\!/_{s} = 0.0118$ KW/Ply/mm

*****Similar to this problem changes in speeds and power***

Step 1: **Calculation of Pulley diameters:**

Assume the driven pulley diameter D = 1000 mm.

W.K.T $\phi = \frac{D}{d} = \frac{N_1}{N_2}$

Case (i): To find the driven pulley speed. (N_2).

Case (ii): To find the driver pulley diameter (d):

$$N_2 = \frac{N_1}{\phi}$$

$$= \frac{1440}{2.5}$$

$$N_2 = 576\text{rpm}$$

Case (iii): To find the driver pulley diameter (d):

$$d = \frac{D}{\phi}$$

$$= \frac{1000}{2.5}$$

$$d = 400\text{mm}$$

From PSGDB 7.54, from recommended series of pulley diameters and tolerances.

$$\left.\begin{array}{l}\text{The standard diameter for}\\ \text{the driver pulley}\end{array}\right\} d = 400\text{mm}$$

Step 2: Calculation of design power in KW.

$$\text{Design power} = \frac{\text{Rated power}(K_w) \times \text{Load correction factor}(K_s)}{\text{Arc of contact factor}(K_\alpha) \times \text{Small pulley factor}(K_d)}$$

Case (i): To find the arc of contact factor ($K\alpha$)

From PSGDB 7.54

$$\text{Arc of contact} = 180° - \left(\frac{D - d}{c}\right) \times 60°$$

$$= 180° - \left(\frac{1000 - 400}{3600}\right) \times 60°$$

$$= 170°$$

From PSGDB 7.54, take the value of $K_\alpha = 1.04$. Corresponding to the arc of contact 170°

$$K_\alpha = 1.04$$

Case (ii): To find the small pulley factor (K_d)

Table: Small pulley factor 'K_d'

Small Pulley diameter	K_d

Upto 100mm	0.5
100 – 200mm	0.6
200 – 300mm	0.7
300 – 400mm	0.8
400 – 750mm	0.9
Over 750mm	1.0

From the above table. We take the K_d value 0.8

$$\therefore K_d = 0.8$$

Case (iii):

To find the design power, KW:

W. K. T. Design power $= \frac{P \times K_s}{K_a \times K_d}$

$$= \frac{12 \times 1.3}{1.04 \times 0.8}$$

$$= 18.75KW$$

Step 3: Selection of belt:

Given: 5 Ply, flat Dunlop belt. Its capacity is given by 0.0118 KW/ply/mm.

Step 4: Load rating correction:

From PSGDB 7.54.

$$\text{Load rating at 'V' m/s} = \text{Load rating at 10 m/s} \times \frac{v}{10}$$

$$\text{Load rating at 16 m/s} = (0.0118 \times 2) \times \frac{16}{10}$$

$$= 0.03776KW/Ply/mm$$

Step 5: Determination of belt width:

$$\text{Width of the belt} = \frac{\text{Design Power}}{\text{Load rating} \times \text{No. of plies.}}$$

$$= \frac{18.75}{0.03776 \times 5}$$

$= 99.31\text{mm}$

From PSGDB 7.52 Specification of transmission belting standard widths.

The standard belt width for 5 Ply belt = 100mm.

Step 6: Determination of Pulley width:

From PSGDB 7.54, Pulley width is given by

Pulley width = Belt width + 18 mm

= 100 + 13 mm

= 113 mm

From PSGDB 7.54, recommended series of width of flat pulleys, mm.

The standard pulley width = 125 mm.

Step 7: Calculation of length of the belt (L):

From PSGDB 7.61,

$$L = 2C + \frac{\pi}{2}(D + d) + \frac{(D - d)^2}{4C}$$

$$= 2 \times 3600 + \pi/2(1000 + 400) + \frac{(1000 - 400)^2}{4 \times 3600}$$

$$= 7200 + 2199.11 + 25$$

$$L = 9424.11\text{mm}$$

20. **A centrifugal pump running at 340rpm is to be driven by a 100 kW motor running at 1440 rpm. The light duty drive is to work for atleast 20 hours every day. The center distance between the motor shaft and the pump shaft is 1200mm. suggest a suitable multiple v belt drive for the application. (Nov/Dec 2018)**

Given data:

C=1m=1000mm
P=100kW
d=300mm
N_1=1000rpm
N_2=375rpm
2ß=40°
A=400 mm^2
$\rho = 1100$ kg/m3

σ =2.1MPa

μ=0.28.

******Similar to this problem change the center distance as C=1200 mm and slight changes in speeds***

Step 1: To find the velocity of the belt 'v':

$$V = \frac{\pi d N1}{60}$$

$$= \frac{\pi x 0.3 x 1000}{60} = 15.71 m/s$$

Step 2: To find the larger pulley diameter 'D'

$$\frac{N_2}{N_1} = \frac{d}{D}$$

$$\frac{375}{1000} = \frac{0.3}{D}$$

D=0.8m

Step 3: To find the number of belts required

For an open belt drive

Case i: To find α:

$$\sin\alpha = \frac{D-d}{2C}$$

$$\sin\alpha = \frac{0.8-0.3}{2x1}$$

α=14.48°

Case ii: To find θ:

$$\theta = (180 - 2\alpha)x\frac{\pi}{180}$$

$$= (180 - 2x14.48)x\frac{\pi}{180}$$

$$\theta = 2.636\ rad$$

Case iii: To find T_1& T_2:

$$\frac{T_1}{T_2} = e^{\mu\,\theta Cosec\,\beta}$$

$$\frac{T_1}{T_2} = e^{0.28x\ 2.636xCosec\ 20°}$$

$$\frac{T_1}{T_2} = 2.158----1$$

Mass of the belt per meter length

m=density x area x Length

=1100x400x10^{-6}x1

=0.44kg/m

Centrifugal tension

$$T_c = mv^2$$

$$= 0.44x15.71^2$$

$$T_c = 108.59N$$

Maximum tension in the belt

$$T = \sigma xa$$

$$= 2.1x10^6x400x10^{-6}$$

T=840N

We know that the tension in the tight side of the belt

$$T = T_1 + T_c$$

$$840 = T_1 + 108.59$$

$$T_1 = 731.49N$$

From equation 1

$$\frac{731.49}{T_2} = 2.158$$

$$T_2 = 338.93N$$

Case iv: To find the power transmitted

$$P = (T_1 - T_2)xV$$

$$= (731.41 - 338.93)x15.71$$

$$P = 6165.86\ W$$

Case v: To find the number of belts

$$Number\ of\ belts = \frac{Total\ power\ transmitted}{Power\ transmitted\ per\ belt}$$

$$= \frac{100x10^3}{6165.86}$$

= 16.22

=17 belts

Step 4: To find the length of the each belt:

$$L = 2C + \left(\pi/2\right)(D+d) + \frac{(D-d)^2}{4C}$$

= 3.852m

21. **The transporter of a heat treatment furnace is driven by a 4.5kW, 1440rpm induction motor through a chain drive with a speed reduction ratio 2.4. The transmission is horizontal with bath type of lubrication. Rating is continuous with 3 shifts per day. Design the complete chain drive. Assume center distance as 500mm and service factor as 1.5. (Nov/Dec 2018)**

Given data:

N = P = 7.5 KW

N_1 = 1400 rpm

i = 10

a_0=600 mm

**** similar to this problem with different data.*

Step 1: Selection of transmission ratio. (i)

$$i = \frac{N_1}{N_2} = 10 \quad \text{given.}$$

Then,

$$\frac{N_1}{10} = N_2$$

$$N_2 = \frac{1400}{10}$$

$$N_2 = 140\text{rpm}$$

Step 2: Selection of no. of teeth on the driver sprocket (z_1).

From PSGDB 7.74

$$Z_1 = 7$$

Step 3: Calculation of no. of teeth on the driven sprocket (Z_2).

From PSGDB 7.74

$$Z_2 = i \times Z_1$$

$$= 10 \times 7$$

$$Z_2 = 70$$

$$Z_{2max} = 100 \text{ to } 120$$

Recommended value of Z_2 should be less than the above value or else the chain may run off the sprocket for a small pull.

$Z_2 = 70$ is satisfactory.

Step 4: Selection of standard pitch (P).

From PSGDB 7.74

$$\text{Centre distance } a = (30 \text{ to } 50)\ P$$

$$\text{Maximum Pitch, } P_{max} = \frac{a}{30} = \frac{600}{30} = 20 \text{ mm}$$

$$\text{Minimum Pitch, } P_{min} = \frac{a}{50} = \frac{600}{50} = 12 \text{ mm}$$

Any standard pitch between 12 mm and 20 mm can be chosen. But to get a quicker solution, it is always preferred to take the standard pitch closer to P_{max}.

From PSGDB 7.72, Standard Pitch P = 15.875 mm.

Step 5: Selection of the chain:

From PSGDB 7.72, assume the chain to be duplex.

$\therefore$ 10 A - 2 / DR50 Chain number is selected.

Step 6: Calculation of total load on the driving side of the chain (P_T):

From PSGDB 7.78,

$P_T = P_t + P_c + P_a$

Case 1: To find the tangential force (P_t)

From PSGDB 7.78

$$P_t = \frac{1020N}{V}$$

Where, v = chain velocity $= \frac{Z_1 \times P \times N_1}{60 \times 1000}$

$$= \frac{7 \times 15.875 \times 1400}{60 \times 1000}$$

$$= 2.59 \text{ m/s}$$

$$\therefore \quad P_t = \frac{1020 \times 7.5}{2.59}$$

$$P_t = 2950.35 \text{ N}$$

Case 2: To find the centrifugal tension (P_c).

From PSGDB 7.78. $P_c = \frac{Wv^2}{g} = mv^2$

Where, m= mass of the chain

From PSGDB 7.72, For the selected chain,

m = 1.78 Kg/m [1Kg m/s²=1N]

$$\therefore \quad P_c = 1.78\ (2.59)^2$$

$$P_c = 11.94 \text{ N}$$

Case 3: To find the tension due to sagging (P_s).

From PSGDB 7.78,

P_s = K. W. a

Where, K = 6 (for horizontal) From PSGDB 7.78

W = m x g = 1.78 x 9.81 = 17.46 N

A = 600 mm = 0.6 m.

$\therefore$ P_s = 6 x 17.46 x 0.6

= 62.82 N

$\therefore \quad P_T = 2950.35 + 11.94 + 6282$

$P_T = 3025.11$ N

Step 7: Calculation of Service factor (K_s).

From PSGDB 7.76

$$Ks = K_1 \cdot K_2 \cdot K_3 \cdot K_4 \cdot K_5 \cdot K_6$$

From PSGDB 7.76 and 7.77.

- $K_1 = 1.25$ for load with mild shocks
- $K_2 = 1$ for adjustable supports.
- $K_3 = 1$ $\because$ we have used $a_p = (30 \text{ to } 50)P$
- $K_4 = 1$ for horizontal drive.
- $K_5 = 1$ for drop lubrication
- $K_6 = 1.25$ for 16 hrs/day running

$\therefore K_s = 1.25 \times 1 \times 1 \times 1 \times 1 \times 1.25$
$= 1.5625$

Step 8: Calculation of design load.

Design load $= P_T \times K_s$

$= 3025.11 \times 1.5625$

$= 4726.73$ N

Step 9: Calculation of working factor of safety (FS_w)

$$FS_w = \frac{Q}{\text{Design load}}$$

Where, Q = Breaking load = 44400 N. From PGSDB 7.72 for the selected chain

$$\therefore \quad FS_W = \frac{44400}{4726.73}$$

$$FS_W = 9.4$$

Step 10: Check for factor of safety.

From PSGDB 7.77, Recommended factor of safety = 12.45

We find $FS_W < 12.45$, the design is not safe.

In order to overcome this issue we have to increase the pitch = 19.05 mm.

$\therefore$ The chain number 12 A -2 / DR 60 is selected.
For this chain, M= 2.90 Kg/m, Q = 63600 N

By the recalculation of step 6 and step 8, step 9.

P_T = 2590.28 N.

Design load = 4047.31 N

FS_w = 15.71

We find FS_w > 12.45 , the design is safe.

Step 11: Check for the bearing stress in the roller.

$$\sigma_{roller} = \frac{P_t \times K_s}{A}$$

Where, $A = 210\ mm^2$ From PSGDB 7.72 for selected chain∴ $\sigma_{roller} = \frac{2459.81*1.5625}{210}$

$= 18.30\ N/mm^2$

From PSGDB 7.77, the allowable bearing stress for the given speed 1400rpm, is 19.75 N/mm².

Induced stress is less than the allowable stress i.e 18.30 < 19.75 N/mm².

∴ The design is safe.

Step 12: Calculation of length of chain (L).

From PSGDB 7.75

$$L = l_p \times P$$

Where no.of links $l_p = 2a_p + \left(\frac{Z_1 + Z_2}{2}\right) + \frac{[(Z_2 - Z_1)/2\pi]^2}{a_p}$

Approximate center distance in multiples of pitches $a_p = \frac{a_0}{P} = \frac{600}{19.05} = 31.50$

$$\therefore \quad l_p = 2 \times 31.50 + \left(\frac{7+70}{2}\right) + \frac{[(70-7)/2\pi]^2}{31.50}$$

$$= 63 + 38.5 + 3.19$$

$$l_p = 104.69$$

$$l_p = 106 \text{ links}$$

$$\therefore \left.\begin{matrix}\text{Actual length}\\ \text{of chain}\end{matrix}\right\} L = 106 \times 19.05$$

$$L = 2019.3\ mm$$

Step 13: Calculation of exact centre distance (a):

From PSGDB 7.75.

$m = 100.54$ $\quad a = \frac{e + \sqrt{e^2 - 8m}}{4} \times P$

Case 1: To find e:

* $e = l_p - \left(\frac{Z_1 + Z_2}{2}\right)$

$= 106 - \left(\frac{7 + 70}{2}\right)$

$e = 67.5$

Case 2: To find m:

* $m = \left(\frac{Z_2 - Z_1}{2\pi}\right)^2$

$= \left(\frac{70 - 7}{2\pi}\right)^2$

$m = 100.54$

$\therefore \quad a = \frac{67.5 + \sqrt{67.5^2 - 8 \times 100.54}}{4} \times 19.05$

$a = 613.18$ mm

From PSGDB 7.75, Decrement in centre distance for an initial sag $= 0.01a$

$= 6.132$ mm

$\therefore$ Exact centre distance $= 613.18 - 6.132$

$= 607.05$ mm.

Step 14: Calculation of sprocket diameters.

Case 1: Smaller sprocket.

PCD of smaller sprocket $d_1 = \frac{P}{\sin\left(\frac{180}{Z_1}\right)}$ From PSGDB 7.78

$= \frac{19.05}{\sin\left(\frac{180}{7}\right)}$

$d_1 = 43.91$ mm.

Sprocket outside diameter $d_{01} = d_1 + 0.8d_r$

$d_r =$ diameter of roller $= 11.90$ mm. From PSGDB 7.72 for selected chain.

$\therefore \quad d_{01} = 43.91 + 0.8 \times 11.90$

$$d_{01} = 53.43 \text{ mm}$$

Case 2: Larger sprocket:

$$d_2 = \frac{P}{\sin\left(\frac{180}{Z_2}\right)} \quad \text{From PSGDB 7.78}$$

$$= \frac{19.05}{\sin\left(\frac{180}{70}\right)}$$

$$d_2 = 424.61 \text{ mm}$$

$$\left.\begin{matrix}\text{Sprocket outside} \\ \text{diameter}\end{matrix}\right\} d_{02} = d_2 + 0.8 d_r$$

$$= 424.61 + 0.8 + 11.90$$

$$d_{02} = 434.13 \text{ mm}$$

22. A workshop crane is lifting a load of 25 kN through a wire rope and hook. The weight of the hook etc., is 15 kN. The rope drum diameter may be taken as 30 times the diameter of the rope. The load is to be lifted with an acceleration of 1m/s². Calculate the diameter of the wire rope. Take a factor of safety of 6 and E for the wire is 80kN/mm². The ultimate stress may be taken as 1800 MPa. The cross-sectional area of the wire rope may be taken 0.38 times the square of the wire rope diameter. (Nov/Dec 2018)

Given data:

Weight to be lifted $= 20 \text{ KN}$

Depth $= 60\text{m}$

$v_2 = v = 4 \text{ m/sec} = 240 \text{ m/min}$

$t = 10 \text{ sec}$

****Similar to this problem*

Step 1: Selection of suitable wire rope.

For hoisting purpose, 6×19 rope is selected. From PSGDB 9.1

Step 2: Calculation of Design load.

Assuming the factor of safety of 15, the design load is calculated.

Design load $= 20 \times 15$

$= 300KN$

Step 3: To find wire rope diameter (d).

From PSGDB 9.5 For design load 300KN, The next standard value.

$d = 25mm$
$m = 2.41\, Kg/m$
$\sigma_u = 1600 \text{ to } 1750\, N/mm^2$

Breaking strength $= 340KN$

Step 4: Sheave diameter (D)

From PSGDB 9.1. We find $D_{min}/d = 27$ for class 4, for velocity upto 50m/min . But the actual speed is 240m/min $\left(\text{i.e} \frac{240}{50} \approx 5 \text{ times } 50\, m/min \right)$. Therefore $\frac{D_{min}}{d}$ has to be modified.

$$\frac{D_{min}}{d} = 27 \times (1.08)^{5-1} = 36.73 \approx 37mm.$$

Sheave diameter $D = 37 \times d$

$= 37 \times 25$

$D = 925\, mm$

Step 5:Calculation of Area of cross section of the rope (A).

From PSGDB 9.1

$A = 0.4 \times \pi/4 \times d^2$

$= 0.4 \times \pi/4 \times 25^2$

$A = 196.35mm^2$

Step 6: To find Wire diameter. (d_w).

$$d_w = \frac{d}{1.5\sqrt{i}}$$

$$= \frac{25}{1.5\sqrt{6 \times 19}}$$

$$= 1.56$$

$$d_w = 2mm$$

Step 7: Weight of the rope. (W_r).

$$W_r \text{ per meter} = 2.41 \times 9.81 = 23.64 \, N/m.$$

$$W_r = 23.64 \times 60 = 1418.53N$$

$$= 1418.53N$$

Step 8: Load calculations

Case 1: Direct load (W_d)

$$W_d = W + W_r = 20 + 1418.53 \times 10^{-3} = 21.42KN$$

Case 2: Bending load (W_b)

$$W_b = \sigma_b \times A = \frac{E_r \times d_w}{D} \times A$$

$$= \frac{0.84 \times 10^5 \times 2}{925} \times 196.35 \qquad \left[\because E_r = 0.84 \times 10^5 \, N/mm^2\right]$$

$$= 35661.41N$$

$$= 35.66KN.$$

Case 3: Acceleration load (W_a)

$$W_a = \left(\frac{W + W_r}{g}\right) a \qquad a = \frac{v_2 - v_1}{t}$$

$$= \left(\frac{20 + 1418.53 \times 10^{-3}}{9.81}\right) \times 0.4 \qquad = \frac{4 - 0}{10}$$

$$= 0.87 \text{ KN} \qquad = 0.4 \, m/s^2$$

$$\therefore \left.\begin{matrix}\text{Effective load on the rope} \\ \text{during acceleration}\end{matrix}\right\} W_{ea} = W_d + W_b + W_a$$

$$= 21.42 + 35.66 + 0.87$$

$= 57.95$

$W_{ea} = 58KN$

Step 9:Working factor of Safety (FS_w).

$$FS_w = \frac{\text{Breaking load}}{W_{ea}}$$

$$= \frac{340}{58}$$

$$FS_w = 5.86$$

Step 10: Check for Safe design

* We find $F_{sw} < n'(6)$. $\therefore$ The design is not safe.
* The safe design can be achieved either by selecting the rope with greater breaking strength.

From PSGDB 9.5, for d=25, take breaking strength = 376 KN and $\sigma_u = 1750 \text{ to } 1900 N/mm^2$

$$\therefore F_{sw} = \frac{376}{58}$$

$$= 6.48$$

Now we find $F_{sw} > n'(6)$. $\therefore$ The design is safe.

23. A motor driven blower is to run at 650rpm driven by an electric motor of 7.5 kW at 1800rpm. Design a suitable V belt drive. (April/May 2019)

Given data:

P = 7.5 KW

N_1 = 1000 rpm

N_2 = 300 rpm

******Similar to this problem assume the missing data's and slight changes in speeds***

Step 1: Selection of belt

From PSGDB 7.58,

For 7.5 KW, B section is selected

Step 2: Selection of pulley diameters. d & D:

d = 150 mm, D = 500 mm given.

Step 3: Selection of centre distance (c) :

C= 925 mm given.

Step 4: Calculation of nominal pitch length (L).

From PSGDB 7.61,

$$L = 2C + \frac{\pi}{2}(D+d) + \frac{(D-d)^2}{4C}$$

$$= 2\times 925 + \frac{\pi}{2}(500+150) + \frac{(500-150)^2}{4\times 925}$$

$$= 2904.12mm.$$

From PSGDB 7.60, For B section.

The next standard length L = 3091 mm.

Step 5: Selection of various modification factors.

Case 1: Length correction factor (F_c)

From PSGDB 7.60 for B section corresponding to 'L'

$$F_c = 1.07$$

Case 2: Correction factor for arc of contact (F_α)

From PSGDB 7.68

Arc of contact angle $= 180^\circ - (\frac{D-d}{C}) \times 60^\circ$

$$= 180^\circ - (\frac{500-150}{925}) \times 60^\circ$$

$$= 157.29^\circ$$

Corresponding to the angle $1579.29^\circ \approx 160^\circ$

F_d = 0.95.

Case 3: Service factor (F_a).

From PSGDB 7.69 F_a = 1.3

Step 6: Calculation of Maximum power capacity (KW).

From PSGDB 7.62, For B section.

$$KW = (0.79S^{-0.09} - \frac{50.8}{d_e} - 1.32\times10^{-4}S^2)S$$

Where, S = Belt speed $= \frac{\pi d N_1}{60}$

$$= \frac{\pi\times0.150\times1000}{60}$$

$$= 7.854 m/s$$

d $_e$ = equivalent pitch diameter; From PSGDB 7.62 $\frac{D}{d} = \frac{500}{150} = 3.33$ Take F_b=1.14

$$= d_p \times F_b$$

$$= 150\times1.14$$

$$= 171 \text{ mm.}$$

$$\therefore \quad KW = (0.79\times7.854^{-0.09} - \frac{50.8}{171} - 1.32\times10^{-4}\times7.84^2)7.84$$

$$= 2.757 KW$$

Step 7: Calculation of number of belts (n_b)

From PSGDB 7.70

$$n_b = \frac{P\times F_a}{K_w\times F_c\times F_d}$$

$$= \frac{7.5\times1.3}{2.757\times1.07\times0.95}$$

$$= 3.48$$

$$n_b = 4 \text{belts.}$$

Step 8: Calculation of actual centre distance. (C_{actual}).

From PSGDB 7.61

$$C_{actual} = A + \sqrt{A^2 - B}$$

$$A = \frac{L}{4} - \pi\left[\frac{D+d}{8}\right]$$

$$= \frac{3091}{4} - \pi\left[\frac{500+150}{8}\right]$$

$$A = 517.5 mm$$

$$B = \frac{(D-d)^2}{8} = \frac{(500-150)^2}{8}$$

$$= 15312.5\ mm^2$$

$$\therefore \quad C_{actual} = 517.5 + \sqrt{517.5^2 - 15312.5}$$

$$= 1020\ mm.$$

Step 9: Calculation of belt tensions (T_1 and T_2).

Power transmitted per belt $= (T_1 - T_2)\,v$

$$\frac{7.5\times10^3}{4} = (T_1 - T_2)7.854$$

$$T_1 - T_2 = 238.73 \quad \text{------}1$$

From PSGDB 7.58 $\Rightarrow$ m = 0.189 Kg/m.

7.70 $\Rightarrow$ 2B = 34°

From step 5: $\Rightarrow$ $\alpha = 157.29° \times \frac{\pi}{180°}$

$$= 2.745\ rad.$$

Tension ratio $\Rightarrow$ $\frac{T_1 - mv^2}{T_2 - mv^2} = e^{\mu\alpha\,cosec\beta}$

$$\frac{T_1 - 0.189(7.854)^2}{T_2 - 0.189(7.854)^2} = e^{0.3\times2.745\times cosec17°}$$

$$T_1 - 16.72T_2 = -184.3 \quad \text{------}2$$

Solving equation 1 and 2

T_2 = 26.9 N , T_1 = 265.64 N

Step 10: Calculation of Stress induced.

$$\text{Stress induced} = \frac{\text{Maximum tension}}{\text{Cross sectional area}}$$

From PSGDB 7.58 Area of B section = 140 mm² $\therefore$ Stress induced = $\frac{265.64}{140}$

$$= 1.897 \text{ N/mm}^2$$

24. Design a chain drive to accurate a compressor from a 10kW electric motor at 960rpm. The compressor speed is to be 350rpm. Minimum center distance should be 0.5m. motor is mounted on an auxiliary bed. Compressor is to work for 8 hours/ day. (April/May 2019)

Given data:

N = P = 7.5 KW

N_1 = 1400 rpm

i = 10

a_0=600 mm

****similar to this problem, change the centre distance and power*

Step 1: Selection of transmission ratio. (i)

$$i = \frac{N_1}{N_2} = 10 \quad \text{given.}$$

Then,

$$\frac{N_1}{10} = N_2$$

$$N_2 = \frac{1400}{10}$$

$$N_2 = 140\text{rpm}$$

Step 2: Selection of no. of teeth on the driver sprocket (z_1).

From PSGDB 7.74

Z_1 = 7

Step 3: Calculation of no. of teeth on the driven sprocket (Z_2).

From PSGDB 7.74

$$Z_2 = i \times Z_1$$

$$= 10 \times 7$$

$$Z_2 = 70$$

$$Z_{2max} = 100 \text{ to } 120$$

Recommended value of Z_2 should be less than the above value or else the chain may run off the sprocket for a small pull.

$Z_2 = 70$ is satisfactory.

Step 4: Selection of standard pitch (P).

From PSGDB 7.74

$$\text{Centre distance } a = (30 \text{ to } 50)\ P$$

$$\text{Maximum Pitch, } P_{max} = \frac{a}{30} = \frac{600}{30} = 20 \text{ mm}$$

$$\text{Minimum Pitch, } P_{min} = \frac{a}{50} = \frac{600}{50} = 12 \text{ mm}$$

Any standard pitch between 12 mm and 20 mm can be chosen. But to get a quicker solution, it is always preferred to take the standard pitch closer to P_{max}.

From PSGDB 7.72, Standard Pitch P = 15.875 mm.

Step 5: Selection of the chain:

From PSGDB 7.72, Assume the chain to be duplex.

$\therefore$ 10 A - 2 / DR50 chain number is selected.

Step 6: Calculation of total load on the driving side of the chain (P_T):

From PSGDB 7.78,

$P_T = P_t + P_c + P_a$

Case 1: To find the tangential force (P_t)

From PSGDB 7.78

$$P_t = \frac{1020N}{V}$$

$$\text{Where, v = chain velocity } = \frac{Z_1 \times P \times N_1}{60 \times 1000}$$

$$= \frac{7 \times 15.875 \times 1400}{60 \times 1000}$$

$$= 2.59 \text{ m/s}$$

$$\therefore \quad P_t = \frac{1020 \times 7.5}{2.59}$$

$$P_t = 2950.35 \text{ N}$$

Case 2: To find the centrifugal tension (P_c).

From PSGDB 7.78. $P_c = \frac{Wv^2}{g} = mv^2$

Where, m= mass of the chain

From PSGDB 7.72, For the selected chain,

m = 1.78 Kg/m [1Kg m/s^2=1N]

$$\therefore \quad P_c = 1.78\ (2.59)^2$$

$$P_c = 11.94 \text{ N}$$

Case 3: To find the tension due to sagging (P_s).

From PSGDB 7.78,

P_s = K. W. a

Where, K = 6 (for horizontal) From PSGDB 7.78

W = m x g = 1.78 x 9.81 = 17.46 N

A = 600 mm = 0.6 m.

∴ P_s = 6 x 17.46 x 0.6

= 62.82 N

∴ P_T = 2950.35 + 11.94 + 6282

P_T = 3025.11 N

Step 7: Calculation of Service factor (K_s).

From PSGDB 7.76

Ks = $K_1 \cdot K_2 \cdot K_3 \cdot K_4 \cdot K_5 \cdot K_6$

From PSGDB 7.76 and 7.77.

- $K_1 = 1.25$ for load with mild shocks
- $K_2 = 1$ for adjustable supports.
- $K_3 = 1$ $\because$ we have used $a_p = (30 \text{ to } 50)P$
- $K_4 = 1$ for horizontal drive.
- $K_5 = 1$ for drop lubrication
- $K_6 = 1.25$ for 16 hrs/day running

$\therefore$ $K_s = 1.25 \times 1 \times 1 \times 1 \times 1 \times 1.25$
$= 1.5625$

Step 8: Calculation of design load.

Design load $= P_T \times K_s$

$= 3025.11 \times 1.5625$

$= 4726.73$ N

Step 9: Calculation of working factor of safety (FS_w)

$$FS_w = \frac{Q}{\text{Design load}}$$

Where, Q = Breaking load = 44400 N. From PGSDB 7.72 for the selected chain

$$\therefore \quad FS_W = \frac{44400}{4726.73}$$

$$FS_W = 9.4$$

Step 10: Check for factor of safety.

From PSGDB 7.77, Recommended factor of safety = 12.45

We find $FS_W < 12.45$, the design is not safe.

In order to overcome this issue we have to increase the pitch = 19.05 mm.

$\therefore$ The chain number 12 A -2 / DR 60 is selected.
For this chain, M= 2.90 Kg/m, Q = 63600 N

By the recalculation of step 6 and step 8, step 9.

$P_T = 2590.28$ N.

Design load = 4047.31 N

$FS_w = 15.71$

We find $FS_w > 12.45$, the design is safe.

Step 11: Check for the bearing stress in the roller.

$$\sigma_{roller} = \frac{P_t \times K_s}{A}$$

Where, $A = 210\ mm^2$ From PSGDB 7.72 for selected chain

$$\therefore \quad \sigma_{roller} = \frac{2459.81*1.5625}{210}$$

= 18.30 N/mm^2

From PSGDB 7.77, the allowable bearing stress for the given speed 1400rpm , is 19.75 N/mm^2.

Induced stress is less than the allowable stress i.e 18.30 < 19.75 N/mm^2. ∴ the design is safe.

Step 12: Calculation of length of chain (L).

From PSGDB 7.75

$$L = l_p \times P$$

$$l_p = 2a_p + \left(\frac{Z_1 + Z_2}{2}\right) + \frac{[(Z_2 - Z_1)/2\pi]^2}{a_p}$$

$$a_p = \frac{a_0}{P} = \frac{600}{19.05} = 31.50$$

$$\therefore \quad l_p = 2 \times 31.50 + \left(\frac{7+70}{2}\right) + \frac{[(70-7)/2\pi]^2}{31.50}$$

$$= 63 + 38.5 + 3.19$$

$$l_p = 104.69$$

$$l_p \approx 106 \text{ links}$$

$$\therefore \left.\begin{array}{l}\text{Actual length}\\ \text{of chain}\end{array}\right\} L = 106 \times 19.05$$

$$L = 2019.3\ mm$$

Step 13:

Calculation of exact centre distance (a): From PSGDB 7.75.

$$a = \frac{e + \sqrt{e^2 - 8m}}{4} \times P$$

$$* \quad e = l_p - \left(\frac{Z_1 + Z_2}{2}\right)$$

$$= 106 - \left(\frac{7 + 70}{2}\right)$$

$$e = 67.5$$

$$* \quad m = \left(\frac{Z_2 - Z_1}{2\pi}\right)^2$$

$$= \left(\frac{70 - 7}{2\pi}\right)^2$$

$$m = 100.54$$

$$\therefore \quad a = \frac{67.5 + \sqrt{67.5^2 - 8 \times 100.54}}{4} \times 19.05$$

$$a = 613.18 \text{ mm}$$

Decrement in centre distance for an initial sag $= 0.01a$

$$= 6.132 \text{ mm}$$

$\therefore$ Exact centre distance $= 613.18 - 6.132$

$$= 607.05 \text{ mm.}$$

Step 14: Calculation of sprocket diameters.

Case 1: Smaller sprocket.

PCD of smaller sprocket $d_1 = \dfrac{P}{\sin\left(\dfrac{180}{Z_1}\right)}$ From PSGDB 7.78

$$= \frac{19.05}{\sin\left(\frac{180}{7}\right)}$$

$$d_1 = 43.91 \text{ mm.}$$

Sprocket outside diameter $d_{01} = d_1 + 0.8d_r$

$d_r =$ diameter of roller $= 11.90$ mm. From PSGDB 7.72 for selected chain.

$$\therefore \quad d_{01} = 43.91 + 0.8 \times 11.90$$

$$d_{01} = 53.43 \text{ mm}$$

Case 2:

Larger sprocket:

$$d_2 = \frac{P}{\sin\left(\frac{180}{Z_2}\right)}$$ From PSGDB 7.78

$$= \frac{19.05}{\sin\left(\frac{180}{70}\right)}$$

$$d_2 = 424.61 \text{ mm}$$

$$\left.\begin{matrix}\text{Sprocket outside}\\ \text{diameter}\end{matrix}\right\} d_{02} = d_2 + 0.8 d_r$$

$$= 424.61 + 0.8 + 11.90$$

$$d_{02} = 434.13 \text{ mm}$$

25. Select a suitable wire rope for a mini hoist carrying a load of 2 tonnes to be lifted from a depth of 100m. A rope speed of 10m/s must be attained in 10 seconds. Assume minimum factor of safety as 10. (April/ May 2019)

Given data:

Weight to be lifted $= 20 \text{ KN}$

Depth $= 60\text{m}$

$v_2 = v = 4 \text{ m/sec} = 240 \text{ m/min}$

$t = 10 \text{ sec}$

**** Refer this Problem*

Step 1: Selection of suitable wire rope.

For hoisting purpose, 6×19 rope is selected. From PSGDB 9.1

Step 2: Calculation of Design load.

Assuming the factor of safety of 15, the design load is calculated.

Design load $= 20 \times 15$

$= 300\text{KN}$

Step 3: To find wire rope diameter (d).

From PSGDB 9.5 For design load 300KN, The next standard value.

$d = 25mm$
$m = 2.41\,Kg/m$
$\sigma_u = 1600 \text{ to } 1750\,N/mm^2$

Breaking strength $= 340KN$

Step 4: Sheave diameter (D)

From PSGDB 9.1. We find $D_{min}/d = 27$ for class 4, for velocity upto 50m/min . But the actual speed is $240m/min\left(i.e\frac{240}{50} \approx 5 \text{ times } 50\,m/min\right)$. Therefore $\frac{D_{min}}{d}$ has to be modified.

$$\frac{D_{min}}{d} = 27\times(1.08)^{5-1} = 36.73 \approx 37mm.$$

Sheave diameter $D = 37\times d$

$= 37\times 25$

$D = 925mm$

Step 5:Calculation of Area of cross section of the rope (A).

From PSGDB 9.1

$A = 0.4\times \pi/4 \times d^2$

$= 0.4\times \pi/4 \times 25^2$

$A = 196.35mm^2$

Step 6: To find Wire diameter. (d_w).

$$d_w = \frac{d}{1.5\sqrt{i}}$$

$$= \frac{25}{1.5\sqrt{6\times 19}}$$

$= 1.56$

$d_w = 2mm$

Step 7: Weight of the rope. (W_r).

$$W_r \text{ per meter} = 2.41 \times 9.81 = 23.64\,\text{N/m}.$$

$$W_r = 23.64 \times 60 = 1418.53\text{N}$$

$$= 1418.53\text{N}$$

Step 8: Load calculations

Case 1: Direct load (W_d)

$$W_d = W + W_r = 20 + 1418.53 \times 10^{-3} = 21.42\text{KN}$$

Case 2: Bending load (W_b)

$$W_b = \sigma_b \times A = \frac{E_r \times d_w}{D} \times A$$

$$= \frac{0.84 \times 10^5 \times 2}{925} \times 196.35 \qquad \left[\because E_r = 0.84 \times 10^5\,\text{N/mm}^2\right]$$

$$= 35661.41\text{N}$$

$$= 35.66\text{KN}.$$

Case 3: Acceleration load (W_a)

$$W_a = \left(\frac{W + W_r}{g}\right) a \qquad a = \frac{v_2 - v_1}{t}$$

$$= \left(\frac{20 + 1418.53 \times 10^{-3}}{9.81}\right) \times 0.4 \qquad = \frac{4 - 0}{10}$$

$$= 0.87\text{ KN} \qquad = 0.4\,\text{m/s}^2$$

$\therefore$ Effective load on the rope during acceleration $\} W_{ea} = W_d + W_b + W_a$

$$= 21.42 + 35.66 + 0.87$$

$$= 57.95$$

$$W_{ea} = 58\text{KN}$$

Step 9: Working factor of Safety (FS_w).

$$FS_w = \frac{\text{Breaking load}}{W_{ea}}$$

$$= \frac{340}{58}$$

$$FS_w = 5.86$$

Step 10: Check for Safe design

* We find $F_{sw} < n'(6)$. $\therefore$ The design is not safe.
* The safe design can be achieved either by selecting the rope with greater breaking strength.

From PSGDB 9.5, for d=25, take breaking strength = 376 KN and $\sigma_u = 1750 \text{ to } 1900\,N/mm^2$

$$\therefore F_{sw} = \frac{376}{58}$$

$$= 6.48$$

Now we find $F_{sw} > n'(6)$. $\therefore$ The design is safe.

ME 6601 - DESIGN OF TRANSMISSION SYSTEMS

QUESTION BANK
UNIT –II

SPUR GEARS AND HELICAL GEARS

PART A

1. **Specify the effects of increasing the pressure angle in gear design?**

   Increasing the pressure angle will increase the beam and surface strengths of tooth. But gear becomes noisy.

2. **Why is gear tooth subjected to dynamic load?**

   Inaccuracy of tooth spacing

   Elasticity of parts

   Deflection of teeth under load

   Dynamic unbalance of rotating masses

3. **State the law of gearing?**

   The law of gearing states that for obtaining a constant velocity ratio, at any instant of teeth the common normal at each point of contact should always pass through a pitch point (fixed point), situated on the line joining the centres of the rotation of the pair of mating gears.

4. **What is pressure angle? What is the effect of increase in pressure angle?**
   - Pressure angle in the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point
   - The increase of the pressure angle results in stronger tooth, because the tooth acting as a beam is wider at the base.

5. **Define module?**

   It is the ratio of the pitch circle diameter to the number of teeth.

6. **Differentiate the double helical and herringbone gears.**

   - When there is groove in between the gears, then the gears are specifically known as double helical gears.

   - When there is no groove in between the gears, then the gears is known as herringbone gears

7. **What are the profiles of a spur gear?**
   - Involute tooth profile
   - Cycloidal tooth profile

8. **What is herringbone gear?**

When there is no groove in between the gears, then the gears is known as herringbone gears.

(Or)

The double helical gears are connecting two parallel shafts are known as herringbone gears. It is used in heavy machinery and gear boxes.

9. Define backlash. What factors influence backlash?

Back lash is the difference between the tooth space and the tooth thickness along the pitch circle.

The factors influencing in backlash are given below,

Module
Pitch line velocity

10. A helical gear has a normal pressure angle of 20°, a helix angle of 45°, and normal module of 4mm and has 20 teeth. Find the pith diameter.

Given data:

$$\Phi = 20°$$
$$\beta = 45°$$
$$m_n = 4mm$$
$$Z = 20$$
$$d = \frac{m_n}{\cos\beta} \times Z$$
$$= \frac{4}{\cos 45} \times 20$$
$$d = 113.14mm$$

11. Why are gear drives superior to belt drives or chain drives? The advantages of gear drives?

- The gear drives possess high load carrying capacity, high compact layout.
- They can transmit power from very small values to several kilowatts.

12. Illustrate the materials for making gears'.

1. Ferrous metals such as carbon steels, alloy steels of nickel, chromium and vanadium.
2. Cast-iron of different grades.
3. Non-ferrous metals such as brass, bronze, etc.
4. Non-metals like phemolic resins nylon, bakelite etc.

Among them steel with proper heat treatment is extensively, employed in many of the engineering applications.

13. **Specify the types of gears-failures**.
    - a) Tooth breakage. b) Pitting of tooth surface. c) Abrasive-wears. d) Seizing of teeth etc.
14. **At what occasions non-metallic gears are employed**.
    - Non-metallic gears are employed 'where we require silent operation and low power transmission. For example, in instruments like pressure gauge and so on.
15. **What is meant by spur-gear?**

Spur-gear is the gear in which teeth are cut at the circumference of a slab called as gear-blank such that the teeth are parallel to gear-axis.

16. **Define the following terms. a**) Tip circle. b), Root circle. c) Pitch circle

a) **Tip circle** or addendum circle is the circle which coincides crests or tops of all teeth.

b) **Root circle** or addendum circle is the circle which coincides with. Roots or bottoms of all teeth.

c) **Pitch circle** is the imaginary circle in which the pair of gears rolls one over the other. This circle can be visible when the pair of gears fast rotating. This will lie between tip circle and root circle.

17. **How are the following terms defined?** a) Pressure angle. b) Module.

a) **Pressure angle** (a) is the angle making by the line of action common- tangent to the pitch circles of mating parts.

b) **Module m** is the ratio of pitch circle diameter to the number d of gear teeth, and is usually represented in millimetres

18. **. Write short notes on backlash of gears.**

Backlash can be defined as the play between a mating pair of gear assembled condition

19. **Define form factor?**

Form factor is a constant, employed in the design of *gear* which, design the shape and the number of teeth.

20. **Why dedendum Value is more than addendum value?**

In order to get clearance between the teeth of one gear and bottom surface of mating gear so as to avoid interference, dedendum is having more value than addendum.

21. **What are the effects of increasing and decreasing the pressure angle in gear design? (April/May 2017)**

- Increasing the pressure angle will increase the beam and surface strengths of tooth. But gear becomes noisy.
- Decreasing the pressure angle will increase the minimum number of teeth required on the pinion to avoid interference/ undercutting

22. Differentiate the double helical and herringbone gears. (April/May 2017)

- When there is groove in between the gears, then the gears are specifically known as double helical gears.

- When there is no groove in between the gears, then the gears is known as herringbone gears

23. What is meant by stub tooth in gear drives? (Nov/Dec 2017)

Teeth in which the working depth is less than 2.000 divided by the normal diametral pitch.

24. Define virtual number of teeth in helical gears. (Nov/Dec 2017)

The equivalent number of teeth (also called virtual number of teeth), Zv, is defined as the number of teeth in a gear of radius.

$$Z_v = \frac{Z}{\cos^3 \varphi}$$

25. Specify the types of gear failures. (April/May 2018)

i. Tooth breakage (due to static and dynamic loads)
ii. Tooth wear (or surface deterioration)- Abrasion, Pitting and Scoring or Seizure

26. In what ways helical gears are different from spur gears? (April/May 2018)

Helical gears produce less noise than spur gears. They have a greater load capacity than equivalent spur gears

27. State the advantages of toothed gears over the other types of transmission systems. (Nov/Dec 2018)

i. Since there is no slip, so exact velocity ratio is obtained
ii. It is capable of transmitting larger power
iii. It is more efficiency and effective means of power transmission

28. Why pinion is made harder than the gear? (Nov/Dec 2018)

Because the teeth of pinion undergo more number of cycles than those of gear and hence quicker wear.

29. State the law of gearing (April/May 2019)

The law of gearing states that for obtaining a constant velocity ratio, at any instant of teeth the common normal at each of contact should always pass through a pitch point, situated on the line joining the centres of rotation of the pair of mating gears.

30. What is meant by virtual number of teeth? (April/May 2019)

The number of teeth on virtual spur gear in the normal plane is known as virtual number of teeth (z_{eq})

$$Z_{eq} = \frac{z}{\cos^3 \beta}$$

Where, z= actual number of teeth on a helical gear

β= Helix angle

PART B

1. Design a spur gear drive for a stone crusher where the gears are made of C40 steel. The pinion is transmitting 30 KW at 1200rpm. The gear ratio is 3. Taking the working life of the gears as 7500hrs.

Given data:

Material = C40 steel

$P = 30KW$

$N_1 = 1200rpm$

$i = 3$

Gear life = 7500hrs.

Step 1: To find Gear ratio (i):

i = 3 Given.

Step 2: Selection of Material:

Pinion and gear are made of C40 steel

∗ Assume surface hardness > 350

Step 3: To find the gear life in number of cycles (N):

N = Gear life in mins × N_1

= (7500 × 60) mins × 1200

N = 54 × 10^7 cycles.

Step 4: Calculation of initial design torque [M_t]:

From PSGDB 8.15, table 13.

$$[M_t] = M_t \cdot K \cdot K_d$$

From PSGDB 8.15 , table 13 Initially assume for symmetric scheme, take $K \cdot K_d = 1.3$

$$M_t = \frac{60 \times P}{2\pi N_1}$$

$$= \frac{60 \times 30 \times 10^3}{2 \times \pi \times 1200}$$

$$238.73 N.m$$

$$\therefore [M_t] = M_t \cdot K \cdot K_d$$

$$= 238.73 \times 1.3$$

$$[M_t] = 310.34 Nm.$$

Step 5: Calculation of E_{eq}, , [σ_b] and [σ_C]:

Case (i): To find equivalent young's Modulus:

From PSGDB 8.14 , table 9 For C40 steel take

$E_{eq} = 2.15 \times 10^5$ N/mm² .

Case (ii): To find design bending stress [σ_b]:

From PSGDB 8.18 , table 18, Rotation in one direction

$$[\sigma_b] = \frac{1.4 \cdot K_{b1}}{n \cdot K_\sigma} \times \sigma_{-1}$$

* Life factor for bending K_{b1}

  K_{b1} = 0.7 for HB > 350 and N ≥ 25 × 10^7 cycles

  From PSGDB 8.20 table 22.

* Factor of safety (n):

  n = 2, for steel tempered From PSGDB 8.19 , table 20

* Stress concentration factor (K_σ):

  K_σ = 1.5 for steel From PSGDB 8.19 , table 21

* Tensile strength (σ_u):

  σ_u = 630 N/mm² for C40 steel From PSGDB 8.19

* Endurance limit stress in bending (σ_{-1}):

  $\sigma_{-1} = 0.35\ \sigma_u + 120$, for alloy steel. From PSGDB 8.19 table 19

= (0.35 × 630) + 120

σ_{-1} = 340.5 N/mm²

$$W \cdot K \cdot T \Rightarrow [\sigma_b] = \frac{1.4 \times K_{b1}}{n \times K_\sigma} \times \sigma_{-1}$$

$$= \frac{1.4 \times 0.7}{2 \times 1.5} \times 340.5$$

$$[\sigma_b] = 111.23\, N/mm^2$$

Case (iii): To find the design contact stress $[\sigma_c]$:

$[\sigma_c] = C_R \cdot HRC \cdot K_{C1}$ From PSGDB 8.16

- C_R = co efficient depending upon surface hardness.

  C_R = 265 , for C40 steel hardened and tempered From PSGDB 8.16 , table 16

- Rockwell Hardness number (HRC):

  HRC = 40 to 55 , for C40 steel From PSGDB 8.16 , table 16

- Life factor (K_u):

  $K_u = 0.585$, for $HB > 350$ and $N \geq 25 \times 10^7$ cycles. From PSGDB 8.17 , table 17

$$\therefore [\sigma_c] = 265 \times 55 \times 0.585$$

$$= 8526.375\, Kgf/cm^2$$

$$[\sigma_c] = 852.64\, N/mm^2$$

Step 6: Calculation of centre distance (a):

$$a \geq (b+1)\sqrt[3]{\left[\frac{0.74}{\sigma_c^{-2}}\right]^2 \times \frac{E_{eq}[M_t]}{i\varphi}}$$ From PSGDB 8.13 table 8 for designing.

$\varphi = b/a = 0.3$, for initial calculation From PSGDB 8.14 table 10

$$a \geq (3+1)\sqrt[3]{\left[\frac{0.74}{852.64}\right]^2 \times \frac{2.15 \times 10^5 \times 310.34 \times 10^3}{3 \times 0.3}}$$

$\geq 152.89mm.$

Take a = 155mm

Step 7: Selection of Z_1 and Z_2:

- Number of teeth on pinion Z_1 = 17 (Assume)
- Number of teeth on gear (Z_2)

$Z_2 = i \times Z_1$ From PSGDB 8.1 and table 4(8.3) $i = Z_2/Z_1 > 1$

$$= 3 \times 17$$

$$Z_2 = 51$$

Step 8: Calculationof module (m):

From PSGDB 8.22 table 26

$$m = \frac{2a}{Z_1 + Z_2}$$

$$= \frac{2 \times 155}{17 + 51}$$

$$m = 4.56mm$$

From PSGDB 8.2, table 1. Choice 1. Take the nearest higher standard module,

$$m = 5mm.$$

Step 9: Revision of centre distance:

From PSGDB 8.22 , table 26.

$$a = \frac{m(Z_1 + Z_2)}{2}$$

$$= \frac{5(17 + 51)}{2}$$

$$a = 170mm$$

Step 10: Calculation of b , d_1 , v and φ_P

Case 1: To find the face width (b):

From PSGDB 8.14 , table 10.

$$b = \varphi \times a$$

$$= 0.3 \times 170$$

$$b = 51mm$$

Case 2: To find the Pitch circle diameter (d_1)

From PSGDB 8.22 , table 26

$$d_1 = m \times Z_1$$

$$= 5 \times 17$$

$$d_1 = 85mm$$

Case 3: To find the pitch line velocity (v):

$$v = \frac{\pi d_1 N_1}{60}$$

$$= \frac{\pi \times 85 \times 10^{-3} \times 1200}{60}$$

$$= 5.34\, m/sec$$

Case 4: To find φ_P:

$$\varphi_P = \frac{b}{d_1}$$

$$\frac{51}{85}$$

$$\varphi_P = 0.6$$

Step 11: Selection of Quality of gear:

From PSGDB 8.3 , table 2

For pitch line velocity 5.34 m/sec. Is Quality 8 gears are selected.

Step 12: Revision of design torque [M_t]:

$$[M_t] = M_t \cdot K \cdot K_d$$

Revised $K = 1.03$, From PSGDB 8.15 , table 14, for $\varphi_p = 0 \cdot 6$ bearings are close to gears and symmetrical.

Revised $K_d = 1.4$. From PSGDB 8.16 , table 15, for IS Quality 8 , HB > 350 , and $v = 5.34\,m/sec$

$$[M_t] = 238.73 \times 1.03 \times 1.4$$

$$= 344.24 Nm$$.

Step 13: Check for bending:

Calculation of induced bending stress (σ_b):

$$\sigma_b = \frac{i \pm 1}{amby}[M_t] \leq [\sigma_b]$$ From PSGDB 8.13A table 8

- $y = \text{Form factor} = 0.366$, for $Z_1 = 17$, From PSGDB 8.18 table 18

$$\sigma_b = \frac{3+1}{170 \times 5 \times 51 \times 0.366} \times 344.24 \times 10^3$$

$$= 86.78\,N/mm^2$$

We find $\sigma_b < [\sigma_b]$, $\therefore$ the design is safe.

Step 14: Check for wear strength (σ_c):

From PSGDB 8.13 table 8 , for checking.

$$\sigma_c = 0.74 \times \frac{i+1}{a}\sqrt{\frac{i+1}{ib} \times E_{eq}[M_t]}$$

$$= 0.74 \times \frac{3+1}{170}\sqrt{\frac{3+1}{3 \times 51} \times 2.15 \times 10^5 \times 344.24 \times 10^3}$$

$$= 765.9\,N/mm^2$$

We find $\sigma_c < [\sigma_c]$. $\therefore$ The design is safe.

Step 15: Basic dimensions of Pinion and gear:

From PSGDB 8.22 table 26

- Module: $m = 5mm$
- Number of teeth: $Z_1 = 17$

$Z_2 = 51$

* Pitch circle diameter:

$$d_1 = m \times Z_1 = 5 \times 17 = 85mm$$

$$d_2 = m \times Z_2 = 5 \times 51 = 255mm$$

* Centre distance:

$$a = 170mm$$

* Face width:

$$b = 51mm$$

* Height factor:

$f_0 = 1$, for full depth teeth,

* Bottom clearance:

$$C = 0.25m = 0.25 \times 5 = 1.25mm$$.

* Tooth depth:

$$h = 2.25m = 2.25 \times 5 = 11.25mm$$

* Tip diameter:

$$d_{a1} = (Z_1 + 2f_0)m$$

$$= (17 + 2 \times 1)5$$

$$= 95mm.$$

$$d_{a2} = (Z_2 + 2f_0)m$$

$$= (51 + 2 \times 1)5$$

$$= 265mm.$$

* Root diameter:

$$d_{f1} = (Z_1 - 2f_0)m - 2C$$

$$= (17 - 2 \times 1)5 - 2 \times 1.25$$

$$= 72.5mm.$$

$$d_{f2} = (Z_2 - 2f_0)m - 2C$$

$$=(51-2\times1)5-2\times1.25$$

$$=242.5\text{mm.}$$

2. **Design of helical gear drive to connect an electric motor to a reciprocating pump. Gears are overhanging in their shafts. Motor speed = 1440rpm. Speed reduction ratio = 5, motor power = 37 KW, pressure angle = 20° , helix angle = 25°.**

Given data:

$$N_1 = 1440\text{rpm}$$
$$i = 5$$
$$P = 37\text{KW}$$
$$\phi = 20° = \alpha_n$$
$$\beta = 25°$$

Step 1: selection of Material.

Generally we assume C45 steel for both pinion and gear.

$[\sigma_b] = 180\,\text{N/mm}^2$, 250 BHN.

Step 2: Calculation of number of teeth Z_1 & Z_2:

No. of teeth on pinion $Z_1 = 20$ (assume)

No. of teeth on gear $Z_2 = i \times Z_1$

$$= 5\times20$$

$$= 100$$

Step 3: Calculation of tangential load on teeth (F_t):

$$F_t = \frac{P}{v}\times K_0$$

Case 1: To find the Pitch line velocity (v)

$$v = \frac{\pi d_1 N_1}{60}$$

From PSGDB 8.22

$$d_1 = \frac{m_n}{\cos\beta}\times Z_1$$

$$= \frac{m_n}{\cos 25^\circ} \times 20$$

$$d_1 = \frac{m_n}{22.06}$$

$$\therefore v = \frac{\pi \times m_n \times 1440}{60 \times 22.06 \times 1000}$$

$$= 1.66 m_n \; m/sec$$

Case 2: To find K_0:

$K_0 = 1.5$ for medium shock conditions.

$$\therefore F_t = \frac{37 \times 10^3}{1.66 m_n} \times 1.5$$

$$= \frac{33433.73}{m_n}$$

Step 4: Calculation of initial dynamic Load (F_d)

$$F_d = \frac{F_t}{C_v}$$

Case 1: To find velocity factor (C_v)

$$C_v = \frac{6}{6+v} \quad \text{for carefully cut gears, } v < 20\,m/s$$

$$= \frac{6}{6+15}$$

$$C_v = 0.286$$

Case 2: To find initial dynamic load (F_d):

$$F_d = \frac{33433.73}{m_n} \times \frac{1}{0.286}$$

$$\frac{116901.17}{m_n}$$

Step 5: Calculation of beam strength (Fs):

$F_s = [\sigma_b] b y^{-1} P_{cn}$ From PSGDB 8.51 $P_{cn} = \pi m_n$

$$\therefore \; F_s = [\sigma_b] b y^1 \pi m_n$$

Where,

$$b = 10 \times m_n \quad \text{From PSGDB 8.14}$$

$$y^1 = 0.154 - \left(0.912 / Z_{v1} \right)$$ From PSGDB 8.50 , $20°$ full depth system.

$$Z_{v1} = \frac{Z_1}{\cos^3 \beta}$$

$$= \frac{20}{\cos^3 25}$$

$$Z_{v1} = 26.86 \approx 27$$

$$\therefore \; y^1 = 0.154 - \frac{0.912}{26.86}$$

$$= 0.12$$

$$F_s = [\sigma_b] b \, y^1 \pi m_n$$

$$= 180 \times 10 \times m_n \times 0.1143 \times \pi \times m_n$$

$$= 678.58 m_n^2$$

Step 6: Calculation of normal module (m_n):

From PSGDB 8.51

$$F_s \geq F_d$$

$$678.58 m_n^2 \geq \frac{116901.17}{m_n}$$

$$m_n \geq 5.56 \text{mm}$$

From PSGDB 8.2 , table 1. The nearest higher standard module value under choice 1, is

$$m_n = 6 \text{mm}$$

Step 7: Calculation of b , d_1, and v:

Case 1: To find the face width (b)

$$b = 10 \times m_n$$

$$= 10 \times 6$$

$$= 60mm$$

Case 2: To find Pitch circle diameter (d_1)

$$d_1 = \frac{m_n}{\cos\beta} \times Z_1$$

$$= \frac{6}{\cos 25°} \times 20$$

$$d_1 = 124.23mm.$$

Case 3: To find Pitch line velocity (v)

$$v = \frac{\pi d_1 N_1}{60}$$

$$= \frac{\pi \times 124.23 \times 10^{-3} \times 1440}{60}$$

$$= 9.37\, m/s$$

Step 8: Recalculation of Beam strength (F_s)

$$F_s = [\sigma_b] b\, y^1 \pi m_n$$

$$= 180 \times 60 \times 0.12 \times 6 \times \pi$$

$$F_s = 24429.02N$$

Step 9: Calculation of Accurate dynamic load (F_d)

From PSGDB 8.51

$$F_d = F_t + \frac{21v\left(6c.\cos^2\beta + F_t\right)\cos\beta}{21v + \sqrt{6c.\cos^2\beta + F_t}}$$

Case 1: To find (F_t)

$$F_t = \frac{P}{v}$$

$$= \frac{37 \times 10^3}{9.37}$$

$$F_t = 3948.77N$$

Case 2: To find deformation factor (C)

$C = 11860\ e$ From PSGDB 8.53 , table 41 , for 20° FD , steel and steel.

$e = 0.030$, for module upto 6 and carefully cut gears – PSGDB 8.53 table 42

$\therefore\ C = 11860 \times 0.030$

$= 355.8\ N/mm$

Case 3: To find (F_d)

$$F_d = 3948.77 + \frac{21 \times 9.37 \times 10^3 \left(60 \times 355.8 \times \cos^2 25° + 3948.77\right) \cos 25°}{21 \times 9.37 \times 10^3 + \sqrt{60 \times 355.8 \times \cos^2 25° + 3948.77}}$$

$F_d = 23398.68\ N$

Step 10: Check for beam strength or tooth breakage.

We find $F_s > F_d$. $\therefore$ the design is safe

Step 11: Calculation of Maximum wear load (F_w):

Case 1: To find Ratio factor (Q)

From PSGDB 8.51

$$Q = \frac{2i}{i+1} = \frac{2 \times 5}{5+1} = 1.67$$

Case 2: To find Load stress factor (K_w)

From PSGDB 8.51

$$K_w = \frac{\left[f_{es}^2\right] \sin d_n}{1.4} \times \left[\frac{1}{E_1} + \frac{1}{E_2}\right]$$

Assume $f_{es} = 618 N/mm^2$

$$K_w = \frac{618^2 \sin 20}{1.4} \times \left[\frac{1}{2.15 \times 10^5} + \frac{1}{2.15 \times 10^5}\right]$$

$$= 0.867\ N/mm^2$$

Case 3: To find Maximum wear load (F_w).

From PSGDB 8.51

$$F_w = \frac{d_1 \times b \times Q \times K_w}{\cos^2 \beta}$$

$$= \frac{124.23 \times 60 \times 1.67 \times 0.867}{\cos^2 25°}$$

$$F_w = 13138.98N$$

Step 12: Check for wear:

- We find $F_w < F_d$. $\therefore$ the design is not safe.
- In order to increase the wear load, we have to increase the hardness (BHN). So now for steel hardened to 400 BHN, K_W = 2.41 N/mm^2.

$$\therefore\ F_w = 36522.44N$$

$\therefore F_w > F_d$, Design is safe.

Step 13: Calculation of basic dimensions of Pinion and gear.

- Normal module: $m_n = 6mm$
- No. of teeth: $Z_1 = 20$, $Z_2 = 100$
- Pitch circle diameter: $d_1 = 124.23mm$, $d_2 = \frac{m_n}{\cos\beta} \times Z_2$

$$= \frac{6}{\cos 25° \times 100} = 662.03mm.$$

- Centre distance: $a = \frac{m_n}{\cos\beta} \times \left(\frac{Z_1 + Z_2}{2}\right)$

$$= \frac{6}{\cos 25°} \times \left(\frac{20+100}{2}\right)$$

$$a = 397.22mm$$

- Face width: $b = 60mm$
- Height factor: $f_0 = 1$, for 20° full depth teeth.
- Bottom clearance: $C = 0.25m_n$

$$= 0.25 \times 6$$
$$C = 1.5mm$$

* Tip diameter: $d_{a1} = \left(\frac{Z_1}{\cos\beta} + 2f_0\right)m_n$

$$= \left(\frac{20}{\cos 25°} + 2\times1\right)\times6$$

$$d_{a1} = 144.41mm$$

$$d_{a2} = \left(\frac{Z_2}{\cos\beta} + 2f_0\right)m_n$$

$$= \left(\frac{100}{\cos 25°} + 2\times1\right)\times6$$

$$d_{a2} = 674.03mm$$

* Root diameter:

$$d_{f1} = \left(\frac{Z_1}{\cos\beta} - 2f_0\right)m_n - 2C \qquad d_{f2} = \left(\frac{Z_2}{\cos\beta} - 2f_0\right)m_n - 2C$$

$$d_{f1} = \left(\frac{20}{\cos 25°} - 2\times1\right)6 - 2\times1.5 \qquad = \left(\frac{100}{\cos 25°} - 2\times1\right)6 - 2\times1.5$$

$$d_{f1} = 117.41mm \qquad d_{f2} = 647.03mm.$$

* Virtual number of teeth:

$$Z_{v1} = 26.86 = 27$$

$$Z_{v2} = \frac{Z_2}{\cos^3\beta} = \frac{100}{\cos^3 25°}$$

$$Z_{v2} = 134.33 = 135$$

3. **Design a spur gear drive to transmit 8 KW at 720 rpm and the speed ratio is 2. The pinion and wheel are made of the same surface hardened carbon steel with 55 RC and core hardness less than 350 BHN. Ultimate strength is 720 N/mm² and yield strength is 360 N/mm².**

Given data:

$$P = 8\ KW$$

$N_1 = 720$ rpm

$i = 2$

Material = Surface hardened carbon steel

$\sigma_u = 720 N/mm^2$

$\sigma_y = 360 N/mm^2$

$\sigma_b = 240 N/mm^2$ (Assume) why ⇒Allowable static stress $[\sigma_b] = \frac{\sigma_u}{3}$

$$= \frac{720}{3}$$

$$= 240 N/mm^2$$

Step 1: To find Gear ratio (i):

$$i = 2 \text{ given}$$

Step 2: Selection of material.

Both pinion and gear, surface hardened carbon steel

Surface hardness < 350 with 55 RC

Step 3: Calculation of Z_1 and Z_2

No. of teeth on pinion $Z_1 = 20$ (Assume)

Gear $Z_2 = i \times Z_1$ From PSGDB 8.1 table 8.3

$= 2 \times 20$

$Z_2 = 40$

Step 4: Calculation of tangential load (F_t):

$$F_t = \frac{P}{v} \times K_0$$

Where $K_0 = 1.5$, Assume medium shock conditions.

$$* \quad v = \frac{\pi d_1 N_1}{60}$$

From PSGDB 8.22, table 26

$$d_1 = m \times Z_1$$

$$\therefore \quad v = \frac{\pi \times m \times Z_1 \times N_1}{60 \times 1000}$$

$$= \frac{\pi \times m \times 20 \times 720}{60 \times 1000}$$

$$v = 0.754 \text{ mm/sec.}$$

$$\therefore \quad F_t = \frac{8 \times 10^3}{0.754m} \times 1.5$$

$$F_t = \frac{15915.12}{m}$$

Step 5: Calculation of initial dynamic load (F_d).

$$F_d = \frac{F_t}{C_v}$$

From PSGDB 8.51, Assume v = 12 m/sec.

$$C_v = \frac{6}{6+v}$$

$$= \frac{6}{6+12}$$

$$C_v = 0.333$$

$$\therefore \quad F_d = \frac{15915.12}{m} \times \frac{1}{0.333}$$

$$F_d = \frac{47793.15}{m}$$

Step 6: Calculation of beam strength (F_s).

$$F_s = \pi \times m \times b \times [\sigma_b] \times y \quad \text{From PSGDB 8.50}$$

Where, $b = 10 \times m$ From PSGDB 8.14

$y = 0.154 - \left(0.912 / Z_1\right)$ From PSGDB 8.50 , for 20° involute.

$$\therefore \quad y = 0.154 - \left(0.912 / 20\right)$$

$$y = 0.1084$$

$$\therefore \quad F_s = \pi \times m \times 10 \times m \times 240 \times 0.1084$$

$$F_s = 817.32 \times m^2$$

Step 7: Calculation of module 'm':

$$F_s \geq F_d$$

$$817.32m^2 \geq \frac{47793.15}{m}$$

$$m \geq 3.88 \text{ mm.}$$

From PSGDB 8.2, table 1. Choice 1.

The next higher standard module $m = 4$ mm.

Step 8: Calculation of b, d_1 and v:

Case 1: To find face width (b):

$$b = 10 \times m$$

$$= 10 \times 4$$

$$b = 40\text{mm}$$

Case 2: To find Pitch circle diameter (d_1).

$$d_1 = m \times Z_1$$

$$= 4 \times 20$$

$$d_1 = 80\text{mm}$$

Case 3: To find Pitch line velocity (v).

$$v = 0.754 \text{ m} \quad \text{From step 4}$$

$$= 0.754 \times 4$$

$$v = 3.016 \text{ m/s}$$

Step 8: Recalculation of beam strength (F_s).

$$F_s = 817.32 \times m^2 \quad \text{From step 5}$$

$$= 817.32 \times 4^2$$

$$F_s = 13077.12 \text{ N}$$

Step 9: Calculation of accurate dynamic load (F_d)

$$F_d = F_t + \frac{21v(b_c + F_t)}{21v + \sqrt{b_c + F_t}}$$ From PSGDB 8.51

Case 1: To find tangential load (F_t)

$$F_t = \frac{P}{v} = \frac{8\times10^3}{3.016} = 2652.52\text{N}$$

Case 2: To find deformation factor (C)

$$C = 11860\ e$$

From PSGDB 8.53, table 42.

$e = 0.0125$, for precision gears, module upto 4

$$\therefore \quad C = 11860 \times 0.0125$$

$$C = 148.25 \text{ N/mm}^2$$

$$\therefore \quad F_d = 2652.52 + \frac{21\times3.016\times10^3(40\times148.25+2652.52)}{21\times3.016\times10^3 + \sqrt{40\times148.25+2652.52}}$$

$$F_d = 11222.504 \text{ N}$$

Step 11: Check for beam strength.

We find $F_d < F_s$ $\therefore$ the design is safe.

Step 12: Calculation of Maximum wear load (F_w)

From PSGDB 8.51,

$$F_w = d_1 \times b \times Q \times K_w$$

Where $Q = \frac{2i}{i+1} = \frac{2\times2}{2+1} = 1.33$

$$K_w = 0.919 \text{ N/mm}^2$$

$$\therefore \quad F_w = 80\times40\times1.33\times0.919$$

$$F_w = 3911.264 \text{ N}$$

Step 13: Check for wear.

We find $F_w < F_d$. ∴ the design is unsatisfactory

∴ Increasing the value of $K_w = 2.553\ N/mm^2$

$$\therefore\ F_w = 10890.08\ N\ .$$

We find F_w value is closer to F_d, but not in that condition $F_w > F_d$. ∴ Moderately safe. mild wear takes place.

Step 14: Calculation of basic dimensions of pinion and gear.

From PSGDB 8.22, table 26

* Module: $m = 4mm$
* No. of teeth: $Z_1 = 20,\ \ Z_2 = 40$
* Pitch circle diameter: $d_1 = 80mm$

$$d_2 = m \times Z_2 = 4 \times 40 = 160mm$$

* Centre distance: $a = \frac{m(Z_1 + Z_2)}{2} = \frac{4(40+40)}{2} = 120mm$
* Face width: $b = 40mm$
* Height factor: $f_0 = 1$, for 20° full depth
* Bottom clearance: $C = 0.25m = 0.25 \times 4 = 1mm$
* Tip diameter: $d_{a1} = (Z_1 + 2f_0)m = (20 + 2(1))4 = 88\,mm$

$$d_{a2} = (Z2 + 2f_0)m = (40 + 2 \times 1)4 = 168\,mm$$

* Root diameter: $d_{f1} = (Z_1 - 2f_0)m - 2C = (20 - 2\times1)4 - 2(1) = 70\,mm$

$$d_{f2} = (Z_2 - 2f_0)m - 2C = (40 - 2\times1)4 - 2(1) = 150\,mm$$

4. Design of helical gear drive to transmit the power of 14.7 KW , speed ration 6 , pinion speed 1200 rpm , helix angle is 25° select suitable material and design the gear.

Given data:

$P = 14.7KW$

$i = 6$

$N_1 = 1200\ rpm$

$\beta = 25°$

Step 1: Selection of Material.

C45 – Steel material is selected for both pinion and gear.

$$\therefore\ [\sigma_b] = 180\ \text{N/mm}^2$$

Step 2: Calculation of no. of teeth:

Case 1: Calculation of Z_1 & Z_2.

No. of teeth on pinion Z_1 = 20 Assume

Gear $Z_2 = i \times Z_1$

$= 6 \times 20$ 6* 20

$= 120$

Case 2: Calculation of Z_{v1} & Z_{v2}:

From PSGDB 8.22 , table 2b

Virtual no. of teeth on pinion $Z_{v1} = \frac{Z_1}{\cos^3 \beta} = \frac{20}{\cos^3 25°}$

$= 26.86 = 27$

Gear $Z_{v2} = \frac{Z_2}{\cos^3 \beta} = \frac{120}{\cos^3 25°}$

$= 161.19 = 162$

Step 3: Calculation of tangential load on teeth (F_t).

$$F_t = \frac{P}{v} \times K_0$$

Case 1: To find the pitch line velocity (v).

$$v = \frac{\pi d_1 N_1}{60}$$

From PSGDB 8.22

$$d_1 = \frac{m_n}{\cos \beta} \times Z_1$$

$$\therefore \quad v = \frac{\pi \times m_n \times 20 \times 1200}{60 \times \cos 25 \times 100°}$$

$$v = 1.39 m_n \quad m/\sec$$

Case 2: To find K_0

K_0 = 1.5 , for medium shock condition.

$$\therefore \quad F_t = \frac{14.7\times10^3}{1.39\times m_n}\times1.5$$

$$F_t = \frac{15902.83}{m_n}(N)$$

Step 4: Calculation of initial dynamic load (F_d)

$$F_d = \frac{F_t}{C_v}$$

Case 1: To find the velocity factor (C_v)

$C_v = \frac{6}{6+v}$ for carefully cut gears , v < 20 m/s From BSGDB 8.51 Assume v = 15 m/s

$$= \frac{6}{6+15}$$

$$C_v = 0.286$$

Case 2: To find initial dynamic load (F_d)

$$F_d = \frac{15902.83}{m_n}\times\frac{1}{0.286}$$

$$F_d = \frac{55604.3}{m_n}$$

Step 5: calculation of beam strength (F_s)

$$F_s = [\sigma_b]by^1\pi m_n \qquad \text{From PSGDB 8.51}$$

Where,

- $b = 10\times m_n$ From PSGDB 8.14
- $y^1 = 0.154 - \left(0.912/Z_{v1}\right)$ From PSGDB 8.50 , 20° Full depth system.

$$= 0.154 - \left(0.912/27\right)$$

$$y^1 = 0.12$$

$$\therefore \quad F_s = 180 \times 10 \times m_n \times 0.12 \times \pi \times m_n$$

$$= 678.58\, m_n^2$$

Step 6: Calculation of normal module (m_n).

From PSGDB 8.51

$$F_s \geq F_d$$

$$678.58 m_n^2 \geq \frac{55604.3}{m_n}$$

$$m_n \geq 4.34 \text{ mm.}$$

From PSGDB 8.2, table 1, the nearest higher standard module value under choice 1 is;

$$m_n = 5 \text{ mm.}$$

Step 7: Calculation of b_1 d_1 and v:

Case 1: To find the face width (b)

$$b = 10 \times m_n$$

$$= 10 \times 5$$

$$= 50 \text{ mm.}$$

Case 2: To find the Pitch circle diameter (d_1)

$$d_1 = \frac{m_n}{\cos\beta} \times Z_1$$

$$= \frac{5}{\cos 25} \times 20$$

$$d_1 = 110.34 \text{ mm}$$

Case 3: To find the pitch line velocity (v)

$$v = \frac{\pi d_1 N_1}{60}$$

$$\frac{\pi \times 110.34 \times 10^{-3} \times 1200}{60}$$

$$v = 6.93 \text{ m/s}$$

Step 8: Recalculation of beam strength (F_s)

$$F_s = [\sigma_b] \times b \times y^1 \times \pi \times m_n$$

$$= 180 \times 50 \times 0.12 \times \pi \times 5$$

$$F_s = 16964.6 \text{ N}$$

Step 9: Calculation of accurate dynamic load (F_d)

From PSGDB 8.51

$$F_d = F_t + \frac{21v(bc.\cos^2\beta + F_t)\cos\beta}{21v + \sqrt{(bc.\cos^2\beta + F_t)}}$$

Case 1: To find (F_t).

$$F_t = \frac{P}{v}$$

$$= \frac{14.7 \times 10^3}{6.93}$$

$$F_t = 2121.21 \text{ N}$$

Case 2: To find deformation factor (C).

$C = 11860\ e$ From PSGDB 8.53 , table 41 , for 20° FD , steel and steel.

$e = 0.025$ From PSGDB 8.53 table 42 , for module upto 5 and carefully cut gears.

$$\therefore \quad C = 11860 \times 0.025$$

$$= 296.5 \text{ N/mm}^2$$

Case 3: To find (F_d).

$$F_d = 2121.21 + \frac{21 \times 6.93 \times 10^3 (50 \times 296.5 \times \cos^2 25 + 212.21)\cos 25}{21 \times 6.93 \times 10^3 + \sqrt{50 \times 296.5 \times \cos^2 25 + 2121.21}}$$

$$F_d = 15069.29 \text{ N}$$

Step 10: Check for beam strength.

We find $F_s > F_d$ $\therefore$ The design is safe.

Step 11: Calculation of Maximum wear load (F_w):

From PSGDB 8.51.

$$F_w = \frac{d_1 \times b \times Q \times K_w}{\cos^2 \beta}$$

Case 1: To find ratio factor (Q).

From PSGDB 8.51.

$$Q = \frac{2i}{i+1} = \frac{2 \times 6}{6+1} = 1.71$$

Case 2: To find Load stress factor (K_w).

Assume K_w = 0.919 for 20° FD

$$\therefore \quad F_w = \frac{110.34 \times 50 \times 1.71 \times 0.919}{\cos^2 25}$$

$$F_w = 10555.12\ N$$

Step 12: Check for wear.

* We find $F_w < F_d$. $\therefore$ The design is not safe.
* In order to increase the wear load, we have to increase the hardness (BHN). So how for steel hardened to 400 BHN, $K_w = 2.553\ N/mm^2$.

$$\therefore \quad F_w = 29322.33\ N$$

$$\therefore F_w > F_d \text{ , Design is safe.}$$

Step 13: Calculation of basic dimension of pinion and gear.

From PSGDB 8.22 , table 26

* Normal module: $m_n = 5mm$
* No. of teeth: $Z_1 = 20$, $Z_2 = 120$
* Pitch circle diameter: $d_1 = 110.34mm$, $d_2 = \frac{m_n}{\cos\beta} \times Z_2$

$$= \frac{5}{\cos 25} \times 120$$

$$= 662.03mm$$

* Centre distance: $a = \frac{m_n}{\cos\beta} \times \left(\frac{Z_1 + Z_2}{2}\right)$

$$= \frac{5}{\cos 25} * \left(\frac{20+120}{2}\right)$$

* Face width: $b = 50mm$
* Height factor: $f_0 = 1$, for 20°FD
* Bottom clearance: $c = 0.25m_n$

$$= 0.25 \times 5$$

$$= 1.25 \text{ mm}$$

* Tip diameter: $d_{a1} = \left(\frac{Z_1}{\cos\beta} + 2f_0\right)m_n$ $\quad d_{a2} = \left(\frac{Z_2}{\cos\beta} + 2f_0\right)m_n$

$$= \left(\frac{20}{\cos 25} + 2\times1\right)5 \qquad = \left(\frac{120}{\cos 25} + 2\times1\right)5$$

$$= 120.33 \text{ mm.} \qquad = 672 \text{ mm.}$$

* Root diameter: $d_{f1} = \left(\frac{Z_1}{\cos\beta} - 2f_0\right)m_n - 2c$ $\quad d_{f2} = \left(\frac{Z_2}{\cos\beta} - 2f_0\right)m_n - 2c$

$$= \left(\frac{20}{\cos 25} - 2\times1\right)5 - 2\times1.25 \qquad = \left(\frac{120}{\cos 25} - 2\times1\right)5 - 2\times1.25$$

$$= 97.83\text{mm.} \qquad = 649.52\text{mm.}$$

5. Design a spur gear drive required to transmit 45 KW at a pinion speed of 800 rpm. The velocity ratio is 3.5:1. The teeth are 20° involute with 18 teeth on the pinion. Both the pinion and gear are made of steel with a maximum safe static stress of 180 N/mm². Assume medium shock conditions

Given data:

$P = 45KW$

$N_1 = 800rpm$

$i = 3.5$

$\phi = 20°$

$Z_1 = 18$

$[\sigma_b] = 180\,N/mm^2$

Material = steel (for both pinion and gear)

Step 1: Selection of Material

Pinion and Gear = Steel

Assume steel is hardened to 200 BHN (BRINELL HARDNESS NUMBER) from PSGDB 8.16 table 16

Step 2: Calculation of Z_1 and Z_2

Number of Teeth on Pinion $\quad Z_1 = 18$

Number of Teeth on Gear $\quad Z_2 = i \times Z_1$

$$= 3.5 \times 18$$

$$Z_2 = 63$$

Step 3: Calculation of Tangential load (F_t)

Case 1: To find the pitch line velocity (v)

$$v = \frac{\pi d_1 N_1}{60}$$

$$v = \frac{\pi m Z_1 N_1}{60}$$

$$= \frac{\pi \times m \times 18 \times 800}{60 \times 1000}$$

$$= 0.754m \ m/sec$$

$F_t = \frac{P}{v} \times K_0$

$P = 45KW$

$K_0 = 1.5$

$v = \frac{\pi d_1 N_1}{60}$

$d_1 = m \times Z_1$

From PSGDB 8.22

Case 2: To find K_0

$K_0 = 1.5$ for medium shock conditions

Case 3: To find F_t

$$F_t = \frac{P}{v} \times K_0$$

$$F_t = \frac{45 \times 10^3}{0.754m} \times 1.5$$

=89522.5/m

Step 4: Calculation of Initial Dynamic Load (F_d)

Case 1: To find velocity factor (C_v)

$C_v = \frac{6}{6+v}$ for accurately hobbed and generated gears

With $v < 20 \, m/sec$

$$C_v = \frac{6}{6+12}$$

$F_d = \frac{F_t}{C_v}$

$C_v = \frac{6}{6+v}$

From PSGDB 8.51

Assume $v = 12 \, m/sec$

Case 2: To find initial dynamic load (F_d)

$$F_d = \frac{89522.5}{m} \times \frac{1}{0.333}$$

$$F_d = \frac{268836.3}{m}$$

Step 5: Calculation of Beam Strength (F_s)

Case 1: To find form factor (y):

From PSGDB 8.50

$$F_s = [\sigma_b] b \, y \, P_c$$

Where

$P_c = \text{circular pitch} = \frac{\pi d}{z} = \pi m$

$m = d/z$

Finally we write

$F_s = [\sigma_b] b \, y \, \pi \, m$

Where

b = Face width $10 \times m$

y = Form Factor

$= 0.154 - (0.912/Z_1)$ for 20°

Full depth system

$$y = 0.154 - (0.912/Z_1)$$

$$= 0.154 - (0.912/18)$$

$$= 0.1033$$

Case 2: To find the beam strength (F_s)

Lewis equation,

$$F_s = [\sigma_b] b \, y \, \pi \, m$$

$$= 180 \times 10m \times 0.1033 \, \pi \, m$$

$$= 584.15m^2$$

Step 6: Calculation of Module (m):

From PSGDB 8.51

$$F_s \geq F_d$$

$$584.15m^2 \geq \frac{268836.3}{m}$$

$$m \geq 7.72\text{mm}$$

From PSGDB 8.2 table 1, the nearest higher standard module value under choice 1 is 8 mm

Step 7: Calculation of b , d and v

Case 1: To find the face width (b)

$$b = 10 \times m$$
$$= 10 \times 8$$
$$= 80mm$$

Case 2: To find pitch circle diameter (d_1)

$$d_1 = m \times Z_1$$
$$= 8 \times 18$$
$$= 144mm$$

Case 3: To find Pitch line velocity (v)

$$v = \frac{\pi d_1 N_1}{60}$$
$$= \frac{\pi \times 144 \times 10^{-3} \times 800}{60}$$
$$= 6.03\, m/sec$$

Step 8: Recalculation of Beam Strength

Beam Strength $F_s = [\sigma_b]\, b\, y\, \pi\, m$

$$= 180 \times 80 \times 0.1033 \times \pi \times 8$$
$$= 37385.45\, N$$

Step 9: Calculation of accurate dynamic load (F_d)

STEP 9: CALCULATION OF ACCURATE DYNAMIC LOAD (F_d)

Case 1: To find tangential load (F_t)

$$F_t = \frac{P}{v}$$
$$F_t = \frac{45x103}{6.03}$$
$$= 7462.68\ N$$

Case 2: To find Deformation factor (C)

C=11860 e

=11860x0.038

= 450.68 N/mm²

Case 3: To find the accurate dynamic load (F_d)

$$F_d = F_t + \frac{21\, v\, (bc + F_t)}{21\, v + \sqrt{bc + F_t}}$$

$$F_d = 7462.68 + \frac{21x6.03x103\ (80x450.68 + 7462.68)}{21x6.03x103 + \sqrt{80x450.68 + 7462.68}}$$

=50908.19 N

$F_d = F_t + \frac{21\, v\, (bc+F_t)}{21\, v + \sqrt{bc+F_t}}$ from PSGDB 8.51

⬆

We know that $F_t = \frac{P}{v}$ for accurate value eliminate K_0

⬆

C= Deformation factor from PSGDB 8.53 , table 41

⬆

C=11860 e, for 20° FD, steel and steel

⬆

e = 0.038 , for module upto 8 and carefully cut gears from PSGDB 8.53 , table 42

Step 10: Check for Beam strength or Tooth breakage

Since $F_d > F_s\,(50908.19N > 37385.45N)$ the design is unsatisfactory. The dynamic load is greater than the beam strength

In order to reduce the dynamic load F_d, Select the precision gears. Therefore from PSGDB 8.53 , table 42 take $e = 0.019$ for precision gears

Recalculation of deformation factor:

$$C = 11860 \times 0.019 = 225.34$$

Recalculation of dynamic load:

$$F_d = 7462.68 + \frac{21 \times 6.03 \times 10^3 (80 \times 225.34 + 7462.68)}{21 \times 6.03 \times 10^3 + \sqrt{80 \times 225.34 + 7462.68}}$$

$$= 32920.46N$$

Now we find $F_s > F_d\,(37385.45N > 32920.46N)$. It means the gear tooth has adequate beam strength and it will not fail by breakage. Therefore the design is safe.

Step 11: Calculation of maximum wear load (F_w)

Case 1: To find ratio factor (Q)

$$Q = \frac{2i}{i+1} = \frac{2 \times 3.5}{3.5+1} = 1.555$$

Case 2: To find maximum wear load (F_w)

$$F_w = d_1 \times b \times Q \times K_w$$

$$= 144 \times 80 \times 1.555 \times 0.919$$

$$= 16462.6N$$

From PSGDB 8.51

$$F_w = d_1 \times b \times Q \times K_w$$

$$Q = \text{Ratio factor} = \frac{2i}{i+1}$$

$K_w = \text{load stress factor} = 0.919\,N/mm^2$, for steel hardened to 250 BHN

Step 12: Check for wear

Since $F_d > F_w\,(32920.46N > 16462.6N)$ the design is unsatisfactory. That is the dynamic load is greater than the wear load.

In order to increase the wear load (F_w), we have to increase the hardness (BHN). So now for steel hardened to 400BHN, $K_w = 2.553\,N/mm^2$

$$\therefore \quad F_w = d_1 \times b \times Q \times K_w$$

$$=144\times80\times1.555\times2.553$$

$$=45733.42N$$

Now we find $F_w > F_d\left(45733.42N > 32920.46N\right)$. It means the gear tooth is adequate wear capacity and it will not wear out. Therefore the design is satisfactory

Step 13: Basic dimensions of Pinion and gear

From PSGDB 8.22, table 26

Module: $m = 8mm$

Number of teeth: $Z_1 = 18, Z_2 = 63$

Pitch circle diameter: $d_1 = 144mm$

$$d_2 = m\times Z_2 = 8\times63$$

$$d_2 = 504mm$$

Centre distance: $a = m\left(Z_1 + Z_2\right)/2$

$$=8\left(18+63\right)/2$$

$$a = 324mm$$

Face width: $b = 80mm$

Height factor: $f_0 = 1$, for $20°$ full depth teeth

Bottom clearance: $c = 0.25m = 0.25\times8$

$$c = 2mm$$

Tip diameter: $d_{a1} = \left(Z_1 + 2f_0\right)m$ $\quad d_{a2} = \left(Z_2 + 2f_0\right)m$

$=\left(18+2\times1\right)8$ $\quad =\left(63+2\times1\right)8$

$=160mm$ $\quad =520mm$

Root diameter: $d_{f1} = \left(Z_1 - 2f_0\right)m - 2c$ $\quad d_{f2} = \left(Z_2 - 2f_0\right)m - 2c$

$=\left(18-2\times1\right)8-2\times2$ $\quad =\left(63-2\times1\right)8-2\times2$

$=124mm$ $\quad =484mm$

6. Design a pair of helical gears to transmit 10 KW at pinion speed of 1000rpm. The Reduction ratio is 5. Assume suitable materials and stresses.

Given data:

$$N_1 = 1000\text{rpm}$$
$$P = 10\text{KW}$$
$$i = 5$$

Step 1: Selection of Material

Generally we assume C45 steel for both pinion and gear.

$[\sigma_b] = 180\,\text{N/mm}^2$, 400 BHN.

Step 2: Calculation of number of teeth Z_1 and Z_2:

No. of teeth on pinion gear $Z_1 = 20$ (assume)

$$Z_2 = i \times Z_1$$
$$= 5 \times 20$$
$$= 100.$$

Virtual no. of teeth Z_{v1} & Z_{v2}

From PSGDB 8.22 , table 26. Assume $\beta = 25°$

$$Z_{v1} = \frac{Z_1}{\cos^3 \beta} \qquad Z_{v2} = \frac{Z_2}{\cos^3 \beta}$$

$$= \frac{20}{\cos^3 25} \qquad = \frac{100}{\cos^3 25}$$

$$Z_{v1} = 27 \qquad = 134.33\text{mm}.$$

$$Z_{v2} \approx 135\text{mm}$$

Step 3: Calculation of tangential load on teeth (F_t).

$$F_t = \frac{P}{v} \times K_0$$

$K_0 = 1.5$, for medium shock conditions.

Case 1: To find the pitch line velocity (v)

$$v = \frac{\pi d_1 N_1}{60}$$

From PSGDB 8.22, table 26

$$d_1 = \frac{m_n}{\cos\beta} \times Z_1$$

$$\therefore \quad v = \frac{\pi \times m_n \times 20 \times 1000}{60 \times 1000 \times \cos 25°}$$

$$v = 1.16 m_n \ \text{m/sec}$$

$$\therefore \quad F_t = \frac{10 \times 10^3}{1.16 m_n} \times 1.5$$

$$= \frac{12931.03}{m_n}$$

Step 4: Calculation of initial dynamic load (F_d)

$$F_d = \frac{F_t}{C_v}$$

Case 1: To find the velocity factor (Cv)

$C_v = \frac{6}{6+v}$ for carefully cut gears $v < 20\,\text{m/s}$. From PSGDB 8.51 Assume $v = 15\,\text{m/s}$.

$$= \frac{6}{6+15}$$

$$C_v = 0.286.$$

$$\therefore \quad F_d = \frac{12931.03}{m_n} \times \frac{1}{0.286}$$

$$= \frac{45213.41}{m_n}$$

Step 5: Calculation of beam strength (F_s).

$$F_s = [\sigma_b] \times b \times y^1 \times \pi \times m_n$$

Where,

$b = 10 \times m_n$ From PSGDB 8.14

$y^1 = 0.154 - \left(0.912 / Z_{v1}\right)$ From PSGDB 8.50 , 20° FD

$= 0.154 - \frac{0.912}{27}$

$= 0.12$

$\therefore\ F_s = 180 \times 10 \times m_n \times 0.12 \times \pi \times m_n$

$F_s = 678.58\ m_n^2$

Step 6: Calculation of normal module (m_n)

From PSGDB 8.51

$F_s \geq F_d$

$678.58\ m_n^2 \geq \frac{45213.41}{m_n}$

$m_n \geq 4.05mm$.

From PSGDB 8.2, table 1. The nearest higher standard module value under choice 1 is $m_n = 5mm$.

Step 7: Calculation of b , d_1 , and v:

Case 1: To find face width (b).

$b = 10 \times m_n$

$= 10 \times 5$

$= 50mm.$

Case 2: To find Pitch circle diameter (d_1).

$d_1 = \frac{m_n}{\cos\beta} \times Z_1$

$= \frac{5}{\cos 25} \times 20$

$d_1 = 110.34mm$

Case 3: To find Pitch line velocity (v)

$$v = \frac{\pi d_1 N_1}{60}$$

$$= \frac{\pi \times 110.34 \times 1000}{60 \times 1000}$$

$$v = 5.78 \, m/s$$

Step 8: Recalculation of beam strength (F_s)

$F_s = 678.58 \times m_n^2$ From step 5

$$= 678.58 \times 5^2$$

$$F_s = 16964.5N$$

Step 9: Calculation of Accurate dynamic load (F_d)

From PSGDB 8.51

$$F_d = F_t + \frac{21v\left(bc \cdot \cos^2\beta + F_t\right)\cos\beta}{21v + \sqrt{bc \cdot \cos^2\beta + F_t}}$$

Case 1: To find (F_t)

$$F_t = \frac{P}{v}$$

$$= \frac{10 \times 10^3}{5.78}$$

$$F_t = 1730.1N$$

Case 2: To find deformation factor (C)

$C = 11860e$ From PSGDB 8.53 , table 41 , 20° FD.

$e = 0.025$ for module upto 5 and carefully cut gears.

$$\therefore \; C = 296.5 \, N/mm^2$$

$$\therefore \; F_d = 1730.1 + \frac{21 \times 5.78 \times 10^3 \left(50 \times 296.5 \cdot \cos^2 25 + 1730.1\right)\cos 25}{21 \times 5.78 \times 10^3 + \sqrt{50 \times 296.5 \cdot \cos^2 25 + 1730.1}}$$

$$F_d = 1836.98N$$

Step 10: Check for beam strength.

We find $F_s > F_d$, $\therefore$ The design is safe.

Step 11: Calculation of maximum wear load (F_w)

Case 1: To find Ratio factor (Q)

From PSGDB 8.51

$$Q = \frac{2(i)}{i+1} = \frac{2\times5}{5+1} = 1.67 .$$

Case 2: To find Load stress factor (K_w)

$K_w = 2.553\,N/mm^2$. For 20° FD , 400BHN.

Case 3: To find maximum wear load.

From PSGDB 8.51

$$F_w = \frac{d_1 \times b \times Q \times K_w}{\cos^2\beta}$$

$$= \frac{110.34\times50\times1.67\times2.553}{\cos^2 25°}$$

$$F_w = 23521.78N$$

Step 12: Check for wear

We find $F_w > F_d$, $\therefore$ Design is safe.

Step 13: Calculation of basic dimension of pinion and gear.

From PSGDB 8.22 , table 26.

* Normal Module: $m_n = 5mm$
* No. of teeth: $Z_1 = 20$, $Z_2 = 100$
* Pitch circle diameter: $d_1 = 110.34mm$, $d_2 = \frac{m_n}{\cos\beta}\times Z_2$

$$= \frac{5}{\cos 25°}\times100$$

$$= 551.68mm$$

* Centre distance: $a = \frac{m_n}{\cos\beta}\times\left(\frac{Z_1+Z_2}{2}\right)$

$$\frac{5}{\cos 25^\circ} \times \left(\frac{20+100}{2}\right)$$

$$a = 331.01\text{mm}$$

* Face width: $b = 50\text{mm}$
* Height factor: $f_0 = 1$, for 20° FD
* Bottom clearance: $C = 0.25m_n$

$$= 0.25 \times 5$$

$$= 1.25\text{mm}$$

* Tip diameter:

$$d_{a1} = \left(\frac{Z_1}{\cos\beta} + 2f_0\right)m_n \qquad d_{a2} = \left(\frac{Z_2}{\cos\beta} + 2f_0\right)m_n$$

$$= \left(\frac{20}{\cos 25} + 2(1)\right)5 \qquad = \left(\frac{100}{\cos 25} + 2\times1\right)\times5$$

$$d_{a1} = 120.34\text{mm} \qquad d_{a2} = 561.69\text{mm}$$

* Root diameter:

$$d_{f1} = \left(\frac{Z_1}{\cos\beta} - 2f_0\right)m_n - 2C \qquad d_{f2} = \left(\frac{Z_2}{\cos\beta} - 2f_0\right)m_n - 2C$$

$$= \left(\frac{20}{\cos 25} - 2\times1\right)5 - 2\times1.25 \qquad = \left(\frac{100}{\cos 25} - 2\times1\right)5 - 2\times1.25$$

$$d_{f1} = 97.84\text{mm} \qquad d_{f2} = 539.19\text{mm}$$

* Virtual no. of teeth: $Z_{v1} = 27$, $Z_{v2} = 135$

7.A speed reducing unit using spur gear is to be designed. Power to be transmitted is 60hp and is continuous with moderate shaft loads. The speeds of the shafts are 720 rpm and 144 rpm, respectively. The centre distance is kept as small as possible. Select a suitable material and design the gears. Give the details of the gears.

Given data:

$$1\text{hp} = 0.746\text{KW}$$

$$\therefore \quad P = 60\text{hp} = 60 \times 0.746$$

$$= 44.76\text{KW}$$

$P = 45KW$

$N_1 = 720rpm$

$N_2 = 144rpm$

Step 1: To find Gear ratio (i).

$$i = \frac{N_1}{N_2} = \frac{720}{144} = 5$$

Step 2: Selection of material.

Assume, both pinion and gear = Surface hardened carbon steel.

Surface hardness < 350 BAU with 55 RC.

Step 3: Calculation of Z_1 and Z_2:

No. of teeth on pinion $Z_1 = 20$ (Assume).

Gear $Z_2 = i \times Z_1$

$= 5 \times 20$

=100

Step 4: Calculation tangential Load (F_t)

$$F_t = \frac{P}{v} \times K_0$$

Where, $K_0 = 1.5$ Assume medium shock conditions.

$$v = \frac{\pi d_1 N_1}{60}$$

From PSGDB 8.22 , table 26

$d_1 = m \times Z_1$

$$\therefore v = \frac{\pi \times m \times Z_1 \times N_1}{60 \times 1000}$$

$$= \frac{\pi \times m \times 20 \times 720}{60 \times 1000}$$

$v = 0.754m\ m/s$

$$\therefore F_t = \frac{45\times10^3}{0.754m}\times1.5$$

$$F_t = \frac{89522.55}{m}$$

Step 5: Calculation of initial dynamic load (F_d)

$$F_d = \frac{F_t}{C_v}$$

From PSGDB 8.51 , Assume v=12 m/sec.

$$C_v = \frac{6}{6+v} = \frac{6}{6+12} = 0.333$$

$$\therefore\ F_d = \frac{89522.55}{m}\times\frac{1}{0.333}$$

$$F_d = \frac{268836.48}{m}$$

Step 6: Calculation of beam strength (F_s).

$$F_s = \pi\times m\times b\times[\sigma_b]\times y$$ From PSGDB 8.50

Where, $b = 10\times m$ from PSGDB 8.14

$$y = 0.154 - \left(0.912/Z_1\right)$$ From PSGDB 8.50 , for 20° FD

$$= 0.154 - \left(0.912/20\right)$$

$$y = 0.1084.$$

$$\sigma_b = 240\ N/mm^2$$

$$\therefore\ F_s = \pi\times m\times10\times m\times240\times0.1084$$

$$F_s = 817.32\times m^2$$

Step 7: Calculation of module 'm':

$$F_s \geq F_d$$

$$817.32m^2 \geq \frac{268836.48}{m}$$

$$m \geq 6.90 \text{ mm.}$$

From PSGDB 8.2, table 1, choice 1.

The next higher standard module m=8mm

Step 8: Calculation of b , d1 and v.

Facewidth $b = 10 \times m$

$= 10 \times 8$

$= 80\text{mm}$

Pitch circle diameter $d_1 = m \times Z_1$

$= 8 \times 20$

$= 160\text{mm}$

Pitch line velocity $v = 0.754m$ From step 4

$= 0.754 \times 8$

$v = 6.032 \text{ m/s}$

Step 9: Recalculation of beam strength (F_s)

$F_s = 817.32 \times m^2$ From step 6

$= 817.32 \times 8^2$

$F_s = 53308.48 \text{ N}$

Step 10: Calculation of accurate dynamic load (F_d).

$$F_d + F_t + \frac{21v\left(b_c + F_t\right)}{21v + \sqrt{b_c + F_t}}$$ From PSGDB 8.51

Case 1: To find tangential load (F_t)

$$F_t = \frac{P}{v} = \frac{45 \times 10^3}{6.032} = 7460.212\text{N}.$$

Case 2: To find deformation factor (c)

$$c = 11860e$$

From PSGDB 8.53 , table 42

take $e = 0.038$. for precision gears.

$$c = 450.68 N/mm^2$$

$$\therefore \quad F_d = 7460.21 + \frac{21 \times 6.03 \times 10^3 (80 \times 450.68 + 7460.21)}{21 \times 6.03 \times 10^3 + \sqrt{(80 \times 450.68) + 7460.21}}$$

$$F_d = 50903.26 \ N$$

Step 11: Check for beam strength.

We find $F_s > F_d$ $\therefore$ the design is safe.

Step 12: Calculation of maximum wear load (F_w).

From PSGDB 8.51

$$F_w = d_1 \times b \times Q \times K_w$$

Where $Q = \frac{2i}{i+1} = \frac{2 \times 5}{5+1} = 1.67$

$$K_w = 2.553 \ N/mm^2$$

$$\therefore \ F_w = 160 \times 80 \times 1.66 \times 2.553$$

$$F_w = 54246.14N$$

Step 13: Check for wear.

We find $F_w > F_d$ $\therefore$ the design is safe.

Step 14: Calculation of basic dimensions of pinion and gear:

From PSGDB 8.22 , table 26

* Module: $m = 8mm$
* No. of teeth: $Z_1 = 20$, $Z_2 = 100$
* Pitch circle diameter: $d_1 = 160mm.$

  $d_2 = m \times Z_2 = 8 \times 100 = 800mm.$

* Centre distance: $a = \frac{m(Z_1 + Z_2)}{2}$

  $= \frac{8(20+100)}{2}$

  $= 480mm.$

- Face width: $b = 80mm.$
- Height factor: $f_0 = 1$, for $20°$ full depth.
- Bottom clearance: $c = 0.25m = 0.25 \times 8 = 2mm.$
- Tip diameter:

$$d_{a1} = (Z_1 + 2f_0)m \qquad d_{a2} = (Z_2 + 2f_0)m$$

$$= (20 + 2 \times 1)8 \qquad = (100 + (2 \times 1))8$$

$$= 176mm \qquad = 816mm$$

- Root diameter:

$$d_{f1} = (Z_1 + 2f_0)m - 2c \qquad d_{f2} = (Z_2 + 2f_0)m - 2c$$

$$= (20 - 2 \times 1)8 - 2 \times 2 \qquad = (100 - 2 \times 1)8 - 2 \times 2$$

$$= 140mm \qquad = 780mm$$

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12 (b) A pair of helical gears subjected to moderate shock loading is to transmit 30 kW at 1500 rpm of the pinion. The speed reduction ratio is 4 and the helix angle is 20°. The service is continuous and the teeth are 20° FD in the normal plane. For gear life of 10,000 hours, design the gear drive.

Given data:

$P = 30\ kW$

$N_1 = 1500\ rpm$

$i = 4$

$\beta = 20°$

$\phi = 20°$ FD

Gear life = 10,000 hrs.

Step 1:- Gear ratio:-

$i = 4$ (Given).

Step 2:- Selection of Material:-

For both pinion and gear, alloy steel 40Ni2Cr1Mo28 can be selected.

Step 3:- Gear life in cycles:-

Gear life = 10000 hours.

The gear life in terms of number of cycles

$N = 10000 \times 1500 \times 60$

$= 9 \times 10^8$ cycles.

Step 4:- Calculation of initial design torque $[M_t]$:-

$[M_t] = M_t \times K \times K_d$. - From PSGDB 8.15

Where, $M_t = \frac{60P}{2\pi N} = \frac{60 \times 30 \times 10^3}{2 \times \pi \times 1500} = 190.98\ Nm$.

Initially assume $K \cdot K_d = 1.3$. - From PSGDB 8.15

$\therefore [M_t] = 190.98 \times 1.3$

$= 248.28$ N m.

Step 5: Calculation of E_{eq}, $[\sigma_b]$ and $[\sigma_c]$.

Case 1: To find Equivalent Young's modulus. (E_{eq}):

$E_{eq} = 2.15 \times 10^5$ N/mm². - From PSGDB 8.14.

Case 2: To find $[\sigma_b]$.

From PSGDB 8.18.

$$[\sigma_b] = \frac{1.4 \times K_{bl}}{n \cdot K_\sigma} \times \sigma_{-1}$$

* $K_{bl} = 0.7$, for HB > 350, and $N \geq 25 \times 10^7$ - From PSGDB 8.20
* $K_\sigma = 1.5$, for steel hardened - From PSGDB 8.19
* $n = 2.5$, for steel hardened - From PSGDB 8.19
* $\sigma_{-1} = 0.35\,\sigma_u + 120$, - From PSGDB 8.16.

$= 0.35 \times 1550 + 120$ $[\because \sigma_u = 1550$ N/mm²$]$.

$= 662.5$ N/mm².

$$\therefore [\sigma_b] = \frac{1.4 \times 0.7}{2.5 \times 0.7} \times 662.5$$

$= 371$ N/mm².

Case 3: To find design contact stress $[\sigma_c]$.

$[\sigma_c] = C_R \times HRC \times K_{cl}$ - From PSGDB 8.16

$C_R = 26.5$

$HRC = 40$ to 55 } From PSGDB 8.16

$K_{cl} = 0.585$

$[\sigma_c] = 26.5 \times 55 \times 0.585$

$= 852.64$ N/mm²

Step 6:- Calculation of centre distance (a) May/June 2016 (15)

$$a \geq (i+1)\sqrt[3]{\left(\frac{0.74}{[\sigma_c]}\right)^2 \times \frac{E_{eq}[M_t]}{i \times \psi}}$$ - From PSGDB 8.13.

$\psi = b/a = 0.3$ - From PSGDB 8.14.

$$a \geq (4+1)\sqrt[3]{\left(\frac{0.74}{852.64}\right)^2 \times \frac{2.15\times10^5 \times 216.28\times10^3}{4 \times 0.3}}$$

$a \geq 161.19$ mm.

$a = 162$ mm.

Step 7:- Selection of number of teeth on pinion and Gear.

$Z_1 = 20$

$Z_2 = i \times z_1$

$= 4 \times 20$

$= 80$.

Step 8:- Calculation of normal module (m_n).

$$m_n = \frac{2a}{Z_1 + Z_2} \times \cos\beta$$

$$= \frac{2 \times 162}{(20+80)} \times \cos 20^\circ$$

$m_n = 2.51$ mm

From PSGDB 8.2, Choice 1, table 1.

The nearest higher standard module is 3 mm.

Step 9:- Revision of centre distance.

From PSGDB 8.22,

$$a = \left(\frac{m_n}{\cos\beta}\right) \times \left(\frac{Z_1 + Z_2}{2}\right)$$ -

$$= \frac{3}{\cos 20^\circ} \times \frac{20+80}{2}$$

$= 159.63$ mm.

Step 10: Calculation of b, d_1, v and ψ_p:

(i) Face width (b) = $\psi \times a = 0.3 \times 159.63 = 47.88$ mm ≃ 48 mm.

(ii) Axial pitch $p_a = \frac{\pi \times m_n}{\sin\beta} = \frac{\pi \times 3}{\sin 20} = 27.55$ mm. - 8.51.

(iii) pitch diameter of pinion. (d_1): $d_1 = \frac{m_n}{\cos\beta} \times Z_1$ - From PSGDB 8.22

$$= \frac{3}{\cos 20} \times 20$$

$$= 63.85 \text{ mm.}$$

(iv) pitch line velocity (v) = $\frac{\pi d_1 N_1}{60}$

$$= \frac{\pi \times 63.85 \times 10^{-3} \times 1500}{60}$$

$$V = 5.01 \text{ m/s}$$

(v) . $\psi_p = \frac{b}{d_1} = \frac{48}{63.85} = 0.75$

Step 11: Selection of quality of gear.

From PSGDB 8.3, table 2,

For velocity 5.01 m/s, I S quality 8 is selected.

Step 12: Revision of design torque [M_t].

$$[M_t] = M_t \times K \times K_d$$

$K = 1.06$ - From PSGDB 8.15, table 14.

$K_d = 1.2$ - From PSGDB 8.16

$$\therefore [M_t] = 190.98 \times 1.06 \times 1.2$$

$$= 242.93 \text{ Nm.}$$

Step 13:- Check for bending. May/June 2016 (14)

From PSGDB 8.13 [A].

$$\sigma_b = \frac{0.7(i+1)[M_t]}{a \cdot b \cdot m_n \cdot y_v}$$

From PSGDB 8.18, Table 18

$$Z_{v1} = \frac{Z_1}{\cos^3\beta} = \frac{20}{\cos^3 20} = 24.10 \simeq 25 \text{ mm.}$$

$$y_v = 0.427$$

$$\therefore \sigma_b = \frac{0.7(4+1)[242.93 \times 10^3]}{159.63 \times 48 \times 3 \times 0.427}$$

$$\sigma_b = 86.63 \text{ N/mm}^2$$

We find $\sigma_b < [\sigma_b]$. Thus the design is safe.

Step 14:- check for Wear strength.

$$\sigma_c = 0.7 \frac{i+1}{a} \sqrt{\frac{i+1}{ib} \times E_{eq}[M_t]}$$

$$= 0.7 \times \frac{4+1}{159.63} \times \sqrt{\frac{4+1}{4 \times 48} \times 2.15 \times 10^5 \times 242.93 \times 10^3}$$

$$= 808.63 \text{ N/mm}^2.$$

$\sigma_c < [\sigma_c]$. $\therefore$ The design is safe.

Step 15:- Calculation of basic dimensions of pinion and gear.

From PSGDB 8.22

* Normal module:- $m_n = 3$ mm
* Number of teeth:- $Z_1 = 20$, $Z_2 = 80$.
* Pitch circle diameter:-

$$d_1 = 63.85 \text{ mm.}$$

$$d_2 = \frac{m_n}{\cos\beta} \times Z_2 = \frac{3}{\cos 20} \times 80 = 255.4 \text{ mm.}$$

* Centre distance:- $a = 159.63$.
* Height factor:- $f_0 = 1$

* Bottom clearance : $c = 0.25\, m_n = 0.25 \times 3 = 0.75$ mm.

* Tooth depth : $h = 2.25\, m_n = 2.25 \times 3 = 6.75$ mm.

* Tip diameter : $d_{a1} = \left(\frac{z_1}{\cos\beta} + 2f_o\right) m_n$.

$$= \left(\left(\frac{20}{\cos 20}\right) + 2\times1\right) \times 3$$

$$d_{a1} = 69.85 \text{ mm}$$

$$d_{a2} = \left(\frac{z_2}{\cos\beta} + 2f_o\right) m_n$$

$$= \left(\frac{80}{\cos 20} + 2\times1\right) 3$$

$$d_{a2} = 261.4 \text{ mm}$$

* Root diameter :-

$$d_{f1} = \left(\frac{z_1}{\cos\beta} - 2\times f_o\right) m_n - 2c$$

$$= \left(\frac{20}{\cos 20} - 2\times1\right) 3 - 2\times0.75$$

$$d_{f1} = 56.35 \text{ mm}$$

$$d_{f2} = \left(\frac{z_2}{\cos\beta} - 2 f_o\right) m_n - 2c$$

$$= \left(\frac{80}{\cos 20} - 2\times1\right) 3 - 2\times0.75$$

$$d_{f2} = 247.90 \text{ mm.}$$

* Virtual number of teeth :

$$Z_{v1} = 26 \text{ mm.}$$

$$Z_{v2} = \frac{Z_2}{\cos^3\beta}$$

$$= \frac{80}{\cos^3 20}$$

$$= 90.59 \text{ mm.}$$

9.Design a pair of strength spur gear drive for a stone crusher, the pinion and wheel are made of C15 steel and cast iron grade 30b respectively. The pinion is to transmit 22.5 KW power at 900 rpm. The gear ratio is 2.5. Take pressure angle of 20° and working life of gears as 10, 000 hours.

Given data:

$$P = 22.5KW;\ N_1 = 900\,r.p.m;\ i = 2.5;\ \phi = 20°;\ N = 10000\,hrs$$

To find: Design a spur gear

Solution: Since the materials for pinion and wheel are different, therefore we have design the pinion first and check both pinion and wheel.

1. Gear ratio: i = 2.5

2. Material selection:

Pinion: C15 steel, case hardened to 55 RC and core hardness < 350, and

Wheel: C.I grade 30.

3. Gear life: N = 10000 hrs

Gear life in terms of number of cycles is given by

$$N = 10000 \times 60 \times 900 = 54 \times 10^2 \text{cycles}$$

4. Design torque [Mt]:

$$[M_t] = M_t.K.K_d$$

$$M_t = \frac{60 \times P}{2\pi N_1} = \frac{60 \times 22.5 \times 10^3}{2\pi \times 900} = 238.73N - m$$

$$K.K_d = 1.3$$

Design torque $[M_t] = 238.73 \times 1.3 = 310.35N - m$

5. Calculation of Eeq, $|\sigma_b|$ and $|\sigma_1|$:

To find Eeq: For pinion steel and cast iron (> 280 N/mm^2), equivalent Young's modulus,

$$E_{eq} = 1.7 \times 10^5 N/mm^2$$

To find $|\sigma_b|$: The design bending stress $[\sigma_p]$ is given by

$$[\sigma_b] = \frac{1.4 \times K_{b1}}{n.K_\sigma} \times \sigma_{-1},$$ assuming rotation in one direction only.

For steel (HB ≤ 350) and N≥ 10^7, K_{b1} = 1.

For steel case hardened, factor of safety n = 2

For steel case hardened, stress concentration factor, $K_\sigma = 1.2$

For forged steel, $\sigma_{-1} = 0.25(\sigma_u + \sigma_y) + 50.$

For C15, $\sigma_u = 490 N/mm^2$ and $\sigma_y = 240 N/mm^2$

$$\sigma_{-1} = 0.25(490+240)+50 = 232.5 N/mm^2$$
$$[\sigma_b] = \frac{1.4\times1}{2\times1.2}\times232.5 = 135.625 N/mm^2$$

(iii) To find $|\sigma_c|$: The design contact stress $|\sigma_c|$ is given by

$$[\sigma_c] = C_R.HRC.K_{ct}$$

Where,

C_R =22, for C 15 steel

HRC = 55 to 63, for C 15 steel

K_{cl} = 0.585, for HB > 350, n ≥ 25 × 10^7

$$[\sigma_c] = 22\times63\times0.585 = 810.81 N/mm^2$$

6. Calculation of centre distance (a):

We know that,

$$a \ge (i+1)\sqrt[3]{\left(\frac{0.74}{[\sigma_c]}\right)^2\times\frac{E_{eq}[M_t]}{i\Psi}}$$
$$\Psi = \frac{b}{a} = 0.3$$
$$a \ge (2.5+1)\sqrt[3]{\left(\frac{0.74}{810.81}\right)^2\times\frac{1.7\times10^3\times310.35\times10^3}{2.5\times0.3}}$$
$$\ge 135.94 mm \text{ or } a = 136 mm$$

7. To find z_1 and z_2:

(i) For 20° full depth system, select z_1 = 18.

(ii) $z_2 = i\times z_1 = 2.5\times18 = 45$

8. Calculation of module (m):

We know that,

$$m = \frac{2a}{z_1+z_2} = \frac{2\times136}{18+45} = 4.32 \text{ mm}$$

The nearest higher standard module, m = 5 mm

9. Revision of centre distance:

New centre distance, $a = \frac{m(z_1+z_2)}{2} = \frac{5(18+45)}{2} = 157.5 mm$

10. Calculation of b, d_p, v and Ψ_p :

Face width (b): $b = \Psi.a = 0.3 \times 157.5 = 47.25 \text{ mm}$

Pitch diameter of pinion (d_1): $d_1 = m.z_1 = 5 \times 18 = 90 \text{mm}$

Pitch line velocity (v): $v = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 90 \times 10^{-3} \times 900}{60} = 4.24 \text{m/s}$

$$\psi_p = \frac{b}{d_1} = \frac{47.25}{90} = 0.525$$

11. Selection of quality of gear:

For *v* = 4.24 m/s, IS quality 8 gears are selected.

12. Revision of design torque $[M_t]$:

Revise K: For $\psi_p = 0.525, K = 1.03$

Revise K_d: for IS quality 8 and *v* = 4.24 m/s, K_d = 1.4,

Revise $[M_t]:[M_t] = M_t.K.K_d \quad = 238.73 \times 1.03 \times 1.4 = 344.24 \text{N} - \text{m}$

13. Check for bending:

Calculation of induced bending stress, σ_p :

Where,

$$\sigma_p = \frac{(i+1)}{a.m.b.y}[M_t]$$

$y =$ Form factor = 0.377, for $z_1 = 18$

$$\sigma_p = \frac{(2.5+1) \times 344.24 \times 10^3}{157.5 \times 5 \times 47.25 \times 0.377} = 58.89 \text{N/mm}^2$$

We find $\sigma_b < [\sigma_B]$. Therefore the design is satisfactory.

14. Check for wear strength:

Calculation of induced contact stress, σ_c

$$\sigma_c = 0.74 \frac{i+1}{a} \sqrt{\frac{i+1}{ib} \times E_{eq} [M_t]}$$

$$= 0.74 \left(\frac{2.5+1}{157.5}\right) \sqrt{\left(\frac{2.5+1}{2.5 \times 47.25}\right) \times 1.7 \times 10^5 \times 344.24 \times 10^3}$$

$$= 684.76 \text{N/mm}^2$$

We find $\sigma_c < |\sigma_c|$. Therefore the design is safe and satisfactory.

15. Check of wheel:

(i) Calculation of $|\sigma_b|_{wheel}$ and $|\sigma_c|_{wheel}$:

Wheel material: CI grade 30.

Wheel speed: $N_2 = \frac{N_1}{i} = \frac{900}{2.5} = 360 \text{r.p.m}$

Life of wheel $= 10{,}000 \times 60 \times 360 = 21.6 \times 10^7$ cycles

To find $|\sigma_b|_{wheel}$ **:** The design bending stress for wheel is given by

$$[\sigma_b]_{wheel} = \frac{1.4 \times K_{b1}}{n.K_a} \times \sigma_{-1}$$, assuming rotation in one direction only.

For cast iron wheel, $K_{b1} = \sqrt[9]{\frac{10^7}{N}} = \sqrt[9]{\frac{10^7}{21.6 \times 10^7}} = 0.918$

For cast iron, n = 2.

For cast iron, $\sigma_{-1} = 0.45\sigma_u$

For cast iron, $\sigma_u = 290 N/mm^2$

$$\sigma_{-1} = 0.45 \times 290 = 130.5 N/mm^2$$
$$[\sigma_b]_{wheel} = \frac{1.4 \times 0.918}{2 \times 1.2} \times 130.5 = 69.88 N/mm^2$$

To find $|\sigma_c|_{wheel}$: The wheel design contact stress for wheel is given by

$$|\sigma_c|_{wheel} = C_B.HB.K_{cl}$$

Where,

C_B = 2.3, for cast iron grade 30

HB = 200 to 260, for cast iron

$$K_{cl} = \sqrt[6]{\frac{10^7}{N}} = \sqrt[6]{\frac{10^7}{21.6 \times 10^7}} = 0.879, \text{for cast iron}$$
$$[\sigma_c]_{wheel} = 2.3 \times 260 \times 0.879 = 525.64 N/mm^2$$

(ii) Check for bending:

Calculation of induced bending stress for wheel σ_{b2}

$$\sigma_{b1} \times y_1 = \sigma_{b2} \times y_2$$

Where σ_{b1} and σ_{b2} = Induced bending stress in the pinion and wheel respectively, and

y_1 and y_2 = Form factors for pinion and wheel respectively.

y_2 = 0.471, for z_2 = 45.

$$\sigma_{b1} = 85.89 N/mm^2 \text{ and } y_1 = 0.377$$
$$85.89 \times 0.377 = \sigma_{b2} \times 0.471$$
$$\sigma_{b2} = 68.75 N/mm^2$$

We find $\sigma_{b2} < [\sigma_b]_{wheel}$. Therefore the design is satisfactory.

(iii) Check for wear strength: Since contact area is same, therefore $\sigma_{c,wheel} = \sigma_{c\,pinion} = 684.76 N/mm^2$. Here $\sigma_{c\ wheel} > [\sigma_c]_{wheel}$. It means, wheel does not have the required wear resistance. So, in order to decrease the

induced contact stress, increase the face width (b)value or in order to increase the design contact stress, increase the surface hardness, say to 340 HB. Increasing the surface hardness will give
$[\sigma_c] = 2.3 \times 340 \times 0.879 = 687.34\,N/mm^2$. Now we find $\sigma_c < [\sigma_c]$. So the design is safe and satisfactory.

16. Calculation of basic dimensions of pinion and wheel:

Module: m = 5mm
Face width: b = 47.25 mm
Height factor: $f_0 = 1$ for full depth teeth.
Bottom clearance: c = 0.25m = 0.25 × 5 = **1.25 mm**
Tooth depth: h = 2.25 m=2.25 × **5 = 11.25 mm**
Pitch circle diameter: $d_1 = m.z_1 = 5 \times 18 = 90$ mm and $d_2 = m.z_2 = 5 \times 45 = 225$ mm
Tip diameter:
$d_{a1} = (z_1 + 2f_0)m = (18 + 2 \times 1)5 = 100$ mm; and
$d_{a2} = (z_2 + 2f_0)m = (45 + 2 \times 1)5 = 235\,mm$
Root diameter:
$df_1 = (z_1 - 2f_0)m - 2c$
$= (18 - 2 \times 1)5 - 2 \times 1.25 = 77.5$ mm; and
$df_2 = (z_2 - 2f_0)m - 2c$
$= (45 - 2 \times 1)5 - 2 \times 1.25 = 212.5$ mm

10.**A hardened steel worm rotates at 1440 rpm and transmits 12 kW to a phosphor Bronze gear. The speed of the worm gear should be 60 rpm. Design the worm gear drive if an efficiency of at least 82% is desired.**

Given data:

$N_{max} = 1440$ rpm, $N_{min} = 60$ rpm $p = 12kW$ $\eta_{desired} = 82\%$

Gear ratio required, $i = \frac{1440}{60} = 24$

1. Material selection: Worm – Hardened steel, and

Worm – Phosphor bronze

2. Selection of z_1 and z_2:

For η = 85%, z_1 = 3
Then, $z_2 = i \times z_1 = 24 \times 3 = 72$.

3. Calculation of q and γ:

Diameter factor: $q = \frac{d_1}{m_x} = 11$

Lead angle: $\gamma = \tan^{-1}\left(\frac{z_1}{q}\right) = \tan^{-1}\left(\frac{3}{11}\right) = 15.25°$

4. Calculation of F_1 in terms m_x:

Tangential load, $F_t = \frac{P}{v} \times K_0$

Where

$$v = \frac{\pi d_2 N_2}{60 \times 1000} = \frac{\pi (z_2 \times m_x) \times N_2}{60 \times 1000}$$

$$= \frac{\pi \times 72 \times m_x \times 60}{60 \times 1000} = 0.226 m_x \text{ m/s}$$

$K_0 = 1.25$, assuming medium shock

$$F_t = \frac{12 \times 10^3}{0.226 m_x} \times 1.25 = \frac{66371.68}{m_x}$$

5. Calculation of dynamic load (F_d):

Dynamic load, $F_d = \frac{F_t}{c_v}$

$$c_v = \frac{6}{6+v}, \; v = 5 \text{ m/s is assumed.}$$

$$= \frac{6}{6+5} = 0.545$$

$$F_d = \frac{66371.68}{m_x} \times \frac{1}{0.545} = \frac{121681.4}{m_x}$$

6. Calculation of beam strength (F_s) in terms of axial module:

Beam strength, $F_s = \pi \times m_x \times b \times [\sigma_b] \times y$

Where

$$b = 0.75 d_1$$

$$= 0.75 \times q m_x = 0.75 \times 11 m_x = 8.25 m_x$$

$$[\sigma_b] = 80 \text{ N/mm}^2$$

$y = 0.125$, assuming $\alpha = 20°$

$$F_x = \pi \times m_x \times 8.25 m_x \times 80 \times 0.125 = 259.18 m_x^2$$

7. Calculation of axial module (m_x):

We know that,

$$259.18 m_x^2 \geq \frac{121681.4}{m_x}$$

$$m_x \geq 7.77 \text{ mm}$$

The nearest higher standard axial pitch is 8 mm.

8. b = 66 mm, d_2 = 576 mm; v = 1.808 m/s

9. Fs = 259.18m_x^2 = 16587. 52 N

10. Dynamic load, $F_d = \frac{F_t}{c_v}$

$$c_v = \frac{6}{6+v} = \frac{6}{6+1.808} = 0.768 \text{ and}$$

$$F_t = \frac{66371.68}{m_x} = \frac{66371.68}{8} = 8296.46N$$

$$F_d = \frac{8296.46}{0.768} = 10802.68N$$

11. Check for beam strength: We find $F_d < F_s$. It means that the gear tooth has adequate beam strength and will not fail by breakage. Thus the design is satisfactory.

12. Calculation of maximum wear load (F_w):

Wear load, $F_w = d_2 \times b \times K_w$

Where

$$K_w = 0.56N/mm^2$$

$$F_w = 576 \times 66 \times 0.56 = 21288.96N$$

13. Check for wear: We find $F_d < F_w$. It means that the gear tooth has adequate wear capacity and will not wear out. Thus the design is safe and satisfactory.

14. Check for efficiency: We know that,

$$\eta_{actual} = 0.95\frac{\tan\gamma}{\tan(\gamma+\rho)}$$

Where

$$\rho = \text{Frictional angle} = \tan^{-1}\mu$$

$$= \tan^{-1}(0.03) = 1.7°$$

$$\eta = 0.95 \times \frac{\tan 15.25°}{\tan(15.25° + 1.7°)} = 0.8498 \text{ or } 84.98\%$$

We find that the actual efficiency is greater than the desired efficiency. Thus the design is satisfactory.

15. Calculation of basic dimensions of worm and worm gears:

Axial module: $m_x = 8$ mm
Number of starts: $z_1 = 3$
Number of teeth on worm wheel: $z_2 = 72$
Face width of worm wheel: b = 66 mm
Length of worm: $L \geq (12.5 + 0.09z_2)m_x$, $L \geq (12.5 + 0.09 \times 72)8 = 151.84mm$
Centre distance: $a = 0.5m_x(q + z_2) = 0.5 \times 8(11 + 72) = 332mm$
Height factor: $f_0 = 1$
Bottom clearance: $c = 0.25m_x = 0.25 \times 8 = 2mm$

Pitch diameter: $d_1 = q \times m_x = 11 \times 8 = 88mm$
$d_2 = z_2 \times m_x = 72 \times 8 = 576mm$

Tip diameter: $d_{a1} = d_1 + 2f_0.m_x = 88 + 2 \times 1 \times 8 = 104$ mm
$d_{a2} = (z_2 + 2f_0)m_x = (72 + 2 \times 1)8 = 592$ mm

Root diameter: $d_{f1} = d_1 - 2f_0.m_x - 2.c = 88 - 2 \times 1 \times 8 - 2 \times 2 = 68mm$
$d_{f2} = (z_2 - 2f_0)m_x - 2.c = (72 - 2 \times 1)8 - 2 \times 2 = 556$ mm

11. Design a spur gear drive required to transmit 45 KW at a pinion speed of 800 rpm. The velocity ratio is 3.5:1. The teeth are 20° involute with 18 teeth on the pinion. Both the pinion and gear are made of steel with a maximum safe static stress of 180 N/mm². Assume medium shock conditions (April/May 2017)

Given data:

$P = 45KW$

$N_1 = 800 rpm$

$i = 3.5$

$\phi = 20°$

$Z_1 = 18$

$[\sigma_b] = 180 N/mm^2$

Material = steel (for both pinion and gear)

Step 1: Selection of Material

Pinion and Gear = Steel

Assume steel is hardened to 200 BHN (BRINELL HARDNESS NUMBER) from PSGDB 8.16 table 16

Step 2: Calculation of Z_1 and Z_2

Number of Teeth on Pinion $Z_1 = 18$

Number of Teeth on Gear $Z_2 = i \times Z_1$

$= 3.5 \times 18$

$Z_2 = 63$

Step 3: Calculation of Tangential load (F_t)

Case 1: To find the pitch line velocity (v)

$$v = \frac{\pi d_1 N_1}{60}$$

$$v = \frac{\pi m Z_1 N_1}{60}$$

$$= \frac{\pi \times m \times 18 \times 800}{60 \times 1000}$$

$$= 0.754 m\, m/sec$$

Case 2: To find K_0

$K_0 = 1.5$ for medium shock conditions

Case 3: To find F_t

> $F_t = \frac{P}{v} \times K_0$
>
> $P = 45KW$
>
> $K_0 = 1.5$
>
> $v = \frac{\pi d_1 N_1}{60}$
>
> $d_1 = m \times Z_1$
>
> From PSGDB 8.22

$$F_t = \frac{P}{v} \times K_0$$

$$F_t = \frac{45 \times 10^3}{0.754m} \times 1.5$$

$$= 89522.5/m$$

Step 4: Calculation of Initial Dynamic Load (F_d)

Case 1: To find velocity factor (C_v)

$C_v = \frac{6}{6+v}$ for accurately hobbed and generated gears

With $v < 20\,m/sec$

$$C_v = \frac{6}{6+12}$$

Case 2: To find initial dynamic load (F_d)

$$F_d = \frac{89522.5}{m} \times \frac{1}{0.333}$$

$$F_d = \frac{268836.3}{m}$$

> $F_d = \frac{F_t}{C_v}$
>
> $C_v = \frac{6}{6+v}$
>
> From PSGDB 8.51 Assume $v = 12\,m/sec$

Step 5: Calculation of Beam Strength (F_s)

> From PSGDB 8.50
>
> $$F_s = [\sigma_b]\, b\, y\, P_c$$
>
> Where
>
> $P_c = \text{circular pitch} = \frac{\pi d}{z} = \pi m$
>
> $m = d/z$
>
> Finally we write
>
> $F_s = [\sigma_b]\, b\, y\, \pi\, m$
>
> Where

Case 1: To find form factor (y):

> b = Face width $10 \times m$
>
> y = Form Factor
>
> $= 0.154 - (0.912/Z_1)$ for 20°
>
> Full depth system

$$y = 0.154 - (0.912/Z_1)$$

$$= 0.154 - (0.912/18)$$

$$= 0.1033$$

Case 2: To find the beam strength (F_s)

Lewis equation,

$$F_s = [\sigma_b] b\ y\ \pi\ m$$

$$= 180 \times 10m \times 0.1033\ \pi\ m$$

$$= 584.15m^2$$

Step 6: Calculation of Module (m):

From PSGDB 8.51

$$F_s \geq F_d$$

$$584.15m^2 \geq \frac{268836.3}{m}$$

$$m \geq 7.72mm$$

From PSGDB 8.2 table 1, the nearest higher standard module value under choice 1 is 8 mm

Step 7: Calculation of b , d and v

> Case 1: To find the face width (b)
>
> $$b = 10 \times m$$
>
> $$= 10 \times 8$$
>
> $$= 80mm$$
>
> Case 2: To find pitch circle diameter (d_1)

$$d_1 = m \times Z_1$$
$$= 8 \times 18$$
$$= 144 mm$$

Case 3: To find Pitch line velocity (v)

$$v = \frac{\pi d_1 N_1}{60}$$
$$= \frac{\pi \times 144 \times 10^{-3} \times 800}{60}$$
$$= 6.03\, m/sec$$

Step 8: Recalculation of Beam Strength

Beam Strength $F_s = [\sigma_b] b\, y\, \pi\, m$

$$= 180 \times 80 \times 0.1033 \times \pi \times 8$$
$$= 37385.45\, N$$

Step 9: Calculation of accurate dynamic load (F_d)

STEP 9: CALCULATION OF ACCURATE DYNAMIC LOAD (F_d)

Case 1: To find tangential load (F_t)

$$F_t = \frac{P}{v}$$
$$F_t = \frac{45x103}{6.03}$$
$= 7462.68$ N

Case 2: To find Deformation factor (C)

C=11860 e

=11860x0.038

= 450.68 N/mm²

Case 3: To find the accurate dynamic load (F_d)

$$F_d = F_t + \frac{21\, v\,(bc + F_t)}{21\, v + \sqrt{bc + F_t}}$$

$$F_d = 7462.68 + \frac{21x6.03x103\ (80x450.68 + 7462.68)}{21x6.03x103 + \sqrt{80x450.68 + 7462.68}}$$

=50908.19 N

$F_d = F_t + \frac{21\, v\,(bc+F_t)}{21\, v + \sqrt{bc+F_t}}$ from PSGDB 8.51

⬆

We know that $F_t = \frac{P}{v}$ for accurate value eliminate K_0

⬆

C= Deformation factor from PSGDB 8.53 , table 41

⬆

C=11860 e, for 20° FD, steel and steel

⬆

e = 0.038 , for module upto 8 and carefully cut gears from PSGDB 8.53 , table 42

Step 10: Check for Beam strength or Tooth breakage

Since $F_d > F_s$ $(50908.19N > 37385.45N)$ the design is unsatisfactory. The dynamic load is greater than the beam strength

In order to reduce the dynamic load F_d, Select the precision gears. Therefore from PSGDB 8.53 , table 42 take $e = 0.019$ for precision gears

Recalculation of deformation factor:

$$C = 11860 \times 0.019 = 225.34$$

Recalculation of dynamic load:

$$F_d = 7462.68 + \frac{21\times 6.03\times 10^3 \left(80\times 225.34 + 7462.68\right)}{21\times 6.03\times 10^3 + \sqrt{80\times 225.34 + 7462.68}}$$

$$= 32920.46\text{N}$$

Now we find $F_s > F_d \left(37385.45\text{N} > 32920.46\text{N}\right)$. It means the gear tooth has adequate beam strength and it will not fail by breakage. Therefore the design is safe.

Step 11: Calculation of maximum wear load (F_w)

Case 1: To find ratio factor (Q)

$$Q = \frac{2i}{i+1} = \frac{2\times 3.5}{3.5+1} = 1.555$$

From PSGDB 8.51

$F_w = d_1 \times b \times Q \times K_w$

$Q = \text{Ratio factor} = \frac{2i}{i+1}$

$K_w = \text{load stress factor} = 0.919\,\text{N/mm}^2$, for steel hardened to 250 BHN

Case 2: To find maximum wear load (F_w)

$$F_w = d_1 \times b \times Q \times K_w$$

$$= 144\times 80\times 1.555\times 0.919$$

$$= 16462.6\text{N}$$

Step 12: Check for wear

Since $F_d > F_w \left(32920.46\text{N} > 16462.6\text{N}\right)$ the design is unsatisfactory. That is the dynamic load is greater than the wear load.

In order to increase the wear load $\left(F_w\right)$, we have to increase the hardness (BHN). So now for steel hardened to 400BHN, $K_w = 2.553\,\text{N/mm}^2$

$$\therefore \quad F_w = d_1 \times b \times Q \times K_w$$

$$= 144\times 80\times 1.555\times 2.553$$

$$= 45733.42\text{N}$$

Now we find $F_w > F_d \left(45733.42\text{N} > 32920.46\text{N}\right)$. It means the gear tooth is adequate wear capacity and it will not wear out. Therefore the design is satisfactory

Step 13: Basic dimensions of Pinion and gear

From PSGDB 8.22, table 26

Module: $m = 8mm$

Number of teeth: $Z_1 = 18, Z_2 = 63$

Pitch circle diameter: $d_1 = 144mm$

$d_2 = m \times Z_2 = 8 \times 63$

$d_2 = 504mm$

Centre distance: $a = m(Z_1 + Z_2)/2$

$= 8(18 + 63)/2$

$a = 324mm$

Face width: $b = 80mm$

Height factor: $f_0 = 1$, for $20°$ full depth teeth

Bottom clearance: $c = 0.25m = 0.25 \times 8$

$c = 2mm$

Tip diameter: $d_{a1} = (Z_1 + 2f_0)m$ $d_{a2} = (Z_2 + 2f_0)m$

$= (18 + 2 \times 1)8$ $= (63 + 2 \times 1)8$

$= 160mm$ $= 520mm$

Root diameter: $d_{f1} = (Z_1 - 2f_0)m - 2c$ $d_{f2} = (Z_2 - 2f_0)m - 2c$

$= (18 - 2 \times 1)8 - 2 \times 2$ $= (63 - 2 \times 1)8 - 2 \times 2$

$= 124mm$ $= 484mm$

12. A general purpose enclosed gear train is based on parallel helical gears, specified life is 36000 hours. Torque at driven shaft is 411 Nm. Driving shaft speed is 475 rpm. Velocity ratio is 4. it is desired to have standard Centre distance. Design a gear drive. (April/May 2017)

Given Data:

Gear life=36000 hours

M_t=411 Nm

N_1 =475 rpm

i= 4

****Similar to this problem, gear life and materials has to be changed, refer the question paper*

STEP 1: CALCULATION OF GEAR RATIO AND VIRTUAL NUMBER OF TEETH

Gear ratio (i)

i=4 (given)

STEP 2: SELECTION OF MATERIAL

Pinion and Gear = C45 steel

STEP 3: CALCULATION OF GEAR LIFE

Given that the gear is to work 36000 hours

Case 1: To find gear life

Gear life=36000 hours

= 2160000 mins

Case 2: To find life in number of cycles (N)

$N=2160000xN_1$

=2160000x475

$=102.6x10^7$ cycles

STEP 4: CALCULATION OF INITIAL DESIGN TORQUE $[M_t]$

$[M_t]=M_t\ K.\ K_d$

=411x 1.3

$[M_t]$=534.3 Nm

STEP 5: CALCULATION OF E_{eq}, $[\sigma_b]$, AND $[\sigma_c]$

Case 1: To find equivalent young's modulus

From PSGDB 8.14,table 9

For C45 steel, take $E_{eq}=2.15x10^5$ N/mm²

Case 2:

1. **To find Endurance limit stress in bending (σ_{-1})**

σ_{-1} =0.35x670+120

= 354.5 N/mm²

2. **To find design bending stress [σ$_b$]**

$$[\sigma_b] = \frac{1.4 \cdot k_{bl}}{n \cdot k_\sigma} \times \sigma_{-1}$$

$$[\sigma_b] = \frac{1.4x0.7}{2x1.5} x\ 354.5$$

$= 115.80\ N/mm^2$

Case 3: To find design contact stress [σ$_c$]

$$[\sigma_c] = C_R.\ HRC.K_{cl}$$

$= 265x55x0.585$

$=8526.375\ kgf/cm^2$

$= 852.64\ N/mm^2$

STEP 6: CALCULATION OF CENTRE DISTANCE (a)

From PSGDB 8.13, table-8- for designing

$$a \geq (i+1) \cdot \sqrt[3]{\left(\frac{0.7}{[\sigma_c]}\right)^2 \times \frac{E_{eq} \cdot [M_t]}{i \cdot \psi}}$$

- ✓ *Where,ψ=b/a = Width to Centre Distance ratio*
- ✓ *Generally assume ψ = 0.3 for Initial Calculation,* From PSGDB 8.14, table-10

$$a \geq (4+1).\sqrt[3]{\left[\frac{0.7}{852.64}\right]^2 x \frac{2.15x10^5 x534.3x10^3}{4x0.3}}$$

$\geq 200.54mm\ or\ a = 201mm$

STEP 7: SELECTION OF z_1 AND z_2

- Number Of Teeth On Pinion $z_1 = 20$
- Number Of Teeth On Gear $z_2 = i \times z_1$

$= 4 \times 20$

$z_2 = 80$

STEP 8: CALCULATION OF NORMAL MODULE (m_n)

From PSGDB 8.22 table 26

$$m_n = \frac{2a}{z_1+z_2} X cos\beta$$

$$= \frac{2x201}{20+80} X cos15^{\circ} \quad \text{(Assume } \beta = 15°\text{)}$$

$$m_n = 3.88mm$$

- ✓ The nearest higher standard module From PSGDB 8.2 table 1 choice 1, take m_n=4mm

STEP 9: REVISION OF CENTRE DISTANCE (a)

From PSGDB 8.22, table 26

Centre distance $a = \left(\frac{m_n}{cos\beta}\right) x \left(\frac{z_1+z_2}{2}\right)$

$$= \left(\frac{4}{cos15°}\right) x \left(\frac{20+80}{2}\right)$$

a= 207.05mm

STEP 10: CALCULATION OF b , d_1,v and Ψ

From PSGDB 8.14, table-10

Case 1: To find the face width (b)

b= Ψ x a

= 0.3 x 207.05

b= 62.12 mm

Case 2: To find pitch circle diameter (d_1)

From PSGDB 8.22, table-26

$d_1 = \frac{m_n}{cos\beta}$ x z_1

$= \frac{4}{cos15°}$ x 20

d_1 = 82.82mm

Case 3: To find pitch line velocity (v)

$$v = \frac{\pi d_1 N_1}{60}$$

$$= \frac{\pi x 82.82x10^{-3} x 475}{60}$$

v = 2.06 m/sec

Case 4: To find Ψ_p

Ψ_p=b/d_1

=62.12/82.82

Ψ_p=0.75

STEP 11: SELECTION OF QUALITY OF GEAR

From PSGDB 8.3, table-2

For pitch line velocity 2.06 m/sec, IS quality 8 gears are selected

STEP 12: REVISION OF DESIGN TORQUE [M_t]

[M_t]=M_t . K. K_d

=411x 1.06x1.1

[M_t]=479.23Nm

STEP 13: CHECK FOR BENDING

Calculation of induced bending stress (σ_b)

$$\sigma_b = 0.7x\frac{i \pm 1}{a.b.m_n.y_v}x[M_t] \leq [\sigma_b]$$

$$= 0.7x\frac{4+1}{207.05x62.12x4x0.402}x479.23x10^3$$

$$= 81.09N/mm^2$$

We find $\sigma_b < [\sigma b]$ ($81.09N/mm^2 <$ 115.80N/mm2). Therefore the design is safe and satisfactory

STEP 14: CHECK FOR WEAR STRENGTH (σ_c)

Calculation of induced contact stress (σ_c)

From PSGDB 8.13 table 8, For checking

$$\sigma_c = 0.7x\frac{i+1}{a}x\sqrt{\frac{i+1}{ixb}xE_{eq}x[M_t]} \leq [\sigma_c]$$

$$= 0.74x\frac{4+1}{207.05}x\sqrt{\frac{4+1}{4x62.12}x2.15x10^5x479.23x10^3}$$

=769.70N/mm² We find $\sigma_c < [\sigma c]$(769.70 N/mm²

< 852.64N/mm²). Therefore the design is safe and satisfactory

STEP 14: BASIC DIMENSIONS OF PINION AND GEAR

- ✓ From PSGDB 8.22, table 26
- Normal Module:mn=4mm
- Number of teeth: z_1=20, z_2=80
- Virtual Number of teeth:z_{v1}=22 , $z_{v2}=\frac{z_2}{cos^3\beta}=\frac{80}{cos^3 15°} \approx 89$
- Pitch circle diameter: d_1=82.82mm

$$d_2=\frac{m_n}{cos\beta} \text{ x } z_2=\frac{4}{cos15°} \text{ x } 80$$

d_2=331.28mm

- Centre distance : a=207.05 mm
- Face width: b=62.12 mm
- Height factor: f_0=1 , for 20° full depth teeth
- Bottom clearance: c=0.25m_n=0.25x4

c=1mm

- Tip diameter:

$$d_{a1} = \left(\frac{z_1}{cos\beta} + 2f_0\right)m_n$$

$$= \left(\frac{20}{cos15°} + 2x1\right)x4$$

=90.82mm

$$d_{a2} = \left(\frac{z_2}{cos\beta} + 2f_0\right)m_n$$

$$= \left(\frac{80}{cos15°} + 2x1\right)x4$$

=339.28mm

- Root diameter:

$$d_{f1} = \left(\frac{z_1}{cos\beta} - 2f_0\right)mn - 2c \qquad = \left(\frac{20}{cos15°} - 2x1\right)x4 - 2x1$$

=72.82mm

$d_{f2} = \left(\frac{z_2}{cos\beta} - 2f_0\right)mn - 2c$ $\quad = \left(\frac{80}{cos15°} - 2x1\right)x4 - 2x1$

=321.28mm

13. **Design a pair of helical gears to transmit 10 KW at pinion speed of 1000rpm. The Reduction ratio is 5. Assume suitable materials and stresses. (Nov/Dec 2017)**

Given data:

$N_1 = 1000rpm$
$P = 10KW$
$i = 5$

Step 1: Selection of Material

Generally we assume C45 steel for both pinion and gear ***(But in this problem we have to change the material as40Ni2 Cr1 Mo28 steel and*** $[\sigma_b] = 450\,N/mm^2$***)***

$[\sigma_b] = 180\,N/mm^2$, 400 BHN.

Step 2: Calculation of number of teeth Z_1 and Z_2:

No. of teeth on pinion gear $Z_1 = 20$ (assume)

$Z_2 = i \times Z_1$

$= 5 \times 20$

$= 100$.

Virtual no. of teeth Z_{v1} & Z_{v2}

From PSGDB 8.22 , table 26. Assume $\beta = 25°$

$Z_{v1} = \frac{Z_1}{\cos^3\beta}$ $\qquad Z_{v2} = \frac{Z_2}{\cos^3\beta}$

$= \frac{20}{\cos^3 25}$ $\qquad = \frac{100}{\cos^3 25}$

$Z_{v1} = 27$ $\qquad = 134.33mm$.

$Z_{v2} \approx 135mm$

Step 3: Calculation of tangential load on teeth (F_t).

$$F_t = \frac{P}{v} \times K_0$$

$K_0 = 1.5$, for medium shock conditions.

Case 1: To find the pitch line velocity (v)

$$v = \frac{\pi d_1 N_1}{60}$$

From PSGDB 8.22, table 26

$$d_1 = \frac{m_n}{\cos\beta} \times Z_1$$

$$\therefore \quad v = \frac{\pi \times m_n \times 20 \times 1000}{60 \times 1000 \times \cos 25°}$$

$$v = 1.16 m_n \ \text{m/sec}$$

$$\therefore \quad F_t = \frac{10 \times 10^3}{1.16 m_n} \times 1.5$$

$$= \frac{12931.03}{m_n}$$

Step 4: Calculation of initial dynamic load (F_d)

$$F_d = \frac{F_t}{C_v}$$

Case 1: To find the velocity factor (Cv)

$C_v = \frac{6}{6+v}$ for carefully cut gears $v < 20\,\text{m/s}$. From PSGDB 8.51 Assume $v = 15\,\text{m/s}$.

$$= \frac{6}{6+15}$$

$$C_v = 0.286.$$

$$\therefore \quad F_d = \frac{12931.03}{m_n} \times \frac{1}{0.286}$$

$$= \frac{45213.41}{m_n}$$

Step 5: Calculation of beam strength (F_s).

$$F_s = [\sigma_b] \times b \times y^1 \times \pi \times m_n$$

Where,

$b = 10 \times m_n$ From PSGDB 8.14

$y^1 = 0.154 - \left(0.912 / Z_{v1}\right)$ From PSGDB 8.50 , 20° FD

$= 0.154 - \dfrac{0.912}{27}$

$= 0.12$

$\therefore \; F_s = 180 \times 10 \times m_n \times 0.12 \times \pi \times m_n$

$F_s = 678.58 \; m_n^2$

Step 6: Calculation of normal module (m_n)

From PSGDB 8.51

$F_s \geq F_d$

$678.58 \; m_n^2 \geq \dfrac{45213.41}{m_n}$

$m_n \geq 4.05\text{mm}$.

From PSGDB 8.2, table 1. The nearest higher standard module value under choice 1 is $m_n = 5\text{mm}$.

Step 7: Calculation of b , d_1 , and v:

Case 1: To find face width (b).

$b = 10 \times m_n$

$= 10 \times 5$

$= 50\text{mm}.$

Case 2: To find Pitch circle diameter (d_1).

$$d_1 = \frac{m_n}{\cos\beta} \times Z_1$$

$$= \frac{5}{\cos 25} \times 20$$

$$d_1 = 110.34 mm$$

Case 3: To find Pitch line velocity (v)

$$v = \frac{\pi d_1 N_1}{60}$$

$$= \frac{\pi \times 110.34 \times 1000}{60 \times 1000}$$

$$v = 5.78 m/s$$

Step 8: Recalculation of beam strength (F_s)

$F_s = 678.58 \times m_n^2$ From step 5

$$= 678.58 \times 5^2$$

$$F_s = 16964.5 N$$

Step 9: Calculation of Accurate dynamic load (F_d)

From PSGDB 8.51

$$F_d = F_t + \frac{21v\left(bc \cdot \cos^2 \beta + F_t\right)\cos\beta}{21v + \sqrt{bc \cdot \cos^2 \beta + F_t}}$$

Case 1: To find (F_t)

$$F_t = \frac{P}{v}$$

$$= \frac{10 \times 10^3}{5.78}$$

$$F_t = 1730.1 N$$

Case 2: To find deformation factor (C)

$C = 11860e$ From PSGDB 8.53 , table 41 , 20° FD.

$e = 0.025$ for module upto 5 and carefully cut gears.

$\therefore C = 296.5 N/mm^2$

$$\therefore\ F_d = 1730.1 + \frac{21\times 5.78\times 10^3\left(50\times 296.5\cdot \cos^2 25 + 1730.1\right)\cos 25}{21\times 5.78\times 10^3 + \sqrt{50\times 296.5\cdot \cos^2 25 + 1730.1}}$$

$$F_d = 1836.98\text{N}$$

Step 10: Check for beam strength.

We find $F_s > F_d$, $\therefore$ The design is safe.

Step 11: Calculation of maximum wear load (F_w)

Case 1: To find Ratio factor (Q)

From PSGDB 8.51

$$Q = \frac{2(i)}{i+1} = \frac{2\times 5}{5+1} = 1.67\ .$$

Case 2: To find Load stress factor (K_w)

$K_w = 2.553\,\text{N/mm}^2$. For 20° FD , 400BHN.

Case 3: To find maximum wear load.

From PSGDB 8.51

$$F_w = \frac{d_1\times b\times Q\times K_w}{\cos^2\beta}$$

$$= \frac{110.34\times 50\times 1.67\times 2.553}{\cos^2 25^\circ}$$

$$F_w = 23521.78\text{N}$$

Step 12: Check for wear

We find $F_w > F_d$, $\therefore$ Design is safe.

Step 13: Calculation of basic dimension of pinion and gear.

From PSGDB 8.22 , table 26.

* Normal Module: $m_n = 5\text{mm}$
* No. of teeth: $Z_1 = 20$, $Z_2 = 100$
* Pitch circle diameter: $d_1 = 110.34\text{mm}$, $d_2 = \frac{m_n}{\cos\beta}\times Z_2$

$$= \frac{5}{\cos 25°} \times 100$$

$$= 551.68\text{mm}$$

* Centre distance: $a = \frac{m_n}{\cos\beta} \times \left(\frac{Z_1 + Z_2}{2}\right)$

$$\frac{5}{\cos 25°} \times \left(\frac{20 + 100}{2}\right)$$

$$a = 331.01\text{mm}$$

* Face width: $b = 50\text{mm}$
* Height factor: $f_0 = 1$, for 20° FD
* Bottom clearance: $C = 0.25 m_n$

$$= 0.25 \times 5$$

$$= 1.25\text{mm}$$

* Tip diameter:

$$d_{a1} = \left(\frac{Z_1}{\cos\beta} + 2f_0\right) m_n \qquad d_{a2} = \left(\frac{Z_2}{\cos\beta} + 2f_0\right) m_n$$

$$= \left(\frac{20}{\cos 25} + 2(1)\right) 5 \qquad = \left(\frac{100}{\cos 25} + 2 \times 1\right) \times 5$$

$$d_{a1} = 120.34\text{mm} \qquad d_{a2} = 561.69\text{mm}$$

* Root diameter:

$$d_{f1} = \left(\frac{Z_1}{\cos\beta} - 2f_0\right) m_n - 2C \qquad d_{f2} = \left(\frac{Z_2}{\cos\beta} - 2f_0\right) m_n - 2C$$

$$= \left(\frac{20}{\cos 25} - 2 \times 1\right) 5 - 2 \times 1.25 \qquad = \left(\frac{100}{\cos 25} - 2 \times 1\right) 5 - 2 \times 1.25$$

$$d_{f1} = 97.84\text{mm} \qquad d_{f2} = 539.19\text{mm}$$

* Virtual no. of teeth: $Z_{v1} = 27$, $Z_{v2} = 135$

14. Design a pair of strength spur gear drive for a stone crusher, the pinion and wheel are made of C15 steel and cast iron grade 30b respectively. The pinion is to transmit 22.5 KW power at 900 rpm. The gear ratio is 2.5. Take pressure angle of 20° and working life of gears as 10, 000 hours. (Nov/Dec 2017)

Given data:

$$P = 22.5KW;\ N_1 = 900\,r.p.m;\ i = 2.5;\ \phi = 20°;\ N = 10000\,hrs$$

To find: Design a spur gear

Solution: Since the materials for pinion and wheel are different, therefore we have design the pinion first and check both pinion and wheel.

1. Gear ratio: i = 2.5

2. Material selection:

Pinion: C15 steel, case hardened to 55 RC and core hardness < 350, and

Wheel: C.I grade 30.

3. Gear life: N = 10000 hrs

Gear life in terms of number of cycles is given by

$$N = 10000 \times 60 \times 900 = 54 \times 10^2\,cycles$$

4. Design torque [Mt]:

$$[M_t] = M_t.K.K_d$$

$$M_t = \frac{60 \times P}{2\pi N_1} = \frac{60 \times 22.5 \times 10^3}{2\pi \times 900} = 238.73N - m$$

$$K.K_d = 1.3$$

Design torque $[M_t] = 238.73 \times 1.3 = 310.35N - m$

5. Calculation of Eeq, $|\sigma_b|$ and $|\sigma_1|$:

To find Eeq: For pinion steel and cast iron (> 280 N/mm²), equivalent Young's modulus, $E_{eq} = 1.7 \times 10^5 N/mm^2$

To find $|\sigma_b|$: The design bending stress $[\sigma_p]$ is given by

$$[\sigma_b] = \frac{1.4 \times K_{b1}}{n.K_\sigma} \times \sigma_{-1},$$ assuming rotation in one direction only.

For steel (HB ≤ 350) and N≥ 10^7, K_{b1} = 1.

For steel case hardened, factor of safety n = 2

For steel case hardened, stress concentration factor, $K_\sigma = 1.2$

For forged steel, $\sigma_{-1} = 0.25(\sigma_u + \sigma_y) + 50.$

For C15, $\sigma_u = 490 N/mm^2$ and $\sigma_y = 240 N/mm^2$

$$\sigma_{-1} = 0.25(490 + 240) + 50 = 232.5 N/mm^2$$

$$[\sigma_b] = \frac{1.4 \times 1}{2 \times 1.2} \times 232.5 = 135.625 N/mm^2$$

(iii) To find $|\sigma_c|$: The design contact stress $|\sigma_c|$ is given by

$$[\sigma_c] = C_R.HRC.K_{ct}$$

Where,

C_R =22, for C 15 steel

HRC = 55 to 63, for C 15 steel

K_{cl} = 0.585, for HB > 350, n ≥ 25 × 10^7

$$[\sigma_c] = 22 \times 63 \times 0.585 = 810.81 N/mm^2$$

6. Calculation of centre distance (a):

We know that,

$$a \geq (i+1)\sqrt[3]{\left(\frac{0.74}{[\sigma_c]}\right)^2 \times \frac{E_{eq}[M_t]}{i\Psi}}$$

$$\Psi = \frac{b}{a} = 0.3$$

$$a \geq (2.5+1)\sqrt[3]{\left(\frac{0.74}{810.81}\right)^2 \times \frac{1.7\times10^3 \times 310.35\times10^3}{2.5\times0.3}}$$

$$\geq 135.94 \text{mm or } a = 136 \text{mm}$$

7. To find z_1 and z_2:

(i) For 20° full depth system, select z_1 = 18.

(ii) $z_2 = i \times z_1 = 2.5 \times 18 = 45$

8. Calculation of module (m):

We know that,

$$m = \frac{2a}{z_1 + z_2} = \frac{2 \times 136}{18 + 45} = 4.32 \text{ mm}$$

The nearest higher standard module, m = 5 mm

9. Revision of centre distance:

New centre distance, $a = \frac{m(z_1 + z_2)}{2} = \frac{5(18+45)}{2} = 157.5\text{mm}$

10. Calculation of b, d_p, v and Ψ_p :

Face width (b): $b = \Psi.a = 0.3 \times 157.5 = 47.25\text{ mm}$

Pitch diameter of pinion (d_1): $d_1 = m.z_1 = 5 \times 18 = 90\text{mm}$

Pitch line velocity (v): $v = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 90 \times 10^{-3} \times 900}{60} = 4.24\text{m/s}$

$\psi_p = \frac{b}{d_1} = \frac{47.25}{90} = 0.525$

11. Selection of quality of gear:

For v = 4.24 m/s, IS quality 8 gears are selected.

12. Revision of design torque $[M_t]$:

Revise K: For $\psi_p = 0.525, K = 1.03$

Revise K_d: for IS quality 8 and v = 4.24 m/s, K_d = 1.4,

Revise $[M_t]$: $[M_t] = M_t.K.K_d \quad = 238.73 \times 1.03 \times 1.4 = 344.24\text{N}-\text{m}$

13. Check for bending:

Calculation of induced bending stress, σ_p :

Where,

$$\sigma_p = \frac{(i+1)}{a.m.b.y}[M_t]$$

$y = \text{Form factor} = 0.377, \text{ for } z_1 = 18$

$$\sigma_p = \frac{(2.5+1) \times 344.24 \times 10^3}{157.5 \times 5 \times 47.25 \times 0.377} = 58.89\text{N/mm}^2$$

We find $\sigma_b < [\sigma_B]$. Therefore the design is satisfactory.

14. Check for wear strength:

Calculation of induced contact stress, σ_c

$$\sigma_c = 0.74\frac{i+1}{a}\sqrt{\frac{i+1}{ib} \times E_{eq}[M_t]}$$

$$= 0.74\left(\frac{2.5+1}{157.5}\right)\sqrt{\left(\frac{2.5+1}{2.5 \times 47.25}\right) \times 1.7 \times 10^5 \times 344.24 \times 10^3}$$

$$= 684.76\text{N/mm}^2$$

We find $\sigma_c < |\sigma_c|$. Therefore the design is safe and satisfactory.

15. Check of wheel:

(i) Calculation of $|\sigma_b|_{wheel}$ and $|\sigma_c|_{wheel}$:

Wheel material: CI grade 30.

Wheel speed: $N_2 = \frac{N_1}{i} = \frac{900}{2.5} = 360 \, r.p.m$

Life of wheel $= 10,000 \times 60 \times 360 = 21.6 \times 10^7$ cycles

To find $|\sigma_b|_{wheel}$**:** The design bending stress for wheel is given by

$[\sigma_b]_{wheel} = \frac{1.4 \times K_{b1}}{n.K_a} \times \sigma_{-1}$, assuming rotation in one direction only.

For cast iron wheel, $K_{b1} = \sqrt[9]{\frac{10^7}{N}} = \sqrt[9]{\frac{10^7}{21.6 \times 10^7}} = 0.918$

For cast iron, n = 2.

For cast iron, $\sigma_{-1} = 0.45 \sigma_u$

For cast iron, $\sigma_u = 290 N/mm^2$

$$\sigma_{-1} = 0.45 \times 290 = 130.5 \, N/mm^2$$

$$[\sigma_b]_{wheel} = \frac{1.4 \times 0.918}{2 \times 1.2} \times 130.5 = 69.88 \, N/mm^2$$

To find $|\sigma_c|_{wheel}$: The wheel design contact stress for wheel is given by

$$|\sigma_c|_{wheel} = C_B.HB.K_{cl}$$

Where,

C_B = 2.3, for cast iron grade 30

HB = 200 to 260, for cast iron

$$K_{cl} = \sqrt[6]{\frac{10^7}{N}} = \sqrt[6]{\frac{10^7}{21.6 \times 10^7}} = 0.879, \text{for cast iron}$$

$$[\sigma_c]_{wheel} = 2.3 \times 260 \times 0.879 = 525.64 \, N/mm^2$$

(ii) Check for bending:

Calculation of induced bending stress for wheel σ_{b2}

$$\sigma_{b1} \times y_1 = \sigma_{b2} \times y_2$$

Where σ_{b1} and σ_{b2} = Induced bending stress in the pinion and wheel respectively, and

y_1 and y_2 = Form factors for pinion and wheel respectively.

$y_2 = 0.471$, for $z_2 = 45$.

$$\sigma_{b1} = 85.89\,N/mm^2 \text{ and } y_1 = 0.377$$
$$85.89 \times 0.377 = \sigma_{b2} \times 0.471$$
$$\sigma_{b2} = 68.75\,N/mm^2$$

We find $\sigma_{b2} < [\sigma_b]_{wheel}$. Therefore the design is satisfactory.

(iii) Check for wear strength: Since contact area is same, therefore $\sigma_{c,wheel} = \sigma_{c\,pinion} = 684.76\,N/mm^2$. Here $\sigma_{c\ wheel} > [\sigma_c]_{wheel}$. It means, wheel does not have the required wear resistance. So, in order to decrease the induced contact stress, increase the face width (b)value or in order to increase the design contact stress, increase the surface hardness, say to 340 HB. Increasing the surface hardness will give $[\sigma_c] = 2.3 \times 340 \times 0.879 = 687.34\,N/mm^2$. Now we find $\sigma_c < [\sigma_c]$. So the design is safe and satisfactory.

16. Calculation of basic dimensions of pinion and wheel:

Module: m = 5mm
Face width: b = 47.25 mm
Height factor: $f_0 = 1$ for full depth teeth.
Bottom clearance: c = 0.25m = 0.25×5 = **1.25 mm**
Tooth depth: h = 2.25 m=2.25 ×**5 = 11.25 mm**
Pitch circle diameter: $d_1 = m.z_1 = 5 \times 18 = 90$ mm and $d_2 = m.z_2 = 5 \times 45 = 225$ mm
Tip diameter:

$$d_{a1} = (z_1 + 2f_0)m = (18 + 2 \times 1)5 = 100 \text{ mm; and}$$
$$d_{a2} = (z_2 + 2f_0)m = (45 + 2 \times 1)5 = 235\,mm$$

Root diameter:

$$df_1 = (z_1 - 2f_0)m - 2c$$
$$= (18 - 2 \times 1)5 - 2 \times 1.25 = 77.5 \text{ mm; and}$$
$$df_2 = (z_2 - 2f_0)m - 2c$$
$$= (45 - 2 \times 1)5 - 2 \times 1.25 = 212.5 \text{ mm}$$

15. **Design a pair of spur gear drive to transmit 20kW at a pinion speed of 1400 rpm. The transmission ratio is 4. Assume 15 Ni2 Cr1 Mo 15 for pinion and C45 for gear. (April/ May 2018)**

Given data:

P= 20 kW
N_1=1400 rpm
i= 4
Material = 15 Ni2 Cr1 Mo 15 for pinion and C45 for gear
pinion and gear)

Step 1: Selection of Material

15 Ni2 Cr1 Mo 15 for pinion and C45 for gear

Assume steel is hardened to 200 BHN (BRINELL HARDNESS NUMBER) from PSGDB 8.16 table 16

Step 2: Calculation of Z_1 and Z_2

Number of Teeth on Pinion $Z_1 = 18$

Number of Teeth on Gear $Z_2 = i \times Z_1$

$= 3.5 \times 18$

$Z_2 = 63$

Step 3: Calculation of Tangential load (F_t)

Case 1: To find the pitch line velocity (v)

$$v = \frac{\pi d_1 N_1}{60}$$

$$v = \frac{\pi m Z_1 N_1}{60}$$

$$= \frac{\pi \times m \times 18 \times 800}{60 \times 1000}$$

$$= 0.754m \; m/sec$$

$F_t = \frac{P}{v} \times K_0$

$P = 45KW$

$K_0 = 1.5$

$v = \frac{\pi d_1 N_1}{60}$

$d_1 = m \times Z_1$

From PSGDB 8.22

Case 2: To find K_0

$K_0 = 1.5$ for medium shock conditions

Case 3: To find F_t

$$F_t = \frac{P}{v} \times K_0$$

$$F_t = \frac{45 \times 10^3}{0.754m} \times 1.5$$

$=89522.5/m$

Step 4: Calculation of Initial Dynamic Load (F_d)

Case 1: To find velocity factor (C_v)

$F_d = \frac{F_t}{C_v}$

$C_v = \frac{6}{6+v}$

From PSGDB 8.51 Assume

$C_v = \frac{6}{6+v}$ for accurately hobbed and generated gears

$v = 12\,m/sec$

With $v < 20\,m/sec$

$C_v = \frac{6}{6+12}$

Case 2: To find initial dynamic load (F_d)

$F_d = \frac{89522.5}{m} \times \frac{1}{0.333}$

$F_d = \frac{268836.3}{m}$

Step 5: Calculation of Beam Strength (F_s)

Case 1: To find form factor (y):

$y = 0.154 - (0.912/Z_1)$

$= 0.154 - (0.912/18)$

$= 0.1033$

Case 2: To find the beam strength (F_s)

Lewis equation,

$F_s = [\sigma_b] b\ y\ \pi\ m$

$= 180 \times 10m \times 0.1033\ \pi\ m$

$= 584.15m^2$

Step 6: Calculation of Module (m):

From PSGDB 8.51

$F_s \geq F_d$

$584.15m^2 \geq \frac{268836.3}{m}$

From PSGDB 8.50

$F_s = [\sigma_b] b\ y\ P_c$

Where

P_c = circular pitch $= \frac{\pi d}{z} = \pi m$

$m = d/z$

Finally we write

$F_s = [\sigma_b] b\ y\ \pi\ m$

Where

b = Face width $10 \times m$

y = Form Factor

$= 0.154 - (0.912/Z_1)$ for 20°

Full depth system

$$m \geq 7.72mm$$

From PSGDB 8.2 table 1, the nearest higher standard module value under choice 1 is 8 mm

Step 7: Calculation of b , d and v

Case 1: To find the face width (b)

$$b = 10 \times m$$
$$= 10 \times 8$$
$$= 80mm$$

Case 2: To find pitch circle diameter (d_1)

$$d_1 = m \times Z_1$$
$$= 8 \times 18$$
$$= 144mm$$

Case 3: To find Pitch line velocity (v)

$$v = \frac{\pi d_1 N_1}{60}$$
$$= \frac{\pi \times 144 \times 10^{-3} \times 800}{60}$$
$$= 6.03\, m/sec$$

Step 8: Recalculation of Beam Strength

Beam Strength $F_s = [\sigma_b] b\ y\ \pi\ m$

$$= 180 \times 80 \times 0.1033 \times \pi \times 8$$
$$= 37385.45\, N$$

Step 9: Calculation of accurate dynamic load (F_d)

STEP 9: CALCULATION OF ACCURATE DYNAMIC LOAD (F_d)

Case 1: To find tangential load (F_t)

$$F_t = \frac{P}{v}$$

$$F_t = \frac{45x103}{6.03}$$

= 7462.68 N

Case 2: To find Deformation factor (C)

C=11860 e

=11860x0.038

= 450.68 N/mm²

Case 3: To find the accurate dynamic load (F_d)

$$F_d = F_t + \frac{21\,v\,(bc + F_t)}{21\,v + \sqrt{bc + F_t}}$$

$$F_d = 7462.68 + \frac{21x6.03x103\,(80x450.68 + 7462.68)}{21x6.03x103 + \sqrt{80x450.68 + 7462.68}}$$

=50908.19 N

$F_d = F_t + \frac{21\,v\,(bc+F_t)}{21\,v+\sqrt{bc+F_t}}$ from PSGDB 8.51

⬆

We know that $F_t = \frac{P}{v}$ for accurate value eliminate K_0

⬆

C= Deformation factor from PSGDB 8.53 , table 41

⬆

C=11860 e, for 20° FD, steel and steel

⬆

e = 0.038 , for module upto 8 and carefully cut gears from PSGDB 8.53 , table 42

Step 10: Check for Beam strength or Tooth breakage

Since $F_d > F_s\,(50908.19N > 37385.45N)$ the design is unsatisfactory. The dynamic load is greater than the beam strength

In order to reduce the dynamic load F_d, Select the precision gears. Therefore from PSGDB 8.53 , table 42 take $e = 0.019$ for precision gears

Recalculation of deformation factor:

$$C = 11860 \times 0.019 = 225.34$$

Recalculation of dynamic load:

$$F_d = 7462.68 + \frac{21\times 6.03\times 10^3\,(80\times 225.34 + 7462.68)}{21\times 6.03\times 10^3 + \sqrt{80\times 225.34 + 7462.68}}$$

$$= 32920.46N$$

Now we find $F_s > F_d\,(37385.45N > 32920.46N)$. It means the gear tooth has adequate beam strength and it will not fail by breakage. Therefore the design is safe.

Step 11: Calculation of maximum wear load (F_w)

From PSGDB 8.51

$$F_w = d_1 \times b \times Q \times K_w$$

$Q = \text{Ratio factor} = \dfrac{2i}{i+1}$

$K_w = \text{load stress factor} = 0.919\,N/mm^2$, for steel hardened to 250 BHN

Case 1: To find ratio factor (Q)

$$Q = \frac{2i}{i+1} = \frac{2\times 3.5}{3.5+1} = 1.555$$

Case 2: To find maximum wear load (F_w)

$$F_w = d_1 \times b \times Q \times K_w$$

$$= 144 \times 80 \times 1.555 \times 0.919$$

$$= 16462.6N$$

Step 12: Check for wear

Since $F_d > F_w\,(32920.46N > 16462.6N)$ the design is unsatisfactory. That is the dynamic load is greater than the wear load.

In order to increase the wear load (F_w), we have to increase the hardness (BHN). So now for steel hardened to 400BHN, $K_w = 2.553\,N/mm^2$

$$\therefore \quad F_w = d_1 \times b \times Q \times K_w$$

$$= 144 \times 80 \times 1.555 \times 2.553$$

$$= 45733.42N$$

Now we find $F_w > F_d\,(45733.42N > 32920.46N)$. It means the gear tooth is adequate wear capacity and it will not wear out. Therefore the design is satisfactory

Step 13: Basic dimensions of Pinion and gear

From PSGDB 8.22, table 26

Module: $m = 8mm$

Number of teeth: $Z_1 = 18,\ Z_2 = 63$

Pitch circle diameter: $d_1 = 144mm$

$d_2 = m \times Z_2 = 8 \times 63$

$d_2 = 504mm$

Centre distance: $a = m(Z_1 + Z_2)/2$

$=8(18+63)/2$

$a=324mm$

Face width: $b=80mm$

Height factor: $f_0=1$, for $20°$ full depth teeth

Bottom clearance: $c=0.25m=0.25\times 8$

$c=2mm$

Tip diameter: $d_{a1}=(Z_1+2f_0)m$ $\quad$ $d_{a2}=(Z_2+2f_0)m$

$=(18+2\times 1)8$ $\quad$ $=(63+2\times 1)8$

$=160mm$ $\quad$ $=520mm$

Root diameter: $d_{f1}=(Z_1-2f_0)m-2c$ $\quad$ $d_{f2}=(Z_2-2f_0)m-2c$

$=(18-2\times 1)8-2\times 2$ $\quad$ $=(63-2\times 1)8-2\times 2$

$=124mm$ $\quad$ $=484mm$

16. Design of helical gear drive to transmit the power of 14.7 KW , speed ratio 6 , pinion speed 1200 rpm , helix angle is 25° select suitable material and design the gear. (April/ May 2018)

Given data:

$P=14.7KW$

$i=6$

$N_1=1200$ rpm

$\beta=25°$

Step 1: Selection of Material.

15 Ni2 Cr1 Mo 15 for pinion and C45 for gear

$\therefore\ [\sigma_b]=180\ N/mm^2$

Step 2: Calculation of no. of teeth:

Case 1: Calculation of Z_1 & Z_2.

No. of teeth on pinion Z_1 = 20 Assume

Gear $Z_2 = i \times Z_1$

$= 6 \times 20$ 6* 20

$= 120$

Case 2: Calculation of Z_{v1} & Z_{v2}:

From PSGDB 8.22 , table 2b

Virtual no. of teeth on pinion $Z_{v1} = \frac{Z_1}{\cos^3 \beta} = \frac{20}{\cos^3 25°}$

$$= 26.86 = 27$$

$$\text{Gear } Z_{v2} = \frac{Z_2}{\cos^3 \beta} = \frac{120}{\cos^3 25°}$$

$$= 161.19 = 162$$

Step 3: Calculation of tangential load on teeth (F_t).

$$F_t = \frac{P}{v} \times K_0$$

Case 1: To find the pitch line velocity (v).

$$v = \frac{\pi d_1 N_1}{60}$$

From PSGDB 8.22

$$d_1 = \frac{m_n}{\cos \beta} \times Z_1$$

$$\therefore \quad v = \frac{\pi \times m_n \times 20 \times 1200}{60 \times \cos 25 \times 100°}$$

$$v = 1.39 m_n \quad m/sec$$

Case 2: To find K_0

K_0 = 1.5 , for medium shock condition.

$$\therefore \quad F_t = \frac{14.7 \times 10^3}{1.39 \times m_n} \times 1.5$$

$$F_t = \frac{15902.83}{m_n}(N)$$

Step 4: Calculation of initial dynamic load (F_d)

$$F_d = \frac{F_t}{C_v}$$

Case 1: To find the velocity factor (C_v)

$C_v = \frac{6}{6+v}$ for carefully cut gears , v < 20 m/s From BSGDB 8.51 Assume v = 15 m/s

$$= \frac{6}{6+15}$$

$$C_v = 0.286$$

Case 2: To find initial dynamic load (F_d)

$$F_d = \frac{15902.83}{m_n} \times \frac{1}{0.286}$$

$$F_d = \frac{55604.3}{m_n}$$

Step 5: calculation of beam strength (F_s)

$F_s = [\sigma_b] b y^1 \pi m_n$ From PSGDB 8.51

Where,

* $b = 10 \times m_n$ From PSGDB 8.14
* $y^1 = 0.154 - \left(0.912/Z_{v1}\right)$ From PSGDB 8.50 , 20° Full depth system.

$$= 0.154 - \left(0.912/27\right)$$

$$y^1 = 0.12$$

$$\therefore \quad F_s = 180 \times 10 \times m_n \times 0.12 \times \pi \times m_n$$

$$= 678.58\, m_n^2$$

Step 6: Calculation of normal module (m_n).

From PSGDB 8.51

$$F_s \geq F_d$$

$$678.58 m_n^2 \geq \frac{55604.3}{m_n}$$

$$m_n \geq 4.34 \text{ mm.}$$

From PSGDB 8.2, table 1, the nearest higher standard module value under choice 1 is;

$$m_n = 5 \text{ mm.}$$

Step 7: Calculation of b_1 d_1 and v:

Case 1: To find the face width (b)

$$b = 10 \times m_n$$

$$= 10 \times 5$$

$$= 50 \text{ mm.}$$

Case 2: To find the Pitch circle diameter (d_1)

$$d_1 = \frac{m_n}{\cos \beta} \times Z_1$$

$$= \frac{5}{\cos 25} \times 20$$

$$d_1 = 110.34 \text{ mm}$$

Case 3: To find the pitch line velocity (v)

$$v = \frac{\pi d_1 N_1}{60}$$

$$\frac{\pi \times 110.34 \times 10^{-3} \times 1200}{60}$$

$$v = 6.93 \text{ m/s}$$

Step 8: Recalculation of beam strength (F_s)

$$F_s = [\sigma_b] \times b \times y^1 \times \pi \times m_n$$

$$= 180 \times 50 \times 0.12 \times \pi \times 5$$

$$F_s = 16964.6 \text{ N}$$

Step 9: Calculation of accurate dynamic load (F_d)

From PSGDB 8.51

$$F_d = F_t + \frac{21v(bc.\cos^2\beta + F_t)\cos\beta}{21v + \sqrt{(bc.\cos^2\beta + F_t)}}$$

Case 1: To find (F_t).

$$F_t = \frac{P}{v}$$

$$= \frac{14.7\times10^3}{6.93}$$

$$F_t = 2121.21\ \text{N}$$

Case 2: To find deformation factor (C).

$C = 11860\ e$ From PSGDB 8.53 , table 41 , for 20° FD , steel and steel.

$e = 0.025$ From PSGDB 8.53 table 42 , for module upto 5 and carefully cut gears.

$$\therefore\quad C = 11860\times0.025$$

$$= 296.5\ \text{N/mm}^2$$

Case 3: To find (F_d).

$$F_d = 2121.21 + \frac{21\times6.93\times10^3(50\times296.5\times\cos^2 25 + 212.21)\cos 25}{21\times6.93\times10^3 + \sqrt{50\times296.5\times\cos^2 25 + 2121.21}}$$

$$F_d = 15069.29\ \text{N}$$

Step 10: Check for beam strength.

We find $F_s > F_d$ $\therefore$ The design is safe.

Step 11: Calculation of Maximum wear load (F_w):

From PSGDB 8.51.

$$F_w = \frac{d_1\times b\times Q\times K_w}{\cos^2\beta}$$

Case 1: To find ratio factor (Q).

From PSGDB 8.51.

$$Q = \frac{2i}{i+1} = \frac{2\times 6}{6+1} = 1.71$$

Case 2: To find Load stress factor (K_w).

Assume K_w = 0.919 for 20° FD

$$\therefore \quad F_w = \frac{110.34\times 50\times 1.71\times 0.919}{\cos^2 25}$$

$$F_w = 10555.12 \text{ N}$$

Step 12: Check for wear.

* We find $F_w < F_d$. $\therefore$ The design is not safe.
* In order to increase the wear load, we have to increase the hardness (BHN). So how for steel hardened to 400 BHN, $K_w = 2.553 \text{ N/mm}^2$.

$$\therefore \quad F_w = 29322.33 \text{ N} .$$

$$\therefore \; F_w > F_d \text{ , Design is safe.}$$

Step 13: Calculation of basic dimension of pinion and gear.

From PSGDB 8.22 , table 26

* Normal module: $m_n = 5\text{mm}$
* No. of teeth: $Z_1 = 20$, $Z_2 = 120$
* Pitch circle diameter: $d_1 = 110.34\text{mm}$, $d_2 = \frac{m_n}{\cos\beta}\times Z_2$

$$= \frac{5}{\cos 25}\times 120$$

$$= 662.03\text{mm}$$

* Centre distance: $a = \frac{m_n}{\cos\beta}\times\left(\frac{Z_1+Z_2}{2}\right)$

$$= \frac{5}{\cos 25} * \left(\frac{20+120}{2}\right)$$

* Face width: $b = 50\text{mm}$
* Height factor: $f_0 = 1$, for 20°FD
* Bottom clearance: $c = 0.25m_n$

$$= 0.25\times 5$$

$$= 1.25 \text{ mm}$$

* Tip diameter: $d_{a1} = \left(\frac{Z_1}{\cos\beta} + 2f_0\right)m_n$ $d_{a2} = \left(\frac{Z_2}{\cos\beta} + 2f_0\right)m_n$

$$= \left(\frac{20}{\cos 25} + 2\times1\right)5 \qquad = \left(\frac{120}{\cos 25} + 2\times1\right)5$$

$$= 120.33\text{ mm.} \qquad = 672\text{ mm.}$$

* Root diameter: $d_{f1} = \left(\frac{Z_1}{\cos\beta} - 2f_0\right)m_n - 2c$ $d_{f2} = \left(\frac{Z_2}{\cos\beta} - 2f_0\right)m_n - 2c$

$$= \left(\frac{20}{\cos 25} - 2\times1\right)5 - 2\times1.25 \qquad = \left(\frac{120}{\cos 25} - 2\times1\right)5 - 2\times1.25$$

$$= 97.83\text{mm.} \qquad = 649.52\text{mm.}$$

17. **A compressor running at 300rpm is driven by a15kW, 1200rpm motor through a $14\frac{1}{2}°$ full depth spur gears. The center distance is 375mm. The motor pinion is to be C30 forged steel hardened (BHN 250) and tempered, and the driven gear is to be of cast iron. Assuming medium shock condition and minimum number of teeth as 18. Design the gear drive completely. (Nov/Dec 2018)**

Given data:

$P = 45KW$

$N_1 = 800rpm$

$i = 3.5$

$\phi = 20°$

$Z_1 = 18$

$[\sigma_b] = 180 N/mm^2$

Material = steel (for both pinion and gear)

Similar to this problem change the material for pinion and gear, and BHN 200 to 250.

Step 1: Selection of Material

Pinion and Gear = Steel

Assume steel is hardened to 200 BHN (BRINELL HARDNESS NUMBER) from PSGDB 8.16 table 16

Step 2: Calculation of Z_1 and Z_2

Number of Teeth on Pinion $Z_1 = 18$

Number of Teeth on Gear $Z_2 = i \times Z_1$

$= 3.5 \times 18$

$Z_2 = 63$

Step 3: Calculation of Tangential load (F_t)

Case 1: To find the pitch line velocity (v)

$$v = \frac{\pi d_1 N_1}{60}$$

$$v = \frac{\pi m Z_1 N_1}{60}$$

$$= \frac{\pi \times m \times 18 \times 800}{60 \times 1000}$$

$$= 0.754m \, m/sec$$

Case 2: To find K_0

$K_0 = 1.5$ for medium shock conditions

Case 3: To find F_t

$$F_t = \frac{P}{v} \times K_0$$

$$F_t = \frac{45 \times 10^3}{0.754m} \times 1.5$$

$$= 89522.5/m$$

$F_t = \frac{P}{v} \times K_0$ $P = 45KW$ $K_0 = 1.5$ $v = \frac{\pi d_1 N_1}{60}$ $d_1 = m \times Z_1$ From PSGDB 8.22

Step 4: Calculation of Initial Dynamic Load (F_d)

Case 1: To find velocity factor (C_v)

$C_v = \frac{6}{6+v}$ for accurately hobbed and generated gears

With $v < 20 \, m/sec$

$$C_v = \frac{6}{6+12}$$

Case 2: To find initial dynamic load (F_d)

$$F_d = \frac{89522.5}{m} \times \frac{1}{0.333}$$

$F_d = \frac{F_t}{C_v}$ $C_v = \frac{6}{6+v}$ From PSGDB 8.51 Assume $v = 12 \, m/sec$

$$F_d = \frac{268836.3}{m}$$

Step 5: Calculation of Beam Strength (F_s)

Case 1: To find form factor (y):

$$y = 0.154 - (0.912/Z_1)$$

$$= 0.154 - (0.912/18)$$

$$= 0.1033$$

Case 2: To find the beam strength (F_s)

Lewis equation,

$$F_s = [\sigma_b] b\ y\ \pi\ m$$

$$= 180 \times 10m \times 0.1033\ \pi\ m$$

$$= 584.15m^2$$

Step 6: Calculation of Module (m):

From PSGDB 8.51

$$F_s \geq F_d$$

$$584.15m^2 \geq \frac{268836.3}{m}$$

$$m \geq 7.72mm$$

From PSGDB 8.2 table 1, the nearest higher standard module value under choice 1 is 8 mm

Step 7: Calculation of b , d and v

From PSGDB 8.50

$$F_s = [\sigma_b] b\ y\ P_c$$

Where

$$P_c = \text{circular pitch} = \frac{\pi d}{z} = \pi m$$

$$m = d/z$$

Finally we write

$$F_s = [\sigma_b] b\ y\ \pi\ m$$

Where

b = Face width $10 \times m$

y = Form Factor

$= 0.154 - (0.912/Z_1)$ for 20°

Full depth system

Case 1: To find the face width (b)

$$b = 10 \times m$$

$$= 10 \times 8$$

$= 80mm$

Case 2: To find pitch circle diameter (d_1)

$$d_1 = m \times Z_1$$

$$= 8 \times 18$$

$$= 144mm$$

Case 3: To find Pitch line velocity (v)

$$v = \frac{\pi d_1 N_1}{60}$$

$$= \frac{\pi \times 144 \times 10^{-3} \times 800}{60}$$

$$= 6.03\, m/sec$$

Step 8: Recalculation of Beam Strength

Beam Strength $F_s = [\sigma_b] b\ y\ \pi\ m$

$$= 180 \times 80 \times 0.1033 \times \pi \times 8$$

$$= 37385.45\, N$$

Step 9: Calculation of accurate dynamic load (F_d)

STEP 9: CALCULATION OF ACCURATE DYNAMIC LOAD (F_d)

Case 1: To find tangential load (F_t)

$$F_t = \frac{P}{v}$$

$$F_t = \frac{45x103}{6.03}$$

$= 7462.68$ N

Case 2: To find Deformation factor (C)

C=11860 e

=11860x0.038

= 450.68 N/mm²

Case 3: To find the accurate dynamic load (F_d)

$$F_d = F_t + \frac{21\ v\ (bc + F_t)}{21\ v + \sqrt{bc + F_t}}$$

$$F_d = 7462.68 + \frac{21x6.03x103\ (80x450.68 + 7462.68)}{21x6.03x103 + \sqrt{80x450.68 + 7462.68}}$$

=50908.19 N

$F_d = F_t + \frac{21\ v\ (bc + F_t)}{21\ v + \sqrt{bc + F_t}}$ from PSGDB 8.51

⬆

We know that $F_t = \frac{P}{v}$ for accurate value eliminate K_0

⬆

C= Deformation factor from PSGDB 8.53 , table 41

⬆

C=11860 e, for 20° FD, steel and steel

⬆

e = 0.038 , for module upto 8 and carefully cut gears from PSGDB 8.53 , table 42

Step 10: Check for Beam strength or Tooth breakage

Since $F_d > F_s$ $(50908.19N > 37385.45N)$ the design is unsatisfactory. The dynamic load is greater than the beam strength

In order to reduce the dynamic load F_d, Select the precision gears. Therefore from PSGDB 8.53 , table 42 take $e = 0.019$ for precision gears

Recalculation of deformation factor:

$$C = 11860 \times 0.019 = 225.34$$

Recalculation of dynamic load:

$$F_d = 7462.68 + \frac{21 \times 6.03 \times 10^3 (80 \times 225.34 + 7462.68)}{21 \times 6.03 \times 10^3 + \sqrt{80 \times 225.34 + 7462.68}}$$

$$= 32920.46N$$

Now we find $F_s > F_d (37385.45N > 32920.46N)$. It means the gear tooth has adequate beam strength and it will not fail by breakage. Therefore the design is safe.

Step 11: Calculation of maximum wear load (F_w)

Case 1: To find ratio factor (Q)

$$Q = \frac{2i}{i+1} = \frac{2 \times 3.5}{3.5+1} = 1.555$$

From PSGDB 8.51

$$F_w = d_1 \times b \times Q \times K_w$$

$$Q = \text{Ratio factor} = \frac{2i}{i+1}$$

$K_w = \text{load stress factor} = 0.919 N/mm^2$, for steel hardened to 250 BHN

Case 2: To find maximum wear load (F_w)

$$F_w = d_1 \times b \times Q \times K_w$$

$$= 144 \times 80 \times 1.555 \times 0.919$$

$$= 16462.6N$$

Step 12: Check for wear

Since $F_d > F_w (32920.46N > 16462.6N)$ the design is unsatisfactory. That is the dynamic load is greater than the wear load.

In order to increase the wear load (F_w), we have to increase the hardness (BHN). So now for steel hardened to 400BHN, $K_w = 2.553 N/mm^2$

$$\therefore \quad F_w = d_1 \times b \times Q \times K_w$$

$$= 144 \times 80 \times 1.555 \times 2.553 = 45733.42N$$

Now we find $F_w > F_d \left(45733.42N > 32920.46N\right)$. It means the gear tooth is adequate wear capacity and it will not wear out. Therefore the design is satisfactory

Step 13: Basic dimensions of Pinion and gear

From PSGDB 8.22, table 26

Module: $m = 8mm$

Number of teeth: $Z_1 = 18, Z_2 = 63$

Pitch circle diameter: $d_1 = 144mm$

$d_2 = m \times Z_2 = 8 \times 63$

$d_2 = 504mm$

Centre distance: $a = m\left(Z_1 + Z_2\right)/2$

$= 8\left(18 + 63\right)/2$

$a = 324mm$

Face width: $b = 80mm$

Height factor: $f_0 = 1$, for $20°$ full depth teeth

Bottom clearance: $c = 0.25m = 0.25 \times 8$

$c = 2mm$

Tip diameter: $d_{a1} = \left(Z_1 + 2f_0\right)m$ $\quad$ $d_{a2} = \left(Z_2 + 2f_0\right)m$

$= \left(18 + 2 \times 1\right)8$ $\quad$ $= \left(63 + 2 \times 1\right)8$

$= 160mm$ $\quad$ $= 520mm$

Root diameter: $d_{f1} = \left(Z_1 - 2f_0\right)m - 2c$ $\quad$ $d_{f2} = \left(Z_2 - 2f_0\right)m - 2c$

$= \left(18 - 2 \times 1\right)8 - 2 \times 2$ $\quad$ $= \left(63 - 2 \times 1\right)8 - 2 \times 2$

$= 124mm$ $\quad$ $= 484mm$

18. Design a carefully cut helical gears to transmit 15kW at 1400rpm to the following specifications. Speed reduction is 3. Pressure angle is 20˚, Helix angle 15˚. The material for both the gears is C45 steel. Allowable static stress is 180 N/mm², endurance limit is 800 N/mm². Young's modulus of the material =2x10⁵ N/mm². Assume minimum number of teeth as 20 and medium shock conditions, v= 15m/s. (Nov/Dec 2018)

Given data:

$$N_1 = 1440\text{rpm}$$
$$i = 5$$
$$P = 37\text{KW}$$
$$\phi = 20° = \alpha_n$$
$$\beta = 25°$$

****** similar to this problem, change the speed as 1400rpm and Helix angle15˚***

Step 1: selection of Material.

Generally we assume C45 steel for both pinion and gear.

$$[\sigma_b] = 180\,\text{N/mm}^2\text{ , 250 BHN.}$$

Step 2: Calculation of number of teeth Z_1 & Z_2:

No. of teeth on pinion $Z_1 = 20$ (assume)

No. of teeth on gear $Z_2 = i \times Z_1$

$$= 5 \times 20$$
$$= 100$$

Step 3: Calculation of tangential load on teeth (F_t):

$$F_t = \frac{P}{v} \times K_0$$

Case 1: To find the Pitch line velocity (v)

$$v = \frac{\pi d_1 N_1}{60}$$

From PSGDB 8.22

$$d_1 = \frac{m_n}{\cos\beta} \times Z_1$$

$$= \frac{m_n}{\cos 25°} \times 20$$

$$d_1 = \frac{m_n}{22.06}$$

$$\therefore v = \frac{\pi \times m_n \times 1440}{60 \times 22.06 \times 1000}$$

$$= 1.66 m_n \ {}^{m}\!/_{sec}$$

Case 2: To find K_0:

$$K_0 = 1.5 \text{ for medium shock conditions.}$$

$$\therefore F_t = \frac{37 \times 10^3}{1.66 m_n} \times 1.5$$

$$= \frac{33433.73}{m_n}$$

Step 4: Calculation of initial dynamic Load (F_d)

$$F_d = \frac{F_t}{C_v}$$

Case 1: To find velocity factor (C_v)

$$C_v = \frac{6}{6+v} \quad \text{for carefully cut gears, } v < 20\,m/s$$

$$= \frac{6}{6+15}$$

$$C_v = 0.286$$

Case 2: To find initial dynamic load (F_d):

$$F_d = \frac{33433.73}{m_n} \times \frac{1}{0.286}$$

$$\frac{116901.17}{m_n}$$

Step 5: Calculation of beam strength (Fs):

$F_s = [\sigma_b] b y^{-1} P_{cn}$ From PSGDB 8.51 $P_{cn} = \pi m_n$

$\therefore \ F_s = [\sigma_b] b y^1 \pi m_n$

Where,

$b = 10 \times m_n$ From PSGDB 8.14

$y^1 = 0.154 - \left(0.912 / Z_{v1} \right)$ From PSGDB 8.50 , $20°$ full depth system.

$$Z_{v1} = \frac{Z_1}{\cos^3 \beta}$$

$$= \frac{20}{\cos^3 25}$$

$$Z_{v1} = 26.86 \approx 27$$

$$\therefore \ y^1 = 0.154 - \frac{0.912}{26.86}$$

$$= 0.12$$

$$F_s = [\sigma_b] b \, y^1 \pi m_n$$

$$= 180 \times 10 \times m_n \times 0.1143 \times \pi \times m_n$$

$$= 678.58 m_n^2$$

Step 6: Calculation of normal module (m_n):

From PSGDB 8.51

$$F_s \geq F_d$$

$$678.58 m_n^2 \geq \frac{116901.17}{m_n}$$

$$m_n \geq 5.56 \text{mm}$$

From PSGDB 8.2 , table 1. The nearest higher standard module value under choice 1, is

$$m_n = 6mm$$

Step 7: Calculation of b , d_1, and v:

Case 1: To find the face width (b)

$$b = 10 \times m_n$$

$$= 10 \times 6$$

$$= 60mm$$

Case 2: To find Pitch circle diameter (d_1)

$$d_1 = \frac{m_n}{\cos\beta} \times Z_1$$

$$= \frac{6}{\cos 25^\circ} \times 20$$

$$d_1 = 124.23mm.$$

Case 3: To find Pitch line velocity (v)

$$v = \frac{\pi d_1 N_1}{60}$$

$$= \frac{\pi \times 124.23 \times 10^{-3} \times 1440}{60}$$

$$= 9.37 \, m/s$$

Step 8: Recalculation of Beam strength (F_s)

$$F_s = [\sigma_b] b \, y^1 \pi m_n$$

$$= 180 \times 60 \times 0.12 \times 6 \times \pi$$

$$F_s = 24429.02N$$

Step 9: Calculation of Accurate dynamic load (F_d)

From PSGDB 8.51

$$F_d = F_t + \frac{21v\left(6c.\cos^2\beta + F_t\right)\cos\beta}{21v + \sqrt{6c.\cos^2\beta + F_t}}$$

Case 1: To find (F_t)

$$F_t = \frac{P}{v}$$

$$= \frac{37 \times 10^3}{9.37}$$

$$F_t = 3948.77N$$

Case 2: To find deformation factor (C)

$C = 11860\ e$ From PSGDB 8.53 , table 41 , for 20° FD , steel and steel.

$e = 0.030$, for module upto 6 and carefully cut gears – PSGDB 8.53 table 42

$\therefore\ C = 11860 \times 0.030$

$= 355.8\ N/mm$

Case 3: To find (F_d)

$$F_d = 3948.77 + \frac{21 \times 9.37 \times 10^3 \left(60 \times 355.8 \times \cos^2 25° + 3948.77\right) \cos 25°}{21 \times 9.37 \times 10^3 + \sqrt{60 \times 355.8 \times \cos^2 25° + 3948.77}}$$

$F_d = 23398.68\ N$

Step 10: Check for beam strength or tooth breakage.

We find $F_s > F_d$. $\therefore$ the design is safe

Step 11: Calculation of Maximum wear load (F_w):

Case 1: To find Ratio factor (Q)

From PSGDB 8.51

$$Q = \frac{2i}{i+1} = \frac{2 \times 5}{5+1} = 1.67$$

Case 2: To find Load stress factor (K_w)

From PSGDB 8.51

$$K_w = \frac{\left[f_{es}^2\right] \sin d_n}{1.4} \times \left[\frac{1}{E_1} + \frac{1}{E_2}\right]$$

Assume $f_{es} = 618 N/mm^2$

$$K_w = \frac{618^2 \sin 20}{1.4} \times \left[\frac{1}{2.15\times10^5} + \frac{1}{2.15\times10^5}\right]$$

$$= 0.867\,N/mm^2$$

Case 3: To find Maximum wear load (F_w).

From PSGDB 8.51

$$F_w = \frac{d_1 \times b \times Q \times K_w}{\cos^2\beta}$$

$$= \frac{124.23\times60\times1.67\times0.867}{\cos^2 25°}$$

$$F_w = 13138.98N$$

Step 12: Check for wear:

* We find $F_w < F_d$. $\therefore$ the design is not safe.
* In order to increase the wear load, we have to increase the hardness (BHN). So now for steel hardened to 400 BHN, K_W = 2.41 N/mm^2.

$$\therefore F_w = 36522.44N$$

$\therefore F_w > F_d$, Design is safe.

Step 13: Calculation of basic dimensions of Pinion and gear.

* Normal module: $m_n = 6mm$
* No. of teeth: $Z_1 = 20$, $Z_2 = 100$
* Pitch circle diameter: $d_1 = 124.23mm$, $d_2 = \frac{m_n}{\cos\beta}\times Z_2$

$$= \frac{6}{\cos 25°\times100} = 662.03mm.$$

* Centre distance: $a = \frac{m_n}{\cos\beta}\times\left(\frac{Z_1+Z_2}{2}\right)$

$$= \frac{6}{\cos 25°}\times\left(\frac{20+100}{2}\right)$$

$$a = 397.22mm$$

* Face width: $b = 60mm$
* Height factor: $f_0 = 1$, for 20° full depth teeth.

* Bottom clearance: $C = 0.25m_n$

$= 0.25 \times 6$

$C = 1.5mm$

* Tip diameter: $d_{a1} = \left(\frac{Z_1}{\cos\beta} + 2f_0\right)m_n$

$= \left(\frac{20}{\cos 25°} + 2 \times 1\right) \times 6$

$d_{a1} = 144.41mm$

$d_{a2} = \left(\frac{Z_2}{\cos\beta} + 2f_0\right)m_n$

$= \left(\frac{100}{\cos 25°} + 2 \times 1\right) \times 6$

$d_{a2} = 674.03mm$

* Root diameter:

$d_{f1} = \left(\frac{Z_1}{\cos\beta} - 2f_0\right)m_n - 2C$ $d_{f2} = \left(\frac{Z_2}{\cos\beta} - 2f_0\right)m_n - 2C$

$d_{f1} = \left(\frac{20}{\cos 25°} - 2 \times 1\right)6 - 2 \times 1.5$ $= \left(\frac{100}{\cos 25°} - 2 \times 1\right)6 - 2 \times 1.5$

$d_{f1} = 117.41mm$ $d_{f2} = 647.03mm.$

* Virtual number of teeth:

$Z_{v1} = 26.86 = 27$

$Z_{v2} = \frac{Z_2}{\cos^3\beta} = \frac{100}{\cos^3 25°}$

$Z_{v2} = 134.33 = 135$

19. Design a spur gear drive to transmit 22 kW at 900rpm, speed reduction is 2.5. materials for pinion and wheel are C15 steel and cast-iron grade 30 respectively. Take pressure angle of 20° and working life of the gears as 10,000 hours. (April/May 2019)

Given data:

$P = 22.5KW;\ N_1 = 900 r.p.m;\ i = 2.5;\ \phi = 20°;\ N = 10000 hrs$

To find: Design a spur gear

Solution: Since the materials for pinion and wheel are different, therefore we have design the pinion first and check both pinion and wheel.

1. Gear ratio: i = 2.5

2. Material selection:

Pinion: C15 steel, case hardened to 55 RC and core hardness < 350, and

Wheel: C.I grade 30.

3. Gear life: N = 10000 hrs

Gear life in terms of number of cycles is given by

$$N = 10000 \times 60 \times 900 = 54 \times 10^2 \text{cycles}$$

4. Design torque [Mt]:

$$[M_t] = M_t.K.K_d$$
$$M_t = \frac{60 \times P}{2\pi N_1} = \frac{60 \times 22.5 \times 10^3}{2\pi \times 900} = 238.73 N - m$$
$$K.K_d = 1.3$$

Design torque $[M_t] = 238.73 \times 1.3 = 310.35 N - m$

5. Calculation of Eeq, $|\sigma_b| \text{and} |\sigma_1|$:

To find Eeq: For pinion steel and cast iron (> 280 N/mm²), equivalent Young's modulus, $E_{eq} = 1.7 \times 10^5 N/mm^2$

To find $|\sigma_b|$: The design bending stress $[\sigma_p]$ is given by

$[\sigma_b] = \frac{1.4 \times K_{b1}}{n.K_\sigma} \times \sigma_{-1}$, assuming rotation in one direction only.

For steel (HB ≤ 350) and N≥ 10^7, K_{b1} = 1.

For steel case hardened, factor of safety n = 2

For steel case hardened, stress concentration factor, $K_\sigma = 1.2$

For forged steel, $\sigma_{-1} = 0.25(\sigma_u + \sigma_y) + 50.$

For C15, $\sigma_u = 490 N/mm^2$ and $\sigma_y = 240 N/mm^2$

$$\sigma_{-1} = 0.25(490 + 240) + 50 = 232.5 N/mm^2$$
$$[\sigma_b] = \frac{1.4 \times 1}{2 \times 1.2} \times 232.5 = 135.625 N/mm^2$$

(iii) To find $|\sigma_c|$: The design contact stress $|\sigma_c|$ is given by

$$[\sigma_c] = C_R.HRC.K_{ct}$$

Where,

C_R =22, for C 15 steel

HRC = 55 to 63, for C 15 steel

K_{cl} = 0.585, for HB > 350, n ≥ 25 × 10^7

$$[\sigma_c] = 22 \times 63 \times 0.585 = 810.81 N/mm^2$$

6. Calculation of centre distance (a):

We know that,

$$a \geq (i+1)\sqrt[3]{\left(\frac{0.74}{[\sigma_c]}\right)^2 \times \frac{E_{eq}[M_t]}{i\Psi}}$$

$$\Psi = \frac{b}{a} = 0.3$$

$$a \geq (2.5+1)\sqrt[3]{\left(\frac{0.74}{810.81}\right)^2 \times \frac{1.7\times10^3 \times 310.35\times10^3}{2.5\times0.3}}$$

$$\geq 135.94 mm \text{ or } a = 136 mm$$

7. To find z_1 and z_2:

(i) For 20° full depth system, select z_1 = 18.

(ii) $z_2 = i \times z_1 = 2.5 \times 18 = 45$

8. Calculation of module (m):

We know that,

$$m = \frac{2a}{z_1 + z_2} = \frac{2\times136}{18+45} = 4.32 mm$$

The nearest higher standard module, m = 5 mm

9. Revision of centre distance:

New centre distance, $a = \frac{m(z_1+z_2)}{2} = \frac{5(18+45)}{2} = 157.5 mm$

10. Calculation of b, d_p, v and Ψ_p :

Face width (b): $b = \Psi.a = 0.3 \times 157.5 = 47.25$ mm

Pitch diameter of pinion (d_1): $d_1 = m.z_1 = 5 \times 18 = 90 mm$

Pitch line velocity (v): $v = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 90 \times 10^{-3} \times 900}{60} = 4.24 m/s$

$$\psi_p = \frac{b}{d_1} = \frac{47.25}{90} = 0.525$$

11. Selection of quality of gear:

For υ = 4.24 m/s, IS quality 8 gears are selected.

12. Revision of design torque $[M_t]$:

Revise K: For $\psi_p = 0.525, K = 1.03$

Revise K_d: for IS quality 8 and υ = 4.24 m/s, K_d = 1.4,

Revise $[M_t]:[M_t] = M_t.K.K_d \quad = 238.73 \times 1.03 \times 1.4 = 344.24 N-m$

13. Check for bending:

Calculation of induced bending stress, σ_p :

Where,

$$\sigma_p = \frac{(i+1)}{a.m.b.y}[M_t]$$

$y =$ Form factor = 0.377, for $z_1 = 18$

$$\sigma_p = \frac{(2.5+1) \times 344.24 \times 10^3}{157.5 \times 5 \times 47.25 \times 0.377} = 58.89 N/mm^2$$

We find $\sigma_b < [\sigma_B]$. Therefore the design is satisfactory.

14. Check for wear strength:

Calculation of induced contact stress, σ_c

$$\sigma_c = 0.74 \frac{i+1}{a} \sqrt{\frac{i+1}{ib} \times E_{eq} [M_t]}$$

$$= 0.74 \left(\frac{2.5+1}{157.5}\right) \sqrt{\left(\frac{2.5+1}{2.5 \times 47.25}\right) \times 1.7 \times 10^5 \times 344.24 \times 10^3}$$

$$= 684.76 N/mm^2$$

We find $\sigma_c < |\sigma_c|$. Therefore the design is safe and satisfactory.

15. Check of wheel:

(i) Calculation of $|\sigma_b|_{wheel}$ and $|\sigma_c|_{wheel}$:

Wheel material: CI grade 30.

Wheel speed: $N_2 = \frac{N_1}{i} = \frac{900}{2.5} = 360 r.p.m$

Life of wheel $= 10,000 \times 60 \times 360 = 21.6 \times 10^7$ cycles

To find $|\sigma_b|_{wheel}$ **:** The design bending stress for wheel is given by

$$[\sigma_b]_{wheel} = \frac{1.4 \times K_{b1}}{n.K_a} \times \sigma_{-1}$$, assuming rotation in one direction only.

For cast iron wheel, $K_{b1} = \sqrt[9]{\frac{10^7}{N}} = \sqrt[9]{\frac{10^7}{21.6 \times 10^7}} = 0.918$

For cast iron, n = 2.

For cast iron, $\sigma_{-1} = 0.45\sigma_u$

For cast iron, $\sigma_u = 290\,N/mm^2$

$$\sigma_{-1} = 0.45 \times 290 = 130.5\,N/mm^2$$
$$[\sigma_b]_{wheel} = \frac{1.4 \times 0.918}{2 \times 1.2} \times 130.5 = 69.88\,N/mm^2$$

To find $|\sigma_c|_{wheel}$: The wheel design contact stress for wheel is given by

$$|\sigma_c|_{wheel} = C_B.HB.K_{cl}$$

Where,

C_B = 2.3, for cast iron grade 30

HB = 200 to 260, for cast iron

$$K_{cl} = \sqrt[6]{\frac{10^7}{N}} = \sqrt[6]{\frac{10^7}{21.6 \times 10^7}} = 0.879, \text{for cast iron}$$
$$[\sigma_c]_{wheel} = 2.3 \times 260 \times 0.879 = 525.64\,N/mm^2$$

(ii) Check for bending:

Calculation of induced bending stress for wheel σ_{b2}

$$\sigma_{b1} \times y_1 = \sigma_{b2} \times y_2$$

Where σ_{b1} and σ_{b2} = Induced bending stress in the pinion and wheel respectively, and

y_1 and y_2 = Form factors for pinion and wheel respectively.

y_2 = 0.471, for z_2 = 45.

$$\sigma_{b1} = 85.89\,N/mm^2 \text{ and } y_1 = 0.377$$
$$85.89 \times 0.377 = \sigma_{b2} \times 0.471$$
$$\sigma_{b2} = 68.75\,N/mm^2$$

We find $\sigma_{b2} < [\sigma_b]_{wheel}$. Therefore the design is satisfactory.

(iii) Check for wear strength: Since contact area is same, therefore $\sigma_{c,wheel} = \sigma_{c\,pinion} = 684.76\,N/mm^2$. Here $\sigma_{c\ wheel} > [\sigma_c]_{wheel}$. It means, wheel does not have the required wear resistance. So, in order to decrease the induced contact stress, increase the face width (b)value or in order to increase the design contact stress, increase the surface hardness, say to 340 HB. Increasing the surface hardness will give $[\sigma_c] = 2.3 \times 340 \times 0.879 = 687.34\, N/mm^2$. Now we find $\sigma_c < [\sigma_c]$. So the design is safe and satisfactory.

16. Calculation of basic dimensions of pinion and wheel:

Module: m = 5mm
Face width: b = 47.25 mm
Height factor: $f_0 = 1$ for full depth teeth.
Bottom clearance: c = 0.25m = 0.25×5 = **1.25 mm**
Tooth depth: h = 2.25 m=2.25 ×**5 = 11.25 mm**
Pitch circle diameter: $d_1 = m.z_1 = 5 \times 18 = 90$ mm and $d_2 = m.z_2 = 5 \times 45 = 225$ mm
Tip diameter:
$d_{a1} = (z_1 + 2f_0)m = (18 + 2 \times 1)5 = 100$ mm; and
$d_{a2} = (z_2 + 2f_0)m = (45 + 2 \times 1)5 = 235$ mm
Root diameter:
$df_1 = (z_1 - 2f_0)m - 2c$
$= (18 - 2 \times 1)5 - 2 \times 1.25 = 77.5$ mm; and
$df_2 = (z_2 - 2f_0)m - 2c$
$= (45 - 2 \times 1)5 - 2 \times 1.25 = 212.5$ mm

20. A pair of helical gears is to be designed to transmit 30kWat a pinion speed of 1500rpm. The velocity ratio is 3. Selecting 15Ni2Cr1Mo15 steel as the material. Determine the dimensions of the gears. (April/May 2019)

Given data:

$N_1 = 1000 rpm$
$P = 10 KW$
$i = 5$

**** similar to this problem*

Step 1: Selection of Material

Generally we assume C45 steel for both pinion and gear.

$[\sigma_b] = 180\, N/mm^2$, 400 BHN.

Step 2: Calculation of number of teeth Z_1 and Z_2:

No. of teeth on pinion gear $Z_1 = 20$ (assume)

$$Z_2 = i \times Z_1$$
$$= 5 \times 20$$

$$=100.$$

Virtual no. of teeth Z_{v1} & Z_{v2}

From PSGDB 8.22 , table 26. Assume $\beta = 25°$

$$Z_{v1} = \frac{Z_1}{\cos^3\beta} \qquad Z_{v2} = \frac{Z_2}{\cos^3\beta}$$

$$= \frac{20}{\cos^3 25} \qquad = \frac{100}{\cos^3 25}$$

$$Z_{v1} = 27 \qquad = 134.33\text{mm}.$$

$$Z_{v2} \approx 135\text{mm}$$

Step 3: Calculation of tangential load on teeth (F_t).

$$F_t = \frac{P}{v} \times K_0$$

$K_0 = 1.5$, for medium shock conditions.

Case 1: To find the pitch line velocity (v)

$$v = \frac{\pi d_1 N_1}{60}$$

From PSGDB 8.22, table 26

$$d_1 = \frac{m_n}{\cos\beta} \times Z_1$$

$$\therefore \quad v = \frac{\pi \times m_n \times 20 \times 1000}{60 \times 1000 \times \cos 25°}$$

$$v = 1.16 m_n \ \ \text{m/sec}$$

$$\therefore \quad F_t = \frac{10 \times 10^3}{1.16 m_n} \times 1.5$$

$$= \frac{12931.03}{m_n}$$

Step 4: Calculation of initial dynamic load (F_d)

$$F_d = \frac{F_t}{C_v}$$

Case 1: To find the velocity factor (Cv)

$C_v = \frac{6}{6+v}$ for carefully cut gears $v < 20\,m/s$. From PSGDB 8.51 Assume $v = 15\,m/s$.

$$= \frac{6}{6+15}$$

$$C_v = 0.286.$$

$$\therefore \quad F_d = \frac{12931.03}{m_n} \times \frac{1}{0.286}$$

$$= \frac{45213.41}{m_n}$$

Step 5: Calculation of beam strength (F_s).

$$F_s = [\sigma_b] \times b \times y^1 \times \pi \times m_n$$

Where,

$b = 10 \times m_n$ From PSGDB 8.14

$y^1 = 0.154 - \left(0.912 / Z_{v1}\right)$ From PSGDB 8.50 , 20° FD

$$= 0.154 - \frac{0.912}{27}$$

$$= 0.12$$

$$\therefore \quad F_s = 180 \times 10 \times m_n \times 0.12 \times \pi \times m_n$$

$$F_s = 678.58\ m_n^2$$

Step 6: Calculation of normal module (m_n)

From PSGDB 8.51

$$F_s \geq F_d$$

$$678.58\ m_n^2 \geq \frac{45213.41}{m_n}$$

$m_n \geq 4.05mm$.

From PSGDB 8.2, table 1. The nearest higher standard module value under choice 1 is $m_n = 5mm$.

Step 7: Calculation of b , d_1 , and v:

Case 1: To find face width (b).

$$b = 10 \times m_n$$

$$= 10 \times 5$$

$$= 50mm.$$

Case 2: To find Pitch circle diameter (d_1).

$$d_1 = \frac{m_n}{\cos\beta} \times Z_1$$

$$= \frac{5}{\cos 25} \times 20$$

$$d_1 = 110.34mm$$

Case 3: To find Pitch line velocity (v)

$$v = \frac{\pi d_1 N_1}{60}$$

$$= \frac{\pi \times 110.34 \times 1000}{60 \times 1000}$$

$$v = 5.78 m/s$$

Step 8: Recalculation of beam strength (F_s)

$F_s = 678.58 \times m_n^2$ From step 5

$$= 678.58 \times 5^2$$

$$F_s = 16964.5N$$

Step 9: Calculation of Accurate dynamic load (F_d)

From PSGDB 8.51

$$F_d = F_t + \frac{21v\left(bc \cdot \cos^2\beta + F_t\right)\cos\beta}{21v + \sqrt{bc \cdot \cos^2\beta + F_t}}$$

Case 1: To find (F_t)

$$F_t = \frac{P}{v}$$

$$= \frac{10\times10^3}{5.78}$$

$$F_t = 1730.1N$$

Case 2: To find deformation factor (C)

$C = 11860e$ From PSGDB 8.53 , table 41 , 20° FD.

$e = 0.025$ for module upto 5 and carefully cut gears.

$\therefore\ C = 296.5 N/mm^2$

$$\therefore\ F_d = 1730.1 + \frac{21\times5.78\times10^3\left(50\times296.5\cdot\cos^2 25 + 1730.1\right)\cos 25}{21\times5.78\times10^3 + \sqrt{50\times296.5\cdot\cos^2 25 + 1730.1}}$$

$F_d = 1836.98N$

Step 10: Check for beam strength.

We find $F_s > F_d$, $\therefore$ The design is safe.

Step 11: Calculation of maximum wear load (F_w)

Case 1: To find Ratio factor (Q)

From PSGDB 8.51

$$Q = \frac{2(i)}{i+1} = \frac{2\times5}{5+1} = 1.67 .$$

Case 2: To find Load stress factor (K_w)

$K_w = 2.553 N/mm^2$. For 20° FD , 400BHN.

Case 3: To find maximum wear load.

From PSGDB 8.51

$$F_w = \frac{d_1\times b\times Q\times K_w}{\cos^2\beta}$$

$$= \frac{110.34\times50\times1.67\times2.553}{\cos^2 25°}$$

$$F_w = 23521.78N$$

Step 12: Check for wear

We find $F_w > F_d$, $\therefore$ Design is safe.

Step 13: Calculation of basic dimension of pinion and gear.

From PSGDB 8.22 , table 26.

* Normal Module: $m_n = 5mm$
* No. of teeth: $Z_1 = 20$, $Z_2 = 100$
* Pitch circle diameter: $d_1 = 110.34mm$, $d_2 = \frac{m_n}{\cos\beta} \times Z_2$

$$= \frac{5}{\cos 25°} \times 100$$

$$= 551.68mm$$

* Centre distance: $a = \frac{m_n}{\cos\beta} \times \left(\frac{Z_1 + Z_2}{2}\right)$

$$\frac{5}{\cos 25°} \times \left(\frac{20+100}{2}\right)$$

$$a = 331.01mm$$

* Face width: $b = 50mm$
* Height factor: $f_0 = 1$, for 20° FD
* Bottom clearance: $C = 0.25m_n$

$$= 0.25 \times 5$$

$$= 1.25mm$$

* Tip diameter:

$$d_{a1} = \left(\frac{Z_1}{\cos\beta} + 2f_0\right)m_n \qquad d_{a2} = \left(\frac{Z_2}{\cos\beta} + 2f_0\right)m_n$$

$$= \left(\frac{20}{\cos 25} + 2(1)\right)5 \qquad = \left(\frac{100}{\cos 25} + 2 \times 1\right) \times 5$$

$$d_{a1} = 120.34mm \qquad d_{a2} = 561.69mm$$

* Root diameter:

$$d_{f1} = \left(\frac{Z_1}{\cos\beta} - 2f_0\right)m_n - 2C \qquad d_{f2} = \left(\frac{Z_2}{\cos\beta} - 2f_0\right)m_n - 2C$$

$$= \left(\frac{20}{\cos 25} - 2\times1\right)5 - 2\times1.25 \qquad = \left(\frac{100}{\cos 25} - 2\times1\right)5 - 2\times1.25$$

$$d_{f1} = 97.84\text{mm} \qquad d_{f2} = 539.19\text{mm}$$

* Virtual no. of teeth: $Z_{v1} = 27$, $Z_{v2} = 135$

ME-6601 DESIGN OF TRANSMISSION SYSTEMS

UNIT-III BEVEL GEAR AND WORM GEARS

(PART-A)

1. **What is virtual number of teeth in bevel gear?**

An imaginary spur gear considered in a plane perpendicular to the tooth of the bevel gear at the larger end is called as virtual spur gear.

The number of teeth z_v on this imaginary spur gear is called virtual number of teeth in bevel gears.

$$z_v = \frac{z}{\cos\delta}$$

Where, z= Actual number of teeth on the bevel gear
δ = Pitch angle

2. **Mention the advantages of worm gear drive?**
   - The worm gear drives can be used for speed ratios as high as 300:1
   - The operation is smooth and silent
3. **State the advantages of herringbone gear?**

It eliminates the existence of axial thrust load in the helical gears. Because, in herringbone gears, the thrust force of the right hand is balanced by that of the left hand helix.

4. **What is zerol bevel gear?**

Spiral bevel gear with curved teeth but with a zero degree spiral angle is known as zerol bevel gear.

5. **What is the difference between an angular gear and miter gear?**
   - When the bevel gears connect two shafts whose axes intersect at an angle other than a right angle, then they are known as **angular bevel gears.**
   - When equal bevel gears (having equal teeth and equal pitch angles) connect two shafts whose axes intersect at right angle, then they are known as **miter gears.**
6. **What kind contact occurred between worm and wheel? How does it differs from other gears?**
   - The worm gears are used to transmit power between two non-intersecting, non-parallel shafts.
   - The worm gears drives are compact, smooth and silent in operation.
7. **How bevel gears are manufactured?**

Bevel gears are not interchangeable. Because they are designed and manufactured in pairs.

8. **What is helical angle of worm?**

Helical angle is the angle between the tangent to the thread helix on the pitch cylinder and axis of the worm. The worm helix angle is the complement of worm lead angle, that is **ß=90°-γ**

9. What is a crown gear?

A bevel gear having a pitch angle 90° and a plane for its pitch surface is known as crown gear.

10. Write some applications of worm gear drive?

Worm gear drives are widely used as a speed reducer in materials handling equipment, machine tools and automobiles.

11. When do we use worm-gears?

When we require to transmit power between nonparallel and non-intersecting shafts and very high velocity ratio, of about 100, worm gears, can be employed. Also worm-gears provide self-locking facility.

12. Write some applications of worm gear drive.

Worm gear, drive find wide applications like milling machine indexing head, table fan and steering rod of automobile and so on.

13. **What is a bevel gear?**

Bevel gear is the type of gear for which the teeth are cut on conical surface in contrast with spur and helical gears for which the teeth are cut on cylindrical surfaces. The structure of bevel gear is similar to and uniformly truncated frustum of a cone.

14. When do we use bevel gears?

When the power is to be transmitted in an angular, direction, i.e., between the shafts whose axes intersecting at an angle, bevel gears are employed.

15. How are bevel gears classified?

Bevel gears are classified in two ways

1. Based on the shape of teeth.
   - Straight bevel gears.
   - Spiral bevel gears
2. Based on the included angle between the shaft axes, called as shaft angle
   - External gears < 90 deg)
   - Internal gears > 90 deg)
   - Crown gears 90 deg

16. **What is a crown gear?**

A crown gear is a type of bevel gear whose shaft angle is 90 degree and angle of pinion is not equal to the pitch angle of gear. Let Shaft angle

17. What is the specific feature of mitre gear?

Mitre gear is the special type of crown gear In which the **shaft,** 90 deg and the pitch angles of pinion and gear are equal and each angle to 45 deg.

18. Fill in the blanks of the following

a) Bevel gears having shaft, Angle of, 90deg are known as........

b) When the spiral angle of a.bevel gear is zero, it is called as...

Answers

a) Crown gears. b) Zerol bevel gear.

19. Define the following terms

a) Cone distance or pitch cone radius.

b) Face angle.

a) Cone distance or pitch cone radius is the slant length of pitch cone, i.e., distance between the apex and the extreme point of tooth of bevel gear.

b) Face angle is the angle subtended by the face of the teeth at the cone centre. It is equal to the pitch angle plus addendum angle. It is also called as tip angle.

20. In which gear-drive, self-locking is available?

Self-locking is available in worm-gear drive.

21. What is known as formative number of teeth on bevel gears? (April/May 2017)

The formative of equivalent number of teeth for a bevel gear may be defined as the number of teeth that can be generated on the surface of a cylinder having a radius of curvature at a point at the tip of minor axis of an ellipse obtained by taking a section of the gear in the normal plane.

22. Write the conditions of self-locking of worm gears in terms of lead and pressure angles. (April/May 2017)

The conditions of self-locking of worm gears in terms of lead and pressure angles can be

If $\mu \geq cos\alpha.tan\gamma$

23. What are the disadvantages of worm gear drive? (Nov/Dec 2017)

- ❖ Manufacturing cost is heavy as compared with manufacturing cost of bevel gear
- ❖ Cost of raw material to manufacture the worm and worm gear set will be quite high
- ❖ Worm and worm gear set will have heavy power losses.
- ❖ Efficiency will be low
- ❖ If speed reduction ratio is large, worm teeth sliding action will create lots of heat
- ❖ Lubrication scheduled must be strictly maintained for healthiness of worm and worm gear as this unit requires much lubrication for smooth working of gearbox.

24. What is Meant by miter gears? (Nov/Dec 2017)

When equal bevel gears (having equal teeth and equal pitch angles) connect two shafts whose axes intersect at right angle, then they are known as **miter gears**

25. **When do we use bevel gears? (April/May 2018)**

Bevel gears are used to transmit power between two intersecting shafts

26. **In which gear drive, self-locking is available? (April/May 2018)**

Worm and Worm Wheel

27. **List the forces acting on bevel gears. (Nov/Dec 2018)**

i. Tangential force
ii. Axial force
iii. Radial force

28. **What is irreversibility in worm gear? (Nov/Dec 2018)**

The worm gear drives are irreversible. It means that the motion cannot be transmitted from worm wheel to the worm. This property of irreversible is advantageous in load hoisting applications like cranes and lifts.

29. What is crown and miter gear? (April/May 2019)

A bevel gear having a pitch angle 90° and a plane for its pitch surface is known as crown gear.

When equal bevel gears (having equal teeth and equal pitch angles) connect two shafts whose axes intersect at right angle, then they are known as **miter gears**

30. Define the pitch and lead of worm gears. (April/May 2019)

It is distance measured along the normal to the threads between two corresponding points on two adjacent threads of the worm.

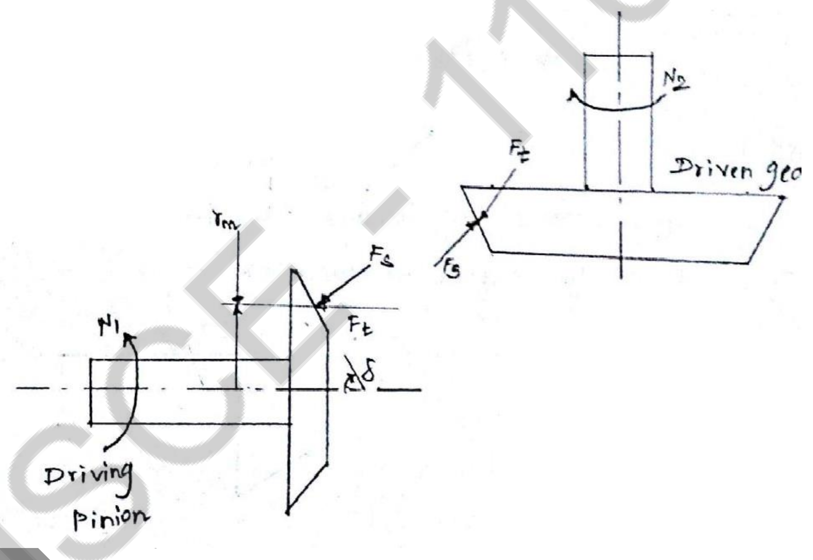
PART B

1. Derive expressions for determining the forces acting on a bevel gear with suitable illustrations.

In force analysis of bevel gears, it is assumed that the resultant tooth force between two meshing gears is concentrated at the midpoint along the face width of the tooth. The forces acting at the centre of the tooth are shown in figure.

The components of the resultant force are,

(a) Tangential or useful component (F_t).
(b) Separating Force (F_s): It is resolved into two components. They are (i) Axial force (F_a) (ii) Radial force (F_r)



(i) Components of the tooth force on the pinion:

To find F_t: The tangential force can be determined using the familiar relationship.

$$F_t = \frac{2M_t}{d_{1av}} = \frac{M_t}{r_m}$$

Where, $M_t = \text{Transmitted torque} = \dfrac{60 \times P}{2\pi N}$

$P = \text{Power transmitted}$

$N = \text{speed of the gear}$

d_{1av} = Average diameter of the pinion , at midpoint along the face width.

$$= Z_1 \cdot m_{av}$$

$$= Z_1 \left(M_t - \frac{b.\text{Sin}\,\delta}{Z_1} \right)$$

$$r_m = \left(\frac{d_1}{2} - \frac{b\ \text{Sin}\delta_1}{2} \right)$$

= Mean radius of the pinion at midpoint along the face width.

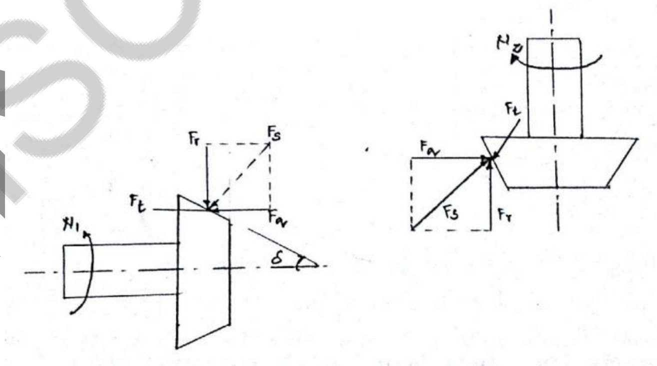
To find F_s: The analysis is similar to that of spur gears and the separating force can be determined using the relation

Separating force $F_s = F_t \times \tan\alpha$ (1)

Where, α = Pressure angle

To find F_r and F_a:

The separating force is further resolved into radial and axial forces, as shown in figure below.



From the geometry of the figure, we can write

Radial force, $F_r = F_s \times \cos\ \delta$

Axial force, $F_a = F_s \times \text{Sin}\ \delta$

Substituting equation (1) in the above equations,

$$F_r = F_t\ \tan\alpha \cdot \cos\ \delta$$

$$F_a = F_t\ \tan\alpha \cdot \text{Sin}\ \delta$$

The above derived expressions are used to determine the components of the tooth force on the pinion.

(ii) Components of the tooth force on the gear:

From the figure a and b, the following conclusions can be made for the right angle bevel gears:

* The radial component on the gear is equal to the axial component on the pinion but in opposite direction.

$$(F_r)\text{gear} = -(F_a)\text{pinion}$$

* Similarly, the axial component on the gear is equal to the radial component on the pinion, but in opposite direction.

$$(F_a)\text{gear} = -(F_r)\text{pinion}$$

Note: The three forces F_t, F_r and F_a are perpendicular to each other and can be used to determine the bearing loads by using the methods of statics.

2. A hardened steel worm rotates at 1440rpm and transmits 12KW to a phosphor bronze gear. The speed of the worm wheel should be 60 ± 3%rpm. Design a worm gear drive if an efficiency of atleast 82% is desired.

Given data:

$N_1 = 1440\text{rpm}$

$P = 12\text{KW}$

$N_2 = 60 \pm 3\%\text{rpm}$

$\eta_{\text{desired}} = 82\%$

Step 1: To find gear ratio (i) :

$$i = \frac{N_1}{N_2} \pm 3\%$$

$$= \frac{1440}{60} \pm 3\%$$

$$= 24 \pm 0.72$$

$$\text{take } i = 24$$

Step 2: Selection of Material:

Worm = Hardened steel

Worm wheel = Phosphor bronze

Step 3: Calculation of Z_1 and Z_2:

From PSGDB 8.46, table 37.

For $\eta = 82\%$, $Z_1 = 3$

$$Z_2 = i \times Z_1$$

$$= 24 \times 3$$

$$Z_2 = 72$$

Step 4: Calculation of q and H:

Case 1: To find diameter factor (q):

From PSGDB 8.43, table 35, and PSGDB 8.44

$$d_1 = \frac{q}{m_x}$$

Initially we assume $q = 11$

Case 2: To find Lead angle (H):

From PSGDB 8.43 , table 35

$$\tan H = \frac{Z_1}{q}$$

$$H = \tan^{-1}\left(\frac{3}{11}\right)$$

$$H = 15.25^\circ$$

Step 5: Calculation of 'F_t' in terms of 'm_x':

Tangential Load $F_t = \frac{P}{v} \times K_0$

Case 1: To find the velocity 'v':

$$v = \frac{\pi d_2 N_2}{60 \times 1000}$$

From PSGDB 8.43 , table 35

$$d_2 = Z_2 \times m_x$$

$$\therefore\ v = \frac{\pi \times Z_2 \times m_x \times N_2}{60 \times 1000}$$

$$= \frac{\pi \times 72 \times m_x \times 60}{60 \times 1000}$$

$$v = 0.226 m_x \ \mathrm{m/s}$$

Case 2: to find shock factor (K_0):

Assume medium shock,

$$K_0 = 1.5$$

$$\therefore\ F_t = \frac{12 \times 10^3}{0.226 m_x} \times 1.5$$

$$F_t = \frac{79646.02}{m_x}$$

Step 6: Calculation of dynamic load: (F_d)

$$F_d = \frac{F_t}{C_v}$$

Case 1: To find velocity factor (C_v) :

From PSGDB 8.51 , assume $v = 5 m/s$

$$C_v = \frac{6}{6+v}$$

$$= \frac{6}{6+5}$$

$$C_v = 0.545$$

Case 2: To find (F_d):

$$F_d = \frac{79646.02}{m_x} \times \frac{1}{0.545}$$

$$= \frac{1460177.70}{m_x}$$

Step 7: Calculation of beam strength (F_s) **in terms of** (m_x)

From PSGDB 8.51

$$F_s = \pi \times m_x \times b \times [\sigma_b] \times y^1$$

Where ,

$b = 0.75 d_1$ From PSGDB 8.48 , table 38

$= 0.75 \times q \times m_x$

$= 0.75 \times 11 \times m_x$

$= 8.25 m_x$

$y^1 = 0.125$ From PSGDB 8.52 , Assume $\alpha = 20°$

Form factor $y = 0.392$

$$\therefore y^1 = \frac{y}{\pi}$$

$$\frac{0.392}{\pi}$$

$= 0.125$

$[\sigma_b] = 80\,N/mm^2$ From PSGDB 8.45 , table 33

$$\therefore F_s = \pi \times m_x \times 8.25 m_x \times 80 \times 0.125$$

$$= 259.18\, m_x^2$$

Step 8: Calculation of Axial module (m_x)

W . K . T $F_s \geq F_d$

$$259.18 \times m_x^2 \geq \frac{146017.70}{m_x}$$

$$m_x \geq 8.26 mm$$

From PSGDB 8.2 , Table 1.

The nearest higher standard axial module

$$m_x = 10mm.$$

Step 9: Calculation of b, d_2 and v:

Case 1: To find the face width (b):

$$b = 8.25m_x \quad \text{From step 7}$$

$$= 82.5mm$$

Case 2: To find pitch diameter of the worm wheel (d_2)

$$d_2 = Z_2 \times m_x \quad \text{From step 5 case 1.}$$

$$= 72 \times 10$$
$$= 720mm$$

Case 3: To find the pitch line velocity of worm wheel (v)

$$v = 0.226\ m_x \quad \text{From step 5, case 1.}$$

$$= 0.226 \times 10$$

$$v = 2.26\,m/s$$

Step 10: Recalculation of beam strength.

$$F_s = 259.18m_x^2 \quad \text{From step 7}$$

$$= 259.18 \times 10^2$$

$$F_s = 25918N$$

Step 11: Recalculation of dynamic load (F_α)

$$F_d = \frac{F_t}{C_v}$$

$$C_v = \frac{6}{6+v} = \frac{6}{6+2.26} = 0.726$$

$$F_t = \frac{79646.02}{m_x} = \frac{79646.02}{10} = 7964.602N \quad \text{From step 5 case 2}$$

$$\therefore \quad F_d = \frac{7964.602}{0.726}$$

$$F_d = 10970.53N$$

Step 12: Check for beam strength.

We find $F_d < F_s$. the design is safe.

Step 13: Check for Maximum wear load (F_N):

From PSGDB 8.52

$$F_w = d_2 \times b \times K_w$$

$K_w = 0.56\,N/mm^2$ From PSGDB 8.54 , table 43

$$F_w = 720 \times 82.5 \times 0.56$$

$$F_w = 33264N$$

Step 14: Check for efficiency.

$$\eta_{actual} = 0.95 \times \frac{\tan H}{\tan(H+e)}$$ From PSGDB 8.49

Where, $\rho = \tan^{-1} M$, Assume $M = 0.03$ From PSGDB 8.49

$$\rho = \tan^{-1}(0.03)$$

$$= 1.7°$$

$$\eta_{actual} = 0.95 \times \frac{\tan 15.25}{\tan(15.25 + 1.7)}$$

$= 0.8498$ We find that the actual efficiency is greater than the desired efficiency. $\therefore$ The design is safe.

$$\eta_{actual} = 84.98\%$$

Step 15: Calculation of basic dimensions of worm and worm gears.

From PSGDB 8.43 , table 35

Axial module: $m_x = 10mm$

No. of starts: $Z_1 = 3$

No. of teeth on the worm wheel: $Z_2 = 72$

Face width of the worm wheel: $b = 82.5\text{mm}$

Length of the worm: $L \geq (12.5 + 0.09Z_2)m_x$

$= (12.5 + 0.09 \times 72)10$

$= 189.8\text{mm}$

Take $L = 190\text{mm}$

Centre distance: $a = 0.5m_x(q + Z_2)$

$a = 0.5 \times 10(11 + 72)$

$a = 415\text{mm}$

Height factor: $f_0 = 1$

Bottom clearance: $C = 0.25m_x = 0.25 \times 10 = 2.5\text{mm.}$

Pitch diameter: $d_1 = q \times m_x = 11 \times 10 = 110\text{mm}$

$d_2 = 720\text{mm}$

Tip diameter: $d_{a1} = d_1 + 2f_0 \times m_x = 110 + 2 \times 1 \times 10 = 130\text{mm}$

$d_{a2} = (Z_2 + 2f_0)m_x = (72 + 2 \times 1)10 = 740\text{mm}$

Root diameter: $d_{f1} = d_1 - 2f_0 \times m_x - 2C$

$= 110 - 2 \times 1 \times 10 - 2 \times 2.5$

$= 85\text{mm}$

$d_{f2} = (Z_2 - 2f_0)m_x - 2C$

$= (72 - 2 \times 1) \times 10 - 2 \times 2.5$

$= 695\text{mm.}$

3. Design a pair of strength bevel gears for two shafts whose axis are at right angles. The power transmitted is 25 KW. The speed of pinion is 300 rpm and the gear is 120 rpm.

Given data:

$P = 25KW$

$N_1 = 300rpm$

$N_2 = 120rpm$

Step 1: Selection of Material:

From PSGDB Pg No. 1.40. Both pinion and gears C45 steel is selected.

Step 2: Calculation of no. of teeth, virtual number of teeth and pitch angles:

$$\frac{N_1}{N_2} = \frac{300}{120} \Rightarrow i$$

$$\therefore i = 2.5.$$

Case(1): Calculation of no. of teeth Z_1 & Z_2

$$Z_1 = 20 \quad \text{Assume}$$

$$Z_2 = i \times Z_1$$

$$= 2.5 \times 20$$

$$= 50$$

Case 2: Calculation of virtual no. of teeth Z_{v1} & Z_{v2}

From PSGDB 8.39

$$z_{V1} = \frac{z_1}{\cos \delta_1}$$

$$= \frac{20}{\cos 21.8°} = 21.54 \approx 22$$

$$Z_{v2} = \frac{Z_2}{\cos \delta_2} = \frac{50}{\cos 68.2}$$

$$= 134.64 \approx 135$$

From PSGDB 8.39

$$\delta_1 = 90° - \delta_2$$

$$\tan \delta_2 = i$$

$$\delta_2 = \tan^{-1} 2.5 = 68.2°$$

$$\therefore \quad \delta_1 = 21.8°$$

Step 4: Calculation of tangential load (F_t).

$$F_t = \frac{P}{v} \times K_0$$

Where

- $K_0 = 1.5$ for medium shock conditions.
- $v = \frac{\pi d_1 N_1}{60}$

From PSGDB 8.38 , table 31

$$d_1 = m_t \times Z_1$$

$$\therefore \quad v = \frac{\pi \times m_t \times 20 \times 300}{60 \times 1000}$$

$$v = 0.314\ m_t\ \text{m/s}$$

$$\therefore \quad F_t = \frac{25 \times 10^3}{0.314 m_t} \times 1.5$$

$$F_t = \frac{119366.21}{m_t}$$

Step 5: Calculation of initial dynamic load (Fd).

$$F_d = \frac{F_t}{C_v}$$

From PSGDB 8.52

$$C_v = \frac{5.5}{5.5 + \sqrt{v}} \quad \text{, assuming v = 5 m/s}$$

$$= \frac{5.5}{5.5 + \sqrt{5}}$$

$$C_v = 0.711$$

$$\therefore \quad F_d = \frac{119366.21}{m_t} \times \frac{1}{0.711}$$

$$F_d = \frac{167895.48}{m_t}$$

Step 6: Calculation of beam strength (F_s)

From PSGDB 8.52,

$$F_s = \pi \times m_t \times [\sigma_b] \times b \times y^1 \left(\frac{R-b}{R} \right)$$

Where,

$b = 10 \times m_t$ From PSGDB 8.38, table 31

$[\sigma_b] = 180\ N/mm^2$, for C45 steel

$y^1 = 0.154 - \frac{0.912}{Z_{v1}}$ From PSGDB 8.50, 20 FD

$= 0.154 - \frac{0.912}{22}$

$y^1 = 0.112$

R = cone radius $= 0.5m + \sqrt{Z_1^2 + Z_2^2}$ From PSGDB 8.38 , table 31

$$= 0.5 \times m_t \sqrt{20^2 + 50^2}$$

$$R = 26.93m_t$$

$$\therefore\ F_s = m_t \times \pi \times 10 \times m_t \times 180 \times 0.112 \times \left[\frac{26.93m_t - 10m_t}{26.93m_t} \right]$$

$$= 398.16\ m_t^2$$

Step 7: Calculation of transverse module (m_t):

From PSGDB 8.51

$$F_s \geq F_d$$

$$398.16\ m_t^2 \geq \frac{167895.48}{m_t}$$

$$m_t \geq 7.5$$

From PSGDB 8.2 , table 1, choice 1. The next nearest higher standard module $m_t = 8mm$.

Step 8: Calculation of b_1 d_1 and v:

- Face width $b = 10 \times m_t$

$= 10 \times 8$

$= 80$ mm

Pitch circle diameter, $d_1 = m_t \times Z_1$

$= 8 \times 20$

$d_1 = 160$ mm

Pitch line velocity $v = \frac{\pi d_1 N_1}{60}$

$= \frac{\pi \times 160 \times 300}{60 \times 1000}$

$v = 2.51$ m/s

Step 9: Recalculation of beam strength:

$F_s = 398.16\ m_t^2$ From step 6

$= 398.16 \times 8^2$

$F_s = 25482.24$ N

Step 10: Calculation of accurate dynamic load (F_d).

From PSGDB 8.51

$$F_d = F_t + \frac{21v(bc + F_t)}{21v + \sqrt{bc + F_t}}$$

Where,

- $F_t = \frac{P}{v}$

$= \frac{25 \times 10^3}{2.51}$

$= 9960.16$N 7961.78N

- $C = 11860\ e$ From PSGDB 8.53, table 41, for 20° FD

$e = 0.019$, for module upto 8, precision gears. Table 42

$$\therefore C = 11860 \times 0.019 = 225.34 N/mn$$

$$\therefore F_d = 9960.16 + \frac{21 \times 2.51 \times 10^3 (80 \times 225.34 + 9960.16)}{21 \times 2.51 \times 10^3 + \sqrt{80 \times 225.34 + 9960.16}}$$

$$F_d = 37858.96 \text{ N}$$

Step 11: Check for beam strength.

We find $F_d > F_s$. Design is not safe.

In order to overcome this issue, increase the module 10mm.

$\therefore \; F_d = 30415.23$ N

& $F_s = 39816$N

$\therefore \; F_s > F_d$. Design is safe.

Step 12: Calculation of maximum wear load. (F_w)

$$F_w = \frac{0.75 \times d_1 \times b \times Q^1 \times K_w}{\cos \delta_1}$$ From PSGDB 8.51

* $Q^1 = \frac{2Z_{v2}}{Z_{v1} + Z_{v2}}$ From PSGDB 8.51

$$= \frac{2 \times 135}{22 + 135}$$

$$Q^1 = 1.72$$

* $K_w = 2.553 \text{ N/mm}^2$, for steel hardened to 400 BHN,

$$\therefore \quad F_w = \frac{0.75 \times 200 \times 100 \times 1.72 \times 2.553}{\cos 21.8°}$$

$$F_w = 70940.66 N$$

Step 13: Check for wear:

$F_w > F_d$. Design is safe

Step 14: Calculation of basic dimensions of pinion and gear.

From PSGDB 8.38 , table 31.

* Transverse module: $m_t = 10$mm

* Number of teeth: $Z_1 = 20$, $Z_2 = 50$
* Pitch circle diameters: $d_1 = 200mm$

  $d_2 = 500mm.$
* Cone distance: $R = 26.93 \times 10 = 269.3mm$
* Face width: $b = 100mm$
* Pitch angles: $\delta_1 = 21.8°$, $\delta_2 = 68.2°$
* Tip diameter: $d_{a1} = m_t(Z_1 + 2\cos\delta_1)$ $\quad$ $d_{a2} = m_t(Z_2 + 2\cos\delta_2)$

  $= 10(20 + 2\cos 21.8°)$ $\quad$ $= 10(50 + 2\cos 68.2°)$

  $d_{a1} = 218.56mm$ $\quad$ $d_{a2} = 507.43mm$
* Height factor: $f_0 = 1$
* Clearance: $c = 0.2$
* Addendum angle: $\tan\theta_{a1} = \tan\theta_{a2} = \frac{m_t \times f_0}{R_1}$

  $= \frac{10 \times 1}{269.3}$

  $= 0.037$

  $\theta_{a1} = \theta_{a2} = 2.13°$
* Deddendum angle:

  $\tan\theta_{f1} = \tan\theta_{f2} = \frac{m_t(f_0 + c)}{R_1}$

  $= \frac{10(1 + 0.2)}{269.3}$

  $= 0.045$

  $\theta_{f1} = \theta_{f2} = 2.55°$
* Tip angle: $\delta_{a1} = \delta_1 + \theta_{a1}$ $\quad$ $\delta_{a2} = \delta_2 + \theta_{a2}$

  $= 21.8 + 2.13$ $\quad$ $= 68.2 + 2.13$

  $\delta_{a1} = 23.93°$ $\quad$ $\delta_{a2} = 70.33°$
* Root angle: $\delta_{f1} = \delta_1 + \theta_{f1}$ $\quad$ $\delta_{f2} = \delta_2 + \theta_{f2}$

$= 21.8 + 2.55$ $\quad\quad$ $= 68.2 + 2.55$

$\delta_{f1} = 19.25°$ $\quad\quad$ $\delta_{f2} = 65.65°$

* Virtual number of teeth:

$$Z_{v1} = 22 \;,\; Z_{v2} = 135$$

4. Design a worm gear drive to transmit 22.5KW at a worm speed of 1440 rpm. velocity ration is 24:1. An efficiency of atleast 85% is desired. The temperature raise should be restricted to 40°C. Determine the required cooling area.

Given data:

$P = 22.5KW$

$N_1 = 1440$ rpm

$i = 24$

$\eta_{desired} = 85\%$

$\Delta_t = t_0 - t_a = 40°c$

Step 1: To find gear ration (i):

$i = 24$ (given)

Step 2: Selection of material.

Assume,

Worm = Hardened steel

Worm wheel = Phosphor bronze.

Step 3: Calculation of Z_1 and Z_2

From PSGDB 8.46 , table 37

For, $\eta = 85\%$, Z_1=3 $\quad\quad$ $\frac{N_1}{N_2} = i$

$\therefore Z_2 = i \times Z_1$ $\quad\quad$ $\frac{1440}{N_2} = 24$

$= 24 \times 3$ $\quad\quad$ $N_2 = 60$rpm.

$Z_2 = 72.$

Step 4: Calculation of q and H:

Case 1: To find diameter factor (q).

From PSGDB 8.43 , table 35, and PSGDB 8.44

$$q = m_x \times d_1$$

Initially we assume q = 11.

Case 2: To find Lead angle (H) .

From PSGDB 8.43 , table 35

$$\tan H = \frac{Z_1}{q}$$

$$H = \tan^{-1}\left(\frac{3}{11}\right)$$

$$H = 15.25°$$

Step 5: Calculation of 'F_t' in terms of 'm_x'

Tangential load $$F_t = \frac{P}{v} \times K_0$$

Case 1: To find the velocity 'v':

$$v = \frac{\pi d_2 N_2}{60 \times 1000}$$

From PSGDB 8.43 , table 35

$$d_2 = Z_2 \times m_x$$

$$\therefore \quad v = \frac{\pi \times Z_2 \times m_x \times N_2}{60 \times 1000}$$

$$= \frac{\pi \times 72 \times m_x \times 60}{60 \times 1000}$$

$$v = 0.226\ m_x\ \text{m/s}$$

Case 2: To find tangential load

Assume medium shock

$$K_0 = 1.5\ .$$

$$\therefore \quad F_t = \frac{22.5\times10^3}{0.226m_x}\times1.5$$

$$\frac{149336.28}{m_x}$$

Step 6: Calculation of dynamic load (F_d)

$$F_d = \frac{F_t}{C_v}$$

Case 1: To find velocity factor (C_v).

From PSGDB 8.51 , assume v= 5 m/s

$$C_v = \frac{6}{6+v}$$

$$= \frac{6}{6+5}$$

$$C_v = 0.545$$

Case 2: To find (F_d).

$$F_d = \frac{149336.28}{m_x}\times\frac{1}{0.545}$$

$$= \frac{274011.53}{m_x}$$

Step 7: Calculation of beam strength (F_s) in terms of (m_x).

From PSGDB 8.51

$$F_s = \pi\times m_x\times b\times[\sigma_b]\times y^1$$

Where,

* $b = 0.75d_1$ From PSGDB 8.48 , table 38

$$= 0.75\times q\times m_x$$

$$= 0.75\times11\times m_x$$

$$= 8.25m_x$$

* $y^1 = 0.125$ From PSGDB 8.52 ,

Assume α=20°

Form factor $y = 0.392$

$$y^1 = \frac{y}{\pi}$$ From PSGDB 8.53 , table 40

$$= \frac{0.392}{\pi} = 0.125$$

* $[\sigma_b] = 80\ N/mm^2$ From PSGDB 8.45 , table 33

$$\therefore \quad F_s = \pi \times m_x \times 8.25 \times m_x \times 80 \times 0.125$$

$$= 259.18\ m_x^2$$

Step 8: Calculation of Axial module (m_x).

From PSGDB 8.51

$$F_s \geq F_d$$

$$259.18\ m_x^2 \geq \frac{274011.53}{m_x}$$

$$m_x \geq 10.18\ mm.$$

PSGDB 8.2 , table 1.

The next nearest higher standard axial module

$$m_x = 12\ mm.$$

Step 9: calculation of b , d_e and v:

Case 1: To find face width (b).

$$b = 8.25\ m_x$$ From step 7

$$= 8.25 \times 12$$

$$b = 99\ mm$$

Case 2: To find Pitch diameter of the worm wheel (d_2).

$d_2 = Z_2 \times m_x$ From step 5 , case 1.

$= 72 \times 12$

$d_2 = 864$ mm

Case 3: To find the Pitch line velocity of worm wheel (v)

$v = 0.226\ m_x$ From step 5 , case 1.

$= 0.226 \times 12$

$v = 2.712\ m/s$

Step 10: Recalculation of beam strength.

$F_s = 259.18\ m_x^2$ From step 7.

$= 259.18 \times 12^2$

$F_s = 37321.92$ N

Step 11: Recalculation of dynamic load (F_d).

$$F_d = \frac{F_t}{C_v}$$

* $C_v = \dfrac{6}{6+v} = \dfrac{6}{6+2.712} = 0.688$

* $F_t = \dfrac{149336.28}{m_x} = \dfrac{149336.28}{12} = 12444.69$ N From step 5 , case 2.

$\therefore\ F_d = \dfrac{12444.69}{0.688}$

$F_d = 18088.21$ N

Step 12: Check for beam strength.

We find $F_s > F_d$. $\therefore$ the design is safe.

Step 13: Check for maximum wear load (F_w).

From PSGDB 8.52

$F_w = d_2 \times 6 \times K_w$

* $K_w = 0.56\ N/mm^2$ From PSGDB 8.54 , table 43.

$F_w = 864 \times 99 \times 0.56$

$F_w = 47900.16\ N$.

We find $F_w > F_d$. $\therefore$ the design is safe.

Step 14: Check for efficiency.

$$\eta_{actual} = 0.95 \times \frac{\tan H}{\tan(H+\rho)}$$ From PSGDB 8.49

Where , $\rho = \tan^{-1}(H)$ Assume H = 0.03From PSGDB 8.49

$$\rho = \tan^{-1}(0.03)$$

$$= 1.7°$$

$$\eta_{actual} = 0.95 \times \frac{\tan 15.25}{\tan(15.25+1.7)}$$

$$= 0.85$$

$$\eta_{actual} = 85\%$$

We find that the actual efficiency is equal to the desired efficiency.

$\therefore$ The design is safe.

Step 15: To find cooling area (A).

In order to avoid overheating, we have to find cooling area.

Heat generated (i.e Power loss) = Heat emitted into the atmosphere.

From PSGDB 8.52 $(1-\eta)\times \text{Input Power} = K_t A(t_0 - t_a)$.

Assume $K_t = 10 w/m^2 °c$

$$(1-0.85)\times 22.5\times 10^3 = 10\times A \times 40$$

Required cooling Area , $A = 8.44\ m^2$

Step 16: Calculation of basic dimension of worm and worm wheel.

From PSGDB 8.43 , table 35.

Axial module: $m_x = 12\ mm$

No. of starts: $Z_1 = 3$

No. of teeth on worm wheel: $Z_2 = 72$

Face width: $b = 99$ mm.

Length of the worm: $L \geq (12.5 + 0.09 Z_2) m_x$

$= (12.5 + 0.09 \times 72) 12$

$= 227.76$ mm.

$L \approx 228$ mm.

Centre distance: $a = 0.5\, m_x (q + Z_2)$

$= 0.5 \times 12 (11 + 72)$

$a = 498$ mm.

Height factor: $f_0 = 1$.

Bottom clearance: $c = 0.25 m_x = 0.25 \times 12 = 3.0$

$c = 3$ mm.

Pitch diameter: $d_1 = q \times m_x = 11 \times 12 = 132$ mm.

$d_2 = 864$ mm

Tip diameter: $d_{a1} = d_1 + 2f_0 \times m_x$

$= 132 + 2 \times 1 \times 12$

$= 156$ mm.

$d_{a2} = Z_2 + 2f_0 \times m_x$

$= (72 + 2(1))12$

$= 888$ mm.

Root diameter: $d_{f1} = d_1 - 2f_0 \times m_x - 2c$

$= 132 - 2 \times 1 \times 12 - 2 \times 3$

$d_{f2} = (Z_2 - 2f_0) m_x - 2c$

$= (72 - 2 \times 1) 12 - 2 \times 3$

$d_{f2} = 834$ mm

5. **Design a straight bevel gear drive between two shafts at right angles to each other. Speed of the pinion shaft is 360 rpm and the speed of the gear. Wheel shaft is 120 rpm. Pinion is of steel and wheel of cast iron. Each gear is expected to work 2 hours / day for 10 years. The drive transmits 9.35 KW.**

Given data: $\theta = 90°$; $N_1 = 360\text{rpm}$; $N_2 = 120\text{rpm}$;

$P = 9.37\text{KW}$

To find: Design the bevel gear drive.

Solution: Since the materials of pinion and gear are different, we have to design the pinion first and check the gear.

1. Gear ratio: $i = \frac{N_1}{N_2} = \frac{360}{120} = 3$

Pitch angles: $\tan\delta_2 = i = 3$ or $\delta_2 = \tan^{-1}(3) = 71.56°$ from PSGDB 8.39

Then, $\delta_1 = 90° - \delta_2 = 90° - 71.56° = 18.44°$

2. Material selection: Pinion – C45 Steel, $\sigma_u = 700\,\text{N/mm}^2$ and $\sigma_y = 360\,\text{N/m}^2$

Gear – CI grade 35, $\sigma_u = 350\,\text{N/mm}^2$

3. Gear life in hours
$= (2\,\text{hours/day}) \times (365\,\text{days/year} \times 10\text{years}) = 7300\text{hours}$

∴ Gear life in cycles, $N = 7300 \times 360 \times 60 = 15.768 \times 10^7\,\text{cycles}$

4. Calculation of initial design torque [M_t]:

We know that, $[M_t] = M_t \times K \times K_d$

Where $M_t = \frac{60 \times P}{2\pi N_1} = \frac{60 \times 9.37 \times 10^3}{2\pi \times 360} = 248.6\text{N}-\text{m}$ and

$K \cdot K_d = 1.3$, initially assumed.

∴ $[M_t] = 248.6 \times 1.3 = 323.28\text{N}-\text{m}$

5. Calculation of E_{eq}, $[\sigma_b]$ and $[\sigma_c]$:

To find E_{eq}: $E_{eq} = 1.7 \times 10^5\,\text{N/mm}^2$ From PSGDB 8.14

To find $[\sigma_{b1}]$: We know that the design bending stress for pinion,

$$[\sigma_{b1}] = \frac{1.4K_{b1}}{n \cdot K_\sigma} \times \sigma_{-1}, \text{ for rotation in one direction}$$

Where $K_{b1} = 1$, for $HB \leq 350$ and $N \geq 10^7$ From PSGDB 8.20, table 22

$K_\sigma = 1.5$, for steel pinion From PSGDB 8.19, table 21

$n = 2.5$, steel hardened table 20, PSGDB 8.19

$\sigma_{-1} = 0.25(\sigma_u + \sigma_y) + 50$, for forged steel - From PSGDB 8.19, table 19

$= 0.25(700 + 360) + 50 = 315\,N/mm^2$

$$[\sigma_{b1}] = \frac{1.4 \times 1}{2.5 \times 1.5} \times 315 = 117.6\,N/mm^2$$

To find $[\sigma_{c1}]$: We know that the design contact stress for pinion,

$[\sigma_{c1}] = C_R \cdot HRC \times K_{cl}$ From PSGDB 8.16

Where $C_R = 23$ From PSGDB 8.16, table 16

$HRC = 40$ to 55 From PSGDB 8.16, table 16

$K_{cl} = 1$, for steel pinion, $HB \leq 350$ and $N \geq 10^7$ From PSGDB 8.16, table 17

$\therefore [\sigma_{c1}] = 23 \times 50 \times 1 = 1150\,N/mm^2$

6. Calculation of cone distance (R):

We know that, $R \geq \psi_y \sqrt{i^2+1}\sqrt[3]{\left[\frac{0.72}{(\psi_y - 0.5)[\sigma_c]}\right]^2 \times \frac{E_{eq}[M_t]}{i}}$ From PSGDB 8.13

Where $\psi_y = R/b = 3$, initially assumed.

$$\therefore \quad R \geq 3\sqrt{3^2+1}\sqrt[3]{\left[\frac{0.72}{(3-0.5)1150}\right]^2 \times \frac{1.7 \times 10^5 \times 323.28 \times 10^3}{3}}$$

≥ 99.36

or $R = 100mm$.

7. Assume $Z_1 = 20$; Then $Z_2 = i \times Z_1 = 3 \times 20 = 60$

Virtual number of teeth: $Z_{v1} = \dfrac{Z_1}{\cos\delta_1} = \dfrac{20}{\cos 18.44^\circ} \approx 22$; and

From PSGDB 8.39 $Z_{v2} = \dfrac{Z_2}{\cos\delta_2} = \dfrac{60}{\cos 71.56^\circ} \approx 190$.

8. Calculation of transverse module (m_t):

We know that, $m_t = \dfrac{R}{0.5\sqrt{Z_1^2 + Z_2^2}}$ From PSGDB 8.38, table 31

$$= \frac{100}{0.5\sqrt{20^2 + 60^2}} = 3.162mm$$

From PSGDB 8.2 , table 1. Under choice 1. The nearest higher standard transverse module is 4mm.

9. Revision of cone distance (R):

We know that, $R = 0.5m_t\sqrt{Z_1^2 + Z_2^2} = 0.5 \times 4\sqrt{20^2 + 60^2} = 126.49mm$

10. Calculation of b, m_{av}, d_{1av}, v and ψ_y :

Face width (b): $b = \dfrac{R}{\psi_y} = \dfrac{126.49}{3} = 42.16mm$ From PSGDB 8.38

Average module (m_{av}):

$$m_{av} = m_t - \frac{b\sin\delta_1}{Z_1} = 4 - \frac{42.16 \times \sin 18.44^\circ}{20} \text{ PSGDB 8.38}$$

$$= 3.333$$

Average pcd of pinion (d_{1av}): $d_{1av} = m_{av} \times Z_1 = 3.333 \times 20 = 66.66mm$

Pitch line velocity (v):

$$v = \frac{\pi \times d_{1av} \times N_1}{60} = \frac{\pi \times 66.66 \times 10^{-3} \times 360}{60} = 1.256\, m/s$$

$$\psi_y = \frac{b}{d_{1av}} = \frac{42.16}{66.66} = 0.632$$

11. IS quality 6 bevel gear is assumed From PSGDB 8.3 , table 2
12. Revision of design torque $[M_t]$:

We know that, $$[M_t] = M_t \times K \times K_d$$

Where $$K = 1.1$$

$$K_d = 1.35$$

$$\therefore \quad [M_t] = 248.6 \times 1.1 \times 1.35 = 369.28 N - m$$

13. Check for bending of pinion: We know that the induced bending stress,

$$\sigma_{b1} = \frac{R\sqrt{i^2+1}[M_t]}{(R-0.5b)^2 \times b \times m_t \times y_{v1}}$$ From PSGDB 8.13 [A]

Where $y_{v1} = 0.402$, for $Z_{v1} = 22$

$$\therefore \quad \sigma_b = \frac{126.49\sqrt{3^2+1} \times 369.28 \times 10^3}{(126.49 - 0.5 \times 42.16)^2 \times 42.16 \times 4 \times 0.402} = 196.09 N/mm^2$$

We find $\sigma_{b1} > [\sigma_{b1}]$. Thus the design is unsatisfactory.

Trial 2: Now we will try with increased transverse module 5mm. Repeating from step 9 again, we get

$$R = 0.5 \times m_t \times \sqrt{Z_1^2 + Z_2^2} = 0.5 \times 5 \times \sqrt{20^2 + 60^2} = 158.11mm$$

$$b = \frac{R}{\psi_y} = \frac{158.11}{3} = 52.7mm$$

$$m_{av} = m_t - \frac{b \sin \delta_1}{Z_1} = 5 - \frac{52.7 \times \sin 18.44}{20} = 4.166mm$$

$$d_{1av} = m_{av} \times Z_1 = 4.166 \times 20 = 83.33mm$$

$$v = \frac{\pi \times d_{1av} \times N_1}{60} = \frac{\pi \times 83.33 \times 10^{-3} \times 360}{60} = 1.57 m/s$$

$$\psi_y = \frac{b}{d_{1av}} = \frac{52.7}{83.33} = 0.632$$

IS quality 6 bevel gear is assumed.

$$K = 1.1; \qquad K_d = 1.35$$

$$M_t = 248.6 \times 1.1 \times 1.35 = 369.28 N-m$$

$$\therefore \quad \sigma_{b1} = \frac{158.11\sqrt{3^2+1} \times 369.28 \times 10^3}{(158.11 - 0.5 \times 52.7)^2 \times 52.7 \times 5 \times 0.402} = 100.4 N/mm^2$$

Now we find $\sigma_{b1} < [\sigma_{b1}]$, thus the design is satisfactory.

14. Check for wearing of pinion: We know that the induced contact stress,

$$\sigma_{c1} = \left(\frac{0.72}{R - 0.5b}\right)\left[\frac{\sqrt{(i^2+1)^3}}{ib} \times E_{eq} \times [M_t]\right]^{\frac{1}{2}} \quad \text{From PSGBD 8.13}$$

$$= \left(\frac{0.72}{158.11 - 0.5 \times 52.7}\right)\left[\frac{\sqrt{(3^2+1)^3}}{3 \times 52.7} \times 1.7 \times 10^5 \times 369.28 \times 10^3\right]^{\frac{1}{2}}$$

$$= 612.33 N/mm^2$$

We find $\sigma_{C1} < [\sigma_{C1}]$. Thus the design is satisfactory for pinion.

15. Check for gear (i.e., wheel): Gear material: CI grade 30.

First we have to calculate $[\sigma_{b2}]$ and $[\sigma_{C2}]$.

Gear life of wheel, $N = \frac{N_{pinion}}{3} = \frac{15.768 \times 10^7}{3} = 5.256 \times 10^7$ cycles

To find $[\sigma_{b2}]$: We know that the design bending stress for gear,

$$[\sigma_{b2}] = \frac{1.4 \times K_{b1}}{n \times K_\sigma} \times \sigma_{-1}$$

Where $K_{b1} = \sqrt[9]{\frac{10^7}{N}} = \sqrt[9]{\frac{107}{5.256 \times 10^7}} = 0.832$

$K_\sigma = 1.2$

$n = 2$

$\sigma_{-1} = 0.45\sigma_u = 0.45 \times 350 = 157.5 N/mm^2$

$$\therefore \quad [\sigma_{b2}] = \frac{1.4 \times 0.832}{2 \times 1.2} \times 157.5 = 76.44 N/mm^2$$

To find $[\sigma_{b2}]$: We know that the design contact stress for gear,

$$[\sigma_{C2}] = C_B \times HB \times K_{cl}$$

Where $C_B = 2.3$

$$HB = 200 \text{ to } 260$$

$$K_{cl} = \sqrt[6]{\frac{10^7}{N}} = \sqrt[6]{\frac{10^7}{5.256 \times 10^7}} = 0.758$$

$\therefore$ $[\sigma_{C2}] = 2.3 \times 260 \times 0.758 = 453.284\,\text{N/mm}^2$

(a) Check for bending of gear: The induced bending stress for gear can be calculated using the relation

$$\sigma_{b1} \times y_{v1} = \sigma_{b2} \times y_{v2}$$

Where $y_{v1} = 0.402$, for $Z_{v1} = 22$

$y_{v2} = 0.520$, for $Z_{v2} = 190$

$\therefore$ $100.4 \times 0.402 = \sigma_{b2} \times 0.520$

or $\sigma_{b2} = 77.6\,\text{N/mm}^2$

We find σ_{b2} is almost equal to $[\sigma_{b2}]$. Thus the design is okay and it can be accepted.

(b) Check for wearing of gear: Since the contact area is same,

$$\sigma_{c2} = \sigma_{c1} = 612.33\,\text{N/mm}^2$$

We find $\sigma_{c2} > [\sigma_{c2}]$. It means the gear does not have adequate beam strength. In order to increase the wear strength of the gear, surface hardness may be raised to 360 BHN. Then we get

$$[\sigma_{b2}] = 2.3 \times 360 \times 0.758 = 627.62\,\text{N/mm}^2$$

Now we find $\sigma_{b2} > [\sigma_{b2}]$, thus the design is safe and satisfactory.

6. The input to the worm gear shaft is 18KW at 600rpm. Speed ratio is 20. The worm is to be of hardened steel and the wheel is made of chilled phosphor bronze. Considering wear and strength, design worm and worm wheel.

Given data:

$N_1 = 600\text{rpm}$

$P = 18\text{KW}$

$i = 20$

Step 1: To find gear ratio (i)

$$i = \frac{N_1}{N_2} = 20(\text{given})$$

$$20 = \frac{600}{N_2}$$

$N_2 = 30\text{rpm}.$

Step 2: Selection of Materail:

Worm = Hardened steel

Worm wheel = Phosphor bronze

Step 3: Calculation of Z_1 and Z_2:

From PSGDB 8.46 , table 37

For $\eta = 80\%$, $Z_1 = 3$

$Z_2 = i \times Z_1 = 20 \times 3$

$Z_2 = 60$

Step 4: Calculation of q and H:

Case 1: To find diameter factor (q)

From PSGDB 8.43 , table 35, and PSGDB 8.44

$$d_1 = \frac{q}{m_x}$$

Initially we assume q= 11.

Case 2: To find Lead angle (H)

From PSGDB 8.43, table 35

$$\tan H = \frac{Z_1}{q}$$

$$H = \tan^{-1}\left(3/11\right)$$

$$H = 15.25°$$

Step 5: Calculation of (F_7) in terms of (m_x).

$$F_t = \frac{P}{v} \times K_0$$

Case 1: To find the velocity 'v'

$$v = \frac{\pi d_2 N_2}{60 \times 1000}$$

From PSGDB 8.42 , table 35

$$d_2 = Z_2 \times m_x$$

$$\therefore \quad v = \frac{\pi \times 60 \times m_x \times 30}{60 \times 1000}$$

$$v = 0.094 m_x \text{ m/s.}$$

Case 2: To find shock factor (K_0)

Assume medium shock, K_0 = 1.5

$$\therefore \quad F_t = \frac{18 \times 10^3}{0.094 m_x} \times 1.5$$

$$F_t = \frac{287234.04}{m_x}$$

Step 6: Calculation of dynamic load (F_d).

$$F_d = \frac{F_t}{C_v}$$

Case 1: To find velocity factor (C_v):

From PSGDB 8.51, assume $v = 5 \text{m/s}$

$$C_v = \frac{6}{6+v}$$

$$= \frac{6}{6+5}$$

$$C_v = 0.545$$

$$\therefore \quad F_d = \frac{287234.04}{m_x} \times \frac{1}{0.545}$$

$$= \frac{527034.94}{m_x}$$

Step 7: Calculation of beam strength (F_s)

From PSGDB8.51,

$$F_s = \pi \times m_x \times b \times [\sigma_b] \times y^1$$

Where,

$b = 0.75 d_1$ From PSGDB 8.48, table 38

$$= 0.75 \times q \times m_x$$

$$= 0.75 \times 11 \times m_x$$

$$= 8.25 m_x$$

$y^1 = 0.125$ From PSGDB 8.52 , Assume $\alpha = 20°$

$[\sigma_b] = 110 N/mm^2$ From PSGDB 8.45, table 33

$$\therefore \; F_s = \pi \times m_x \times 8.25 m_x \times 110 \times 0.125$$

$$= 356.37 m_x^2$$

Step 8: Calculation of axial module (m_x).

$$F_s \geq F_d$$

$$356.37 m_x^2 \geq \frac{527034.94}{m_x}$$

$$m_x \geq 11.4 mm.$$

From PSGDB 8.2, table 1. The next nearest higher standard module m_x = 12mm.

Step 9: Calculation of b , d_2 and v:

From step 7 $\Rightarrow$ $b = 8.25 \times m_x$

$= 8.25 \times 12$

$b = 99mm$

From step 5, Case 1 $\Rightarrow$ $d_2 = Z_2 \times m_x = 60 \times 12$

$= 720mm$

From step 5, Case 1 $\Rightarrow v = 0.094 \times m_x = 0.094 \times 12$

$= 1.13\,m/s$

Step 10: Recalculation of beam strength (F_s)

$F_s = 356.37 \times m_x^2$ From step 7.

$= 356.37 \times 12^2$

$F_s = 51317.28N$

Step 11: Recalculation of dynamic load (F_d).

$$F_d = \frac{F_t}{C_v}$$

$$C_v = \frac{6}{6+v} = \frac{6}{6+1.13} = 0.84$$

$$F_t = \frac{287234.04}{m_x} = \frac{287234.04}{12} = 23936.17N$$

$\therefore F_d = 28495.44N$

We find $F_s > F_d$. The design is safe.

Step 12: Check for maximum wear load (F_w):

From PSGDB 8.52

$F_w = d_2 \times b \times K_w$

$K_w = 0.88\,N/mm^2$ From PSGDB 8.54 , table 43.

$$F_w = 720 \times 99 \times 0.88 = 62726.4N$$

We find $F_w > F_d$ $\therefore$ The design is safe.

Step 13: Check for efficiency.

$$\eta_{actual} = 0.95 \times \frac{\tan H}{\tan(H+e)}$$ From PSGDB 8.49

Where, $e = \tan^{-1}(M)$, Assume $M = 0.05$

$$e = \tan^{-1}(0.05) = 2.86°$$

$$\therefore \eta_{actual} = 0.95 \times \frac{\tan 15.25}{\tan(15.25 + 2.86)}$$

=0.792

$\eta_{actual} = 79.2\%$

We find that the actual efficiency is greater than the desired efficiency.

$\therefore$ The design is safe.

Step 14: Calculation of basic dimensions.

* Axial module: $M_x = 12mm$
* No. of starts: $Z_1 = 3$
* No. of teeth on the worm wheel: $Z_2 = 60$
* Face width of worm wheel: $b = 99mm$
* Length of the worm: $L \geq (12.5 + 0.09Z_2)m_x$

  $= (12.5 + 0.09 \times 60)12$

  $= 214.8mm$

  $L \square 215mm$
* Centre distance: $a = 0.5m_x(q + Z_2)$

  $= 0.5 \times 12(11 + 60)$

  $a = 426mm$.
* Height factor: $f_0 = 1$
* Bottom clearance: $C = 0.25m_x = 0.25 \times 12 = 3mm$

* Pitch diameter: $d_1 = q \times m_x = 11 \times 12 = 132mm$

$$d_2 = 720mm$$

* Tip diameter:

$d_{a1} = d_1 + 2f_0 \times m_x$ $d_{a2} = (Z_2 + 2f_0)m_x$

$= 132 + 2 \times 1 \times 12$ $= (63 + 2(1))12$

$= 156mm$ $= 744mm$

* Root diameter:

$d_{f1} = d_1 - 2f_0 \times m_x - 2c$ $d_{f2} = (Z_2 - 2f_0)m_x - 2c$

$= 132 - 2 \times 1 \times 12 - 2 \times 3$ $= (60 - 2 \times 1)12 - 2 \times 3$

$d_{f1} = 102mm$ $= 690mm$

7. Design a bevel gear device to transmit 3.5KW speed ratio =4. Driving shaft speed =200rpm. The drive is non-reversible. Pinion is of steel and wheel of CI. Assume a life of 25,000hrs.

Given data:

$P = 3.5KW$
$i = 4$
$N_2 = 200rpm.$
$\text{Material} \Rightarrow \text{Pinion} - \text{Steel}$
$\text{Wheel} - \text{CI}$

The materials of pinion and gear are different, we have to design the pinion first and check the gear.

Step 1: Gear ratio & Pitch angles:

$$i = \frac{N_1}{N_2} = 4$$

$$\frac{N_1}{200} = 4$$

$$N_1 = 800rpm.$$

Pitch angles:

From PSGDB 8.34

$$\tan\delta_2 = i = 4$$

$$\therefore\ \delta_2 = \tan^{-1}(4) = 75.96°$$

$$\delta_1 = 90 - \delta_2 = 90 - 75.96$$

$$= 14.04°$$

Step 2: Material selection:

Pinion: Steel, $\sigma_u = 700 N/mm^2 \quad \sigma_y = 360 N/m^2$

Gear: CI grad 35 - $\sigma_u = 350 N/mm^2$

Step 3: Gear life in cycles:

Gear life in hours $= 25000 hrs.$

Gear life in cycles $N = 25000 \times 800 \times 60$

$= 12 \times 10^8$ cycles.

Step 4: Calculation of initial design torque $[M_t]$:

From PSGDB 8.44,

$$[M_t] = M_t \times K \times K_d$$

$$M_t = \frac{60 \times P}{2\pi N_1} = \frac{60 \times 3.5 \times 10^3}{2 \times \pi \times 800} = 41.778 N.m$$

$K.K_d = 1.3$ initially assume.

$$\therefore\ [M_t] = 41.778 \times 1.3$$

$$= 54.31 Nm.$$

Step 5: Calculation of E_{eq}, $[\sigma_b]$ and $[\sigma_c]$:

To find E_{eq}.

$E_{eq} = 1.7 \times 10^5 N/mm^2$ From PSGDB8.14

To find $[\sigma_{b1}]$ Design bending stress for pinion.

$$[\sigma_{b1}] = \frac{1.4K_{b1}}{n \times K_\sigma} \times \sigma - 1$$ From PSGDB 8.18 rotation in one direction

$K_{b1} = 1$,for $HB \le 350$ and $N\,Z10^{7'}$ From PSGDB 8.20, table 22.

$K_\sigma = 1.5,$ for steel pinion. From PSGDB 8.19, table 21

$n = 2.5$ steel hardened From PSGDB 8.19, table 20

$\sigma_{-1} = \left(0.25\left(\sigma_u + \sigma_y\right) + 50\right)$, for forged steel. From PSGDB 8.19, table 19

$$\sigma_{-1} = 0.25(700 + 360) + 50$$

$$= 315 N / mm^2.$$

$$\therefore [\sigma_{b1}] = \frac{1.4 \times 1}{2.5 \times 1.5} \times 315 = 117.6 N / mm^2.$$

To find $[\sigma_{c1}]$:

$$[\sigma_{c1}] = C_R.HRC \times K_{C1}$$

$$C_R = 23$$

$$HRC = 40 to 55$$

$$K_{C1} = 1$$

$$[\sigma_{c1}] = 23 \times 55 \times 1$$

$$= 1265 N / mm^2$$

Step 6: Calculation of cone distance (R).

$$R \ge \varphi_y \sqrt{i^2 + 1} \sqrt[3]{\left[\frac{0.72}{(\varphi_y - 0.5)[\sigma_c]}\right]^2 \times \frac{E_{eq}[M_t]}{i}}$$

$$\varphi_y = R/6 = 3$$

$$R \ge 3\sqrt{4^2 + 1} \sqrt{\left[\frac{0.72}{(3 - 0.5) \times 1265}\right]^2 \times \left[\frac{1.7 \times 10^5 \times 54.31 \times 10^3}{4}\right]}$$

$$\ge 135.29 mm.$$

$R = 136\text{mm}.$

Step 7: Selection of No. of teeth on pinion and gear.

$Z_1 = 20$

$Z_2 = i \times Z_1$

4×20

$= 80$

Virtual no. of teeth: $Z_{v1} = \dfrac{Z_1}{\cos\delta_1} = \dfrac{20}{\cos 14.04} = 20.61 \approx 21$

From PSGDB 8.22. $Z_{v2} = \dfrac{Z_2}{\cos\delta_2} = \dfrac{80}{\cos 75.96°} = 329.76 \approx 330$

Step 8: Calculation of transverse module: (m_t).

$$M_t = \frac{R_1}{0.5\sqrt{Z_1^2 + Z_2^2}}$$ From PSGDB 8.38 table 31

$$= \frac{136}{0.5\sqrt{20^2 + 80^2}}$$

$$= 3.29\text{mm}$$

From PSGDB 8.2, table 1, choice 1.

The nearest next higher standard transverse module m_t =4mm.

Step 9: Revision of cone distance: (R)

$$R = 0.5 m_t \sqrt{Z_1^2 + Z_2^2}$$

$$= 0.5 \times 4\sqrt{20^2 + 80^2}$$

$$= 164.92\text{mm}.$$

Step 10: Calculation of b, m_{av} , d_{1av}, v and φ_y :

(i) To find b: $b = \dfrac{R}{\varphi_y} = \dfrac{164.92}{3} = 54.97 \approx 55\text{mm}$ PSGDB 8.38

(ii) Average module $m_{av} = m_t - \dfrac{b\sin\delta_1}{Z_1}$ PSGDB 8.38

$$= 4 - \frac{55 SIN 14.04}{20}$$

$$= 3.33 mm.$$

(iii) Average PCD of pinion: $d_{1av} = m_{av} \times Z_1$

$$= 3.33 \times 20$$

$$= 66.66 mm.$$

(iv) Pitch line velocity $v = \frac{\pi d_{1av} \times N_1}{60} = \frac{\pi \times 66.66 \times 800}{60 \times 1000} = 2.79 \, m/s.$

(v) $\varphi_y = \frac{b}{d_{1av}} = \frac{55}{66.66} = 0.83$

Step 11: Selection of Quality of gears.

Is Quality 8 bevel gear is selected. From PSGDB 8.3 table 2.

Step 12: Revision of design torque $[M_t]$:

$$[M_t] = M_t \times K \times K_d$$

$K = 1.1$ From PSGDB 8.15

$K_d = 1.45$ From PSGDB 8.16 table 15.

$$\therefore \; [M_t] = 41.778 \times 1.1 \times 1.45$$

$$= 66.64 Nm.$$

Step 13: Check for bending of pinion.

$$\sigma_{b1} = \frac{R\sqrt{l^2+1}\,[M_t]}{(R-0.5b)^2 \times b \times m_t \times y_{v1}}$$ From PSGDB 8.13[A].

$y_{v1} = 0.402$ for $Z_{v1} = 21$

$$\therefore \; \sigma_{b1} = \frac{164.92\sqrt{4^2+1} \times 66.64 \times 10^3}{(164.92 - 0.5 \times 55)^2 \times 55 \times 4 \times 0.402}$$

$$\sigma_{b1} = 27.13 N/mm^2$$

We find $\sigma_{b1} < [\sigma_{b1}]$. $\therefore$ the design is safe.

Step 14: Check for wearing of pinion.

$$\sigma_{c1} = \left(\frac{0.72}{R-0.56}\right)\left[\frac{\sqrt{(i^2+1)^3}}{i\times b}\times E_{eq}\times[M_t]\right]^{1/2}$$ From PSGDB 8.13

$$= \left(\frac{0.72}{164.92-0.5\times 55}\right)\left[\frac{\sqrt{(4^2+1)^3}}{4\times 55}\times 1.7\times 10^5\times 66.64\times 10^3\right]^{1/2}$$

$$\sigma_{C1} = 314.77 N/mm^2$$

We find $\sigma_{C1} < [\sigma_C]$. Thus the design is satisfactory for pinion.

Step 15: Cheek for gear.

Gear material: CI grade 30.

First we have to calculate $[\sigma_{b2}]$ and $[\sigma_{c2}]$.

Gear life of wheel $N = \frac{N_{pinion}}{3}$

$$= \frac{12\times 10^8}{3}$$

$$= 4\times 10^8$$

To find $[\sigma_{b2}]$:

$$[\sigma_{b2}] = \frac{1.4\times K_{b1}}{h\times K_\sigma}\times\sigma_{-1}$$

Where,

$$K_{b1} = \sqrt[9]{\frac{10^7}{N}} = \sqrt[9]{\frac{10^7}{4\times 10^8}} = 0.66$$

$$K_\sigma = 1.2$$

$$n = 2$$

$$\sigma_{-1} = 0.45\sigma_u = 0.45\times 350 = 157.5 N/mm^2.$$

$$\therefore\ [\sigma_{b2}] = \frac{1.4\times 0.66}{2\times 1.2}\times 157.5$$

$= 60.64 N.mm^2.$

To find $[\sigma_{c2}]$:

$$[\sigma_{c2}] = C_B \times HB \times K_{cl}.$$

Where $C_B = 2.3,$ $HB = 200$ to 260

$$K_{cl} = \sqrt[b]{\frac{10^7}{N}} = \sqrt[b]{\frac{10^7}{4\times10^8}} = 0.54$$

$$\therefore [\sigma_{c2}] = 2.3 \times 260 \times 0.54$$

$$= 322.92 N/mm^2.$$

Case 1: Check for bending of gear.

$$\sigma_{b1} \times y_{v1} = \sigma_{b2} \times y_{v2}$$

$y_{v1} = 0.402$ for $Z_{v1} = 21$

$y_{v2} = 0.521$ for $Z_{v2} = 330$

$$27.13 \times 0.402 = \sigma_{b2} \times 0.521$$

$$\sigma_{b2} = 20.93 N/mm^2.$$

$\sigma_{b2} < [\sigma_{b2}].$ $\therefore$ design is safe.

Case 2: Check for wearing of gear.

Since the contact area is same.

$\therefore$ $\sigma_{c2} = \sigma_{c1} = 314.77 N/mm^2$

We find $\sigma_{c2} < [\sigma_{c2}]$. It means the gear having the adequate beam strength.

$\therefore$ The design is safe and satisfactory.

8. Design a worm gear drive to transmit 20KW at 1440rpm. Speed of worm wheel is 60rpm.

Given data:

$P = 20KW$

$N_1 = 1440rpm.$

$N_2 = 60rpm.$

Step 1: To find gear ratio (i).

$i = \frac{N_1}{N_2}$

$= \frac{1440}{60}$

$i = 24$

Step 2: Selection of Material:

Worm = Hardened steel

Worm wheel = Phosphor bronze.

Step 3: Calculation of Z1 and Z2:

From PSGDB 8.46 , table 37.

For $\eta = 80\%$, $Z_1 = 3$

$Z_2 = i \times Z_1 = 24 \times 3$

$= 72$

Step 4: Calculation of q and H:

Case 1: To find diameter factor (q):

From PSGDB 8.43 , table 35 and PSGDB 8.44

$$d_1 = \frac{q}{m_x}$$

Initially we assume q=11

Case 2: To find Lead angle (H).

From PSGDB 8.43 , table 35

$$\tan H = \frac{Z_1}{q} = \frac{3}{11}$$

$$H = 15.25°$$

Step 5: Calculation of (F_t) in terms of (m_x).

$$F_t = \frac{P}{v} \times \kappa_0$$

Case 1: To find the velocity 'v'

$$v = \frac{\pi d_2 N_2}{60 \times 1000}$$

From PSGDB 8.43 , table 35.

$$d_2 = z_2 \times m_x$$

$$v = \frac{\pi \times z_2 \times m_x \times N_2}{60 \times 1000}$$

$$= \frac{\pi \times 72 \times m_x \times 60}{60 \times 1000}$$

$$v = 0.226 m_x \ m/s$$

Case 2: To find shock factor (K_0).

Assume medium shock , $K_0 = 1.5$

$$\therefore F_t = \frac{20 \times 10^3}{0.226 m_x} \times 1.5$$

$$F_t = \frac{132629.12}{m_x}$$

Step 6: Calculation of dynamic load (F_d).

$$F_d = \frac{F_t}{C_v}$$

Case 1: To find velocity factor (C_v):

From PSGDB 8.51, Assume $v = 5 m/s$

$$C_v = \frac{6}{6+v} = \frac{6}{6+5} = 0.545.$$

$$\therefore\ F_d = \frac{132629.12}{m_x} \times \frac{1}{0.545}$$

$$F_d = \frac{243356.18}{m_x}$$

Step 7: Calculation of beam strength (F_s).

From PSGDB 8.51

$$F_s = \pi \times m_x \times b \times [\sigma_b] \times y^1$$

Where,

* $b = 0.75d_1$ From PSGDB 8.48 , table 38

$= 0.75 \times q \times m_x$

$= 8.25m_x$

* $y^1 = 0.125$ From PSGDB 8.52 , Assume $\alpha = 20^\circ$
* $[\sigma_b] = 110\,N/mm^2$ From PSGDB 8.45 , table 33.

$\therefore\ F_s = \pi \times m_x \times 8.25 \times m_x \times 110 \times 0.125$

$= 356.37m_x^2$

Step 8: Calculation of axial module: (m_x)

$$F_s \geq F_d$$

$$356.37m_x^2 \geq \frac{243356.18}{m_x}$$

$$m_x \geq 8.81mm.$$

From PSGDB 8.2 , table 1.The next nearest higher standard module m_x = 10 mm

Step 9: Calculation of b , d_2 and v:

From step 7 ⇒ $b = 8.25m_x = 8.25 \times 10$

From step 5, case 1 ⇒ $d_2 = Z_2 \times m_x = 72 \times 10$

$= 720\text{mm.}$

From step 5, case 1 ⇒ $v = 0.226 \times m_x = 0.226 \times 10$

$= 2.26 \, m/s$.

Step 10: Recalculation of beam strength (F_s).

$F_s = 356.37 \times m_x^2$ From step 7

$= 356.37 \times 10^2$

$F_s = 35637$ N

Step 11: Recalculation of dynamic load (F_d).

$$F_d = \frac{F_t}{C_v}$$

* $C_v = \dfrac{6}{6+v} = \dfrac{6}{6+2.26} = 0.73$

* $F_t = \dfrac{132629.12}{m_x} = \dfrac{132629.12}{10} = 13262.91$ N

$$\therefore \quad F_d = \frac{13262.91}{0.73}$$

$F_d = 18168.37$ N

We find $F_s > F_d$. The design is safe.

Step 12: Check for Maximum wear load (F_w)

From PSGDB 8.52,

$F_w = d_2 \times b \times K_w$

* $K_w = 0.88 \, N/mm^2$ From PSGDB 5.54 , table 43.

$F_w = 720 \times 82.5 \times 0.88$

$= 52272$ N

We find $F_w > F_d$. ∴ The design is safe.

Step 13: Check for efficiency.

$$\eta_{actual} = 0.95 \times \frac{\tan H}{\tan(H+e)}$$ From PSGDB 8.49.

Where,

$e = \tan^{-1} M$, take $M = 0.04$ From PSGDB 8.49, Graph.

$$e = \tan^{-1}(0.04) = 2.29°$$

$$\eta_{actual} = 0.95 \times \frac{\tan 15.25}{\tan(15.25 + 2.29)}$$

$$= 0.82$$

$$= 82\%$$

We find that the actual efficiency is greater than the desired efficiency. $\therefore$The design is safe.

Step 14: Calculation of basic dimensions.

From PSGDB 8.43 , table 35.

* Axial module: $m_x = 10mm.$
* No. of starts: $Z_1 = 3$
* No. of teeth on the worm wheel: $Z_2 = 72$
* Face width of the worm wheel: $b = 82.5mm.$
* Length of the worm: $L \geq (12.5 + 0.09Z_2) m_{x.}$

$$= (12.5 + 0.09 \times 72)10$$

$$= 189.8$$

$$L \square 190mm$$

* Centre distance: $a = 0.5m_x(q + Z_2).$

$$= 0.5 \times 10(11 + 72)$$

$$a = 415mm$$

* Height factor: $f_0 = 1$
* Bottom clearance: $c = 0.25m_x = 0.25 \times 10 = 2.5mm.$
* Pitch diameter: $d_1 = q \times m_x = 11 \times 10 = 110mm.$

$$d_2 = 720mm.$$

* Tip diameter:

$d_{a1} = d_1 + 2f_0 \times m_x$ $\qquad$ $d_{a2} = (Z_2 + 2f_0)m_x$

$= 110 + 2 \times 1 \times 10$ $\qquad$ $(72 + 2 \times 1)10$

$= 130mm.$ $\qquad$ $= 740mm.$

* Root diameter:

$d_{f1} = d_1 - 2f_0 \times m_x - 2$ $\qquad$ $d_{f2} = (Z_2 - 2f_0)m_x - 2c$

$= 110 - 2 \times 1 \times 10 - 2 \times 2.5$ $\qquad$ $= (72 - 2 \times 1)10 - 2 \times 2.5$

$= 85mm.$ $\qquad$ $= 715mm.$

9. Design a bevel gear drive to transmit 3.5 KW with dividing shaft is 200 rpm. Speed ratio required is 4. The drive is no – reversible pinion is made of steel and wheel made of steel and wheel made of CI. Assume lite at 25000 hrs.

Given data:

$P = 3.5KW$
$i = 4$
$N_2 = 200rpm.$
Material $\Rightarrow$ Pinion – Steel
Wheel – CI

The materials of pinion and gear are different, we have to design the pinion first and check the gear.

Step 1: Gear ratio & Pitch angles:

$$i = \frac{N_1}{N_2} = 4$$

$$\frac{N_1}{200} = 4$$

$$N_1 = 800rpm.$$

Pitch angles:

From PSGDB 8.34

$\tan\delta_2 = i = 4$

$\therefore\ \delta_2 = \tan^{-1}(4) = 75.96°$

$\delta_1 = 90 - \delta_2 = 90 - 75.96$

$= 14.04°$

Step 2: Material selection:

Pinion: Steel, $\sigma_u = 700 N/mm^2 \quad \sigma_y = 360 N/m^2$

Gear: CI grad 35 - $\sigma_u = 350 N/mm^2$

Step 3: Gear life in cycles:

Gear life in hours $= 25000 hrs.$

Gear life in cycles $N = 25000 \times 800 \times 60$

$= 12 \times 10^8$ cycles.

Step 4: Calculation of initial design torque $[M_t]$:

From PSGDB 8.44,

$$[M_t] = M_t \times K \times K_d$$

$$M_t = \frac{60 \times P}{2\pi N_1} = \frac{60 \times 3.5 \times 10^3}{2 \times \pi \times 800} = 41.778 N.m$$

$K.K_d = 1.3$ initially assume.

$\therefore\ [M_t] = 41.778 \times 1.3$

$= 54.31$ Nm.

Step 5: Calculation of E_{eq}, $[\sigma_b]$ and $[\sigma_c]$:

To find E_{eq}.

$E_{eq} = 1.7 \times 10^5 N/mm^2$ From PSGDB8.14

To find $[\sigma_{b1}]$ Design bending stress for pinion.

$[\sigma_{b1}] = \frac{1.4 K_{b1}}{n \times K_\sigma} \times \sigma - 1$ From PSGDB 8.18 rotation in one direction

$K_{b1} = 1$, for HB ≤ 350 and $N Z 10^{7,}$ From PSGDB 8.20, table 22.

$K_\sigma = 1.5,$ for steel pinion. From PSGDB 8.19, table 21

$n = 2.5$ steel hardened From PSGDB 8.19, table 20

$\sigma_{-1} = \left(0.25\left(\sigma_u + \sigma_y\right) + 50\right)$, for forged steel. From PSGDB 8.19, table 19

$\sigma_{-1} = 0.25(700 + 360) + 50$

$= 315 N/mm^2.$

$\therefore [\sigma_{b1}] = \frac{1.4 \times 1}{2.5 \times 1.5} \times 315 = 117.6 N/mm^2.$

To find $[\sigma_{c1}]$:

$[\sigma_{c1}] = C_R.HRC \times K_{C1}$

$C_R = 23$

$HRC = 40 to 55$

$K_{C1} = 1$

$[\sigma_{c1}] = 23 \times 55 \times 1$

$= 1265 N/mm^2$

Step 6: Calculation of cone distance (R).

$$R \geq \varphi_y \sqrt{i^2 + 1} \sqrt[3]{\left[\frac{0.72}{(\varphi_y - 0.5)[\sigma_c]}\right]^2 \times \frac{E_{eq}[M_t]}{i}}$$

$\varphi_y = R/_6 = 3$

$$R \geq 3\sqrt{4^2 + 1} \sqrt{\left[\frac{0.72}{(3 - 0.5) \times 1265}\right]^2 \times \left[\frac{1.7 \times 10^5 \times 54.31 \times 10^3}{4}\right]}$$

$\geq 135.29 mm.$

$R = 136 mm.$

Step 7: Selection of No. of teeth on pinion and gear.

$Z_1 = 20$

$$Z_2 = i \times Z_1$$

$$4 \times 20$$

$$= 80$$

Virtual no. of teeth: $Z_{v1} = \frac{Z_1}{\cos\delta_1} = \frac{20}{\cos 14.04} = 20.61 \approx 21$

From PSGDB 8.22. $Z_{v2} = \frac{Z_2}{\cos\delta_2} = \frac{80}{\cos 75.96°} = 329.76 \approx 330$

Step 8: Calculation of transverse module: (m_t).

$$M_t = \frac{R_1}{0.5\sqrt{Z_1^2 + Z_2^2}}$$ From PSGDB 8.38 table 31

$$= \frac{136}{0.5\sqrt{20^2 + 80^2}}$$

$$= 3.29mm$$

From PSGDB 8.2, table 1, choice 1.

The nearest next higher standard transverse module m_t =4mm.

Step 9: Revision of cone distance: (R)

$$R = 0.5m_t\sqrt{Z_1^2 + Z_2^2}$$

$$= 0.5 \times 4\sqrt{20^2 + 80^2}$$

$$= 164.92mm.$$

Step 10: Calculation of b, m_{av} , d_{1av}, v and φ_y :

(i) To find b: $b = \frac{R}{\varphi_y} = \frac{164.92}{3} = 54.97 \approx 55mm$ PSGDB 8.38

(ii) Average module $m_{av} = m_t - \frac{b\sin\delta_1}{Z_1}$ PSGDB 8.38

$$= 4 - \frac{55SIN14.04}{20}$$

$$= 3.33mm.$$

(iii) Average PCD of pinion: $d_{1av} = m_{av} \times Z_1$

$$= 3.33 \times 20$$

$$= 66.66\text{mm.}$$

(iv) Pitch line velocity $v = \frac{\pi d_{1av} \times N_1}{60} = \frac{\pi \times 66.66 \times 800}{60 \times 1000} = 2.79\, m/s$.

(v) $\varphi_y = \frac{b}{d_{1av}} = \frac{55}{66.66} = 0.83$

Step 11: Selection of Quality of gears.

Is Quality 8 bevel gear is selected. From PSGDB 8.3 table 2.

Step 12: Revision of design torque $[M_t]$:

$$[M_t] = M_t \times K \times K_d$$

$K = 1.1$ From PSGDB 8.15

$K_d = 1.45$ From PSGDB 8.16 table 15.

$$\therefore \; [M_t] = 41.778 \times 1.1 \times 1.45$$

$$= 66.64\text{Nm.}$$

Step 13: Check for bending of pinion.

$$\sigma_{b1} = \frac{R\sqrt{i^2+1}\,[M_t]}{(R-0.5b)^2 \times b \times m_t \times y_{v1}}$$ From PSGDB 8.13[A].

$y_{v1} = 0.402$ for $Z_{v1} = 21$

$$\therefore \; \sigma_{b1} = \frac{164.92\sqrt{4^2+1} \times 66.64 \times 10^3}{(164.92 - 0.5 \times 55)^2 \times 55 \times 4 \times 0.402}$$

$$\sigma_{b1} = 27.13\,\text{N/mm}^2$$

We find $\sigma_{b1} < [\sigma_{b1}]$. $\therefore$ the design is safe.

Step 14: Check for wearing of pinion.

$$\sigma_{c1} = \left(\frac{0.72}{R-0.56}\right)\left[\frac{\sqrt{(i^2+1)^3}}{i \times b} \times E_{eq} \times [M_t]\right]^{1/2}$$ From PSGDB 8.13

$$= \left(\frac{0.72}{164.92 - 0.5 \times 55}\right)\left[\frac{\sqrt{\left(4^2+1\right)^3}}{4 \times 55} \times 1.7 \times 10^5 \times 66.64 \times 10^3\right]^{1/2}$$

$$\sigma_{C1} = 314.77 N/mm^2$$

We find $\sigma_{C1} < [\sigma_C]$. Thus the design is satisfactory for pinion.

Step 15: Cheek for gear.

Gear material: CI grade 30.

First we have to calculate $[\sigma_{b2}]$ and $[\sigma_{c2}]$.

Gear life of wheel $N = \frac{N_{pinion}}{3}$

$$= \frac{12 \times 10^8}{3}$$

$$= 4 \times 10^8$$

To find $[\sigma_{b2}]$:

$$[\sigma_{b2}] = \frac{1.4 \times K_{b1}}{h \times K_\sigma} \times \sigma_{-1}$$

Where,

$$K_{b1} = \sqrt[9]{\frac{10^7}{N}} = \sqrt[9]{\frac{10^7}{4 \times 10^8}} = 0.66$$

$$K_\sigma = 1.2$$

$$n = 2$$

$$\sigma_{-1} = 0.45\sigma_u = 0.45 \times 350 = 157.5 N/mm^2.$$

$$\therefore [\sigma_{b2}] = \frac{1.4 \times 0.66}{2 \times 1.2} \times 157.5$$

$$= 60.64 N.mm^2.$$

To find $[\sigma_{c2}]$:

$$[\sigma_{c2}] = C_B \times HB \times K_{cl}.$$

Where $C_B = 2.3,\quad HB = 200 \text{ to } 260$

$$K_{cl} = \sqrt[b]{\frac{10^7}{N}} = \sqrt[b]{\frac{10^7}{4\times10^8}} = 0.54$$

$$\therefore [\sigma_{c2}] = 2.3\times260\times0.54$$

$$= 322.92 N/mm^2.$$

Case 1: Check for bending of gear.

$$\sigma_{b1}\times y_{v1} = \sigma_{b2}\times y_{v2}$$

$y_{v1} = 0.402$ for $Z_{v1} = 21$

$y_{v2} = 0.521$ for $Z_{v2} = 330$

$$27.13\times0.402 = \sigma_{b2}\times0.521$$

$$\sigma_{b2} = 20.93 N/mm^2.$$

$\sigma_{b2} < [\sigma_{b2}]$. $\therefore$ design is safe.

Case 2: Check for wearing of gear.

Since the contact area is same.

$$\therefore\ \sigma_{c2} = \sigma_{c1} = 314.77 N/mm^2$$

We find $\sigma_{c2} < [\sigma_{c2}]$. It means the gear having the adequate beam strength.

$\therefore$ The design is safe and satisfactory.

10. A hardened steel worm rotates at 1440 rpm and transmits 12 kW to a phosphor Bronze gear. The speed of the worm gear should be 60 rpm. Design the worm gear drive if an efficiency of at least 82% is desired.

Given data:

$N_{max} = 1440$ rpm, $N_{min} = 60$ rpm $p = 12kW$ $\eta_{desired} = 82\%$

Gear ratio required, $i = \frac{1440}{60} = 24$

1. Material selection: Worm – Hardened steel, and

Worm – Phosphor bronze

2. Selection of z_1 and z_2:

For η = 85%, z_1 = 3
Then, $z_2 = i \times z_1 = 24 \times 3 = 72$.

3. Calculation of q and γ:

Diameter factor: $q = \frac{d_1}{m_x} = 11$

Lead angle: $\gamma = \tan^{-1}\left(\frac{z_1}{q}\right) = \tan^{-1}\left(\frac{3}{11}\right) = 15.25°$

4. Calculation of F_1 in terms m_x:

Tangential load, $F_t = \frac{P}{v} \times K_0$

Where

$$v = \frac{\pi d_2 N_2}{60 \times 1000} = \frac{\pi(z_2 \times m_x) \times N_2}{60 \times 1000}$$
$$= \frac{\pi \times 72 \times m_x \times 60}{60 \times 1000} = 0.226 m_x \text{ m/s}$$

$K_0 = 1.25$, assuming medium shock

$$F_t = \frac{12 \times 10^3}{0.226 m_x} \times 1.25 = \frac{66371.68}{m_x}$$

5. Calculation of dynamic load (F_d):

Dynamic load, $F_d = \frac{F_t}{c_v}$

$$c_v = \frac{6}{6+v}, \; v = 5 \text{ m/s is assumed.}$$
$$= \frac{6}{6+5} = 0.545$$
$$F_d = \frac{66371.68}{m_x} \times \frac{1}{0.545} = \frac{121681.4}{m_x}$$

6. Calculation of beam strength (F_s) in terms of axial module:

Beam strength, $F_s = \pi \times m_x \times b \times [\sigma_b] \times y$

Where

$$b = 0.75 d_1$$
$$= 0.75 \times q m_x = 0.75 \times 11 m_x = 8.25 m_x$$
$$[\sigma_b] = 80 \text{ N/mm}^2$$
$$y = 0.125, \text{ assuming } \alpha = 20°$$
$$F_x = \pi \times m_x \times 8.25 m_x \times 80 \times 0.125 = 259.18 m_x^2$$

7. Calculation of axial module (m_x):

We know that,

$$259.18 m_x^2 \geq \frac{121681.4}{m_x}$$
$$m_x \geq 7.77\,\text{mm}$$

The nearest higher standard axial pitch is 8 mm.

8. b = 66 mm, d_2 = 576 mm; v = 1.808 m/s

9. Fs = $259.18 m_x^2$ = 16587. 52 N

10. Dynamic load, $F_d = \frac{F_t}{c_v}$

$$c_v = \frac{6}{6+v} = \frac{6}{6+1.808} = 0.768 \text{ and}$$
$$F_t = \frac{66371.68}{m_x} = \frac{66371.68}{8} = 8296.46\text{N}$$
$$F_d = \frac{8296.46}{0.768} = 10802.68\text{N}$$

11. Check for beam strength: We find $F_d < F_s$. It means that the gear tooth has adequate beam strength and will not fail by breakage. Thus the design is satisfactory.

12. Calculation of maximum wear load (F_w):

Wear load, $F_w = d_2 \times b \times K_w$

Where

$$K_w = 0.56\text{N}/\text{mm}^2$$
$$F_w = 576 \times 66 \times 0.56 = 21288.96\text{N}$$

13. Check for wear: We find $F_d < F_w$. It means that the gear tooth has adequate wear capacity and will not wear out. Thus the design is safe and satisfactory.

14. Check for efficiency: We know that,

$$\eta_{actual} = 0.95 \frac{\tan\gamma}{\tan(\gamma+\rho)}$$

Where

$$\rho = \text{Frictional angle} = \tan^{-1}\mu$$
$$= \tan^{-1}(0.03) = 1.7°$$
$$\eta = 0.95 \times \frac{\tan 15.25°}{\tan(15.25° + 1.7°)} = 0.8498 \text{ or } 84.98\%$$

We find that the actual efficiency is greater than the desired efficiency. Thus the design is satisfactory.

15. Calculation of basic dimensions of worm and worm gears:

Axial module: m_x = 8 mm
Number of starts: z_1 = 3
Number of teeth on worm wheel: z_2 = 72
Face width of worm wheel: b = 66 mm
Length of worm: $L \geq (12.5+0.09z_2)m_x$, $L \geq (12.5+0.09\times72)8 = 151.84\text{mm}$
Centre distance: $a = 0.5m_x(q+z_2) = 0.5\times8(11+72) = 332\text{mm}$
Height factor: f_0 = 1
Bottom clearance: $c = 0.25m_x = 0.25\times8 = 2\text{mm}$

Pitch diameter: $d_1 = q\times m_x = 11\times8 = 88\text{mm}$
$d_2 = z_2\times m_x = 72\times8 = 576\text{mm}$

Tip diameter: $d_{a1} = d_1 + 2f_0.m_x = 88+2\times1\times8 = 104\text{ mm}$
$d_{a2} = (z_2+2f_0)m_x = (72+2\times1)8 = 592\text{ mm}$

Root diameter: $d_{f1} = d_1 - 2f_0.m_x - 2.c = 88-2\times1\times8-2\times2 = 68\text{mm}$
$d_{f2} = (z_2-2f_0)m_x - 2.c = (72-2\times1)8-2\times2 = 556\text{ mm}$

11. A hardened steel worm rotates at 1440rpm and transmits 12KW to a phosphor bronze gear. The speed of the worm wheel should be 60 ± 3%rpm. Design a worm gear drive if an efficiency of at least 82% is desired. (April/May 2017)

Given data:

$N_1 = 1440\text{rpm}$

$P = 12\text{KW}$

$N_2 = 60\pm3\%\text{rpm}$

$\eta_{desired} = 82\%$

******Similar to this problem, power has to be changed to 20HP=15 kW and efficiency is 80%***

Step 1: To find gear ratio (i) :

$$i = \frac{N_1}{N_2}\pm3\%$$

$$= \frac{1440}{60}\pm3\%$$

$$= 24\pm0.72$$

$$\text{take } i = 24$$

Step 2: Selection of Material:

Worm = Hardened steel

Worm wheel = Phosphor bronze

Step 3: Calculation of Z_1 and Z_2:

From PSGDB 8.46, table 37.

For $\eta = 82\%$, $Z_1 = 3$

$$Z_2 = i \times Z_1$$

$$= 24 \times 3$$

$$Z_2 = 72$$

Step 4: Calculation of q and H:

Case 1: To find diameter factor (q):

From PSGDB 8.43, table 35, and PSGDB 8.44

$$d_1 = \frac{q}{m_x}$$

Initially we assume $q = 11$

Case 2: To find Lead angle (H):

From PSGDB 8.43 , table 35

$$\tan H = \frac{Z_1}{q}$$

$$H = \tan^{-1}\left(\frac{3}{11}\right)$$

$$H = 15.25°$$

Step 5: Calculation of 'F_t' in terms of 'm_x':

Tangential Load $F_t = \frac{P}{v} \times K_0$

Case 1: To find the velocity 'v':

$$v = \frac{\pi d_2 N_2}{60 \times 1000}$$

From PSGDB 8.43 , table 35

$d_2 = Z_2 \times m_x$

$$\therefore \ v = \frac{\pi \times Z_2 \times m_x \times N_2}{60 \times 1000}$$

$$= \frac{\pi \times 72 \times m_x \times 60}{60 \times 1000}$$

$$v = 0.226 m_x \ m/s$$

Case 2: to find shock factor (K_0):

Assume medium shock,

$K_0 = 1.5$

$$\therefore \ F_t = \frac{12 \times 10^3}{0.226 m_x} \times 1.5$$

$$F_t = \frac{79646.02}{m_x}$$

Step 6: Calculation of dynamic load: (F_d)

$$F_d = \frac{F_t}{C_v}$$

Case 1: To find velocity factor (C_v) :

From PSGDB 8.51 , assume $v = 5 m/s$

$$C_v = \frac{6}{6+v}$$

$$= \frac{6}{6+5}$$

$$C_v = 0.545$$

Case 2: To find (F_d):

$$F_d = \frac{79646.02}{m_x} \times \frac{1}{0.545}$$

$$= \frac{1460177.70}{m_x}$$

Step 7: Calculation of beam strength (F_s) in terms of (m_x)

From PSGDB 8.51

$$F_s = \pi \times m_x \times b \times [\sigma_b] \times y^1$$

Where ,

$b = 0.75 d_1$ From PSGDB 8.48 , table 38

$= 0.75 \times q \times m_x$

$= 0.75 \times 11 \times m_x$

$= 8.25 m_x$

$y^1 = 0.125$ From PSGDB 8.52 , Assume $\alpha = 20°$

Form factor $y = 0.392$

$$\therefore \; y^1 = \frac{y}{\pi}$$

$$\frac{0.392}{\pi}$$

$$=0.125$$

$[\sigma_b] = 80\, N/mm^2$ From PSGDB 8.45 , table 33

$$\therefore F_s = \pi \times m_x \times 8.25 m_x \times 80 \times 0.125$$

$$= 259.18\, m_x^2$$

Step 8: Calculation of Axial module (m_x)

W . K . T $F_s \geq F_d$

$$259.18 \times m_x^2 \geq \frac{146017.70}{m_x}$$

$$m_x \geq 8.26mm$$

From PSGDB 8.2 , Table 1.

The nearest higher standard axial module

$$m_x = 10mm.$$

Step 9: Calculation of b, d_2 and v:

Case 1: To find the face width (b):

$$b = 8.25m_x \quad \text{From step 7}$$

$$= 82.5mm$$

Case 2: To find pitch diameter of the worm wheel (d_2)

$$d_2 = Z_2 \times m_x \quad \text{From step 5 case 1.}$$

$$= 72 \times 10$$
$$= 720mm$$

Case 3: To find the pitch line velocity of worm wheel (v)

$$v = 0.226\ m_x \quad \text{From step 5, case 1.}$$

$$= 0.226 \times 10$$

$$v = 2.26\,m/s$$

Step 10: Recalculation of beam strength.

$$F_s = 259.18m_x^2 \qquad \text{From step 7}$$

$$= 259.18 \times 10^2$$

$$F_s = 25918N$$

Step 11: Recalculation of dynamic load (F_α)

$$F_d = \frac{F_t}{C_v}$$

$$C_v = \frac{6}{6+v} = \frac{6}{6+2.26} = 0.726$$

$$F_t = \frac{79646.02}{m_x} = \frac{79646.02}{10} = 7964.602N \quad \text{From step 5 case 2}$$

$$\therefore \quad F_d = \frac{7964.602}{0.726}$$

$$F_d = 10970.53N$$

Step 12: Check for beam strength.

We find $F_d < F_s$. the design is safe.

Step 13: Check for Maximum wear load (F_N):

From PSGDB 8.52

$$F_w = d_2 \times b \times K_w$$

$K_w = 0.56\,N/mm^2$ From PSGDB 8.54 , table 43

$$F_w = 720 \times 82.5 \times 0.56$$

$$F_w = 33264N$$

Step 14: Check for efficiency.

$$\eta_{actual} = 0.95 \times \frac{\tan H}{\tan(H+e)}$$ From PSGDB 8.49

Where, $\rho = \tan^{-1} M$, Assume $M = 0.03$ From PSGDB 8.49

$$\rho = \tan^{-1}(0.03)$$

$$= 1.7°$$

$$\eta_{actual} = 0.95 \times \frac{\tan 15.25}{\tan(15.25 + 1.7)}$$

$= 0.8498$ We find that the actual efficiency is greater than the desired efficiency. $\therefore$ The design is safe.

$$\eta_{actual} = 84.98\%$$

Step 15: Calculation of basic dimensions of worm and worm gears.

From PSGDB 8.43 , table 35

Axial module: $m_x = 10mm$

No. of starts: $Z_1 = 3$

No. of teeth on the worm wheel: $Z_2 = 72$

Face width of the worm wheel: $b = 82.5mm$

Length of the worm: $L \geq (12.5 + 0.09Z_2)m_x$

$= (12.5 + 0.09 \times 72)10$

$= 189.8mm$

Take $L = 190mm$

Centre distance: $a = 0.5m_x(q + Z_2)$

$a = 0.5 \times 10(11 + 72)$

$a = 415mm$

Height factor: $f_0 = 1$

Bottom clearance: $C = 0.25m_x = 0.25 \times 10 = 2.5mm.$

Pitch diameter: $d_1 = q \times m_x = 11 \times 10 = 110mm$

$d_2 = 720mm$

Tip diameter: $d_{a1} = d_1 + 2f_0 \times m_x = 110 + 2 \times 1 \times 10 = 130mm$

$d_{a2} = (Z_2 + 2f_0)m_x = (72 + 2 \times 1)10 = 740mm$

Root diameter: $d_{f1} = d_1 - 2f_0 \times m_x - 2C$

$= 110 - 2 \times 1 \times 10 - 2 \times 2.5$

$= 85mm$

$d_{f2} = (Z_2 - 2f_0)m_x - 2C$

$= (72 - 2 \times 1) \times 10 - 2 \times 2.5$

$= 695mm.$

12. Design a bevel gear drive to transmit 3.5 KW with dividing shaft is 200 rpm. Speed ratio required is 4. The drive is no – reversible pinion is made of steel and wheel made of steel and wheel made of CI. Assume lite at 25000 hrs. (April/May 2017)

Given data:

$P = 3.5KW$
$i = 4$
$N_2 = 200rpm.$
Material $\Rightarrow$ Pinion – Steel
Wheel – CI

The materials of pinion and gear are different, we have to design the pinion first and check the gear.

Step 1: Gear ratio & Pitch angles:

$$i = \frac{N_1}{N_2} = 4$$

$$\frac{N_1}{200} = 4$$

$$N_1 = 800rpm.$$

Pitch angles:

From PSGDB 8.34

$\tan\delta_2 = i = 4$

$\therefore \ \delta_2 = \tan^{-1}(4) = 75.96°$

$\delta_1 = 90 - \delta_2 = 90 - 75.96$

$= 14.04°$

Step 2: Material selection:

Pinion: Steel, $\sigma_u = 700N/mm^2 \quad \sigma_y = 360N/m^2$

Gear: CI grad 35 - $\sigma_u = 350N/mm^2$

Step 3: Gear life in cycles:

Gear life in hours $= 25000hrs.$

Gear life in cycles $N = 25000 \times 800 \times 60$

$= 12 \times 10^8$ cycles.

Step 4: Calculation of initial design torque [M_t]:

From PSGDB 8.44,

$$[M_t] = M_t \times K \times K_d$$

$$M_t = \frac{60 \times P}{2\pi N_1} = \frac{60 \times 3.5 \times 10^3}{2 \times \pi \times 800} = 41.778 N.m$$

$K.K_d = 1.3$ initially assume.

$$\therefore \quad [M_t] = 41.778 \times 1.3$$

$$= 54.31 \text{ Nm.}$$

Step 5: Calculation of E_{eq} , [σ_b] and [σ_c]:

To find E_{eq}.

$E_{eq} = 1.7 \times 10^5 N/mm^2$ From PSGDB8.14

To find [σ_{b1}] Design bending stress for pinion.

$[\sigma_{b1}] = \frac{1.4K_{b1}}{n \times K_\sigma} \times \sigma - 1$ From PSGDB 8.18 rotation in one direction

$K_{b1} = 1$,for HB ≤ 350 and N Z10^{7}, From PSGDB 8.20, table 22.

$K_\sigma = 1.5$, for steel pinion. From PSGDB 8.19, table 21

$n = 2.5$ steel hardened From PSGDB 8.19, table 20

$\sigma_{-1} = \left(0.25\left(\sigma_u + \sigma_y\right) + 50\right)$, for forged steel. From PSGDB 8.19, table 19

$\sigma_{-1} = 0.25(700 + 360) + 50$

$= 315 N/mm^2.$

$$\therefore [\sigma_{b1}] = \frac{1.4 \times 1}{2.5 \times 1.5} \times 315 = 117.6 N/mm^2.$$

To find $[\sigma_{c1}]$:

$$[\sigma_{c1}] = C_R.HRC \times K_{C1}$$

$$C_R = 23$$

$$HRC = 40 to 55$$

$$K_{C1} = 1$$

$$[\sigma_{c1}] = 23 \times 55 \times 1$$

$$= 1265 N/mm^2$$

Step 6: Calculation of cone distance (R).

$$R \geq \varphi_y \sqrt{i^2+1} \sqrt[3]{\left[\frac{0.72}{(\varphi_y - 0.5)[\sigma_c]}\right]^2 \times \frac{E_{eq}[M_t]}{i}}$$

$$\varphi_y = R/6 = 3$$

$$R \geq 3\sqrt{4^2+1} \sqrt{\left[\frac{0.72}{(3-0.5)\times 1265}\right]^2 \times \left[\frac{1.7\times 10^5 \times 54.31 \times 10^3}{4}\right]}$$

$$\geq 135.29 mm.$$

$$R = 136 mm.$$

Step 7: Selection of No. of teeth on pinion and gear.

$$Z_1 = 20$$

$$Z_2 = i \times Z_1$$

$$4 \times 20$$

$$= 80$$

Virtual no. of teeth: $Z_{v1} = \frac{Z_1}{\cos\delta_1} = \frac{20}{\cos 14.04} = 20.61 \approx 21$

From PSGDB 8.22. $Z_{v2} = \frac{Z_2}{\cos\delta_2} = \frac{80}{\cos 75.96°} = 329.76 \approx 330$

Step 8: Calculation of transverse module: (m_t).

$$M_t = \frac{R_1}{0.5\sqrt{Z_1^2 + Z_2^2}}$$ From PSGDB 8.38 table 31

$$= \frac{136}{0.5\sqrt{20^2 + 80^2}}$$

$$= 3.29\text{mm}$$

From PSGDB 8.2, table 1, choice 1.

The nearest next higher standard transverse module m_t =4mm.

Step 9: Revision of cone distance: (R)

$$R = 0.5m_t\sqrt{Z_1^2 + Z_2^2}$$

$$= 0.5 \times 4\sqrt{20^2 + 80^2}$$

$$= 164.92\text{mm.}$$

Step 10: Calculation of b, m_{av} , d_{1av}, v and φ_y :

(vi) To find b: $b = \frac{R}{\varphi_y} = \frac{164.92}{3} = 54.97 \approx 55\text{mm}$ PSGDB 8.38

(vii) Average module $m_{av} = m_t - \frac{b\sin\delta_1}{Z_1}$ PSGDB 8.38

$$= 4 - \frac{55\text{SIN}14.04}{20}$$

$$= 3.33\text{mm.}$$

(viii) Average PCD of pinion: $d_{1av} = m_{av} \times Z_1$

$$= 3.33 \times 20$$

$$= 66.66\text{mm.}$$

(ix) Pitch line velocity $v = \frac{\pi d_{1av} \times N_1}{60} = \frac{\pi \times 66.66 \times 800}{60 \times 1000} = 2.79\,{}^{m}/_{s}$.

(x) $\varphi_y = \frac{b}{d_{1av}} = \frac{55}{66.66} = 0.83$

Step 11: Selection of Quality of gears.

Is Quality 8 bevel gear is selected. From PSGDB 8.3 table 2.

Step 12: Revision of design torque [M_t]:

$$[M_t] = M_t \times K \times K_d$$

$K = 1.1$ From PSGDB 8.15

$K_d = 1.45$ From PSGDB 8.16 table 15.

$\therefore \ [M_t] = 41.778 \times 1.1 \times 1.45$

$= 66.64 Nm.$

Step 13: Check for bending of pinion.

$$\sigma_{b1} = \frac{R\sqrt{l^2+1}[M_t]}{(R-0.5b)^2 \times b \times m_t \times y_{v1}}$$ From PSGDB 8.13[A].

$y_{v1} = 0.402$ for $Z_{v1} = 21$

$$\therefore \ \sigma_{b1} = \frac{164.92\sqrt{4^2+1} \times 66.64 \times 10^3}{(164.92 - 0.5 \times 55)^2 \times 55 \times 4 \times 0.402}$$

$\sigma_{b1} = 27.13 N/mm^2$

We find $\sigma_{b1} < [\sigma_{b1}]$. $\therefore$ the design is safe.

Step 14: Check for wearing of pinion.

$$\sigma_{c1} = \left(\frac{0.72}{R-0.56}\right)\left[\frac{\sqrt{(i^2+1)^3}}{i \times b} \times E_{eq} \times [M_t]\right]^{1/2}$$ From PSGDB 8.13

$$= \left(\frac{0.72}{164.92 - 0.5 \times 55}\right)\left[\frac{\sqrt{(4^2+1)^3}}{4 \times 55} \times 1.7 \times 10^5 \times 66.64 \times 10^3\right]^{1/2}$$

$\sigma_{C1} = 314.77 N/mm^2$

We find $\sigma_{C1} < [\sigma_C]$. Thus the design is satisfactory for pinion.

Step 15: Cheek for gear.

Gear material: CI grade 30.

First we have to calculate $[\sigma_{b2}]$ and $[\sigma_{c2}]$.

Gear life of wheel $N = \frac{N_{pinion}}{3}$

$$= \frac{12\times10^8}{3}$$

$$= 4\times10^8$$

To find $[\sigma_{b2}]$:

$$[\sigma_{b2}] = \frac{1.4\times K_{b1}}{h\times K_\sigma}\times\sigma_{-1}$$

Where,

$$K_{b1} = \sqrt[9]{\frac{10^7}{N}} = \sqrt[9]{\frac{10^7}{4\times10^8}} = 0.66$$

$$K_\sigma = 1.2$$

$$n = 2$$

$$\sigma_{-1} = 0.45\sigma_u = 0.45\times350 = 157.5N/mm^2.$$

$$\therefore\ [\sigma_{b2}] = \frac{1.4\times0.66}{2\times1.2}\times157.5$$

$$= 60.64N.mm^2.$$

To find $[\sigma_{c2}]$:

$$[\sigma_{c2}] = C_B\times HB\times K_{cl}.$$

Where $C_B = 2.3,$ $HB = 200$ to 260

$$K_{cl} = \sqrt[b]{\frac{10^7}{N}} = \sqrt[b]{\frac{10^7}{4\times10^8}} = 0.54$$

$$\therefore[\sigma_{c2}] = 2.3\times260\times0.54$$

$$= 322.92N/mm^2.$$

Case 1: Check for bending of gear.

$$\sigma_{b1}\times y_{v1} = \sigma_{b2}\times y_{v2}$$

$y_{v1} = 0.402$ for $Z_{v1} = 21$

$y_{v2} = 0.521$ for $Z_{v2} = 330$

$$27.13 \times 0.402 = \sigma_{b2} \times 0.521$$

$$\sigma_{b2} = 20.93 N/mm^2.$$

$$\sigma_{b2} < [\sigma_{b2}]. \qquad \therefore \text{ design is safe.}$$

Case 2: Check for wearing of gear.

Since the contact area is same.

$$\therefore \ \sigma_{c2} = \sigma_{c1} = 314.77 N/mm^2$$

We find $\sigma_{c2} < [\sigma_{c2}]$. It means the gear having the adequate beam strength.

$\therefore$ The design is safe and satisfactory.

13. Design a worm gear drive to transmit 22.5KW at a worm speed of 1440 rpm. velocity ration is 24:1. An efficiency of at least 85% is desired. The temperature raise should be restricted to 40°C. Determine the required cooling area. (Nov/Dec 2017)

Given data:

$$P = 22.5 KW$$

$$N_1 = 1440 \text{ rpm}$$

$$i = 24$$

$$\eta_{desired} = 85\%$$

$$\Delta_t = t_0 - t_a = 40°c$$

Step 1: To find gear ration (i):

$$i = 24 \quad \text{(given)}$$

Step 2: Selection of material.

Assume,

Worm = Hardened steel

Worm wheel = Phosphor bronze.

Step 3: Calculation of Z_1 and Z_2

From PSGDB 8.46 , table 37

For, $\eta = 85\%$, $Z_1 = 3$ $\qquad \frac{N_1}{N_2} = i$

$\therefore \quad Z_2 = i \times Z_1$

$= 24 \times 3$

$Z_2 = 72.$

$\frac{1440}{N_2} = 24$

$N_2 = 60\text{rpm}.$

Step 4: Calculation of q and H:

Case 1: To find diameter factor (q).

From PSGDB 8.43 , table 35, and PSGDB 8.44

$$q = m_x \times d_1$$

Initially we assume q = 11.

Case 2: To find Lead angle (H) .

From PSGDB 8.43 , table 35

$$\tan H = \frac{Z_1}{q}$$

$$H = \tan^{-1}\left(\frac{3}{11}\right)$$

$$H = 15.25°$$

Step 5: Calculation of 'F_t' in terms of 'm_x'

Tangential load $F_t = \frac{P}{v} \times K_0$

Case 1: To find the velocity 'v':

$$v = \frac{\pi d_2 N_2}{60 \times 1000}$$

From PSGDB 8.43 , table 35

$$d_2 = Z_2 \times m_x$$

$$\therefore \quad v = \frac{\pi \times Z_2 \times m_x \times N_2}{60 \times 1000}$$

$$= \frac{\pi \times 72 \times m_x \times 60}{60 \times 1000}$$

$$v = 0.226\ m_x\ m/s$$

Case 2: To find tangential load

Assume medium shock

$$K_0 = 1.5\ .$$

$$\therefore\ F_t = \frac{22.5 \times 10^3}{0.226 m_x} \times 1.5$$

$$\frac{149336.28}{m_x}$$

Step 6: Calculation of dynamic load (F_d)

$$F_d = \frac{F_t}{C_v}$$

Case 1: To find velocity factor (C_v).

From PSGDB 8.51 , assume v= 5 m/s

$$C_v = \frac{6}{6+v}$$

$$= \frac{6}{6+5}$$

$$C_v = 0.545$$

Case 2: To find (F_d).

$$F_d = \frac{149336.28}{m_x} \times \frac{1}{0.545}$$

$$= \frac{274011.53}{m_x}$$

Step 7: Calculation of beam strength (F_s) in terms of (m_x).

From PSGDB 8.51

$$F_s = \pi \times m_x \times b \times [\sigma_b] \times y^1$$

Where,

* $b = 0.75 d_1$ From PSGDB 8.48 , table 38

$= 0.75 \times q \times m_x$

$= 0.75 \times 11 \times m_x$

$= 8.25 m_x$

* $y^1 = 0.125$ From PSGDB 8.52 ,

Assume α=20°

Form factor $y = 0.392$

$y^1 = \frac{y}{\pi}$ From PSGDB 8.53 , table 40

$= \frac{0.392}{\pi} = 0.125$

* $[\sigma_b] = 80 \text{ N/mm}^2$ From PSGDB 8.45 , table 33

$$\therefore \quad F_s = \pi \times m_x \times 8.25 \times m_x \times 80 \times 0.125$$

$$= 259.18\ m_x^2$$

Step 8: Calculation of Axial module (m_x).

From PSGDB 8.51

$$F_s \geq F_d$$

$$259.18\ m_x^2 \geq \frac{274011.53}{m_x}$$

$$m_x \geq 10.18 \text{ mm.}$$

PSGDB 8.2 , table 1.

The next nearest higher standard axial module

$$m_x = 12 \text{ mm.}$$

Step 9: calculation of b , d_e and v:

Case 1: To find face width (b).

$$b = 8.25\ m_x \quad \text{From step 7}$$

$$= 8.25 \times 12$$

$$b = 99 \text{ mm}$$

Case 2: To find Pitch diameter of the worm wheel (d_2).

$d_2 = Z_2 \times m_x$ From step 5 , case 1.

$= 72 \times 12$

$d_2 = 864 \text{ mm}$

Case 3: To find the Pitch line velocity of worm wheel (v)

$v = 0.226\ m_x$ From step 5 , case 1.

$= 0.226 \times 12$

$v = 2.712 \text{ m/s}$

Step 10: Recalculation of beam strength.

$F_s = 259.18\ m_x^2$ From step 7.

$= 259.18 \times 12^2$

$F_s = 37321.92 \text{ N}$

Step 11: Recalculation of dynamic load (F_d).

$$F_d = \frac{F_t}{C_v}$$

* $C_v = \frac{6}{6+v} = \frac{6}{6+2.712} = 0.688$

* $F_t = \frac{149336.28}{m_x} = \frac{149336.28}{12} = 12444.69 \text{ N}$ From step 5 , case 2.

$\therefore F_d = \frac{12444.69}{0.688}$

$F_d = 18088.21 \text{ N}$

Step 12: Check for beam strength.

We find $F_s > F_d$. $\therefore$ the design is safe.

Step 13: Check for maximum wear load (F_w).

From PSGDB 8.52

$$F_w = d_2 \times 6 \times K_w$$

* $K_w = 0.56\ N/mm^2$ From PSGDB 8.54 , table 43.

$F_w = 864 \times 99 \times 0.56$

$F_w = 47900.16\ N$.

We find $F_w > F_d$. $\therefore$ the design is safe.

Step 14: Check for efficiency.

$\eta_{actual} = 0.95 \times \dfrac{\tan H}{\tan(H+\rho)}$ From PSGDB 8.49

Where , $\rho = \tan^{-1}(H)$ Assume H = 0.03From PSGDB 8.49

$\rho = \tan^{-1}(0.03)$

$= 1.7°$

$\eta_{actual} = 0.95 \times \dfrac{\tan 15.25}{\tan(15.25 + 1.7)}$

$= 0.85$

$\eta_{actual} = 85\%$

We find that the actual efficiency is equal to the desired efficiency.

$\therefore$ The design is safe.

Step 15: To find cooling area (A).

In order to avoid overheating, we have to find cooling area.

Heat generated (i.e Power loss) = Heat emitted into the atmosphere.

From PSGDB 8.52 $(1-\eta) \times \text{Input Power} = K_t A(t_0 - t_a)$.

Assume $K_t = 10 w/m^2 °c$

$(1-0.85) \times 22.5 \times 10^3 = 10 \times A \times 40$

Required cooling Area , $A = 8.44\ m^2$

Step 16: Calculation of basic dimension of worm and worm wheel.

From PSGDB 8.43 , table 35.

- Axial module: $m_x = 12$ mm
- No. of starts: $Z_1 = 3$
- No. of teeth on worm wheel: $Z_2 = 72$
- Face width: $b = 99$ mm.
- Length of the worm: $L \geq (12.5 + 0.09 Z_2) m_x$

$$= (12.5 + 0.09 \times 72) 12$$

$$= 227.76 \text{ mm.}$$

$$L \square 228 \text{ mm.}$$

- Centre distance: $a = 0.5\, m_x (q + Z_2)$

$$= 0.5 \times 12 (11 + 72)$$

$$a = 498 \text{ mm.}$$

- Height factor: $f_0 = 1.$
- Bottom clearance: $c = 0.25 m_x = 0.25 \times 12 = 3.0$

$$c = 3 \text{ mm.}$$

- Pitch diameter: $d_1 = q \times m_x = 11 \times 12 = 132$ mm.

$$d_2 = 864 \text{ mm}$$

- Tip diameter: $d_{a1} = d_1 + 2 f_0 \times m_x$ $\qquad d_{a2} = Z_2 + 2 f_0 \times m_x$

$$= 132 + 2 \times 1 \times 12 \qquad = (72 + 2(1)) 12$$

$$= 156 \text{ mm.} \qquad = 888 \text{ mm.}$$

- Root diameter: $d_{f1} = d_1 - 2 f_0 \times m_x - 2c$

$$= 132 - 2 \times 1 \times 12 - 2 \times 3$$

$$d_{f2} = (Z_2 - 2 f_0) m_x - 2c$$

$$= (72 - 2 \times 1) 12 - 2 \times 3$$

$$d_{f2} = 834 \text{ mm}$$

14. Design a straight bevel gear drive between two shafts at right angles to each other. Speed of the pinion shaft is 360 rpm and the speed of the gear. Wheel shaft is 120 rpm. Pinion is of steel and wheel of cast iron. Each gear is expected to work 2 hours / day for 10 years. The drive transmits 9.35 KW. (Nov/Dec 2017)

Given data: $\theta = 90°$; $N_1 = 360\text{rpm}$; $N_2 = 120\text{rpm}$;
$P = 9.37\text{KW}$

To find: Design the bevel gear drive.

Solution: Since the materials of pinion and gear are different, we have to design the pinion first and check the gear.

1. **Gear ratio:** $i = \frac{N_1}{N_2} = \frac{360}{120} = 3$

Pitch angles: $\tan\delta_2 = i = 3$ or $\delta_2 = \tan^{-1}(3) = 71.56°$ from PSGDB 8.39

Then, $\delta_1 = 90° - \delta_2 = 90° - 71.56° = 18.44°$

2. **Material selection:** Pinion – C45 Steel, $\sigma_u = 700\,\text{N/mm}^2$ and $\sigma_y = 360\,\text{N/m}^2$

Gear – CI grade 35, $\sigma_u = 350\,\text{N/mm}^2$

3. **Gear life in hours** $= (2\,\text{hours/day}) \times (365\,\text{days/year} \times 10\text{years}) = 7300\text{hours}$

$\therefore$ Gear life in cycles, $N = 7300 \times 360 \times 60 = 15.768 \times 10^7\,\text{cycles}$

4. **Calculation of initial design torque [M_t]:**

We know that, $[M_t] = M_t \times K \times K_d$

Where $M_t = \frac{60 \times P}{2\pi N_1} = \frac{60 \times 9.37 \times 10^3}{2\pi \times 360} = 248.6\text{N}-\text{m}$ and

$K \cdot K_d = 1.3$, initially assumed.

$\therefore$ $[M_t] = 248.6 \times 1.3 = 323.28\text{N}-\text{m}$

5. **Calculation of E_{eq}, $[\sigma_b]$ and $[\sigma_c]$:**

To find E_{eq}: $E_{eq} = 1.7 \times 10^5\,\text{N/mm}^2$ From PSGDB 8.14

To find $[\sigma_{b1}]$: We know that the design bending stress for pinion,

$$[\sigma_{b1}] = \frac{1.4K_{b1}}{n \cdot K_\sigma} \times \sigma_{-1},\ \text{for rotation in one direction}$$

Where $K_{b1} = 1$, for $HB \le 350$ and $N \ge 10^7$ From PSGDB 8.20, table 22

$K_\sigma = 1.5$, for steel pinion From PSGDB 8.19, table 21

$n = 2.5$, steel hardened table 20, PSGDB 8.19

$\sigma_{-1} = 0.25(\sigma_u + \sigma_y) + 50$, for forged steel - From PSGDB 8.19, table 19

$= 0.25(700 + 360) + 50 = 315\,N/mm^2$

$$[\sigma_{b1}] = \frac{1.4 \times 1}{2.5 \times 1.5} \times 315 = 117.6\,N/mm^2$$

To find $[\sigma_{c1}]$: We know that the design contact stress for pinion,

$[\sigma_{c1}] = C_R \cdot HRC \times K_{cl}$ From PSGDB 8.16

Where $C_R = 23$ From PSGDB 8.16, table 16

$HRC = 40$ to 55 From PSGDB 8.16, table 16

$K_{cl} = 1$, for steel pinion, $HB \le 350$ and $N \ge 10^7$ From PSGDB 8.16, table 17

$\therefore\ [\sigma_{c1}] = 23 \times 50 \times 1 = 1150\,N/mm^2$

6. Calculation of cone distance (R):

We know that, $R \ge \psi_y \sqrt{i^2 + 1}\sqrt[3]{\left[\frac{0.72}{(\psi_y - 0.5)[\sigma_c]}\right]^2 \times \frac{E_{eq}[M_t]}{i}}$ From PSGDB 8.13

Where $\psi_y = R/b = 3$, initially assumed.

$$\therefore\quad R \ge 3\sqrt{3^2 + 1}\sqrt[3]{\left[\frac{0.72}{(3 - 0.5)1150}\right]^2 \times \frac{1.7 \times 10^5 \times 323.28 \times 10^3}{3}}$$

≥ 99.36

or $R = 100mm$.

7. Assume $Z_1 = 20$ **; Then** $Z_2 = i \times Z_1 = 3 \times 20 = 60$

Virtual number of teeth: $Z_{v1} = \dfrac{Z_1}{\cos\delta_1} = \dfrac{20}{\cos 18.44^\circ} \approx 22$; and

From PSGDB 8.39 $Z_{v2} = \dfrac{Z_2}{\cos\delta_2} = \dfrac{60}{\cos 71.56^\circ} \approx 190$.

8. Calculation of transverse module (m_t):

We know that, $m_t = \dfrac{R}{0.5\sqrt{Z_1^2 + Z_2^2}}$ From PSGDB 8.38, table 31

$$= \frac{100}{0.5\sqrt{20^2 + 60^2}} = 3.162\text{mm}$$

From PSGDB 8.2 , table 1. Under choice 1. The nearest higher standard transverse module is 4mm.

9. Revision of cone distance (R):

We know that, $R = 0.5 m_t \sqrt{Z_1^2 + Z_2^2} = 0.5 \times 4\sqrt{20^2 + 60^2} = 126.49\text{mm}$

10. Calculation of b, m_{av}, d_{1av}, v and ψ_y :

Face width (b): $b = \dfrac{R}{\psi_y} = \dfrac{126.49}{3} = 42.16\text{mm}$ From PSGDB 8.38

Average module (m_{av}):

$$m_{av} = m_t - \frac{b \sin\delta_1}{Z_1} = 4 - \frac{42.16 \times \sin 18.44^\circ}{20} \text{ PSGDB 8.38}$$

$$= 3.333$$

Average pcd of pinion (d_{1av}): $d_{1av} = m_{av} \times Z_1 = 3.333 \times 20 = 66.66\text{mm}$

Pitch line velocity (v):

$$v = \frac{\pi \times d_{1av} \times N_1}{60} = \frac{\pi \times 66.66 \times 10^{-3} \times 360}{60} = 1.256\,\text{m/s}$$

$$\psi_y = \frac{b}{d_{1av}} = \frac{42.16}{66.66} = 0.632$$

11. IS quality 6 bevel gear is assumed From PSGDB 8.3 , table 2

12. Revision of design torque $[M_t]$:

We know that, $[M_t] = M_t \times K \times K_d$

Where $K = 1.1$

$K_d = 1.35$

$\therefore \quad [M_t] = 248.6 \times 1.1 \times 1.35 = 369.28 N-m$

13. Check for bending of pinion: We know that the induced bending stress,

$$\sigma_{b1} = \frac{R\sqrt{i^2+1}[M_t]}{(R-0.5b)^2 \times b \times m_t \times y_{v1}}$$ From PSGDB 8.13 [A]

Where $y_{v1} = 0.402$, for $Z_{v1} = 22$

$$\therefore \quad \sigma_b = \frac{126.49\sqrt{3^2+1} \times 369.28 \times 10^3}{(126.49 - 0.5 \times 42.16)^2 \times 42.16 \times 4 \times 0.402} = 196.09\, N/mm^2$$

We find $\sigma_{b1} > [\sigma_{b1}]$. Thus the design is unsatisfactory.

Trial 2: Now we will try with increased transverse module 5mm. Repeating from step 9 again, we get

$$R = 0.5 \times m_t \times \sqrt{Z_1^2 + Z_2^2} = 0.5 \times 5 \times \sqrt{20^2 + 60^2} = 158.11mm$$

$$b = \frac{R}{\psi_y} = \frac{158.11}{3} = 52.7mm$$

$$m_{av} = m_t - \frac{b \sin \delta_1}{Z_1} = 5 - \frac{52.7 \times \sin 18.44}{20} = 4.166mm$$

$$d_{1av} = m_{av} \times Z_1 = 4.166 \times 20 = 83.33mm$$

$$v = \frac{\pi \times d_{1av} \times N_1}{60} = \frac{\pi \times 83.33 \times 10^{-3} \times 360}{60} = 1.57\, m/s$$

$$\psi_y = \frac{b}{d_{1av}} = \frac{52.7}{83.33} = 0.632$$

IS quality 6 bevel gear is assumed.

$K = 1.1; \quad K_d = 1.35$

$$M_t = 248.6 \times 1.1 \times 1.35 = 369.28\,N-m$$

$$\therefore \quad \sigma_{b1} = \frac{158.11\sqrt{3^2+1} \times 369.28 \times 10^3}{(158.11 - 0.5 \times 52.7)^2 \times 52.7 \times 5 \times 0.402} = 100.4\,N/mm^2$$

Now we find $\sigma_{b1} < [\sigma_{b1}]$, thus the design is satisfactory.

14. **Check for wearing of pinion:** We know that the induced contact stress,

$$\sigma_{c1} = \left(\frac{0.72}{R-0.5b}\right)\left[\frac{\sqrt{(i^2+1)^3}}{ib} \times E_{eq} \times [M_t]\right]^{\frac{1}{2}}$$ From PSGBD 8.13

$$= \left(\frac{0.72}{158.11 - 0.5 \times 52.7}\right)\left[\frac{\sqrt{(3^2+1)^3}}{3 \times 52.7} \times 1.7 \times 10^5 \times 369.28 \times 10^3\right]^{\frac{1}{2}}$$

$$= 612.33\,N/mm^2$$

We find $\sigma_{C1} < [\sigma_{C1}]$. Thus the design is satisfactory for pinion.

15. **Check for gear (i.e., wheel):** Gear material: CI grade 30.

First we have to calculate $[\sigma_{b2}]$ and $[\sigma_{C2}]$.

Gear life of wheel, $N = \frac{N_{pinion}}{3} = \frac{15.768 \times 10^7}{3} = 5.256 \times 10^7$ cycles

To find $[\sigma_{b2}]$: We know that the design bending stress for gear,

$$[\sigma_{b2}] = \frac{1.4 \times K_{b1}}{n \times K_\sigma} \times \sigma_{-1}$$

Where $K_{b1} = \sqrt[9]{\frac{10^7}{N}} = \sqrt[9]{\frac{107}{5.256 \times 10^7}} = 0.832$

$K_\sigma = 1.2$

$n = 2$ & $\sigma_{-1} = 0.45\sigma_u = 0.45 \times 350 = 157.5\,N/mm^2$

$$\therefore \quad [\sigma_{b2}] = \frac{1.4 \times 0.832}{2 \times 1.2} \times 157.5 = 76.44\,N/mm^2$$

To find $[\sigma_{b2}]$: We know that the design contact stress for gear,

$$[\sigma_{C2}] = C_B \times HB \times K_{cl}$$

Where $C_B = 2.3$

$$HB = 200 \text{ to } 260$$

$$K_{cl} = \sqrt[6]{\frac{10^7}{N}} = \sqrt[6]{\frac{10^7}{5.256 \times 10^7}} = 0.758$$

$\therefore$ $[\sigma_{C2}] = 2.3 \times 260 \times 0.758 = 453.284\, N/mm^2$

(c) Check for bending of gear: The induced bending stress for gear can be calculated using the relation

$$\sigma_{b1} \times y_{v1} = \sigma_{b2} \times y_{v2}$$

Where $y_{v1} = 0.402$, for $Z_{v1} = 22$

$y_{v2} = 0.520$, for $Z_{v2} = 190$

$\therefore$ $100.4 \times 0.402 = \sigma_{b2} \times 0.520$

or $\sigma_{b2} = 77.6\, N/mm^2$

We find σ_{b2} is almost equal to $[\sigma_{b2}]$. Thus the design is okay and it can be accepted.

(d) Check for wearing of gear: Since the contact area is same,

$$\sigma_{c2} = \sigma_{c1} = 612.33\, N/mm^2$$

We find $\sigma_{c2} > [\sigma_{c2}]$. It means the gear does not have adequate beam strength. In order to increase the wear strength of the gear, surface hardness may be raised to 360 BHN. Then we get

$$[\sigma_{b2}] = 2.3 \times 360 \times 0.758 = 627.62\, N/mm^2$$

Now we find $\sigma_{b2} > [\sigma_{b2}]$, thus the design is safe and satisfactory.

15. Design a bevel gear drive to transmit 7 kW at 1600 rpm for the following data. Gear ratio= 3, Material for pinion and gear -C45 steel, Life 10,000 Hours. (April/May 2018)

Given data: $N_1 = 360\text{rpm}$; $N_2 = 120\text{rpm}$; $P = 9.37\text{KW}$

******similar to this problem, Change the power to be N_1=1600 rpm and the Material for pinion and gear -C45 steel,***

To find: Design the bevel gear drive.

Solution: Since the materials of pinion and gear are different, we have to design the pinion first and check the gear.

16. Gear ratio: $i = \frac{N_1}{N_2} = \frac{360}{120} = 3$

Pitch angles: $\tan\delta_2 = i = 3$ or $\delta_2 = \tan^{-1}(3) = 71.56°$ from PSGDB 8.39

Then, $\delta_1 = 90° - \delta_2 = 90° - 71.56° = 18.44°$

17. Material selection: Pinion – C45 Steel, $\sigma_u = 700\,\text{N/mm}^2$ and $\sigma_y = 360\,\text{N/m}^2$

Gear – CI grade 35, $\sigma_u = 350\,\text{N/mm}^2$

18. Gear life in hours

$= (2\,\text{hours/day}) \times (365\,\text{days/year} \times 10\text{years}) = 7300\text{hours}$

$\therefore$ Gear life in cycles, $N = 7300 \times 360 \times 60 = 15.768 \times 10^7$ cycles

19. Calculation of initial design torque $[M_t]$:

We know that, $[M_t] = M_t \times K \times K_d$

Where $M_t = \frac{60 \times P}{2\pi N_1} = \frac{60 \times 9.37 \times 10^3}{2\pi \times 360} = 248.6\text{N} - \text{m}$ and

$K \cdot K_d = 1.3$, initially assumed.

$\therefore \quad [M_t] = 248.6 \times 1.3 = 323.28\text{N} - \text{m}$

20. Calculation of E_{eq}, $[\sigma_b]$ and $[\sigma_c]$:

To find E_{eq}: $E_{eq} = 1.7 \times 10^5\,\text{N/mm}^2$ From PSGDB 8.14

To find $[\sigma_{b1}]$: We know that the design bending stress for pinion,

$$[\sigma_{b1}] = \frac{1.4K_{b1}}{n \cdot K_\sigma} \times \sigma_{-1},$$ for rotation in one direction

Where $K_{b1} = 1$, for $HB \le 350$ and $N \ge 10^7$ From PSGDB 8.20, table 22

$K_\sigma = 1.5$, for steel pinion From PSGDB 8.19, table 21

$n = 2.5$, steel hardened table 20, PSGDB 8.19

$\sigma_{-1} = 0.25(\sigma_u + \sigma_y) + 50$, for forged steel - From PSGDB 8.19, table 19

$= 0.25(700 + 360) + 50 = 315 \, N/mm^2$

$$[\sigma_{b1}] = \frac{1.4 \times 1}{2.5 \times 1.5} \times 315 = 117.6 \, N/mm^2$$

To find $[\sigma_{c1}]$: We know that the design contact stress for pinion,

$[\sigma_{c1}] = C_R \cdot HRC \times K_{cl}$ From PSGDB 8.16

Where $C_R = 23$ From PSGDB 8.16, table 16

$HRC = 40$ to 55 From PSGDB 8.16, table 16

$K_{cl} = 1$, for steel pinion, $HB \le 350$ and $N \ge 10^7$ From PSGDB 8.16, table 17

$\therefore \; [\sigma_{c1}] = 23 \times 50 \times 1 = 1150 \, N/mm^2$

21. Calculation of cone distance (R):

We know that, $R \ge \psi_y \sqrt{i^2+1} \sqrt[3]{\left[\frac{0.72}{(\psi_y - 0.5)[\sigma_c]}\right]^2 \times \frac{E_{eq}[M_t]}{i}}$ From PSGDB 8.13

Where $\psi_y = R/b = 3$, initially assumed.

$$\therefore \quad R \ge 3\sqrt{3^2+1} \sqrt[3]{\left[\frac{0.72}{(3-0.5)1150}\right]^2 \times \frac{1.7 \times 10^5 \times 323.28 \times 10^3}{3}}$$

≥ 99.36

or $R = 100mm$.

22. Assume $Z_1 = 20$; Then $Z_2 = i \times Z_1 = 3 \times 20 = 60$

Virtual number of teeth: $Z_{v1} = \dfrac{Z_1}{\cos\delta_1} = \dfrac{20}{\cos 18.44°} \approx 22$; and

From PSGDB 8.39 $Z_{v2} = \dfrac{Z_2}{\cos\delta_2} = \dfrac{60}{\cos 71.56°} \approx 190$.

23. Calculation of transverse module (m_t):

We know that, $m_t = \dfrac{R}{0.5\sqrt{Z_1^2 + Z_2^2}}$ From PSGDB 8.38, table 31

$$= \frac{100}{0.5\sqrt{20^2 + 60^2}} = 3.162\text{mm}$$

From PSGDB 8.2 , table 1. Under choice 1. The nearest higher standard transverse module is 4mm.

24. Revision of cone distance (R):

We know that, $R = 0.5 m_t \sqrt{Z_1^2 + Z_2^2} = 0.5 \times 4\sqrt{20^2 + 60^2} = 126.49\text{mm}$

25. Calculation of b, m_{av}, d_{1av}, v and ψ_y :

Face width (b): $b = \dfrac{R}{\psi_y} = \dfrac{126.49}{3} = 42.16\text{mm}$ From PSGDB 8.38

Average module (m_{av}):

$$m_{av} = m_t - \frac{b\sin\delta_1}{Z_1} = 4 - \frac{42.16 \times \sin 18.44°}{20} \text{ PSGDB 8.38}$$

$$= 3.333$$

Average pcd of pinion (d_{1av}): $d_{1av} = m_{av} \times Z_1 = 3.333 \times 20 = 66.66\text{mm}$

Pitch line velocity (v):

$$v = \frac{\pi \times d_{1av} \times N_1}{60} = \frac{\pi \times 66.66 \times 10^{-3} \times 360}{60} = 1.256\,\text{m/s}$$

$$\psi_y = \frac{b}{d_{1av}} = \frac{42.16}{66.66} = 0.632$$

26. IS quality 6 bevel gear is assumed From PSGDB 8.3 , table 2

27. Revision of design torque $[M_t]$:

We know that, $[M_t] = M_t \times K \times K_d$

Where $K = 1.1$

$K_d = 1.35$

$\therefore \quad [M_t] = 248.6 \times 1.1 \times 1.35 = 369.28 N - m$

28. Check for bending of pinion: We know that the induced bending stress,

$$\sigma_{b1} = \frac{R\sqrt{i^2+1}[M_t]}{(R-0.5b)^2 \times b \times m_t \times y_{v1}}$$ From PSGDB 8.13 [A]

Where $y_{v1} = 0.402$, for $Z_{v1} = 22$

$$\therefore \quad \sigma_b = \frac{126.49\sqrt{3^2+1} \times 369.28 \times 10^3}{(126.49 - 0.5 \times 42.16)^2 \times 42.16 \times 4 \times 0.402} = 196.09 N/mm^2$$

We find $\sigma_{b1} > [\sigma_{b1}]$. Thus the design is unsatisfactory.

Trial 2: Now we will try with increased transverse module 5mm. Repeating from step 9 again, we get

$$R = 0.5 \times m_t \times \sqrt{Z_1^2 + Z_2^2} = 0.5 \times 5 \times \sqrt{20^2 + 60^2} = 158.11 mm$$

$$b = \frac{R}{\psi_y} = \frac{158.11}{3} = 52.7 mm$$

$$m_{av} = m_t - \frac{b \sin \delta_1}{Z_1} = 5 - \frac{52.7 \times \sin 18.44}{20} = 4.166 mm$$

$$d_{1av} = m_{av} \times Z_1 = 4.166 \times 20 = 83.33 mm$$

$$v = \frac{\pi \times d_{1av} \times N_1}{60} = \frac{\pi \times 83.33 \times 10^{-3} \times 360}{60} = 1.57 m/s$$

$$\psi_y = \frac{b}{d_{1av}} = \frac{52.7}{83.33} = 0.632$$

IS quality 6 bevel gear is assumed.

$K = 1.1; \quad K_d = 1.35$

$$M_t = 248.6 \times 1.1 \times 1.35 = 369.28 N-m$$

$$\therefore \quad \sigma_{b1} = \frac{158.11\sqrt{3^2+1} \times 369.28 \times 10^3}{(158.11 - 0.5 \times 52.7)^2 \times 52.7 \times 5 \times 0.402} = 100.4 \, N/mm^2$$

Now we find $\sigma_{b1} < [\sigma_{b1}]$, thus the design is satisfactory.

29. Check for wearing of pinion: We know that the induced contact stress,

$$\sigma_{c1} = \left(\frac{0.72}{R - 0.5b}\right)\left[\frac{\sqrt{(i^2+1)^3}}{ib} \times E_{eq} \times [M_t]\right]^{\frac{1}{2}} \qquad \text{From PSGBD 8.13}$$

$$= \left(\frac{0.72}{158.11 - 0.5 \times 52.7}\right)\left[\frac{\sqrt{(3^2+1)^3}}{3 \times 52.7} \times 1.7 \times 10^5 \times 369.28 \times 10^3\right]^{\frac{1}{2}}$$

$$= 612.33 \, N/mm^2$$

We find $\sigma_{C1} < [\sigma_{C1}]$. Thus the design is satisfactory for pinion.

30. Check for gear (i.e., wheel): Gear material: CI grade 30.

First we have to calculate $[\sigma_{b2}]$ and $[\sigma_{C2}]$.

Gear life of wheel, $N = \frac{N_{pinion}}{3} = \frac{15.768 \times 10^7}{3} = 5.256 \times 10^7$ cycles

To find $[\sigma_{b2}]$: We know that the design bending stress for gear,

$$[\sigma_{b2}] = \frac{1.4 \times K_{b1}}{n \times K_\sigma} \times \sigma_{-1}$$

Where $K_{b1} = \sqrt[9]{\frac{10^7}{N}} = \sqrt[9]{\frac{107}{5.256 \times 10^7}} = 0.832$

$K_\sigma = 1.2$

$n = 2$

$\sigma_{-1} = 0.45\sigma_u = 0.45 \times 350 = 157.5 \, N/mm^2$

$$\therefore \quad [\sigma_{b2}] = \frac{1.4 \times 0.832}{2 \times 1.2} \times 157.5 = 76.44\,N/mm^2$$

To find $[\sigma_{b2}]$: We know that the design contact stress for gear,

$$[\sigma_{C2}] = C_B \times HB \times K_{cl}$$

Where $C_B = 2.3$

$$HB = 200 \text{ to } 260$$

$$K_{cl} = \sqrt[6]{\frac{10^7}{N}} = \sqrt[6]{\frac{10^7}{5.256 \times 10^7}} = 0.758$$

$$\therefore \quad [\sigma_{C2}] = 2.3 \times 260 \times 0.758 = 453.284\,N/mm^2$$

(e) Check for bending of gear: The induced bending stress for gear can be calculated using the relation

$$\sigma_{b1} \times y_{v1} = \sigma_{b2} \times y_{v2}$$

Where $y_{v1} = 0.402$, for $Z_{v1} = 22$

$$y_{v2} = 0.520, \quad \text{for } Z_{v2} = 190$$

$$\therefore \quad 100.4 \times 0.402 = \sigma_{b2} \times 0.520$$

or $\sigma_{b2} = 77.6\,N/mm^2$

We find σ_{b2} is almost equal to $[\sigma_{b2}]$. Thus the design is okay and it can be accepted.

(f) Check for wearing of gear: Since the contact area is same,

$$\sigma_{c2} = \sigma_{c1} = 612.33\,N/mm^2$$

We find $\sigma_{c2} > [\sigma_{c2}]$. It means the gear does not have adequate beam strength. In order to increase the wear strength of the gear, surface hardness may be raised to 360 BHN. Then we get

$$[\sigma_{b2}] = 2.3 \times 360 \times 0.758 = 627.62\,N/mm^2$$

Now we find $\sigma_{b2} > [\sigma_{b2}]$, thus the design is safe and satisfactory.

16. The input to the worm gear shaft is 18KW at 600rpm. Speed ratio is 20. The worm is to be of hardened steel and the wheel is made of chilled phosphor bronze. Considering wear and strength, design worm and worm wheel. (April/May 2018)

Given data:

$$N_1 = 600rpm$$

$$P = 18KW$$

$$i = 20$$

Step 1: To find gear ratio (i)

$$i = \frac{N_1}{N_2} = 20(given)$$

$$20 = \frac{600}{N_2}$$

$$N_2 = 30rpm.$$

Step 2: Selection of Materail:

Worm = Hardened steel

Worm wheel = Phosphor bronze

Step 3: Calculation of Z_1 and Z_2:

From PSGDB 8.46 , table 37

For $\eta = 80\%$, $Z_1 = 3$

$$Z_2 = i \times Z_1 = 20 \times 3$$

$$Z_2 = 60$$

Step 4: Calculation of q and H:

Case 1: To find diameter factor (q)

From PSGDB 8.43 , table 35, and PSGDB 8.44

$$d_1 = \frac{q}{m_x}$$

Initially we assume q= 11.

Case 2: To find Lead angle (H)

From PSGDB 8.43, table 35

$$\tan H = \frac{Z_1}{q}$$

$$H = \tan^{-1}\left(3/11\right)$$

$$H = 15.25°$$

Step 5: Calculation of (F_7) in terms of (m_x).

$$F_t = \frac{P}{v} \times K_0$$

Case 1: To find the velocity 'v'

$$v = \frac{\pi d_2 N_2}{60 \times 1000}$$

From PSGDB 8.42 , table 35

$$d_2 = Z_2 \times m_x$$

$$\therefore \quad v = \frac{\pi \times 60 \times m_x \times 30}{60 \times 1000}$$

$$v = 0.094 m_x \ m/s.$$

Case 2: To find shock factor (K_0)

Assume medium shock, K_0 = 1.5

$$\therefore \quad F_t = \frac{18 \times 10^3}{0.094 m_x} \times 1.5$$

$$F_t = \frac{287234.04}{m_x}$$

Step 6: Calculation of dynamic load (F_d).

$$F_d = \frac{F_t}{C_v}$$

Case 1: To find velocity factor (C_v):

From PSGDB 8.51, assume $v = 5 m/s$

$$C_v = \frac{6}{6 + v}$$

$$= \frac{6}{6+5}$$

$$C_v = 0.545$$

$$\therefore \quad F_d = \frac{287234.04}{m_x} \times \frac{1}{0.545}$$

$$= \frac{527034.94}{m_x}$$

Step 7: Calculation of beam strength (F_s)

From PSGDB8.51,

$$F_s = \pi \times m_x \times b \times [\sigma_b] \times y^1$$

Where,

$b = 0.75d_1$ From PSGDB 8.48, table 38

$$= 0.75 \times q \times m_x$$

$$= 0.75 \times 11 \times m_x$$

$$= 8.25m_x$$

$y^1 = 0.125$ From PSGDB 8.52 , Assume $\alpha = 20°$

$[\sigma_b] = 110 N/mm^2$ From PSGDB 8.45, table 33

$$\therefore F_s = \pi \times m_x \times 8.25m_x \times 110 \times 0.125$$

$$= 356.37m_x^2$$

Step 8: Calculation of axial module (m_x).

$$F_s \geq F_d$$

$$356.37m_x^2 \geq \frac{527034.94}{m_x}$$

$$m_x \geq 11.4mm.$$

From PSGDB 8.2, table 1. The next nearest higher standard module m_x = 12mm.

Step 9: Calculation of b , d_2 and v:

From step 7 $\Rightarrow$ $b = 8.25 \times m_x$

$= 8.25 \times 12$

$b = 99mm$

From step 5, Case 1 $\Rightarrow$ $d_2 = Z_2 \times m_x = 60 \times 12$

$= 720mm$

From step 5, Case 1 $\Rightarrow v = 0.094 \times m_x = 0.094 \times 12$

$= 1.13\,m/s$

Step 10: Recalculation of beam strength (F_s)

$F_s = 356.37 \times m_x^2$ From step 7.

$= 356.37 \times 12^2$

$F_s = 51317.28N$

Step 11: Recalculation of dynamic load (F_d).

$$F_d = \frac{F_t}{C_v}$$

$$C_v = \frac{6}{6+v} = \frac{6}{6+1.13} = 0.84$$

$$F_t = \frac{287234.04}{m_x} = \frac{287234.04}{12} = 23936.17N$$

$\therefore\ F_d = 28495.44N$

We find $F_s > F_d$. The design is safe.

Step 12: Check for maximum wear load (F_w):

From PSGDB 8.52

$F_w = d_2 \times b \times K_w$

$K_w = 0.88\,N/mm^2$ From PSGDB 8.54 , table 43.

$F_w = 720 \times 99 \times 0.88 = 62726.4N$

We find $F_w > F_d$ $\therefore$ The design is safe.

Step 13: Check for efficiency.

$$\eta_{actual} = 0.95 \times \frac{\tan H}{\tan(H+e)}$$ From PSGDB 8.49

Where, $e = \tan^{-1}(M)$, Assume $M = 0.05$

$$e = \tan^{-1}(0.05) = 2.86°$$

$$\therefore \quad \eta_{actual} = 0.95 \times \frac{\tan 15.25}{\tan(15.25 + 2.86)}$$

=0.792

$\eta_{actual} = 79.2\%$

We find that the actual efficiency is greater than the desired efficiency.

∴ The design is safe.

Step 14: Calculation of basic dimensions.

* Axial module: $M_x = 12mm$
* No. of starts: $Z_1 = 3$
* No. of teeth on the worm wheel: $Z_2 = 60$
* Face width of worm wheel: $b = 99mm$
* Length of the worm: $L \geq (12.5 + 0.09Z_2)m_x$

  $= (12.5 + 0.09 \times 60)12$

  $= 214.8mm$

  $L \approx 215mm$
* Centre distance: $a = 0.5m_x(q + Z_2)$

  $= 0.5 \times 12(11 + 60)$

  $a = 426mm$.
* Height factor: $f_0 = 1$
* Bottom clearance: $C = 0.25m_x = 0.25 \times 12 = 3mm$
* Pitch diameter: $d_1 = q \times m_x = 11 \times 12 = 132mm$

  $d_2 = 720mm$
* Tip diameter:

$d_{a1} = d_1 + 2f_0 \times m_x$ $\quad$ $d_{a2} = (Z_2 + 2f_0)m_x$

$= 132 + 2\times1\times12$ $\quad$ $= (63 + 2(1))12$

$= 156mm$ $\quad$ $= 744mm$

* Root diameter:

$d_{f1} = d_1 - 2f_0 \times m_x - 2c$ $\quad$ $d_{f2} = (Z_2 - 2f_0)m_x - 2c$

$= 132 - 2\times1\times12 - 2\times3$ $\quad$ $= (60 - 2\times1)12 - 2\times3$

$d_{f1} = 102mm$ $\quad$ $= 690mm$

17.**Design a pair of bevel gears to transmit 10kW at a pinion speed of 1440rpm. Required transmission ratio is 4. Material of gears is 15Ni 2Cr 1Mo 15 steel (BHN 400). The tooth profiles of the gears are of 20° composite form. Assume minimum number of teeth as 20, v=5m/s and medium shock conditions. (Nov/Dec 2018)**

Given data:

$P = 25KW$

$N_1 = 300rpm$

$N_2 = 120rpm$

**** Similar to this problem, change the power, speeds and material.*

Step 1: Selection of Material:

From PSGDB Pg No. 1.40. Both pinion and gears C45 steel is selected.

Step 2: Calculation of no. of teeth, virtual number of teeth and pitch angles:

$$\frac{N_1}{N_2} = \frac{300}{120} \Rightarrow i$$

$$\therefore i = 2.5.$$

Case(1): Calculation of no. of teeth Z_1 & Z_2

$$Z_1 = 20 \quad \text{Assume}$$

$$Z_2 = i \times Z_1$$

$$= 2.5 \times 20$$

$$= 50$$

Case 2: Calculation of virtual no. of teeth Z_{v1} & Z_{v2}

From PSGDB 8.39

$$z_{V1} = \frac{z_1}{\cos\delta_1}$$

$$= \frac{20}{\cos 21.8°} = 21.54 \approx 22$$

$$Z_{v2} = \frac{Z_2}{\cos\delta_2} = \frac{50}{\cos 68.2}$$

$$= 134.64 \approx 135$$

From PSGDB 8.39

$$\delta_1 = 90° - \delta_2$$

$$\tan\delta_2 = i$$

$$\delta_2 = \tan^{-1} 2.5 = 68.2°$$

$$\therefore \ \delta_1 = 21.8°$$

Step 4: Calculation of tangential load (F_t).

$$F_t = \frac{P}{v} \times K_0$$

Where

- $K_0 = 1.5$ for medium shock conditions.
- $v = \frac{\pi d_1 N_1}{60}$

From PSGDB 8.38 , table 31

$$d_1 = m_t \times Z_1$$

$$\therefore \ v = \frac{\pi \times m_t \times 20 \times 300}{60 \times 1000}$$

$$v = 0.314\ m_t \ \text{m/s}$$

$$\therefore \ F_t = \frac{25 \times 10^3}{0.314 m_t} \times 1.5$$

$$F_t = \frac{119366.21}{m_t}$$

Step 5: Calculation of initial dynamic load (Fd).

$$F_d = \frac{F_t}{C_v}$$

From PSGDB 8.52

$$C_v = \frac{5.5}{5.5+\sqrt{v}}$$, assuming v = 5 m/s

$$= \frac{5.5}{5.5+\sqrt{5}}$$

$$C_v = 0.711$$

$$\therefore \quad F_d = \frac{119366.21}{m_t} \times \frac{1}{0.711}$$

$$F_d = \frac{167895.48}{m_t}$$

Step 6: Calculation of beam strength (F_s)

From PSGDB 8.52,

$$F_s = \pi \times m_t \times [\sigma_b] \times b \times y^1 \left(\frac{R-b}{R}\right)$$

Where,

$b = 10 \times m_t$ From PSGDB 8.38, table 31

$[\sigma_b] = 180\ N/mm^2$, for C45 steel

$y^1 = 0.154 - \frac{0.912}{Z_{v1}}$ From PSGDB 8.50, 20 FD

$= 0.154 - \frac{0.912}{22}$

$y^1 = 0.112$

R = cone radius $= 0.5m + \sqrt{Z_1^2 + Z_2^2}$ From PSGDB 8.38 , table 31

$$= 0.5 \times m_t \sqrt{20^2 + 50^2}$$

$$R = 26.93 m_t$$

$$\therefore \quad F_s = m_t \times \pi \times 10 \times m_t \times 180 \times 0.112 \times \left[\frac{26.93m_t - 10m_t}{26.93m_t}\right]$$

$$= 398.16\ m_t^2$$

Step 7: Calculation of transverse module (m_t):

From PSGDB 8.51

$$F_s \geq F_d$$

$$398.16\ m_t^2 \geq \frac{167895.48}{m_t}$$

$$m_t \geq 7.5$$

From PSGDB 8.2 , table 1, choice 1. The next nearest higher standard module $m_t = 8mm$.

Step 8: Calculation of b_1 d_1 and v:

* Face width $b = 10 \times m_t$

$= 10 \times 8$

$= 80\ mm$

Pitch circle diameter, $d_1 = m_t \times Z_1$

$= 8 \times 20$

$d_1 = 160\ mm$

Pitch line velocity $v = \frac{\pi d_1 N_1}{60}$

$$= \frac{\pi \times 160 \times 300}{60 \times 1000}$$

$v = 2.51\ m/s$

Step 9: Recalculation of beam strength:

$F_s = 398.16\ m_t^2$ From step 6

$$= 398.16 \times 8^2$$

$$F_s = 25482.24\ N$$

Step 10: Calculation of accurate dynamic load (F_d).

From PSGDB 8.51

$$F_d = F_t + \frac{21v(bc+F_t)}{21v+\sqrt{bc+F_t}}$$

Where,

* $F_t = \frac{P}{v}$

$= \frac{25\times10^3}{2.51}$

$= 9960.16N \quad 7961.78N$

* $C = 11860$ e From PSGDB 8.53 , table 41, for 20° FD
$e = 0.019$, for module upto 8 , precision gears. Table 42

$\therefore C = 11860 \times 0.019 = 225.34 N/mn$

$$\therefore F_d = 9960.16 + \frac{21\times2.51\times10^3(80\times225.34+9960.16)}{21\times2.51\times10^3+\sqrt{80\times225.34+9960.16}}$$

$F_d = 37858.96$ N

Step 11: Check for beam strength.

We find $F_d > F_s$. Design is not safe.

In order to overcome this issue, increase the module 10mm.

$\therefore \ F_d = 30415.23$ N

& $F_s = 39816N$

$\therefore \ F_s > F_d$. Design is safe.

Step 12: Calculation of maximum wear load. (F_w)

$$F_w = \frac{0.75\times d_1\times b\times Q^1\times K_w}{\cos\delta_1}$$ From PSGDB 8.51

* $Q^1 = \frac{2Z_{v2}}{Z_{v1}+Z_{v2}}$ From PSGDB 8.51

$= \frac{2\times135}{22+135}$

$Q^1 = 1.72$

* $K_w = 2.553\ \text{N/mm}^2$, for steel hardened to 400 BHN,

$$\therefore \quad F_w = \frac{0.75 \times 200 \times 100 \times 1.72 \times 2.553}{\cos 21.8°}$$

$$F_w = 70940.66\text{N}$$

Step 13: Check for wear:

$F_w > F_d$. Design is safe

Step 14: Calculation of basic dimensions of pinion and gear.

From PSGDB 8.38 , table 31.

* Transverse module: $m_t = 10\text{mm}$
* Number of teeth: $Z_1 = 20$, $Z_2 = 50$
* Pitch circle diameters: $d_1 = 200\text{mm}$

  $d_2 = 500\text{mm.}$
* Cone distance: $R = 26.93 \times 10 = 269.3\text{mm}$
* Face width: $b = 100\text{mm}$
* Pitch angles: $\delta_1 = 21.8°$, $\delta_2 = 68.2°$
* Tip diameter: $d_{a1} = m_t(Z_1 + 2\cos\delta_1)$ $\quad d_{a2} = m_t(Z_2 + 2\cos\delta_2)$

  $= 10(20 + 2\cos 21.8°)$ $\quad = 10(50 + 2\cos 68.2°)$

  $d_{a1} = 218.56\text{mm}$ $\quad d_{a2} = 507.43\text{mm}$
* Height factor: $f_0 = 1$
* Clearance: $c = 0.2$
* Addendum angle: $\tan\theta_{a1} = \tan\theta_{a2} = \dfrac{m_t \times f_0}{R_1}$

$$= \frac{10 \times 1}{269.3}$$

$$= 0.037$$

$$\theta_{a1} = \theta_{a2} = 2.13°$$

* Deddendum angle:

$$\tan\theta_{f1} = \tan\theta_{f2} = \frac{m_t(f_0 + c)}{R_1}$$

$$= \frac{10(1+0.2)}{269.3}$$

$$= 0.045$$

$$\theta_{f1} = \theta_{f2} = 2.55°$$

* Tip angle: $\delta_{a1} = \delta_1 + \theta_{a1}$ $= 21.8 + 2.13$ $\delta_{a1} = 23.93°$; $\delta_{a2} = \delta_2 + \theta_{a2}$ $= 68.2 + 2.13$ $\delta_{a2} = 70.33°$

* Root angle: $\delta_{f1} = \delta_1 + \theta_{f1}$ $= 21.8 + 2.55$ $\delta_{f1} = 19.25°$; $\delta_{f2} = \delta_2 + \theta_{f2}$ $= 68.2 + 2.55$ $\delta_{f2} = 65.65°$

* Virtual number of teeth:

$$Z_{v1} = 22 \ , \ Z_{v2} = 135$$

18. A hardened steel worm rotates at 1440rpm and transmits 12KW to a phosphor bronze gear. The speed of the worm wheel should be 60 ± 3%rpm. Design a worm gear drive if an efficiency of at least 82% is desired. Assume q=1, medium shock conditions, v=5m/s, pressure angle 20° (Nov/Dec 2018)

Given data:

$N_1 = 1440\text{rpm}$

$P = 12\text{KW}$

$N_2 = 60 \pm 3\%\text{rpm}$

$\eta_{desired} = 82\%$

Step 1: To find gear ratio (i) :

$$i = \frac{N_1}{N_2} \pm 3\%$$

$$= \frac{1440}{60} \pm 3\%$$

$$= 24 \pm 0.72$$

$$\text{take } i = 24$$

Step 2: Selection of Material:

Worm = Hardened steel

Worm wheel = Phosphor bronze

Step 3: Calculation of Z_1 and Z_2:

From PSGDB 8.46, table 37.

For $\eta = 82\%$, $Z_1 = 3$

$$Z_2 = i \times Z_1$$

$$= 24 \times 3$$

$$Z_2 = 72$$

Step 4: Calculation of q and H:

Case 1: To find diameter factor (q):

From PSGDB 8.43, table 35, and PSGDB 8.44

$$d_1 = \frac{q}{m_x}$$

Initially we assume $q = 11$

Case 2: To find Lead angle (H):

From PSGDB 8.43 , table 35

$$\tan H = \frac{Z_1}{q}$$

$$H = \tan^{-1}\left(\frac{3}{11}\right)$$

$$H = 15.25°$$

Step 5: Calculation of 'F_t' in terms of 'm_x':

Tangential Load $F_t = \frac{P}{v} \times K_0$

Case 1: To find the velocity 'v':

$$v = \frac{\pi d_2 N_2}{60 \times 1000}$$

From PSGDB 8.43 , table 35

$$d_2 = Z_2 \times m_x$$

$$\therefore \; v = \frac{\pi \times Z_2 \times m_x \times N_2}{60 \times 1000}$$

$$= \frac{\pi \times 72 \times m_x \times 60}{60 \times 1000}$$

$$v = 0.226 m_x \; {}^{m}\!/_{s}$$

Case 2: to find shock factor (K_0):

Assume medium shock,

$$K_0 = 1.5$$

$$\therefore \; F_t = \frac{12 \times 10^3}{0.226 m_x} \times 1.5$$

$$F_t = \frac{79646.02}{m_x}$$

Step 6: Calculation of dynamic load: (F_d)

$$F_d = \frac{F_t}{C_v}$$

Case 1: To find velocity factor (C_v) :

From PSGDB 8.51 , assume $v = 5 m/s$

$$C_v = \frac{6}{6 + v}$$

$$= \frac{6}{6 + 5}$$

$$C_v = 0.545$$

Case 2: To find (F_d):

$$F_d = \frac{79646.02}{m_x} \times \frac{1}{0.545}$$

$$= \frac{1460177.70}{m_x}$$

Step 7: Calculation of beam strength (F_s) **in terms of** (m_x)

From PSGDB 8.51

$$F_s = \pi \times m_x \times b \times [\sigma_b] \times y^1$$

Where ,

$b = 0.75d_1$ From PSGDB 8.48 , table 38

$= 0.75 \times q \times m_x$

$= 0.75 \times 11 \times m_x$

$= 8.25m_x$

$y^1 = 0.125$ From PSGDB 8.52 , Assume $\alpha = 20°$

Form factor $y = 0.392$

$$\therefore y^1 = \frac{y}{\pi}$$

$$\frac{0.392}{\pi}$$

$=0.125$

$[\sigma_b] = 80\,N/mm^2$ From PSGDB 8.45 , table 33

$$\therefore F_s = \pi \times m_x \times 8.25m_x \times 80 \times 0.125$$

$$= 259.18\ m_x^2$$

Step 8: Calculation of Axial module (m_x)

W . K . T $F_s \geq F_d$

$$259.18 \times m_x^2 \geq \frac{146017.70}{m_x}$$

$$m_x \geq 8.26mm$$

From PSGDB 8.2 , Table 1.

The nearest higher standard axial module

$$m_x = 10mm.$$

Step 9: Calculation of b, d_2 and v:

Case 1: To find the face width (b):

$$b = 8.25m_x \quad \text{From step 7}$$

$$= 82.5mm$$

Case 2: To find pitch diameter of the worm wheel (d_2)

$$d_2 = Z_2 \times m_x \quad \text{From step 5 case 1.}$$

$$= 72 \times 10$$
$$= 720mm$$

Case 3: To find the pitch line velocity of worm wheel (v)

$$v = 0.226\ m_x \quad \text{From step 5, case 1.}$$

$$= 0.226 \times 10$$

$$v = 2.26 m/s$$

Step 10: Recalculation of beam strength.

$$F_s = 259.18m_x^2 \qquad \text{From step 7}$$

$$= 259.18 \times 10^2$$

$$F_s = 25918N$$

Step 11: Recalculation of dynamic load (F_α)

$$F_d = \frac{F_t}{C_v}$$

$$C_v = \frac{6}{6+v} = \frac{6}{6+2.26} = 0.726$$

$$F_t = \frac{79646.02}{m_x} = \frac{79646.02}{10} = 7964.602N$$ From step 5 case 2

$$\therefore \ F_d = \frac{7964.602}{0.726}$$

$$F_d = 10970.53N$$

Step 12: Check for beam strength.

We find $F_d < F_s$. the design is safe.

Step 13: Check for Maximum wear load (F_N):

From PSGDB 8.52

$$F_w = d_2 \times b \times K_w$$

$K_w = 0.56 N/mm^2$ From PSGDB 8.54 , table 43

$$F_w = 720 \times 82.5 \times 0.56$$

$$F_w = 33264N$$

Step 14: Check for efficiency.

$$\eta_{actual} = 0.95 \times \frac{\tan H}{\tan(H+e)}$$ From PSGDB 8.49

Where, $\rho = \tan^{-1} M$, Assume $M = 0.03$ From PSGDB 8.49

$$\rho = \tan^{-1}(0.03)$$

$$= 1.7°$$

$$\eta_{actual} = 0.95 \times \frac{\tan 15.25}{\tan(15.25 + 1.7)}$$

$= 0.8498$ We find that the actual efficiency is greater than the desired efficiency. $\therefore$ The design is safe.

$$\eta_{actual} = 84.98\%$$

Step 15: Calculation of basic dimensions of worm and worm gears.

From PSGDB 8.43 , table 35

Axial module: $m_x = 10mm$

No. of starts: $Z_1 = 3$

No. of teeth on the worm wheel: $Z_2 = 72$

Face width of the worm wheel: $b = 82.5mm$

Length of the worm: $L \geq (12.5 + 0.09Z_2)m_x$

$= (12.5 + 0.09 \times 72)10$

$= 189.8mm$

Take $L = 190mm$

Centre distance: $a = 0.5m_x(q + Z_2)$

$a = 0.5 \times 10(11 + 72)$

$a = 415mm$

Height factor: $f_0 = 1$

Bottom clearance: $C = 0.25m_x = 0.25 \times 10 = 2.5mm.$

Pitch diameter: $d_1 = q \times m_x = 11 \times 10 = 110mm$

$d_2 = 720mm$

Tip diameter: $d_{a1} = d_1 + 2f_0 \times m_x = 110 + 2 \times 1 \times 10 = 130mm$

$d_{a2} = (Z_2 + 2f_0)m_x = (72 + 2 \times 1)10 = 740mm$

Root diameter: $d_{f1} = d_1 - 2f_0 \times m_x - 2C$

$= 110 - 2 \times 1 \times 10 - 2 \times 2.5$

$= 85mm$

$d_{f2} = (Z_2 - 2f_0)m_x - 2C$

$= (72 - 2 \times 1) \times 10 - 2 \times 2.5$

$= 695mm.$

19. Design a bevel gear drive to transmit 7kW at 1600rpm for the following data. Gear ratio=3, Material for pinion and gear= C45 steel, Life 10,000 hours. (April/May 2019)

Given data:

$P = 3.5KW$
$i = 4$
$N_2 = 200rpm.$
$\text{Material} \Rightarrow \text{Pinion} - \text{Steel}$
$\text{Wheel} - \text{CI}$

The materials of pinion and gear are different, we have to design the pinion first and check the gear.

**** similar to the problem*

Step 1: Gear ratio & Pitch angles:

$$i = \frac{N_1}{N_2} = 4$$

$$\frac{N_1}{200} = 4 \quad N_1 = 800rpm.$$

Pitch angles:

From PSGDB 8.34

$\tan\delta_2 = i = 4$

$\therefore \ \delta_2 = \tan^{-1}(4) = 75.96°$

$\delta_1 = 90 - \delta_2 = 90 - 75.96$

$= 14.04°$

Step 2: Material selection:

Pinion: Steel, $\sigma_u = 700N/mm^2 \quad \sigma_y = 360N/m^2$

Gear: CI grad 35 - $\sigma_u = 350N/mm^2$

Step 3: Gear life in cycles:

Gear life in hours $= 25000hrs.$

Gear life in cycles $N = 25000 \times 800 \times 60$

$$= 12 \times 10^8 \text{ cycles.}$$

Step 4: Calculation of initial design torque $[M_t]$:

From PSGDB 8.44,

$$[M_t] = M_t \times K \times K_d$$

$$M_t = \frac{60 \times P}{2\pi N_1} = \frac{60 \times 3.5 \times 10^3}{2 \times \pi \times 800} = 41.778 \text{N.m}$$

$K.K_d = 1.3$ initially assume.

$$\therefore \quad [M_t] = 41.778 \times 1.3$$

$$= 54.31 \text{ Nm.}$$

Step 5: Calculation of E_{eq}, $[\sigma_b]$ and $[\sigma_c]$:

To find E_{eq}.

$E_{eq} = 1.7 \times 10^5 \text{N/mm}^2$ From PSGDB8.14

To find $[\sigma_{b1}]$ Design bending stress for pinion.

$[\sigma_{b1}] = \frac{1.4 K_{b1}}{n \times K_\sigma} \times \sigma - 1$ From PSGDB 8.18 rotation in one direction

$K_{b1} = 1$,for HB ≤ 350 and N Z$10^{7,}$ From PSGDB 8.20, table 22.

$K_\sigma = 1.5$, for steel pinion. From PSGDB 8.19, table 21

$n = 2.5$ steel hardened From PSGDB 8.19, table 20

$\sigma_{-1} = \left(0.25\left(\sigma_u + \sigma_y\right) + 50\right)$, for forged steel. From PSGDB 8.19, table 19

$$\sigma_{-1} = 0.25(700 + 360) + 50$$

$$= 315 \text{N/mm}^2.$$

$$\therefore [\sigma_{b1}] = \frac{1.4 \times 1}{2.5 \times 1.5} \times 315 = 117.6 \text{N/mm}^2.$$

To find $[\sigma_{c1}]$:

$$[\sigma_{c1}] = C_R.\text{HRC} \times K_{C1}$$

$C_R = 23$

$HRC = 40 to 55$

$K_{C1} = 1$

$[\sigma_{c1}] = 23 \times 55 \times 1$

$= 1265 N/mm^2$

Step 6: Calculation of cone distance (R).

$$R \geq \varphi_y \sqrt{i^2+1}\sqrt[3]{\left[\frac{0.72}{(\varphi_y - 0.5)[\sigma_c]}\right]^2 \times \frac{E_{eq}[M_t]}{i}}$$

$$\varphi_y = R/6 = 3$$

$$R \geq 3\sqrt{4^2+1}\sqrt{\left[\frac{0.72}{(3-0.5)\times 1265}\right]^2 \times \left[\frac{1.7\times10^5 \times 54.31\times10^3}{4}\right]}$$

$$\geq 135.29 mm.$$

$R = 136 mm.$

Step 7: Selection of No. of teeth on pinion and gear.

$Z_1 = 20$

$Z_2 = i \times Z_1$

4×20

$= 80$

Virtual no. of teeth: $Z_{v1} = \dfrac{Z_1}{\cos\delta_1} = \dfrac{20}{\cos 14.04} = 20.61 \approx 21$

From PSGDB 8.22. $Z_{v2} = \dfrac{Z_2}{\cos\delta_2} = \dfrac{80}{\cos 75.96°} = 329.76 \approx 330$

Step 8: Calculation of transverse module: (m_t).

$$M_t = \frac{R_1}{0.5\sqrt{Z_1^2 + Z_2^2}}$$ From PSGDB 8.38 table 31

$$= \frac{136}{0.5\sqrt{20^2 + 80^2}}$$

$$= 3.29mm$$

From PSGDB 8.2, table 1, choice 1.

The nearest next higher standard transverse module m_t =4mm.

Step 9: Revision of cone distance: (R)

$$R = 0.5m_t\sqrt{Z_1^2 + Z_2^2}$$

$$= 0.5 \times 4\sqrt{20^2 + 80^2}$$

$$= 164.92mm.$$

Step 10: Calculation of b, m_{av} , d_{1av}, v and φ_y :

(vi) To find b: $b = \frac{R}{\varphi_y} = \frac{164.92}{3} = 54.97 \approx 55mm$ PSGDB 8.38

(vii) Average module $m_{av} = m_t - \frac{b \sin \delta_1}{Z_1}$ PSGDB 8.38

$$= 4 - \frac{55 SIN 14.04}{20}$$

$$= 3.33mm.$$

(viii) Average PCD of pinion: $d_{1av} = m_{av} \times Z_1$

$$= 3.33 \times 20$$

$$= 66.66mm.$$

(ix) Pitch line velocity $v = \frac{\pi d_{1av} \times N_1}{60} = \frac{\pi \times 66.66 \times 800}{60 \times 1000} = 2.79 \, m/s.$

(x) $\varphi_y = \frac{b}{d_{1av}} = \frac{55}{66.66} = 0.83$

Step 11: Selection of Quality of gears.

Is Quality 8 bevel gear is selected. From PSGDB 8.3 table 2.

Step 12: Revision of design torque [M_t]:

$$[M_t] = M_t \times K \times K_d$$

$K = 1.1$ From PSGDB 8.15

$K_d = 1.45$ From PSGDB 8.16 table 15.

$\therefore \ [M_t] = 41.778 \times 1.1 \times 1.45$

$= 66.64 Nm.$

Step 13: Check for bending of pinion.

$$\sigma_{b1} = \frac{R\sqrt{1^2+1}[M_t]}{(R-0.5b)^2 \times b \times m_t \times y_{v1}}$$ From PSGDB 8.13[A].

$y_{v1} = 0.402$ for $Z_{v1} = 21$

$$\therefore \ \sigma_{b1} = \frac{164.92\sqrt{4^2+1} \times 66.64 \times 10^3}{(164.92 - 0.5 \times 55)^2 \times 55 \times 4 \times 0.402}$$

$$\sigma_{b1} = 27.13 N/mm^2$$

We find $\sigma_{b1} < [\sigma_{b1}]$. $\therefore$ the design is safe.

Step 14: Check for wearing of pinion.

$$\sigma_{c1} = \left(\frac{0.72}{R-0.56}\right)\left[\frac{\sqrt{(i^2+1)^3}}{i \times b} \times E_{eq} \times [M_t]\right]^{1/2}$$ From PSGDB 8.13

$$= \left(\frac{0.72}{164.92 - 0.5 \times 55}\right)\left[\frac{\sqrt{(4^2+1)^3}}{4 \times 55} \times 1.7 \times 10^5 \times 66.64 \times 10^3\right]^{1/2}$$

$$\sigma_{C1} = 314.77 N/mm^2$$

We find $\sigma_{C1} < [\sigma_C]$. Thus the design is satisfactory for pinion.

Step 15: Check for gear.

Gear material: CI grade 30.

First we have to calculate $[\sigma_{b2}]$ and $[\sigma_{c2}]$.

Gear life of wheel $N = \frac{N_{pinion}}{3}$

$$=\frac{12\times10^{8}}{3}$$

$$=4\times10^{8}$$

To find $[\sigma_{b2}]$:

$$[\sigma_{b2}]=\frac{1.4\times K_{b1}}{h\times K_{\sigma}}\times\sigma_{-1}$$

Where,

$$K_{b1}=\sqrt[9]{\frac{10^{7}}{N}}=\sqrt[9]{\frac{10^{7}}{4\times10^{8}}}=0.66$$

$$K_{\sigma}=1.2$$

$$n=2$$

$$\sigma_{-1}=0.45\sigma_{u}=0.45\times350=157.5N/mm^{2}.$$

$$\therefore\ [\sigma_{b2}]=\frac{1.4\times0.66}{2\times1.2}\times157.5$$

$$=60.64N.mm^{2}.$$

To find $[\sigma_{c2}]$:

$$[\sigma_{c2}]=C_{B}\times HB\times K_{cl}.$$

Where $C_{B}=2.3,$ $HB=200\text{ to }260$

$$K_{cl}=\sqrt[b]{\frac{10^{7}}{N}}=\sqrt[b]{\frac{10^{7}}{4\times10^{8}}}=0.54$$

$$\therefore[\sigma_{c2}]=2.3\times260\times0.54$$

$$=322.92N/mm^{2}.$$

Case 1: Check for bending of gear.

$$\sigma_{b1}\times y_{v1}=\sigma_{b2}\times y_{v2}$$

$y_{v1}=0.402$ for $Z_{v1}=21$

$y_{v2}=0.521$ for $Z_{v2}=330$

$27.13 \times 0.402 = \sigma_{b2} \times 0.521$

$\sigma_{b2} = 20.93 N/mm^2.$

$\sigma_{b2} < [\sigma_{b2}].$ $\therefore$ design is safe.

Case 2: Check for wearing of gear.

Since the contact area is same.

$\therefore \sigma_{c2} = \sigma_{c1} = 314.77 N/mm^2$

We find $\sigma_{c2} < [\sigma_{c2}]$. It means the gear having the adequate beam strength.

$\therefore$ The design is safe and satisfactory.

20. A hardened steel worm rotates at 1440rpm and transmits 12KW to a phosphor bronze gear. The speed of the worm wheel should be 60 ± 3%rpm. Design a worm gear drive if an efficiency of at least 82% is desired. (April/May 2019)

Given data:

$N_1 = 1440 rpm$

$P = 12 KW$

$N_2 = 60 \pm 3\% rpm$

$\eta_{desired} = 82\%$

Step 1: To find gear ratio (i) :

$$i = \frac{N_1}{N_2} \pm 3\%$$

$$= \frac{1440}{60} \pm 3\%$$

$$= 24 \pm 0.72$$

$$\text{take } i = 24$$

Step 2: Selection of Material:

Worm = Hardened steel

Worm wheel = Phosphor bronze

Step 3: Calculation of Z_1 and Z_2:

From PSGDB 8.46, table 37.

For $\eta = 82\%$, $Z_1 = 3$

$$Z_2 = i \times Z_1$$

$$= 24 \times 3$$

$$Z_2 = 72$$

Step 4: Calculation of q and H:

Case 1: To find diameter factor (q):

From PSGDB 8.43, table 35, and PSGDB 8.44

$$d_1 = \frac{q}{m_x}$$

Initially we assume $q = 11$

Case 2: To find Lead angle (H):

From PSGDB 8.43 , table 35

$$\tan H = \frac{Z_1}{q}$$

$$H = \tan^{-1}\left(\frac{3}{11}\right)$$

$$H = 15.25°$$

Step 5: Calculation of 'F_t' in terms of 'm_x':

Tangential Load $F_t = \frac{P}{v} \times K_0$

Case 1: To find the velocity 'v':

$$v = \frac{\pi d_2 N_2}{60 \times 1000}$$

From PSGDB 8.43 , table 35

$d_2 = Z_2 \times m_x$

$$\therefore \; v = \frac{\pi \times Z_2 \times m_x \times N_2}{60 \times 1000}$$

$$= \frac{\pi \times 72 \times m_x \times 60}{60 \times 1000}$$

$$v = 0.226 m_x \; m/s$$

Case 2: to find shock factor (K_0):

Assume medium shock,

$$K_0 = 1.5$$

$$\therefore \; F_t = \frac{12 \times 10^3}{0.226 m_x} \times 1.5$$

$$F_t = \frac{79646.02}{m_x}$$

Step 6: Calculation of dynamic load: (F_d)

$$F_d = \frac{F_t}{C_v}$$

Case 1: To find velocity factor (C_v) :

From PSGDB 8.51 , assume $v = 5 m/s$

$$C_v = \frac{6}{6+v}$$

$$= \frac{6}{6+5}$$

$$C_v = 0.545$$

Case 2: To find (F_d):

$$F_d = \frac{79646.02}{m_x} \times \frac{1}{0.545}$$

$$= \frac{1460177.70}{m_x}$$

Step 7: Calculation of beam strength (F_s) **in terms of** (m_x)

From PSGDB 8.51

$$F_s = \pi \times m_x \times b \times [\sigma_b] \times y^1$$

Where ,

$b = 0.75 d_1$ From PSGDB 8.48 , table 38

$= 0.75 \times q \times m_x$

$= 0.75 \times 11 \times m_x$

$= 8.25 m_x$

$y^1 = 0.125$ From PSGDB 8.52 , Assume $\alpha = 20°$

Form factor $y = 0.392$

$\therefore\ y^1 = \frac{y}{\pi}$

$\frac{0.392}{\pi}$

$= 0.125$

$[\sigma_b] = 80\, N/mm^2$ From PSGDB 8.45 , table 33

$\therefore F_s = \pi \times m_x \times 8.25 m_x \times 80 \times 0.125$

$= 259.18\, m_x^2$

Step 8: Calculation of Axial module (m_x)

W . K . T $F_s \geq F_d$

$$259.18 \times m_x^2 \geq \frac{146017.70}{m_x}$$

$$m_x \geq 8.26 mm$$

From PSGDB 8.2 , Table 1.

The nearest higher standard axial module

$$m_x = 10 mm.$$

Step 9: Calculation of b, d_2 and v:

Case 1: To find the face width (b):

$b = 8.25 m_x$ From step 7

$= 82.5 mm$

Case 2: To find pitch diameter of the worm wheel (d_2)

$d_2 = Z_2 \times m_x$ From step 5 case 1.

$= 72 \times 10$

$= 720mm$

Case 3: To find the pitch line velocity of worm wheel (v)

$v = 0.226\ m_x$ From step 5, case 1.

$= 0.226 \times 10$

$v = 2.26\,m/s$

Step 10: Recalculation of beam strength.

$F_s = 259.18 m_x^2$ From step 7

$= 259.18 \times 10^2$

$F_s = 25918N$

Step 11: Recalculation of dynamic load (F_α)

$$F_d = \frac{F_t}{C_v}$$

$$C_v = \frac{6}{6+v} = \frac{6}{6+2.26} = 0.726$$

$$F_t = \frac{79646.02}{m_x} = \frac{79646.02}{10} = 7964.602N$$ From step 5 case 2

$$\therefore\ F_d = \frac{7964.602}{0.726}$$

$$F_d = 10970.53N$$

Step 12: Check for beam strength.

We find $F_d < F_s$. the design is safe.

Step 13: Check for Maximum wear load (F_N):

From PSGDB 8.52

$F_w = d_2 \times b \times K_w$

$K_w = 0.56\,N/mm^2$ From PSGDB 8.54 , table 43

$F_w = 720 \times 82.5 \times 0.56$

$F_w = 33264N$

Step 14: Check for efficiency.

$$\eta_{actual} = 0.95 \times \frac{\tan H}{\tan(H+e)}$$ From PSGDB 8.49

Where, $\rho = \tan^{-1} M$, Assume $M = 0.03$ From PSGDB 8.49

$\rho = \tan^{-1}(0.03)$

$= 1.7°$

$$\eta_{actual} = 0.95 \times \frac{\tan 15.25}{\tan(15.25+1.7)}$$

$= 0.8498$ We find that the actual efficiency is greater than the desired efficiency. $\therefore$ The design is safe.

$\eta_{actual} = 84.98\%$

Step 15: Calculation of basic dimensions of worm and worm gears.

From PSGDB 8.43 , table 35

Axial module: $m_x = 10mm$

No. of starts: $Z_1 = 3$

No. of teeth on the worm wheel: $Z_2 = 72$

Face width of the worm wheel: $b = 82.5mm$

Length of the worm: $L \geq (12.5 + 0.09Z_2)m_x$

$= (12.5 + 0.09 \times 72)10$

$= 189.8mm$

Take $L = 190mm$

Centre distance: $a = 0.5m_x(q + Z_2)$

$a = 0.5 \times 10(11 + 72)$

$a = 415mm$

Height factor: $f_0 = 1$

Bottom clearance: $C = 0.25m_x = 0.25 \times 10 = 2.5mm.$

Pitch diameter: $d_1 = q \times m_x = 11 \times 10 = 110mm$

$$d_2 = 720\text{mm}$$

Tip diameter: $d_{a1} = d_1 + 2f_0 \times m_x = 110 + 2 \times 1 \times 10 = 130\text{mm}$

$$d_{a2} = (Z_2 + 2f_0)m_x = (72 + 2 \times 1)10 = 740\text{mm}$$

Root diameter: $d_{f1} = d_1 - 2f_0 \times m_x - 2C$

$$= 110 - 2 \times 1 \times 10 - 2 \times 2.5$$

$$= 85\text{mm}$$

$$d_{f2} = (Z_2 - 2f_0)m_x - 2C$$

$$= (72 - 2 \times 1) \times 10 - 2 \times 2.5$$

$$= 695\text{mm}.$$

ME-6601 DESIGN OF TRANSMISSION SYSTEMS

UNIT-IV DESIGN OF GEAR BOXES
(PART-A)

1. What are the preferred numbers?

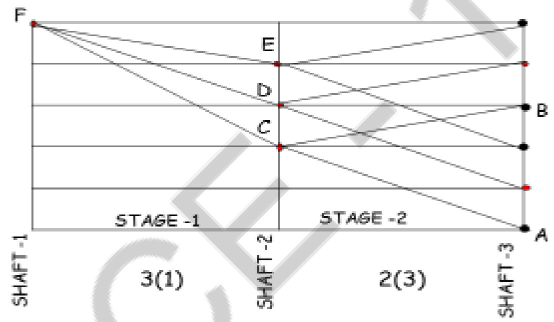
Preferred numbers are conventionally rounded of values derived from geometric series. There are five basic series, denoted as R5, R10, R20, R40 and R80 series.

2. Specify four types of gear boxes?

- Sliding mesh gear box
- Constant mesh gear box
- Synchromesh gear box
- Planetary gear box

3. Draw the ray diagram for a six speed gear box?

A typical ray diagram for a 6 speed gear box, for the preferred structural formula 3(1) 2(3), is shown in figure below.



4. In which gear drive, self-locking is available?

Self-locking is available in worm gear drive.

5. Define progression ratio?

When the spindle speeds are arranged in geometric progression, then the ratio between the two adjacent speeds is known as **step ratio or progression ratio.**

6. Write the significance of structural formula.

- STRUCTURAL FORMULAE:

No.of Stages: {(p_1 (X_1) . p_2 (X_2). P_3 (X_3) }

1st stage. 2nd stage 3rd stage

Note:Where $X_1 = 1$ $X_2 = p_1$ $X_3 = p_1 p_2$

7. What is multispeed gear box?

The gear box containing variable spindle speeds are known as multispeed gear box

8. **What is R20 series?**

R20 is one of the series of five basic geometric series. The symbol 'R' is used as a tribute to French engineer Charles Renard, whom introduced the preferred number first.

9. **Differentiate ray diagram and structural diagram.**

The structural diagram is a kinematic layout that shows the arrangement of gears in a gear box
The speed diagram, also known as ray diagram, is a graphical representation of the structural formula.

10. **List out two methods used for changing speeds in gear boxes?**

Sliding mesh gear box
Constant mesh gear box

11. **What purpose does the housing of gear-box serve?**

Gear-box -housing or casing is used as container inside which, the gears, shafts, bearings and other components are "mounted.' Also it prevents the entry of dust inside the housing and reduces noise of operation.
That is, the housing Safe-guard the inner components.

12. **What is the function of spacers in a gear-box?(or) What are spacers as applied to a gear-box?**

Spacers are sleeve like components, which are mounted, in shafts in-between gears and bearings or one gear and another gear in order to maintain the distance between them so as to avoid interruption between them.

13. **Fill in the blanks of the following.**

(a) The number of gears employed in a gear-box is kept to the minimum by arranging the Speed of the spindle is .... series.
(b) In a gear- box, -for a set of gears, if the centre distance and module are same, then the sum of teeth of engaging pair will be .....

Answers

a) Geometric series.

b) Equal.

14. What is a speed diagram? (or) What is the structural diagram-of - &.gear-box

Speed diagram or structural diagram is the graphical representation different speeds of output shaft, motor shaft and intermediate shafts.

15. For what purpose we are using gear-box?

Since the gear-box is provided with number of gears of different size arranged is different forms, we can get number of output speeds by operated motor at single speed.

16. Name the types of speed reducers.

a) Single reduction speed reduces.

b) Multi reduction speed reducers.

17. What does the ray-diagram of gear-box indicate?

The ray-diagram or kinematic arrangement of a gear box indicates arrangement of various gears in various shafts of the gear box in order to obtain the different output speeds from the single speed of the motor.

18. What is step ratio?,

Step ratio is the ratio of one speed of the shaft to its previous lower speed Since the spindle speeds are arranged in geometric progression, the ratios adjacent speeds (i.e., step ratios) are constant.

19. What is the functions of spacers in gear box?

The functions of spacers in gear box is to provide the necessary distance between the gears and the bearings

20. What are the methods of lubrications in speed reducers?

- ❖ Splash or spray lubricating methods
- ❖ Pressure lubricating methods

21. Why geometric progression is selected for arranging the speeds in gear box? (April/May 2017)

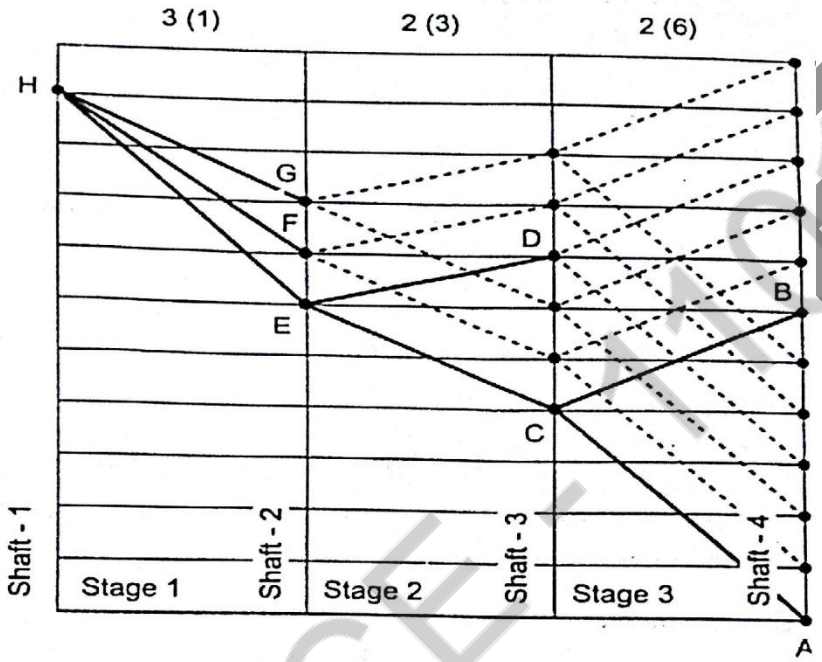
- ❖ The speed loss is minimum , if geometric progression is used
- ❖ The number of gears to be employed is minimum, if geometric progression is used
- ❖ Geometric progression provides more even the range of spindle speeds at each step

- The layout is comparatively very compact, if geometric progression is used

22. What does the ray diagram of gear box indicate? (April/May 2017)

The ray diagram is a graphical representation of the drive arrangement in general form. It serves to determine the specific values of all the transmission ratios and speeds of all the shafts in the drive.

23. Draw the ray diagram of 12 speed gear box. (Nov/Dec 2017)



Ray diagram for 12 speed gear box

24. Write any two principles to be followed to obtain optimum design in gear box. (Nov/Dec 2017)

- Reliability of the system (Gear box)
- Component reliability (Gear pair)

25. For what purpose we are using gear box? (April/May 2018)

Gear boxes are required wherever the variable spindle speeds is necessary

26. What is speed diagram? (April/ May 2018)

The speed diagram is also known as ray diagram, is a graphical representation of the drive arrangement in general form. It serves to determine the specific values of all the transmission ratios and speeds of all the shafts in the drive.

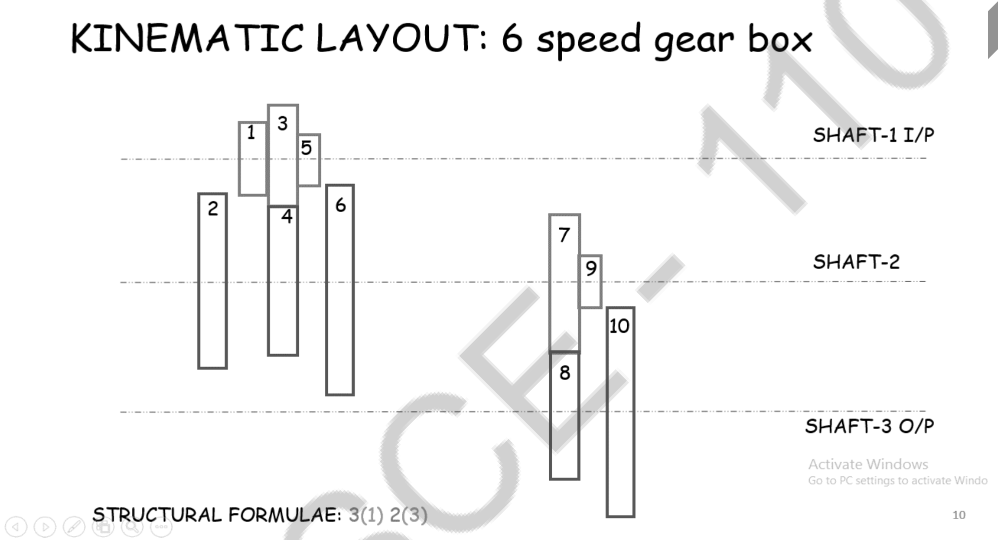
27. Define progression ratio. (Nov/Dec 2018)

When the spindle speeds are arranged in geometric progression, the the ratio between the two adjacent speeds is known as step ratio or progression ratio.

28. **List out the all possible arrangements to achieve 16 speed gear box. (Nov/Dec 2018)**
    i. 4 X 2 X 2 Scheme
    ii. 2 X 4 X 2 Scheme
    iii. 2 X 2 X 4 Scheme

29. What is torque converter? (April/May 2019)
   (i) A torque converter is a type of fluid coupling which transfers rotating power from a prime mover, like an internal combustion engine, to a rotating driven load.
   (ii) In a vehicle with an automatic transmission, the torque converter connects the power source to the load.

30. **Draw the kinematic layout for the 6-speed gear box. (April/May 2019)**



PART B

1. **A sixteen speed gear box is required to furnish output speeds in the range of 100 to 560rpm. Sketch the kinematic arrangement and draw the speed diagram.**

   Given data:

   $M = 16$

   $N_{min} = 100 rpm$

   $N_{max} = 560 rpm$

   Step 1: Selection of spindle speeds.

$$\frac{N_{max}}{N_{min}} = \phi^{16-1}$$

$$\frac{560}{100} = \phi^{15}$$

$$\phi = (5.6)^{1/15}$$

$$\phi = 1.12$$

We find $\phi = 1.12$ is the standard ratio, it satisfies the requirement.

Select the spindle speeds using the series of preferred numbers.

PSGDB 7.20

Basic series R20 $(\phi = 1.12)$

Spindle speeds are 100, 112, 125, 140, 160, 180, 200, 224, 250, 280, 315, 355, 400, 450, 500, 560 rpm.

Step 2: To find the structural formulae.

$$16 \text{ Speeds} = 4(1)2(4)2(8)$$

Step 3: Construct the speed diagram for 16 speed gear box.

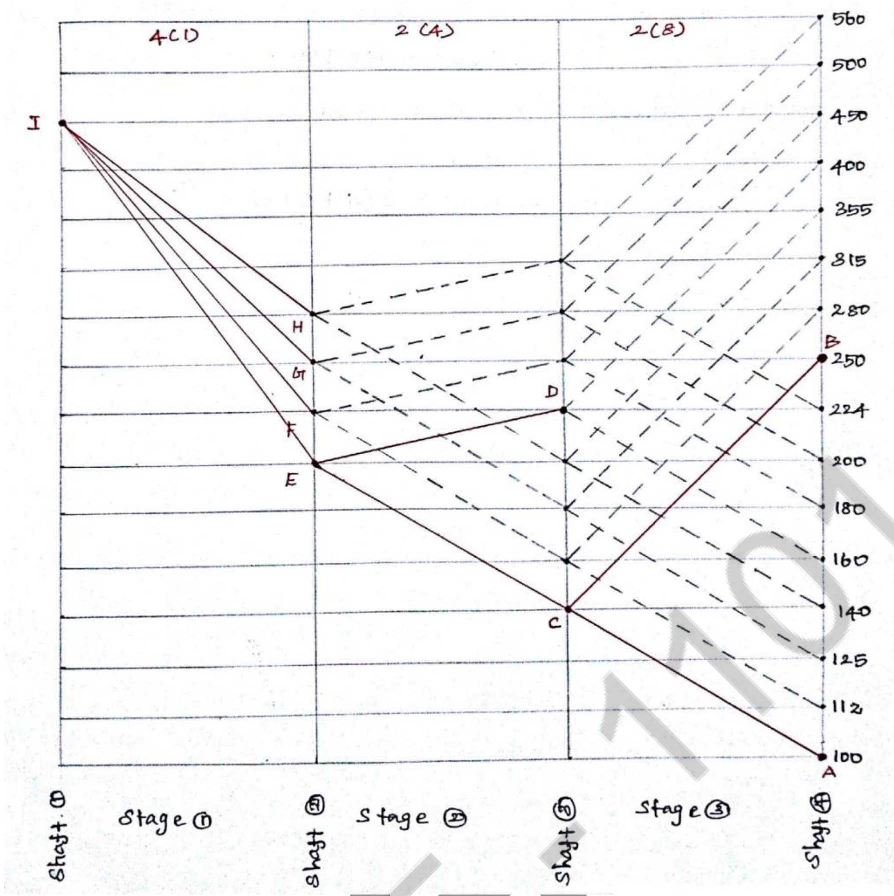
* Structural formula $= 4(1)\ 2(4)\ 2(8)$
* No. of stages $= 3\{P_1(X_1) \cdot P_2(X_2) \cdot P_3(X_3)\}$

  $P_1 = 4$, $P_2 = 2$, $P_3 = 2$

  Note: $X_1 = 1$, $X_2 = P_1 = 4$, $X_3 = P_1 \times P_2 = 4 \times 2 = 8.$
* No. of shafts = No. of stages +1

  $= 3 + 1$

  $= 4$ (Draw 4 vertical lines)
* No. of speeds $= 16$ (Draw 16 horizontal lines).



Stage 3:

$$\frac{N_{min}}{N_{input}} \geq \frac{1}{4} \qquad \frac{N_{max}}{N_{input}} \leq 2$$

$$\frac{100}{140} = 0.714 > \frac{1}{4} \qquad \frac{250}{140} = 1.78 < 2$$

$$\therefore \quad N_{input} = 140\text{rpm.}$$

Stage 2:

$$\frac{N_{min}}{N_{input}} \geq \frac{1}{4} \qquad \frac{N_{max}}{N_{input}} \leq 2$$

$$\frac{140}{200} = 0.7 \geq \frac{1}{4} \qquad \frac{224}{200} = 1.12 \leq 2$$

$$\therefore \quad N_{input} = 200\text{rpm.}$$

Stage 1:

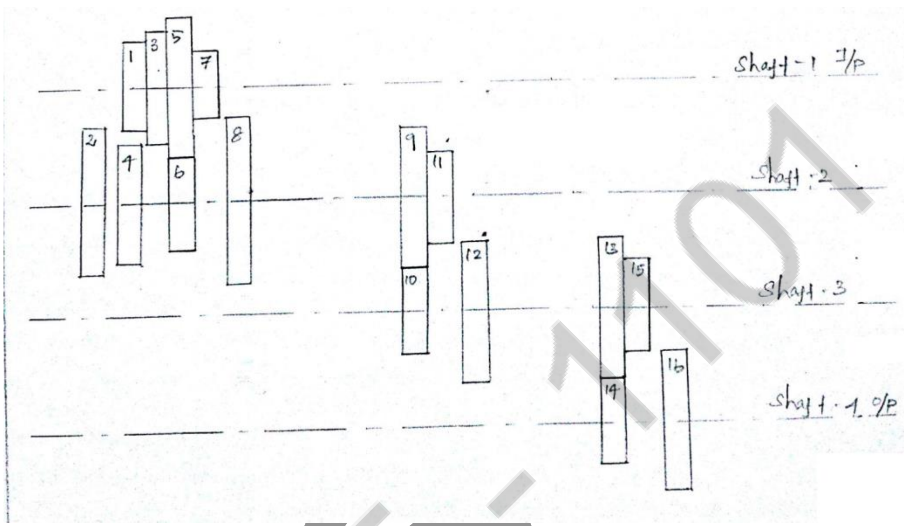
$$\frac{200}{450} = 0.44 \geq \frac{1}{4} \qquad \frac{280}{450} = 0.622 \leq 2$$

$$\therefore \quad N_{input} = 450\text{rpm}.$$

Step 4: Kinematic Layout - 16 Speed gear box

No. of shafts = 4

No. of Gears = 2 (4 + 2 + 2) = 16



2. Design a nine speed gear box for a machine to provide speeds ranging from 100rpm to 1500rpm. The input is from a motor of 5KW at 1440rpm. Assume any alloy steel for the gears.

Given data:

$$\eta = 9$$
$$N\min = 100\text{rpm}.$$
$$N\max = 1500\text{rpm}.$$
$$P = 5KW$$
$$N_{input} = 1440\text{rpm}.$$

Note: In this problem the given max speed is 1500rpm. But as per R 20 series am taken the 9th speed 1400rpm. If you want to take 1500rpm as the 9th speed also correct. No issues. Anyhow maximum cases we should follow the standard values.

Step 1: Selection of spindle speeds:

$$\frac{N_{max}}{N_{min}} = \phi^{n-1}$$

$$\frac{1500}{100} = \phi^{9-1}$$

$$15 = \phi^8$$

$$\phi = 1.403.$$

* We find $\phi = 1.403$ is not a standard ratio. So let us find out whether multiples of standard ratio 1.12 or 1.06 come close to 1.403.

  $1.12 \times 1.12 \times 1.12 = 1.405$ Skip 2 speeds.

* $\phi = 1.12$ Satisfies the requirement. Therefore the spindle speeds from R 20 series skipping 2 speeds, are.

From PSGDB 7.20,

100, 140, 200, 280, 400, 560, 800, 1000, 1400 rpm.

Step 2: To find the structural formula:

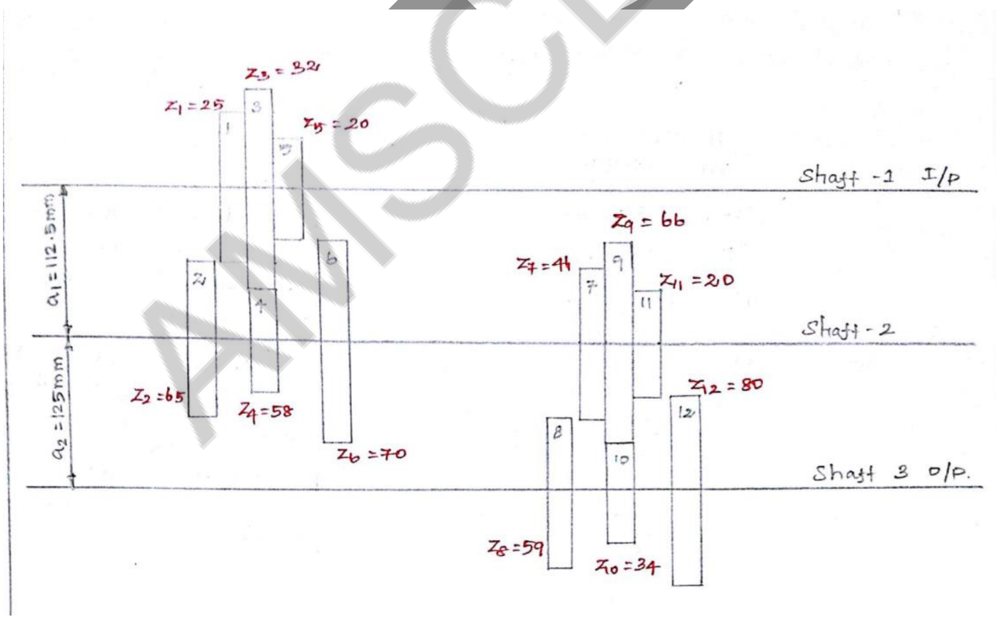
9 speeds = 3(1) 3(3)

Step 3: Kinematic diagram for 9 speeds.

Structural formula =3(1) 3(3).

No. of shafts = No. of stages +1 = 3 (3 horizontal lines).

No. of gears = 2(P_1+P_2) = 2(3+3) = 12 gears.



Step 3: Ray diagram for 9 speed.

Structural formula = 9 speeds = 3(1) 3(3).

No. of shafts =3 (3 vertical lines)

Speeds = 9 (9 horizontal lines)

For stage 2

$$\frac{N_{min}}{N_{I/p}} \geq 1/4 \qquad \frac{N_{max}}{N_{I/p}} \leq 2$$

$$\frac{100}{400} \geq 1/4 \Rightarrow \frac{N_{min}}{N_{I/p}} = 1/4 \geq 1/4 \qquad \frac{N_{max}}{N_{I/p}} = \frac{800}{400} = 2 \leq 2$$

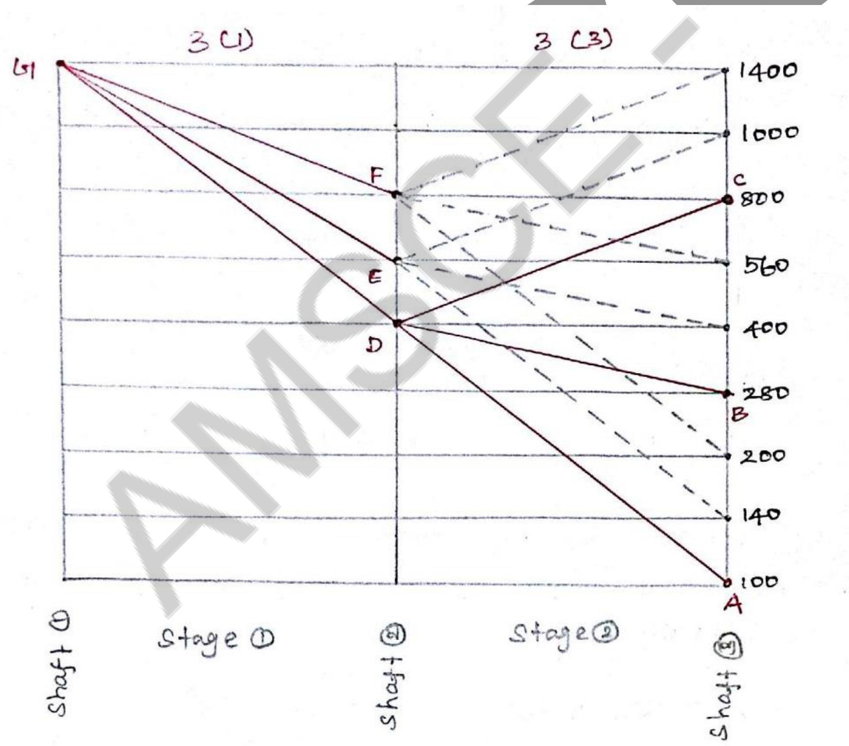
$$\therefore N_{I/p} = 400\text{rpm.}$$

For stage 1

$$\frac{N_{min}}{N_{I/p}} \geq 1/4 \qquad \frac{N_{max}}{N_{I/p}} \leq 2$$

$$\frac{N_{min}}{N_{I/p}} = \frac{400}{1400} = 0.29 \geq 1/4 \qquad \frac{N_{max}}{N_{I/p}} = \frac{800}{1400} = 0.57 \leq 2$$

$$\therefore N_{I/p} = 1400\text{rpm.}$$



Step 4: Calculation of no. of teeth on all the gears.

Let $Z_1, Z_2, Z_3 \ldots Z_{12}$ = No. of teeth of the gears 1, 2, 3.... 12 respectively.

$N_1, N_2, N_3 \ldots N_{12}$ = No. of speed of the gears 1, 2, 3.... 12 respectively.

We know that , $\frac{Z_1}{Z_2} = \frac{N_2}{N_1}$

Case 1: consider stage 2.

First pair:

* Gears 11 and 12
* From the ray diagram consider Ray DA.
* Maximum speed reduction from 400 rpm to 1000 rpm .

$$Z_{11} = 20 \ (\text{driver}).$$

$$\therefore \quad \frac{Z_{11}}{Z_{12}} = \frac{N_{12}}{N_{11}}$$

$$\frac{20}{Z_{12}} = \frac{100}{400}$$

$$Z_{12} = 80$$

$$Z_{11} = 20 \ , \quad Z_{12} = 80$$

Second Pair:

* Gears 7, 8 & Ray DB
* Minimum speed reduction 400 to 280 rpm.

$$\frac{Z_7}{Z_8} = \frac{N_8}{N_7}$$

$$\frac{Z_7}{Z_8} = \frac{280}{400}$$

$$Z_7 = 0.7 Z_8 \qquad 1$$

Note: The centre distance between the shafts are fixed and same. ∴ The sum of number of teeth of mating gears should be equal.

$$\therefore \quad Z_{11} + Z_{12} = Z_7 + Z_8 = 100$$

$$0.7 Z_8 + Z_8 = 100$$

$$Z_8 = 58.82 \approx 59$$

$$\therefore \quad Z_7 = 41 \ , \quad Z_8 = 59$$

Third Pair:

* Gears 9 & 10, Ray DC
* Speed increase from 400 to 800 rpm.

$$\frac{Z_9}{Z_{10}} = \frac{N_{10}}{N_9}$$

$$\frac{Z_9}{Z_{10}} = \frac{800}{400}$$

$$Z_9 = 2Z_{10}$$

$$\text{W.K.T} \quad Z_{11} + Z_{12} = 100 = Z_9 + Z_{10}$$

$$2Z_{10} + Z_{10} = 100$$

$$Z_{10} = 33.33 \approx 34$$

$$\therefore \quad Z_9 = 66$$

$$Z_{10} = 34$$

Case 2: Consider stage 1:

First Pair:

* Gears 5 and 6 , Ray GD.
* Maximum speed reduction from 1400 to 400 rpm.

$$\frac{Z_5}{Z_6} = \frac{N_6}{N_5}$$

$$Z_5 = 20 \quad (\text{driver})$$

$$\frac{20}{Z_6} = \frac{400}{1400}$$

$$Z_6 = 70$$

$$Z_5 = 20, \quad Z_6 = 70.$$

Second Pair:

* Gears 1 and 2 , Ray GE
* Speed reduction from 1400 to 560 rpm.

$$\frac{Z_1}{Z_2} = \frac{N_2}{N_1}$$

$$\frac{Z_1}{Z_2} = \frac{560}{1400}$$

$$Z_1 = 0.4Z_2$$

We know that $Z_5 + Z_6 = 90 = Z_1 + Z_2$

$$0.4Z_2 + Z_2 = 90$$

$$Z_2 = 64.28 \approx 65$$

$$Z_1 = 25$$

Third Pair

* Gears 3 and 4, Ray GF
* Speed reduction from 1400 to 800 rpm.

$$\frac{Z_3}{Z_4} = \frac{N_4}{N_3}$$

$$\frac{Z_3}{Z_4} = \frac{800}{1400} \Rightarrow Z_3 = 0.57Z_4$$

$$\text{W.K.T} \Rightarrow Z_5 + Z_6 = 90 = Z_3 + Z_4$$

$$0.57Z_4 + Z_4 = 90$$

$$Z_4 = 57.32 \approx 58$$

$$Z_3 = 32$$

Step 5: Select suitable material.

Take, 40 Ni 2 Cr 1M028 (Hardened and tempered).

Material constant M=100 , [i]=55 N/mm^2 .

Step 6: Calculation of module (m).

Case 1: To find the torque (T)

Calculate the torque for the gear (12) has the lowest speed of 100 rpm using the relation.

$$T_{12} = \frac{60P}{2\pi N}$$

$$= \frac{60 \times 5 \times 10^3}{2 \times \pi \times 100}$$

$$T_{12} = 477.46 \text{ Nm.}$$

Case 2: To find the tangential force on gear 12.

From PSGDB 8.57 , table 46

$$F_{t12} = \frac{T}{r} = \frac{2T_{12}}{Z_{12}m}$$

$$= \frac{2 \times 477.46 \times 10^3}{80 \times m}$$

$$F_{t12} = \frac{11944}{m}$$

Case 3: To find the module (m).

$$m = \sqrt{\frac{F_{t12}}{(\varphi_m . M)}}$$

Where $\varphi_m = b/_m = 10$ From PSGDB 8.1 and 8.14 , (table 12).

M=100

$$m = \sqrt{\frac{11944/m}{10 \times 100}}$$

$$m = \sqrt{\frac{11944}{1000 \times m}}$$

$$m^2 = \frac{11.944}{m}$$

$$m = 2.28 \text{ mm.}$$

From PSGDB 8.2 , table 1. Choice 1.

The next nearest higher standard module m= 2.5 mm

Step 7: Calculation of centre distance in all stages.

From PSGDB 8.22 , table 26

$$a = \left(\frac{Z_x + Z_y}{2}\right) m.$$

Z_x and Z_y = No. of teeth on the gear pair in engagement in each stage.

Case 1:

Centre distance for stage 1.

$$a_1 = \left(\frac{Z_3 + Z_4}{2}\right) m.$$

$$= \left(\frac{32 + 58}{2}\right) 2.5$$

$$a_1 = 112.5 \text{ mm}$$

Case 2:

Centre distance for stage 2.

$$a_2 = \left(\frac{Z_9 + Z_{10}}{2}\right) m$$

$$= \left(\frac{66 + 34}{2}\right) 2.5$$

$$a_2 = 125 \text{ mm}$$

Step 8: Calculation of face width. (b)

W.K.T. $b = 10 \times m$

$= 10 \times 2.5$

$b = 25 \text{ mm}$

Step 9: Calculation of Length of the shafts.

$$L = 25 + 10 + 7b + 20 + 7b + 10 + 25$$
$$= 90 + 14b$$
$$= 90 + 14 \times 25$$
$$L = 440\text{mm}$$

* Bearing width = 25 mm.
* Gear & Bearing clearance = 10 mm.
* Adjacent group distance = 20 mm
* If two pair gear group = 4b

* Three pair gear group = 7b

Step 10: Design of shafts.

Case 1: Design of spindle (or) output shafts.

(i) To find normal load on gear 12 (F_n)

$$F_n = F_{t12} / \cos\alpha$$

$$F_n = \frac{4777.6}{\cos 20°} \qquad [\because \alpha = 20° FD]$$

$$F_n = 5084.22 \text{ N.}$$

(ii) To find maximum bending moment (M).

$$M = \frac{(F_n . L)}{4}$$

$$= \frac{5084.22 \times 440}{4}$$

$$M = 5.59 \times 10^5 \text{ Nmm.}$$

(iii) To find the equivalent torque.

$$T_{eq} = \sqrt{M^2 + T_{12}^2}$$

$$= \sqrt{(5.59 \times 10^5)^2 + (477.46 \times 10^3)^2}$$

$$T_{eq} = 7.35 \times 10^5 \text{ Nmm}$$

(iv) To find the diameter of the spindle.

$$d_s = \sqrt[3]{\frac{16 T_{eq}}{\pi . [\tau]}}$$

$$= \sqrt[3]{\frac{16 \times 7.35 \times 10^5}{\pi \times 55}}$$

$$d_s = 40.82 \text{ mm}$$

From R_{20} series, the standard diameter

$$d_s = 45 \text{ mm} .$$

Case 2: Design of other shafts.

(a) Diameter of shaft 1.

$$\text{Input speed} = 1400 \text{ rpm.}$$

$$\text{Torque } T = \frac{60P}{2\pi N}$$

$$= \frac{60 \times 5 \times 10^3}{2 \times \pi \times 1400}$$

$$T = 34.10 \text{ Nm}$$

$$\text{W.K.T } T = 0.2 d_{s1}^3 [\tau]$$

$$34.10 \times 10^3 = 0.2 \times d_{s1}^3 \times 55$$

$$d_{s1} = 14.58 \text{ mm}.$$

From R_{20} series, the standard diameter

$$d_{s1} = 16 \text{ mm}.$$

(b) Diameter of shaft 2.

$$\text{Input speed} = 400 \text{ rpm.}$$

$$\text{Torque } T = \frac{60P}{2\pi N}$$

$$= \frac{60 \times 5 \times 10^3}{2 \times \pi \times 400}$$

$$T = 119.37 \text{ Nm}$$

$$\text{W.K.T } T = 0.2 d_{s2}^3 [\tau]$$

$$119.37 \times 10^3 = 0.2 \times d_{s2}^3 \times 55$$

$$d_{s2} = 22.14 \text{ mm}.$$

From R_{20} series, the standard diameter

$$d_{s2} = 25 \text{ mm}.$$

3. Design of 12 speed gear box for a lathe. The minimum and maximum speeds are 100 and 1200 rpm. Power is 5 KW from 1440 rpm induction motor.

Given data:

$n = 12$ speeds

$N\min = 100 rpm.$

$N\max = 1200 rpm.$

$P = 5 KW$

$N_{input} = 1440 rpm.$

Step 1: Selection of spindle speeds.

$$\frac{N_{max}}{N_{min}} = \phi^{n-1}$$

$$\frac{1200}{100} = \phi^{12-1}$$

$$\phi = 1.25 .$$

Therefore the spindle speeds from R10 series.

From PSGDB 7.20.

100, 125, 160, 200, 250, 315, 400, 500, 630, 800, 1000 and 1200rpm.

Step 2: To find the structural formula.

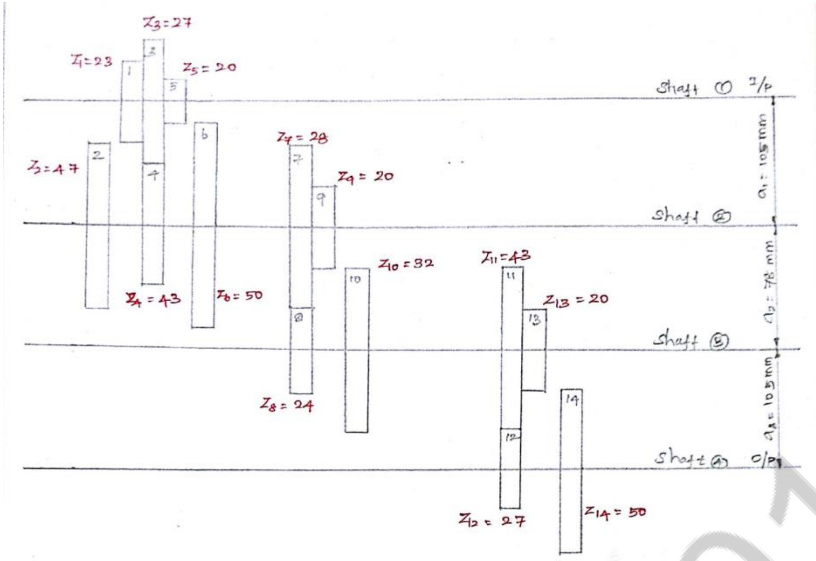
12 speeds = 3(1) 2(3) 2(6)

Step 3: Kinematic diagram for 12 speeds.

Structural formula = 3(1) 2(3) 2(6)

No. of shafts = No. of stages +1 = 3+1=4 (4 horizontal lines)

No. of gears = $2(P_1+P_2+P_3)$ =2(3+2+2) = 14 gears.



Step 3: Ray diagram for 12 speed.

Structural formula = 3(1) 2(3) 2(6)

No. of shafts = 4 (4 vertical lines)

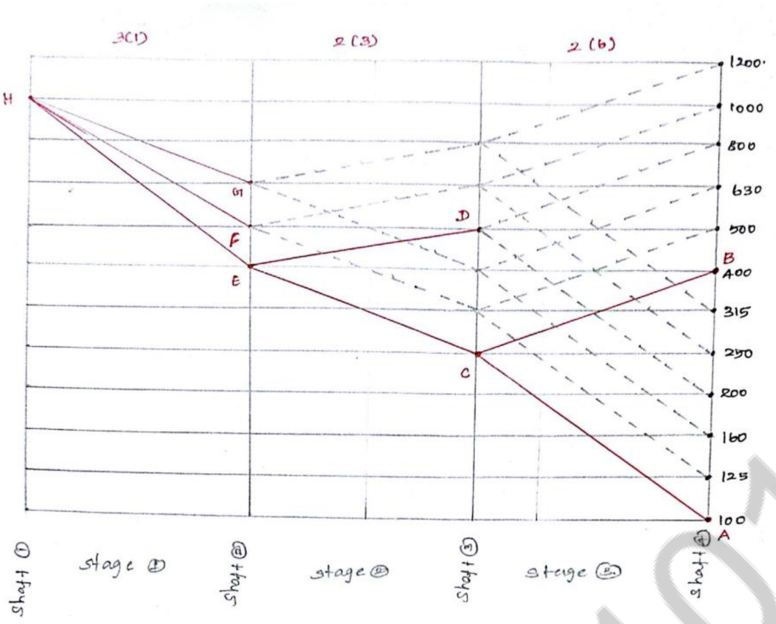
Speeds = 12 (12 horizontal lines)

For stage 3:

$$\frac{N_{min}}{N_{I/P}} \geq 1/4 \qquad \frac{N_{max}}{N_{I/P}} \leq 2$$

$$\frac{100}{250} = 0.4 \geq 1/4 \qquad \frac{400}{250} = 1.6 \leq 2$$

$N_{I/P} = 250$ rpm .



For stage 2:

$$\frac{N_{min}}{N_{I/P}} \geq 1/4 \qquad \frac{N_{max}}{N_{I/P}} \leq 2$$

$$\frac{250}{400} = 0.625 \geq 1/4 \qquad \frac{500}{400} = 1.25 \leq 2$$

$$N_{I/P} = 400 \text{ rpm}.$$

For stage 1:

$$\frac{N_{min}}{N_{I/P}} \geq 1/4 \qquad \frac{N_{max}}{N_{I/P}} \leq 2$$

$$\frac{400}{1000} = 0.4 \geq 1/4 \qquad \frac{630}{1000} = 0.63 \leq 2$$

$$\therefore \; N_{I/P} = 1000 \text{ rpm}.$$

Step 4: Calculation of no. of teeth on all the gears.

Let $Z_1, Z_2, Z_3 \ldots Z_{14}$ = No. of teeth of the gears 1, 2, 3…. 14 respectively.

$N_1, N_2, N_3 \ldots N_{14}$ = No. of speed of the gears 1, 2, 3…. 14 respectively.

We know that, $\frac{Z_1}{Z_2} = \frac{N_2}{N_1}$

Case 1: Consider stage 3

First Pair:

* Gears 13 and 14 , Ray CA
* Speed reduction from 250 to 100 rpm.

$$\frac{Z_{13}}{Z_{14}} = \frac{N_{14}}{N_{13}} \quad , \qquad Z_{13} = 20 \ \ (\text{driver})$$

$$\frac{20}{Z_{14}} = \frac{100}{250}$$

$$Z_{14} = 50$$

$$Z_{13} = 20 \ , \ Z_{14} = 50$$

Second Pair:

* Gears 11 and 12 , Ray CB
* Speed increase from 250 to 400 rpm.

$$\therefore \frac{Z_{11}}{Z_{12}} = \frac{N_{12}}{N_{11}}$$

$$\frac{Z_{11}}{Z_{12}} = \frac{400}{250}$$

$$Z_{11} = 1.6\ Z_{12}$$

W.K.T. $\quad Z_{13} + Z_{14} = 70 = Z_{11} + Z_{12}$

$$\therefore \ 1.6Z_{12} + Z_{12} = 70$$

$$Z_{12} = 26.92 \approx 27$$

$$Z_{11} = 43$$

$$Z_{12} = 27 \ , \ Z_{11} = 43$$

Case 2: consider stage 2:

First Pair:

* Gears 9 and 10, Ray EC

* Speed reduction from 400 to 250 rpm.

$$\frac{Z_9}{Z_{10}} = \frac{N_{10}}{N_9} \qquad Z_9 = 20 \quad (\text{driver})$$

$$\frac{20}{40} = \frac{250}{400}$$

$$Z_{10} = 32$$

$$Z_9 = 20 \quad , \quad Z_{10} = 32$$

Second Pair:

* Gears 7 and 8, Ray ED
* Speed increase 400 to 500 rpm.

$$\frac{Z_7}{Z_8} = \frac{N_8}{N_7}$$

$$\frac{Z_7}{Z_8} = \frac{500}{400}$$

$$Z_7 = 1.25 Z_8$$

W.K.T. $Z_9 + Z_{10} = 52 = Z_7 + Z_8$

$$1.25 Z_8 + Z_9 = 52$$

$$Z_8 = 23.11 \approx 24$$

$$Z_7 = 28$$

$$Z_7 = 28 \quad , \quad Z_8 = 24$$

Case 3: consider stage 1:

First Pair:

* Gears 5 and 6, Ray HE
* Speed reduction 1000 to 400 rpm.

$$\frac{Z_5}{Z_6} = \frac{N_6}{N_5}$$

$$\frac{20}{Z_6} = \frac{400}{1000} \qquad Z_5 = 20 \quad (\text{driver})$$

$Z_6 = 50$

$Z_5 = 20$, $Z_6 = 50$

Second Pair:

* Gears 1 and 2, Ray HF
* Speed reduction from 1000 to 500 rpm.

$$\frac{Z_1}{Z_2} = \frac{N_2}{N_1}$$

$$\frac{Z_1}{Z_2} = \frac{500}{1000}$$

$$Z_1 = 0.5Z_2$$

W. K. T. $Z_5 + Z_6 = 70 = Z_1 + Z_2$

$$1.5Z_2 = 70$$

$$Z_2 = 46.7 \approx 47$$

$$Z_1 = 23$$

$Z_1 = 23$, $Z_2 = 47$

Third Pair:

* Gears 3 and 4, Ray HG
* Speed reduction from 1000 to 630 rpm.

$$\frac{Z_3}{Z_4} = \frac{N_4}{N_3}$$

$$\frac{Z_3}{Z_4} = \frac{630}{1000}$$

$$Z_3 = 0.63Z_4$$

W. K. T. $Z_3 + Z_4 = Z_5 + Z_6 = 70$

$$1.63Z_4 = 70$$

$$Z_4 = 42.94 \approx 43$$

$Z_3 = 27.$

$Z_3 = 27$, $Z_4 = 43$

Step 5: Selection of material,

40N: 2cr 1MO 28 (Hardened and tempered) material is selected.

Material constant M=100 , $[\tau] = 55N/mm^2$

Step 6: Calculation of module (m)

Case 1: To find the torque (T)

Calculate the torque for the gear (14) has the lowest speed of 100 rpm, using the relation.

$$T_{14} = \frac{60P}{2\pi N}$$

$$= \frac{60 \times 5 \times 10^3}{2 \times \pi \times 100}$$

$$T_{14} = 477.46 \text{ Nm.}$$

Case 2: To find the tangential force on gear 14.

From PSGDB 8.57 , table 46.

$$F_{t14} = \frac{T}{r} = \frac{2T_{14}}{Z_{14} \times m}$$

$$= \frac{2 \times 477.46 \times 10^3}{50 \times m}$$

$$F_{t14} = \frac{19098.4}{m}$$

Case 3: To find the module (m).

$$m = \sqrt{\frac{F_{t14}}{\varphi m.m}}$$

Where , $\varphi_m = {}^b\!/_m = 10$ From PSGDB 8.1 , and 8.14 (table 12).

$$m = \sqrt{\frac{19098.4/m}{10 \times 100}}$$

$$m^2 = \frac{19.098}{m}$$

$$m = 2.67 \text{ mm.}$$

From PSGDB 8.2 , table 1, choice 1.

The next nearest higher standard module

$$m = 3\text{mm}.$$

Step 7: Calculation of centre distance in all stages. From PSGDB 8.22 , table 26.

$$a = \left(\frac{Z_x + Z_y}{2}\right)m$$

Z_x and Z_y No. of teeth on the gear pair in engagement is each stage.

Case 1: Centre distance for stage 1.

$$a_1 = \left(\frac{Z_3 + Z_4}{2}\right)m$$

$$= \left(\frac{27 + 43}{2}\right)3$$

$$a_1 = 105\text{mm}.$$

Case 2: Centre distance for stage 2.

$$a_2 = \left(\frac{Z_9 + Z_8}{2}\right)m$$

$$= \left(\frac{28 + 24}{2}\right)3$$

$$a_2 = 78\text{mm}.$$

Case 3: Centre distance for stage 3.

$$a_3 = \left(\frac{Z_{11} + Z_{12}}{2}\right)m$$

$$= \left(\frac{43 + 27}{2}\right)3$$

$$a_1 = 105\text{mm}.$$

Step 8: Calculation of Face width (b).

$$\text{W.K.T} \Rightarrow b = 10 \times m$$

$$= 10 \times 3$$

$$b = 30\text{mm}$$

Step 9: Calculation of Length of the shafts.

$$L = 25 + 10 + 7b + 20 + 4b + 20 + 4b + 10 + 25$$

$$= 110 + 15b$$

$$= 110 + 15 \times 30$$

$$L = 450 \text{ mm}$$

Step 10: Design of shafts.

Case 1: Design of spindle (or) output shafts.

(i) To find normal load on gear 14 (F_n)

$$F_n = \frac{F_{t14}}{\cos\alpha} \quad [\alpha = 20°\text{FD}]$$

$$= \frac{6366.13}{\cos 20}$$

$$F_n = 6774.7 \text{ N}$$

(ii) To find maximum bending moment (M).

$$M = \frac{(F_n.L)}{4}$$

$$= \frac{6774.7 \times 450}{4}$$

$$M = 7.62 \times 10^5 \text{ Nmm.}$$

(iii) To find the equivalent torque. (T_{eq})

$$T_{eq} = \sqrt{M^2 + T_4^2}$$

$$= \sqrt{(7.62 \times 10^5)^2 + (477.46 \times 10^3)^2}$$

$$= 8.99 \times 10^5 \text{ Nmm}$$

(iv) To find the diameter of the spindle (d_s)

$$d_s = \sqrt[3]{\frac{16T_{eq}}{\pi[\tau]}}$$

$$= \sqrt[3]{\frac{16 \times 8.99 \times 10^5}{\pi \times 55}}$$

$$d_s = 43.66 \text{ mm}$$

From R_{10} series, The standard diameter.

$$d_s = 50\text{mm} .$$

Case 2: Design of other shafts.

(a) Diameter of shaft 1.

Input speed = 1000 rpm.

$$\text{Torque } T = \frac{60P}{2\pi N}$$

$$= \frac{60 \times 5 \times 10^3}{2 \times \pi \times 1000}$$

$$= 47.75 \text{ Nm} .$$

W.K.T $T = 0.2d_1^3[\tau]$.

$$47.75 \times 10^3 = 0.2 \times d_{s1}^3 \times 55$$

$$d_{s1} = 16.31 \text{ mm.}$$

From R_{10} series., The standard diameter d_{s1}=20 mm.

(b) Diameter of shaft 2.

Input speed = 400 rpm.

$$\therefore T = \frac{60 \times 5 \times 10^3}{2 \times \pi \times 400}$$

$$= 119.36 \text{ Nm} .$$

$$\text{W.K.T} \Rightarrow \quad T = 0.2 d_2^3 [\tau].$$

$$119.36 \times 10^3 = 0.2 \times d_{s2}^3 [55]$$

$$d_{s2} = 22.14 \text{ mm.}$$

From R_{10} series., The standard diameter d_{s2}=25 mm.

(c) Diameter of shaft 3.

Input speed = 250 rpm.

$$\therefore \quad T = \frac{60 \times 5 \times 10^3}{2 \times \pi \times 250}$$

$$T = 190.98 \text{ Nm} \ .$$

$$\text{W.K.T} \ \Rightarrow \ T = 0.2 d_3^3 [\tau].$$

$$190.98 \times 10^3 = 0.2 \times d_{s3}^3 [55]$$

$$d_{s3} = 25.89 \text{ mm.}$$

From R_{10} series. The standard diameter d_{s3}=31.5 mm.

4. A nine speed box, used as a head stock gear box of a turret lathe, is to provide a speed range of 180 rpm to 1800 rpm. Using standard step ratio, draw the speed diagram, and the kinematic layout showing number of teeth in all gears.

Given data:

$$n = 9$$
$$N_{min} = 180 \text{rpm}$$
$$N_{max} = 1800 \text{rpm}$$

Step 1:- selection of spindle speeds

Determine the progression ratio (ϕ) using the relation

$$\frac{N_{max}}{N_{min}} = \phi^{n-1}$$
$$\frac{1800}{180} = \phi^{9-1}$$
$$\phi = (10)^{1/8}$$
$$\phi = 1.333$$

✓ We find ϕ =1.333 is not a standard ratio. So let us find out whether multiplies of standard ratio 1.12 or 1.06 come close to 1.333

- ✓ For example we can write, $1.12 \times 1.12 = 1.2544$ & $1.12 \times 1.12 \times 1.12 = 1.405$

Then $1.06 \times 1.06 \times 1.06 \times 1.06 = 1.338$ skip 4 speeds

So we take ϕ= 1.06, because satisfies the requirement, select the standard spindle speeds using the series of preferred numbers

Take Step Ratio from R40 series ϕ=1.06

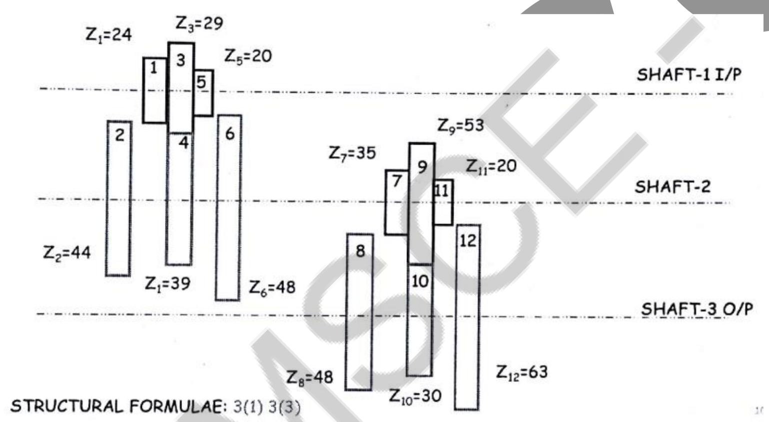
Spindle Speeds are 180, 236, 315, 425, 560, 750, 1000, 1320 and 1800rpm

Step 2: To find the structural formulae

Structural formulae: 3(1) 3(3)

Step 3: Construct the kinematic arrangement for 9 speed gear box

- ✓ Structural formulae: 3(1) 3(3)
- ✓ P1 = 3 p2 = 3 Note: where $X_1 = 1$; $X_2 = p_1 = 3$
- ✓ No. of shafts = No. of stages +1 (2+1=3 shafts) (so draw 3 horizontal lines)
- ✓ To find the no. of gears by using
  No. of gears $= 2(p_1 + p_2)\{[2(3+3)] = 12\,\text{gears}\}$



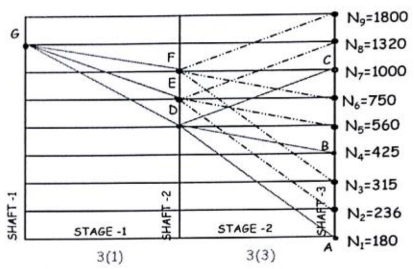
Step 4:- Construct the ray diagram for 9 speed gear box

- ➢ Structural formulae: 3(1) 3(3)

No .of stages: $2\{(p_1(X_1).p_2(X_2))\}$

$p_1 = 3$ $p_2 =$ Note: Where $X_1 =$ $X_2 = p_1 = 3$

- ✓ No. of shafts = No. of stages +1 (2+1= 3 shafts) (so draw 3 vertical lines)
- ✓ No. of speeds = 9 (Draw 9 horizontal lines)



Step 5: Calculation of No. of teeth

- Calculation of numbers of teeth on all the gears

Let Z_1, Z_2, Z_3, ...Z_{12} = Number of teeth of the gears 1, 2, 3, …12 respectively

Formulae given $\frac{z_1}{z_2} = \frac{N_2}{N_1}$

Take stage – 2

- Consider the first pair of gear 11 and 12
- From ray diagram consider ray DA
- Maximum speed reduction 560rpm to 180rpm

We know that, $Z_{min} \geq 17$, assume Z_{11} = 20 (driver)

$$\frac{z_{11}}{z_{12}} = \frac{N_{12}}{N_{11}}$$
$$\frac{20}{z_{12}} = \frac{180}{560}$$
$$z_{12} = 62.22 \approx 63$$

Z_{11} = 20, Z_{12} = 63

Take stage – 2

- Consider the second pair of gear 7 and 8
- From ray diagram consider ray DB
- Maximum speed reduction 560rpm to 425rpm

We know that,

$$\frac{z_7}{z_8} = \frac{N_8}{N_7}$$
$$\frac{z_7}{z_8} = \frac{425}{560}$$
$$z_7 = 0.76 z_8 \qquad --(i)$$

NOTE: The centre distance between the shafts are fixed and same. The sum of number of teeth of mating gears should be equal.

So we can write

$$z_7 + z_8 = z_{11} + z_{12} = 20 + 63 = 83 \qquad \text{(ii)}$$

Solving equations (i) and (ii), we get

$$z_8 = 47.16 \approx 48$$
$$z_7 = 83 - 48 = 35$$
$$z_7 = 35 \quad z_8 = 48$$

Take stage – 2

- Consider the third pair of gear 9 and 10
- From ray diagram consider ray DC
- Speed increase from 560rpm to 1000rpm

We know that,

$$\frac{z_9}{z_{10}} = \frac{N_{10}}{N_9}$$
$$\frac{z_9}{z_{10}} = \frac{1000}{560}$$
$$Z_9 = 1.786 Z_{10} \qquad --\text{(iii)}$$

So we can write

$$Z_9 + Z_{10} = Z_{11} + Z_{12} = 20 + 63 = 83 \qquad --\text{(iv)}$$

Solving equation (iii) and (iv), we get

$$Z_{10} = 29.79 \approx 30$$
$$Z_9 = 83 - 30 = 53$$
$$Z_9 = 53 \quad Z_{10} = 30$$

Take stage -1

- Consider the first pair of gear 5 and 6
- From ray diagram consider ray GD
- Maximum speed reduction 1320rpm to 560rpm

We know that, $Z_{min} \geq 17$ ∴ assume Z_5 = 20 (Driver)

$$\frac{z_5}{z_6} = \frac{N_6}{N_5}$$
$$\frac{20}{z_{12}} = \frac{1320}{560}$$
$$z_6 = 47.14 \approx 48$$

Take stage – 1

- Consider the first pair of gear 5 and 6

- From ray diagram consider ray GD
- Maximum speed reduction 1320rpm to 560rpm

We know that,

$$\frac{z_1}{z_2} = \frac{N_2}{N_1}$$
$$\frac{z_1}{z_2} = \frac{750}{1320}$$
$$Z_1 = 0.57z_2 \qquad ---(v)$$

NOTE: The centre distance between the shafts are fixed and same. The sum of number of teeth of mating gears should be equal.

So we can write

$$z_1 + z_2 = z_5 + z_6 = 20 + 48 = 68 \qquad ---(vi)$$

Solving equations (v) and (vi), we get

$$z_2 = 43.3 \approx 44$$
$$z_1 = 68.44 = 24$$
$$Z_1 = 24 \quad Z_2 = 44$$

Take stage – 1

- Consider the third pair of gear 3 and 4
- From ray diagram consider ray GF
- Speed increase from 1320rpm to 1000 rpm

We know that,

$$\frac{z_3}{z_4} = \frac{N_4}{N_3}$$
$$\frac{z_3}{z_4} = \frac{1000}{1320}$$
$$Z_3 = 0.76z_4 \qquad --(vii)$$

Solving equations (iii) and (iv), we get

$$Z_4 = 38.64 \approx 39$$
$$Z_3 = 68 - 39 = 29$$
$$Z_3 = 29 \quad Z_4 = 39$$

5. **A gear box is to give 18 speeds for a spindle of a milling machine. Maximum and minimum speeds of the spindle are to be around 650 and 35 rpm respectively. Find the speed ratios which will give the desired**

speeds and draw the structural diagram and kinematic arrangement of the drive.

Given data:

$$n = 18$$
$$N_{min} = 35rpm$$
$$N_{max} = 650rpm$$

Step 1: Selection of Spindle speeds

Determine the progression ratio (ϕ) using the relation

$$N_{max}/N_{min} = \phi^{n-1}$$

$$650/35 = \phi^{18-1}$$

$$\phi = (18.571)^{1/17}$$

$$\phi = 1.87$$

We find $\phi = 1.87$ is not a standard ratio. So let us find out whether multiples of standard ratio 1.12 OR 1.06 come close to 1.87

For example we can write $1.12 \times 1.12 = 1.2544$

Then $1.06 \times (1.06 \times 1.06) = 1.91$ ... Skip 2 speeds

So we take $\phi = 1.06$, because satisfies the requirement. Select the standard spindle speeds using the series of preferred numbers From PSGDB 7.20, 7.19

Step ratio from R40 series $\phi = 1.06$

∴ Spindle speeds are 35.5, 42.5, 50, 60, 71, 85, 100, 118, 140, 170, 200, 236, 280, 335, 400, 475, 560 and 670 rpm

Step 2: To find the Structural Formulae

Structural formulae: $2(1)3(2)3(6)$

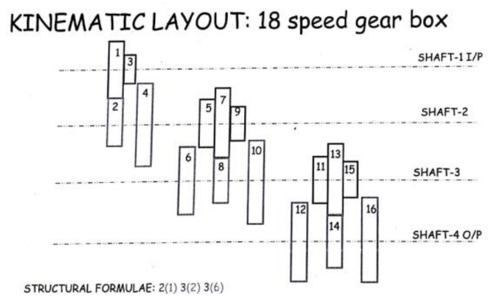
Step 3: Construct the Kinematic arrangement for 18 speed gear box

Structural formulae: $2(1)3(2)3(6)$

No. of shafts = No. of stages + 1 (3 + 1 = 4 shafts) (so draw 4 horizontal lines)

To find the no. of gears by using

No. of gears $=2(p_1+p_2+p_3)\{[2(2+3+3)]=16\text{gears}\}$



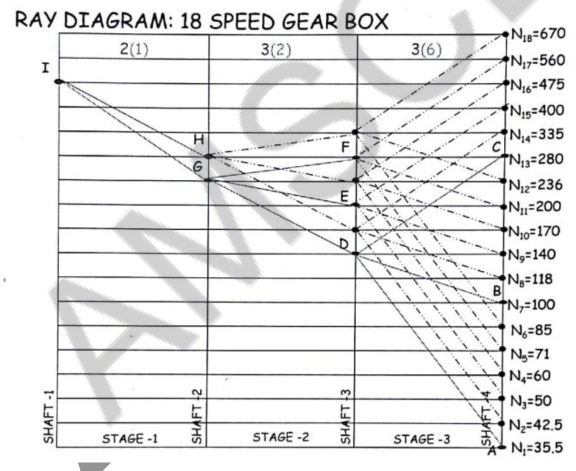
Step 4: Construct the ray diagram for 18 speed gear box

Structural formulae: $2(1)3(2)3(6)$

Note: Where $X_1=1$ $X_2=p_1=2$ $X_3=p_1\cdot p_2=2\times3=6$

No. of shafts = No. of stages +1 (3 + 1 = 4 shafts) (so draw 4 vertical lines)

No. of speeds = 18 (Draw 18 horizontal lines)



6. Draw the speed diagram, and the kinematic layout of the head stock gear box of a turret lathe having arrangement for 9 spindle speeds, ranging from 31.5rpm to 1050rpm. Calculate the no. of teeth on each gear. Minimum number of teeth on a gear is 25. Also calculate the percentage deviation of the obtainable speeds from the calculated ones.

GIVEN DATA:

$n = 9$

$N_{min} = 31.5 rpm$

$N_{max} = 1050 rpm$

$Z_{driver} = 25$

Step 1: Selection of Spindle Speeds

Determine the progression ration (ϕ) using the relation

$$N_{max}/N_{min} = \phi^{n-1}$$

$$1050/31.5 = \phi^{9-1}$$

$$\phi = (33.33)^{1/8}$$

$$\phi = 1.55$$

- ✓ We find $\phi = 1.55$ is not a standard ratio. So let us find out whether multiples of standard ratio 1.12 or 1.25 come close to 1.55
- ✓ For example we can write 1.12 × 1.12 = 1.2544 and 1.12 × 1.12 × 1.12 = 1.405

Then 1.25 × 1.25 = 1.55 ….. skip 1 speed

So we take $\phi = 1.25$, because satisfies the requirement. Select the standard spindle speeds using the series of preferred numbers – From PSGDB 7.20, 7.19

Take step Ratio from R10 series $\phi = 1.25$

∴ Spindle speeds are 31.5, 50, 80, 100, 160, 250, 400, 630, 1000rpm

Step 2: To find the Structural Formulae

Structural Formulae: 3(1) 3(3)

Step 3: Construct the Kinematic arrangement for 9 speed gear box

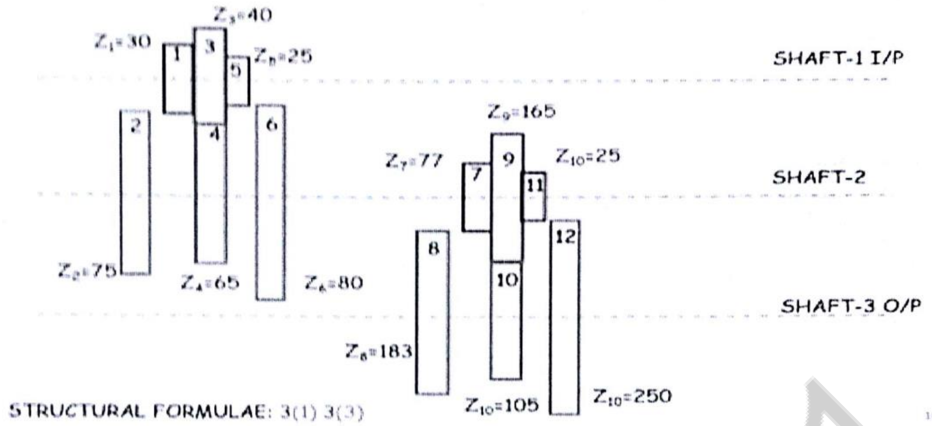
Structural Formulae: 3(1) 3(3)

Note: Where $X_1 = 1$ $X_2 = p_1 = 3$

No. of shafts = No. of stages + 1 (2+1=3 shafts) (so draw 3 horizontal lines)

To find No. of gears = 2(p_1 + p_2) {[2(3+3)]=12gears}

KINEMATIC LAYOUT: 9 speed gear box



Step 4: Construct the Ray diagram for 9 speed gear box

Structural Formulae: 3(1) 3(3)

Note: Where X_1 = 1 X2 = p1 =3

No. of shafts = No. of stages + 1 (2+1=3 shafts) (so draw 3 vertical lines)

No. of speeds = 9 (Draw 9 horizontal lines)

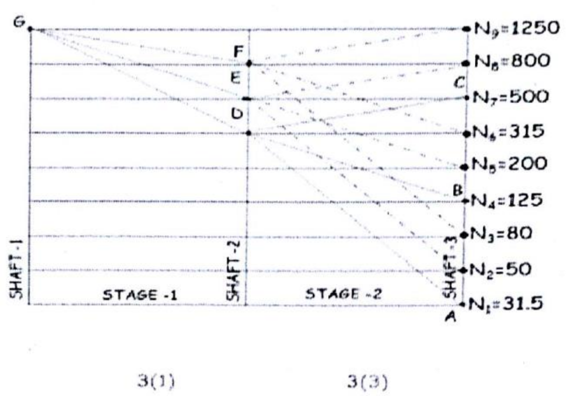
Stage 2:

- ✓ For stage 2 = 3(3) , 3 points with 3 speed gap or 3 speeds on shaft 3, Make the points A, B & C
- ✓ Find input speed for the speeds A=31.5 rpm and C=500rpm by using

$$\frac{31.5}{31.5} = 0.1 \leq \frac{1500}{4315} = 1.58 \leq 2$$ Ratio requirement satisfied,

$\therefore$Input speed for stage 2=315rpm

RAY DIAGRAM: 9 SPEED GEAR BOX



Stage 1:

- ✓ For stage 1=3(1), 3 points with 1 speed gap or 1 speeds on shaft 2, Make the points D, E & F
- ✓ Find input speed for the speeds D=315 rpm and F=800rpm by using

$$\frac{31.5}{1250} = 0.252 \geq \frac{1}{4} \qquad \frac{800}{1250} = 0.64 \leq 2$$, Ratio requirement satisfied,

$\therefore$ Input speed for stage 1=1250 rpm

Step 5: Calculation of Number of Teeth on all the gears

Let, Z_1, Z_2, Z_3 …Z_{12} = Number of teeth of the gears 1, 2, 3…12 respectively

N_1, N_2, N_3… N_{12} = Speeds of the gears 1, 2, 3 …12 respectively

Formulae given $\frac{Z_1}{Z_2} = \frac{N_2}{N_1}$

Take stage-2-Consider the first pair of gear 11, and 12

- From ray diagram consider ray DA
- Maximum speed reduction 315rpm to 31.5rpm

We know that, $Z_{min} \geq 17$, $\therefore$ assume Z_{11} = 25 (Driver)

$$\frac{Z_{11}}{Z_{12}} = \frac{N_{12}}{N_{11}} = \frac{25}{Z_{12}} = \frac{31.5}{315} \qquad Z_{12} = 250$$

Take stage-1-Consider the second pair of gear 1 and 2

- From ray diagram consider ray GE
- Speed reduction 1250rpm to 500rpm

We know that, $\frac{Z_1}{Z_2} = \frac{N_2}{N_1} = \frac{Z_1}{Z_2} = \frac{500}{1250}$

$$Z_1 = 0.4Z_2 \qquad \text{..... (v)}$$

$$Z_1 + Z_2 = Z_5 + Z_6 = 25 + 80 = 105 \qquad \text{.......(vi)}$$

Solving equations (v) and (vi), we get

$$Z_2 = 75$$
$$Z_1 = 105 - 75 = 30$$

Take stage-1-Consider the second pair of gear 3 and 4

- From ray diagram consider ray GF

- Speed reduction 1250rpm to 800rpm

We know that, $\frac{Z_3}{Z_4} = \frac{N_4}{N_3} = \frac{Z_3}{Z_4} = \frac{800}{1250}$

$$Z_3 = 0.64Z_4 \quad ..... \text{(vii)}$$

$$Z_3 + Z_4 = Z_5 + Z_6 = 25 + 80 = 105 \quad ....\text{(viii)}$$

Solving equations (viii) and (vii), we get

$$Z_4 = 64.02 \approx 65$$
$$Z_3 = 105 - 65 = 40$$

Step 6: Calculation of Output Speeds

Let N_1 and N_0 = Input and output speeds of the gears. From the ray diagram input speed N_1 = 1250 rpm

$$N_{01} = N_1 \times \frac{Z_1}{Z_2} \times \frac{Z_7}{Z_8} = 1250 \times \frac{30}{75} \times \frac{78}{197} = 197.96\text{rpm}$$

$$N_{02} = N_1 \times \frac{Z_1}{Z_2} \times \frac{Z_9}{Z_{10}} = 1250 \times \frac{30}{75} \times \frac{168}{107} = 785.05\text{rpm}$$

$$N_{03} = N_1 \times \frac{Z_1}{Z_2} \times \frac{Z_{11}}{Z_{12}} = 1250 \times \frac{30}{75} \times \frac{25}{250} = 50\text{rpm}$$

Take stage-2-Consider the second pair of gear 7 and 8

- From ray diagram consider ray DB
- Speed reduction 315rpm to 125rpm

We know that, $\frac{Z_7}{Z_8} = \frac{N_8}{N_7} = \frac{Z_7}{Z_8} = \frac{125}{315}$

$$Z_7 = 0.4Z_8 \quad ....\text{(i)}$$

$$Z_7 + Z_8 = Z_{11} + Z_{12} = 25 + 250 = 275 \quad ....\text{(ii)}$$

Solving equations (i) and (ii), we get

$$Z_8 = 196.42 \approx 197$$
$$Z_7 = 275 - 197 = 78$$

Take stage-2-Consider the second pair of gear 9 and 10

- From ray diagram consider ray DC
- Speed reduction 315rpm to 500rpm

We know that, $\frac{Z_9}{Z_{10}} = \frac{N_{10}}{N_9} = \frac{Z_9}{Z_{10}} = \frac{500}{315}$

$$Z_9 = 1.59 Z_{10} \quad \ldots.(iii)$$

$$Z_9 + Z_{10} = Z_{11} + Z_{12} = 25 + 250 = 275 \quad \ldots.(iv)$$

Solving equations (iii) and (iv), we get

$$Z_{10} = 106.18 \approx 107$$
$$Z_9 = 275 - 107 = 168$$

Take stage-1-Consider the second pair of gear 5 and 6

- From ray diagram consider ray GD
- Maximum Speed reduction 1250rpm to 315rpm

We know that, $Z_{min} \geq 17$, $\therefore$ assume $Z_5 = 25$ (Driver)

$$\frac{Z_5}{Z_6} = \frac{N_6}{N_5} = \frac{25}{Z_{12}} = \frac{315}{1250}$$

$$Z_6 = 79.37 \approx 80$$

$$N_{04} = N_1 \times \frac{Z_3}{Z_4} \times \frac{Z_7}{Z_8} = 1250 \times \frac{40}{65} \times \frac{78}{197} = 304.57\text{rpm}$$

$$N_{05} = N_1 \times \frac{Z_3}{Z_4} \times \frac{Z_9}{Z_{10}} = 1250 \times \frac{40}{65} \times \frac{168}{107} = 1207.76\text{rpm}$$

$$N_{06} = N_1 \times \frac{Z_3}{Z_4} \times \frac{Z_{11}}{Z_{12}} = 1250 \times \frac{40}{65} \times \frac{25}{250} = 76.92\text{rpm}$$

$$N_{07} = N_1 \times \frac{Z_5}{Z_6} \times \frac{Z_7}{Z_8} = 1250 \times \frac{25}{80} \times \frac{78}{197} = 154.66\text{rpm}$$

$$N_{08} = N_1 \times \frac{Z_5}{Z_6} \times \frac{Z_9}{Z_{10}} = 1250 \times \frac{25}{80} \times \frac{168}{107} = 613.32\text{rpm}$$

$$N_{09} = N_1 \times \frac{Z_5}{Z_6} \times \frac{Z_{11}}{Z_{12}} = 1250 \times \frac{25}{80} \times \frac{25}{250} = 39.06\text{rpm}$$

Step 7: Calculation of % Deviation:

Sl. No	Obtainable speed ($N_{obt,}rpm$)	Calculated speed ($N_{cal,}rpm$)	% deviation=$\frac{N_{Obt}-N_{cal}}{N_{cal}}$x100
1	39.6	31.5	25.71
2	50	50	0
3	76.92	80	-3.85
4	154.66	125	23.72
5	197.96	200	-1.02
6	304.97	315	-3.18
7	613.32	500	22.64
8	785.05	800	-1.868
9	1207.76	1250	-3.38

7.A 6 speed gear box is required to provide output speeds in the range of 125 to 400 rpm, with a step ratio of 1.25 and transmit a power 5kW at 710 rpm. Draw the speed diagram and kinematic diagram. Determine the number of teeth module and face width of all the gears, assuming materials for gears. Determine he length of the gear box along the axis of the gear shaft.

GIVEN DATA:

n= 6

N_{min}=125 rpm

N_{max}= 400 rpm

∅=1.25

P=5 kW

N_{input}= 710 rpm

STEP 1: SELECTION OF SPINDLE SPEEDS

∅=1.25 (given)

we take ∅=1.25, because satisfies the requirement. Select the standard spindle speeds using the series of preferred numbers – From PSGDB 7.20, 7.19

TAKE

STEP RATIO from R10 series ∅=1.25

∴SPINDLE SPEEDS ARE 125, 160,200,250,315 and400

STEP 2: TO FIND THE STRUCTURAL FORMULAE

STRUCTURAL FORMULAE: 3(1) 2(3)

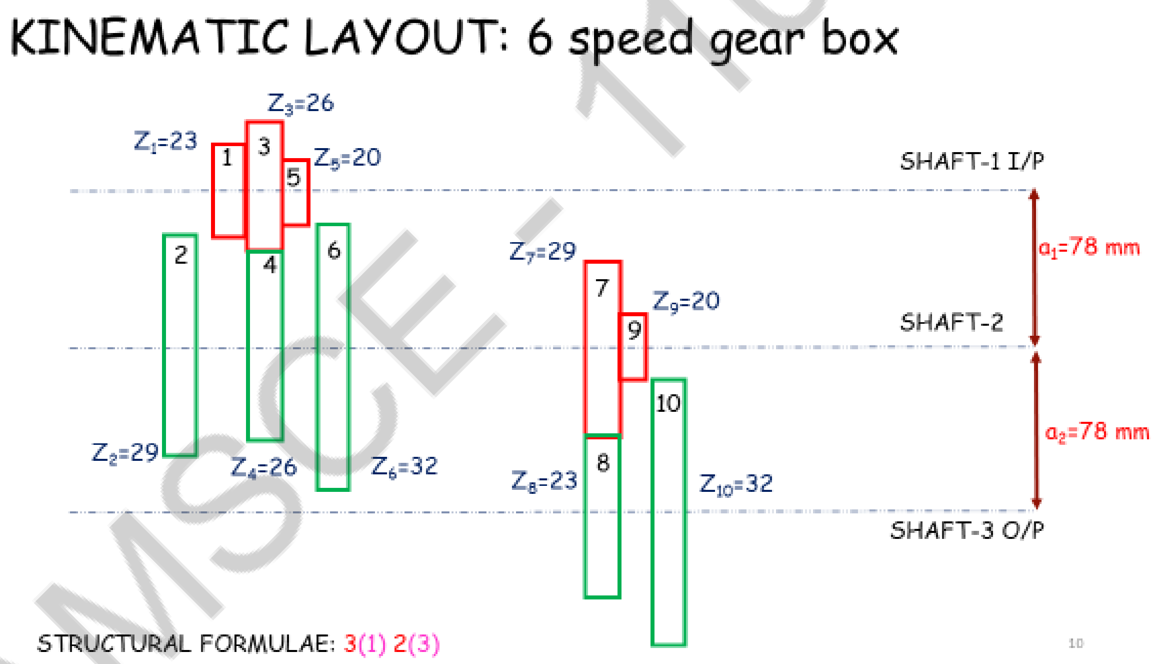
STEP 3: CONSTRUCT THE KINEMATIC ARRANGEMENT FOR 6 SPEED GEAR BOX

STRUCTURAL FORMULAE: 3(1) 2(3)

No.of Stages: 2, {(p_1 (X_1) . p_2 (X_2)}

Note:Where X_1 =1 X_2= p_1 =3

- ✓ **No. of shafts= No. of stages+1 (2+1=3 shafts) (so draw 3 horizontal lines)**
- ✓ **No. of gears= 2(p_1 + p_2) {[2(3+2)]=10 gears}**



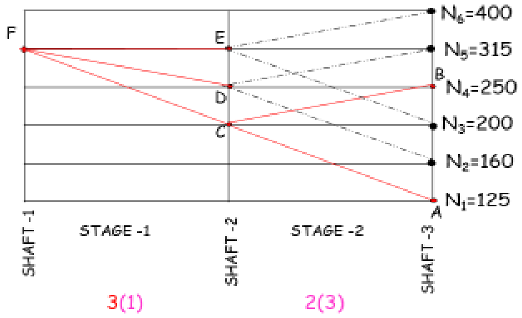
STEP 4: CONSTRUCT THE RAY DIAGRAM FOR 6 SPEED GEAR BOX

STRUCTURAL FORMULAE: 3(1) 2(3)

Note: Where X_1 =1 X_2= p_1 =3

- ✓ **No. of shafts= No. of stages+1 (2+1=3 shafts) (so draw 3 vertical lines)**
- ✓ **No. of speeds=6 (Draw 6 horizontal lines)**

RAY DIAGRAM: 6 SPEED GEAR BOX



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RAY DIAGRAM: 6 SPEED GEAR BOX cont…

- STAGE 2:
  - ✓For stage 2 = 2(3)
  - ✓ 2 points with 3 speed gap
  - ✓Make the points A& B
  - ✓Find input speed for the speeds A=125rpm and B=250rpm by using

$$\frac{125}{200} = 0.625 \geq \frac{1}{4}$$

$$i_{min} = \frac{N_{MIN}}{N_{INPUT}} \geq \frac{1}{4}$$

$$i_{max} = \frac{N_{MAX}}{N_{INPUT}} \leq 2$$

$$\frac{250}{200} = 1.25 \leq 2$$

Ratio requirement satisfied,
∴Input speed for stage 2= 200 rpm
Mark a point C on shaft 2
Join AC, BC

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RAY DIAGRAM: 6SPEED GEAR BOX cont…

- STAGE 1:
  - ✓For stage 1 = 3(1)
  - ✓ 3 points with 1 speed gap
  - ✓Make the points C,D &E,
  - ✓Find input speed for the speeds C=200 rpm and F=315rpm by using

$$\frac{200}{315} = 0.34 \geq \frac{1}{4}$$

$$i_{min} = \frac{N_{MIN}}{N_{INPUT}} \geq \frac{1}{4}$$

$$i_{max} = \frac{N_{MAX}}{N_{INPUT}} \leq 2$$

$$\frac{315}{315} = 1 \leq 2$$

Ratio requirement satisfied,
∴input speed for stage 1 = 315 rpm
Mark a point F on shaft 1
Join CF, DF and EF

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STEP 5: CALCULATION OF NUMBER OF TEETH ON ALL THE GEARS

LET, Z_1, Z_2,Z_3,……Z_{10}= Number of teeth of the gears 1,2,3,…10 respectively

N_1,N_2,N_3,……N_{10}= Speeds of the gears 1,2,3,…10 respectively

Formulae given $\frac{Z_1}{Z_2} = \frac{N_2}{N_1}$

Take stage-2-Consider the first pair of gear 9 and 10

- **From ray diagram consider ray CA**
- **Maximum speed reduction 200rpm to 125rpm**

We know that, $Z_{min} \geq$ 17, ∴ assume Z_9=20 (Driver)

$$\frac{Z_9}{Z_{10}} = \frac{N_{10}}{N_9}$$

$$\frac{20}{Z_{10}} = \frac{125}{200}$$

$$Z_{10}=32$$

Take stage-2-Consider the second pair of gear 7 and 8

- **From ray diagram consider ray CB**
- **Minimum speed increase 200rpm to 250rpm**

We know that,

$$\frac{Z_7}{Z_8} = \frac{N_8}{N_7}$$

$$\frac{Z_7}{Z_8} = \frac{250}{200}$$

$$Z_7=1.25\ Z_8 \qquad ......(i)$$

$$Z_7+ Z_8 =Z_9+ Z_{10} =20+32=52 \qquad ......(ii)$$

Solving equations (i) and (ii) ,we get

Z_8=22.22 ≈23

Z_7=52-23=29

Take stage-1-Consider the first pair of gear 5 and 6

- **From ray diagram consider ray FC**
- **Maximum speed reduction 315rpm to 200rpm**

We know that, $Z_{min} \geq 17$, ∴ assume Z_5=20 (Driver)

$$\frac{Z_5}{Z_6} = \frac{N_6}{N_5}$$

$$\frac{20}{Z_{12}} = \frac{200}{315}$$

$$Z_6 = 31.5 \approx 32$$

Take stage-1-Consider the second pair of gear 1 and 2

- **From ray diagram consider ray FD**
- **Speed reduction 315rpm to 250rpm**

We know that,

$$\frac{Z_1}{Z_2} = \frac{N_2}{N_1}$$

$$\frac{Z_1}{Z_2} = \frac{250}{315}$$

$$Z_1 = 0.79\, Z_2 \qquad \text{......(v)}$$

$$Z_1 + Z_2 = Z_5 + Z_6 = 20+32=52 \qquad \text{......(vi)}$$

Solving equations (v) and (vi) ,we get

$$Z_2 = 29$$

$$Z_1 = 52-29=23$$

Take stage-2-Consider the third pair of gear 3 and 4

- **From ray diagram consider ray FE**
- **Speed from 315rpm to 315 rpm**

We know that,

$$\frac{Z_3}{Z_4} = \frac{N_4}{N_3}$$

$$\frac{Z_3}{Z_4} = \frac{315}{315}$$

$$Z_3 = 1\, Z_4 \qquad \text{......(vii)}$$

$$Z_3 + Z_4 = Z_5 + Z_6 = 20+32=52 \qquad \text{......(viii)}$$

Solving equations (iii) and (iv) ,we get

Z_4=26

Z_3=52-26=26

STEP 6: SELECTION OF MATERIAL:

- **Take 40 Ni 2 Cr 1 Mo 28 (Hardened and Tempered) and material constant M=100**

STEP 7: CALCULATION OF MODULE

Case 1: To Find the Torque

- ✓ **Calculate the torque for the gear 10 has the lowest speed of 125 rpm using the relation,**

$$T = \frac{60\,P}{2\pi N}$$

$$T = \frac{60\,x\,5x10^3}{2\pi\,x\,125}$$

$$T = 381.97\,Nm$$

Case 2: To Find the Tangential force on gear 10:

- ✓ **Calculate the tangential force (F_t) on the gear in terms of module using the relation, From PSGDB 8.57, table 46**

$$F_{t10} = \frac{T}{r} = \frac{2\,T10}{Z10\,.\,m}$$

$$F_{t10} = \frac{2\,x\,381.97\,x\,10^3}{32\,x\,m}$$

$$F_{t10} = \frac{23873.13}{m}$$

Case 3: To Find module:

- **Now calculate the module (*module is defined as the ratio of pitch circle diameter to number of teeth)* using the relation**

$$m = \sqrt{\frac{F_t}{(\psi_m .\ M)}}$$

Where, ψ_m= Ratio between the face width and module = b/m =10, From PSGDB 8.1 and 8.14 (table 12)

M= Material constant = 100 from table 1 (step 6-40 Ni 2 Cr 1 Mo 28

- **Module:**

$$m = \sqrt{\frac{F_{t10}}{(\psi_m \cdot M)}} = \sqrt{\frac{(23873.13/m)}{(10\,x\,100)}} = \sqrt{23.87/m}$$

OR

$$m^2 = 23.87/m$$

∴ **Module m=2.88mm**

From PSGDB 8.2 choice 1 the nearest higher standard module is 3 mm

STEP 8: CALCULATION OF CENTRE DISTANCE IN ALL STAGES

By using the relation

$a = \left(\frac{Z_x+Z_y}{2}\right) m$ **From PSGDB 8.22, Table 26**

Zx and Zy = Number of teeth on the gear pair in engagement in each stage.

- ✓ **Centre distance in stage 1,** $a1 = \left(\frac{Z_3+Z_4}{2}\right) m = \left(\frac{26+26}{2}\right) x\,3$ **= 78mm**
- ✓ **Centre distance in stage 2,** $a2 = \left(\frac{Z_7+Z_8}{2}\right) m = \left(\frac{29+23}{2}\right) x\,3$ **= 78 mm**

STEP 9: CALCULATION OF FACE WIDTH

b = 10 x m

We know that module m=3 mm

∴ Face width b= 10 x3 =30 mm

STEP 10: CALCULATION OF LENGTH OF THE SHAFTS

L= 25+10+7b+20+4b+10+25

= 90 + 11b

= 90 + (11x30)

L= 420 mm

8. Design the layout of a 12 speed gear box far a milling machine having an output of speeds ranging from 25 to 600 rpm. Power is applied to the gear box from a 2.25 KW induction motor at 1440 rpm. Construct the speed diagram using standard speed ratio. Calculate the number of teeth on each gear and sketch the arrangement of the gear box.

Given data:

n = 12

N_{min} = 25rpm

N_{max} = 1440 rpm

P = 2.25KW

1. Selection of spindle speeds:

We know that,

$$\phi^{n-1} = \frac{N_{max}}{N_{min}}$$
$$\phi^{12-1} = \frac{600}{25}$$
$$\phi = 1.335$$

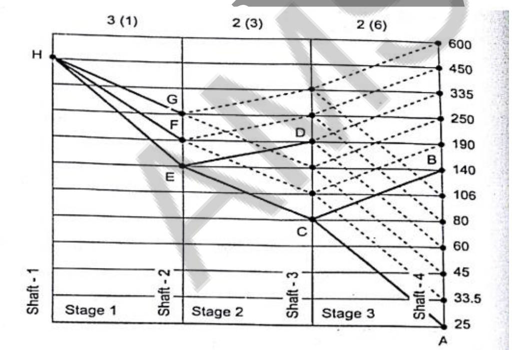
We can write, $1.06 \times (1.6 \times 1.06 \times 1.06 \times 1.06) = 1.338$

So, ϕ = 1.06 satisfies the requirement. Therefore the spindle from R 40 series skipping four speeds, are given as

25, 33.5, 45, 60, 80, 106, 140, 190, 250, 250, 335, 450 and 600 rpm.

2. Ray diagram: The ray diagram is constructed, as shown in fig.

Structural formula: 3(1) 2(3) 2(6)



Ray diagram for 12 speed gear box

Step 3:

$$\frac{N_{min}}{N_{input}} = \frac{25}{80} = 0.31 > \frac{1}{4} \text{ and}$$
$$\frac{N_{max}}{N_{input}} = \frac{140}{80} = 1.75 < 2$$

Step 2:

$$\frac{N_{min}}{N_{input}} = \frac{80}{140} = 0.57 > \frac{1}{4}$$

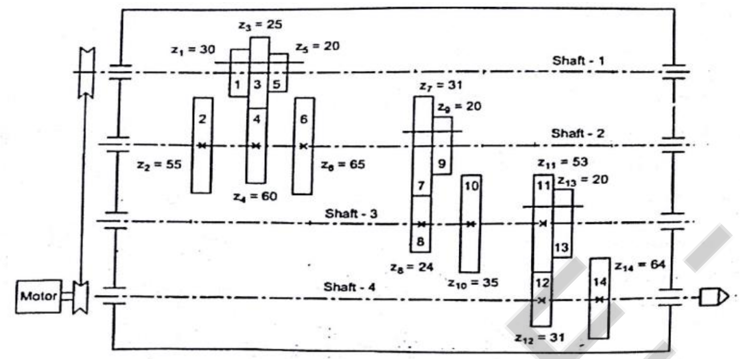
$$\frac{N_{max}}{N_{input}} = \frac{190}{140} = 1.36 < 2$$

Step 1:

$$\frac{N_{min}}{N_{input}} = \frac{140}{450} = 0.311 > \frac{1}{4}$$

$$\frac{N_{max}}{N_{input}} = \frac{250}{450} = 0.56 < 2$$

3. Kinematic arrangement: The kinematic arrangement for the given 12 speed gear box is constructed, as shown in fig.



Kinematic arrangement for 12 speed gear box

4. Calculation of number of teeth on alt gears: The number of teeth on all gears are calculate as below, following the procedure used

Stage 3:

First pair: Consider the ray that gives, maximum reduction i.e, from 80 r. p. m to 25 r. p. m. The corresponding gears are 13 and 14 on shaft 4.

We know that, $Z_{min} \geq 17$. Therefore assume z_{13}= 20 (driver)

$$\frac{z_{13}}{z_{14}} = \frac{N_{14}}{N_{13}} \text{ or } \frac{20}{z_{14}} = \frac{25}{80}; \qquad \therefore z_{14} = 64$$

Second pair: Consider the other ray that gives speed increase form 80 r. p. m. To 140r. p. m. The corresponding gears are 11 and 12.

$$\frac{z_{11}}{z_{12}} = \frac{N_{12}}{N_{11}} = \frac{140}{80} \text{ or } z_{11} = 1.75 z_{12} \qquad --(i)$$

We also know that the sun of number of teeth of mating gears should be equal.

$$z_{11} + z_{12} = z_{13} + z_{14} = 20 + 64 = 84 \qquad --(ii)$$

On solving equations (i) and (ii), we get

$$z_{12} = 30.5 \approx 31 \text{ and } z_{11} = 84 - 31 = 53$$

Stage 2:

First pair: Consider the ray that gives maximum reduction from 140 r.p.m to 8 r.p.m. The corresponding gears are 9 and 10. Assume $z_9 = 20$ (driver).

$$\frac{z_9}{z_{10}} = \frac{N_{10}}{N_9} \text{ or } \frac{20}{z_{10}} = \frac{80}{140}; \qquad z_{10} = 35$$

Second pair: Consider the other ray that gives speed increase from 140 r.p.m to 190 r.p.m. The corresponding gears are 7 and 8.

$$\frac{z_7}{z_8} = \frac{N_8}{N_7} = \frac{190}{140} \text{ or } \quad z_7 = 1.357 z_8 \qquad --\text{(iii)}$$

$$z_7 + z_8 = z_9 + z_{10} = 20 + 35 = 55 \qquad --\text{(iv)}$$

On solving equation (iii) and (iv), we get

$$z_8 = 23.3 = 24 \text{ and } z_7 = 55 - 24 = 31$$

Stage 1:

First pair: Consider the ray that gives maximum from 450 r.p.m to 140 r.p.m. The corresponding gears are 5 and 6. Assume $z_5 = 20$ (driver)

$$\frac{z_5}{z_6} = \frac{N_6}{N_5} \text{ or } \frac{20}{z_6} = \frac{140}{450}; \qquad z_6 = 64.28 = 65$$

Second pair: Consider the ray that gives speed reduction from r.p.m to 190 r.p.m. The corresponding gears are 3 and 4.

$$\frac{z_3}{z_4} = \frac{N_4}{N_3} = \frac{190}{450} \text{ or } z_3 = 0.422 z_4 \qquad --\text{(v)}$$

$$z_3 + z_4 = z_5 + z_6 = 20 + 65 = 85 \qquad --\text{(vi)}$$

On solving the equations (v) and (vi), we get

$$z_4 = 59.77 \approx 60 \text{ and } z_3 = 85 - 60 = 25$$

Third pair: Consider the ray that gives speed reduction from 450 r.p.m to 250 r.p.m. The corresponding gears are 1 and 2.

$$\frac{z_1}{z_2} = \frac{N_2}{N_1} = \frac{250}{450} \text{ or } z_1 = 0.555 z_2 \qquad --\text{(vii)}$$

$$z_1 + z_2 = z_3 + z_4 = 60 + 25 = 85 \qquad --\text{(viii)}$$

On solving the equations (vii) and (viii), we get

$$z_2 = 54.66 \approx 55 \text{ and } z_1 = 85 - 55 = 30$$

9. Sketch the arrangement of six speed gear box for a minimum speed of 460 rpm. Draw the speed diagram and kinematic arrangement showing number of teeth in all gears. Check whether all the speeds obtained through the selected gears are within ± 2% of standard speeds. The drive is for an electric motor giving 2.25kW at 1440rpm.

Given data;

$$n = 6$$
$$N_{min} = 460rpm$$
$$N_{max} = 1400rpm$$
$$p = 2.25kW$$
$$N_{input} = 1440rpm$$

Step 1: Selection of spindle speeds

Determine the progression ratio (ϕ) using the relation

$$\frac{N_{max}}{N_{min}} = \phi^{n-1}$$
$$\frac{1400}{460} = \phi^{6-1}$$
$$\phi = 1.25$$

So we take , because satisfies the requirement . Select the standard spindle speeds using the series of preferred numbers .

Take , step ratio from R10 series $\phi = 1.25$

Standard spindle speeds are 500, 630, 800, 1250 and 1600 rpm

Step 2: Structural Formulae

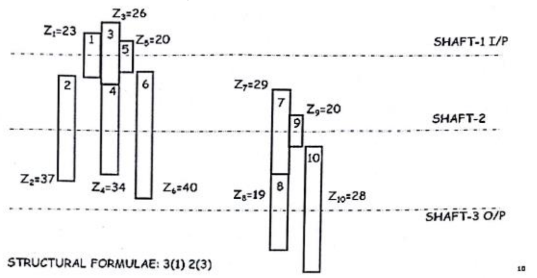
Structural Formulae 3(1) 2(3)

Step 3: Construct the kinematic arrangement for 6 speed gearbox

Structural formulae 3(1) 2(3)

Note: Where $x_1 = {}_1x_2 = p_1 = 3$

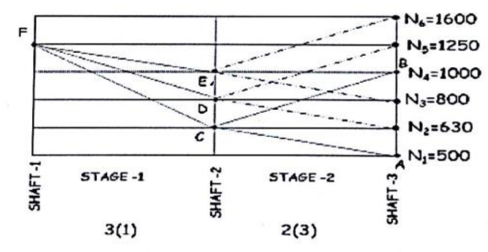
- ✓ To find the no. of gears $= 2(p_1 + p_2)\{[2(3+2)] = 10 \text{ gears}\}$



Step 4: construct The Ray Diagram for 6 speed Gear box

Structural formulae 3(1) 2(3)

- ✓ No. of shafts = No. of stages +1 (2+1=3)(so draw 3 vertical lines)
- ✓ No. of speeds = 6 (Draw 6 horizontal lines)



Stage 2:

- ✓ For stage 2 = 2(3), 2 points with 3 speed gap
- ✓ Find input speed for the speeds A = 500 rpm and B = 1000rpm by using

$$\frac{630}{1250} = 0.504 \geq \frac{1}{4} \frac{1000}{1250} = 0.8 \leq 2$$

Ratio requirement satisfied. Input speed for stage 1 = 1250 rpm

Step 5: Calculation of number of teeth on all the gears

Let Z_1, Z_2, Z_3,.....Z_{10} = Number of teeth of the gears 1, 2, 3, ...10 respectively

N_1, N_2, N_3, ...N_{10} = Speeds of the gears 1, 2, 3, ...10 respectively

Formulae given $\frac{z_1}{z_2} = \frac{N_1}{N_2}$

Take stage – 2 consider the first pair of gear 9 and 10

- From ray diagram consider ray CA
- Maximum speed reduction 630 rpm to 460 rpm

We know that, $Z_{min} \geq 17$, ∴ assume Z_9 =20 (Driver)

$$\frac{z_9}{z_{10}} = \frac{N_{10}}{N_9} = \frac{20}{z_{10}} = \frac{460}{630}$$

$$Z_{10} = 28$$

Take stage – 2 consider the speed pair of gear 7 and 8

- From ray diagram consider ray CB
- Speed increase 630 rpm to 1000 rpm

We know that,

$$\frac{z_7}{z_6} = \frac{N_6}{N_5} = \frac{20}{}$$

Take stage – 1 consider the second pair of gear 1 and 2

- From ray diagram consider ray FD
- Speed reduction 1250 rpm to 800rpm

We know that, $\frac{z_1}{z_2}=\frac{N_2}{N_1}=\frac{z_1}{z_2}=\frac{800}{1250}$

$$z_1 = 0.64z_2 \qquad --(v)$$
$$z_1+z_2=z_5+z_6=20+40=60 \qquad --(vi)$$

Solving equations (v) and (vi), we get

$$z_2 = 36.58 \approx 37,\ zl1 = 60-37 = 23$$

Take stage – 2 – consider the third pair of gear 3 and 4

- From ray diagram consider ray FE
- Speed from 1250 rpm to 100rpm

We know that, $\frac{z_3}{z_4}=\frac{N_4}{N_3}=\frac{z_3}{z_4}=\frac{1000}{1250}$

$$z_3 = 0.8z_4 \qquad --(vii)$$
$$z_3+z_4=z_5+z_6=20+40=60 \qquad --(viii)$$

Solving equations (iii) and (iv) we get

$$z_4 = 33.33 \approx 34$$
$$z_3 = 60-34 = 26$$

Step 6:- Calculation of output speeds

Let N_1 and N_0 = Input and output speeds of the gears. From the ray diagram input speed N_1 = 1250rpm

$$N_{01}=N_1\times\frac{Z_1}{Z_2}X\frac{Z_7}{Z_8}=1250\times\frac{23}{37}\times\frac{29}{19}=1186\text{rpm}$$
$$N_{02}=N_1\times\frac{Z_1}{Z_2}X\frac{Z_9}{Z_{10}}=1250\times\frac{23}{37}\times\frac{20}{28}=555.02\text{rpm}$$
$$N_{03}=N_1\times\frac{Z_3}{Z_4}X\frac{Z_7}{Z_8}=1250\times\frac{26}{34}\times\frac{29}{19}=1459\text{rpm}$$
$$N_{04}=N_1\times\frac{Z_3}{Z_4}X\frac{Z_9}{Z_{10}}=1250\times\frac{26}{34}\times\frac{20}{28}=682.77\text{rpm}$$
$$N_{05}=N_1\times\frac{Z_5}{Z_6}X\frac{Z_7}{Z_8}=1250\times\frac{20}{40}\times\frac{29}{19}=954\text{rpm}$$
$$N_{06}=N_1\times\frac{Z_5}{Z_6}X\frac{Z_9}{Z_{10}}=1250\times\frac{20}{40}\times\frac{20}{28}=446.42\text{rpm}$$

Sl. No	Obtainable speed ($N_{obt,}\,rpm$)	Calculated speed ($N_{cal,}\,rpm$)	% deviation=$\frac{N_{obt}-N_{cal}}{N_{cal}}$x100
1	446.42	500	-10.92
2	1186	630	88.05
3	1459	800	82.375
4	954	1000	-4.6
5	555.02	1250	-55.59
6	682.77	1600	-57.32

10. A sixteen speed gear box is required to furnish output speeds in the range of 100 to 560rpm. Sketch the kinematic arrangement and draw the speed diagram.

Given data:

$M = 16$

$N_{min} = 100rpm$

$N_{max} = 560rpm$

Step 1: Selection of spindle speeds.

$$\frac{N_{max}}{N_{min}} = \phi^{16-1}$$

$$\frac{560}{100} = \phi^{15}$$

$$\phi = (5.6)^{1/15}$$

$$\phi = 1.12$$

We find $\phi = 1.12$ is the standard ratio, it satisfies the requirement.

Select the spindle speeds using the series of preferred numbers.

PSGDB 7.20

Basic series R20 $(\phi = 1.12)$

Spindle speeds are 100, 112, 125, 140, 160, 180, 200, 224, 250, 280, 315, 355, 400, 450, 500, 560 rpm.

Step 2: To find the structural formulae.

$$16 \text{ Speeds} = 4(1)2(4)2(8)$$

Step 3: Construct the speed diagram for 16 speed gear box.

- Structural formula $= 4(1)\ 2(4)\ 2(8)$
- No. of stages $= 3\{P_1(X_1)\cdot P_2(X_2)\cdot P_3(X_3)\}$

$$P_1 = 4,\ P_2 = 2,\ P_3 = 2$$

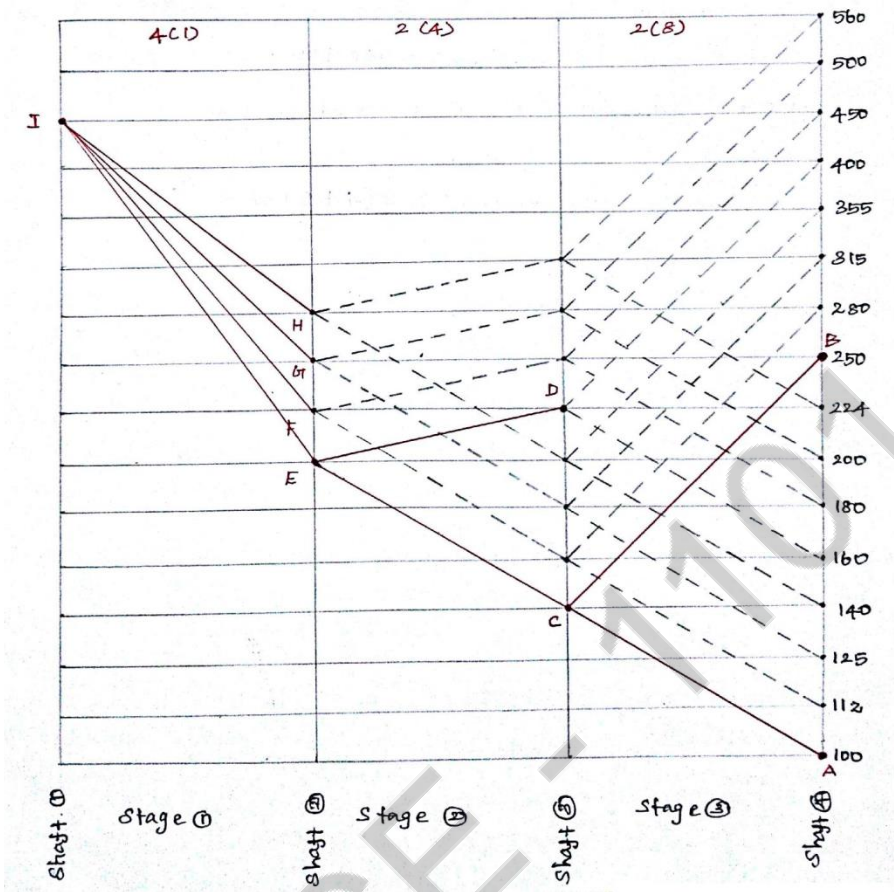
Note: $X_1 = 1,\ X_2 = P_1 = 4,\ X_3 = P_1 \times P_2 = 4 \times 2 = 8.$

- No. of shafts = No. of stages +1

$$= 3 + 1$$

$$= 4 \ (\text{Draw 4 vertical lines})$$

* No. of speeds $= 16$ (Draw 16 horizontal lines).



Stage 3:

$$\frac{N_{min}}{N_{input}} \geq \frac{1}{4} \qquad \frac{N_{max}}{N_{input}} \leq 2$$

$$\frac{100}{140} = 0.714 > \frac{1}{4} \qquad \frac{250}{140} = 1.78 < 2$$

$$\therefore \; N_{input} = 140\text{rpm.}$$

Stage 2:

$$\frac{N_{min}}{N_{input}} \geq \frac{1}{4} \qquad \frac{N_{max}}{N_{input}} \leq 2$$

$$\frac{140}{200} = 0.7 \geq \frac{1}{4} \qquad \frac{224}{200} = 1.12 \leq 2$$

$$\therefore \; N_{input} = 200\text{rpm.}$$

Stage 1:

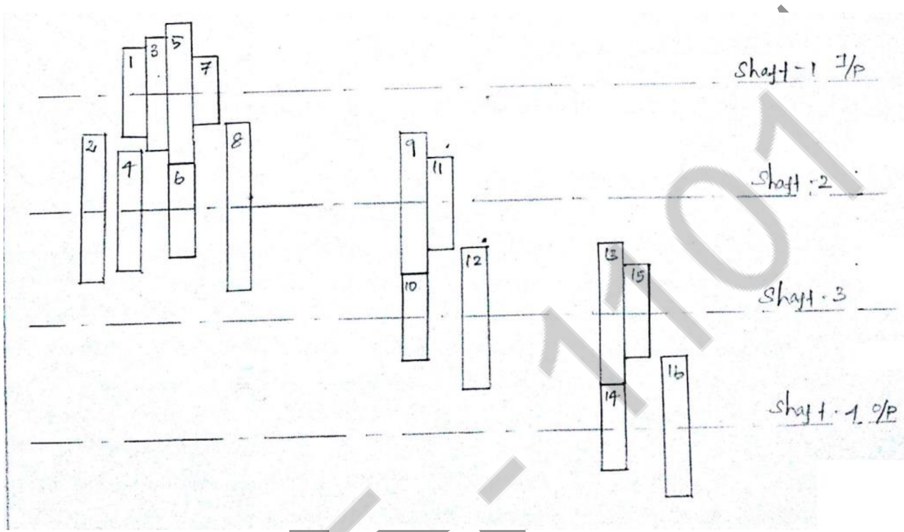
$\frac{200}{450} = 0.44 \geq \frac{1}{4}$ $\frac{280}{450} = 0.622 \leq 2$

$\therefore \ N_{input} = 450rpm.$

Step 4: Kinematic Layout - 16 Speed gear box

No. of shafts = 4

No. of Gears = 2 (4 + 2 + 2) = 16



11. Design a nine speed gear box for a machine to provide speeds ranging from 100rpm to 1500rpm. The input is from a motor of 5KW at 1440rpm. Assume any alloy steel for the gears. (April/May 2017)

Given data:

$\eta = 9$

$N\min = 100rpm.$

$N\max = 1500\,rpm.$

$P = 5KW$

$N_{input} = 1440rpm.$

Note: In this problem the given max speed is 1500rpm. But as per R 20 series am taken the 9th speed 1400rpm. If you want to take 1500rpm as the 9th speed also correct. No issues. Anyhow maximum cases we should follow the standard values.

Step 1: Selection of spindle speeds:

$$\frac{N_{max}}{N_{min}} = \phi^{n-1}$$

$$\frac{1500}{100} = \phi^{9-1}$$

$$15 = \phi^8$$

$$\phi = 1.403.$$

* We find $\phi = 1.403$ is not a standard ratio. So let us find out whether multiples of standard ratio 1.12 or 1.06 come close to 1.403.

  $1.12 \times 1.12 \times 1.12 = 1.405$ Skip 2 speeds.

* $\phi = 1.12$ Satisfies the requirement. Therefore the spindle speeds from R 20 series skipping 2 speeds, are.

From PSGDB 7.20,

100, 140, 200, 280, 400, 560, 800, 1000, 1400 rpm.

Step 2: To find the structural formula:

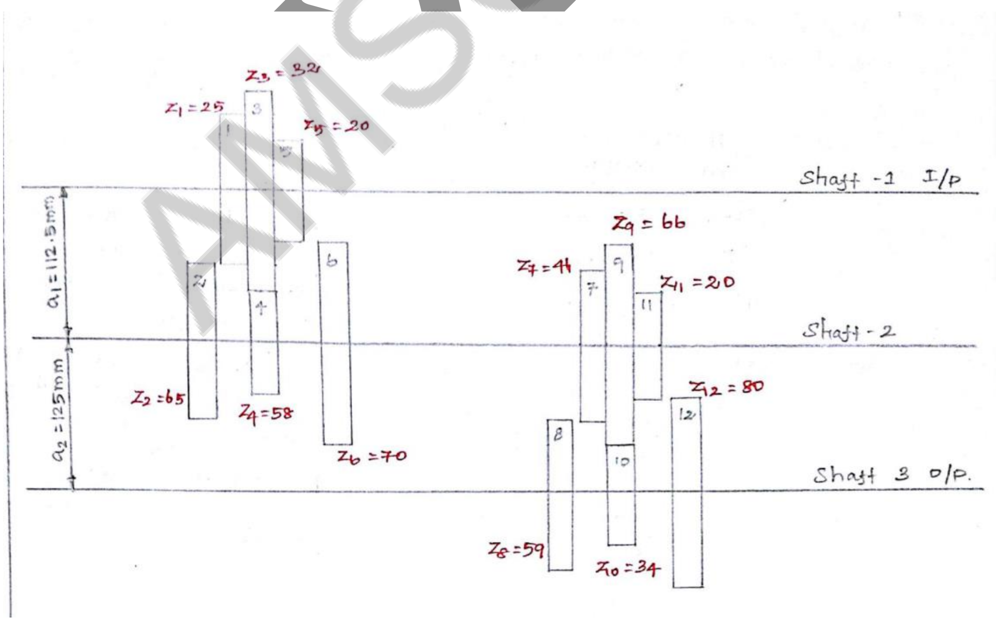
9 speeds = 3(1) 3(3)

Step 3: Kinematic diagram for 9 speeds.

Structural formula =3(1) 3(3).

No. of shafts = No. of stages +1 = 3 (3 horizontal lines).

No. of gears = $2(P_1+P_2)$ = 2(3+3) = 12 gears.



Step 3: Ray diagram for 9 speed.

Structural formula = 9 speeds = 3(1) 3(3).

No. of shafts =3 (3 vertical lines)

Speeds = 9 (9 horizontal lines)

For stage 2

$$\frac{N_{min}}{N_{I/p}} \geq 1/4 \qquad \frac{N_{max}}{N_{I/p}} \leq 2$$

$$\frac{100}{400} \geq 1/4 \Rightarrow \frac{N_{min}}{N_{I/p}} = 1/4 \geq 1/4 \qquad \frac{N_{max}}{N_{I/p}} = \frac{800}{400} = 2 \leq 2$$

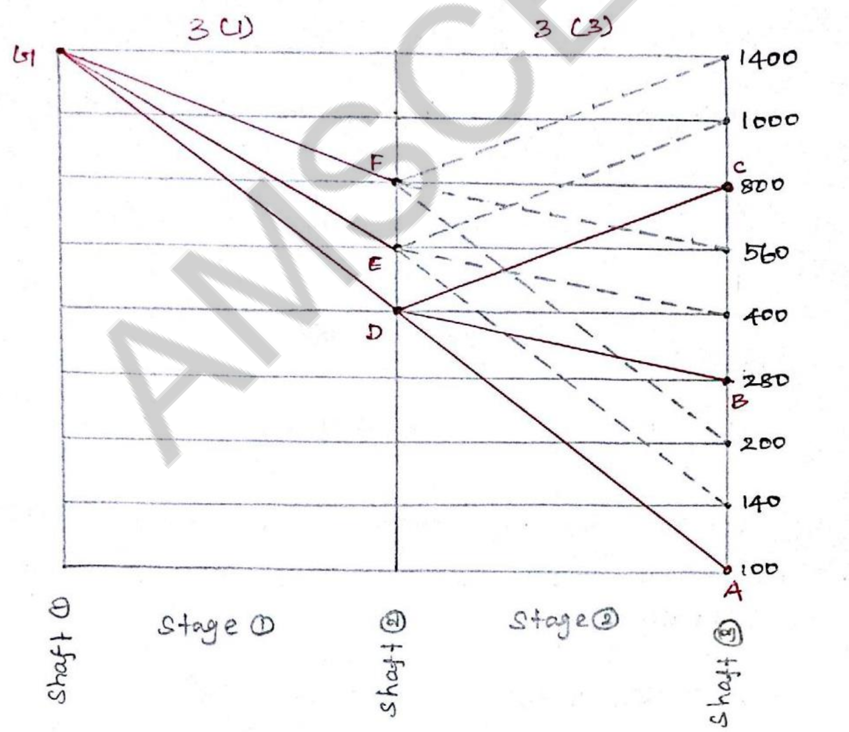
$$\therefore N_{I/p} = 400\text{rpm.}$$

For stage 1

$$\frac{N_{min}}{N_{I/p}} \geq 1/4 \qquad \frac{N_{max}}{N_{I/p}} \leq 2$$

$$\frac{N_{min}}{N_{I/p}} = \frac{400}{1400} = 0.29 \geq 1/4 \qquad \frac{N_{max}}{N_{I/p}} = \frac{800}{1400} = 0.57 \leq 2$$

$$\therefore N_{I/p} = 1400\text{rpm.}$$



Step 4: Calculation of no. of teeth on all the gears.

Let $Z_1, Z_2, Z_3 \ldots Z_{12}$ = No. of teeth of the gears 1, 2, 3…. 12 respectively.

$N_1, N_2, N_3 \ldots N_{12}$ = No. of speed of the gears 1, 2, 3…. 12 respectively.

We know that , $\frac{Z_1}{Z_2} = \frac{N_2}{N_1}$

Case 1: consider stage 2.

First pair:

* Gears 11 and 12
* From the ray diagram consider Ray DA.
* Maximum speed reduction from 400 rpm to 1000 rpm .

$$Z_{11} = 20 \text{ (driver)}.$$

$$\therefore \quad \frac{Z_{11}}{Z_{12}} = \frac{N_{12}}{N_{11}}$$

$$\frac{20}{Z_{12}} = \frac{100}{400}$$

$$Z_{12} = 80$$

$$Z_{11} = 20, \quad Z_{12} = 80$$

Second Pair:

* Gears 7, 8 & Ray DB
* Minimum speed reduction 400 to 280 rpm.

$$\frac{Z_7}{Z_8} = \frac{N_8}{N_7}$$

$$\frac{Z_7}{Z_8} = \frac{280}{400}$$

$$Z_7 = 0.7 Z_8 \qquad 1$$

Note: The centre distance between the shafts are fixed and same. ∴ The sum of number of teeth of mating gears should be equal.

$$\therefore \quad Z_{11} + Z_{12} = Z_7 + Z_8 = 100$$

$$0.7 Z_8 + Z_8 = 100$$

$$Z_8 = 58.82 \approx 59$$

$$\therefore \quad Z_7 = 41 \ , \quad Z_8 = 59$$

Third Pair:

* Gears 9 & 10, Ray DC
* Speed increase from 400 to 800 rpm.

$$\frac{Z_9}{Z_{10}} = \frac{N_{10}}{N_9}$$

$$\frac{Z_9}{Z_{10}} = \frac{800}{400}$$

$$Z_9 = 2Z_{10}$$

$$\text{W.K.T} \quad Z_{11} + Z_{12} = 100 = Z_9 + Z_{10}$$

$$2Z_{10} + Z_{10} = 100$$

$$Z_{10} = 33.33 \approx 34$$

$$\therefore \quad Z_9 = 66$$

$$Z_{10} = 34$$

Case 2: Consider stage 1:

First Pair:

* Gears 5 and 6 , Ray GD.
* Maximum speed reduction from 1400 to 400 rpm.

$$\frac{Z_5}{Z_6} = \frac{N_6}{N_5}$$

$$Z_5 = 20 \quad (\text{driver})$$

$$\frac{20}{Z_6} = \frac{400}{1400}$$

$$Z_6 = 70$$

$$Z_5 = 20, \quad Z_6 = 70.$$

Second Pair:

* Gears 1 and 2 , Ray GE

* Speed reduction from 1400 to 560 rpm.

$$\frac{Z_1}{Z_2} = \frac{N_2}{N_1}$$

$$\frac{Z_1}{Z_2} = \frac{560}{1400}$$

$$Z_1 = 0.4 Z_2$$

We know that $Z_5 + Z_6 = 90 = Z_1 + Z_2$

$$0.4Z_2 + Z_2 = 90$$

$$Z_2 = 64.28 \approx 65$$

$$Z_1 = 25$$

Third Pair

* Gears 3 and 4, Ray GF
* Speed reduction from 1400 to 800 rpm.

$$\frac{Z_3}{Z_4} = \frac{N_4}{N_3}$$

$$\frac{Z_3}{Z_4} = \frac{800}{1400} \Rightarrow Z_3 = 0.57 Z_4$$

$$\text{W.K.T} \Rightarrow Z_5 + Z_6 = 90 = Z_3 + Z_4$$

$$0.57Z_4 + Z_4 = 90$$

$$Z_4 = 57.32 \approx 58$$

$$Z_3 = 32$$

12. Design a 12 speed gear box. The required speed range is 100 to 355 rpm. Draw the ray diagram, kinematic arrangement (April/May 2017)

GIVEN DATA:

n= 12

N_{min}=100 rpm

N_{max}= 355 rpm

Step 1: SELECTION OF SPINDLE SPEEDS

Determine the progression ratio (∅) using the relation

$N_{max}/N_{min}=\emptyset^{n-1}$

$355/100=\emptyset^{12-1}$

$\emptyset=(3.55)^{1/11}$

∅=1.122

For the calculated ∅=1.122 ,select the standard spindle speeds using the series of preferred numbers – From PSGDB 7.20, 7.19

TAKE

STEP RATIO from R20 series ∅=1.122

∴SPINDLE SPEEDS ARE 100, 112, 125, 140, 160,180, 200, 224, 250, 280, 315,AND 355

Step 2: CONSTRUCT THE KINEMATIC ARRANGEMENT FOR 12 SPEED GEAR BOX

- STRUCTURAL FORMULAE: 3(1) 2(3) 2(6)

No.of Stages: 3, {(p_1 (X_1) . p_2 (X_2) . p_3 (X_3)}

1st stage. 2nd stage . 3rd stage

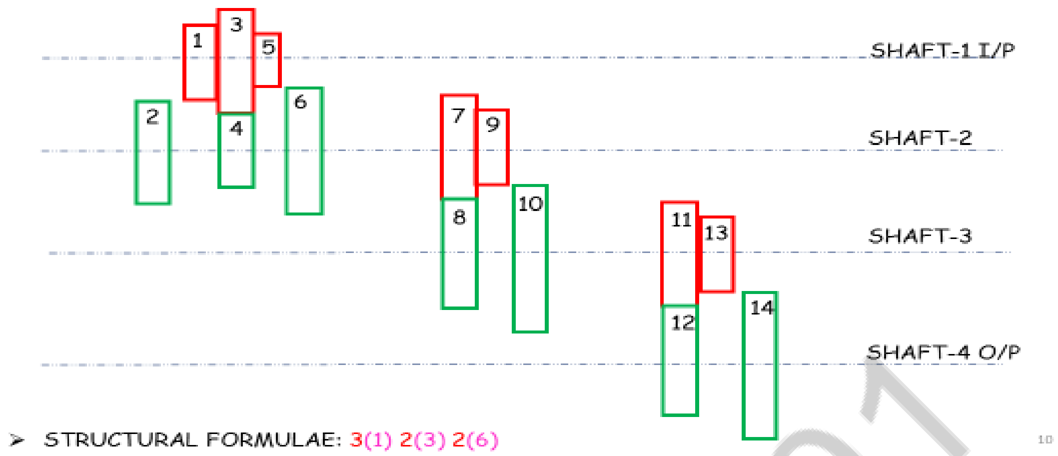
p_1 =3 p_2 =2 p_3 =2

Note:Where X_1 =1 X_2= p_1 =3 X_3 = $p_1.p_2$ =(3.2)=6

- ✓ No. of shafts= No. of stages+1 (3+1=4 shafts) (so draw FOUR horizontal lines)
- ✓ To find the no. of gears by using

No. of gears= 2(p_1 + p_2+p_3) {[2(3+2+2)]=14 gears}

KINEMATIC LAYOUT: 12 speed gear box



Step 3: CONSTRUCT THE RAY DIAGRAM FOR 12SPEED GEAR BOX

STRUCTURAL FORMULAE: 3(1) 2(3) 2(6)

No.of Stages: 2, $\{(p_1 (X_1) \,.\, p_2 (X_2)\} \,.\, p_3 (X_3)\}$

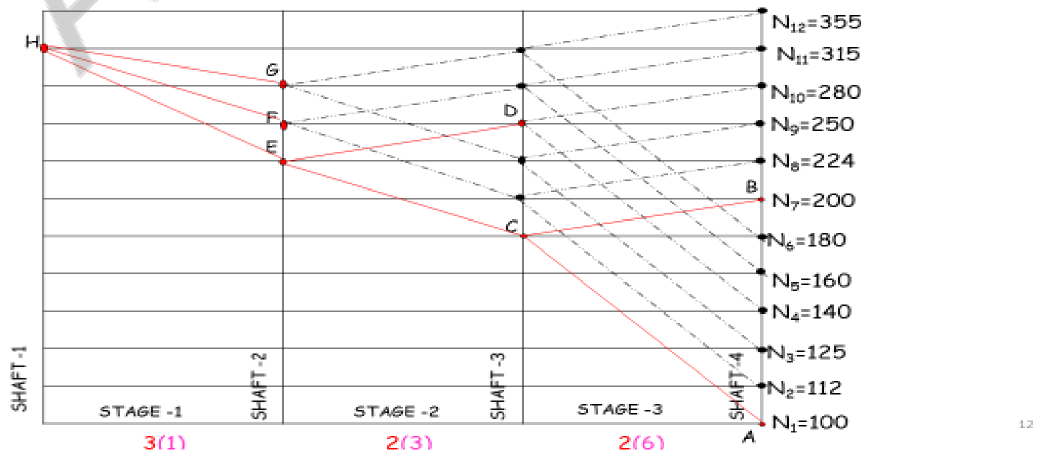
1st stage. 2nd stage . 3rd stage

$p_1 = 3 \quad p_2 = 2 \quad p_3 = 2$

Note:Where $X_1 = 1 \qquad X_2 = p_1 = 3 \qquad X_3 = p_1.p_2 = (3.2) = 6$

- ✓ No. of shafts= No. of stages+1 (3+1=4 shafts) (so draw 4 vertical lines)
- ✓ No. of speeds=12 (Draw 12 horizontal lines)

RAY DIAGRAM: 12 SPEED GEAR BOX



13. Draw the speed diagram, and the kinematic layout of the head stock gear box of a turret lathe having arrangement for 9 spindle speeds, ranging from 31.5rpm to 1050rpm. Calculate the no. of teeth on each gear. Minimum number of teeth on a gear is 25. Also calculate the percentage deviation of the obtainable speeds from the calculated ones. (Nov/Dec 2017)

GIVEN DATA:

$$n = 9$$
$$N_{min} = 31.5rpm$$
$$N_{max} = 1050rpm$$
$$Z_{driver} = 25$$

Step 1: Selection of Spindle Speeds

Determine the progression ration (ϕ) using the relation

$$N_{max}/N_{min} = \phi^{n-1}$$

$$1050/31.5 = \phi^{9-1}$$

$$\phi = (33.33)^{1/8}$$

$$\phi = 1.55$$

- ✓ We find $\phi = 1.55$ is not a standard ratio. So let us find out whether multiples of standard ratio 1.12 or 1.25 come close to 1.55
- ✓ For example we can write 1.12 × 1.12 = 1.2544 and 1.12 × 1.12 × 1.12 = 1.405

Then 1.25 × 1.25 = 1.55 ..... skip 1 speed

So we take $\phi = 1.25$, because satisfies the requirement. Select the standard spindle speeds using the series of preferred numbers – From PSGDB 7.20, 7.19

Take step Ratio from R10 series $\phi = 1.25$

∴ Spindle speeds are 31.5, 50, 80, 100, 160, 250, 400, 630, 1000rpm

Step 2: To find the Structural Formulae

Structural Formulae: 3(1) 3(3)

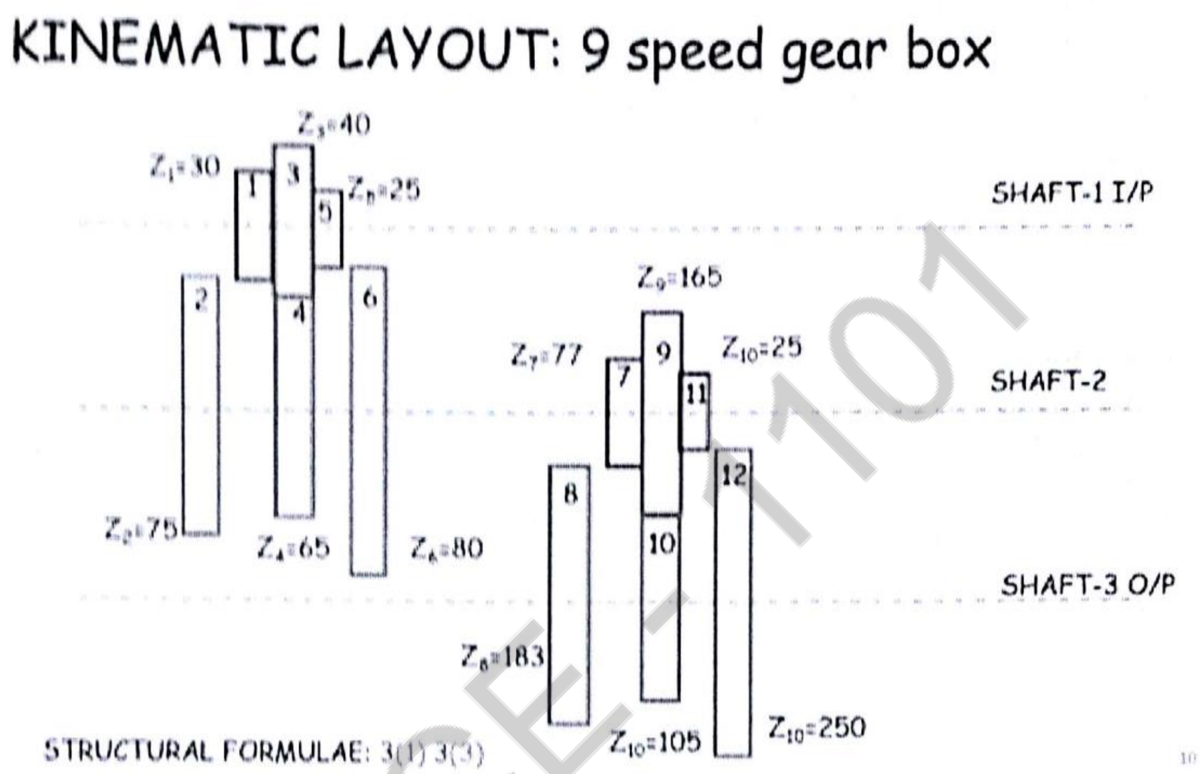
Step 3: Construct the Kinematic arrangement for 9 speed gear box

Structural Formulae: 3(1) 3(3)

Note: Where $X_1 = 1\ X_2 = p_1 = 3$

No. of shafts = No. of stages + 1 (2+1=3 shafts) (so draw 3 horizontal lines)

To find No. of gears = 2(p_1 + p_2) {[2(3+3)]=12gears}



Step 4: Construct the Ray diagram for 9 speed gear box

Structural Formulae: 3(1) 3(3)

Note: Where X_1 = 1 X2 = p1 =3

No. of shafts = No. of stages + 1 (2+1=3 shafts) (so draw 3 vertical lines)

No. of speeds = 9 (Draw 9 horizontal lines)

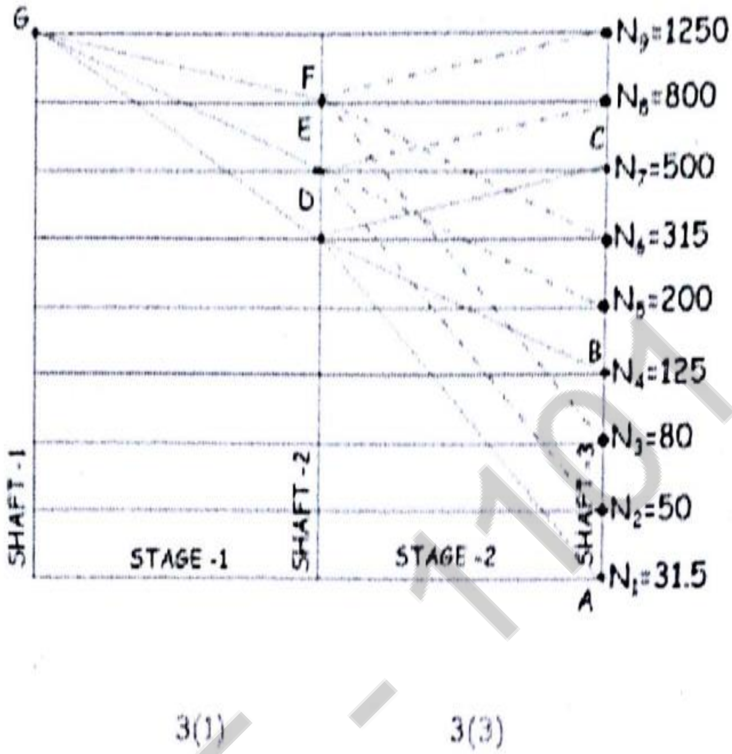
Stage 2:

- ✓ For stage 2 = 3(3) , 3 points with 3 speed gap or 3 speeds on shaft 3, Make the points A, B & C
- ✓ Find input speed for the speeds A=31.5 rpm and C=500rpm by using

$$\frac{31.5}{31.5} = 0.1 \leq \frac{1500}{4315} = 1.58 \leq 2$$ Ratio requirement satisfied,

$\therefore$Input speed for stage 2=315rpm

RAY DIAGRAM: 9 SPEED GEAR BOX



Stage 1:

- ✓ For stage 1=3(1), 3 points with 1 speed gap or 1 speeds on shaft 2, Make the points D, E & F
- ✓ Find input speed for the speeds D=315 rpm and F=800rpm by using

$\frac{31.5}{1250} = 0.252 \geq \frac{1}{4}$ $\frac{800}{1250} = 0.64 \leq 2$, Ratio requirement satisfied,

$\therefore$ Input speed for stage 1=1250 rpm

Step 5: Calculation of Number of Teeth on all the gears

Let, Z_1, Z_2, Z_3 ...Z_{12} = Number of teeth of the gears 1, 2, 3...12 respectively

N_1, N_2, N_3... N_{12} = Speeds of the gears 1, 2, 3 ...12 respectively

Formulae given $\frac{Z_1}{Z_2} = \frac{N_2}{N_1}$

Take stage-2-Consider the first pair of gear 11, and 12

- From ray diagram consider ray DA
- Maximum speed reduction 315rpm to 31.5rpm

We know that, $Z_{min} \geq 17$, $\therefore$ assume Z_{11} = 25 (Driver)

$$\frac{Z_{11}}{Z_{12}} = \frac{N_{12}}{N_{11}} = \frac{25}{Z_{12}} = \frac{31.5}{315} \qquad Z_{12} = 250$$

Take stage-1-Consider the second pair of gear 1 and 2

- From ray diagram consider ray GE
- Speed reduction 1250rpm to 500rpm

We know that, $\frac{Z_1}{Z_2} = \frac{N_2}{N_1} = \frac{Z_1}{Z_2} = \frac{500}{1250}$

$$Z_1 = 0.4Z_2 \qquad ..... (v)$$

$$Z_1 + Z_2 = Z_5 + Z_6 = 25 + 80 = 105 \qquad ....(vi)$$

Solving equations (v) and (vi), we get

$$Z_2 = 75$$
$$Z_1 = 105 - 75 = 30$$

Take stage-1-Consider the second pair of gear 3 and 4

- From ray diagram consider ray GF
- Speed reduction 1250rpm to 800rpm

We know that, $\frac{Z_3}{Z_4} = \frac{N_4}{N_3} = \frac{Z_3}{Z_4} = \frac{800}{1250}$

$$Z_3 = 0.64Z_4 \qquad ..... (vii)$$

$$Z_3 + Z_4 = Z_5 + Z_6 = 25 + 80 = 105 \qquad ....(viii)$$

Solving equations (viii) and (vii), we get

$$Z_4 = 64.02 \approx 65$$
$$Z_3 = 105 - 65 = 40$$

Step 6: Calculation of Output Speeds

Let N_1 and N_0 = Input and output speeds of the gears. From the ray diagram input speed N_1 = 1250 rpm

$$N_{01} = N_1 \times \frac{Z_1}{Z_2} \times \frac{Z_7}{Z_8} = 1250 \times \frac{30}{75} \times \frac{78}{197} = 197.96 \text{rpm}$$

$$N_{02} = N_1 \times \frac{Z_1}{Z_2} \times \frac{Z_9}{Z_{10}} = 1250 \times \frac{30}{75} \times \frac{168}{107} = 785.05\text{rpm}$$

$$N_{03} = N_1 \times \frac{Z_1}{Z_2} \times \frac{Z_{11}}{Z_{12}} = 1250 \times \frac{30}{75} \times \frac{25}{250} = 50\text{rpm}$$

Take stage-2-Consider the second pair of gear 7 and 8

- From ray diagram consider ray DB
- Speed reduction 315rpm to 125rpm

We know that, $\frac{Z_7}{Z_8} = \frac{N_8}{N_7} = \frac{Z_7}{Z_8} = \frac{125}{315}$

$$Z_7 = 0.4Z_8 \quad \ldots.(i)$$

$$Z_7 + Z_8 = Z_{11} + Z_{12} = 25 + 250 = 275 \quad \ldots.(ii)$$

Solving equations (i) and (ii), we get

$$Z_8 = 196.42 \approx 197$$
$$Z_7 = 275 - 197 = 78$$

Take stage-2-Consider the second pair of gear 9 and 10

- From ray diagram consider ray DC
- Speed reduction 315rpm to 500rpm

We know that, $\frac{Z_9}{Z_{10}} = \frac{N_{10}}{N_9} = \frac{Z_9}{Z_{10}} = \frac{500}{315}$

$$Z_9 = 1.59Z_{10} \quad \ldots.(iii)$$

$$Z_9 + Z_{10} = Z_{11} + Z_{12} = 25 + 250 = 275 \quad \ldots.(iv)$$

Solving equations (iii) and (iv), we get

$$Z_{10} = 106.18 \approx 107$$
$$Z_9 = 275 - 107 = 168$$

Take stage-1-Consider the second pair of gear 5 and 6

- From ray diagram consider ray GD
- Maximum Speed reduction 1250rpm to 315rpm

We know that, $Z_{min} \geq 17$, $\therefore$ assume $Z_5 = 25$ (Driver)

$$\frac{Z_5}{Z_6} = \frac{N_6}{N_5} = \frac{25}{Z_{12}} = \frac{315}{1250}$$

$$Z_6 = 79.37 \approx 80$$

$$N_{04} = N_1 \times \frac{Z_3}{Z_4} \times \frac{Z_7}{Z_8} = 1250 \times \frac{40}{65} \times \frac{78}{197} = 304.57\text{rpm}$$

$$N_{05} = N_1 \times \frac{Z_3}{Z_4} \times \frac{Z_9}{Z_{10}} = 1250 \times \frac{40}{65} \times \frac{168}{107} = 1207.76\text{rpm}$$

$$N_{06} = N_1 \times \frac{Z_3}{Z_4} \times \frac{Z_{11}}{Z_{12}} = 1250 \times \frac{40}{65} \times \frac{25}{250} = 76.92\text{rpm}$$

$$N_{07} = N_1 \times \frac{Z_5}{Z_6} \times \frac{Z_7}{Z_8} = 1250 \times \frac{25}{80} \times \frac{78}{197} = 154.66\text{rpm}$$

$$N_{08} = N_1 \times \frac{Z_5}{Z_6} \times \frac{Z_9}{Z_{10}} = 1250 \times \frac{25}{80} \times \frac{168}{107} = 613.32\text{rpm}$$

$$N_{09} = N_1 \times \frac{Z_5}{Z_6} \times \frac{Z_{11}}{Z_{12}} = 1250 \times \frac{25}{80} \times \frac{25}{250} = 39.06\text{rpm}$$

Step 7: Calculation of % Deviation:

Sl. No	Obtainable speed ($N_{obt,}\, rpm$)	Calculated speed ($N_{cal,}\, rpm$)	% deviation= $\frac{N_{obt}-N_{cal}}{N_{cal}}$x100
1	39.6	31.5	25.71
2	50	50	0
3	76.92	80	-3.85
4	154.66	125	23.72
5	197.96	200	-1.02
6	304.97	315	-3.18
7	613.32	500	22.64
8	785.05	800	-1.868
9	1207.76	1250	-3.38

14. Design of 12 speed gear box for a lathe. The minimum and maximum speeds are 100 and 1200 rpm. Power is 5 KW from 1440 rpm induction motor.
(Nov/Dec 2017)

Given data:

$n = 12$ speeds
$N\min = 100$rpm.
$N\max = 1200$ rpm.
$P = 5$KW
$N_{input} = 1440$rpm.

****** Similar problem. We have to change the speed range of 100 to 355 rpm and also solved in April/May 2017***

Step 1: Selection of spindle speeds.

$$\frac{N_{max}}{N_{min}} = \phi^{n-1}$$

$$\frac{1200}{100} = \phi^{12-1}$$

$$\phi = 1.25 .$$

Therefore the spindle speeds from R10 series.

From PSGDB 7.20.

100, 125, 160, 200, 250, 315, 400, 500, 630, 800, 1000 and 1200rpm.

Step 2: To find the structural formula.

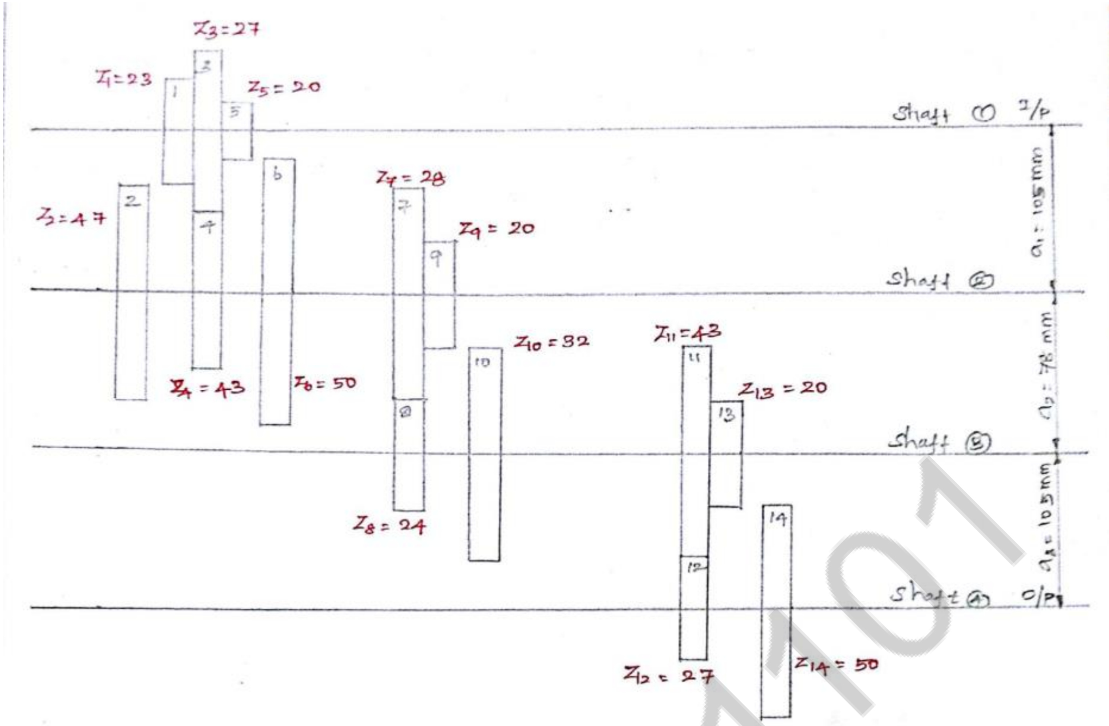
12 speeds = 3(1) 2(3) 2(6)

Step 3: Kinematic diagram for 12 speeds.

Structural formula = 3(1) 2(3) 2(6)

No. of shafts = No. of stages +1 = 3+1=4 (4 horizontal lines)

No. of gears = 2(P_1+P_2+P_3) =2(3+2+2) = 14 gears.



Step 3: Ray diagram for 12 speed.

Structural formula = 3(1) 2(3) 2(6)

No. of shafts = 4 (4 vertical lines)

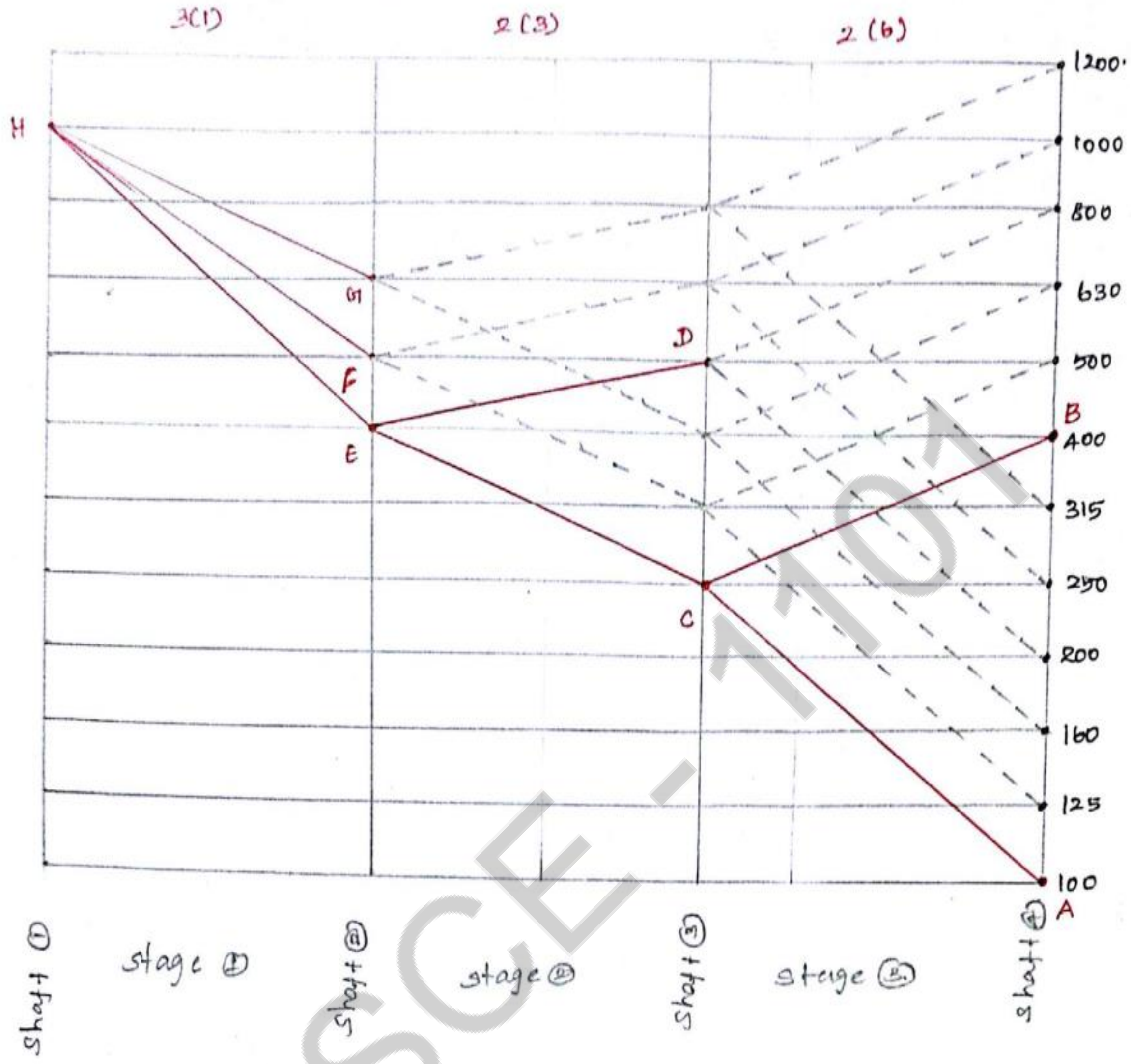
Speeds = 12 (12 horizontal lines)

For stage 3:

$$\frac{N_{min}}{N_{I/P}} \geq 1/4 \qquad \frac{N_{max}}{N_{I/P}} \leq 2$$

$$\frac{100}{250} = 0.4 \geq 1/4 \qquad \frac{400}{250} = 1.6 \leq 2$$

$$N_{I/P} = 250 \text{ rpm}.$$



For stage 2:

$$\frac{N_{min}}{N_{I/P}} \geq 1/4 \qquad \frac{N_{max}}{N_{I/P}} \leq 2$$

$$\frac{250}{400} = 0.625 \geq 1/4 \qquad \frac{500}{400} = 1.25 \leq 2$$

$$N_{I/P} = 400 \text{ rpm}.$$

For stage 1:

$$\frac{N_{min}}{N_{I/P}} \geq 1/4 \qquad \frac{N_{max}}{N_{I/P}} \leq 2$$

$$\frac{400}{1000} = 0.4 \geq \frac{1}{4} \qquad \frac{630}{1000} = 0.63 \leq 2$$

$$\therefore \; N_{I/P} = 1000 \text{ rpm} .$$

Step 4: Calculation of no. of teeth on all the gears.

Let $Z_1, Z_2, Z_3 \ldots Z_{14}$ = No. of teeth of the gears 1, 2, 3.... 14 respectively.

$N_1, N_2, N_3 \ldots N_{14}$ = No. of speed of the gears 1, 2, 3.... 14 respectively.

We know that, $\frac{Z_1}{Z_2} = \frac{N_2}{N_1}$

Case 1: Consider stage 3

First Pair:

* Gears 13 and 14 , Ray CA
* Speed reduction from 250 to 100 rpm.

$$\frac{Z_{13}}{Z_{14}} = \frac{N_{14}}{N_{13}} \quad , \quad Z_{13} = 20 \;\; (\text{driver})$$

$$\frac{20}{Z_{14}} = \frac{100}{250}$$

$$Z_{14} = 50$$

$$Z_{13} = 20 \;\; , \;\; Z_{14} = 50$$

Second Pair:

* Gears 11 and 12 , Ray CB
* Speed increase from 250 to 400 rpm.

$$\therefore \; \frac{Z_{11}}{Z_{12}} = \frac{N_{12}}{N_{11}}$$

$$\frac{Z_{11}}{Z_{12}} = \frac{400}{250}$$

$$Z_{11} = 1.6\, Z_{12}$$

$$\text{W.K.T.} \qquad Z_{13} + Z_{14} = 70 = Z_{11} + Z_{12}$$

$$\therefore \; 1.6 Z_{12} + Z_{12} = 70$$

$$Z_{12} = 26.92 \approx 27$$

$$Z_{11} = 43$$

$$Z_{12} = 27 \;\;,\;\; Z_{11} = 43$$

Case 2: consider stage 2:

First Pair:

- Gears 9 and 10, Ray EC
- Speed reduction from 400 to 250 rpm.

$$\frac{Z_9}{Z_{10}} = \frac{N_{10}}{N_9} \qquad\qquad Z_9 = 20 \quad (\text{driver})$$

$$\frac{20}{40} = \frac{250}{400}$$

$$Z_{10} = 32$$

$$Z_9 = 20 \qquad , \qquad Z_{10} = 32$$

Second Pair:

- Gears 7 and 8, Ray ED
- Speed increase 400 to 500 rpm.

$$\frac{Z_7}{Z_8} = \frac{N_8}{N_7}$$

$$\frac{Z_7}{Z_8} = \frac{500}{400}$$

$$Z_7 = 1.25Z_8$$

W.K.T. $Z_9 + Z_{10} = 52 = Z_7 + Z_8$

$$1.25Z_8 + Z_9 = 52$$

$$Z_8 = 23.11 \approx 24$$

$$Z_7 = 28$$

$$Z_7 = 28 \qquad , \qquad Z_8 = 24$$

Case 3: consider stage 1:

First Pair:

* Gears 5 and 6, Ray HE
* Speed reduction 1000 to 400 rpm.

$$\frac{Z_5}{Z_6} = \frac{N_6}{N_5}$$

$$\frac{20}{Z_6} = \frac{400}{1000} \qquad Z_5 = 20 \ \text{(driver)}$$

$$Z_6 = 50$$

$$Z_5 = 20 \qquad , \qquad Z_6 = 50$$

Second Pair:

* Gears 1 and 2, Ray HF
* Speed reduction from 1000 to 500 rpm.

$$\frac{Z_1}{Z_2} = \frac{N_2}{N_1}$$

$$\frac{Z_1}{Z_2} = \frac{500}{1000}$$

$$Z_1 = 0.5Z_2$$

W. K. T. $\quad Z_5 + Z_6 = 70 = Z_1 + Z_2$

$$1.5Z_2 = 70$$

$$Z_2 = 46.7 \approx 47$$

$$Z_1 = 23$$

$$Z_1 = 23 \qquad , \qquad Z_2 = 47$$

Third Pair:

* Gears 3 and 4, Ray HG
* Speed reduction from 1000 to 630 rpm.

$$\frac{Z_3}{Z_4} = \frac{N_4}{N_3}$$

$$\frac{Z_3}{Z_4} = \frac{630}{1000}$$

$$Z_3 = 0.63Z_4$$

W. K. T. $\quad Z_3 + Z_4 = Z_5 + Z_6 = 70$

$$1.63Z_4 = 70$$

$$Z_4 = 42.94 \approx 43$$

$$Z_3 = 27.$$

$$Z_3 = 27 \quad , \quad Z_4 = 43$$

Step 5: Selection of material,

40N: 2cr 1MO 28 (Hardened and tempered) material is selected.

Material constant M=100 , $[\tau] = 55 N / mm^2$

Step 6: Calculation of module (m)

Case 1: To find the torque (T)

Calculate the torque for the gear (14) has the lowest speed of 100 rpm, using the relation.

$$T_{14} = \frac{60P}{2\pi N}$$

$$= \frac{60 \times 5 \times 10^3}{2 \times \pi \times 100}$$

$$T_{14} = 477.46 \text{ Nm.}$$

Case 2: To find the tangential force on gear 14.

From PSGDB 8.57 , table 46.

$$F_{t14} = \frac{T}{r} = \frac{2T_{14}}{Z_{14} \times m}$$

$$= \frac{2 \times 477.46 \times 10^3}{50 \times m}$$

$$F_{t14} = \frac{19098.4}{m}$$

Case 3: To find the module (m).

$$m = \sqrt{\frac{F_{t14}}{\varphi m.m}}$$

Where , $\varphi_m = b/m = 10$ From PSGDB 8.1 , and 8.14 (table 12).

$$m = \sqrt{\frac{19098.4/m}{10 \times 100}}$$

$$m^2 = \frac{19.098}{m}$$

$$m = 2.67 \text{ mm.}$$

From PSGDB 8.2 , table 1, choice 1.

The next nearest higher standard module

$$m = 3mm.$$

Step 7: Calculation of centre distance in all stages. From PSGDB 8.22 , table 26.

$$a = \left(\frac{Z_x + Z_y}{2}\right)m$$

Z_x and Z_y No. of teeth on the gear pair in engagement is each stage.

Case 1: Centre distance for stage 1.

$$a_1 = \left(\frac{Z_3 + Z_4}{2}\right)m$$

$$= \left(\frac{27 + 43}{2}\right)3$$

$$a_1 = 105mm.$$

Case 2: Centre distance for stage 2.

$$a_2 = \left(\frac{Z_9 + Z_8}{2}\right)m$$

$$=\left(\frac{28+24}{2}\right)3$$

$$a_2 = 78mm.$$

Case 3: Centre distance for stage 3.

$$a_3 = \left(\frac{Z_{11}+Z_{12}}{2}\right)m$$

$$=\left(\frac{43+27}{2}\right)3$$

$$a_1 = 105mm.$$

Step 8: Calculation of Face width (b).

$$W.K.T \Rightarrow b = 10\times m$$

$$= 10\times 3$$

$$b = 30mm$$

Step 9: Calculation of Length of the shafts.

$$L = 25+10+7b+20+4b+20+4b+10+25$$

$$= 110+15b$$

$$= 110+15\times 30$$

$$L = 450 \text{ mm}$$

Step 10: Design of shafts.

Case 1: Design of spindle (or) output shafts.

(v) To find normal load on gear 14 (F_n)

$$F_n = \frac{F_{t14}}{\cos\alpha} \quad [\alpha = 20°FD]$$

$$= \frac{6366.13}{\cos 20}$$

$$F_n = 6774.7 \text{ N}$$

(vi) To find maximum bending moment (M).

$$M = \frac{(F_n.L)}{4}$$

$$= \frac{6774.7 \times 450}{4}$$

$$M = 7.62 \times 10^5 \text{ Nmm.}$$

(vii) To find the equivalent torque. (T_{eq})

$$T_{eq} = \sqrt{M^2 + T_4^2}$$

$$= \sqrt{(7.62 \times 10^5)^2 + (477.46 \times 10^3)^2}$$

$$= 8.99 \times 10^5 \text{ Nmm}$$

(viii) To find the diameter of the spindle (d_s)

$$d_s = \sqrt[3]{\frac{16T_{eq}}{\pi[\tau]}}$$

$$= \sqrt[3]{\frac{16 \times 8.99 \times 10^5}{\pi \times 55}}$$

$$d_s = 43.66 \text{ mm}$$

From R_{10} series, The standard diameter.

$$d_s = 50\text{mm} \ .$$

Case 2: Design of other shafts.

(d) Diameter of shaft 1.

Input speed = 1000 rpm.

$$\text{Torque } T = \frac{60P}{2\pi N}$$

$$= \frac{60 \times 5 \times 10^3}{2 \times \pi \times 1000}$$

$$= 47.75 \text{ Nm} \ .$$

$$\text{W.K.T} \quad T = 0.2 d_1^3 [\tau].$$

$$47.75\times10^3 = 0.2\times d_{s1}^3\times55$$

$$d_{s1} = 16.31 \text{ mm.}$$

From R_{10} series., The standard diameter d_{s1}=20 mm.

(e) Diameter of shaft 2.

Input speed = 400 rpm.

$$\therefore\ T = \frac{60\times5\times10^3}{2\times\pi\times400}$$

$$= 119.36 \text{ Nm}\ .$$

$$\text{W.K.T} \Rightarrow\quad T = 0.2d_2^3[\tau].$$

$$119.36\times10^3 = 0.2\times d_{s2}^3[55]$$

$$d_{s2} = 22.14 \text{ mm.}$$

From R_{10} series., The standard diameter d_{s2}=25 mm.

(f) Diameter of shaft 3.

Input speed = 250 rpm.

$$\therefore\ T = \frac{60\times5\times10^3}{2\times\pi\times250}$$

$$T = 190.98 \text{ Nm}\ .$$

$$\text{W.K.T} \Rightarrow\quad T = 0.2d_3^3[\tau].$$

$$190.98\times10^3 = 0.2\times d_{s3}^3[55]$$

$$d_{s3} = 25.89 \text{ mm.}$$

From R_{10} series. The standard diameter d_{s3}=31.5 mm.

15. Design of 12 speed gear box for a lathe. The minimum and maximum speeds are 100 and 1200 rpm. Power is 5 KW from 1440 rpm induction motor. (April/May 2018)

Given data:

$n = 12$ speeds

$N_{min} = 100rpm.$

$N_{max} = 1200 rpm.$

$P = 5KW$

$N_{input} = 1440rpm.$

Step 1: Selection of spindle speeds.

$$\frac{N_{max}}{N_{min}} = \phi^{n-1}$$

$$\frac{1200}{100} = \phi^{12-1}$$

$$\phi = 1.25 .$$

Therefore the spindle speeds from R10 series.

From PSGDB 7.20.

100, 125, 160, 200, 250, 315, 400, 500, 630, 800, 1000 and 1200rpm.

Step 2: To find the structural formula.

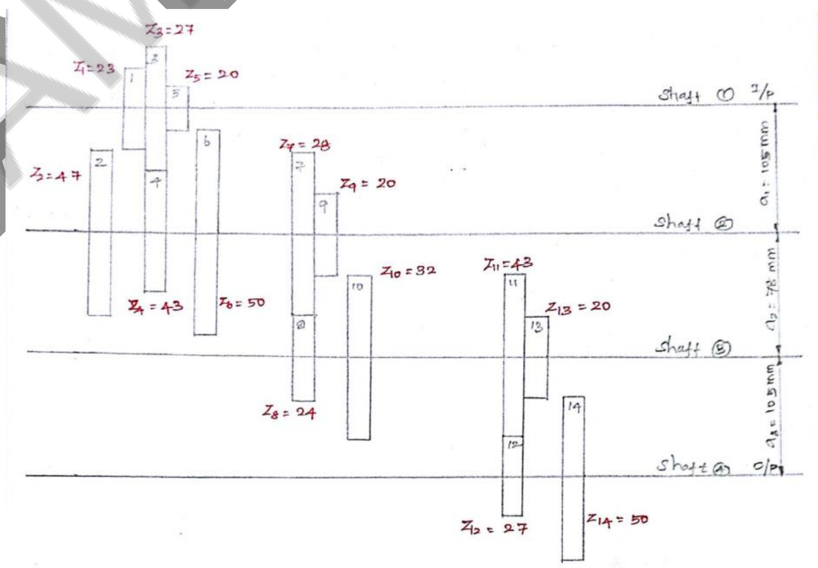
12 speeds = 3(1) 2(3) 2(6)

Step 3: Kinematic diagram for 12 speeds.

Structural formula = 3(1) 2(3) 2(6)

No. of shafts = No. of stages +1 = 3+1=4 (4 horizontal lines)

No. of gears = $2(P_1+P_2+P_3)$ =2(3+2+2) = 14 gears.



Step 3: Ray diagram for 12 speed.

Structural formula = 3(1) 2(3) 2(6)

No. of shafts = 4 (4 vertical lines)

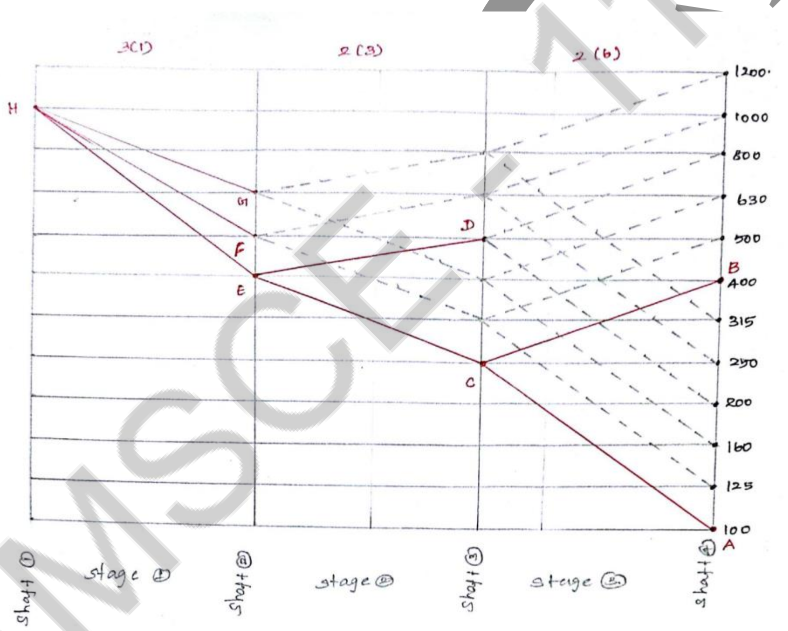
Speeds = 12 (12 horizontal lines)

For stage 3:

$$\frac{N_{min}}{N_{I/P}} \geq 1/4 \qquad \frac{N_{max}}{N_{I/P}} \leq 2$$

$$\frac{100}{250} = 0.4 \geq 1/4 \qquad \frac{400}{250} = 1.6 \leq 2$$

$$N_{I/P} = 250 \text{ rpm}.$$



For stage 2:

$$\frac{N_{min}}{N_{I/P}} \geq 1/4 \qquad \frac{N_{max}}{N_{I/P}} \leq 2$$

$$\frac{250}{400} = 0.625 \geq 1/4 \qquad \frac{500}{400} = 1.25 \leq 2$$

$$N_{I/P} = 400 \text{ rpm}.$$

For stage 1:

$$\frac{N_{min}}{N_{I/P}} \geq 1/4 \qquad \frac{N_{max}}{N_{I/P}} \leq 2$$

$$\frac{400}{1000} = 0.4 \geq 1/4 \qquad \frac{630}{1000} = 0.63 \leq 2$$

$$\therefore \; N_{I/P} = 1000 \text{ rpm} .$$

Step 4: Calculation of no. of teeth on all the gears.

Let $Z_1, Z_2, Z_3 \ldots Z_{14}$ = No. of teeth of the gears 1, 2, 3…. 14 respectively.

$N_1, N_2, N_3 \ldots N_{14}$ = No. of speed of the gears 1, 2, 3…. 14 respectively.

We know that, $\frac{Z_1}{Z_2} = \frac{N_2}{N_1}$

Case 1: Consider stage 3

First Pair:

* Gears 13 and 14 , Ray CA
* Speed reduction from 250 to 100 rpm.

$$\frac{Z_{13}}{Z_{14}} = \frac{N_{14}}{N_{13}} \quad , \quad Z_{13} = 20 \;\; (\text{driver})$$

$$\frac{20}{Z_{14}} = \frac{100}{250}$$

$$Z_{14} = 50$$

$$Z_{13} = 20 \;\; , \; Z_{14} = 50$$

Second Pair:

* Gears 11 and 12 , Ray CB
* Speed increase from 250 to 400 rpm.

$$\therefore \; \frac{Z_{11}}{Z_{12}} = \frac{N_{12}}{N_{11}}$$

$$\frac{Z_{11}}{Z_{12}} = \frac{400}{250}$$

$$Z_{11} = 1.6 \, Z_{12}$$

W.K.T. $Z_{13} + Z_{14} = 70 = Z_{11} + Z_{12}$

$$\therefore\ 1.6Z_{12} + Z_{12} = 70$$

$$Z_{12} = 26.92 \approx 27$$

$$Z_{11} = 43$$

$$Z_{12} = 27\ ,\ Z_{11} = 43$$

Case 2: consider stage 2:

First Pair:

* Gears 9 and 10, Ray EC
* Speed reduction from 400 to 250 rpm.

$$\frac{Z_9}{Z_{10}} = \frac{N_{10}}{N_9} \qquad Z_9 = 20\ \ (\text{driver})$$

$$\frac{20}{40} = \frac{250}{400}$$

$$Z_{10} = 32$$

$$Z_9 = 20 \quad , \quad Z_{10} = 32$$

Second Pair:

* Gears 7 and 8, Ray ED
* Speed increase 400 to 500 rpm.

$$\frac{Z_7}{Z_8} = \frac{N_8}{N_7}$$

$$\frac{Z_7}{Z_8} = \frac{500}{400}$$

$$Z_7 = 1.25Z_8$$

W.K.T. $Z_9 + Z_{10} = 52 = Z_7 + Z_8$

$$1.25Z_8 + Z_9 = 52$$

$$Z_8 = 23.11 \approx 24$$

$$Z_7 = 28$$

$Z_7 = 28 \quad , \quad Z_8 = 24$

Case 3: consider stage 1:

First Pair:

- Gears 5 and 6, Ray HE
- Speed reduction 1000 to 400 rpm.

$$\frac{Z_5}{Z_6} = \frac{N_6}{N_5}$$

$$\frac{20}{Z_6} = \frac{400}{1000} \qquad Z_5 = 20 \ \ (\text{driver})$$

$$Z_6 = 50$$

$$Z_5 = 20 \quad , \quad Z_6 = 50$$

Second Pair:

- Gears 1 and 2, Ray HF
- Speed reduction from 1000 to 500 rpm.

$$\frac{Z_1}{Z_2} = \frac{N_2}{N_1}$$

$$\frac{Z_1}{Z_2} = \frac{500}{1000}$$

$$Z_1 = 0.5Z_2$$

W. K. T. $\quad Z_5 + Z_6 = 70 = Z_1 + Z_2$

$$1.5Z_2 = 70$$

$$Z_2 = 46.7 \approx 47$$

$$Z_1 = 23$$

$$Z_1 = 23 \quad , \quad Z_2 = 47$$

Third Pair:

- Gears 3 and 4, Ray HG
- Speed reduction from 1000 to 630 rpm.

$$\frac{Z_3}{Z_4} = \frac{N_4}{N_3}$$

$$\frac{Z_3}{Z_4} = \frac{630}{1000}$$

$$Z_3 = 0.63Z_4$$

W. K. T. $Z_3 + Z_4 = Z_5 + Z_6 = 70$

$$1.63Z_4 = 70$$

$$Z_4 = 42.94 \approx 43$$

$$Z_3 = 27.$$

$$Z_3 = 27 \quad , \quad Z_4 = 43$$

Step 5: Selection of material,

40N: 2cr 1MO 28 (Hardened and tempered) material is selected.

Material constant M=100 , $[\tau] = 55N/mm^2$

Step 6: Calculation of module (m)

Case 1: To find the torque (T)

Calculate the torque for the gear (14) has the lowest speed of 100 rpm, using the relation.

$$T_{14} = \frac{60P}{2\pi N}$$

$$= \frac{60 \times 5 \times 10^3}{2 \times \pi \times 100}$$

$$T_{14} = 477.46 \text{ Nm.}$$

Case 2: To find the tangential force on gear 14.

From PSGDB 8.57 , table 46.

$$F_{t14} = \frac{T}{r} = \frac{2T_{14}}{Z_{14} \times m}$$

$$= \frac{2 \times 477.46 \times 10^3}{50 \times m}$$

$$F_{t14} = \frac{19098.4}{m}$$

Case 3: To find the module (m).

$$m = \sqrt{\frac{F_{t14}}{\varphi m.m}}$$

Where , $\varphi_m = b/m = 10$ From PSGDB 8.1 , and 8.14 (table 12).

$$m = \sqrt{\frac{19098.4/m}{10 \times 100}}$$

$$m^2 = \frac{19.098}{m}$$

$$m = 2.67 \text{ mm.}$$

From PSGDB 8.2 , table 1, choice 1.

The next nearest higher standard module

$$m = 3\text{mm}.$$

Step 7: Calculation of centre distance in all stages. From PSGDB 8.22 , table 26.

$$a = \left(\frac{Z_x + Z_y}{2}\right)m$$

Z_x and Z_y No. of teeth on the gear pair in engagement is each stage.

Case 1: Centre distance for stage 1.

$$a_1 = \left(\frac{Z_3 + Z_4}{2}\right)m$$

$$= \left(\frac{27 + 43}{2}\right)3$$

$$a_1 = 105\text{mm}.$$

Case 2: Centre distance for stage 2.

$$a_2 = \left(\frac{Z_9 + Z_8}{2}\right)m$$

$$= \left(\frac{28+24}{2}\right)3$$

$$a_2 = 78mm.$$

Case 3: Centre distance for stage 3.

$$a_3 = \left(\frac{Z_{11} + Z_{12}}{2}\right)m$$

$$= \left(\frac{43+27}{2}\right)3$$

$$a_1 = 105mm.$$

Step 8: Calculation of Face width (b).

$$W.K.T \Rightarrow b = 10 \times m$$

$$= 10 \times 3$$

$$b = 30mm$$

Step 9: Calculation of Length of the shafts.

$$L = 25 + 10 + 7b + 20 + 4b + 20 + 4b + 10 + 25$$

$$= 110 + 15b$$

$$= 110 + 15 \times 30$$

$$L = 450 \text{ mm}$$

Step 10: Design of shafts.

Case 1: Design of spindle (or) output shafts.

(ix) To find normal load on gear 14 (F_n)

$$F_n = \frac{F_{t14}}{\cos\alpha} \quad [\alpha = 20°FD]$$

$$= \frac{6366.13}{\cos 20}$$

$$F_n = 6774.7 \text{ N}$$

(x) To find maximum bending moment (M).

$$M = \frac{(F_n.L)}{4}$$

$$= \frac{6774.7 \times 450}{4}$$

$$M = 7.62 \times 10^5 \text{ Nmm.}$$

(xi) To find the equivalent torque. (T_{eq})

$$T_{eq} = \sqrt{M^2 + T_4^2}$$

$$= \sqrt{(7.62 \times 10^5)^2 + (477.46 \times 10^3)^2}$$

$$= 8.99 \times 10^5 \text{ Nmm}$$

(xii) To find the diameter of the spindle (d_s)

$$d_s = \sqrt[3]{\frac{16 T_{eq}}{\pi[\tau]}}$$

$$= \sqrt[3]{\frac{16 \times 8.99 \times 10^5}{\pi \times 55}}$$

$$d_s = 43.66 \text{ mm}$$

From R_{10} series, The standard diameter.

$$d_s = 50\text{mm} .$$

Case 2: Design of other shafts.

(g) Diameter of shaft 1.

Input speed = 1000 rpm.

$$\text{Torque } T = \frac{60P}{2\pi N}$$

$$= \frac{60 \times 5 \times 10^3}{2 \times \pi \times 1000}$$

$$= 47.75 \text{ Nm} .$$

W.K.T $T = 0.2d_1^3[\tau]$.

$47.75\times10^3 = 0.2\times d_{s1}^3\times55$

$d_{s1} = 16.31$ mm.

From R_{10} series., The standard diameter d_{s1}=20 mm.

(h) Diameter of shaft 2.

Input speed = 400 rpm.

$\therefore\ T = \dfrac{60\times5\times10^3}{2\times\pi\times400}$

$= 119.36$ Nm .

W.K.T $\Rightarrow$ $T = 0.2d_2^3[\tau]$.

$119.36\times10^3 = 0.2\times d_{s2}^3[55]$

$d_{s2} = 22.14$ mm.

From R_{10} series., The standard diameter d_{s2}=25 mm.

(i) Diameter of shaft 3.

Input speed = 250 rpm.

$\therefore\ T = \dfrac{60\times5\times10^3}{2\times\pi\times250}$

$T = 190.98$ Nm .

W.K.T $\Rightarrow$ $T = 0.2d_3^3[\tau]$.

$190.98\times10^3 = 0.2\times d_{s3}^3[55]$

$d_{s3} = 25.89$ mm.

From R_{10} series. The standard diameter d_{s3}=31.5 mm.

16. A gear box is to give 18 speeds for a spindle of a milling machine. Maximum and minimum speeds of the spindle are to be around 650 and 35 rpm respectively. Find the speed ratios which will give the desired speeds and draw the structural diagram and kinematic arrangement of the drive. (April/May 2018)

Given data:

$$n = 18$$
$$N_{min} = 35rpm$$
$$N_{max} = 650rpm$$

Step 1: Selection of Spindle speeds

Determine the progression ratio (ϕ) using the relation

$$N_{max}/N_{min} = \phi^{n-1}$$

$$650/35 = \phi^{18-1}$$

$$\phi = (18.571)^{1/17}$$

$$\phi = 1.87$$

We find $\phi = 1.87$ is not a standard ratio. So let us find out whether multiples of standard ratio 1.12 OR 1.06 come close to 1.87

For example we can write $1.12 \times 1.12 = 1.2544$

Then $1.06 \times (1.06 \times 1.06) = 1.91$ ... Skip 2 speeds

So we take $\phi = 1.06$, because satisfies the requirement. Select the standard spindle speeds using the series of preferred numbers From PSGDB 7.20, 7.19

Step ratio from R40 series $\phi = 1.06$

$\therefore$ Spindle speeds are 35.5, 42.5, 50, 60, 71, 85, 100, 118, 140, 170, 200, 236, 280, 335, 400, 475, 560 and 670 rpm

Step 2: To find the Structural Formulae

Structural formulae: $2(1)3(2)3(6)$

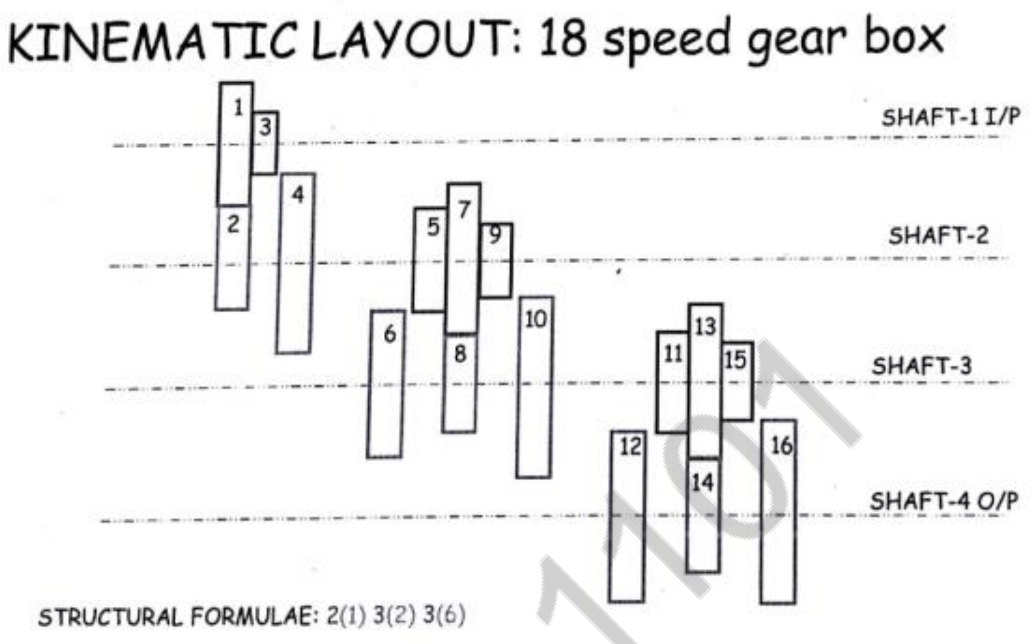
Step 3: Construct the Kinematic arrangement for 18 speed gear box

Structural formulae: $2(1)3(2)3(6)$

No. of shafts = No. of stages + 1 (3 + 1 = 4 shafts) (so draw 4 horizontal lines)

To find the no. of gears by using

No. of gears $= 2(p_1 + p_2 + p_3)\{[2(2+3+3)] = 16\text{gears}\}$



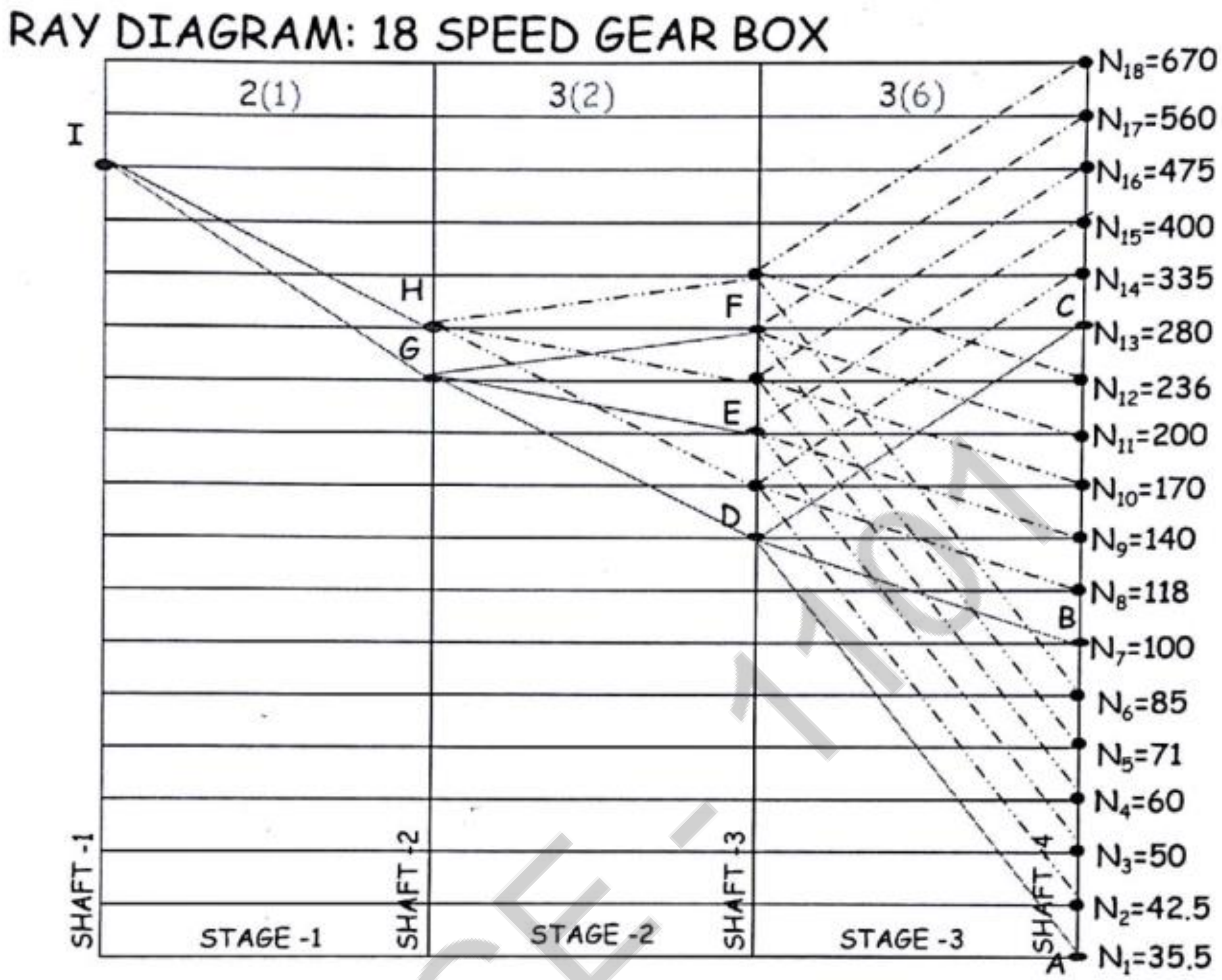
Step 4: Construct the ray diagram for 18 speed gear box

Structural formulae: $2(1)3(2)3(6)$

Note: Where $X_1 = 1$ $X_2 = p_1 = 2$ $X_3 = p_1 \cdot p_2 = 2 \times 3 = 6$

No. of shafts = No. of stages +1 (3 + 1 = 4 shafts) (so draw 4 vertical lines)

No. of speeds = 18 (Draw 18 horizontal lines)



17. A nine-speed gear box used as a headstock gear box of a turret lathe is to provide a speed range of 18 rpm to 1800 rpm. Using standard step ratio draw the speed diagram, and the kinematic layout. Also find and fix the number of teeth on all the gears. (Nov/Dec 2018)

Given data:

$$n = 9$$
$$N_{min} = 180 rpm$$
$$N_{max} = 1800 rpm$$

Step 1:- selection of spindle speeds

Determine the progression ratio (ϕ) using the relation

$$\frac{N_{max}}{N_{min}} = \phi^{n-1}$$
$$\frac{1800}{180} = \phi^{9-1}$$
$$\phi = (10)^{1/8}$$
$$\phi = 1.333$$

- ✓ We find ϕ =1.333 is not a standard ratio. So let us find out whether multiplies of standard ratio 1.12 or 1.06 come close to 1.333
- ✓ For example we can write, $1.12\times1.12=1.2544$ & $1.12\times1.12\times1.12=1.405$

Then $1.06\times1.06\times1.06\times1.06=1.338$ skip 4 speeds

So we take ϕ= 1.06, because satisfies the requirement, select the standard spindle speeds using the series of preferred numbers

Take Step Ratio from R40 series ϕ=1.06

Spindle Speeds are 180, 236, 315, 425, 560, 750, 1000, 1320 and 1800rpm

Step 2: To find the structural formulae

Structural formulae: 3(1) 3(3)

Step 3: Construct the kinematic arrangement for 9 speed gear box

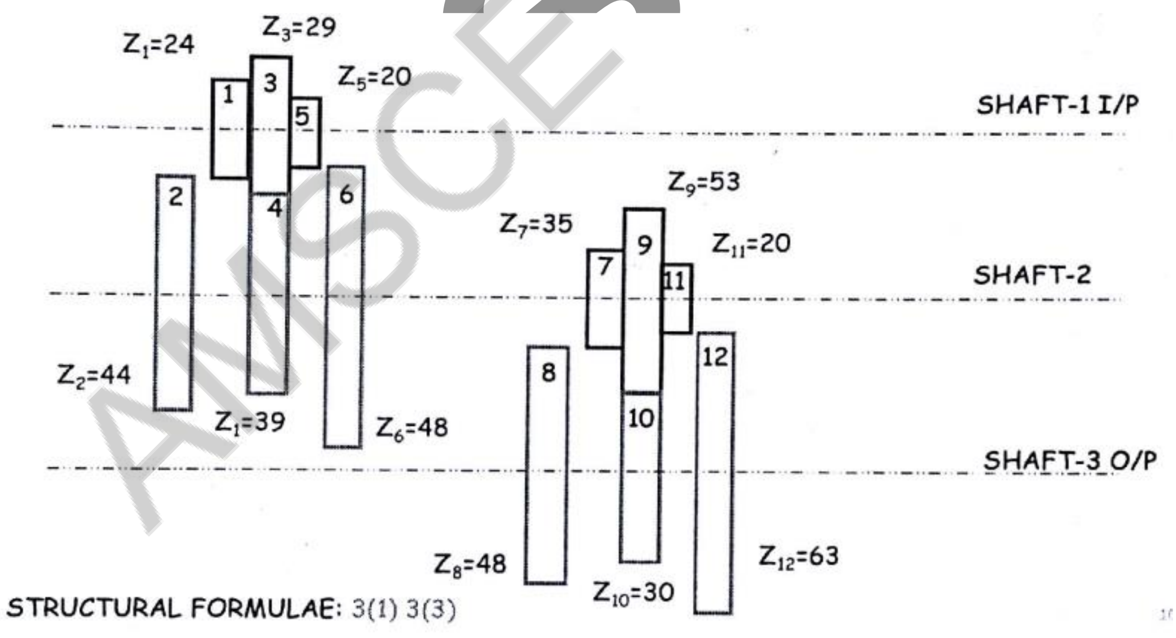
Structural formulae: 3(1) 3(3)

P1 = 3 p2 = 3 Note: where X_1 = 1; $X_2 = p_1$ = 3

No. of shafts = No. of stages +1 (2+1=3 shafts) (so draw 3 horizontal lines)

To find the no. of gears by using

No. of gears $=2(p_1+p_2)\{[2(3+3)]=12\,\text{gears}\}$



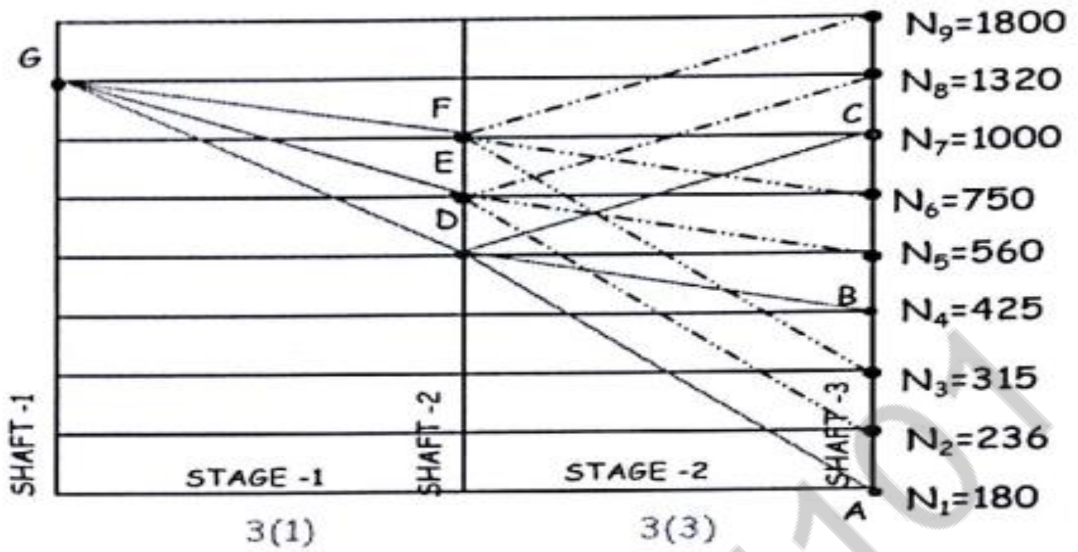
Step 4:- Construct the ray diagram for 9 speed gear box

- ➢ Structural formulae: 3(1) 3(3)

No .of stages: $2\{(p_1(X_1).p_2(X_2)\}$

$p_1=3$ $p_2=$ Note: Where X_1 = $X_2 = p_1$ = 3

- ✓ No. of shafts = No. of stages +1 (2+1= 3 shafts) (so draw 3 vertical lines)
- ✓ No. of speeds = 9 (Draw 9 horizontal lines)



Step 5: Calculation of No. of teeth

- Calculation of numbers of teeth on all the gears

Let Z_1, Z_2, Z_3, ...Z_{12} = Number of teeth of the gears 1, 2, 3, ...12 respectively

Formulae given $\frac{z_1}{z_2} = \frac{N_2}{N_1}$

Take stage – 2

- Consider the first pair of gear 11 and 12
- From ray diagram consider ray DA
- Maximum speed reduction 560rpm to 180rpm

We know that, $Z_{min} \geq 17$, assume Z_{11} = 20 (driver)

$$\frac{z_{11}}{z_{12}} = \frac{N_{12}}{N_{11}}$$
$$\frac{20}{z_{12}} = \frac{180}{560}$$
$$z_{12} = 62.22 \approx 63$$

Z_{11} = 20, Z_{12} = 63

Take stage – 2

- Consider the second pair of gear 7 and 8
- From ray diagram consider ray DB
- Maximum speed reduction 560rpm to 425rpm

We know that,

$$\frac{z_7}{z_8} = \frac{N_8}{N_7}$$
$$\frac{z_7}{z_8} = \frac{425}{560}$$
$$z_7 = 0.76 z_8 \qquad --(i)$$

NOTE: The centre distance between the shafts are fixed and same. The sum of number of teeth of mating gears should be equal.

So we can write

$$z_7 + z_8 = z_{11} + z_{12} = 20 + 63 = 83 \qquad (ii)$$

Solving equations (i) and (ii), we get

$$z_8 = 47.16 \approx 48$$
$$z_7 = 83 - 48 = 35$$
$$z_7 = 35 \quad z_8 = 48$$

Take stage – 2

- Consider the third pair of gear 9 and 10
- From ray diagram consider ray DC
- Speed increase from 560rpm to 1000rpm

We know that,

$$\frac{z_9}{z_{10}} = \frac{N_{10}}{N_9}$$
$$\frac{z_9}{z_{10}} = \frac{1000}{560}$$
$$Z_9 = 1.786 Z_{10} \qquad --(iii)$$

So we can write

$$Z_9 + Z_{10} = Z_{11} + Z_{12} = 20 + 63 = 83 \qquad --(iv)$$

Solving equation (iii) and (iv), we get

$$Z_{10} = 29.79 \approx 30$$
$$Z_9 = 83 - 30 = 53$$
$$Z_9 = 53 \quad Z_{10} = 30$$

Take stage -1

- Consider the first pair of gear 5 and 6
- From ray diagram consider ray GD

- Maximum speed reduction 1320rpm to 560rpm

We know that, $Z_{min} \geq 17$ $\therefore$ assume Z_5 = 20 (Driver)

$$\frac{z_5}{z_6} = \frac{N_6}{N_5}$$
$$\frac{20}{z_{12}} = \frac{1320}{560}$$
$$z_6 = 47.14 \approx 48$$

Take stage – 1

- Consider the first pair of gear 5 and 6
- From ray diagram consider ray GD
- Maximum speed reduction 1320rpm to 560rpm

We know that,

$$\frac{z_1}{z_2} = \frac{N_2}{N_1}$$
$$\frac{z_1}{z_2} = \frac{750}{1320}$$
$$Z_1 = 0.57z_2 \qquad ---(v)$$

NOTE: The centre distance between the shafts are fixed and same. The sum of number of teeth of mating gears should be equal.

So we can write

$$z_1 + z_2 = z_5 + z_6 = 20 + 48 = 68 \qquad ---(vi)$$

Solving equations (v) and (vi), we get

$$z_2 = 43.3 \approx 44$$
$$z_1 = 68.44 = 24$$
$$Z_1 = 24 \quad Z_2 = 44$$

Take stage – 1

- Consider the third pair of gear 3 and 4
- From ray diagram consider ray GF
- Speed increase from 1320rpm to 1000 rpm

We know that,

$$\frac{z_3}{z_4} = \frac{N_4}{N_3}$$

$$\frac{z_3}{z_4} = \frac{1000}{1320}$$

$$Z_3 = 0.76 z_4 \qquad --(vii)$$

Solving equations (iii) and (iv), we get

$$Z_4 = 38.64 \approx 39$$

$$Z_3 = 68 - 39 = 29$$

$$Z_3 = 29 \quad Z_4 = 39$$

18. Sketch the speed diagram and the kinematic layout for an 18 speed gear box the following data. Motor speed =1440rpm, minimum output speed 16 rpm, maximum output speed= 800rpm, arrangement 2X3X3. List the speeds of all the shafts when the output speed is 16 rpm. (Nov/Dec 2018)

Given data:

$$n = 18$$

$$N_{min} = 35\text{rpm}$$

$$N_{max} = 650\text{rpm}$$

**** Similar to this problem, Change the minimum and maximum speed*

Step 1: Selection of Spindle speeds

Determine the progression ratio (ϕ) using the relation

$$N_{max}/N_{min} = \phi^{n-1}$$

$$650/35 = \phi^{18-1}$$

$$\phi = (18.571)^{1/17}$$

$$\phi = 1.87$$

We find $\phi = 1.87$ is not a standard ratio. So let us find out whether multiples of standard ratio 1.12 OR 1.06 come close to 1.87

For example we can write $1.12 \times 1.12 = 1.2544$

Then $1.06 \times (1.06 \times 1.06) = 1.91$ ... Skip 2 speeds

So we take $\phi = 1.06$, because satisfies the requirement. Select the standard spindle speeds using the series of preferred numbers From PSGDB 7.20, 7.19

Step ratio from R40 series $\phi = 1.06$

$\therefore$ Spindle speeds are 35.5, 42.5, 50, 60, 71, 85, 100, 118, 140, 170, 200, 236, 280, 335, 400, 475, 560 and 670 rpm

Step 2: To find the Structural Formulae

Structural formulae: $2(1)3(2)3(6)$

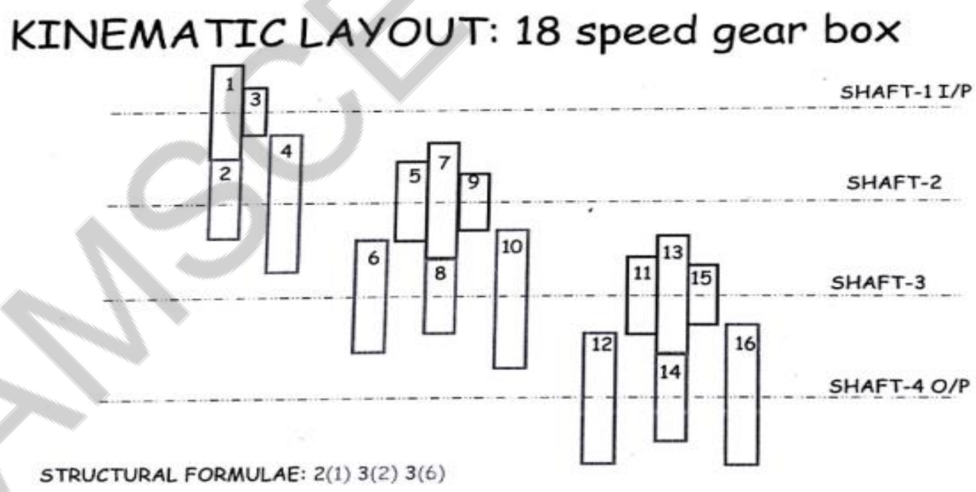
Step 3: Construct the Kinematic arrangement for 18 speed gear box

Structural formulae: $2(1)3(2)3(6)$

No. of shafts = No. of stages + 1 (3 + 1 = 4 shafts) (so draw 4 horizontal lines)

To find the no. of gears by using

No. of gears $= 2(p_1 + p_2 + p_3)\{[2(2+3+3)] = 16\text{gears}\}$



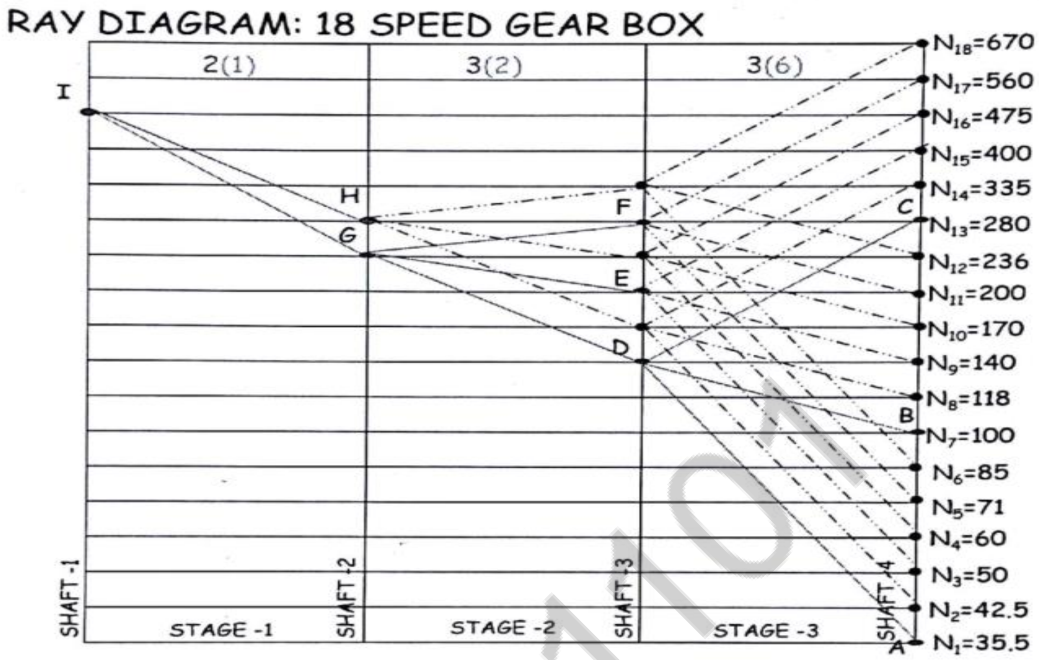
Step 4: Construct the ray diagram for 18 speed gear box

Structural formulae: $2(1)3(2)3(6)$

Note: Where $X_1 = 1$ $X_2 = p_1 = 2$ $X_3 = p_1 \cdot p_2 = 2 \times 3 = 6$

No. of shafts = No. of stages +1 (3 + 1 = 4 shafts) (so draw 4 vertical lines)

No. of speeds = 18 (Draw 18 horizontal lines)



19. Design a gear box with 12 speed from a source of motor with a speed of 1600rpm. The required range is from 160rpm to 2000rpm. (April/ May 2019)

Given data:

n = 12

N_{min} = 25rpm

N_{max} = 1440 rpm

P = 2.25KW

*** Change the Speed range

1. Selection of spindle speeds:

We know that,

$$\phi^{n-1} = \frac{N_{max}}{N_{min}}$$

$$\phi^{12-1} = \frac{600}{25}$$

$$\phi = 1.335$$

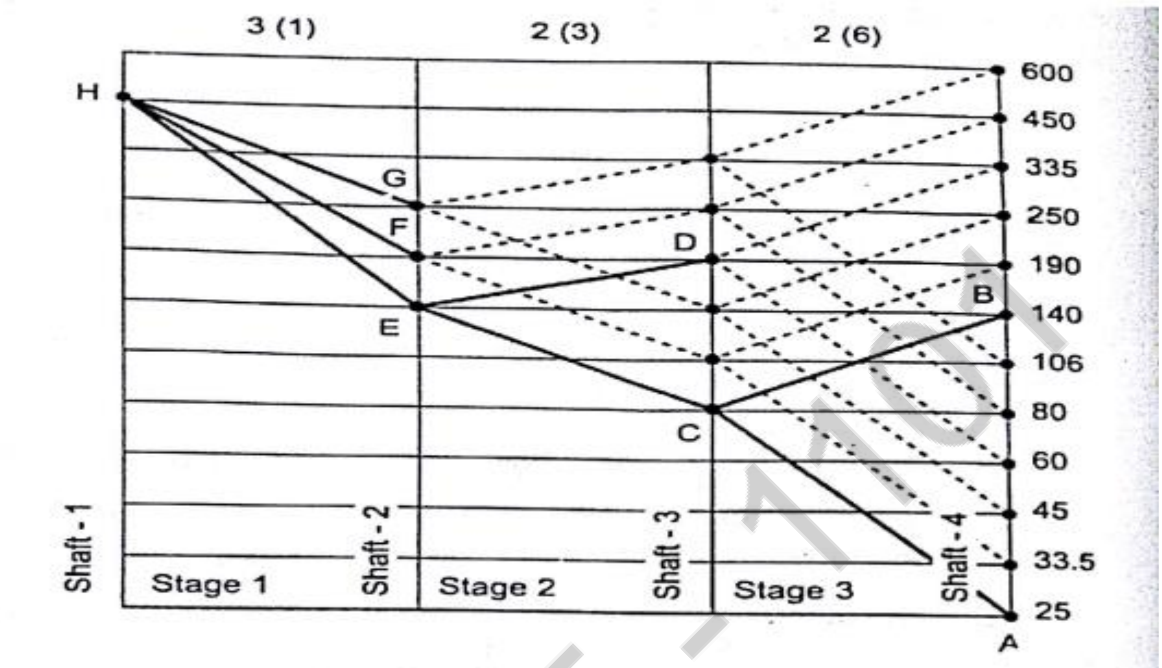
We can write, $1.06 \times (1.6 \times 1.06 \times 1.06 \times 1.06) = 1.338$

So, ϕ = 1.06 satisfies the requirement. Therefore the spindle from R 40 series skipping four speeds, are given as

25, 33.5, 45, 60, 80, 106, 140, 190, 250, 250, 335, 450 and 600 rpm.

2. Ray diagram: The ray diagram is constructed, as shown in fig.

Structural formula: 3(1) 2(3) 2(6)



Ray diagram for 12 speed gear box

Step 3:

$$\frac{N_{min}}{N_{input}} = \frac{25}{80} = 0.31 > \frac{1}{4} \text{ and}$$

$$\frac{N_{max}}{N_{input}} = \frac{140}{80} = 1.75 < 2$$

Step 2:

$$\frac{N_{min}}{N_{input}} = \frac{80}{140} = 0.57 > \frac{1}{4}$$

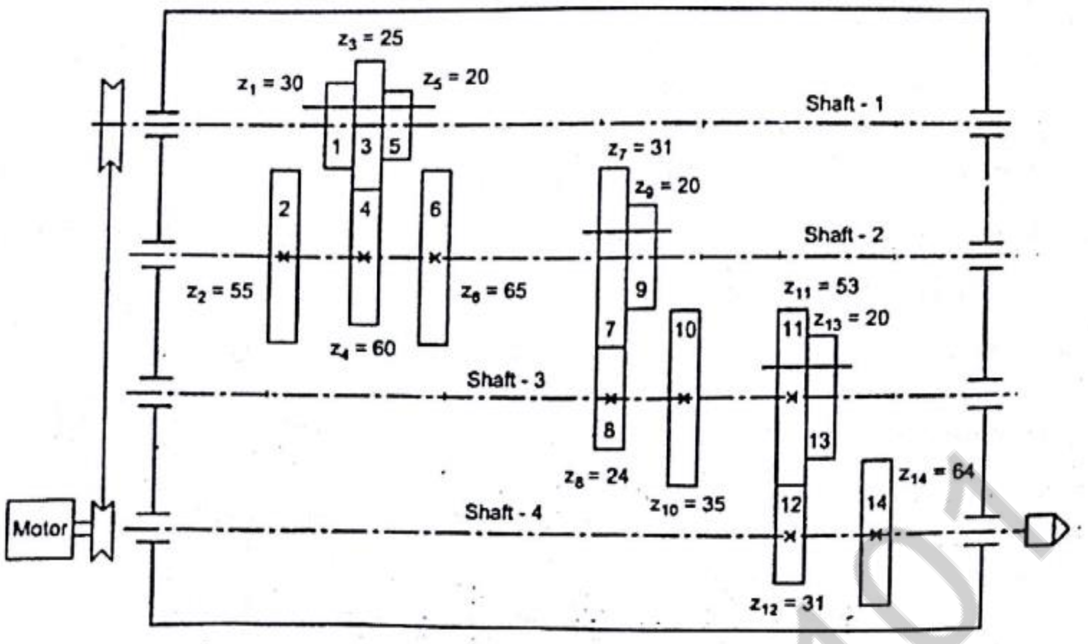
$$\frac{N_{max}}{N_{input}} = \frac{190}{140} = 1.36 < 2$$

Step 1:

$$\frac{N_{min}}{N_{input}} = \frac{140}{450} = 0.311 > \frac{1}{4}$$

$$\frac{N_{max}}{N_{input}} = \frac{250}{450} = 0.56 < 2$$

3. Kinematic arrangement: The kinematic arrangement for the given 12 speed gear box is constructed, as shown in fig.



Kinematic arrangement for 12 speed gear box

4. Calculation of number of teeth on alt gears: The number of teeth on all gears are calculate as below, following the procedure used

Stage 3:

First pair: Consider the ray that gives, maximum reduction i.e, from 80 r. p. m to 25 r. p. m. The corresponding gears are 13 and 14 on shaft 4.

We know that, $Z_{min} \geq 17$. Therefore assume z_{13}= 20 (driver)

$$\frac{z_{13}}{z_{14}} = \frac{N_{14}}{N_{13}} \text{ or } \frac{20}{z_{14}} = \frac{25}{80}; \qquad \therefore z_{14} = 64$$

Second pair: Consider the other ray that gives speed increase form 80 r. p. m. To 140r. p. m. The corresponding gears are 11 and 12.

$$\frac{z_{11}}{z_{12}} = \frac{N_{12}}{N_{11}} = \frac{140}{80} \text{ or } z_{11} = 1.75 z_{12} \qquad --(i)$$

We also know that the sun of number of teeth of mating gears should be equal.

$$z_{11} + z_{12} = z_{13} + z_{14} = 20 + 64 = 84 \qquad --(ii)$$

On solving equations (i) and (ii), we get

$$z_{12} = 30.5 \approx 31 \text{ and } z_{11} = 84 - 31 = 53$$

Stage 2:

First pair: Consider the ray that gives maximum reduction from 140 r.p.m to 8 r.p.m. The corresponding gears are 9 and 10. Assume z_9 = 20 (driver).

$$\frac{z_9}{z_{10}} = \frac{N_{10}}{N_9} \text{ or } \frac{20}{z_{10}} = \frac{80}{140}; \qquad z_{10} = 35$$

Second pair: Consider the other ray that gives speed increase from 140 r.p.m to 190 r.p.m. The corresponding gears are 7 and 8.

$$\frac{z_7}{z_8} = \frac{N_8}{N_7} = \frac{190}{140} \text{ or } \quad z_7 = 1.357 z_8 \qquad --(iii)$$

$$z_7 + z_8 = z_9 + z_{10} = 20 + 35 = 55 \qquad --(iv)$$

On solving equation (iii) and (iv), we get

$$z_8 = 23.3 = 24 \text{ and } z_7 = 55 - 24 = 31$$

Stage 1:

First pair: Consider the ray that gives maximum from 450 r.p.m to 140 r.p.m. The corresponding gears are 5 and 6. Assume z_5 = 20 (driver)

$$\frac{z_5}{z_6} = \frac{N_6}{N_5} \text{ or } \frac{20}{z_6} = \frac{140}{450}; \qquad z_6 = 64.28 = 65$$

Second pair: Consider the ray that gives speed reduction from r.p.m to 190 r.p.m. The corresponding gears are 3 and 4.

$$\frac{z_3}{z_4} = \frac{N_4}{N_3} = \frac{190}{450} \text{ or } z_3 = 0.422 z_4 \qquad --(v)$$

$$z_3 + z_4 = z_5 + z_6 = 20 + 65 = 85 \qquad --(vi)$$

On solving the equations (v) and (vi), we get

$$z_4 = 59.77 \approx 60 \text{ and } z_3 = 85 - 60 = 25$$

Third pair: Consider the ray that gives speed reduction from 450 r.p.m to 250 r.p.m. The corresponding gears are 1 and 2.

$$\frac{z_1}{z_2} = \frac{N_2}{N_1} = \frac{250}{450} \text{ or } z_1 = 0.555 z_2 \qquad --(vii)$$

$$z_1 + z_2 = z_3 + z_4 = 60 + 25 = 85 \qquad --(viii)$$

On solving the equations (vii) and (viii), we get

$$z_2 = 54.66 \approx 55 \text{ and } z_1 = 85 - 55 = 30$$

20. A 9-speed box, used as a head stock gear box of a turret lathe, is to provide a speed range of 180 rpm to 1800 rpm. (April/ May 2019)

Given data:

$$n = 9$$
$$N_{min} = 180 rpm$$
$$N_{max} = 1800 rpm$$

Step 1:- selection of spindle speeds

Determine the progression ratio (ϕ) using the relation

$$\frac{N_{max}}{N_{min}} = \phi^{n-1}$$
$$\frac{1800}{180} = \phi^{9-1}$$
$$\phi = (10)^{1/8}$$
$$\phi = 1.333$$

- ✓ We find ϕ =1.333 is not a standard ratio. So let us find out whether multiplies of standard ratio 1.12 or 1.06 come close to 1.333
- ✓ For example we can write, $1.12 \times 1.12 = 1.2544 \ \& \ 1.12 \times 1.12 \times 1.12 = 1.405$

Then $1.06 \times 1.06 \times 1.06 \times 1.06 = 1.338$ skip 4 speeds

So we take ϕ= 1.06, because satisfies the requirement, select the standard spindle speeds using the series of preferred numbers

Take Step Ratio from R40 series ϕ=1.06

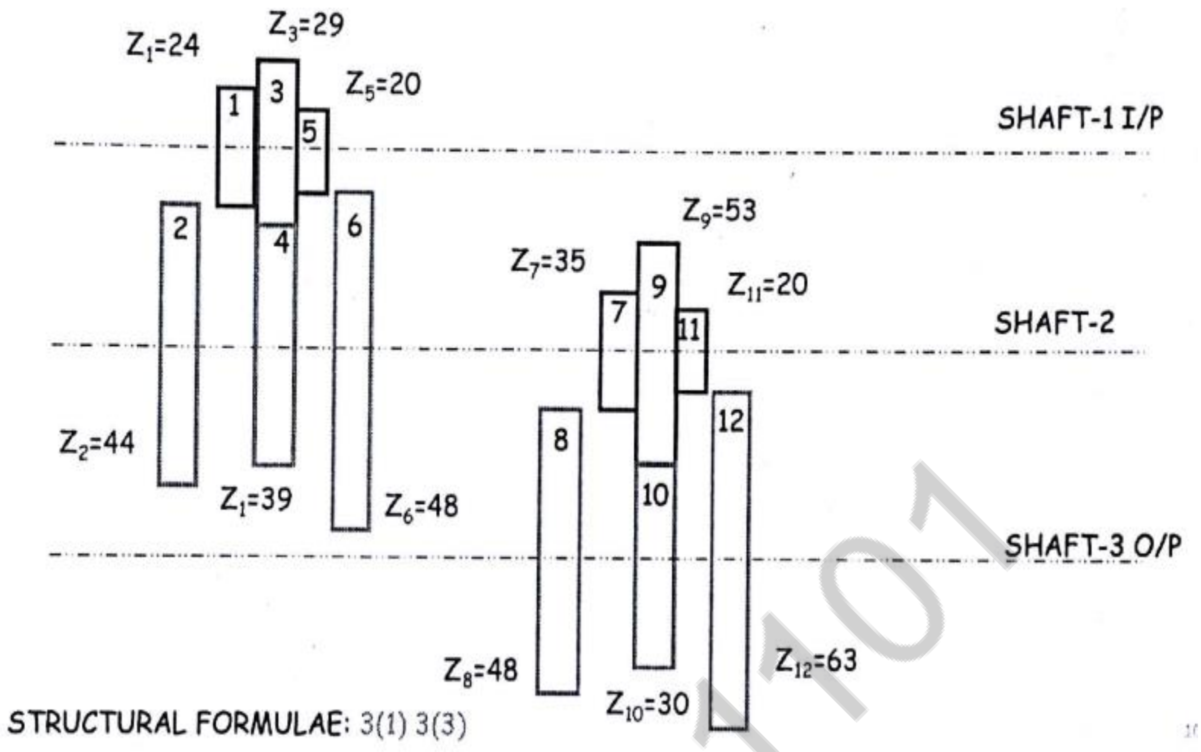
Spindle Speeds are 180, 236, 315, 425, 560, 750, 1000, 1320 and 1800rpm

Step 2: To find the structural formulae

Structural formulae: 3(1) 3(3)

Step 3: Construct the kinematic arrangement for 9 speed gear box

- ✓ Structural formulae: 3(1) 3(3)
- ✓ P1 = 3 p2 = 3 Note: where X_1 = 1; X_2 = p_1 = 3
- ✓ No. of shafts = No. of stages +1 (2+1=3 shafts) (so draw 3 horizontal lines)
- ✓ To find the no. of gears by using
  No. of gears $= 2(p_1 + p_2)\{[2(3+3)] = 12 \text{ gears}\}$



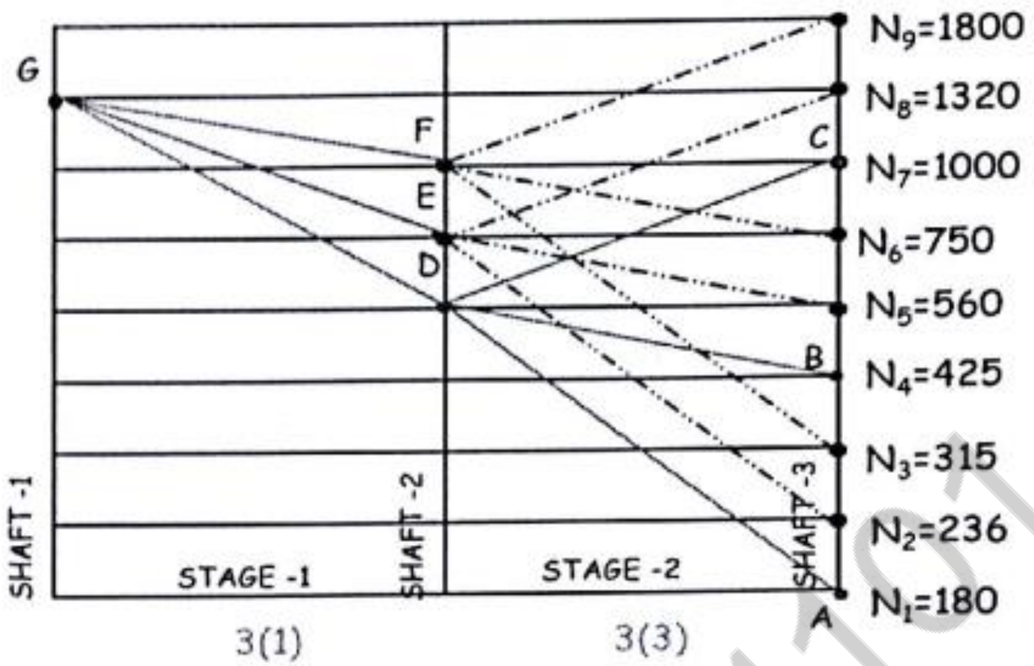
Step 4:- Construct the ray diagram for 9 speed gear box

- Structural formulae: 3(1) 3(3)

No .of stages: $2\{(p_1(X_1).p_2(X_2)\}$

$p_1 = 3$ $p_2 =$ Note: Where $X_1 =$ $X_2 = p_1 = 3$

- ✓ No. of shafts = No. of stages +1 (2+1= 3 shafts) (so draw 3 vertical lines)
- ✓ No. of speeds = 9 (Draw 9 horizontal lines)



Step 5: Calculation of No. of teeth

- Calculation of numbers of teeth on all the gears

Let Z_1, Z_2, Z_3, ...Z_{12} = Number of teeth of the gears 1, 2, 3, …12 respectively

Formulae given $\frac{z_1}{z_2} = \frac{N_2}{N_1}$

Take stage – 2

- Consider the first pair of gear 11 and 12
- From ray diagram consider ray DA
- Maximum speed reduction 560rpm to 180rpm

We know that, $Z_{min} \geq 17$, assume Z_{11} = 20 (driver)

$$\frac{z_{11}}{z_{12}} = \frac{N_{12}}{N_{11}}$$
$$\frac{20}{z_{12}} = \frac{180}{560}$$
$$z_{12} = 62.22 \approx 63$$

$Z_{11} = 20$, $Z_{12} = 63$

Take stage – 2

- Consider the second pair of gear 7 and 8
- From ray diagram consider ray DB
- Maximum speed reduction 560rpm to 425rpm

We know that,

$$\frac{z_7}{z_8} = \frac{N_8}{N_7}$$
$$\frac{z_7}{z_8} = \frac{425}{560}$$
$$z_7 = 0.76 z_8 \qquad --(i)$$

NOTE: The centre distance between the shafts are fixed and same. The sum of number of teeth of mating gears should be equal.

So we can write

$$z_7 + z_8 = z_{11} + z_{12} = 20 + 63 = 83 \qquad (ii)$$

Solving equations (i) and (ii), we get

$$z_8 = 47.16 \approx 48$$
$$z_7 = 83 - 48 = 35$$
$$z_7 = 35 \quad z_8 = 48$$

Take stage – 2

- Consider the third pair of gear 9 and 10
- From ray diagram consider ray DC
- Speed increase from 560rpm to 1000rpm

We know that,

$$\frac{z_9}{z_{10}} = \frac{N_{10}}{N_9}$$
$$\frac{z_9}{z_{10}} = \frac{1000}{560}$$
$$Z_9 = 1.786 Z_{10} \qquad --(iii)$$

So we can write

$$Z_9 + Z_{10} = Z_{11} + Z_{12} = 20 + 63 = 83 \qquad --(iv)$$

Solving equation (iii) and (iv), we get

$$Z_{10} = 29.79 \approx 30$$
$$Z_9 = 83 - 30 = 53$$
$$Z_9 = 53 \quad Z_{10} = 30$$

Take stage -1

- Consider the first pair of gear 5 and 6
- From ray diagram consider ray GD
- Maximum speed reduction 1320rpm to 560rpm

We know that, $Z_{min} \geq 17$ ∴ assume Z_5 = 20 (Driver)

$$\frac{z_5}{z_6} = \frac{N_6}{N_5}$$
$$\frac{20}{z_{12}} = \frac{1320}{560}$$
$$z_6 = 47.14 \approx 48$$

Take stage – 1

- Consider the first pair of gear 5 and 6
- From ray diagram consider ray GD
- Maximum speed reduction 1320rpm to 560rpm

We know that,

$$\frac{z_1}{z_2} = \frac{N_2}{N_1}$$
$$\frac{z_1}{z_2} = \frac{750}{1320}$$
$$Z_1 = 0.57z_2 \qquad ---(v)$$

NOTE: The centre distance between the shafts are fixed and same. The sum of number of teeth of mating gears should be equal.

So we can write

$$z_1 + z_2 = z_5 + z_6 = 20 + 48 = 68 \qquad ---(vi)$$

Solving equations (v) and (vi), we get

$$z_2 = 43.3 \approx 44$$
$$z_1 = 68.44 = 24$$
$$Z_1 = 24 \quad Z_2 = 44$$

Take stage – 1

- Consider the third pair of gear 3 and 4
- From ray diagram consider ray GF
- Speed increase from 1320rpm to 1000 rpm

We know that,

$$\frac{z_3}{z_4} = \frac{N_4}{N_3}$$

$$\frac{z_3}{z_4} = \frac{1000}{1320}$$

$$Z_3 = 0.76 z_4 \qquad --(vii)$$

Solving equations (iii) and (iv), we get

$$Z_4 = 38.64 \approx 39$$
$$Z_3 = 68 - 39 = 29$$
$$Z_3 = 29 \quad Z_4 = 39$$

21. Design an 18 speed gear box from a source of 1000 rpm. Maximum and minimum speeds are to be around 650rpm and 35rpm respectively. (April/ May 2019)

Given data:

$$n = 18$$
$$N_{min} = 35rpm$$
$$N_{max} = 650rpm$$

Step 1: Selection of Spindle speeds

Determine the progression ratio (ϕ) using the relation

$$N_{max}/N_{min} = \phi^{n-1}$$

$$650/35 = \phi^{18-1}$$

$$\phi = (18.571)^{1/17}$$

$$\phi = 1.87$$

We find $\phi = 1.87$ is not a standard ratio. So let us find out whether multiples of standard ratio 1.12 OR 1.06 come close to 1.87

For example we can write $1.12 \times 1.12 = 1.2544$

Then $1.06 \times (1.06 \times 1.06) = 1.91$ … Skip 2 speeds

So we take $\phi = 1.06$, because satisfies the requirement. Select the standard spindle speeds using the series of preferred numbers From PSGDB 7.20, 7.19

Step ratio from R40 series $\phi = 1.06$

$\therefore$ Spindle speeds are 35.5, 42.5, 50, 60, 71, 85, 100, 118, 140, 170, 200, 236, 280, 335, 400, 475, 560 and 670 rpm

Step 2: To find the Structural Formulae

Structural formulae: $2(1)3(2)3(6)$

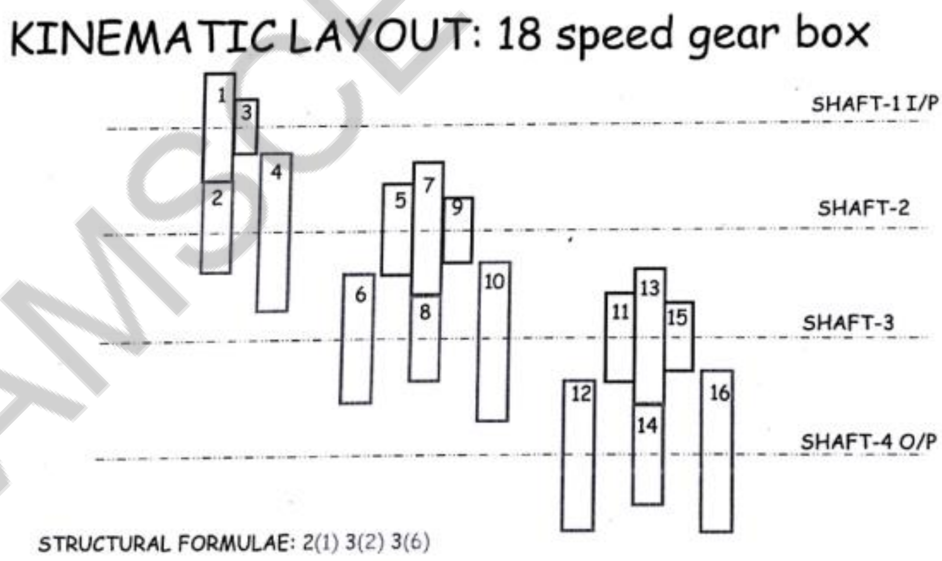
Step 3: Construct the Kinematic arrangement for 18 speed gear box

Structural formulae: $2(1)3(2)3(6)$

No. of shafts = No. of stages + 1 (3 + 1 = 4 shafts) (so draw 4 horizontal lines)

To find the no. of gears by using

No. of gears $= 2(p_1 + p_2 + p_3)\{[2(2+3+3)] = 16\text{gears}\}$



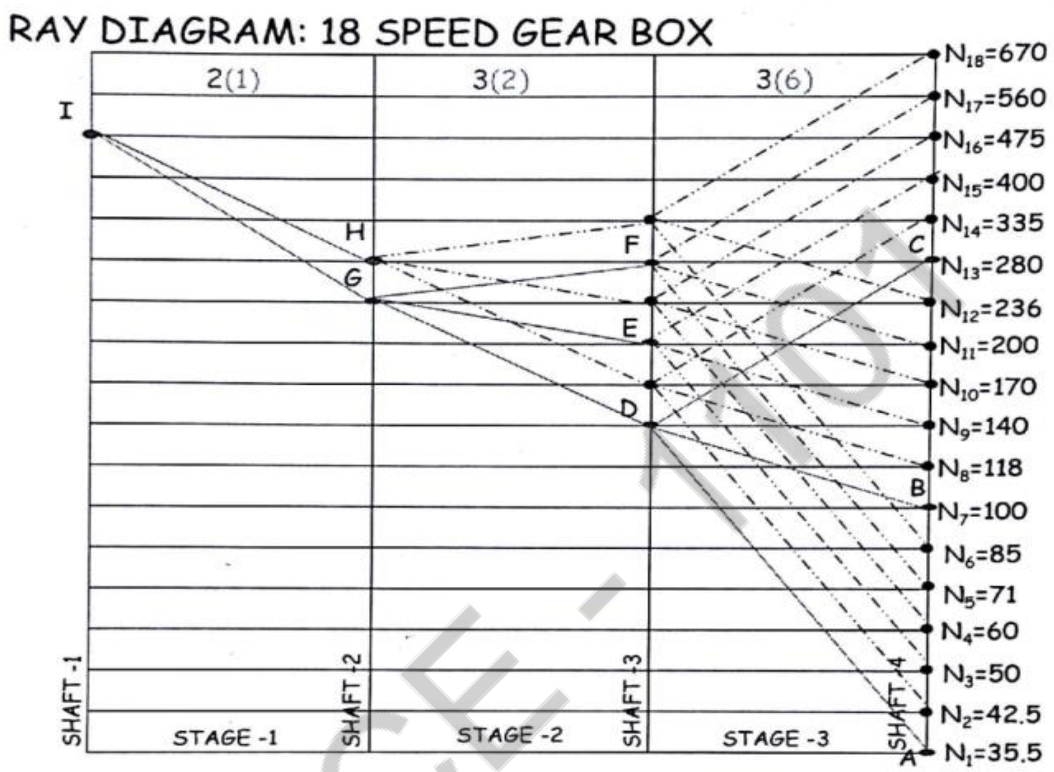
Step 4: Construct the ray diagram for 18 speed gear box

Structural formulae: $2(1)3(2)3(6)$

Note: Where $X_1 = 1$ $X_2 = p_1 = 2$ $X_3 = p_1 \cdot p_2 = 2 \times 3 = 6$

No. of shafts = No. of stages +1 (3 + 1 = 4 shafts) (so draw 4 vertical lines)

No. of speeds = 18 (Draw 18 horizontal lines)



ME-6601 DESIGN OF TRANSMISSION SYSTEMS

UNIT-V CLUTCHES AND BRAKES

(PART-A)

1. Differentiate between uniform pressure and uniform wear theories adopted in the design of clutches?

In clutches, the value of normal pressure, axial load for the given clutch is limited by the rate of wear that can be tolerated in the brake links. Moreover the assumption of uniform wear rate gives a lower calculated clutch capacity than assumption of uniform pressure. Hence clutches are usually designed on the basis of uniform wear.

2. In a hoisting machinery, what are the different energies absorbed by a brake system?

- Kinetic energy of translation: $KE = \frac{1}{2}mv^2$
- Kinetic energy of rotation: $KE = \frac{1}{2}I\omega^2$
- Potential or gravitational energy: $P.E = W * x$

Total energy absorbed: $E_T = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + W * x$

3. If a multidisc clutch has 6 discs in driving shaft and 7 discs in driven shaft, then how many number of contact surfaces it will have?

Given data: $n_1 = 6;\ n_2 = 7$

Number of pair of contact surfaces, $n = n_1 + n_2 - 1$

$$= 6 + 7 - 1$$

$$n = 12$$

4. Why in automobiles, braking action when travelling in reverse is not as effective as when moving forward?

When an automobile moves forward, the braking force acting in the opposite direction to the direction of motion of the vehicle. Whereas in reverse travelling the braking force acts in the same direction to the direction of motion of the vehicle. So it requires more braking force to applying brake.

5. Name the profile of cam that gives no jerk?

Circle- arc cam gives no jerk. Because the derivative of acceleration of cam is zero.

6. What is meant by positive clutch?

Positive clutches means to have interlocking engaging surfaces to form a rigid mechanical junction.

7. What is the function of clutch in a transmission system?

- To connect and disconnect the shafts at will.
- To start or stop a machine (or a rotating element) without starting and stopping the prime mover.
- To maintain constant speed, torque and power.
- For automatic disconnect, quick start and stop, gradual starts, non-reversing and over-running functions

8. What is the significance of pressure angle in cam design?

The pressure angle is very important in cam design as it represents steepness of the cam profile. If the pressure angle is too large, a reciprocating follower will jam in its bearings

9. Mention few application of cams.

The cam can be a simple tooth, as is used to deliver pulses of power to a stream hammer, for example, or an eccentric disc or other shape that produces a smooth reciprocating (back and forth) motion in the follower, which is a lever making constant with the cam.
Also it is used in IC engines for value opening and closing

10. What do you mean by self-energizing brake?

When the moment of applied force and the moment of the frictional force are in the same direction, then frictional force helps in applying the brake. This type of brake is called us self-energizing brake.

11. What is a clutch and where it is used?

Clutch is machine, component used as temporary coupling: and is used mainly in automobiles for engaging and disengaging the driving shaft where periodical engagement is required.

12. What is meant by positive clutch?

A positive clutch transmits power from driving shaft to the driven shaft by jaws or teeth is called positive clutch. No slipping is there.

13. By what means, power is transmitted by clutches?

In clutches, power transmission is achieved through

(a) Interlocking (b) Friction (c) Wedging

14. Why are cone clutches better than disc clutches?

Since the cone discs are having large frictional areas and they can transmit a larger torque than disc clutches with, the same oil diameter and actuating force and hence cone clutches are preferred over disk clutches. But usually cone clutches are mainly used in low peripheral s applications.

15. What factors should be considered when designing friction clutches?

- ❖ The friction materials for the clutch should have high co-efficient of friction and-they should not be affected by moisture and oil.
- ❖ May be light in weight.
- ❖ The design is in such a way that the engagement should be made without shock and fast
- ❖ Disengagement without drag.

16. Why should the generated heat be dissipated in clutch operation?

In order to save the friction plates and materials from melting by the heat produced during operation, the generated heat should be dissipated.

17. Name the two theories applied for the design of friction clutches.

1. Uniform Pressure theory
2. Uniform wear theory

18. Name four materials used for lining of friction surfacing clutches.

- ❖ Wood
- ❖ Leather
- ❖ Asbestos based friction materials
- ❖ Powdered metal friction materials

19. State the advantages of cam mechanisms.

Cams are used for transmitting desired motion to a follower by direct contact. Cam mechanisms are used in the operation of IC engine valves.

20. Why should the temperature rise be kept within the permissible range in brakes?

Otherwise the brake drawn will be overheated and hence the brake shoes may be damaged due to overheating.

21. Differentiate a brake and a dynamometer. (April/May 2017)

A dynamometer is a brake incorporating a device to measure the frictional resistance applied.

22. Double shoe brakes are preferred than single shoe brake. Why? (April/May 2017)

In a single shoe brake normal force introduces transverse loading on the shaft on which the brake drum is mounted two shoes are often used to provide braking torque.

23. Write the difference between dry and wet clutch. (Nov/Dec 2017)

- When a clutch operates in the absence of a lubricant, then that the clutch is known as dry clutch. In dry clutch the torque capacity is high but the heat dissipating capacity is low
- When the clutch operates 'wet' (i.e., with lubrication), then torque capacity is low but the heat dissipating capacity is high

24. What is meant by self-energizing brakes? (Nov/Dec 2017

When the moment of applied force and the moment of the frictional force are in the same direction, then frictional force helps in applying the brake. This type of brake is called us self-energizing brake.

25. What are the types of brakes used in modern vehicles? (April/May 2018)

Disc brakes, drum brakes and internally expanding brakes

26. How does the function of a brake differ from that of a clutch? (April/May 2018)

A clutch connects two moving members of a machine, whereas a brake connects a moving member to a stationary member.

27. Name few commonly used friction materials. (Nov/Dec 2018)

Wood, Cork, Leather, Asbestos based friction materials, and powdered metal friction materials.

28. What do you meant by self-locking brake? (Nov/Dec 2018)

When the moment of applied force and the moment of the frictional force are in the same direction, then frictional force helps in applying the brake. This type of brake is called us self-energizing brake.

29. How does the function of a brake differ from that of a clutch? (April/May 2019)

Brake is a mechanical device by means of which motion of a body is retarded for slowing down or to bring it to rest, by applying frictional resistance

30. Why are cone clutches better than disc clutches? (April/May 2019)

i. In disc clutches, friction lined flat plates are used
ii. In cone clutches, friction lined frustum of cone is used

PART B

1. An automobile engine has an output of 80KW at 3000rpm. The mean diameter of the clutch is 200mm with a permissible pressure of 0.2 N/mm². Friction lining is of asbestos with M=0.22 . What should be the inner diameter of the disc? Take both the sides of the plates with friction lining as effective. There are 8 springs and axial deflection in spring is limited to 10 mm. Given G=80KN/mm² spring index may be taken as b.

Given data:

$P = 80KW$

$N = 3000rpm$

$d1 = 200mm$

$P\max = 0.2\,N/mm^2$

$M = 0.22$

$No.ofSprings = 8$

$C = 6$

$G = 80\,KN/mm^2$

$\text{Axial deflection} = 10mm.$

Step 1: To find the inside diameter of the plate.

Case: 1: To find the torque transmitted.

$$T = \frac{60P}{2\pi N}$$

$$= \frac{60 \times 80 \times 10^3}{2 \times \pi \times 3000}$$

$$= 254.65 N.m$$

Case 2: To find the axial force acting on the friction faces.

$$W = A \times P \qquad \frac{R}{b} = 4$$

$$= 2\pi Rb \cdot P \qquad b = \frac{R}{4}$$

$$= 2\pi \times R \times \frac{R}{4} \times 0.2 \qquad P = P_{max}$$

$$W = 0.314\ R^2$$

Case 3: To find the mean radius of the friction lining (R)

$$T = M \cdot W \cdot R \cdot n$$

$$254.65 \times 10^3 = 0.22 \times 0.314 \times R^2 \times R \times 2 \qquad [\because n = 2]$$

$$R = 122.61 mm$$

$$W \cdot K \cdot T \Rightarrow R_1 = \frac{r_1 + r_2}{2}$$

$$122.61 = \frac{100 + r_2}{2}$$

$$r_2 = 145.22 mm$$

Case 4: To find the inside diameter of the Plate.

$$d_2 = 2 \times r_2$$

$$= 2 \times 145.22$$

$$d_2 = 290.44 mm$$

Step 2: To find the axial force to engage the clutch.

$$W = 0.314 \times R^2$$

$$= 4720.43 N$$

Step 3: To find the spring wire diameter. (d)

In order to allow for adjustment and for Maximum torque, the spring is designed for an overload of 25%

$$\therefore \left.\begin{array}{l}\text{Total load on} \\ \text{the springs}\end{array}\right\} = 1.25 \times W$$

$$= 1.25 \times 4720.43$$

$$= 5900.54 N$$

Since there are 8 springs, therefore the maximum load on each spring.

$$W_s = \frac{5900.54}{8}$$

$$=737.56N$$

We know that Wahl's stress factor

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

$$= \frac{4\times6-1}{4\times6-4} + \frac{0.615}{6}$$

$$= \frac{23}{20} + \frac{0.615}{6}$$

$$K = 1.2525$$

Maximum stress induced in the wire (σ_s) ,

Assume $\sigma_s = 600mpa$

$$\sigma_s = K \times \frac{8W_sC}{\pi d^2}$$

$$600 = 1.2525 \times \frac{8\times737.56\times6}{\pi\times d^2}$$

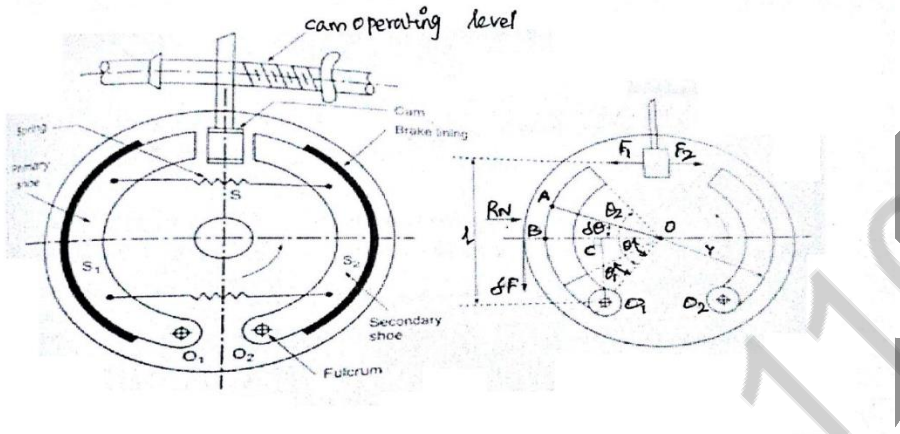
$$d^2 = 23.524$$

$$d = 4.85mm$$

From PSGDB 13.1, We shall take a standard wire of size SWG 6 having diameter. 4.88mm.

2. Derive an expression to determine the braking torque for an internal expanding shoe brake.

The figure shows an internal shoe automatic brake. It consists of two semi-circular shoes S1 and S2 which are lined with a frictional material such as ferrodo. When brakes are applied, can rotates which pushes the shoes outwards to press the brake lining against the rim of the drum. As soon as the brakes are off, the shoes are pushed inside by the spring.



(a) Internal expanding brake (b) Forces on the brake.

It may be noted that for the anticlockwise direction the left side shoe is known as primary or leading shoe, while the right hand shoe is known as trailing or secondary shoe.

Determination of pressure and Brake torque:

Consider the forces on the brake when the drum rotates in anticlockwise direction as shown in figure.

Let P_1 = Maximum intensity of normal pressure

P_N = Normal pressure

r = Internal radius of the drum

b = width of the brake lining

T_B = Braking torque

F_1 = Force exerted by the cam on the loading or primary shoe.

F_2 = Force exerted by the cam on the trailing or secondary shoe.

R_N = Normal force.

F = Frictional force.

M= Co-efficient of friction between shoe and drum.

M_N = Moment of normal force

M_F = Moment of Frictional force.

Consider a small element AB of brake lining subtending an angle $\delta\theta$ at the centre of the drum. Join 0_1 to 0. It is assumed that the pressure distribution on the shoe is nearly uniform. However the shoe wears out more at the free end. The rate of wear of the shoe lining varies directly as the perpendicular distance from 0_1 to B ie 0_1 c.

From the geometry of the figure b.

$$O_1C = 00, \sin\theta$$

and normal pressure at B,

$$P_N \alpha \sin\theta \text{ or } P_N = P_1 \sin\theta$$

Normal force acting on the element ,

$$\delta R_N = \text{Normal pressure} \times \text{Area of the element}$$

$$= P_N \times (b \cdot r \cdot \delta\theta) = P_1 \sin\theta b \cdot r \cdot \delta\theta$$

Friction force on the element

$$\delta F = M \cdot \delta R_N = MP_1 \sin\theta \cdot b \cdot r \cdot \delta\theta$$

Braking torque due to the element about 0

$$\delta T_B = \delta F \cdot r$$

$$= MP_1 \cdot \sin\theta \times b \times r \times \delta\theta \times r$$

$$= MP_1 \sin\theta \cdot br^2 \delta\theta$$

Total braking torque for whole shoe about 0

$$T_B = MP_1 br^2 \int_{\theta_1}^{\theta_2} \sin\theta . d\theta$$

$$= MP_1 br^2 \left[-\cos\theta \right]_{\theta_1}^{\theta_2}$$

$$T_B = MP_1 br^2 \left[\cos\theta_1 - \cos\theta_2 \right]$$

3. A Power of 20 KW is to be transmitted through a cone clutch at 500 rpm. For uniform wear condition, find the main dimensions of clutch and shaft. Also determine the axial force required to engage the clutch. Assume co-efficient of friction as 0.25, the maximum normal pressure on the friction surface is not to exceed 0.08 Mpa, and take the design stress for the shaft materials as 40 Mpa.

Given data:

$P = 20KW$
$N = 500rpm$
$M = 0.25$
$P_{max} = 0.08mpa$
$P_{shaft} = 40mpa.$

Step 1: To find the Torque transmitted.

$$T = \frac{60P}{2\pi N}$$

$$= \frac{60 \times 20 \times 10^3}{2 \times \pi \times 500}$$

$$T = 382 \text{ Nm.}$$

Step 2: To find b, R, r_1 and r_2.

$b = \frac{R}{2}$, semi cone angle α=15°

For cone clutch $\frac{r_1 - r_2}{b} = \sin\alpha$

$$b = \frac{r_1 - r_2}{\sin\alpha}$$

Mean radius $R = \frac{r_1 + r_2}{2}$

$$\therefore \frac{r_1 - r_2}{\sin\alpha} = \frac{r_1 + r_2}{4}$$

$$\frac{r_1 - r_2}{\sin 15} = \frac{r_1 + r_2}{4}$$

By solving the above equation $r_1 = 1.139\ r_2$ 1

Design Torque $[T] = T \times K_s$

Assume $K_s = 2.5$

$[T] = 382 \times 2.5$

$= 955$ Nm.

For uniform wear,

$[T] = 2\pi M P_{max} P_1^2 b$

$955 \times 10^3 = 2 \times \pi \times 0.25 \times 0.08 \times R^2 \times \frac{R}{2}$

$R = 247.7$mm.

Also, $R = \frac{r_1 + r_2}{2}$

$247.7 = \frac{r_1 + r_2}{2}$

$r_1 + r_2 = 495.4$ 2

Solving 1 and 2

Inner radius $r_1 = 264$mm.

Outer radius $r_2 = 231.6$mm.

Face width $b = \frac{R}{2}$

$= \frac{247.7}{2}$

$b = 123.85$mm.

Step 3: To find axial force required to engage the clutch (W).

$$W = 2\pi c (r_1 - r_2).$$

$$= 2\pi \times P_{max} r_2 (r_1 - r_2) \quad [\because c = P_{max} \times r_2] .$$

$$= 2 \times \pi \times 0.08 \times 10^6 \times 0.2316(0.264 - 0.2316).$$

$$= 3771.84 \text{ N}.$$

Step 4: To find the diameter of the shaft:

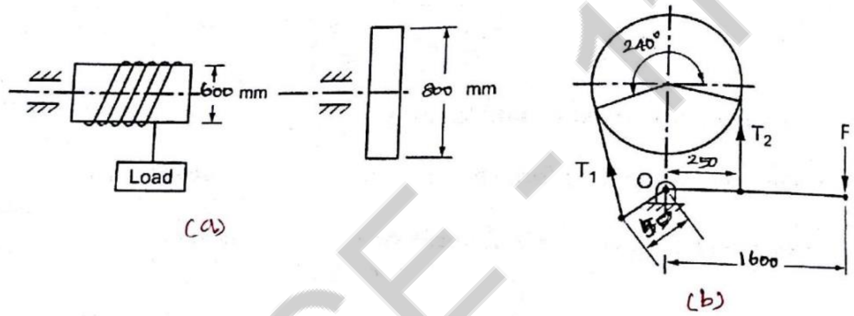
$$[T] = \pi/16 \times d_s^3 \times P_{shaft}$$

$$955 = \pi/16 \times d_s^3 \times 40 \times 10^6$$

$$d_s = 0.0495\,m\ .$$

$$d_s = 49.5\ mm.$$

4.Design a differential band brake for a winch lifting a load of 20 KN through a steel wire rope wound around a barrel of 600mm diameter. The brake drum keyed to the barrel shaft, is 800mm diameter and the angle of lap of the band over the drum is about 240° operating arms of the brake are 50mm and 250mm. The length of operating level is 1.6m.



Given data:

$$Load = 20\ KN$$

$$Barrel diameter = 600mm.$$

$$\Theta = 240° = 240 \times \pi/180 = 4.188rad.$$

$$M = 0.25.$$

Step 1: Calculation of braking torque. (T_B)

$$T_B = Load \times Barrel\ radius.$$

$$= 20 \times 10^3 \times \left(\frac{0.6}{2}\right)$$

$$= 6000\ Nm$$

Step 2: Brake drum diameter. (D).

$$D = 800mm \quad (given).$$

Step 3: Calculation of T1 and T2:

Tension ratio , $\frac{T_1}{T_2} = e^{M\Theta} = e^{0.25\times4.188}$

$$T_1 = 2.849\ T_2$$

$$T_B = (T_1 - T_2)\times r$$

$$6000 = (T_1 - T_2)\times\left(\frac{0.8}{2}\right)$$

$$T_1 - T_2 = 15000$$

$$2.849T_2 - T_1 = 15000$$

$$T_2 = 3897.12\ N$$

$$\therefore\quad T_1 = 11102.88\ N.$$

Step 4: Thickness of band. (t).

$$t = 0.005\ D$$

$$= 0.005\times800$$

$$t = 4mm.$$

Step 5: Calculation of band width. (w).

$$\sigma_t = \frac{T_1}{w\times t} \le [\sigma_t]$$

$$\therefore\ 50 = \frac{11102.88}{w\times4} \qquad \because [\sigma_t] = 50\ N/mm^2\ \text{is assumed}\ .$$

$$w = 55.51$$

$$w = 56\ mm$$

Step 6: Check for bearing pressure.

$$P_{max} = \frac{T_1}{w.r.}$$

$$= \frac{11102.88}{56\times\left(\frac{800}{2}\right)}$$

$$= 0.495\ \text{N/mm}^2$$

For steel band on steel drum. $[P] = 1.5\text{N}/\text{mm}^2$

We find $P_{max} < [P]$, $\therefore$ the design is safe.

Step 7: Calculation of the force to be applied at the end of the lever.

Refer figure (b) , taking moments about O , we get.

$$F \times 1600 + T_1 \times 50 = T_2 \times 250$$

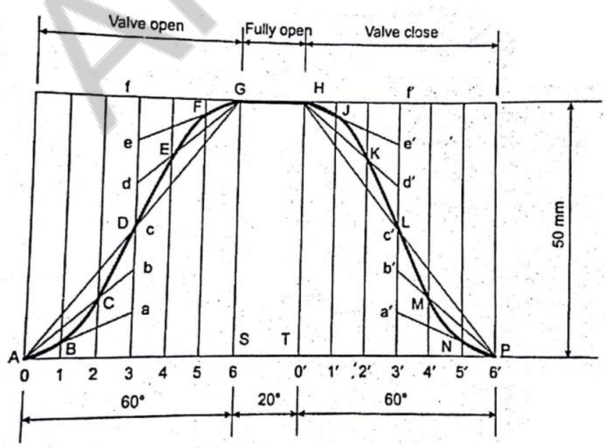
$$F \times 1600 + 11102.88 \times 50 = 3897.12 \times 250$$

$$F = 261.96\ \text{N.}$$

5. **Design a cam for operating the exhaust valve of an oil engine. It is required to give equal uniform acceleration and retardation during opening and closing or the valve, each of which corresponding to 60° of cam rotation. The valve should remain in the fully open position for 20° of cam rotation. The lift valve is 50mm and the least radius of the cam is 50mm, the follower is provided with a roller of 50mm diameter and its line of stroke passes through the axis of the cam.**

   Construction: First of all, the displacement diagram, as shown in Fig.1., is drawn as discussed in the following steps:

1. Draw a horizontal line ASTP such that AS represents the angular displacement of the cam during opening (i.e., outstroke) of the valve (equal to 60°) to some suitable scale. The line ST represents the dwell period of 20° i.e., the period during which the valve remains fully open and TP represents the angular displacement during closing (i.e., return stroke) of the valve which is equal to 60°.



2. Divide AS and TP into any number of equal even parts (say six).

3. Draw vertical lines through points 0, 1, 2, 3, etc., and equal to the lift of the valve (i.e., 50 mm).
4. Divide the vertical lines 3 f and 3 'f' into six equal parts as shown by the points a, b, c, .. and a', b', c' in Fig.1.
5. Since the valve moves with equal uniform acceleration and retardation, therefore the displacement diagram for opening and closing of a valve consists of double parabola.
6. Complete the displacement diagram as shown in Fig.1. Now, the profile of the cam, with a roller follower when its line of stroke passes through the axis of the cam, as shown in Fig.2 is drawn in the usual way.

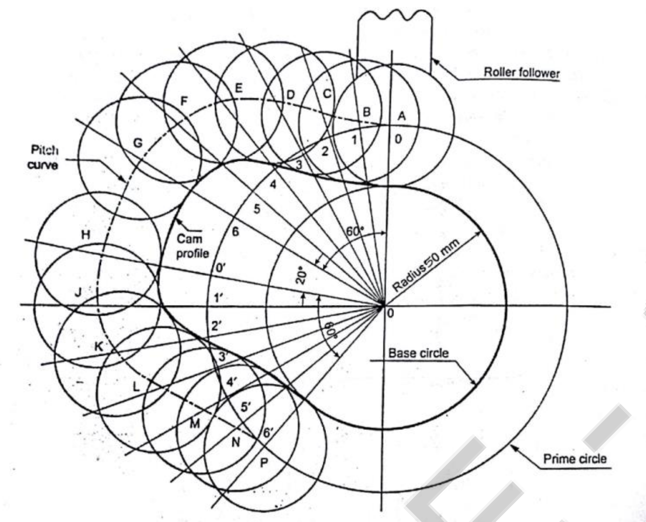


Fig. 2

6. Explain with a neat sketch the working of a single plate clutch. Derive an expression for the torque to be transmitted by clutch assuming

i. Uniform pressure condition and

ii. Uniform wear condition

Consider two friction surfaces held together by an axial thrust W, as shown in Fig. (A)

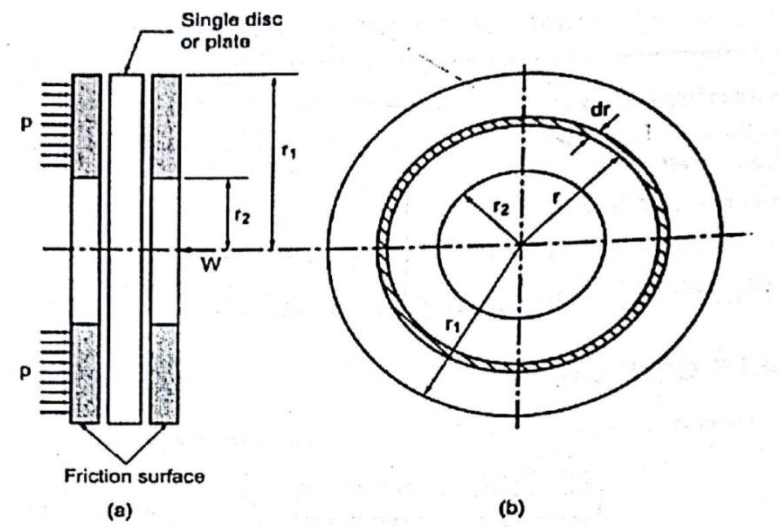


Fig.* (A) *Forces on a single disc or plate clutch

Let T = Torque transmitted by the clutch,

P = Intensity of axial pressure acting on contact surfaces,

r_1 = External radius of friction surface,

r_2 = Internal radius of friction surface, and

μ = Coefficient of friction.

Consider an elementary ring of radius r and thickness dr as shown in Fig. 10.3 (b).

Area of the elemental ring = $2\pi r.dr$

Normal or axial force on the ring, $\delta W = \text{Pressure} \times \text{Area} = p \times 2\pi r \cdot dr$

and the frictional force on the ring acting tangentially at radius r is given by

$$F_r = \mu \cdot \delta W = \mu p \times 2\pi r \cdot dr$$

$\therefore$ Frictional torque acting on the ring, $T_r = F_r \times r$

$$T_r = \mu p \times 2\pi r \cdot dr \times r$$

$$= 2\pi \mu p r^2 dr$$

The design of friction clutch is done based on any one of the following assumptions:

(i) When there is a uniform pressure, and
(ii) When there is a uniform wear.

(i) Considering uniform pressure:

Area of the friction surface, $A = \pi\left(r_1^2 - r_2^2\right)$

Uniform intensity of pressure (p) is given by

$$p = \frac{W}{A} = \frac{W}{\pi\left(r_1^2 - r_2^2\right)}$$

Total frictional torque acting on the friction surface or on the clutch is obtained by integrating the equation of the frictional torque on the elementary ring within the limits from r_2 to r_1.

$$\therefore \quad T = \int_{r_2}^{r_1} 2\pi\mu p r^2 dr = 2\pi\mu p\left[\frac{r^3}{3}\right]_{r_2}^{r_1} = 2\pi\mu p\left[\frac{r_1^3 - r_2^3}{3}\right]$$

Substituting the value of p from equation (1)

$$T = 2\pi\mu \times \frac{W}{\pi\left(r_1^2 - r_2^2\right)}\left[\frac{r_1^3 - r_2^3}{3}\right]$$

or

$$T = \frac{2}{3} \times \mu W\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right] = \mu W R$$

where R = Mean radius of friction surface

$$R = \frac{2}{3}\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

(ii) Considering uniform wear: For uniform wear, the intensity of pressure varies inversely with the distance. Therefore,

$$p \cdot r = \text{constant} = C \quad \text{or} \quad p = \frac{C}{r}$$

So

$$p_1 \cdot r_1 = p_2 \cdot r_2 = C$$

Where p_1 and p_2 are intensities of pressure at radii r_1 and r_2 respectively.

We know that normal or axial force on the elementary ring,

$$\delta W = p 2\pi r dr = 2\pi(p \cdot r)dr = 2\pi C dr \qquad \left[\because p = \frac{C}{r}\right]$$

$\therefore$ Total force acting on the friction surface,

$$W = \int_{r_2}^{r_1} 2\pi C dr = 2\pi C[r]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

or

$$C = \frac{W}{2\pi(r_1 - r_2)}$$

We know that the frictional torque acting on the elementary ring,

$$T_r = 2\pi\mu p r^2 dr = 2\pi\mu \times \frac{C}{r} \times r^2 \cdot dr \qquad \left[\because p = \frac{C}{r}\right]$$

$$= 2\pi\mu \cdot C \cdot r \cdot dr$$

$\therefore$ Total frictional torque on the friction surface,

$$T = \int_{r_2}^{r_1} 2\pi\mu \cdot C \cdot r \cdot dr = 2\pi\mu C\left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\mu C\left[\frac{r_1^2 - r_2^2}{2}\right]$$

$$T = \pi\mu C\left[r_1^2 - r_2^2\right]$$

Substituting the value of C from equation (4)

$$T = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)} \times (r_1^2 - r_2^2)$$

$$T = \frac{1}{2} \times \mu \cdot W(r_1 + r_2) = \mu.W.R$$

Where R = Mean radius of the friction surface

or

$$R = \frac{r_1 + r_2}{2}$$

Note:

1. In general, total frictional torque acting on the friction surface or on the clutch is given by

$$T = n \cdot \mu \cdot W \cdot R$$

Where n = Number of pairs of friction or contact surfaces.

2. For single disc plate clutch, n=2. Since both the sides of the disc are in contact.

7. A multiplate clutch with both sides effective transmits 30KW at 360rpm. Inner and Outer radii of the clutch discs are 100mm and 200 mm respectively. The effective co-efficient of friction is 0.25. An axial load of 600N is applied. Assuming uniform wear conditions, find the number of discs required and the maximum intensity of pressure developed.

Given data:

$P = 30KW$

$N = 360rpm.$

$r_2 = 100mm.$

$r_1 = 200mm$

$M = 0.25$

$w = 660N.$

Assuming uniform wear, axial force exerted is given by,

$$w = 2\pi c(r_1 - r_2).$$

$$600 = 2\times\pi\times P_{max}\times r_2(r_1 - r_2) \quad [\because c = P_{max}\times r_2]$$

$$600 = 2\times\pi\times P_{max}\times 100(200-100)$$

$$600 = 2\times\pi\times P_{max}\times 0.1(0.2-0.1)$$

$$P_{max} = 9.55\times 10^3 \text{ N/m}^2$$

Torque transmitted by a single friction surface is given by.

$$T = M\times w\left(\frac{r_1 + r_2}{2}\right)$$

$$= 0.25\times 600\left(\frac{0.2+0.1}{2}\right)$$

$$\left.\begin{array}{l}\text{torque required}\\ \text{per surface}\end{array}\right\} T = 22.5\text{Nm}$$

The total torque is required can be calculated as given below

Power P=2πNT/60

$30\times10^3 = 2\times\pi\times360\times T/60$

T=795.77 Nm

No.of friction surfaces required= total torque required/ torque required per surface

= 795.77/22.5

=35.36

=36

Total number of plates= no. of pairs of contact surface+1

=36+1=37

8. **A 50 Kg wheel , 0.5 m in diameter turning at 150 rpm is stationary bearing is brought to rest by pressing a brake shoe radially against the rim with a force of 100N. If the radius of gyration of wheel 0.2m. How many revolutions will the wheel make before coming to rest? Assume that the co-efficient of friction between shoe and rim has the steady value of 0.25.**

Given data:

$m = 50Kg$

$D = 0.5m.$

$N = 150rpm.$

$F = 100N.$

$K = 0.2m.$

$M = 0.25$

Take, l=0.28m

$\theta = 60^\circ$ From PSGDB 7.129

Step 1: To find Braking torque.

$$\theta = 60 \times \frac{\pi}{180}$$

$$\theta = 1.05 rad.$$

Tension ratio is given by

$$\frac{T_1}{T_2} = e^{M\theta} = e^{0.25\times1.05}$$

$$T_1 = 1.3T_2 \quad 1$$

Moments about the fulcrum 0,

$$F \times l = T_1 \times a$$

$$100 \times 0.28 = T_1 \times 0.25$$

$$T_1 = 112Nm.$$

From eqn 1 $T_1 = 1.3T_2$

$$112 = 1.3T_2$$

$$T_2 = 86.15Nm.$$

Braking torque is given by $T_B = (T_1 - T_2)r$

$$= (112 - 86.15)0.25$$

$$= 6.46Nm.$$

Step 2: No. of turns of flywheel before it comes to rests. (n):

We know that kinetic energy of flywheel.

$$K.E = \frac{1}{2} \times mK^2 \times w^2$$

$$= \frac{1}{2} \times 50 \times (0.2)^2 \times \left(\frac{2 \times \pi \times 150}{60}\right)^2$$

$$K.E = 246.74Nm.$$

This kinetic energy is used to overcome the work done due to braking torque (T_B).

$$K.E = T_B \times w$$

$$246.74 = 6.46 \times 2 \times \pi \times n.$$

$n = 6.08$ revolutions.

9.A multiple clutch, steel on bronze is to transmit 6KW power at 750 rpm. The inner radius of contact surface is 4 cm and outer radius is 7cm. The clutch plates operate in oil, so the co – efficient of friction is 0.1. The average pressure is 0.35 N/mm^2. Determine (i) The total number of steel and bronze friction discs. (ii) Actual axial force required. (iii) Actual average pressure (iv)Actual maximum pressure

Given data:

$p = 6kW$
$N = 750rpm$
$r_2 = 4cm = 40mm$
$r_1 = 7cm = 70mm$
$M = 0.1$
$P_{avg} = 0.35N/mm^2$

Step 1:- To find P_1 and P_2.

$$P_{avg} = \frac{P_1 + P_2}{2} = 0.35$$
$$P_1 + P_2 = 0.7 \qquad ---(1)$$
$$P_1 r_1 = p_2 r_2 = c$$
$$\frac{P_1}{P_2} = \frac{r_1}{r_2} = \frac{40}{70}$$
$$P_1 = \frac{4}{7} \times P_2 \qquad ---(2)$$

Solving (1) and (2)

$$P_2 = 0.4454\ N/mm^2 = P_{max}$$
$$P_1 = 0.2545$$

Step 2:- To find 'c'

$$c = P_{max} \times r_2$$
$$= 0.4454 \times 40$$
$$c = 17.82\ N/mm$$

Step 3:- To find axial force:

$$W = 2\pi c(r_1 - r_2)$$
$$= 2 \times \pi \times 17.82(70 - 40)$$
$$W = 3358.99\ N$$

Step 4:- To find the torque transmitted by a single friction surface:

$$T = M \times W \times \left(\frac{r_1 + r_2}{2}\right)$$
$$= 0.1 \times 3358.99 \times \left(\frac{70 + 40}{2}\right)$$
$$T = 18474.45\ N/mm$$

Step 5:- To find total torque

$$P = \frac{2\pi NT}{60}$$
$$6 \times 10^3 = \frac{2 \times \pi \times 750 \times T}{60}$$
$$T = 76.39\ Nm$$

Step 6:- To find number of friction surface required

$$n = \frac{76.39}{18.48} = 4.13 \approx 5$$

Step 7:- To find total number of plates

Total number of plates = 5+1 =6 surface

Step 8:- To find the actual torque (T_1)

$$n = \frac{T}{T_1}$$

$$5 = \frac{76.39}{T_1}$$

$$T_1 = 15.278 \text{ Nm}$$

Step 9:- To find the actual axial force (W)

$$T_1 = MW\left(\frac{r_1 + r_2}{2}\right)$$

$$15.278 \times 10^3 = 0.1 \times W\left(\frac{70+40}{2}\right)$$

$$W = 2777.82N$$

Step 10:- To find actual average pressure:

$$W = 2\pi c(r_1 - r_2)$$

$$2777.82 = 2 \times \pi \times c(70 - 40)$$

$$c = 14.73 \text{ N / mm}$$

$$\Rightarrow c = P_{max} \times r_2$$

$$14.73 = P_{max} \times 40$$

$$\left.\begin{matrix}\text{Actual maximum} \\ \text{pressure}\end{matrix}\right\} P_{max} = 0.368 \text{ N / mm}^2$$

Step 11:- To find actual average pressure:

$$P_1 = \frac{4}{7} P_2$$

$$P_1 = \frac{4}{7} \times 0.3684 = 0.2105 \text{ N / mm}^2$$

$$P_{avg} = \frac{P_1 + P_2}{2} = \frac{0.2105 + 0.3684}{2}$$

$$P_{avg} = 0.2895 \text{ N / mm}^2$$

10. A single shoe brake is shown in figure. The diameter of drum is 250 mm and angle of contact is 90°. If the operating force of 750 N is applied at the end the of the lever and M = 0.35, Determine the torque that may be transmitted by the brake.

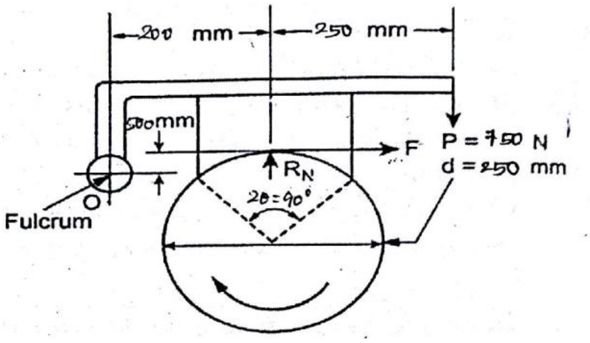
Given data:

$$d = 250 \text{ mm or } r = 125 \text{mm}$$

$$20 = 90° = \pi/2 \text{ rad}$$

$$P = 750N$$

$$M = 0.35$$



Since 20 > 90° therefore equivalent co – efficient of friction

$$M = \frac{4M\sin\theta}{2\theta + \sin 2\theta}$$
$$= \frac{4 \times 0.35 \times \sin 45^\circ}{\pi/2 + \sin 90^\circ}$$
$$= 0.385$$

Taking moments about the fulcrum 0, we get,

$$750(250 + 200) + F \times 500 = R_{IN} \times 200 = \frac{F}{\mu'} \times 200$$
$$337500 + F(500) = \frac{F}{0.385} \times 200 = 520F$$
$$F = \frac{337500}{19.48}$$
$$F = 17325.46N$$

Torque transmitted by the block brake $\} T_B = F - r$

$$= 17325.46\ N - 0.125$$
$$= 2165.68\ NM$$

11. A multi plate clutch with both sides effective transmits 30KW at 360rpm. Inner and Outer radii of the clutch discs are 100mm and 200 mm respectively. The effective co-efficient of friction is 0.25. An axial load of 600N is applied. Assuming uniform wear conditions, find the number of discs required and the maximum intensity of pressure developed. (April/May 2017)

Given data:

$P = 30KW$

$N = 360rpm.$

$r_2 = 100mm.$

$r_1 = 200mm$

$M = 0.25$

$w = 660N.$

Assuming uniform wear, axial force exerted is given by,

$$w = 2\pi c(r_1 - r_2).$$

$$600 = 2\times\pi\times P_{max}\times r_2(r_1 - r_2)\ [\because c = P_{max}\times r_2]$$

$$600 = 2\times\pi\times P_{max}\times 100(200-100)$$

$$600 = 2\times\pi\times P_{max}\times 0.1(0.2-0.1)$$

$$P_{max} = 9.55\times 10^3\ N/m^2$$

Torque transmitted by a single friction surface is given by.

$$T = M\times w\left(\frac{r_1 + r_2}{2}\right)$$

$$= 0.25\times 600\left(\frac{0.2+0.1}{2}\right)$$

$$\left.\begin{matrix}\text{torque required}\\ \text{per surface}\end{matrix}\right\} T = 22.5Nm$$

The total torque is required can be calculated as given below

Power P=2πNT/60

$30x10^3$= 2xπx360xT/60

T=795.77 Nm

No.of friction surfaces required= total torque required/ torque required per surface

= 795.77/22.5

=35.36

=36

Total number of plates= no. of pairs of contact surface+1

=36+1=37

12. A single shoe brake is shown in figure. The diameter of drum is 250 mm and angle of contact is 90°. If the operating force of 750 N is applied at the end that of the lever and M = 0.35, Determine the torque that may be transmitted by the brake. (April/May 2017)

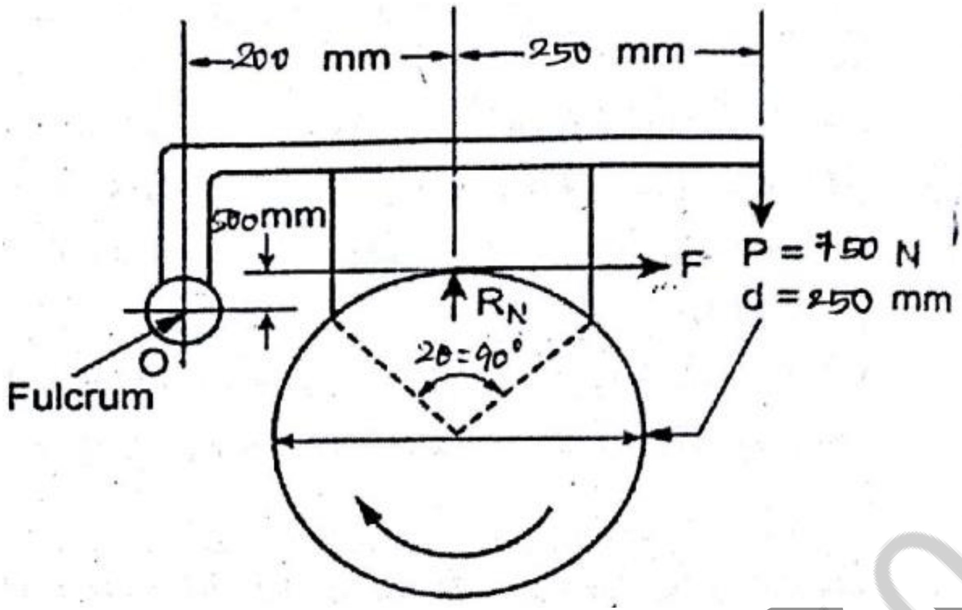
Given data:

$$d = 250\ mm\ or\ r = 125mm$$

$$20 = 90° = \pi/2\ rad$$

$$P = 750N$$

$$M = 0.35$$



Since 20 > 90° therefore equivalent co – efficient of friction

$$M = \frac{4M\sin\theta}{2\theta + \sin 2\theta}$$
$$= \frac{4 \times 0.35 \times \sin 45°}{\pi/2 + \sin 90°}$$
$$= 0.385$$

Taking moments about the fulcrum 0, we get,

$$750(250 + 200) + F \times 500 = R_{IN} \times 200 = \frac{F}{\mu'} \times 200$$

$$337500 + F(500) = \frac{F}{0.385} \times 200 = 520F$$

$$F = \frac{337500}{19.48}$$

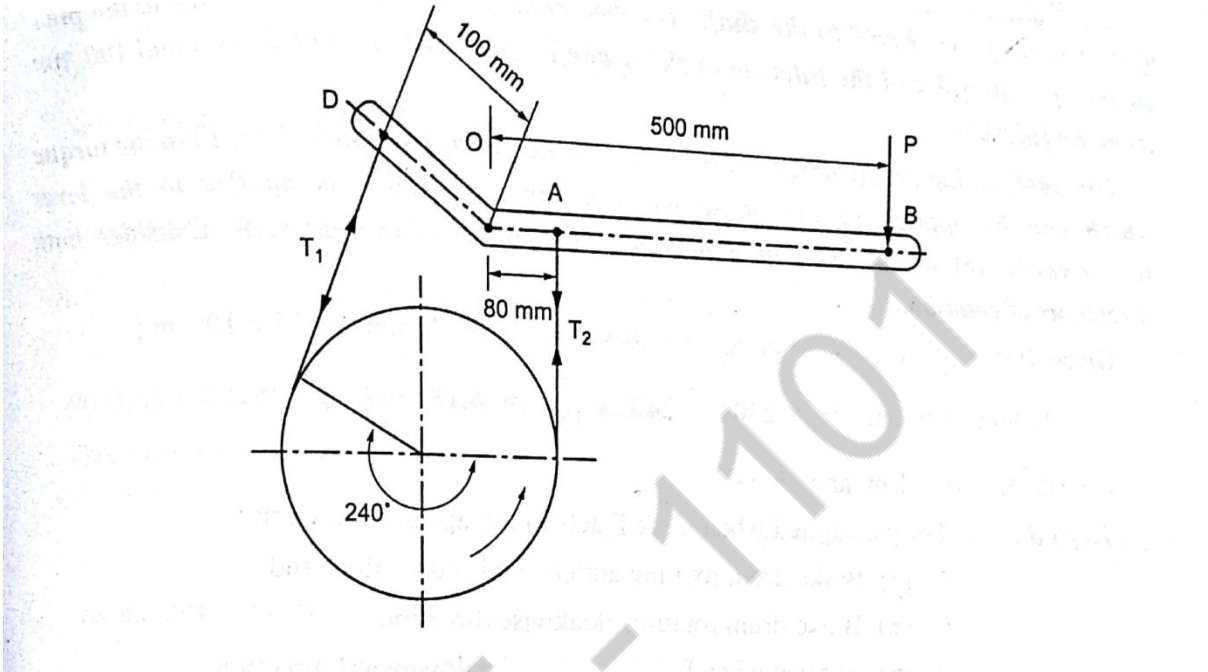
$$F = 17325.46N$$

Torque transmitted by the block brake $\left.\right\} T_B = F - r$

$$= 17325.46\ N - 0.125$$
$$= 2165.68\ NM$$

13. A differential band brake is operated by the lever of length 500mm. the brake drum has a diameter of 500mm and the maximum torque on the drum is 100Nm. The band brake embraces 2/3rd of the circumference. One end of the band is attached to a pin 100mm from the fulcrum and the other end to the pin 80mm from the fulcrum and on the other side of it when operating force is also acting. Coefficient of friction 0.3. Find the operating force. Design the steel band, shaft, and key. The permissible

stresses may be taken as 70MPa in tension, 50MPa in shear and 20MPa i9n bearing. The bearing pressure for the brake lining should not exceed 0.2N/mm^2. (Nov/Dec 2017)



Given Data:

d=500mm, T_B =1kN-m, μ=0.3, a=OB=500mm, b=OA=80mm, c=OD=100mm, θ=240°=240xπ/180=4.188 rad

Solution:

To find the operating force

We know that the tension ratio is given by

$$\frac{T_1}{T_2} = e^{\mu x\theta}$$

$$\frac{T_1}{T_2} = e^{0.3x4.188}$$

T_1=3.5136 T_2 -----1

Braking torque is given by

$$T_B = (T_1 - T_2)x\, r$$

$$1x10^3 = (T_1 - T_2)x\, 0.25$$

$$(T_1 - T_2) = 4000 \quad \text{-------2}$$

Solving the equations 1 & 2

$$T_1 = 5593.6\ N\ and \qquad T_2 = 1593.6N$$

Taking moments about the fulcrum O, we get

$$Px\ 0.500 = T_1 x 0.100 - T_2 x 80x10^{-3}$$

$$Px\ 0.500 = 5593.6x0.100 - 1593.6x80x10^{-3}$$

Operating force $P = 863.34\ N$

14. A Power of 20 KW is to be transmitted through a cone clutch at 500 rpm. For uniform wear condition, find the main dimensions of clutch and shaft. Also determine the axial force required to engage the clutch. Assume co-efficient of friction as 0.25, the maximum normal pressure on the friction surface is not to exceed 0.08 Mpa, and take the design stress for the shaft materials as 40 Mpa. (Nov/Dec 2017)

Given data:

$P = 20KW$
$N = 500rpm$
$M = 0.25$
$P_{max} = 0.08mpa$
$P_{shaft} = 40mpa.$

Step 1: To find the Torque transmitted.

$$T = \frac{60P}{2\pi N}$$

$$= \frac{60 \times 20 \times 10^3}{2 \times \pi \times 500}$$

$$T = 382\ Nm.$$

Step 2: To find b, R, r_1 and r_2.

$b = \frac{R}{2}$, semi cone angle α=15° For cone clutch $\frac{r_1 - r_2}{b} = \sin\alpha$

$$b = \frac{r_1 - r_2}{\sin\alpha}$$

Mean radius $R = \frac{r_1 + r_2}{2}$

$$\therefore \quad \frac{r_1 - r_2}{\sin\alpha} = \frac{r_1 + r_2}{4}$$

$$\frac{r_1 - r_2}{\sin 15} = \frac{r_1 + r_2}{4}$$

By solving the above equation $r_1 = 1.139\ r_2$ 1

Design Torque $[T] = T \times K_s$

Assume $K_s = 2.5$

$$[T] = 382 \times 2.5$$

$$= 955\ \text{Nm.}$$

For uniform wear,

$$[T] = 2\pi M P_{max} P_1^2 b$$

$$955 \times 10^3 = 2 \times \pi \times 0.25 \times 0.08 \times R^2 \times \frac{R}{2}$$

$$R = 247.7\text{mm.}$$

Also, $R = \dfrac{r_1 + r_2}{2}$

$$247.7 = \frac{r_1 + r_2}{2}$$

$r_1 + r_2 = 495.4$ 2

Solving 1 and 2

Inner radius $r_1 = 264\text{mm.}$

Outer radius $r_2 = 231.6\text{mm.}$

Face width $b = \dfrac{R}{2}$

$$= \frac{247.7}{2}$$

$$b = 123.85\text{mm.}$$

Step 3: To find axial force required to engage the clutch (W).

$$W = 2\pi c(r_1 - r_2).$$

$$= 2\pi \times P_{max} r_2 (r_1 - r_2) \quad [\because c = P_{max} \times r_2] .$$

$$= 2 \times \pi \times 0.08 \times 10^6 \times 0.2316(0.264 - 0.2316).$$

$$= 3771.84 \text{ N.}$$

Step 4: To find the diameter of the shaft:

$$[T] = \pi/16 \times d_s^3 \times P_{shaft}$$

$$955 = \pi/16 \times d_s^3 \times 40 \times 10^6$$

$$d_s = 0.0495 \text{ m} .$$

$$d_s = 49.5 \text{ mm.}$$

15. A single disc clutch having one pair of contacting surface is required to transmit 10kW at 720 rpm under normal operating condition. Due to space limitation the outer diameter should be limited to 250 mm. the coefficient of friction is 0.25 and the permissible intensity of pressure is 0.5 N/mm² . Use (a) Uniform pressure theory (b) Uniform wear theory and determine the clutch dimensions. (April/May 2018)

Given Data:

P=10kW

N=720rpm

μ=0.25, $p_a = 0.5\ MPa$, , D=2$R_o = 250\ mm$

For Single plate, z=1

Assumption: Use the hardened steel palte for clutch with single disk

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{60x10x10^3}{2x\pi x720}$$

T= 132.64 Nm

1. Uniform Pressure Theory

$$T = \frac{1\pi\mu x p_a}{12} x(D^3 - d^3)$$

$$132.64x1000 = \frac{1x\pi x0.25x0.5}{12}x(250^3 - d^3)$$

d=226.18 mm

2. Uniform Wear Theory

$$T = \frac{\pi\mu x p_a d}{12}x(D^2 - d^2)$$

$$132.64x1000 = \frac{\pi x0.25x0.5xd}{12}x(250^2 - d^2)$$

Rearranging the terms, we get

$$d\ (250^2 - d^2) = 270211.9$$

d^3-($250^2 d + 270211.9$= 0

By solving the above equation d= 224.66= 225 mm

16. A single block brake as shown in Figure, has the drum diameter 250 mm. The angle of contact is 90° and the coefficient of friction between the drum and lining is 0.35. if the torque transmitted by the brake is 80000 Nmm, find the force required to operate the brake. (April/May 2018)

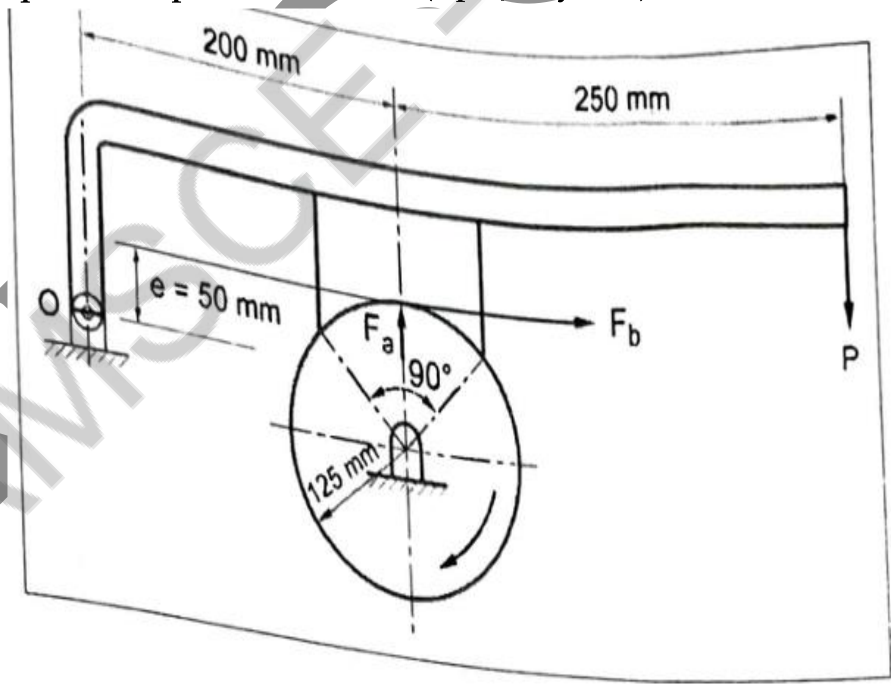


Fig.1

Given data: $d = 250$ mm or $r = 125$ mm $= 0.125$ m; $2\theta = 90° = 90° \times \frac{\pi}{180°} = \frac{\pi}{2}$ rad; $\mu = 0.35$; $T_B = 80{,}000$ N.mm or 80 N.m; $a = 200$ mm $= 0.2$ m; $b = 250$ mm $= 0.25$ m; $c = 50$ mm $= 0.05$ m; $l = 450$ mm $= 0.45$ m.

Solution: Since the angle of contract is more than 40°, therefore equivalent coefficient of friction

$$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta} = \frac{4 \times 0.35 \times \sin 45°}{\left(2 \times \frac{\pi}{2}\right) + \sin 90°} = 0.239$$

Then the braking torque is given by

$$T_B = \mu' R_N r$$

or $$80 = 0.239 \times R_N \times 0.125 \quad \text{or} \quad R_N = 2677.82 \text{ N}$$

When the rotation of drum is clockwise:

Taking moment about fulcrum, we get

$$P \cdot l = R_N \cdot a + \mu' R_N \times c$$

or $$P \times 0.45 = (2677.82 \times 0.2) + (0.239 \times 2677.82 \times 0.05)$$

or $$P = \mathbf{1261 \text{ N}}$$ **Ans.**

17. A single plate clutch transmits 25kW at 900rpm. The maximum pressure intensity between the plates is 85 kN/m². The ratio of radii is 1.25. both the sides of the plates are effective and the coefficient of friction 0.25. Determine (i) The inner diameter of the plate and (ii) The axial force to engage the clutch. Assume theory of uniform wear. (Nov/Dec 2018)

Given Data : $P = 25\text{ kW} = 25 \times 10^3\text{ W}$; $N = 900\text{ r.p.m.}$; $r_1/r_2 = 1.25$; $n = 2$; $p_{max} = 85\text{ kN/m}^2 = 85 \times 10^3\text{ N/m}^2$; $\mu = 0.25$.

Solution : (i) The inner diameter of the plate :

We know that the power transmitted, $P = \dfrac{2\pi N T}{60}$

$$25 \times 10^3 = \frac{2\pi \times 900 \times T}{60}$$

or $T = 265.26$ N-m

Since the intensity of pressure is maximum at the inner radius (r_2),

$\therefore \quad p_{max} \cdot r_2 = C$ or $C = 85 \times 10^3\, r_2$ N/mm

and the axial thrust transmitted to the frictional surface,

$$W = 2\pi C(r_1 - r_2) = 2\pi \times 85 \times 10^3\, r_2 (1.25 r_2 - r_2) \quad \ldots [\because r_1 = 1.25 r_2]$$

$$= 1.335 \times 10^5 (r_2)^2$$

The mean radius for uniform wear is given by

$$R = \frac{r_1 + r_2}{2} = \frac{1.25 r_2 + r_2}{2} = 1.125 r_2$$

Torque transmitted, $T = n \cdot \mu \cdot W \cdot R$

$$265.26 = 2 \times 0.25 \times 1.335 \times 10^5 (r_2)^2 \times 1.125 r_2$$

$$= 75.104 \times 10^3 r_2^3$$

or $r_2 = 0.1523$ m or 152.3 mm

and $r_1 = 1.25 r_2 = 1.25 \times 152.3 =$ **190.375 mm Ans.**

(ii) The axial force to engage the clutch :

$$W = 2\pi C(r_1 - r_2) = 1.335 \times 10^5 (r_2)^2 = 1.335 \times 10^5 (0.1523)^2$$

$=$ **3096.57 N Ans.**

18. Determine the capacity and the main dimensions of a double block brake for the following data. The brake sheave is mounted on the cast iron drum shaft. The hoist with its load weighs 45kN and moves downwards with a velocity of 1.15 m/s. the pitch diameter of the hoist drum is 1.2m. the hoist must be stopped within a distance of 3.25m. The kinetic energy of the drum maybe neglected. Assume sintered metal block shoe, equal friction force on each shoe, continuous service and poor heat condition. (Nov/Dec 2018)

Given Data : Load = 45 kN ; v = 1.15 m/s ; D = 1.25 m ; x = 3.25 m.

To find : Capacity and main dimensions of a double block brake.

Solution :

1. Calculation of the total energy absorbed by the brake :

The various sources of energy to be absorbed are :

(a) Kinetic energy of translation $= \frac{1}{2} mv^2 = \frac{1}{2} m(v_1^2 - v_2^2)$

where v = Velocity at the time of applying the brake, and

v_1 and v_2 = Initial and final velocities of the load.

(b) Potential energy = Weight × Vertical distance = W × x

(c) Kinetic energy of rotation $= \frac{1}{2} I\omega^2$

∴ Total energy, $E_T = \frac{1}{2} m(v_1^2 - v_2^2) + W \cdot x + \frac{1}{2} I\omega^2$

Neglecting the kinetic energy of the drum,

$$E_T = \frac{1}{2} m(v_1^2 - v_2^2) + W \cdot x$$

Initial velocity of load, v_1 = 1.15 m/s ... [Given]

and final velocity of load, v_2 = 0

$$\therefore E_T = \frac{1}{2} \times \frac{45000}{9.81} (1.15^2 - 0^2) + (45000 \times 3.25)$$

$= \mathbf{149.283\ kN\text{-}m}$ **Ans.**

2. Calculation of braking torque (or torque capacity) :

Braking torque in terms of energy absorbed is given by

$$T_B = \frac{60\, E_T}{\pi \times N_1 \times t}$$

where N_1 = Initial speed of brake drum, and

t = Time of application of brake.

Given that, the distance travelled by the load, $x = 3.25$ m

But we know that, $x = \frac{1}{2}(v_1 + v_2)\, t$

or $3.25 = \frac{1}{2}(1.15 + 0) \times t$

or Time of application of brake, $t = 5.652$ sec

Initial speed of brake drum, $N_1 = \frac{60 \times v_1}{\pi D} \quad \left[\because v_1 = \frac{\pi D N_1}{60}\right]$

$$= \frac{60 \times 1.15}{\pi \times 1.25} = 17.57 \text{ r.p.m.}$$

$\therefore$ Braking torque, $T_B = \frac{60 \times 149.283 \times 10^3}{\pi \times 17.57 \times 5.652} =$ **28.71 kN-m Ans.**

3. Calculation of initial braking power :

We know that, Braking power, $P = \frac{2\pi N_1 T_B}{60}$

$$= \frac{2\pi \times 17.57 \times 28.71 \times 10^3}{60} = \mathbf{52.82\ kW}$$ **Ans.**

4. Selection of brake drum diameter : Assume a brake drum diameter = 1.5 m

5. Selection of brake drum and block shoe material : *A cast iron brake drum* and sintered metal block shoe may be chosen, from Table 11.1.

From Table 11.1, safe value of coefficient of friction, $\mu = 0.15$.

6. Selection of induced bearing pressure : From Table 11.2, for continuous service, poor heat condition,

$pv = 1.05$ (MPa) m/s is selected.

$\therefore \quad pv \leq 1.05$ (MPa) m/s

or $\quad p \leq \frac{1.05}{1.15} \leq 0.913$ MPa

But from Table 11.1, $p_{max} = 2.8$ MPa

Therefore let us use, bearing pressure $p = 2.5$ MPa.

7. Calculation of projected area of the shoe :

We know that induced bearing pressure, $p = \frac{R_N}{A}$

where R_N = Normal force, and

A = Projected area of the shoe.

To find R_N : Assume equal friction force on each shoe.

$$\text{Braking torque} = F \times \frac{D}{2} \times 2$$

$$28.71 \times 10^3 = F \times \frac{1.5}{2} \times 2$$

or $$\text{Friction force, } F = 19140 \text{ N}$$

$\therefore$ $$\text{Normal reaction, } R_N = \frac{F}{\mu} = \frac{19140}{0.15} = 127.6 \text{ kN}$$

Therefore, projected area of the shoe, $A = \frac{R_N}{p} = \frac{127.6 \times 10^3}{2.5 \times 10^6} = 0.051 \text{ m}^2$

8. Calculation of breadth and width of the shoe : Assuming breadth (b) of the block shoe s twice its width (w).

$\therefore$ Projected area of the shoe, $A = \text{Breadth} \times \text{Width} = 2\,w \times w = 2\,w^2$

$$0.051 = 2\,w^2$$

or $$\text{Width, } W = 0.15968 \text{ m or } \mathbf{159.68 \text{ mm}}$$ **Ans.**

and $$\text{Breadth, } b = 2\,w = 2 \times 159.68 = \mathbf{319.37 \text{ mm}}$$ **Ans.**

19. A multiple disc clutch is to be designed for a machine tool driven by an electric motor of 12.5 kW running at 1440 rpm the frequency of clutch engagement is 6/ hr and the machine tool is to operate continuously 8hrs/day. Determine the appropriate values for disc inside diameter, outside diameter, total number of discs and clamping force. (April/May 2019)

Given Data : P = 22 kW = 22×10^3 W ; N = 1440 r.p.m. ; d_2 = 130 mm or r_2 = 65 mm ; $p_{max} = 0.1\ \text{N/mm}^2 = 0.1 \times 10^6\ \text{N/m}^2$.

To find : Design the clutch (*i.e.*, determine the outside diameter of disc, total number of discs, and clamping force).

☺ ***Solution :*** Assume uniform wear.

1. Outside diameter of disc (d_1) : We know that the torque transmitted,

$$T = \frac{P \times 60}{2\pi N} = \frac{22 \times 10^3 \times 60}{2 \times \pi \times 1440} = 145.89\ \text{N-m}$$

$\therefore$ Design torque, $[T] = T \times k_s$

where Service factor, $k_s = k_1 + k_2 + k_3 + k_4$

From Table 10.2, $k_1 = 0.5$ (for electric motor)

From Table 10.3, $k_2 = 1.25$ (for machine tools)

From Table 10.4, $k_3 = 0.38$ (for 1440 r.p.m.)

From Table 10.5, $k_4 = 0.75$ (assuming 32 engagements / shift)

$\therefore$ $k_s = 0.5 + 1.25 + 0.38 + 0.75 = 2.88$

Design torque, $[T] = 145.89 \times 2.88 = 420.16$ N-m.

We know that maximum intensity of pressure (p_{max}) is at the inner radius (r_2). Therefore,

$$p_{max} \cdot r_2 = C \text{ or } C = 0.1 \times 10^6 \times 65 \times 10^{-3} = 6500\ \text{N/m}$$

For uniform wear, axial force exerted is given by

$$W = 2\pi C (r_1 - r_2)$$
$$= 2\pi \times 6500 (r_1 - 65 \times 10^{-3}) = 40840.7 (r_1 - 0.065) \quad \ldots(i)$$

We know that number of pairs of contact surfaces,

$$n = n_1 + n_2 - 1 = 3 + 2 - 1 = 4$$

$$\therefore \text{Torque transmitted, } [T] = n \cdot \mu \cdot W \left(\frac{r_1 + r_2}{2}\right)$$

$$420.16 = 4 \times 0.3 \times 40840.7 (r_1 - 0.065) \left(\frac{r_1 + 0.065}{2}\right)$$

or $$420.16 = 24504.42 (r_1^2 - 4.225 \times 10^{-3})$$

or $$r_1 = 0.14618\ \text{m or } 146.18\ \text{mm}$$

$\therefore$ Outer diameter of the disc, d_1 = **292.37 mm Ans.**

2. Total number of disc :

Total number of disc = Number of pairs of contact surface + 1 = $n + 1$

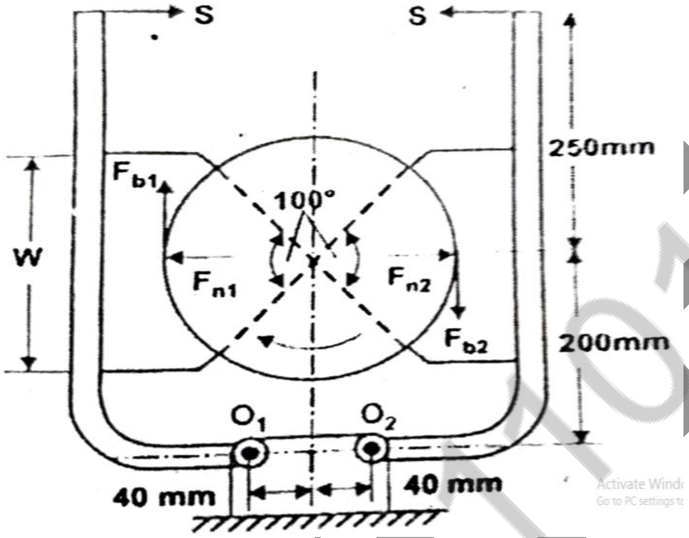
= 4 + 1 = **5 Ans.**

3. Clamping force (or axial force exerted) :

Substituting the value of r_1 in equation (i), we get

Axial force exerted, W = 40840.7 (0.14618 − 0.065) = **3315.45 N Ans.**

20. A double shoe brake as shown in the figure is capable of absorbing a torque of 1500N-m. The diameter of the brake drum is 400mm and the angle of contact for each shoe is 100°. If the coefficient of friction between the brake drum and lining is 0.4, find (i) The spring force necessary to set the brake and (ii) The width of the brake shoe, if the bearing pressure on the lining material is not exceed 0.3 N/mm² (April/May 2019)



Given Data : S = 3500 N ; d = 350 mm or r = 175 mm = 0.175 m ; $2\theta = 120° = 120 \times \frac{\pi}{180} = 2.094$ rad ; $\mu = 0.35$; $p = 0.3$ N/mm².

To find : (i) Torque absorbed by the brake, and

(ii) Width of the brake shoe.

☺ *Solution :* Since angle of contact (2θ) is greater than 40°, therefore, the equivalent coefficient of friction is given by

$$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta} = \frac{4 \times 0.35 \times \sin 60°}{2.094 + \sin 120°} = 0.409$$

(i) Torque absorbed by the brake (T_B) :

Consider the left hand side brake shoe :

Taking moments about O_2, we get

$$S\,(25 + 20) = R_{N2} \times 20 - F_2\,(17.5 - 4)$$

$$45\,S = R_{N2} \times 20 - F_2 \times 13.5 \qquad \left[\because F_2 = \mu' R_{N2} ;\; R_{N2} = \frac{F_2}{\mu'} = \frac{F_2}{0.409}\right]$$

$$= \left[\frac{20}{0.409} - 13.5\right] F_2$$

$$45 \times 3500 = 35.4\,F_2$$

or $$F_2 = 4449.2\text{ N}$$

Consider right hand side brake shoe :

Taking moments about fulcrum O_1, we get

$$S(20+25) = R_{N1} \times 20 + F_1(17.5-4)$$

$$45\,S = F_1\left[\frac{20}{0.409} + 13.5\right] \qquad \left[\because F_1 = \mu' R_{N1};\ R_{N1} = \frac{F_1}{\mu'} = \frac{F_1}{0.409}\right]$$

$$45 \times 3500 = 62.39\,F_1$$

or $$F_1 = 2524.1 \text{ N}$$

Braking torque T_B is given by, $T_B = (F_1 + F_2)\,r$

$$= (2524.1 + 4449.2) \times 0.175$$

$$= \mathbf{1220.32 \text{ N-m}}$$ **Ans.**

(ii) Width of the brake shoe (b) :

Let b = Width of the brake shoes in mm

We know that projected bearing area for one shoe,

$$A = 2\,r\,b\sin\theta \qquad \text{... [From equation (11.7)]}$$

$$= 2 \times 175 \times b \times \sin 60° = 303.11\,b \text{ mm}^2$$

∴ Normal force on the right hand side of the shoe,

$$R_{N1} = \frac{F_1}{\mu'} = \frac{2524.1}{0.409} = 6171.39 \text{ N}$$

and normal force on the left hand side of the shoe,

$$R_{N2} = \frac{F_2}{\mu'} = \frac{4449.2}{0.409} = 10878.24 \text{ N}$$

Since the maximum normal force is on the left hand side of the shoe, therefore we have t design the shoe for R_{N2} *i.e.*, the maximum normal force.

We know that the bearing pressure on the lining material,

$$p = \frac{R_{N2}}{A} \qquad \text{... [From equation (11.8}$$

$$0.3 = \frac{10878.24}{303.116}$$

or breadth of the block shoe, b = **119.63 mm Ans.**