UNIT – I (SNME)

PART – A

1) Write an expression of volumetric strain for a rectangular bar subjected to an axial load P. (Nov/Dec 2018)

$$e_v = \frac{\delta l}{l}(1-2\mu)$$

2) What does the radius of mohr's circle refer to? (May/June 2017)

The radius of mohr's circle refers to the maximum shear stress.

3) Define principle plane (May /June 2016)

The plane which have no shear stress are known as principle planes.

4) Obtain the relation between E and K (May/June 2016) (Apr/May 2018)

$$E = 3K\left(1-\frac{2}{m}\right) = 3K\left(1-2\mu\right)$$

$$E \rightarrow \text{Young's modulus } \left(N/mm^2\right)$$

$$K \rightarrow \text{Bulk modulus } \left(N/mm^2\right)$$

$$\frac{1}{m} = \mu \rightarrow \text{poisson's ratio}$$

5) Differentiate elasticity and elastic limit (Nov/Dec 2015)

Elasticity

The body which regains its original position on the removal of the force that property is known as Elasticity

Elastic limit

There is always a limiting values of load upto which the strain totally disappears on the removal of load the stress corresponding to this load is known as Elastic limit

6) What is principle of super position? (Nov/Dec 2015)

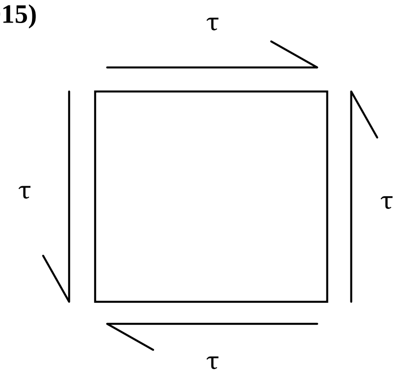
In some cases, interior cross section of a body subjected to external axial forces. In such cases, the forces are split up and their effects are considered on individual section. The total deformation is equal to the algebraic sum of the deformation individual section. This principle of finding the resultant deformation is known as principle of super position.

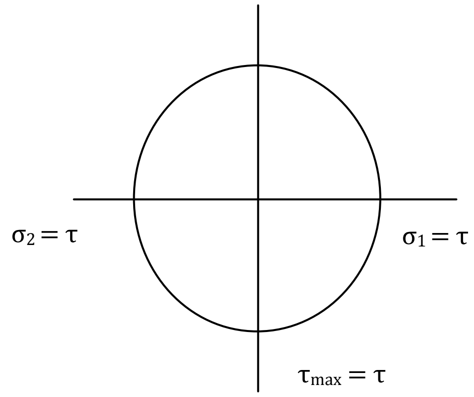
$$\delta l = \frac{P_1 l_1 + P_2 l_2 + P_3 l_3 + ...}{AE}$$

7) What do you mean by thermal stresses?(Apr/ May 2015) (Apr/ May 2019)

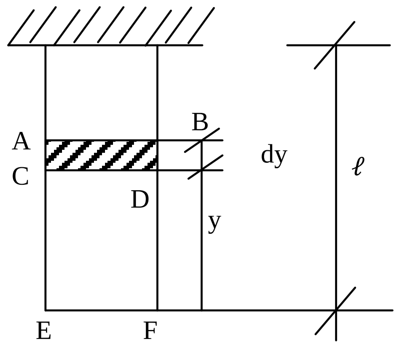
When the temperature varies, the bar will tends to expands or contracts, but the same is prevented by external forces or by fixing the bar ends, the temperature stress will be produced in that bar.

8) Draw the Mohr's circle for the state of pure shear in a strained body and mark all salient points in it (Apr/ May 2015)





9) Derive a relation for change in length of a bar hanging freely under its own weight (May / June 2017) (Nov/ Dec 2014)



A bar of length - l (meter)

area – A (m^2)

Fixed at one end ρ – kg/m^3

Force acting down at CD= weight of bar CDEF = Aρy × 9.81

$$\sigma = \frac{\text{Force at CD}}{A} = \frac{\cancel{A}\, y\rho \times 9.81}{\cancel{A}}$$

$$\sigma = 9.81\rho y \ \ N/m^2$$

$$\sigma \ \alpha \ y \ [\text{stress is directly propotional to } y]$$

$$\text{Strain in length } dy = \frac{\sigma}{E} = \frac{9.81\rho y}{E}$$

$$\text{Elangation in } dy = \frac{9.81\ \rho y}{lE} dy$$

$$\text{Total elangation of bar}(Sl) = \int_0^{\ell} \frac{9.81\ \rho y}{lE} dy$$

$$= \frac{9.81\ \rho y}{lE} \frac{y^2}{2}\Bigg]_0^{\ell} = \frac{9.81\rho\ell^2}{2E}$$

10) Write the relationship between shear modulus & young's modulus of elasticity (Nov/ Dec 2014)

$$E = 2G\left(1+\frac{1}{m}\right) = 2G(1+\mu)$$

11) Define young's modulus (Nov/ Dec 2016)

When a body is stressed within elastic limit, the ratio of stress is constant and that constant is known as Young's modulus.

12) What do you mean by principal planes and principal stress? (Nov/ Dec 2016) (Nov/ Dec 2017) (Apr/ May 2018) (Apr/ May 2019)

Principal plane:

The plane which have no shear stress are known as principal plane

Principal Stress:

The magnitude of normal stress, acting on a principal plane are known as principal stress

13) Define Bulk – modulus. (Nov/Dec 2017)

The ratio of direct stress to volumetric strain

K = Direct stress / Volumetric strain.

14) State Hooke's law

It states when a material is loaded, within its elastics limit, the stress is directly proportional to the strain.

15) Define strain energy

Whenever a body is strained, some amount of energy is absorbed in the body**.** The energy which is absorbed in the body due to straining effect is known as strain energy.

16) Define Poisson's ratio. (Nov/Dec 2018)

When a body is stressed, within it's elastic limit, the ratio of lateral strain to the longitudinal strain is constant for a given material. Poisson's ratio

$$\mu \text{ or } \frac{1}{m} = \text{Lateral strain } / \text{ Longitudinal strain}$$

17) What is compounds bar?

A composite bar composed of two or more different material joined together such that system is elongated or compressed in a single unit.

18) Define strain

When a body is subjected to an external force, there is some change of dimension in the body. Numerically the strain is equal to the ratio change in length to the original length of the body

Strain (e) = change in length / Original length $= \partial L / L$

19) Define stress

When an external forces acts on a body, it undergoes deformation. At the same time the body resists deformation. The magnitude of the resistance force is numerically equal to the applied force. This internal resistance force per unit area is called stress. Stress σ = Force/Area, P/A Unit N/mm^2.

20) Define shear stress and shear strain.

The two equal and opposite force act tangentially on any cross section plane of the body tending to slide one part of the body over the other part. The stress induced is called shear stress and corresponding strain is known as shear strain.

21) Define – Lateral strain.

The strain right to the direction of the applied load is called lateral strain.

22) Define – longitudinal strain

When a body is subjected to axis load P, the length of the body is increased**.** The axial deformation of the length of the body is called longitudinal strain.

23) A rod of diameter 30 mm and length 400 mm was found to eligible 0.35 mm. When it was subjected to a load of 65 KN. Compute the modulus of elasticity of material of this rod.

$$\delta l = \frac{P\ell}{AE} \Rightarrow E = \frac{P\ell}{A\delta l} = \frac{65\times10^3\times400}{\frac{\pi}{4}\times30^2\times0.35}$$

$$E = 105.09\times10^3 \ \ N/mm^2$$

24) The Young's modulus and the shear modulus of material are 120GPa and 45GPa respectively. What is it Bulk modulus?

$$E = 120\times10^9 \ N/m^2 \qquad G = 45\times10^9 \ N/m^2$$
$$= 120\times10^3 \ N/mm^2 \qquad = 45\times10^3 \ N/mm^2$$

$$E = \frac{9KG}{3K+G}$$
$$120\times10^3 = \frac{9K\times45\times10^3}{3K+45\times10^3}$$
$$120\times10^3\left[3K+45\times10^3\right] = 9K\times45\times10^3$$
$$3K+45\times10^3 = 3.375\ K$$
$$45\times10^3 = 0.375\ K$$
$$K = 120\times10^3 \ N/mm^2$$

PART – B

1) A steel rod of 3cm diameter and 5m long is connected to two grips and the rod is maintained at a temperature of 95^{0}C. Determine the stress and pull exerted when the temperature falls to 30^{0}C, if (i) the ends do not yield and (ii) the ends yield by 0.12cm. Take E=2x10^{5}MN/m^2 and α=12x10^{-6}/^{0}C (Apr/May 2019)

d=30mm

$A = \left(\pi/4\right) x d^2 = 225\pi \text{ mm}^2$

$L = 5000 mm$

$T_1 = 95^0C$

$T_2 = 30^0C$

$T = T_1 - T_2 = 65^0C$

(i) when the ends do not yield

stress = αTE = 156N/mm^2

Pull in the rod = stress x area = 156x225π = 110269.9 N

(ii) When the ends yield by 0.12cm (δ=1.2mm)

$$\text{stress} = \frac{(\alpha TL\text{-}\delta)}{L} xE = 108\text{N/mm}^2$$

Pull in the rod = stress x area = 108x225π = 76340.7 N

2) An elemental cube is subjected to tensile stresses of 30 N/mm^2 and 10 N/mm^2 acting on two mutually perpendicular planes and a shear stress of 10 N/mm^2 on these planes. Draw the Mohr's circle of stresses and hence or otherwise determine the magnitude and directions of principal stresses and also the greatest shear stress. (Apr/May 2019)

Major tensile stress (σ_1) = 30N/mm^2

Minor tensile stress (σ_2) = 10 N/mm^2

Shear stress (τ) = 10 N/mm^2

Location of principle planes,

θ = Angle, which one of the principle planes makes with the stress of 10 N/mm^2

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2} = \frac{2\times 10}{30-10} = 1$$

$$2\theta = \tan^{-1}(1) = 45° \text{ or } 225°$$

$$\theta = 22°5' \text{ or } 112° 5'$$

Principle stress

$$\text{Major principle stress} = \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{30+10}{2} + \sqrt{\left(\frac{30-10}{2}\right)^2 + 10^2}$$

$$= 20 + 14.14 = 34.14 \text{ N/mm}^2$$

Minor principle stress = $\frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$

$$= \frac{30+10}{2} - \sqrt{\left(\frac{30-10}{2}\right)^2 + 10^2}$$

$$= 20 - 14.14 = 5.86 \text{ N/mm}^2$$

3) A reinforced short concrete column 250mm x 250mm in section is reinforced with 8 steel bars. The total area of steel bars is 2500 mm^2**. The column carries a load of 390kN. If the modulus of elasticity of steel is 15 times that of concrete. Find the stresses in concrete and steel. (Nov/Dec 2018)**

$E_s = 15E_c$

$A_s = 2500 \text{ mm}^2$

Area of concrete column = 250x250 = 62500mm^2

$A_c = 62500 - 2500 = 60000 \text{ mm}^2$

P = 390000N

i)

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_s = \sigma_c \times \frac{E_s}{E_c} = 15\sigma_c$$

$$\boxed{\sigma_s = 15\ \sigma_c} \quad ...1$$

$P = \sigma_s A_s + \sigma_c A_c$

$390000 = 15\sigma_c \times 2500 + 60000\sigma_c$

$390000 = 97500\,\sigma_c$

$\sigma_c = 4 \text{N/mm}^2$

$\sigma_s = 60 \text{N/mm}^2$

4) The stresses at a point in a bar are 200 N/mm^2 (tensile) and 100 N/mm^2 (compressive). Determine the resultant stress in magnitude and direction on a plane inclined at 60^0 to the axis of major stress. Also determine the maximum shear stress in the material at the point. (Nov/Dec 2018)

Major Principal stress, σ_1=200 N/mm^2

Minor Principal stress, σ_2= - 100 N/mm^2

Angle inclined with major principal stress = 60^0

Angle inclined with minor principal stress θ = 90^0 - 60^0 = 30^0

Normal stress:

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta$$
$$\sigma_n = \frac{200 + (-100)}{2} + \frac{200 - (-100)}{2}\cos(2x30)$$
$$\sigma_n = 125 N/mm^2$$

Shear stress:

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta$$
$$\sigma_t = \frac{200 - (-100)}{2}\sin(2x30)$$
$$\sigma_t = 129.9 N/mm^2$$

Resultant stress:

$$\sigma_R = \sqrt{\left(\sigma_n{}^2 + \sigma_t{}^2\right)} = \sqrt{\left(125^2 + 129.9^2\right)} = 180.27 N/mm^2$$

Maximum shear stress:

$$(\sigma_t)_{max} = \frac{\sigma_1 - \sigma_2}{2}$$
$$(\sigma_t)_{max} = \frac{200 - (-100)}{2} = 150 N/mm^2$$
$$\tan\phi = \frac{\sigma_t}{\sigma_n} = 1.04$$
$$\phi = 46^0 6'$$

5) At a point in a strained material the principal stresses are 100 N/mm^2 (tensile) and 60 N/mm^2 (compressive). Determine the normal stress, shear stress and resultant stress on a plane inclined at 50^0 to the axis of major principal stress. Also determine the maximum shear stress at the point. (Nov/Dec 2017)

Major Principal stress, σ_1=100 N/mm^2

Minor Principal stress, σ_2= - 60 N/mm^2

Angle inclined with major principal stress = 50^0

Angle inclined with minor principal stress θ = 90^0 - 50^0 = 40^0

Normal stress:

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta$$
$$\sigma_n = \frac{100 + (-60)}{2} + \frac{100 - (-60)}{2}\cos(2x40)$$
$$\sigma_n = 33.89 N/mm^2$$

Shear stress:

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta$$

$$\sigma_t = \frac{100-(-60)}{2}\sin(2x40)$$

$$\sigma_t = 78.785 N/mm^2$$

Resultant stress:

$$\sigma_R = \sqrt{\left(\sigma_n{}^2 + \sigma_t{}^2\right)} = \sqrt{\left(33.89^2 + 78.785^2\right)} = 85.765 N/mm^2$$

Maximum shear stress:

$$(\sigma_t)_{max} = \frac{\sigma_1 - \sigma_2}{2}$$

$$(\sigma_t)_{max} = \frac{100-(-60)}{2} = 80 N/mm^2$$

6) A solid steel bar 40mm diameter 2m long passes centrally through a copper tube of internal diameter 40mm, thickness of metal 5mm and length 2m. The ends of the bar and tube are brazed together and a tensile load of 150kN is applied axially to the compound bar. Assume E_c= 100GN/m^2 and E_s= 200GN/m^2 Find the stresses and load sheared by the steel and copper section (Apr/May 2018)

$$d_s = 40 mm$$

$$t = 5mm$$

$$d_c = 40 mm$$

$$D_c = d_c + 2t = 40 + 2x5 = 50 mm$$

$$P = 150000N$$

$$\ell = 2m$$

$$E_s = 2\times10^5 N/mm^2$$

$$E_c = 1\times10^5 \ N/mm^2$$

$$A_s = \frac{\pi}{4} d_s{}^2$$

$$= \frac{\pi}{4}\times 40^2 = 1256.64 \ mm^2$$

$$A_c = \frac{\pi}{4}\left(D_c{}^2 - d_c{}^2\right) = \frac{\pi}{4}\left(50^2 - 40^2\right)$$

$$= 706.86 \ mm^2$$

i)

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_s = \sigma_c \times \frac{E_s}{E_c} = \frac{2\times10^5}{1\times10^5}\sigma_c$$

$$\boxed{\sigma_s = 2\ \sigma_c} \qquad ...1$$

$$P = \sigma_s A_s + \sigma_c A_c$$
$$150000 = 2\sigma_c \times 1256.64 + 706.86\sigma_c$$
$$150000 = 2120.58\sigma_c$$
$$\sigma_c = 70.74 N/mm^2$$
$$\sigma_s = 141.47 N/mm^2$$

7) At a point within a body subjected to two mutually perpendicular directions, the tensile stresses are 80N/mm^2 and 40N/mm^{2}respectively. Each stress is accompanied by shear stress of 60N/mm^2. Determine the normal stress, shear stress and resultant stress on an oblique plane inclined at an angle of 45^0 with the axis of minor tensile stress. (Apr/May 2018)

Major tensile stress σ_1=80N/mm^2

Minor tensile stress σ_2=40N/mm^2

Shear stress τ=60N/mm^2

Angle incline with minor axis (θ)= 45^0

Normal Stress:

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta + \tau \sin 2\theta$$
$$\sigma_n = \frac{80+40}{2} + \frac{80-40}{2}\cos(2x45) + 60\sin(2x45)$$
$$\sigma_n = 120 N/mm^2$$

Shear stress:

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta - \tau \cos 2\theta$$
$$\sigma_t = \frac{80-40}{2}\sin(2x45) - 60\cos(2\,x\,45)$$
$$\sigma_t = 20 N/mm^2$$

Resultant stress:

$$\sigma_R = \sqrt{\left(\sigma_n{}^2 + \sigma_t{}^2\right)} = \sqrt{\left(120^2 + 20^2\right)} = 121.65 N/mm^2$$

8) The bar shown in fig. Q.11(a) is subjected to a tensed load of 100 KN of the stress in middle portion is limited to 150N/mm^2. Determine the diameter of the middle portion. Find also the length of the middle portion if the total elongation of the bar is to be 0.2 mm young modules is 2.1 x 10^5 N/mm^2

(May / June 2017) 13-Marks

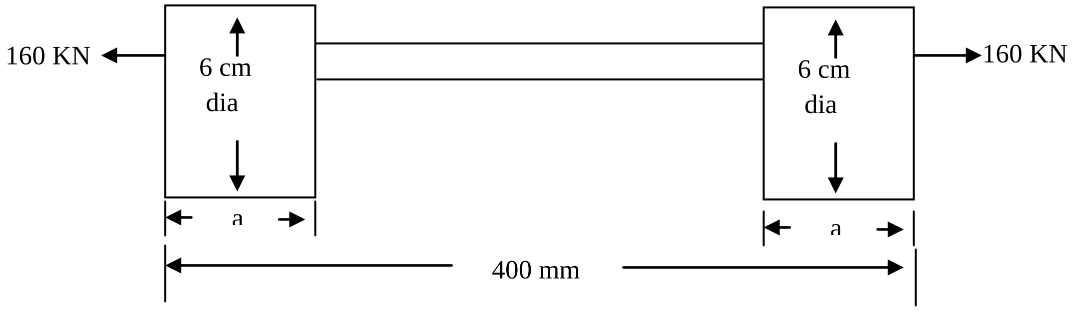


Fig.Q.11(a)

Given: P = 100 KN = 100 x 10^3 N

Stress at middle portion, σ_2 = 150 N/mm^2

Total elongation δL = 0.2 mm

Young modulus, E = 2.1 x 10^5 N/mm^2

Total length, L = 400 mm

To find:

i) Diameter of the middle portion, D_2

ii) Length of the middle portion, L_2

Solution

Stress at the middle portion, $\sigma_2 = \frac{\text{Load}}{\text{Area}} = \frac{p}{A_2}$

$$150 = \frac{p}{\frac{\pi}{4} D_2^2} = \frac{100 \times 10^3}{\frac{\pi}{4} \times D_2^2}$$

Diameter of middle portion, D_2 = 29.14 mm

Let,

Length of first portion = L_1

Length of middle portion = L_2

Length of last portion = L_3

We know that

Total elongation, $\delta L = \frac{p}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right]$

$$= 0.2 = \frac{100\times10^3}{2.1\times10^5}\left[\frac{L_1}{\frac{\pi}{4}D_1^2}+\frac{L_2}{\frac{\pi}{4}D_2^2}+\frac{L_3}{\frac{\pi}{4}D_3^2}\right]$$

$$\Rightarrow 0.2 = \frac{100\times10^3}{2.1\times10^5}\left[\frac{L_1}{\frac{\pi}{4}(60)^2}+\frac{L_2}{\frac{\pi}{4}(29.14)^2}+\frac{L_3}{\frac{\pi}{4}(60)^2}\right]$$

$$\Rightarrow 0.2 = \frac{100\times10^3}{2.1\times10^5}\left[\frac{L_1}{2826}+\frac{L_2}{666.57}+\frac{L_3}{2826}\right]$$

$$\Rightarrow 0.2 = 0.476\left[\frac{L_1+L_3}{2826}+\frac{L_2}{666.57}\right]$$

$$0.2 = 0.476\left[\frac{(400-L_2)}{2826}+\frac{L_2}{666.57}\right]$$

$$0.2 = 0.476\left[\frac{400}{2826}-\frac{L_2}{2826}+\frac{L_2}{666.57}\right]$$

$0.2 = 0.0673 – 1.684 \times 10^{-4} L_2 + 7.141 \times 10^{-4} L_2$

$0.2 = 0.0673 + 5.457 \times 10^{-4} L_2$

$$\frac{0.2-0.0673}{5.457\times10^{-4}} = L_2$$

$L_2 = 243.17$ mm

Result

1) Diameter of middle portion, $D_2 = 29.14$ mm

2) Length of middle portion, $L_2 = 243.17$ mm

9) A bar of 30 mm diameter is subjected to a pull of 60KN. The measured extension on gauge length of 200 mm is 0.1 mm and change in diameter is 0.004 mm.

Calculate **(May 2017) 13 Marks**

(i) Young's modulus

(ii) Poisson's ratio and

(iii) Bulk modulus

Given:

Diameter, d = 30 mm
Pull, p = 60 KN = 60×10^3 N
Length, L = 200 mm
Change in Length, $\delta L = 0.1$ mm
Change in diameter, $\delta d = 0.004$ mm

To Find:

(i) Young's modulus
(ii) Poisson's ratio and
(iii) Bulk modulus

Solution: we know that

$$\text{Poisson's ratio} = \frac{1}{m} = \frac{\text{Lateral strain}}{\text{longitudinal strain}} = \frac{e_t}{e_\ell} \rightarrow (1)$$

$$\text{Lateral strain } = e_t = \frac{\delta b}{b}(\text{or})\frac{\delta d}{d}(\text{or})\frac{\delta t}{t}$$

$$e_t = \frac{\delta d}{d} = \frac{0.004}{30} = 1.333\times10^{-4}$$

$$\text{Longitudinal strain, } e_\ell = \frac{\delta L}{L} = \frac{0.1}{200} = 5\times10^{-4}$$

Substitute e_t and e_ℓ in equation (1)

$$\frac{1}{m} = \frac{1.333\times10^{-4}}{5\times10^{-4}} = 0.26$$

$$\boxed{\text{Poisson's ratio} = \frac{1}{m} = 0.26}$$

$$\text{young's modulus, } E = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{\sigma}{e_\ell}$$

$$\text{stress} = \sigma = \frac{\text{Load}}{\text{Area}} = \frac{p}{A}$$

$$E = \frac{60\times10^3}{\frac{\pi}{4}d^2\times5\times10^{-4}}$$

$$= \frac{60\times10^3}{\frac{\pi}{4}(50)^2\times5\times10^{-4}} = \frac{60\times10^3}{0.353}$$

$$\boxed{E = 1.69\,x\,10^5\ N/mm^2}$$

We know that,

$$E = 3k\left(1-\frac{2}{m}\right)$$

Young's modulus, $1.69\times10^5 = 3k[1-2(0.26)]$

$$\boxed{\text{Bulk modulus} = k = 1.17\times10^5\ N/mm^2}$$

Results:

(i) Poisson's ratio = $\frac{1}{m}$ =0.26

(ii) Young's modulus = E = 1.69 x 10^5 N/mm^2

(iii) Bulk modulus = k = 1.17 x 10^5 N/mm^2

10) A steel bar 20mm in diameter 2m long is subjected to an axial pull of 50 KN. If E= 2×10^5 N/mm^2 and m = 3. Calculate the change in the i) Length ii) diameter iii) Volume (8 mark)

(May / June 2016)

Given data:

$$d = 20 \text{ mm} \qquad \ell = 2\text{m} = 2000\,\text{mm} \qquad p = 50\text{KN} = 50\times10^3 \qquad E = 2\times10^5 \text{ N/mm}^2$$

$$m = 3$$

$$\text{i) } E = \frac{\sigma}{e} = \frac{P/A}{\delta\ell/l}$$

$$e = \frac{\sigma}{E}$$

$$\sigma = \frac{P}{A} = \frac{50\times10^3}{\pi/4\times20^2} = \frac{50\times10^3}{314.16} = 159.15 \text{ N/mm}^2$$

$$e = \frac{\sigma}{E} = \frac{159.15}{2\times10^5} = 7.957\times10^{-4}$$

$$e = \frac{\delta\ell}{\ell}$$

$$\delta\ell = 7.96\times10^{-4}\times2000 = 1.59\,\text{mm}$$

$$\text{Change in legnth } \boxed{\delta\ell = 1.59 \text{ mm}}$$

$$\Rightarrow \mu = \frac{1}{m} = \text{poisson's ratio} = \frac{1}{3} = 0.33$$

$$\mu = \frac{\text{Lateral strain}}{\text{Linear strain}} = \frac{(\delta d/d)}{(\delta\ell/\ell)}$$

$$0.33 = \frac{\delta d/d}{7.96\times10^{-4}}$$

$$\delta d/d = 2.6268\times10^{-4}$$

$$\delta d = 2.6268\times10^{-4}\times20 = 5.25\times10^{-3}\,\text{mm}$$

$$\text{change in dialmeter } \boxed{\delta d = 5.25\times10^{-3}}$$

$$\delta v/v = \delta\ell/\ell - 2\delta d/d$$

$$\frac{\delta v}{v} = 7.96\times10^{-4} - 2\times2.63\times10^{-4}$$

$$\frac{\delta v}{v} = 2.7\times10^{-4}$$

$$\delta V = 2.7\times10^{-4}\times\frac{\pi}{4}\times20^2\times2000$$

$$\text{change in volume } \boxed{\delta V = 169.65 \text{ mm}^3}$$

11) A mild steel bar 20 mm in diameter and 40 cm long is encase in a tube whose external diameter is 30 and internal diameter is 25 mm. The composite bar is heated through 80°C. Calculate the stress induced in each metal α for steel is 11.2×10^{-6} per °C; α for brass is 11.2×10^{-6} per °C. E for steel is 2×10^{5} N/mm^2 and E for brass is 1×10^{5} N/mm^2 (8mark)

(May /June 2016)

Given

$$d_s = 20\text{mm} \qquad \ell_s = 40\,\text{cm} = 400\,\text{mm} = \ell_b = \ell$$

$$D_b = 30\text{ mm} \qquad d_b = 25\text{mm} \qquad \Delta t = 80°C$$

$$\alpha_s = 11.2\times10^{-6}\,/\,°C \qquad \alpha_b = 16.5\times10^{-6}\,/\,°C$$

$$E_s = 2\times10^{5}\text{ N/mm}^2 \qquad E_b = 1\times10^{5}\text{ N/mm}^2$$

$$A_s = \frac{\pi}{4}d_s^{\,2} = 314.16\text{ mm}^2$$

$$A_b = \frac{\pi}{4}\left(D_b^{\,2} - d_b^2\right) = 215.98\text{ mm}^2$$

Under equilibrium condition,

Compression in brass is equal to tension in steel i.e,

Load on brass(P_b) = load on steel (P_s)

$$\sigma_b A_b = \sigma_s A_s$$

$$\sigma_s = \sigma_b \times \frac{A_b}{A_s} = \sigma_b \times \frac{215.98}{314.16} = 0.687\sigma_b$$

$$\boxed{\sigma_s = 0.687\ \sigma_b} \qquad ...1$$

Actual expansion of steel = Actual expansion of brass

$$\alpha_s\ \Delta t\ \ell_s + \frac{\sigma_s}{E_s}\ell_s = \alpha_b\ \Delta t\ \ell_b - \frac{\sigma_b}{E_b}\ell_b$$

$$\ell_s\left(\alpha_s\Delta t + \frac{\sigma_s}{E_s}\right) = \ell_b\left(\alpha_b\ \Delta t - \frac{\sigma_b}{E_b}\right) \qquad (\ell_s = \ell_b = \ell)$$

$$\frac{\sigma_s}{E_s} + \frac{\sigma_b}{E_b} = \Delta t\left(\alpha_b - \alpha_s\right)$$

$$\frac{0.687\sigma_b}{2\times10^5} + \frac{\sigma_b}{1\times10^5} = \left(16.5\times10^{-5} - 11.2\times10^{-6}\right)\times 80°C$$

$$0.3435\sigma_b + \sigma_b = 5.3\times10^{-6}\times10^{5}\times80° = 42.4$$

$$1.3435\ \sigma_b = 42.4$$

$$\boxed{\sigma_b = 31.56\ \text{N/mm}^2} \qquad \text{substitute in equation 1,}$$

we get $\boxed{\sigma_s = 21.68\ \text{N/mm}^2}$

12) Two steel rods and one copper rod, each of 20 mm diameter together support a load 20 KN as shown in fig i) Find the stresses in the rods. Take E for steel = 210 KN/mm^2 and E for copper = 110 KN /mm^2

(May / June 2016)

$d_c = d_s = 20\,mm$

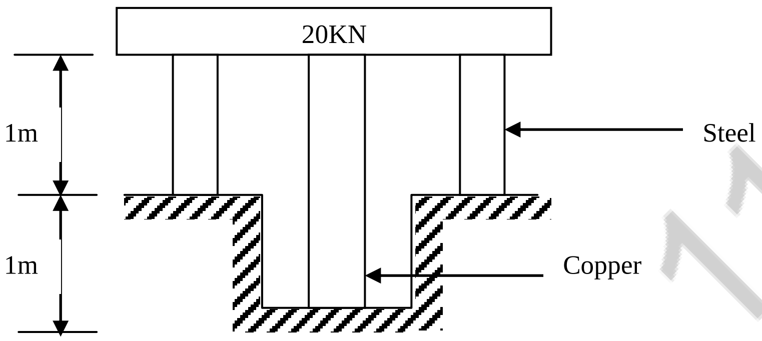
$P = 20\,KN = 20 \times 10^3\,N$

$E_s = 210 \times 10^3\ \ N/mm^2$

$E_c = 110 \times 10^3\ \ N/mm^2$

$\ell_s = 1m = 1000\ mm$

$\ell_c = 2m = 2000\ mm$



$A_s = A_c = \frac{\pi}{4} \times 20^2 = 314.16\ mm^2$

$\text{contraction in steel}\left(\delta\ell_s\right) = \text{contraction in copper}\left(\delta\ell_c\right)$

$$\frac{P_s\ell_s}{A_sE_s} = \frac{\sigma_c\ell_c}{E_c}$$

$$\frac{\sigma_s\ell_s}{E_s} = \frac{\sigma_c\ell_c}{E_c}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{E_s}{E_c} \times \frac{\ell_c}{\ell_s}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{210 \times 10^{-3}}{110 \times 10^{-3}} \times \frac{2000}{1000}$$

$$\boxed{\sigma_s = 3.82\ \sigma_c} \qquad ....1$$

$$P = \sigma_s A_x + \sigma_c A_c$$

$$20 \times 10^3 = 3.82\sigma_c \times 2 \times 314.16\ \sigma_c$$

$$20 \times 10^3 = 2714.34\sigma_c$$

$$\boxed{\sigma_c = 7.37\ N/mm^2} \qquad \text{sub in 1,}$$

we get $\boxed{\sigma_s = 28.15\ N/mm^2}$

13) Direct stress of 140 N/mm^2 tensile and 100 N/mm^2 compression exist on two perpendicular planes at a certain point in a body, They are also accompanied by shear stress on the planes. The greatest principal stress at the point due to these is 160 N/mm^2

1) **What must be the magnitude of the shear stress on the two planes?**
2) **What will be the max. shear stress at the point?** **(May / June 2016)**

$\sigma_x = 140 N/mm^2$ $\qquad \sigma_y = 100 N/mm^2$ $\qquad \sigma_1 = 160 N/mm^2$

1 Shear stress(τ_{xy}):

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left[\frac{\sigma_x - \sigma_y}{2}\right]^2 + \tau_{xy}^{\ 2}}$$

$$160 = \frac{140+(-100)}{2} + \sqrt{\left[\frac{140+(-100)}{2}\right]^2 + \tau_{xy}}$$

$$160 = 20 + \sqrt{120^2 + \tau_{xy}^2}$$

$$140 = \sqrt{120^2 + \tau_{xy}^2}$$

$$140^2 = 120^2 + \tau_{xy}^2$$

$$\boxed{\tau_{xy} = 72.11 \text{ N/mm}^2}$$

Min. principal stress, $\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right) + \tau_{xy}^2}$

$$= 20 - \sqrt{120^2 + (72.11)^2}$$

$$= -119.99 \approx -120 \text{ N/mm}^2$$

$$\sigma_2 = 120 \text{ N/mm}^2 (\text{comp})$$

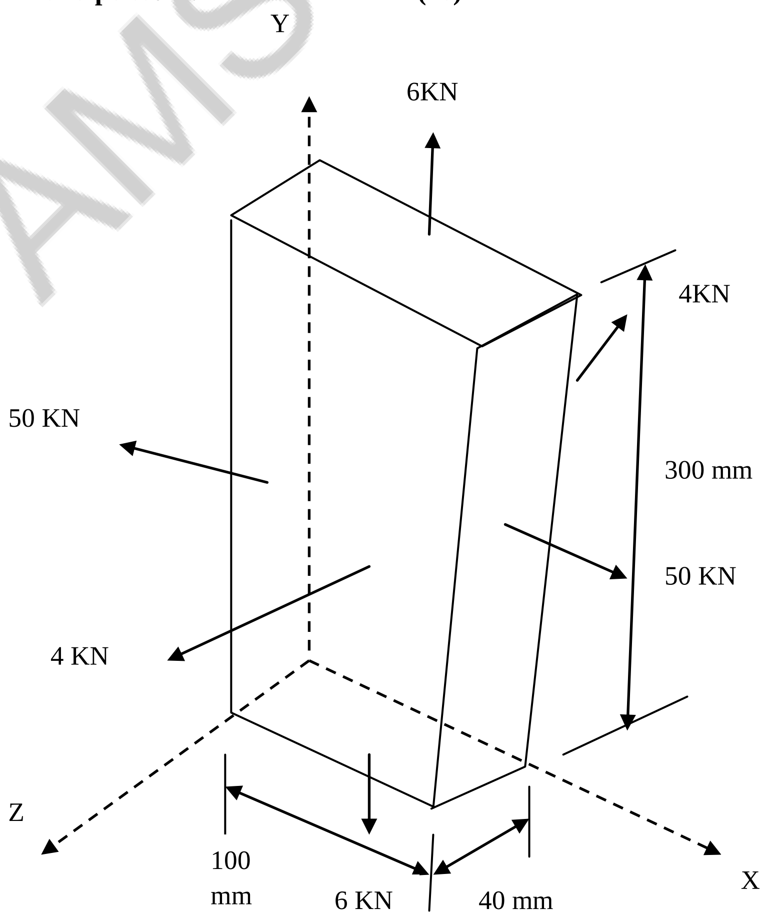
2.Max. shear stress(τ_{max}):

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{160-(-120)}{2}$$

$$\tau_{max} = 140 \text{ N/mm}^2$$

14) A metallic bar 300 mm × 100 mm×40 mm is subjected to a force of 50 KN (tensile), 6 KN (tensile), 4 KN (tensile) along x,y and z direction respectively. Determine the change in the volume of the block. Take E = 2×10^5 N/mm^2 and poisson's ratio = 0.25 (16)

(Nov / Dec 2015)



$x = 100\,\text{mm} \qquad y = 300\text{mm} \qquad z = 40\text{mm}$

$$\sigma_x = \frac{p_x}{A_{yz}} = \frac{50\times10^3}{300\times40} = 4.167\,\text{N/mm}^2$$

$$\sigma_y = \frac{p_y}{A_{zx}} = \frac{6\times10^3}{100\times40} = 1.5\,\text{N/mm}^2$$

$$\sigma_z = \frac{P_z}{A_{xy}} = \frac{4\times10^3}{100\times300} = 0.133\,\text{N/mm}^2$$

$$e_x = \frac{\sigma_x}{E} - \frac{\sigma_y}{mE} - \frac{\sigma_z}{mE}$$

$$= \frac{4.167}{2\times10^5} - \frac{-0.25\times1.5}{2\times10^5} - \frac{0.25\times0.133}{2\times10^5}$$

$$= \frac{1}{2\times10^5}\left(4.167 - 0.25\times1.5 - 0.25\times0.133\right)$$

$$= \frac{3.75875}{2\times10^5} = 1.879\times10^{-5}$$

$$e_y = \frac{\sigma_y}{E} - \frac{\sigma_x}{mE} - \frac{\sigma_z}{mE}$$

$$= \frac{1}{2\times10^5}\left(1.5 - 4.167\times0.25 - 0.133\times0.25\right)$$

$$= \frac{0.425}{2\times10^5} = 2.125\times10^{-6}$$

$$e_y = \frac{\sigma_z}{E} - \frac{\sigma_x}{mE} - \frac{\sigma_z}{mE}$$

$$= \frac{1}{2\times10^5}\left(0.133 - 4.167\times0.25 - 1.5\times0.25\right)$$

$$= \frac{-1.28375}{2\times10^5} = -6.418\times10^{-6}$$

$$e_v = \frac{\delta V}{V} = e_x + e_y + e_z = 1.4497\times10^{-5}$$

$$\delta V = 1.4497\times10^{-5}\times300\times100\times40$$

$$\boxed{\delta V = 17.396 \approx 17.40\ \text{mm}^3}$$

15. A steel rod of 3 cm diameter is enclosed centrally in a hollow copper tube of external diameter 5 cm and internal diameter of 4 cm as shown fig. The composite bar is then subjected to axial pull of 45000 N. If the length of each bar is equal to 15 cm determine i) The stresses in the rod and the tube and ii) load carried by each bar Take E for steel = 2.1× 10^5 N/mm^2 and for copper = 1.1× 10^5 N/mm^2 (16)

(Nov /Dec 2015)

$d_s = 3\text{cm} = 30\,\text{mm}$

$D_c = 5\text{cm} = 50\,\text{mm}$

$d_c = 4\text{cm} = 40\,\text{mm}$

$\rho = 45000\,\text{N}$

$\ell = 15\,\text{cm}$

$E_s = 2.1\times10^5\, N/mm^2$

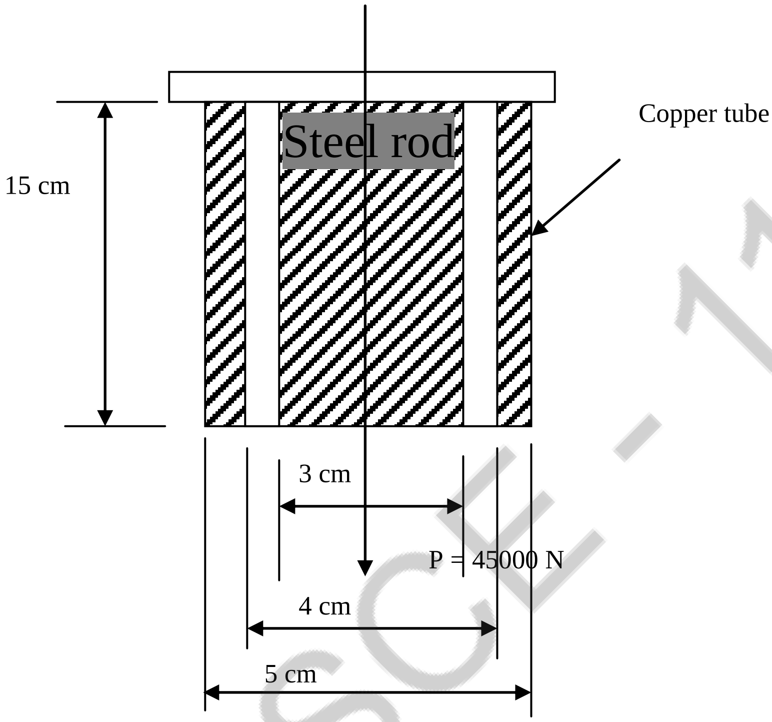
$E_c = 1.1\times10^5\ \ N/mm^2$

$A_s = \frac{\pi}{4} d_s{}^2$

$= \frac{\pi}{4}\times 30^2 = 706.86\ mm^2$

$A_c = \frac{\pi}{4}\left(D_c{}^2 - d_c{}^2\right)$

$= 706.86\ mm^2$



i)

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_s = \sigma_c \times \frac{E_s}{E_c} = \frac{2.1\times10^5}{1.1\times10^5}\sigma_c$$

$$\boxed{\sigma_s = 1.91\ \sigma_c} \qquad ...1$$

$P = \sigma_s A_s + \sigma_c A_c$

$4500 = 1.91\sigma_c \times 706.86 + 706.86\sigma_c$

$4500 = 2056.96\sigma_c$

$\boxed{\sigma_c = 21.88 N/mm^2}$ subs. (1), we get,

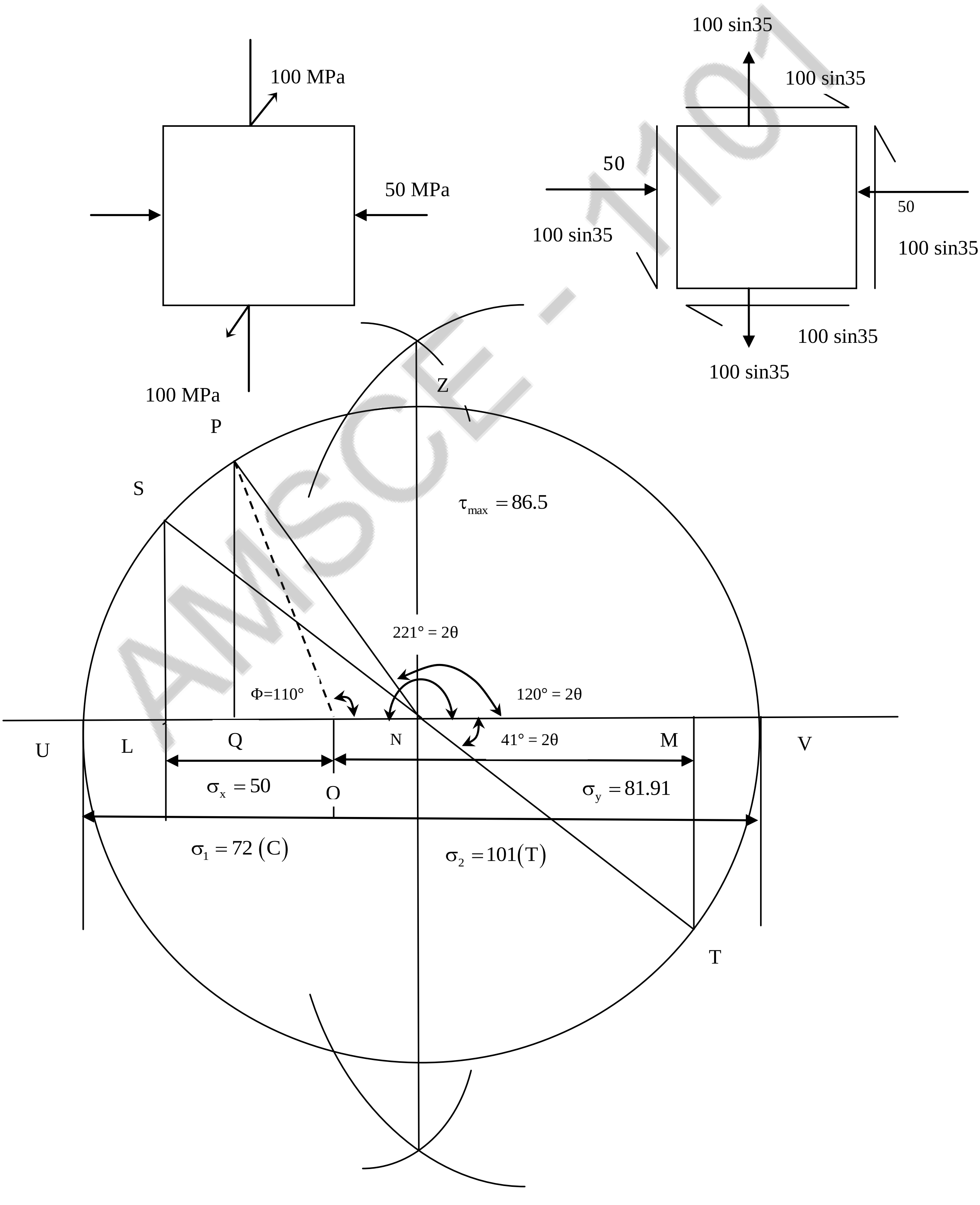
$\boxed{\sigma_s = 41.78 N/mm^2}$

ii)

$$P_c = \sigma_c A_c = 15466.09 \text{ N}$$
$$P_s = \sigma_s A_s = 29532.61 \text{ N}$$

16) At a point in a strained material the resultant intensity of stress across a vertical plane is 100MPa tensile inclined at 35° clockwise to its normal. The normal component of intensity of stress across the horizontal plane is 50 MPa compressive Determine graphically using Mohr's circle method
i) The position of principal planes and stresses across them and
ii) The normal and tangential stresses across a plane which is 60° clockwise to the vertical plane

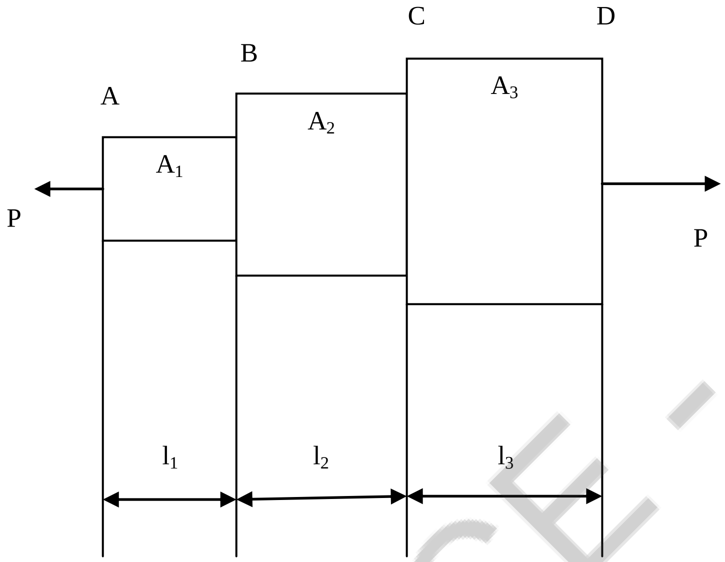
(Apr/ May 2016)



i)

From	ii)
$\sigma_1 = OV = 72\ \text{MPa (Compressive)}$	$\theta = 60°$
$\sigma_2 = OV = 101\ \text{MPa (Tensile)}$	$\sigma_\eta = OQ = 28\,\text{MPa}$
$\tau_{max} = NZ = 86.5\ \text{MPa(shear)}$	$\tau = PQ = 75\ \text{MPa}$
$2\theta = 41°\ \text{or}\ 221°$	$\sigma_r = OP = 80\ \text{MPa}$
$\theta_1 = 20.5°$	$\phi = 110°$
$\theta_2 = 110.5°$	

17) Derive an expression for change in length of a circular bar with uniformly varying diameter and subjected to an axial tensile load P (8 mark) (Nov /Dec 2014)



Tensile stress in AB (σ_1) $= \dfrac{P}{A_1}$

Elongation in AB (δl_1) = $\dfrac{P\ell_1}{A_1 E}$

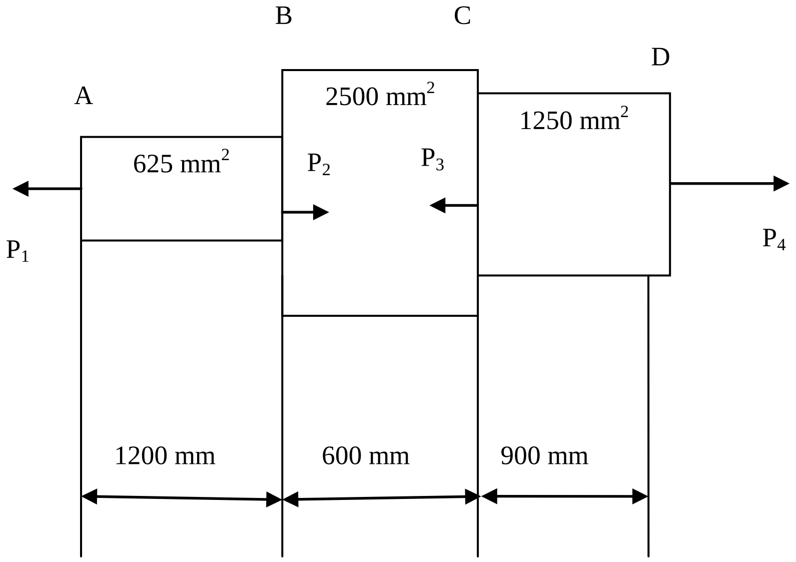
lll^{ly} for BC & CD

Total Elongation (δl) = $\delta l_1 + \delta l_2 + \delta l_2$

$$= \frac{Pl_1}{A_1E} + \frac{Pl_2}{A_2E} + \frac{Pl_3}{A_3E}$$

$$\boxed{\delta l = \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right]}$$

18) A member is subject to point load as shown in Fig Calculate the force P_2, necessary for equilibrium if P_1=45 KN; P_3=450 KN and P_4=130 KN. Determine the total elongation of the member, assuming the modulus of elasticity to be $E = 2.1 \times 10^5\ N/mm^2$ (8 mark) (Nov /Dec 2014)



$P_1 = 45\,KN \qquad P_3 = 450\ KN \qquad P_4 = 130\ KN$

$-P_1 + P_2 - P_3 + P_4 = 0$

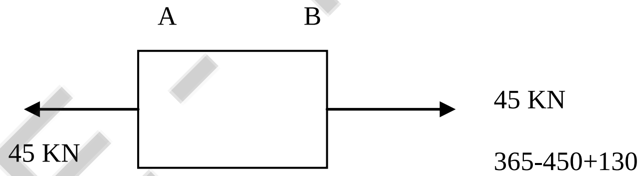
$-45 + P_2 - 450 + 130 = 0$

$P_2 = 365\ KN$

AB

$$\delta l_{AB} = \frac{P_{AB} l_{AB}}{A_{AB} E} = \frac{45 \times 10^3 \times 1200}{625 \times 2.1 \times 10^5}$$

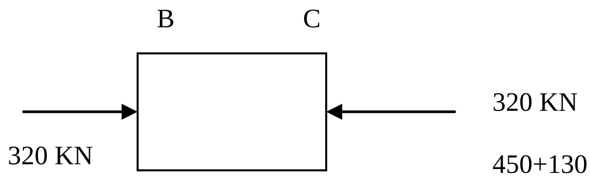
$\delta l_{AB} = 0.414$ mm(Tensile)



BC

$$\delta l_{BC} = \frac{P_{BC} \times l_{BC}}{A_{BC} E} = \frac{320 \times 10^3 \times 600}{2500 \times 2.1 \times 10^5}$$

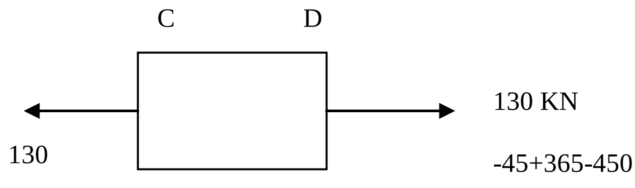
$\delta l_{BC} = 0.3657$ mm (Comp)



CD

$$\delta l_{CD} = \frac{P_{CD} l_{CD}}{A_{CD} E} = \frac{130 \times 10^3 \times 900}{1250 \times 2.1 \times 10^5}$$

$\delta l_{CD} = 0.4457$ mm (Tansile)



$$\boxed{\delta l = \delta l_{AB} - \delta l_{BC} + \delta l_{CD} = 0.4914 \text{ mm}}$$

19) A compound tube consists of a steel tube 140 mm internal diameter and 160 mm external diameter and an outer brass tube 160 mm internal diameter and 180 mm external diameter. The two tubes are of same length. The compound tube carries an axial compression load of 900 KN. Find the stress and the

load carried by each tube and the amount of it shorten. Length of each tube is 140 mm. Take E for steel as $2\times10^5 N/mm^2$ & for brass is $1\times10^5 N/mm^2$ (16 mark) (Nov /Dec 2016) (Nov /Dec 2017)

$D_s = 160 \qquad D_b = 180 \text{ mm} \qquad P = 900\,KN$

$d_s = 140\,mm \qquad d_b = 160 \text{ mm} \qquad = 900\times10^3$

$\ell_s = \ell_b = \ell = 140 \text{ mm} \qquad E_s = 2\times10^5 \text{ N/mm}$

$E_b = 1\times10^5 \text{ N/mm}$

$A_s = \frac{\pi}{4}\left(D_s^2 - d_s^2\right) \qquad A_b = \frac{\pi}{4}\left(D_b^2 - b_b^2\right)$

$= 4712.39mm^2 \qquad = 5340.71 \text{ mm}^2$

$\frac{\sigma_s}{E_s} = \frac{\sigma_b}{E_b}$

$$\boxed{\sigma_s = \sigma_b \frac{E_s}{E_b} = 2\sigma_b} \qquad ...1$$

$P = \sigma_s A_s + \sigma_b A_b$

$900 = \sigma_b \times 4712.39 + 5340.71\,\sigma_b$

$\boxed{\sigma_b = 60.95 \text{ N/mm}^2}$ sub in 1, we get

$\boxed{\sigma_s = 121.91 \text{ N/mm}^2}$

$P_s = \sigma_s A_s = 574468.03 \text{ N}$

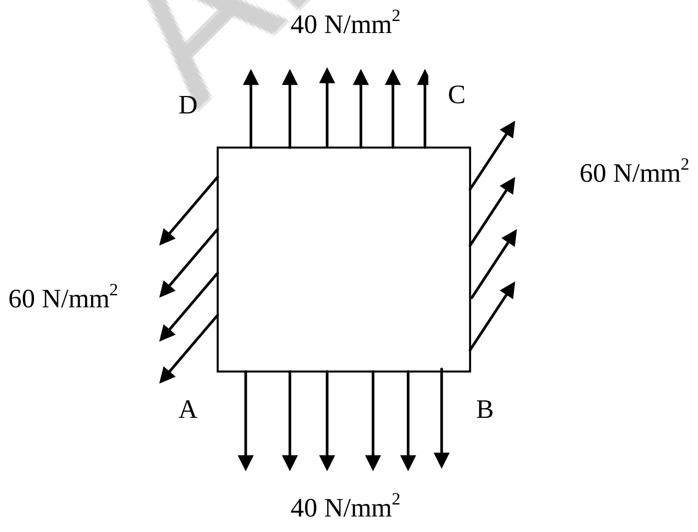
$P_b = \sigma_b A_b = 32516.27 \text{ N}$

20) Two members are connected to carry a tensile force of 80 KN by a lap joint with two number of 20 mm diameter bolt. Find the shear stress induced in the bolt (3)

(Nov / Dec 2016)

$$\tau = \frac{P}{A} = \frac{80\times10^3}{\frac{\pi}{4}\times20^2} = 254.65 \text{ N/mm}^2$$

21) A point in a strained material is subjected to the stress as shown in fig. Locate the principle phone and find the principle stress (7 marks) (Nov / Dec 2017)



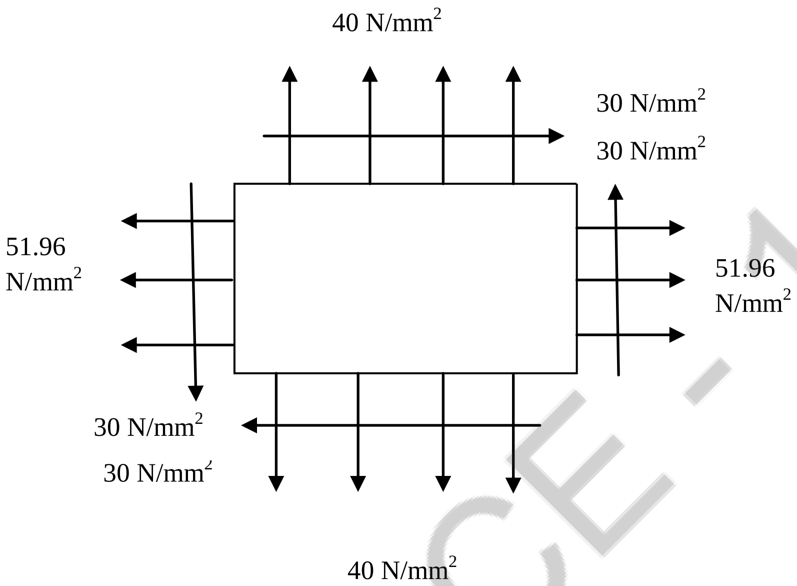
Stress on face AD & BC is not normal It is inclined at an angle 60° with face BC at AD stress can be resolved into two components

Stress normal to face (BC or AD) = 60 sin90°

$= 60 \times 0.866 = 51.96 \text{ N/mm}^2$

Stress normal to face (BC or AD) = 60 cos90°

$= 60 \times 0.5 = 30 \text{ N/mm}^2$



Major tensile stress (σ_1) = 51.9 N/mm^2

Minor tensile stress (σ_2) = 40 N/mm^2

Shear stress (τ) = 30 N/mm^2

Location of principle planes,

θ = Angle, which one of the principle planes makes with the stress of 40 N/mm^2

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2} = \frac{2 \times 30}{51.96 - 40} = 4.999$$
$$2\theta = \tan^{-1}(4.999) = 78°42' \text{ or } 258°42'$$
$$\theta = 39°21' \text{ or } 129°\ 21'$$

Principle stress

Major principle stress = $\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta$

$$= \frac{51.9+40}{2} + \sqrt{\left(\frac{51.9-40}{2}\right)^2 + 30^2}$$

$$= 45.98 + 30.6 = 76.58 \text{ N/mm}^2$$

Minor principle stress = $\frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$

$$= \frac{51.9+40}{2} - \sqrt{\left(\frac{51.9-40}{2}\right)^2 + 30^2}$$

$$= 45.98 - 30.6 = 15.38 \text{ N/mm}^2$$

22) A steel rod of 20 mm diameter passes centrally through a copper tube of 50 mm external diameter and 40 mm internal diameter. The tube is closed at the end by rigid plates of negligible thickness. The nuts are tightened lightly on the projecting part of the rod. If the temperature of the assembly is raised by 50℃. Calculate the stresses developed in copper and steel. Take E for steel as $2\times10^5 \text{ N/mm}^2$ **and copper as** $1\times10^5 \text{ N/mm}^2$ **and as for steel and copper as** $12\times10^{-6}\,°\text{C}$ & $18\times10^{-6}\,°\text{C}$ **(6 mark)**

(Nov / Dec 2016)

$d_s = 20\text{mm}, \quad D_c = 50\text{mm}, \quad A_t = 50°\text{C},$

$A_s = \frac{\pi}{4}\times20^2 = 314.16 \text{ mm}^2, \quad d_c = 40\text{mm},$

$A_c = \frac{\pi}{4}\left(D_c^{\,2} - d_c^{\,2}\right) = A_c = 706.86 \text{ mm}^2$

$E_s = 2\times10^5 \text{ N/mm}^2 \quad \alpha_s = 12\times10^{-6}\,/°\text{C}$

$E_c = 1\times10^5 \text{ N/mm}^2 \quad \alpha_c = 18\times10^{-6}\,/°\text{C}$

$\sigma_s A_s = \sigma_c A_c$

$$\boxed{\sigma_s = \sigma_c \frac{A_c}{A_s} = 2.25\sigma_c} \quad ...1$$

$$\frac{\sigma_c}{E_c} + \frac{\sigma_s}{E_s} = \left(\alpha_c - \alpha_s\right)\Delta t$$

$$\frac{\alpha_c}{1\times10^{-5}} + \frac{2.25\sigma_c}{2\times10^5} = 6\times10^{-6}\times50$$

$$2.215\sigma_c = 6\times10^{-6}\times50\times10^5 = 30$$

$$\boxed{\sigma_c = 14.11 \text{N/mm}^2} \quad \text{sub in 1}$$

$$\boxed{\sigma_s = 31.76 \text{ N/mm}^2}$$

23) A metallic bar 300 mm (x) × 100 mm(y) × 40 mm is subjected to a force of 5 KN tensile, 6KN (tensile) and 4 KN (tensile) along x,y,z direction respectively. Determine the change in volume of the block. Take E =2 × 10^5 N/mm^2 and Poisson's ratio = 0.25 (16 mark)

(Nov /Dec 2014)

Solution

$x = 300mm \qquad y = 100mm \qquad z = 40mm$

$P_x = 5\,KN \qquad P_y = 6\,KN \qquad P_z = 4\,KN$

$$\sigma_x = \frac{P_x}{A_{yz}} = \frac{5\times10^3}{100\times40} = 1.25\ N/mm^2$$

$$\boxed{\sigma_x = 1.25\ N/mm^2}$$

$$\sigma_y = \frac{P_y}{A_{zx}} = \frac{6\times10^3}{30\times40} = 0.5\ N/mm^2$$

$$\boxed{\sigma_y = 0.5\ N/mm^2}$$

$$\sigma_z = \frac{P_z}{A_{xy}} = \frac{4\times10^3}{100\times300} = 0.133\ N/mm^2$$

$$\boxed{\sigma_z = 0.133\ N/mm^2}$$

$$e_x = \frac{\sigma_x}{E} - \frac{\sigma_y}{mE} - \frac{\sigma_z}{mE}$$

$$= \frac{1.25}{2\times10^5} - \frac{0.5\times0.25}{2\times10^5} - \frac{0.133\times0.25}{2\times10^5}$$

$$= \frac{1}{2\times10^5}\left[1.25 - 0.125 - 0.0332\right]$$

$$\boxed{e_x = 5.459\times10^{-6}}$$

$$e_y = \frac{\sigma_y}{E} - \frac{\sigma_x}{mE} - \frac{\sigma_z}{mE}$$

$$= \frac{0.5}{2\times10^5} - \frac{1.25\times0.25}{2\times10^5} - \frac{0.133\times0.25}{2\times10^5}$$

$$= \frac{1}{2\times10^5}\left[0.5 - 0.125 - 0.0332\right]$$

$$\boxed{e_y = 5.459\times10^{-6}}$$

$$e_z = \frac{\sigma_z}{E} - \frac{\sigma_x}{mE} - \frac{\sigma_y}{mE}$$

$$= \frac{0.133}{2\times10^5} - \frac{1.25\times0.25}{2\times10^5} - \frac{0.5\times0.25}{2\times10^5}$$

$$= \frac{1}{2\times10^5}\left[0.133 - 0.3125 - 0.125\right]$$

$$\boxed{e_z = -1.5225\times10^{-6}}$$

$$e_v = \frac{\delta V}{V} = e_x + e_y + e_z$$

$$\frac{\delta V}{V} = 5.459\times10^{-6} \times 7.715\times10^{-7} - 1.5225\times10^{-6}$$

$$\delta V = 4.708\times10^{-6} \times 300\times40\times40$$

$$\delta V = 5.6496\ mm^3$$

Part – C

1) (i) Draw stress strain curve for mild steel and explain the salient points on it. (7)

We have studied in chapter of simple stress and strain, that whenever some external system of forces acts on a body, it undergoes some deformation. If a body is stressed within its elastic limit, the deformation entirely disappears as soon as the forces are removed. It has been also found that beyond the elastic limit, the deformation does not disappear entirely, even after the removal of the forces and there remains some residual deformation. We study this phenomenon, in a greater detail by referring to a tensile test or stress-strain diagram) for a mild steel bar

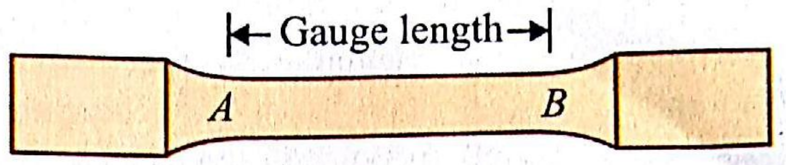


Fig. 11 a (i) Mild Steel Bar

Take a specimen of mild steel bar of uniform section as shown in Fig. 11 a (i). Let this bar be subjected to a gradually increasing pull (as applied by universal testing machine). If we plot the stresses along the vertical axis, and the corresponding strains along the horizontal axis and draw a curve passing through the vicinity of all such points, we shall obtain a graph as shown in Fig. **11 a (ii)**

We see from the graph, that

(1). From points O to A is a straight line, which represents that the stress is linearly proportional to strain.

(2). From A to B, the curve slightly deviates from the straight line but the material still shows behaviour until the curve reaches to point B, which is called elastic limit. Upto this point B if the load is removed the specimen will still come back to its original position. It is thus obvious, that the Hooke's law holds good only up to this limit. When the specimen is stressed beyond the elastic limit, the strain increases more quickly than the stress. This happens, because a sudden of the specimen takes place, without an appreciable increase in the stress (or load). This phenomenon is called yielding. The stress, corresponding to the point B is called the yield stress.

(3) After point B the material shows plastic behaviour. From points C to D the specimen shows perfectly plastic behaviour because specimen deforms without increase in the applied load. It may be noted, that if the load on the specimen is removed, then the elongation from points B to D will not disappear, but will remain as a permanent set.

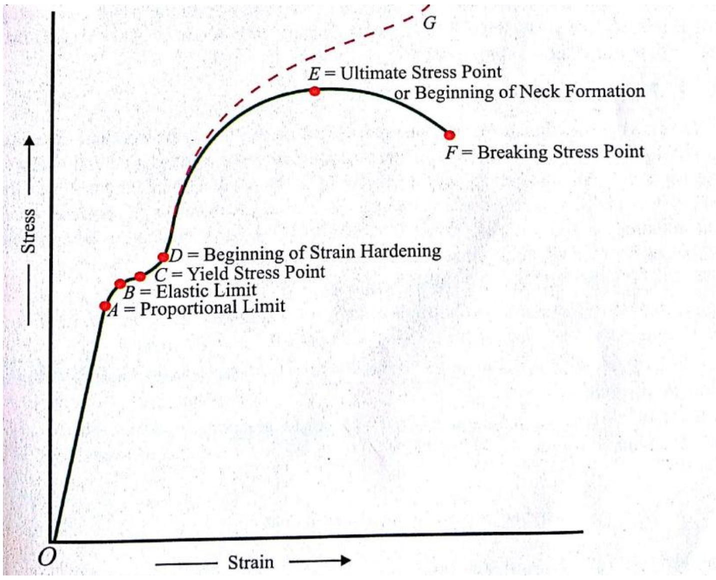


Fig. 11 a (ii) Stress-Strain Graph for a Mild Steel Bar

(4). At point D the specimen regains some strength and higher values of stresses are required, for higher strains. From points D to E is the region of strain hardening. During strain hardening the material undergoes the changes is crystalline structure, resulting in increased resistance of the material to further deformation.

(5). After point E the gradual increase in the length of the specimen is followed with the uniform reduction of its cross-sectional area. The work done during stretching the specimen, is transformed largely into heat and the specimen becomes hot. At point E, the stress attains its maximum value and is known as ultimate stress.

After the specimen has reached the ultimate stress, a neck is formed, which decreases the cross- sectional area of the specimen. From points E to F is the region of necking.

(6). A little consideration will show, that the stress (or load) necessary, to break away the specimen is less than the ultimate stress (or maximum load). The stress is therefore reduced until the specimen breaks away at the stress represented by the point F. At point F, the stress is known as the breaking stress.

Notes:

i) At this point, the elongation of a mild steel specimen is about 2%.

ii) The breaking stress (i e., stress at F which is less than that at E, appears to be somewhat misleading. As the formation of a neck takes place at E, which reduces the cross-sectional area. It causes the specimen suddenly to fail at F. If for each value of the strain between D and F the tensile load is divided by the reduced cross sectional area at the narrowest part of the neck, then the true stress-strain curve will follow the dotted line DG. However, it is an established practice, to calculate strains on the basis of original cross-sectional area of the specimen.

1) (ii) Derive a relation for change in length of a circular bar with uniformly varying diameter, subjected to an axial tensile load 'W' (8)

A bar of different lengths and of different diameters (and hence of different cross-sectional areas) is shown in Fig.12. Let this bar is subjected to an axial load P.

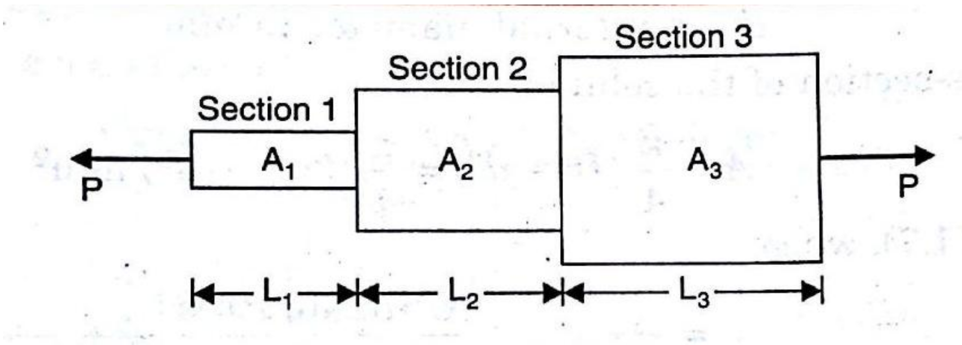


Fig.12

Though each section is subjected to the same axial load P, yet the stresses, strains and change in lengths will be different. The total change in length will be obtained by adding the changes in length of individual section.

Let P = Axial load acting on the bar,

L_1 = Length of section 1,

A_1 = Cross-sectional area of section 1,

L_2,A_2 = Length and cross-sectional area os section 2,

L_3,A_3 = Length and cross-sectional area of section 3, and

E = Young's modulus for the bar.

Then stress for thee section 1,

$$\sigma_1 = \frac{\text{Load}}{\text{Area of section 1}} = \frac{P}{A_1}$$

Similarly stresses for the section 2 and section 3 are given as,

$$\sigma_2 = \frac{P}{A_2} \text{ and } \sigma_3 = \frac{P}{A_3}$$

Using equations (1.5), the strains in different sections are obtained.

$$\therefore \text{ strain of section 1 , } e_1 = \frac{\sigma_1}{E} = \frac{P}{A_1 E} \qquad \left(\because \sigma_1 = \frac{P}{A_1}\right)$$

Similarly the strains of section 2 and section 3 are,

$$e_2 = \frac{\sigma_2}{E} = \frac{P}{A_2 E} \text{ and } e_3 = \frac{\sigma_3}{E} = \frac{P}{A_3 E}$$

But strain`in section 1= $\frac{\text{Change in length of section 1}}{\text{Length of section 1}}$

or $$e_1 = \frac{dL_1}{L_1}$$

where dL_1 = change in length of section 1.

∴ Change in length of section 1, $dL_1 = e_1L_1$

$$= \frac{PL_1}{A_1E} \qquad \left(\because e_1 = \frac{P}{A_1E}\right)$$

Similarly changes in length of section 2 and of section 3 are obtained as:

Change in length of section 2, $dL_2 = e_2L_2$

$$= \frac{PL_2}{A_2E} \qquad \left(\because e_2 = \frac{P}{A_2E}\right)$$

and change in length of section 3, $dL_3 = e_3L_3$

$$= \frac{PL_3}{A_3E} \qquad \left(\because e_3 = \frac{P}{A_3E}\right)$$

∴ Total change in the length of the bar,

$$dL = dL_1 + dL_2 + dL_3 = \frac{PL_1}{A_1E} + \frac{PL_2}{A_2E} + \frac{PL_3}{A_3E}$$

$$= \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right] \qquad ...(1.8)$$

Equation (1.8) is used when the young's modulus of different sections is same. If the Young's modulus of different sections is different , then total change in length of the bar is given by,

$$dL = P\left[\frac{L_1}{E_1A_1} + \frac{L_2}{E_2A_2} + \frac{L_3}{E_3A_3}\right] \qquad ...(1.9)$$

UNIT –II

Part – A

Transverse loading on Beans and stresses in Beam

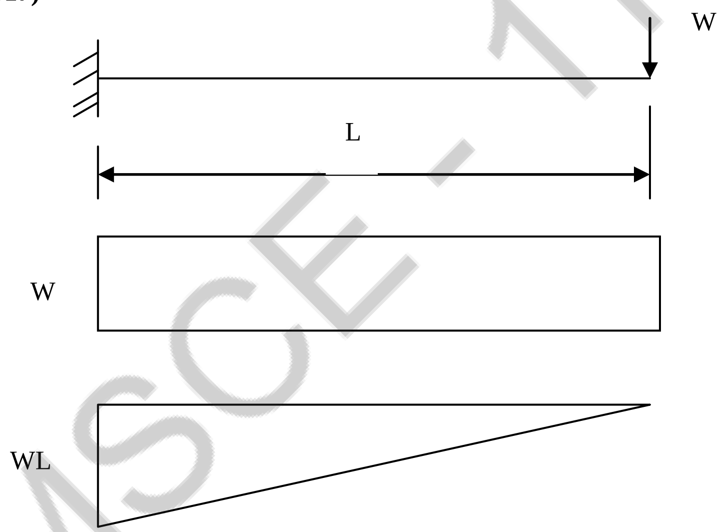
1) What is the ratio of maximum shear stress to the average shear stress in the case of solid circular section? (Apr/May 2019)

$$\frac{\tau_{max}}{\tau_{avg}} = \frac{4}{3}$$

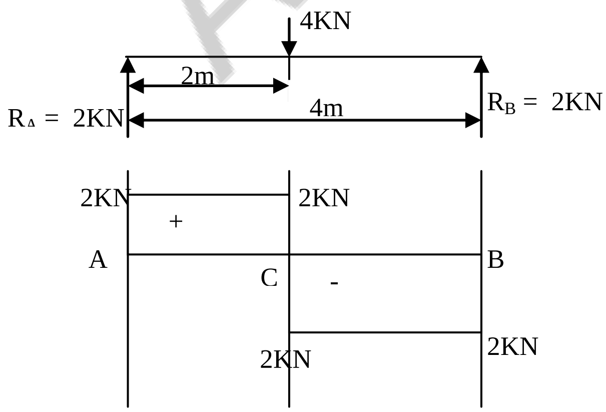
2) What is meant by shear stresses in beams? (Apr/May 2018)

When a beam is subjected to shear force and zero bending moment, then there will be only shear stresses in the beam. These stresses acting across the transverse section of the beam.

3) Draw the shear force and bending moment diagram for a cantilever of length L carrying a point load W at the free end. (Nov/Dec 2017)



4) Draw shear force diagram for a simply supported beam of length 4m carrying a central point load of 4 KN. (May/June 2017)



$$R_A + R_B = 4KN$$

$$M_A = 0 \Rightarrow 4R_B = 4KN \times 2$$

$$R_B = \frac{4\times 2}{4} = 2KN$$

5) Prove that the shear stress distribution over a rectangular section due to shear force is parabolic.

(May/June 2017)

The figure shows a rectangular section of a beam of width b and depth d. Let F is the shear force acting at the section. Consider a level EF at a distance y from the neutral axis.

Shear stress , $\tau = F.\frac{A\bar{y}}{b\times \ell}$

Where A = Area of the section above y (i.e, shaded area ABFE) = $\left(\frac{d}{2} - y\right)\times b$

$\bar{y}$ = Distance of C.G of area A from neutral axis

$$= y + \frac{1}{2}\left(\frac{d}{2} - y\right) = y + \frac{d}{4} - \frac{y}{2} = \frac{y}{2} + \frac{d}{4}$$

$$= \frac{1}{2}\left(y + \frac{d}{2}\right)$$

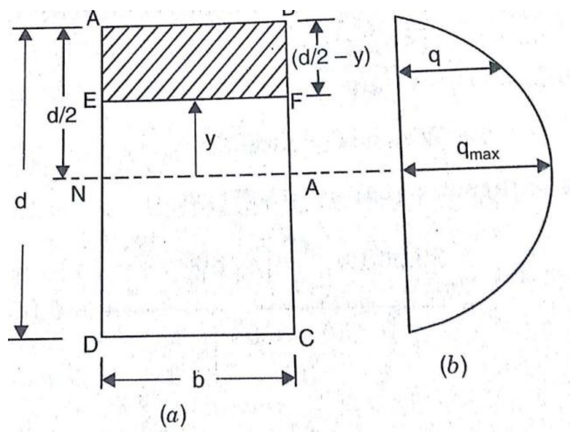
b = actual width of section at EF

I = M.O.I of whole section

Substituting the values in above equation

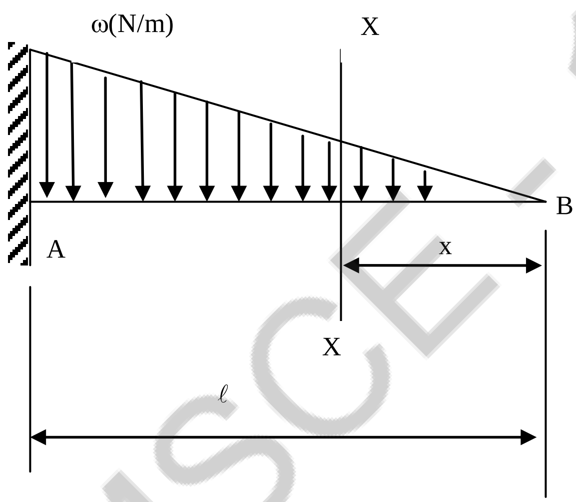
$$\tau = \frac{F\left(\frac{d}{2} - y\right)\times b \times \frac{1}{2}\left(y + \frac{d}{2}\right)}{b\times \ell}$$

$$= \frac{F}{2l}\left[\frac{d^2}{4} - y^2\right]$$ The variation of τ with respect y in parabola.



6) Draw the shear force diagram and bending moment diagram for the cantilever beam carriers uniformly varying load of zero initially at the freed end and w KN/m at the fixed end.

[Nov /Dec 2016]



SFD:

$$\text{SF at XX} = \frac{\omega x}{\ell} \times \frac{x}{2} = \frac{\omega x^2}{2\ell}$$

$$x = 0 \Rightarrow \text{SF at B} = 0$$

$$x = \ell \Rightarrow \text{SF at A} = \frac{\omega \ell}{2}$$

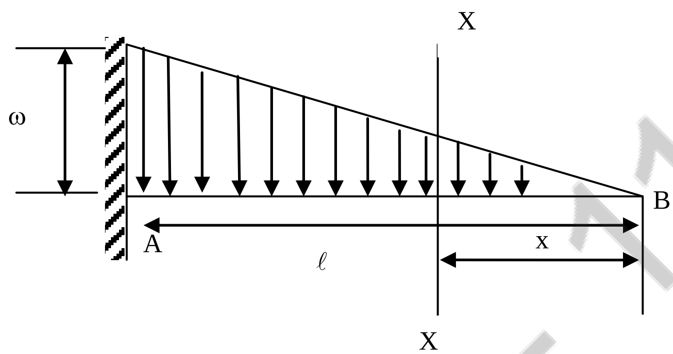
BMD:

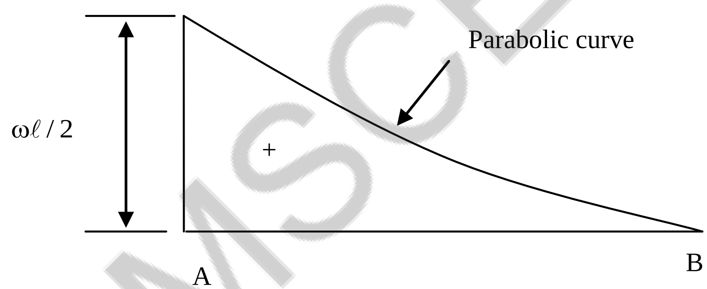
$$\text{BM at XX} = \frac{-\omega x}{\ell} \times \frac{x}{2} \times \frac{x}{3}$$

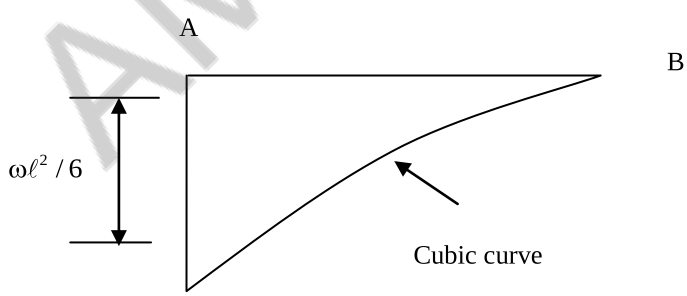
$$= \frac{-\omega x^3}{6\ell}$$

$x = 0 \Rightarrow$ B.M at B = 0

$$x = \ell \Rightarrow \text{B.M at A} = \frac{-\omega \ell^3}{6\ell} = \frac{-\omega \ell^2}{6}$$







7) List out the assumption used to derive the simple bending equation[Nov/ Dec 2015, 2014, 2018]

1) The material is perfectly homogeneous and isotopic. It obeys hooks law.

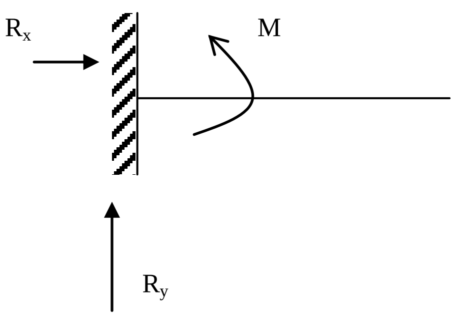
2) Transverse section, which are plane before bending, remains plane after bending

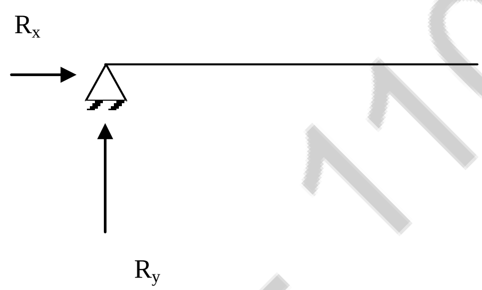
3) The radius of curvature of the beam is very large compared to the cross sectional dimension of the beam.

4) Each layer of the beam is free to expand or contract, independently of the layer above or below it.

8) Discuss the fixed and Hinged support

[May/ June 2016]





Resistance to the moment m = 0
Displacement at (x & y axis)

$u_x = 0$

$u_y = 0$

No resistance to moment
Resistance to Displacement x & y axis

$u_x = 0$

$u_y = 0$

9) What are the advantages of flitched beams [May /June 2016]

* It is used to strengthen the material. Ex. steel bars in concrete beam.

* Less space occupied.

10) What is the type of beams? [Nov / Dec 2015]

a) Cantilever beams

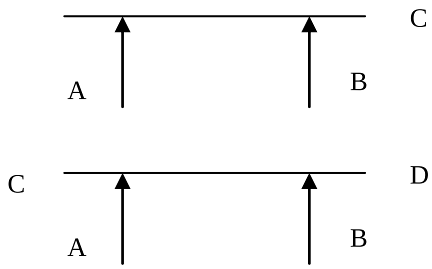
A beam with on end free (B) fixed (A)

b) Simply supported beam (SSB)

A beam is resting freely on supports at is both ends (A&B)

C) Overhanging beam

One or both the end portion beyond the support

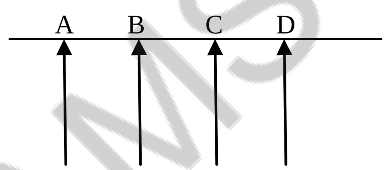


d) Fixed beam

A beam whose both ends are fixed

e) Continuous beam:

A beam which has more than two supports



11) Define a) sheer force b) bending moment [Apr /May 2015]

Sheer Force:

Algebraic sum of the forces acting on either right side or left side of the section

Bending moment

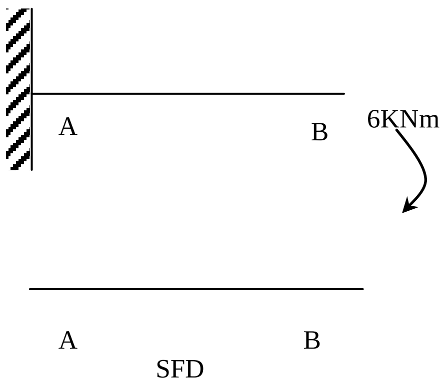
Algebraic sum of moment due to all forces acting on either right or left of the section

12) What is neutral axis of a beam under simple bending? [Apr/ May 2015]

The line of intersection of the neutral layer, with any normal cross section of a beam is known as neutral axis of that section.

To locate the neutral axis of a section, first find out the centroid of the section and then draw a line passing through this centroid and normal to the plane of bending. This line will be the neutral axis of the section.

13) Draw SFD for a 6m cantilever beam carrying a clockwise moment of 6KNm at free end [Nov/ Dec 2014]



No vertical force. So shear force is zero

14) What are flitched beams? (Nov/Dec 2017)

A beam which is constructed by two different materials is known as flitched or composite beam. It is used to reinforced the material and reduced the cost.

15) Mention the assumption made in the theory of simple bending?

Assumption made in the theory of pure Bending

- The material of the beam is homogeneous and isotropic.
- The value of young's modulus of elasticity is same in tension and compression
- The transverse section which were plane before bending remain plane after bending also
- The beam is initially straight and all longitudinal filaments bend into circular arcs with a common centre of curvature.
- The radius of curvature is large as compared to the dimension of the cross section
- Each layer of the beam is free to expand or contract, independently of the layer above or below it.

16) Define point of contra flexure? In which beam it occurs? (Apr/May 2018) (Nov/Dec 2018) (Apr/May 2019)

The point where the bending moments change its sign or zero is called point contra flexure. It occurs in overcharging beam.

17) Write the theory of simple bending equation?

$$\frac{M}{I} = \frac{F}{Y} = \frac{E}{R}$$

M – Maximum bending moments

I – Moments of inertia

F – Maximum stress induced

Y – Distance from the neutral axis

E – Young's modules
R – Radius of curvature

18) Define beam?

BEAM is a structural member which is support along the length and subjected to external loads acting transversely (i.e) perpendicular to the centre line of the beam.

19) What is mean by transverse loading on beam?

If a load is acting on the beam perpendicular to the axis of the beam then it is called transverse loading.

20) What is mean by positive or sagging BM?

BM is said to positive if moment on left side of beam is clockwise or right side of the beam is counter clockwise.

21) What is mean by negative or hogging BM?

BM is said to negative if moment on left side of beam is counter clockwise or right side of the beam is clockwise.

22) When will bending moments is maximum?

BM will be maximum, when shear force change its sign.

23) What are the types of loads?

- Concentrated load or point load
- Uniform distributed load
- Uniform varying load.

24) Define "Section Modulus"

It is the ratio of moment of inertia to the distance of plane from the neutral axis.

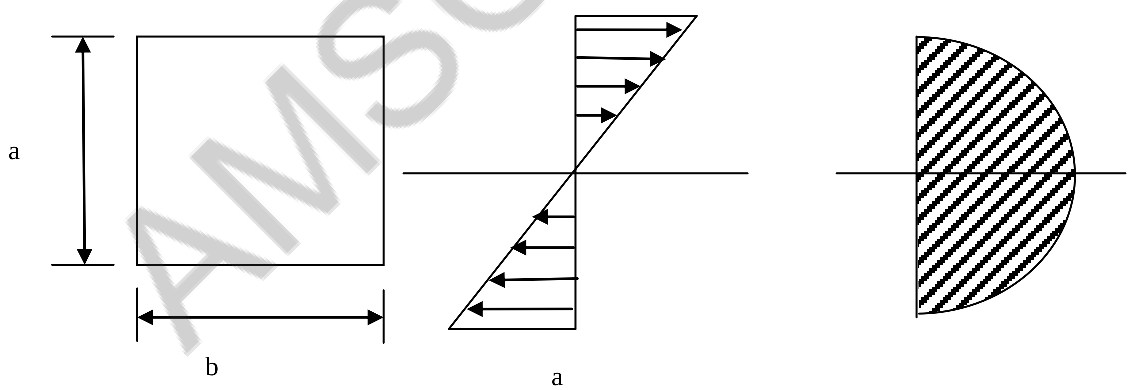
25) What is moment of resistance of the section?

It is product of the section modules and stress at that section

26) Define shear stress distribution

The variation of shear stress along the depth of the beam is called shear stress distribution.

27) Sketch a) the bending stress distribution b) shear stress distribution for a beam of rectangular cross section



28) A rectangular beam of 150 mm wide & 250 mm deep is subjected to a max. shear force of 30KN. Determine i) Avg. shear stress ii) max. shear stress iii) shear stress at a distance of 25 mm above the neutral axis

$$A = b \times d = 150 \times 250 = 37500\,mm^2$$

i) Avg shear stress :

$$q_{avg} = \frac{F}{A} = \frac{30 \times 10^3}{37500} = 0.8 N/mm^2$$

ii) Max shear stress :

$$q_{max} = 1.5 q_{avg} = 1.5 \times 0.8 = 1.2 N/mm^2$$

$$\text{iii) } q = \frac{F}{2I}\left(\frac{d^4}{4} - y^2\right)$$

$$I = \frac{bd^3}{12}$$

$$= \frac{150 \times 250^3}{12}$$

$$= 195312500\,mm^4$$

$$y = 25mm$$

$$q = \frac{30 \times 10^3}{2 \times 195312500}\left(\frac{250^2}{4} - 25^2\right)$$

$$q = 1.152\ N/mm^2$$

PART – B

1) Draw the shear force and bending moment diagram for the overhanging beam carrying uniformly distributed load of 2kN/m over the entire length and a point load of 2kN as shown in fig. Locate the point of contraflexure. (Apr/May 2019)

Sol. First calculate the reactions R_A and R_B.

Taking moments of all forces about A, we get

$$R_B \times 4 = 2 \times 6 \times 3 + 2 \times 6 = 36 + 12 = 48$$

$$\therefore \quad R_B = \frac{48}{4} = 12 \text{ kN}$$

and $$R_A = \text{Total load} - R_B = (2 \times 6 + 2) - 12 = 2 \text{ kN}$$

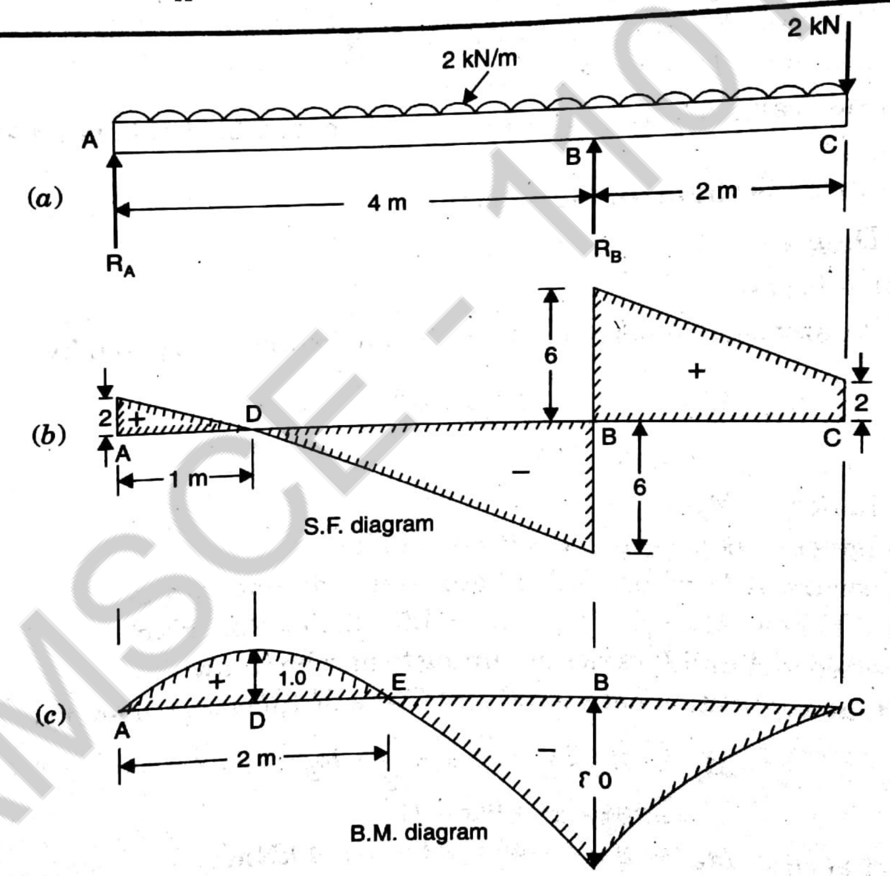


Fig. 6.36

S.F. Diagram

S.F. at A $= +R_A = +2$ kN

(i)The S.F. at any section between A and B at a distance x from A is given by,

$$F_x = +R_A - 2 \times x$$
$$= 2 - 2x \quad ...(i)$$

At A, $x = 0$ hence $F_A = 2 - 2 \times 0 = 2$ kN

At B, $x = 4$ hence $F_A = 2 - 2 \times 4 = -6$ kN

The S.F. between A and B varies according to straight line law. At A, S.F. is positive and at B, S.F. is negative. Hence between A and B, S.F. is zero. The point of zero S.F. is obtained by substituting $F_x = 0$ in equation (i).

$$\therefore \quad 0 = 2 - 2x \quad \text{or} \quad x = \frac{2}{2} = 1 \text{ m}$$

The S.F. is zero at point D. Hence distance of D from A is 1 m.

(ii)The S.F. at any section between B and C at a distance x from A is given by,

$$F_x = +R_A - 2 \times 4 + R_B - 2(x - 4)$$
$$= 2 - 8 + 12 - 2(x - 4) = 6 - 2(x - 4) \quad ...(ii)$$

At B, $x = 4$ hence $F_B = 6 - 2(4 - 4) = +6$ kN

At C, $x = 6$ hence $F_C = 6 - 2(6 - 4) = 6 - 4 = 2$ kN

The S.F. diagram is drawn as shown in Fig. 6.36 (b).

B.M. Diagram

B.M. at A is zero

(i) B.M. at any section between A and B at a distance x from A is given by,

$$M_x = R_A \times x - 2 \times x \times \frac{x}{2} = 2x - x^2 \quad ...(iii)$$

The above equation shows that the B.M. between A and B varies according to parabolic law.

At A, $x = 0$ hence $M_A = 0$

At B, $x = 4$ hence $M_B = 2 \times 4 - 4^2 = -8$ kNm

Max. B.M. is at D where S.F. is zero after changing sign

At D, $x = 1$ hence $M_D = 2 \times 1 - 1^2 = 1$ kNm

The B.M. at C is zero. The B.M. also varies between B and C according to parabolic law. Now the B.M. diagram is drawn as shown in Fig. 6.36 (c).

Point of Contraflexure

This point is at E between A and B, where B.M. is zero after changing its sign. The distance of E from A is obtained by putting $M_x = 0$ in equation (iii).

$$\therefore \quad 0 = 2x - x^2 = x(2 - x)$$
$$2 - x = 0$$

and $x = $ **2 m. Ans.**

2) A timber beam 100mm wide and 200mm deep is to be reinforced by bolting on two steel flitches each 150mm by 12.5mm in section. Calculate the moment of resistance when flitches are attached symmetrically at the top and bottom. Allowable stress in timber is 6 N/mm^2. $E_s = 2\times 10^5$ N/mm^2 and $E_t = 1\times 10^4$ N/mm^2 (Apr/May 2019)

1st Case. *Flitches attached symmetrically at the top and bottom.*

(See Fig. 7.31).

Let suffix 1 represents steel and suffix 2 represents timber.

Width of steel, $b_1 = 150$ mm

Depth of steel, $d_1 = 12.5$ mm

Width of timber, $b_2 = 150$ mm

Depth of timber, $d_2 = 200$ mm

Number of steel plates $= 2$

Max. stress in timber, $\sigma_2 = 6$ N/mm^2

E for steel, $E_1 = E_s = 2 \times 10^5$ N/mm^2

E for timber, $E_2 = E_t = 1 \times 10^4$ N/mm^2

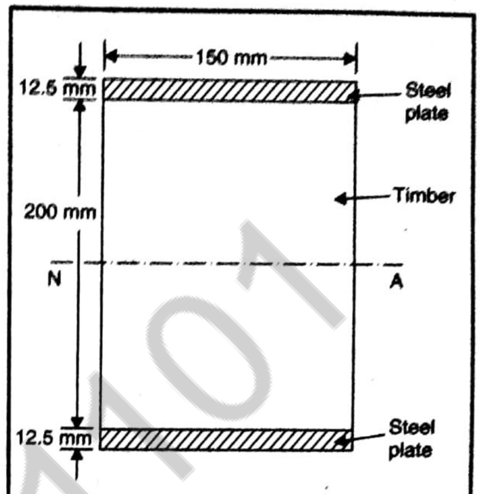


Fig. 7.31

Distance of extreme fibre of timber from N.A.,

$$y_2 = 100 \text{ mm}$$

Distance of extreme fibre of steel from N.A.,

$$y_1 = 100 + 12.5 = 112.5 \text{ mm.}$$

Let $\sigma_1^* =$ Max. stress in steel

$\sigma_1 =$ Stress in steel at a distance of 100 mm from N.A.

Now we know that strain at the common surface is same. The strain at a common distance of 100 mm from N.A. is steel and wood would be same. Hence using equation (7.11), we get

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

$$\therefore \quad \sigma_1 = \frac{E_1}{E_2} \times \sigma_2 = \frac{2\times 10^5}{1\times 10^4} \times 6 = 120 \text{ N/mm}^2.$$

But σ_1 is the stress in steel at a distance of 100 mm from N.A. Maximum stress in steel would be at a distance of 112.5 mm from N.A. As bending stresses are proportional to the distance from N.A.

Hence
$$\frac{\sigma_1}{100} = \frac{\sigma_1^*}{112.5}$$

$$\therefore \quad \sigma_1^* = \frac{112.5}{100} \times \sigma_1 = \frac{112.5}{100} \times 120 = \mathbf{135 \text{ N/mm}^2}. \quad \textbf{Ans.}$$

Now moment of resistance of steel is given by

$$M_1 = \frac{\sigma_1^*}{y_1} \times I_1 \text{ (where } \sigma_1^* \text{ is the maximum stress in steel)}$$

$$= \frac{135}{112.5} \times I_1$$

where I_1 = M.O.I. of two steel plates about N.A.

= 2 × [M.O.I. one steel plate about its C.G. + Area of one steel plate × (Distance between its C.G. and N.A.)^2]

$$= 2 \times \left[\frac{b_1 d_1^3}{12} + b_1 d_1 \times \left(100 + \frac{d_1}{2}\right)^2 \right]$$

$$= 2 \times \left[\frac{150 \times 12.5^3}{12} + 150 \times 12.5 \times \left(100 + \frac{12.5}{2}\right)^2 \right]$$

$$= 2 \times [24414.06 + 21166992.18]$$

$$= 42382812.48 \text{ mm}^4$$

∴ $$M_1 = \frac{135}{112.5} \times 42382812.48$$

$$= 50859374.96 \text{ Nmm} = 50859.375 \text{ Nm}$$

Similarly, $$M_2 = \frac{\sigma_2}{y_2} \times I_2$$

$$= \frac{6}{100} \times \frac{150 \times 200^3}{12}$$

$$= 6000000 \text{ Nmm} = 6000 \text{ Nm}$$

∴ Total moment of resistance is given by,

$$M = M_1 + M_2$$

$$= 50859.375 + 6000 = \mathbf{56859.375 \text{ Nm.}}$$ **Ans.**

3) Draw a shear force and bending moment diagram for a simply supported beam of length 9m and carrying a uniformly distributed load of 10kN/m for a distance of 6m from the left end. Also calculate the maximum bending moment on the section. (Nov/Dec 2018)

Sol. First calculate reactions R_A and R_B.

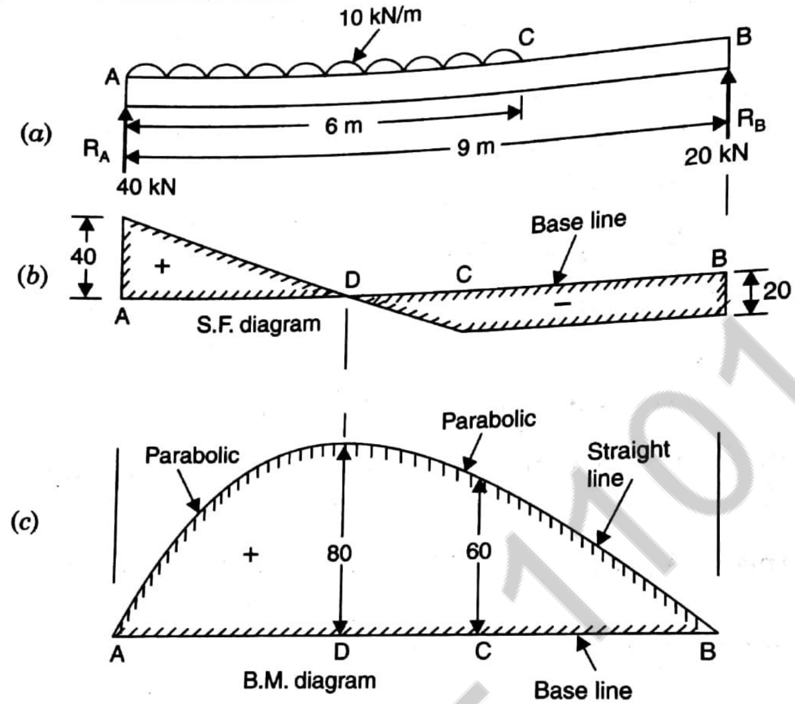


Fig. 6.28

Taking moments of the forces about A, we get

$$R_B \times 9 = 10 \times 6 \times \frac{6}{2} = 180$$

$$\therefore \quad R_B = \frac{180}{9} = 20 \text{ kN}$$

$$\therefore \quad R_A = \text{Total load on beam} - R_B = 10 \times 6 - 20 = 40 \text{ kN.}$$

Shear Force Diagram

Consider any section at a distance x from A between A and C. The shear force at the section is given by,

$$F_x = +R_A - 10x = +40 - 10x \qquad \text{...}(i)$$

Equation (i) shows that shear force varies by a straight line law between A and C.

At A, $x = 0$ hence $\quad F_A = +40 - 0 = 40$ kN

At C, $x = 6$ m hence $\quad F_C = +40 - 10 \times 6 = -20$ kN

The shear force at A is + 40 kN and at C is – 20 kN. Also shear force between A and C varies by a straight line. This means that somewhere between A and C, the shear force is zero. Let the S.F. is zero at x metre from A. Then substituting the value of S.F. (*i.e.*, F_x) equal to zero in equation (i), we get

$$0 = 40 - 10x$$

$$\therefore \qquad x = \frac{40}{10} = 4 \text{ m}$$

Hence shear force is zero at a distance 4 m from A.

The shear force is constant between C and B. This equal to – 20 kN.

Now the shear force diagram is drawn as shown in Fig. 6.28 (b). In the shear force diagram, distance $AD = 4$ m. The point D is at a distance 4 m from A.

B.M. Diagram

The B.M. at any section between A and C at a distance x from A is given by,

$$M_x = R_A \times x - 10 \,.\, x \,.\, \frac{x}{2} = 40x - 5x^2 \qquad \text{...}(ii)$$

Equation (ii) shows that B.M. varies according to parabolic law between A and C.

At A, $x = 0$ hence $\quad M_A = 40 \times 0 - 5 \times 0 = 0$

At C, $x = 6$ m hence $\quad M_C = 40 \times 6 - 5 \times 6^2 = 240 - 180 = +60$ kNm

At D, $x = 4$ m hence $\quad M_D = 40 \times 4 - 5 \times 4^2 = 160 - 80 = +80$ kNm

The bending moment between C and B varies according to linear law.

B.M. at B is zero whereas at C is 60 kNm.

The bending moment diagram is drawn as shown in Fig. 6.28 (c).

Maximum Bending Moment

The B.M. is maximum at a point where shear force changes sign. This means that the point where shear force becomes zero from positive value to the negative or *vice-versa,* the B.M. at that point will be maximum. From the shear force diagram, we know that at point D, the shear force is zero after changing its sign. Hence B.M. is maximum at point D. But the B.M. at D is + 80 kNm.

$\therefore$ Max. B.M. = **+ 80 kN. Ans.**

4) A simply supported wooden beam of span 1.3m having a cross section 150mm wide by 250mm deep carries a point load W at the centre. The permissible stresses are 7 N/mm^2 in bending 1 N/mm^2 in shearing. Calculate the safe load W. (Nov/Dec 2018)

Sol. Given :

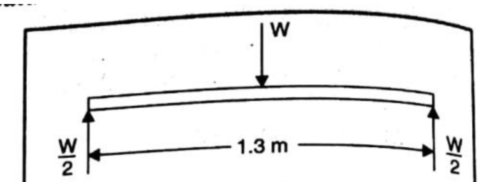


Fig. 8.6

Span, $L = 1.30$ mm

Width, $b = 150$ mm

Depth, $d = 250$ mm

Bending stress, $\sigma = 7$ N/mm^2

Shearing stress, $\tau = 1$ N/mm^2

Maximum B.M., $M = \dfrac{W \times L}{4} = \dfrac{W}{2} \times 1.3$

Nm

$$= \frac{W}{4} \times 1.3 \times 1000 \text{ Nmm} = 325\,W \text{ Nmm}$$

Maximum S.F. $= \dfrac{W}{2}$ N.

(i) Value of W for bending stress consideration

Using bending equation

$$\frac{M}{I} = \frac{\sigma}{y} \quad \text{...}(i)$$

where $M = 325\,W$ Nmm

$$I = \frac{bd^3}{12} = \frac{150 \times 250^3}{12} = 195312500 \text{ mm}^4$$

$\sigma = 7$ N/mm^2

and $y = \dfrac{d}{2} = \dfrac{205}{2} = 125.$

Substituting these values in the above equation (i), we get

$$\frac{325W}{195312500} = \frac{7}{125}$$

$$\therefore \quad W = \frac{7 \times 195312500}{325 \times 125} = 33653.8 \text{ N.}$$

(ii) Value of W for shear stress consideration

Average shear stress,

$$\tau_{avg} = \frac{\text{Shear force}}{\text{Area}} = \frac{\left(\frac{W}{2}\right)}{b \times d} = \frac{W}{2 \times 150 \times 250}$$

[redacted]

Maximum shear stress is given by equation (8.4)

$$\therefore \qquad \tau_{max} = \frac{3}{2} \times \tau_{avg}$$

But

$$\tau_{max} = 1 \text{ N/mm}^2$$

$$\therefore \qquad 1 = \frac{3}{2} \times \frac{W}{2 \times 150 \times 250}$$

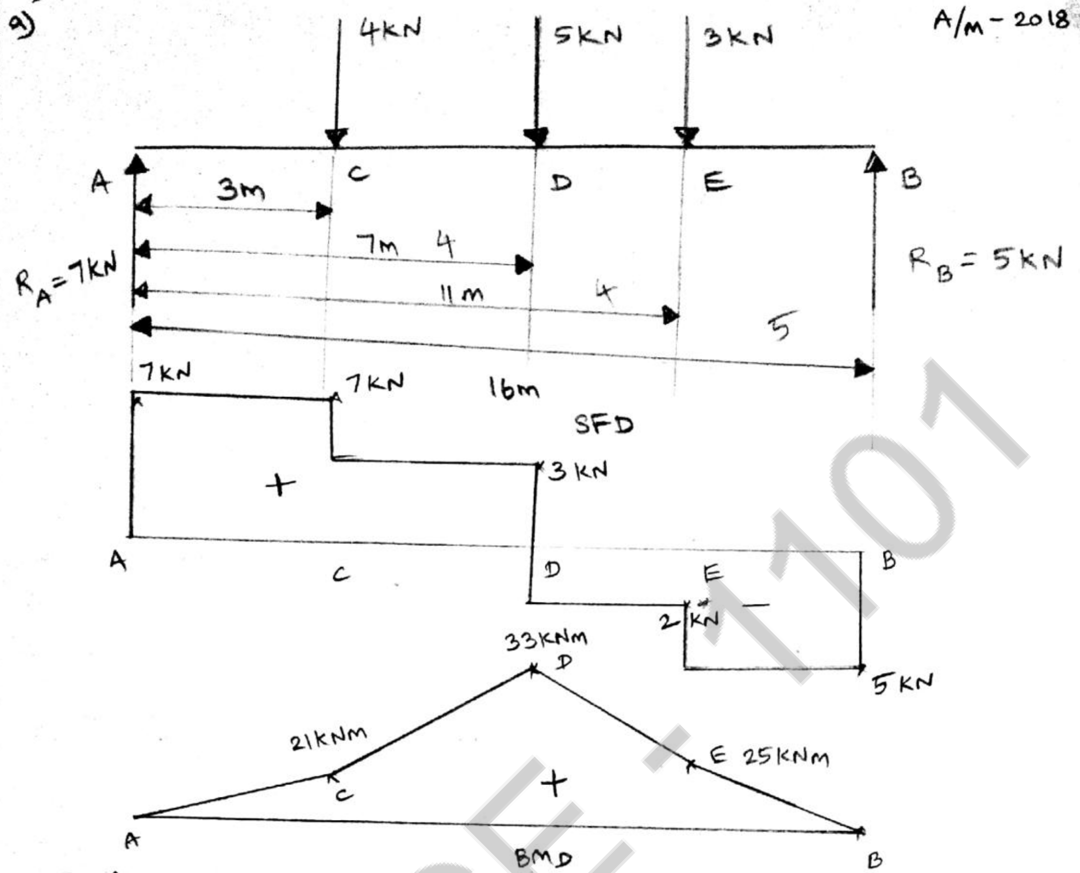
or

$$W = \frac{2 \times 2 \times 150 \times 250}{3} = 50000 \text{ N.}$$

Hence, the safe load is minimum of the two values (*i.e.*, 33653.8 and 50000 N) of *W*. Hence safe load is 33653.8 N. **Ans.**

5) A simply supported beam of 16m effective span carries the concentrated loads of 4kN, 5kN and 3kN at distances 3m, 7m and 11m respectively from the left end support. Calculate maximum shearing force and bending moment. Draw the S.F and B.M diagrams (Apr/May 2018)

12)
a) A/M - 2018



To find,

R_A & R_B, $R_A + R_B = 4 + 5 + 3 = 12\,KN \longrightarrow (1)$

$\Sigma M_A = 0 \Rightarrow -4\times3 - 5\times7 - 3\times11 + R_B \times 16 = 0$

$16 R_B = 80$

$\boxed{R_B = 5KN}$ subs. in (1),

we get, $\boxed{R_A = 7KN}$

<u>SF</u>

SF at B = −5KN

" " E = −5+3 = −2KN

" " D = −2+5 = +3KN

" " C = +3+4 = +7KN

" " A = +7KN

L	R
+ ↑	↓
− ↓	↑

<u>BM</u>

$M_A = M_B = 0$ (End support)

$M_E = 5\times5 = 25\,KNm$

$M_D = 5\times9 - 3\times4 = 33\,KNm$

$M_C = 5\times13 - 3\times8 - 5\times4 = 21\,KNm$

6) A timber beam of rectangular section is support a load of 50kN uniformly distributed over a span of 4.8m when beam is simply supported. If the depth of section is to be twice the breath, and the stress in the timber is not to exceed 7 N/mm^2, find the dimensions of the cross section. (Apr/May 2018)

$W = 50kN$

$l = 4.8m$

$d = 2b$

$\sigma_b = 7N/mm^2$

$Z = \frac{bd^2}{6} = \frac{b(2b)^2}{6} = \frac{2b^3}{3}$

we know that,

SSB with UDL, $M = \frac{wl^2}{8} = \frac{Wl}{8}$

$M = \frac{50x10^3 x4.8}{8} = 30000Nm = 30000000Nmm$

$M = \sigma_{max} xZ$

$30000000 = 7x\frac{2b^3}{3} \Rightarrow b = 185.94 \approx 186mm$

$d = 2b = 372mm$

7) A cantilever of length 2m carries a uniformly distributed load of 2kN/m length over the whole length and a point load of 3kN at the free end. Draw the S.F and B.M diagram for the cantilever. (Nov/Dec 2017)

Sol. Given :

Length, $L = 2.0$ m

U.D.L., $w = 2$ kN/m length

Point load at free end = 3 kN

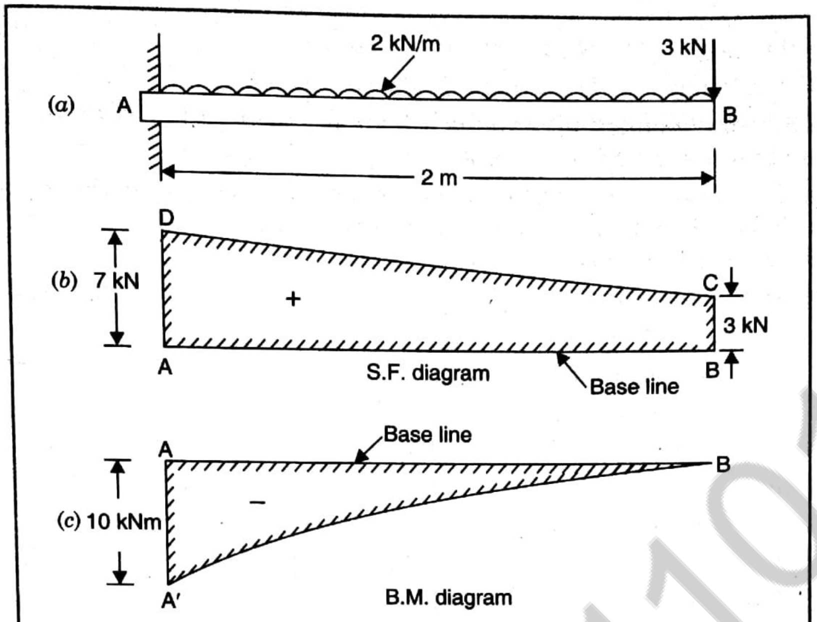


Fig. 6.18

Shear Force Diagram

The shear force at B = 3 kN

Consider any section at a distance x from the free end B. The shear force at the section is given by,

$$F_x = 3.0 + w.x \quad \text{(+ve sign is due to downward force on right portion of the section)}$$

$$= 3.0 + 2 \times x \quad (\because \; w = 2 \text{ kN/m})$$

The above equation shows that shear force follows a straight line law.

At B, $x = 0$ hence $F_B = 3.0$ kN

At A, $x = 2$ m hence $F_A = 3 + 2 \times 2 = 7$ kN.

The shear force diagram is shown in Fig. 6.18 (*b*) in which $F_B = BC = 3$ kN and $F_A = AD$ = 7 kN. The points C and D are joined by a straight line.

Bending Moment Diagram

The bending moment at any section at a distance x from the free end B is given by,

$$M_x = -\left(3x + wx \cdot \frac{x}{2}\right)$$

$$= -\left(3x + \frac{2x^2}{2}\right) \quad (\because \; w = 2 \text{ kN/m})$$

$$= -(3x + x^2) \quad \text{...}(i)$$

(The bending moment will be negative as for the right portion of the section, the moment of loads at x is clockwise).

Equation (i) shows that the B.M. varies according to the parabolic law. From equation (i), we have

At B, $x = 0$ hence $\quad M_B = -(3 \times 0 + 0^2) = 0$

At A, $x = 2$ m hence $\quad M_A = -(3 \times 2 + 2^2) = -10$ kN/m

Now the bending moment diagram is drawn as shown in Fig. 6.18 (c). In this diagram, $AA' = 10$ kNm and points A' and B are joined by a parabolic curve.

8) A beam is simply supported and carries a uniformly distributed load of 40kN/m run over the whole span. The section of the beam is rectangular having depth as 500mm. If the maximum stress in the material of the beam is 120 N/mm^2 and moment of inertia of the section is 7x10^{8}mm^4, find the span of the beam. (Nov/Dec 2017)

$$w = 40kN/m = 40000N/m$$

$$d = 500mm$$

$$\sigma_{\max} = 120N/mm^2$$

$$I = 7x10^8 mm^4$$

$$Z = \frac{I}{y_{\max}}$$

$$y_{\max} = \frac{d}{2} = \frac{500}{2} = 250mm$$

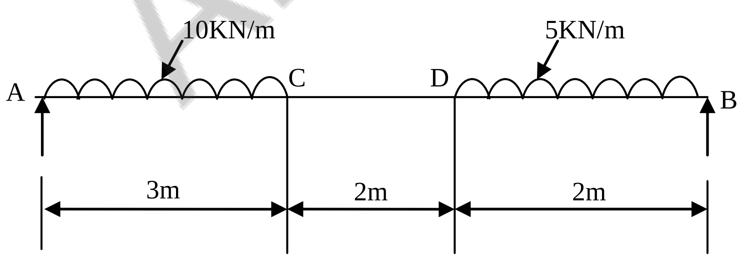
$$Z = \frac{I}{y_{\max}} = \frac{7x10^8}{250} = 28x10^5 mm^3$$

$$M = \frac{wl^2}{8} = \frac{40000xl^2}{8} = 5000l^2 Nm = 5000l^2 x1000 Nmm$$

$$M = \sigma_{\max} xZ$$

$$5000l^2 x1000 = 120x28x10^5 \quad \Rightarrow \quad \text{l=8.197mm} \approx 8.2\text{mm}$$

9) Draw shear force diagram and bending moment diagram for the beam given in fig.2 (May/June 2017)



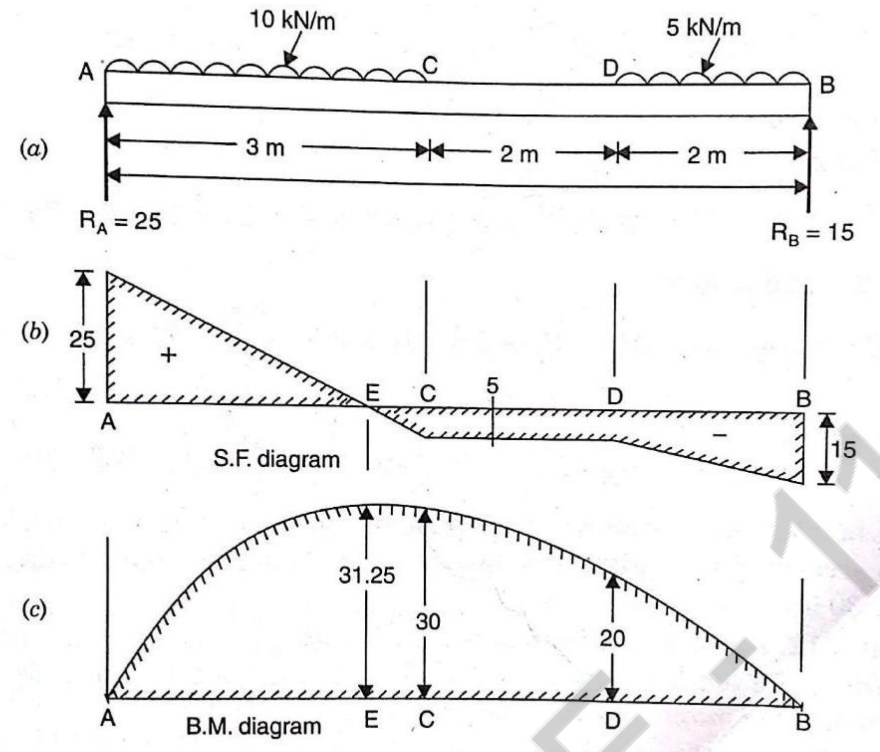


Fig. 2

Solution.

First calculate the reactions R_A and R_B

Taking moments of all forces about A, we get

$$R_B \times 7 = 10 \times 3 \times \frac{3}{2} + 5 \times 2 \times \left(3 + 2 + \frac{2}{2}\right) = 45 + 60 = 105$$

$$\therefore \quad R_B = \frac{105}{7} = 15\text{kN}$$

and R_A = Total load on beam-R_B

= (10 x 3 + 5 x 2) – 15 = 40 – 15 = 25 kN

S.F Diagram

The shear force At A is + 25kN

The shear force at C = R_A – 3 x 10 = +25 – 30 = -5kN

The shear force varies between A and C by a straight line law.

The shear force between C and D is constant and equal to -5kN

The shear force at B is -15kN

The shear force between D and B varies by a straight line law.

The shear force diagram is drawn as shown in Fig.2(b)

The shear force is zero at point E between A and C . Let us find the location of E from A. Let the point E at a be distance x from A.

The shear force at E = R_A – 10 x x= 25 – 10x

But shear force at E = 0

∴ 25 – 10x = 0 or 10x = 25

Or $x = \frac{25}{10} = 2.5\text{m}$

B.M.Diagram

B.M.. at A is zero

B.M. at B is zero

B.M. at C, $M_c = R_A \times 3 - 10 \times 3 \times \frac{3}{2} = 25 \times 3 - 45 = 75 - 45 = 30$ kNm

At E, x = 2.5 and hence

B.M. at E, $M_E = R_A \times 2.5 - 10 \times 2.5 \times \frac{2.5}{2} = 25 \times - 5 \times 6.25$

$= 62.5 - 31.25 = 31.25$ kNm

B.M. at D, $M_D = 25(3+2) - 10 \times 3 \times \left(\frac{3}{2} + 2\right) = 125 - 105 = 20$ kNm

The B.M. between AC and between BD varies according to parabolic law. But B.M. between C and D varies according to straight line law. Now the bending moment diagram is drawn as shown in Fig.2.(c)

10) A beam of square section is used as a beam with one diagonal horizontal. The beam is subjected to a shear force F, at a section. Find the maximum shear in the cross section of the beam and draw shear stress distribution diagram for the section.

Solution:

Given : A square section with its diagonal horizontal.

The beam with horizontal diagonal is shown in Fig.2.(a)

Let 2b = Diagonal of the square, and

F = shear force at the section

Now consider the shaded strip AJK at a distance x from the corner A. From the geometry of the figure, we find that length JK = 2x

$$\therefore \text{Area of AJK,}\quad A = \frac{1}{2} \times 2x.x = x^2$$

and $$\bar{y} = b - \frac{2x}{3}$$

we know that moment of inertia of the section ABCD about the neutral axis,

$$I = 2 \times \frac{2b \times b^3}{12} = \frac{b^4}{3}$$

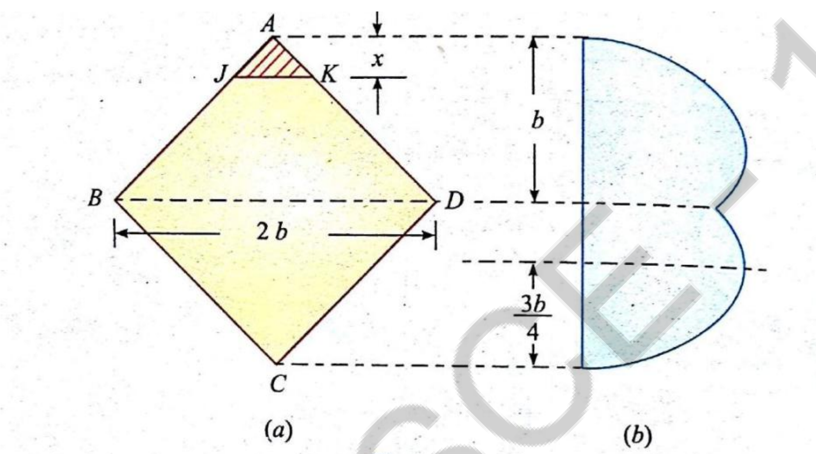


Fig. 2

and shearing stress at any point,

$$\tau = F \times \frac{A\bar{y}}{Ib} = F \times \frac{x^2\left(b - \frac{2x}{3}\right)}{\frac{b^4}{3} \times 2x} \qquad (\text{Here } b = JK = 2x)$$

$$= \frac{F}{2b^4}\left(3bx - 2x^2\right) \qquad ....(i)$$

We also know that when x = 0, τ = 0 and when x = b, then

$$\tau = \frac{F}{2b^2} = \frac{F}{\text{Area}} = \tau_{mean}$$

Now for maximum shear stress, differentiating the equation (i) and equating it to zero

$$\frac{d\tau}{dx} = \frac{d}{dx}\left[\frac{F}{2b^4}\left(3bx - 2x^2\right)\right] = 0$$

$$\therefore \quad 3b - 4x = 0 \text{ or } x = \frac{3b}{4}$$

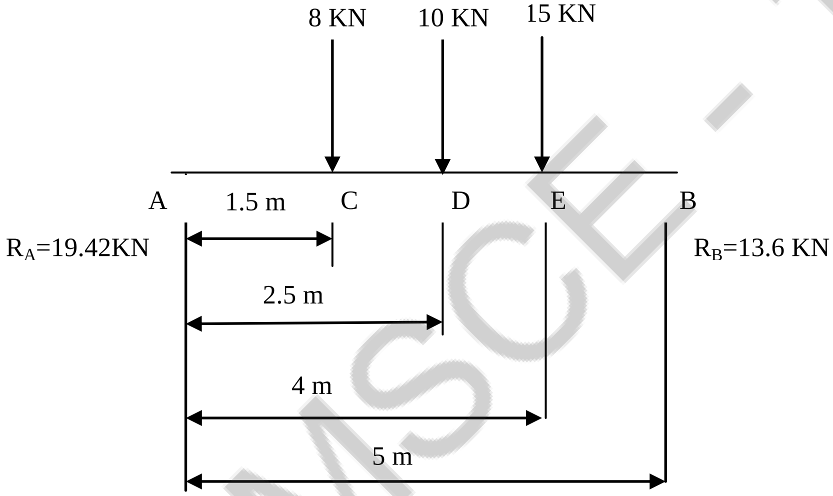
Substituting this value of x in equation (i),

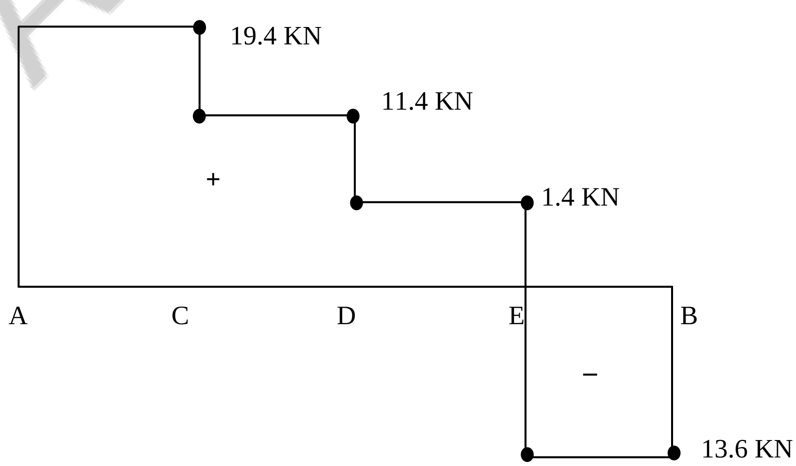
$$\tau_{max} = \frac{F}{2b^4}\left[3b \times \frac{3b}{4} - 2\left(\frac{3b}{4}\right)^2\right] = \frac{F}{2b^4} \times \frac{9b^2}{8}$$

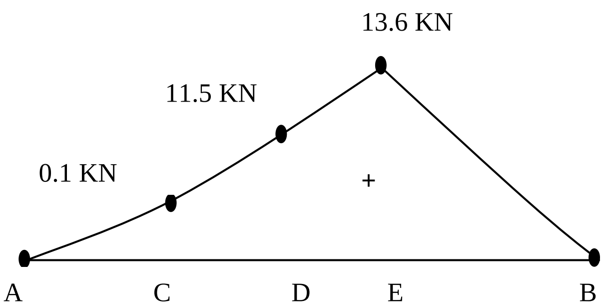
$$= \frac{9}{8} \times \frac{F}{2b^2} = \frac{9}{8} \times \frac{F}{Area} = \frac{9}{8} \times \tau_{mean}$$

Now complete the shear stress distribution diagram as shown in Fig.2(b)

11) A simply supported beam AB of length 5m carries point loads of 8 KN, 10 KN and 15 KN at 1.5 m, 2.50m and 4.0 m respectively from the left hand support. Draw the sheer force diagram and bending moment diagram. (Nov / Dec 2016)







To Find Reac

$$R_A + R_B = 8 + 10 + 15 = 33$$
$$R_A + R_B = 33 \text{ KN} \qquad \text{......1}$$

$$\sum M_A = 0 \Rightarrow \quad R_B \times 5 - 15 \times 4 - 10 \times 2.5 - 8 \times 1.5 = 0$$
$$5R_B = 97$$
$$\therefore \quad R_B = 19.4 \text{KN substitute in } 1$$
$$R_A = 13.6 \text{KN}$$

SFD | BMD

$\text{SF at B} = -13.6 \text{ KN}$

$\text{SF at E} = -13.6 + 15 = 1.4 \text{KN}$

$\text{SF at D} = -13.6 + 15 + 10 = 11.4 \text{KN}$

$\text{SF at C} = -13.6 + 15 + 10 + 8 = 19.4 \text{ KN}$

$\text{SF at A} = 19.4 \text{KN}$

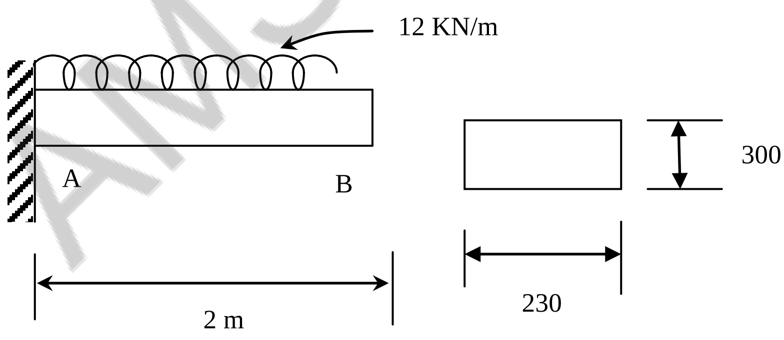
$\text{SSB at supports } M_A = M_B = 0$

$M_E = 13.6 \times 1 = 13.6 \text{KNm}$

$M_D = 13.6 \times 2.5 - 15 \times 1.5 = 11.5 \text{ KNm}$

12) A cantilever beam AB of length 2m carries a uniformly distributed load of 12 KN/m over entire length. Find the shear stress and bending stress, if the size of the beam is 230mm $\times$ 300 mm. [5 mark]

[Nov/ Dec 2016]



Bending stress: **Shear stress:**

$$\frac{\sigma_b}{y} = \frac{M}{I}$$

$$M = \frac{\omega \ell^2}{2}$$

$$M = \frac{12 \times 2^2}{2} = 24 \text{ KNm}$$

$$I = \frac{bd^3}{12} = \frac{230 \times 300^3}{12}$$

$$I = 517500000 \, mm^4$$

$$y = 150mm$$

$$\sigma_b = \frac{M}{I} y$$

$$= \frac{24 \times 10^3 \times 10^2 \times 150}{517500000}$$

$$\boxed{\sigma_b = 6.96 \text{ N/mm}^2}$$

$$F = \omega \ell = 24 \text{ KN}$$

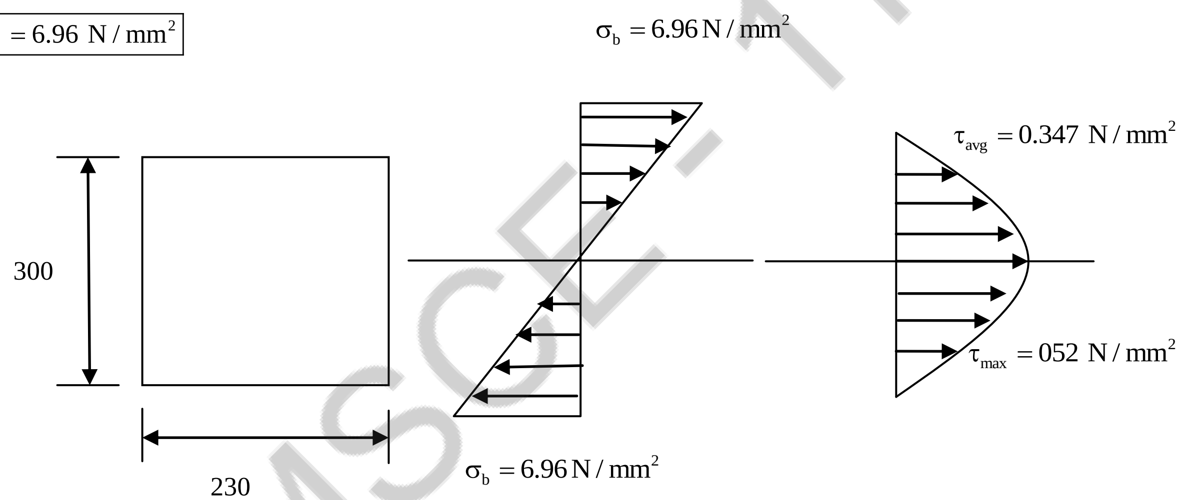
$$\tau_{max} = \frac{3}{2} \tau_{avg}$$

$$\tau_{avg} = \frac{F}{bd}$$

$$= \frac{24 \times 10^3}{230 \times 300}$$

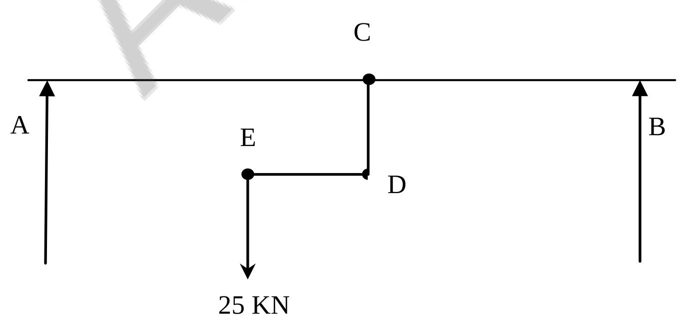
$$\boxed{\tau_{avg} = 0.347 \text{ N/mm}^2}$$

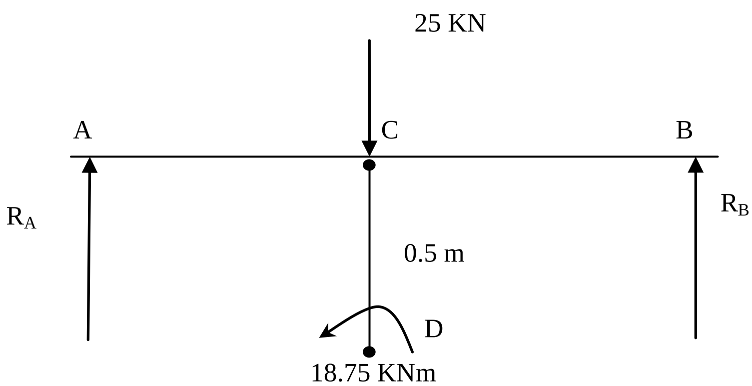
$$\boxed{\tau_{max} = 0.52 \text{ N/mm}^2}$$

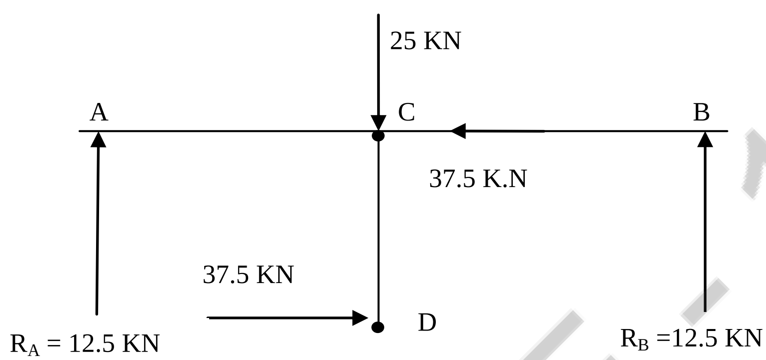


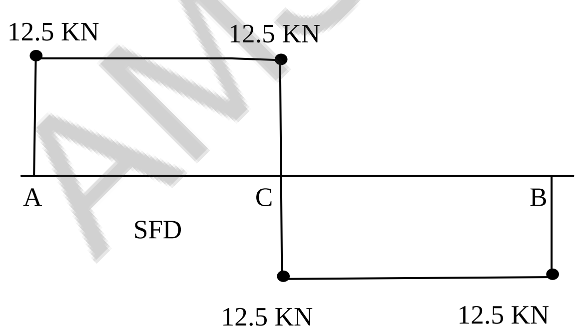
13) Construct the SFD & BMD for the beam as shown in fig (6 mark)

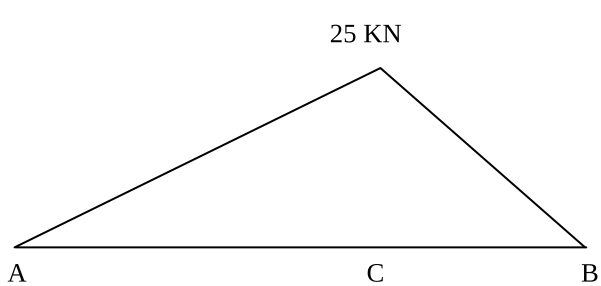
[Nov/ Dec 2016]




25 KN
A
C
B
R_A
R_B
0.5 m
D
18.75 KNm


25 KN
A
C
B
37.5 K.N
37.5 KN
D
R_A = 12.5 KN
R_B =12.5 KN


12.5 KN
12.5 KN
A
C
B
SFD
12.5 KN
12.5 KN


25 KN
A
C
B

$R_A + R_B = 25\,KN \qquad ...1$

$\sum M_A = 0$

$4R_B = 25 \times 2$

$R_B = 12.5\,KN$

$R_A = 12.5\,KN$

SFD

$S.F \text{ at } B = -12.5 \text{ KN}$

$S.F \text{ at } C = -12.5 + 25 \text{ KN}$

$= +12.5 \text{ KN}$

$S.F \text{ at } A = +12.5 \text{ KN}$

BMD

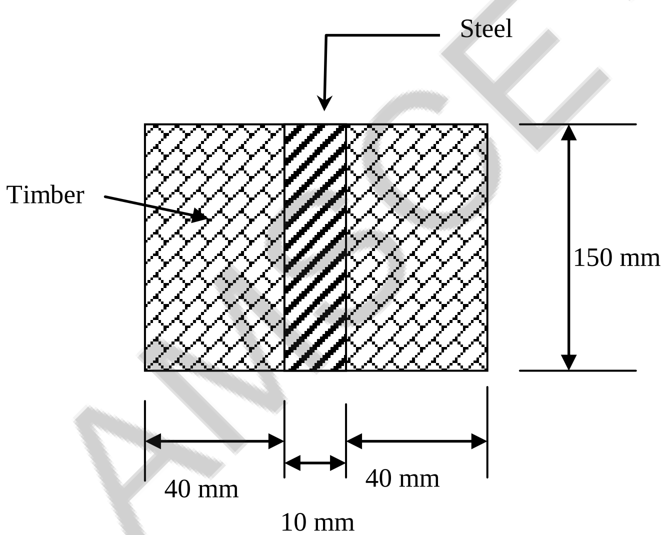
$m_A = m_B = 0$

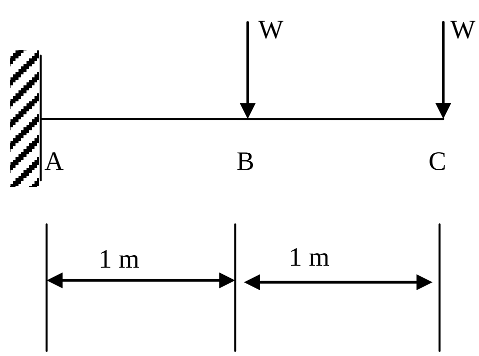
$m_C = 12.5 \times 2 = 25\,KNm$

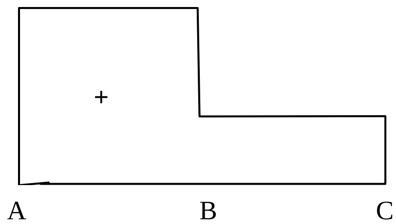
$SF = 0 \text{ at } C \Rightarrow (BM)_{max} = 25 \text{ KNm}$

14) Two timber joist are connected by a steel plate, are used as beam as show in fig find the load W if the permissible stress in steel and timber are 165 N/m^2 and 8.5 N/m^2 respectively (7 mark)

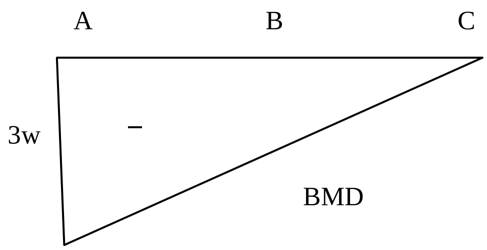
[Nov/ Dec 2016]







SFD



BMD

$\sigma_s = 165\ \text{N/mm}^2 \qquad \sigma_w = 8.5\ \text{N/mm}^2$

$$Z_w = \frac{bd^2}{6} = \frac{80 \times 150^2}{6}$$

$$= 300 \times 10^3\ \text{mm}^3$$

$$Z_s = \frac{bd^2}{6} = \frac{10 \times 150^2}{6}$$

$$= 37500\ \text{mm}^3$$

$$m_w = \sigma_w Z_w$$

$$= 8.5 \times 300 \times 10^3$$

$$= 2.55 \times 10^6\ \text{N mm}$$

$$m_s = \sigma_s Z_s$$

$$= 165 \times 37500$$

$$= 6.18 \times 10^6\ \text{N mm}$$

$$m = m_w + m_s$$

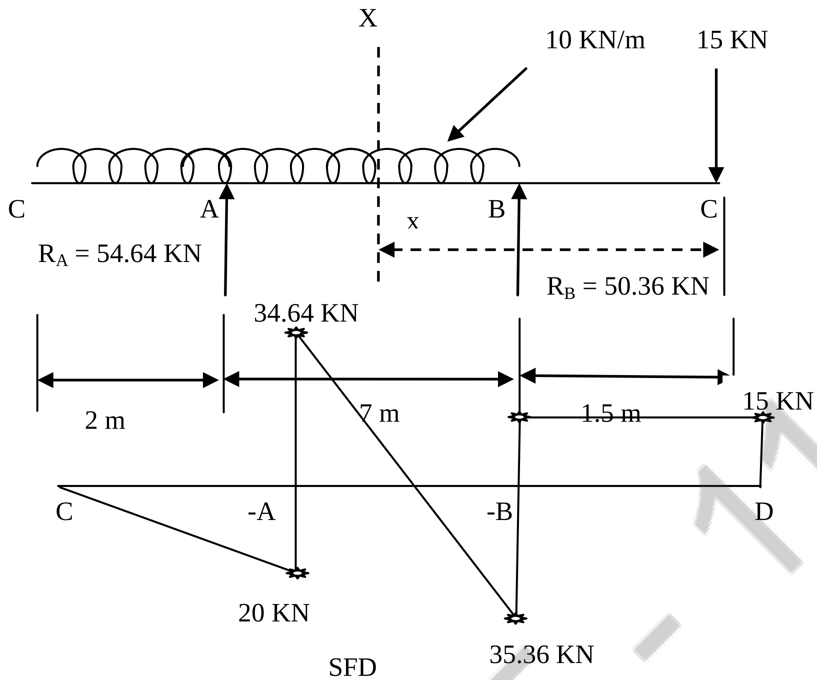
$$3w = m = 8.7 \times 10^6\ \text{N mm}$$

$$W = m / 3000 = 2.91 \times 10^3\ \text{N}$$

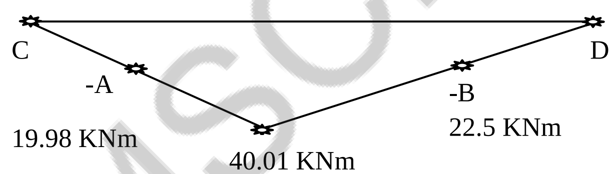
$$\boxed{W = 2.91}$$

15) Draw SRD & BDM and indicates the salient feature of beam loaded in fig

[May / June 2016]



SFD



$R_A \ \& \ R_B$:

$\sum m_A = 0 \Rightarrow$

$$R_A + R_B = 10 \times (2+7) + 15 = 105\,KN \qquad ...1$$

$$R_B \times 7 - 5 \times 7 \times \frac{7}{2} - 15 \times 8.5 + 10 \times 2 \times 1 = 0$$

$$7R_B = 352.5\ KN$$

$$R_B = 50.36\ KN \qquad \text{sub in 1}$$

$$R_A = 54.64\ KN$$

SFD

$$\text{SF at } D = 15\,KN$$
$$\text{SF at } B = +15 - 50.36 = -35.64$$
$$\text{SF at } A = +15 - 50.36 + 10 \times 7 = 34.64 \text{ KN}$$
$$\text{SF at } A = 34.64 - 54.64 = -20 \text{ KN}$$
$$\text{SF at } C = -20 + 10 \times 2 = 0$$

BMD

$$\text{BM at } D = 0$$
$$\text{BM at } B = -15 \times 1.5 = -22.5 \text{ KN}$$
$$\text{BM at } A = -15 \times 8.5 + 50.36 \times 7 - 10 \times 7 \times \frac{7}{2}$$
$$= -19.98 \text{ KNm}$$
$$\text{BM at } C = -15 \times 10.5 + 50.36 \times 9 - 10 \times 9 \times \frac{9}{2} + 54.64 \times 2$$
$$= 0.02 \approx 0$$

$$\text{SF at } XX = +15 - 50.36 + 10 \times (x - 1.5)$$
$$\text{BM at } XX = 15x - 50.6 \times (x - 1.5) + 10 \times (x - 1.5) \times (x - 1.5) \qquad ...2$$
$$\text{SF at } XX = 0$$
$$15 - 50.36 + 10(x - 1.5) = 0$$
$$10(x - 1.5) = 35.36$$
$$x - 1.5 = 3.536$$
$$\boxed{x = 5.036\,m} \quad \text{sub in equation 2}$$
$$\text{B.M at } (x = 5.036) = -40.01 \text{ KNm}$$

16) Find the dimensions of a timber joist, spam 4m to carry a brickwork is 20 KN/m^3. Permissible bending stress in timber is 10N/mm^2. The depth of the joist twice the width (8)

[May/ June 2016]

$$\ell = 4m$$
$$t = 230 \text{ mm} = 0.23m$$
$$h = 3m$$
$$\rho = 20 \text{ KN/m}^3$$
$$(\sigma_b)_{max} = 10 N/mm^2$$
$$d = 2b$$

Wt of bricks wall (W) = $\rho \times t \times h \times \ell$

SSB with UDL, $= 20 \times 0.23 \times 3 \times 4 = 55.2\,KN$

$$M = \frac{\omega \ell^2}{8} = \frac{\omega \ell}{8} = \frac{55.2 \times 4}{8}$$

$$M = 27.6 \text{ KNm} = 27.6 \times 10^6 \text{ Nmm}$$

$$I = \frac{bd^3}{12} = \frac{b \times (2b)^3}{12} = \frac{8b^4}{12}$$

$$y = \frac{d}{2} = \frac{2b}{2} = b$$

$$\text{section modulus}(Z) = \frac{I}{y} = \frac{8b^4}{12} \times \frac{1}{b} = \frac{8b^3}{12}$$

$$M = \sigma_b Z = 10 \times \frac{8b^3}{12}$$

$$10 \times \frac{8b^3}{12} = 27.6 \times 10^6$$

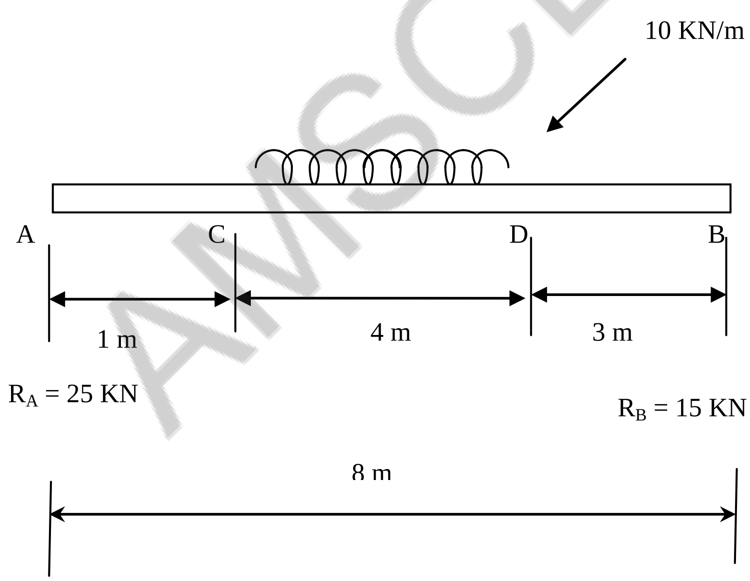
$$b^3 = \frac{27.6 \times 10^6 \times 12}{10 \times 8}$$

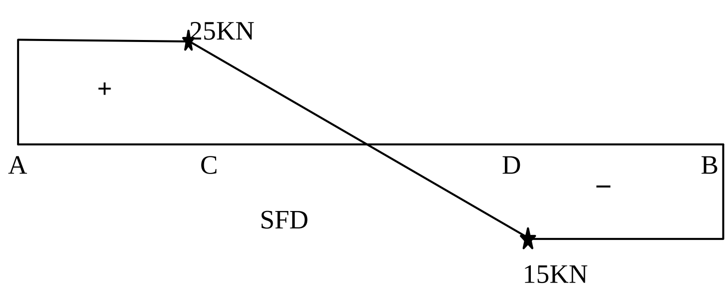
$$b = 160.57 \text{ mm}$$

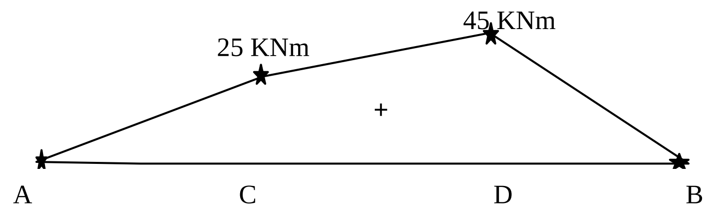
$$d = 2b = 321.14 \text{ mm}$$

17) Draw the sheer force & bending moment diagram for a simply supported beam of length 8m and carrying a UDL of 10KN/m for a distance of 4m as shown in fig (16)

[Nov/ Dec 2015]







R_A & R_B

$$R_A + R_B = 10\times4 = 40 \text{ KN} \qquad ...1$$

$$\sum M_A = 0 \Rightarrow \quad R_B\times8 - 10\times4\times3 = 0$$

$$8R_B = 120$$

$$R_B = 15 \text{ KN} \text{ sub in } 1$$

$$R_A = 25 \text{ KN}$$

SFD

SF at B $= -15$ KN

SF at D $= -15$ KN

SF at C $= -15 + 10\times4 = 25$ KN

SF at A $= +25$ KN

BMD

$M_A = M_B = 0$ (At end of supporb)

BM at D $(M_D) = 15\times3 = 45$ KNm

BM at C $(M_C) = 15\times7 - 10\times4\times2$

$= 25$ KNm

$$10\times\frac{8b^3}{12} = 27.6\times10^6$$

$$b^3 = \frac{27.6\times10^6\times12}{10\times8} = 4140000$$

$$b = 160.57$$

$$d = 2b = 321.14 \text{ mm}$$

18) A Steel plate at width 120 mm and of thickness 20 mm is bent into circular arc of radius 10m. Determine the maximum stress induced and the bending moment which will produce the maximum stress Take $E = 2\times10^5 \text{ N/mm}^2$ **(16)**

[Nov / Dec 2015]

$b = 120mm \qquad t = 20mm \qquad R = 10m$

$E = 2\times10^3 N/mm^2$

$I = \frac{bt^3}{12} = 80000\, mm^4$

$y_{max} = \frac{t}{2} = \frac{20}{2} = 10mm$

$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$

$(\sigma_b) = \frac{E}{R}\, y_{max}$

$\boxed{(\sigma_b) = \frac{2\times10^5\times10^2}{10\times10^3}\times10 = 200\ N/mm^2}$

$\frac{M}{I} = \frac{E}{R}$

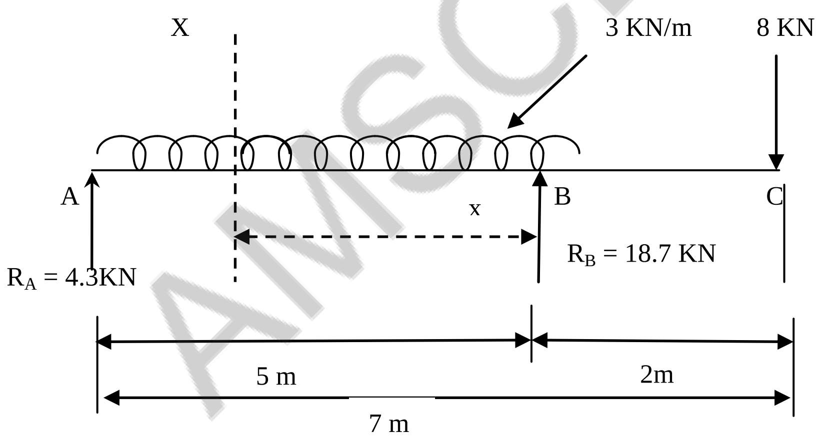
$M = \frac{E}{R} I = \frac{2\times10^5}{10\times10^3}\times80000$

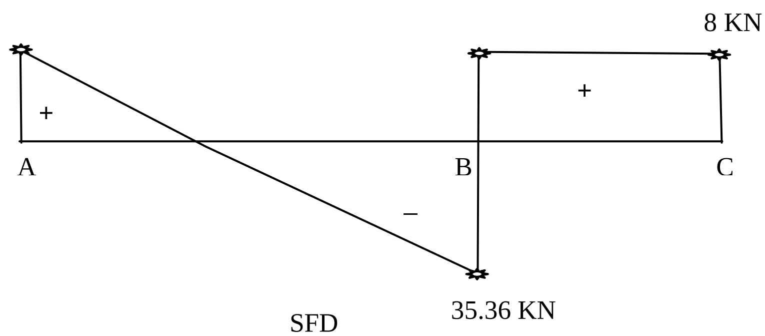
$\boxed{M = 1.6\times10^6\, Nmm}$

19) An overhanging beam ABC of length 7 m is simply supported at A & B over a span of 5m and the portion overhangs by 2m. Draw the shearing force & bending moments diagram and determine the point of contra flexure if it is subjected to UDL of 3 KN/ m over the portion AB and a concentrated load of 8 KN at C.(16)

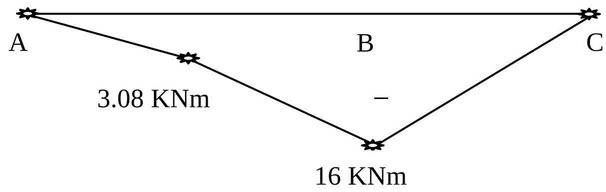
[Apr/ May 2015]

$R_A \& R_B$





SFD



$$R_A + R_B = 3\times5+8 = 23\ KN \qquad ...1$$

$$\sum M_A = 0 \Rightarrow \qquad R_B\times5-3\times5\times\frac{5}{2}-8\times7=0$$

$$5R_B = 93.5$$

$$\boxed{R_B = 18.7\ KN} \quad \text{sub in 1}$$

$$\boxed{R_A = 4.3\ KN}$$

SFD

SF at $C = +8\ KN$

SF at $B = +8-18.7 = -10.7\ KN$

SF at $A = -10.7+3\times5$

$= +4.3\ KN$

BMD

$M_C = +4.3\times7-3\times5\times4.5+18.7\times2$

$M_B = -8\times2 = -16\,KNm$

$M_A = -8\times7+18.7\times5-3\times5\times\frac{5}{2}=0$

Assume

XX section at a distance of x from end B,

SF at $XX = +8\,KN-18.7+3Xx = 0$

$3Xx = 10.7$

$x = 3.57\,m$

BM at $XX = 8\times(x+2)-18.7\times x+3\times x\times\frac{x}{2}$

$(BM)_{max} = -3.08\ KNm$

$(x=3.57)$ B.M at $XX = 0$

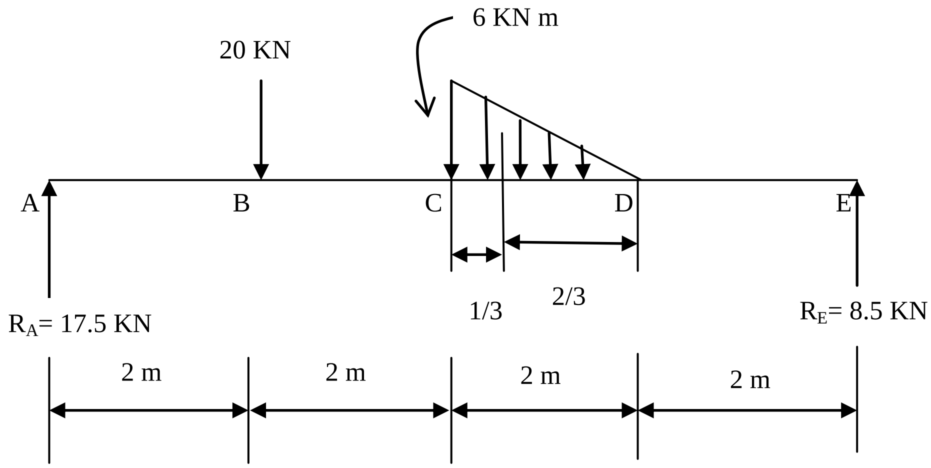
$8x+16-18.7x+1.5x^2 = 0$

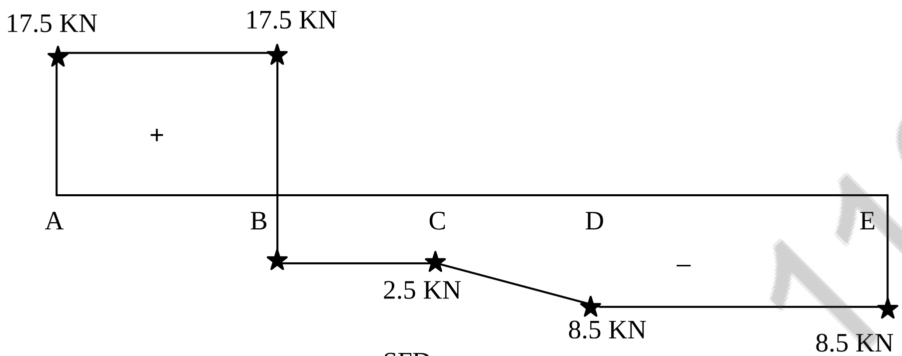
$1.5x^2-10.7x+16=0$

$\boxed{x=5m}$ or $\boxed{x=2.13m}$

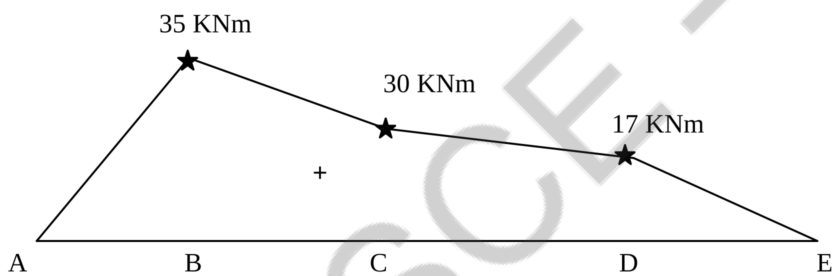
20) Draw SFD & BMD and find the max bending moment for the beam given in fig

[Nov/ Dec 2014]





SFD



R_A & R_E

$$R_A + R_E = 20 + \frac{1}{2}\times 2\times 6 = 26 \text{ KN} \qquad \ldots 1$$

$$M_A = 0 \Rightarrow \qquad R_E\times 8 - \frac{1}{2}\times 2\times 6\left(2 + 2\times\frac{1}{3}\times 2\right) - 20\times 2 = 0$$

$$8R_E = 68$$

$$\boxed{R_E = 8.5\,\text{KN}} \text{ sub in } 1$$

$$\boxed{R_A = 17.5 \text{ KN}}$$

SFD

SF at $E = -8.5$ KN

SF at $D = -8.5$ KN

SF at $C = -8.5 + \frac{1}{2} \times 2 \times 6 = 2.5$ KN

SF at $B = -2.5 + 20 = 17.5$ KN

SF at $A = +17.5$ KN

BMD

$M_A = M_E = 0$

B.M at $D = 8.5 \times 2 = 17$ KNm

B.M at $C = 8.5 \times 4 - \frac{1}{2} \times 2 \times 6 \times \frac{1}{3} \times 2$

$= 30$ KNm

B.M at $B = 8.5 \times 6 - \frac{1}{2} \times 2 \times 6 \times \frac{1}{2} \times 2 + 2$

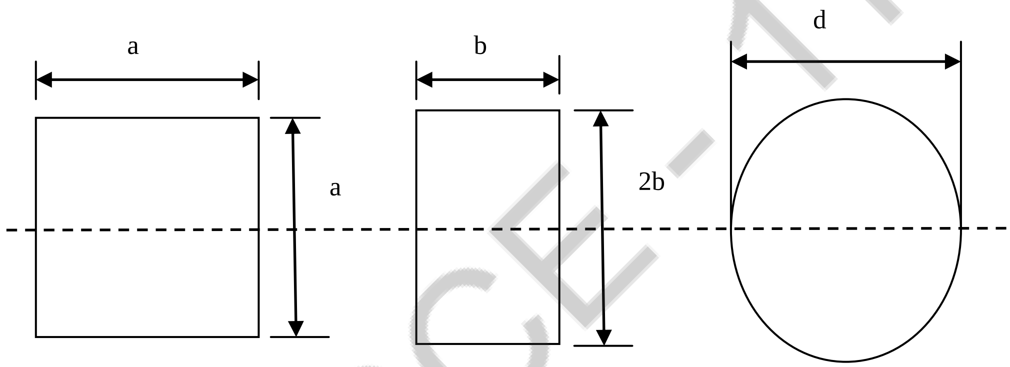
$= 35$ KNm

$SF = 0 \Rightarrow (BM)_{max}$

SF = 0 at point B $\Rightarrow (B.M)_{max}$ at B = 35KNm

21) Three beams here the same length allowable stress and the same bending moment. The cross section of the beams are a square, a rectangular with depth twice the width and a circle. Find the ratio of weight of circular and the rectangular beam with respect to the square beam (16)

[Apr / May 2015]



A= side of square beam

Rectangular beam

b= Width

2b=Depth

d= disc of circular beam

Since all three beams here the same σ & M the modules of section of the three beams must be equal

Square beams

$Z_1 = \frac{bd^2}{6}$

$= \frac{a \times a^2}{6}$

$Z_1 = \frac{a^3}{6}$ ...1

Rectangular beam

$Z_2 = \frac{bd^2}{6}$

$= \frac{b(2b)^2}{6}$

$Z_2 = \frac{4b^2}{6} = \frac{2}{3}b^2$ ...2

Circular beam

$z_3 = \frac{\pi}{32} \times d^3$ ...3

Equating 1 & 2

$$\frac{a^3}{b} = \frac{2b^2}{3}$$

$$a^3 = 6 \times \frac{2}{3} b^2$$

$$a^3 = 4b^3$$

$$b = 0.63a \qquad ...4$$

Equating 1 & 3

$$\frac{a^3}{6} = \frac{\pi}{32} d^3$$

$$a^3 = 6 \times \frac{\pi}{32} \times d^3$$

$$d = 1.19\,a \qquad ...5$$

Weight of all the beams are proportional to the c/s area of their section,

$$\frac{\text{Weight of Square beam}}{\text{Weight of rectangular beam}} = \frac{\text{Area of square beam}}{\text{Area of rectangular beam}}$$

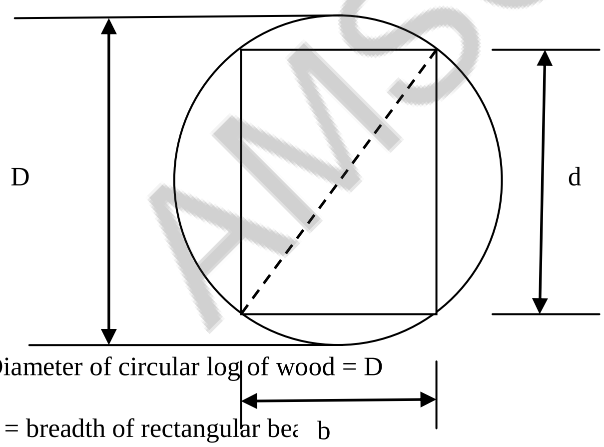
$$\frac{a^2}{2b^2} = \frac{a^2}{2 \times (0.63\,a)^2} = \frac{1}{0.79}$$

$$\frac{\text{Weight of Square beam}}{\text{Weight of circular beam}} = \frac{\text{Area of square beam}}{\text{Area of Circular beam}}$$

$$= \frac{a^2}{\frac{\pi}{4} d^2} = \frac{a^2}{\frac{\pi}{4} \times (1.19a)^2}$$

$$= \frac{1}{1.12}$$

22) Prove that the ratio of depth to width of the strongest beam that can be cut from a circular log of diameter d is 1.414. Hence calculate the depth and width of the strongest beam that can be cut of a cylindrical log of wood whose diameter is 300mm.



Diameter of circular log of wood = D

b = breadth of rectangular beam

d = depth of rectangular beam

$$\left.\begin{array}{l}\text{section}\\ \text{modulus}\end{array}\right\} \qquad Z = \frac{bd^2}{6} \qquad ...1$$

From geometry of the fig ,

$$b^2 + d^2 = D^2$$

$$d^2 = D^2 - b^2 \qquad ..2(\text{substituting equ1})$$

$$Z = \frac{b \times (D^2 \times b^2)}{6}$$

$$= \frac{bD^2 - b^3}{6}$$

For strongest section, Differentiate the above equation and equate it to zero,

$$\frac{dz}{db} = \frac{d}{db}\left(\frac{bD^2 - b^3}{6}\right) = \frac{D^2 - 3b^2}{6}$$

$$\frac{D^2 - 3b^2}{6} = 0$$

$$D^2 - 3b^2 = 0$$

$$3b^2 = D^2$$

$$b = \frac{D}{\sqrt{3}} \qquad ...2 \text{ subtituting in equa 1}$$

$$d^2 = D^2 - \frac{D^2}{3} = \frac{2D^2}{3}$$

$$d = D\sqrt{\frac{2}{3}} \qquad ..4$$

$$\text{Equ } 4 \div \text{Equ } 3$$

$$\frac{d}{b} = \frac{D\sqrt{\frac{2}{3}}}{\frac{D}{\sqrt{3}}} = D\sqrt{\frac{2}{3}} \times \frac{\sqrt{3}}{D}$$

$$\frac{d}{b} = \sqrt{2} = 1.414 \ (\text{Hence it is proved})$$

PART-C

23) A water main of 500mm internal diameter and 20mm thick is full. The water main is of cast iron and is supported at two points 10m apart. Find the maximum stress in the metal. The cast iron and water weigh 72000 N/m^3 and 10000 N/m^3 respectively. (May 2017 – 15 Marks)

Given:

Internal diameter, D_i = 500 mm = 0.5 m

Thickness of pipe, t = 20 mm

∴ outer dia, $D_0 = D_i + 2 \times t = 500 + 2 \times 20$

= 540 mm = 0.54 m

Weight density of cast iron = 72000 N/m^3

Weight density of water = 10000 N/m^3

Internal area of pipe = $\frac{\pi}{4}D_i^2 = \frac{\pi}{4}\times 0.5^2 = 0.1960\text{m}^2$

This is also equal to the area of water section

∴ Area of water section = 0.196 m^2

Outer area of pipe = $\frac{\pi}{4}D_0^2 = \frac{\pi}{4}\times 0.54^2\,\text{m}^2$

Area of pipe section = $\frac{\pi}{4}D_0^2 - \frac{\pi}{4}D_i^2$

$$= \frac{\pi}{4}\left[D_0^2 - D_i^2\right] = \frac{\pi}{4}\left[0.54^2 - 0.5^2\right] = 0.0327\text{m}^2$$

Moment of inertia of pipe section about neutral axis

$$I = \frac{\pi}{64}\left[D_0^4 - D_i^4\right] = \frac{\pi}{64}\left[540^4 - 500^4\right] = 1.105\times 10^9\ \text{mm}^4$$

Weight of pipe for one metre run = weight density of cast iron x volume of pipe

= 72000 x [area of pipe section x Length]

= 72000 x 0.0327 x 1

= 2354 N

Weight of water for me metre run = weight density of water x Volume of water

= 10000 x (Area of water section x length]

= 10000 x 0.196 x 1 = 1960 N

Total weight on the pipe for one metre run

= 2354 + 1960 = 4314 N

Hence the above weight is the U.D.L on pipe.

The maximum bending moment due to U.D.L is w x L^2/8, where w = Rate of U.D.L= 4314 N per metre run.

∴ Maximum bending moment due to U.D.L

$$M = \frac{w\times L^2}{8} = \frac{4314\times 10^2}{8} = 53925\,\text{Nm}$$

$$M = 53925\times 10^3\,\text{Nmm}$$

Now using $\frac{M}{I} = \frac{\sigma}{y}$

$$\sigma = \frac{M}{I} \times y$$

The stress is maximum when y is maximum

$$y = \frac{D_0}{2} = \frac{540}{2} = 270\,\text{mm}$$

y_{max} = 270 mm

$\therefore$ maximum stress $\sigma_{max} = \frac{M}{I} \times y_{max}$

$$= \frac{53925 \times 10^3}{1.105 \times 10^9} \times 270$$

$$= 13.18\,\text{N/mm}^2$$

UNIT-III

TORSION AND SPRINGS

PART-A [2-MARKS]

1) Write down the expression for power transmitted by a shaft. (Apr/May 2019)

$$P = 2\pi NT/60$$

2) Define helical springs. (Apr/May 2019)

A helical spring is a length of wire or bar wound into a helix. There are mainly two types of helical springs (i) close-coiled (ii) open-coiled

3) Give any two functions of spring. (Nov/Dec 2018)

- To absorb shock or impact loading as in carriage springs
- To store energy as in clock springs
- To apply forces to and to control motions as in brakes and clutches
- To measure forces as in spring balance
- To change the variations characteristics of a member as in flexible mounting of motors

4) Write the expression for polar modulus for a solid shaft and for a hollow shaft (Nov/Dec 2017) (Apr/May 2018)

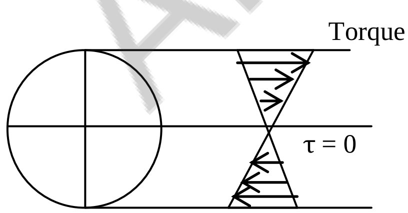
$$J = \frac{\pi}{32} d^4$$

$$J = \frac{\pi}{32}(D^4 - d^4)$$

5) Draw the shear stress distribution of a circular section due to torque. (May/June 2017)

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\tau = \frac{TR}{J}$$



6) What is meant by spring constant? (May/June 2017)

Spring constant (spring index) is the ratio of mean diameter of the spring to the diameter of the wire.

7) Define torsional rigidity. Nov/Dec-2016 , Nov/Dec-2014

From the torsional equation, we know that

$$\frac{T}{J} = \frac{C\theta}{\ell} \Rightarrow \theta = \frac{T\ell}{CJ}$$

Since, c,ℓ, and J are constant for A given shaft, θ(angle of twist) is directly proportional to T(Torque). The term CJ is known as torsional rigidity at it is represented by K.

8) What is a spring? Name the two important types of springs. (Nov/Dec 2016) (Nov/Dec 2017)

Spring is a device which is used to absorb energy by taking very large change in its form without permanent deformation and then release the same when it is required.

The important types of springs are

1) Torsion spring
2) Closed coiled helical spring
3) Open coil helical spring
4) Leaf spring

9) List out the stresses induced in the helical and carriage springs. (May/June 2016)

i) Direct shear stress
ii) Torsional shear stress
iii) Bending stress

10) Draw and discuss the shafts in series and parallel (May/June 2016)

In order to form a composite shaft sometimes two shaft are connected in series. In such case each shaft transmit, the same torque. The angle of twist is the sum of the angle of twist of two shaft connected in series.

When shaft are said to be parallel when the driving torque is applied at the junction of the shaft and the resisting torque is at the other end of the shaft. The angle of twist is same for each shaft.

11) The shearing stress in a solid shaft is not to exceed 40N/mm^2. When the torque transmitted is 2000N.m. Determine the minimum diameter of the shaft (Nov/Dec-2015)

Data

Shear stress(τ) = 40N/mm^2
Torque (T) = 2000N.m = 2000x10^3 N.mm
To find
(i) Minimum diameter of shaft

Solution:

$$T = \frac{\pi}{16} \times \tau \times d^3$$

$$\frac{2000 \times 10^3 \times 16}{\pi \times 40} = D^3$$

$$\boxed{D = 64mm}$$

12) What are the various types of springs? (Nov/Dec 2015)

1) Helical springs
(i) Closed coil helical springs
(ii) Open coil helical springs
2) Leaf springs
(i) Full-elliptic (ii) Semi-elliptic (iii) cantilever
3) Torsion springs
4) Circular springs
5) flat springs

13) What is meant by torsional stiffness? (Apr/May 2015)

It is the radio of Torque (T) to the angle of twist (θ)

$$\text{Torsional stiffness}(q) = \frac{T}{\theta}$$

14) What are the uses of helical springs? (Apr/May 2015)

1) Railway wagons
2) Cycle seating
3) Pistols
4) brakes

15) Differentiate open coiled and closed coiled helical springs. (Nov/Dec 2014) (Apr/May 2018)

	Open coil helical spring	**Closed coil helical spring**
1	Large gap between adjacent coils	Adjacent coils are very close to each other
2	Tensile and compressive loads can carry	Only tensile load can carry
3	Helix angle is considerable	Helix angle is negligible

16) Write down the expression for torque transmitted by hollow shaft.

$$T = \frac{\pi}{16} \times \tau \left[\frac{D^4 - d^4}{D}\right]$$

Where, T – Torque in N mm
τ - shear stress in N/mm^2
D – outer diameter in mm
d – inner diameter in mm

17) Calculate the maximum torque that a shaft of 125 mm diameter can transmit, if the maximum angle of twist is 1° in a length of 1.5m. Take C = 70x10^3 N/mm^2.

Given data:

Diameter, D = 125mm

Angle of twist, Q = $1° \times \frac{\pi}{180} = 0.017$

Length, ℓ = 1.5m = 1500mm

Modulus of rigidity, $C = 70x10^3$ N/mm^2

To find

Maximum Torque, T_{max}

Solution: Torsional equation

$$\frac{T}{J} = \frac{C\theta}{\ell}$$

$$T = \frac{JC\theta}{\ell}$$

$$T = \frac{\frac{\pi}{32}\left[D^4\right]}{\ell} \times C\theta$$

$$= \frac{\frac{\pi}{32}\left[125^4\right]}{1500} \times 70 \times 10^3 \times 0.017$$

$T = T_{max} = 19.01 \times 10^6$ N/mm

18) The stiffness of spring is 10N/mm. What is the axial deformation in the spring when a load of 50N is acting?

Given:

K = 10N/mm^2

W = 50N

$$k = \frac{w}{\delta} \Rightarrow \delta = \frac{w}{k} = \frac{50}{10} = 5\text{mm}$$

19) A helical spring is made of 4mm steel wire with a mean radius of 25mm and number of turns of coil 15. What will be deflection of the spring under a load of 6N. Take C = 80 x 10^3 N/mm^2

Given:

d = 4mm
R = 25mm
n = 15
w = 6N
c = 80 x 10^3 N/mm^2

Solution:

$$\text{Axial deformation, } \delta = \frac{64wR^3n}{cd^4}$$

$$\delta = \frac{64 \times 6 \times 25^3 \times 15}{80 \times 10^3 \times 4^4}$$

$$\delta = 4.39\text{mm}$$

20) Define Torsion / Twisting moment (Nov/Dec 2018)

When a pair of forces of equal magnitude but opposite directions acting on body, it tends to twist the body. It is known as twisting moment or torsion moment or simply as torque. Torque is equal to the product of the force applied and the distance between the point of application of the force and the axis of the shaft.

21) What are the assumptions made in Tortion equation

- The material of the shaft is homogeneous, perfectly elastic and obeys Hooke's law.
- Twist is uniform along the length of the shaft
- The stress does not exceed the limit of proportionality
- The shaft circular in section remains circular after loading
- Strain and deformations are small.

22) Define polar modulus

It is the ratio between polar moment of inertia and radius of the shaft = J/R polar moment of inertia = J Radius = R

23) Why hollow circular shafts are preferred when compared to solid circular shafts?

The torque transmitted by the hollow shaft is greater than the solid shaft.

For same material, length and given torque, the weight of the hollow shaft will be less compared to solid shaft.

24) Write torsion equation

$T/J = C\theta/L = q/R$

T – Torque
θ – angle of twist in radians
J- Modulus of rigidity
L- Length
Q – shear stress
R- Radius

25) What is spring index(C)?

The ratio of mean or pitch diameter to the diameter of wire for the spring is called the spring index.

26) What is solid length?

The length of a spring under the maximum compression is called its solid length. It is the product of total number of coils and the diameter of wire.

$L_s = n_t \times d$ where, n_t = total number of coils.

27) Define spring rate(stiffness).

The spring stiffness or spring constant is defined as the required per unit deflection of the spring, $K = W/\delta$

Where W = load and δ = Deflection

Part-B

1) Two shafts of the same material and same length are subjected to the same torque. If the first shaft is of a solid circular section and the second shaft is of a hollow circular section whose internal diameter is 2/3 of the outside diameter and the maximum shear stress developed in each shaft is the same, compare the weights of the two shafts. (Apr/May 2019)

Solution. Let D_S = Diameter of the solid shaft,

D_H = External diameter of the hollow shaft,

d_H = Internal diameter of the hollow shaft, and

τ = Maximum shear stress developed.

$$d_H = \frac{2}{3} D_H \qquad (Given)$$

The torque transmitted by the solid shaft,

$$T_s = \tau \cdot \frac{\pi}{16} D_S^3$$

and, the torque transmitted by the hollow shaft,

$$T_H = \tau \cdot \frac{\pi}{16}\left[\frac{D_H^4 - d_H^4}{D_H}\right] = \tau \cdot \frac{\pi}{16}\left[\frac{D_H^4 - (2/3\,D_H)^4}{D_H}\right]$$

$$= \tau \cdot \frac{\pi}{16}\left[\frac{D_H^4 - \left(\frac{16}{81}D_H^4\right)}{D_H}\right] = \tau \cdot \frac{\pi}{16} \times \frac{65}{81} D_H^3$$

Since both the torques are equal, therefore equating (*i*) and (*ii*), we get

$$T_S = T_H$$

$$\tau \cdot \frac{\pi}{16} D_S^3 = \tau \cdot \frac{\pi}{16} \cdot \frac{65}{81} D_H^3$$

$\therefore$

$$D_H^3 = 1{\cdot}246\, D_S^3, \text{ or, } D_H = 1{\cdot}08\, D_S$$

We know that,

$$\frac{\text{Weight of solid shaft}}{\text{Weight of hollow shaft}} = \frac{W_S}{W_H}$$

$$= \frac{A_S \times l_S \times w_S}{A_H \times l_H \times w_H} = \frac{A_S}{A_H}$$

$$\left[\begin{array}{l} \therefore\ l_S = l_H \\ w_S = w_H \text{ where, } w \text{ stands for weight density} \end{array}\right]$$

$$= \frac{\frac{\pi}{4} D_S^2}{\frac{\pi}{4}(D_H^2 - d_H^2)} = \frac{D_S^2}{[D_H^2 - \left(\frac{2}{3} D_H\right)^2]} = \frac{D_S^2}{D_H^2\left(1 - \frac{4}{9}\right)}$$

$$= \frac{D_S^2}{\frac{5}{9} \times (1{\cdot}08\, D_S)^2} = \frac{\mathbf{1{\cdot}543}}{\mathbf{1}} \quad \textbf{(Ans.)}$$

2) A closely coiled helical spring of mean diameter 20cm is made of 3cm diameter rod and has 16 turns. A weight of 3kN is dropped on this spring. Find the height by which the should be dropped before striking the spring so that the spring may be compressed by 18cm. Take C=8 x 10^4 N/mm^2 (Apr/May 2019) (Nov/Dec 2015)

D = 20cm = 200mm

R = $\frac{D}{2}$ = 100mm

d = 3cm = 30mm

n=16

W = 3kN = 3000N

δ = 18cm = 180mm

C = 8x10^4 N/mm^2

h = Height through which the weight W is dropped

W = Gradually applied load which produces the compression of spring equal to 180mm

$\delta = \frac{64WR^3n}{Cd^4} = \frac{64xWx100^3x16}{8x10^4x30^4}$

W = 11390N

Work done by falling weight on spring = W(h+δ)=3000(h+180) $\rightarrow 1$

Energy stored in the spring = $\frac{1}{2}$W x δ = $\frac{1}{2}$x11390x180=1025100Nmm $\rightarrow 2$

Equate 1 & 2, we get

3000(h+180) = 1025100

h= 161.7mm

3) A hollow shaft is to transmit 300kW power at 80rpm. If the shear stress is not to exceed 60 N/mm^2 and the internal diameter is 0.6 of the external diameter, find the external and internal diameters assuming that the maximum torque is 1.4 times the mean. (Nov/Dec 2017) (Nov/Dec 2018)

P = 300kW = 300000W

N = 80rpm

τ = 60N/mm^2

D = 0.6d

$T_{max} = 1.4T_{mean}$

$$P = \frac{2\pi N T_{mean}}{60} \Rightarrow T_{mean} = 35809.8\text{Nm}$$

$T_{max} = 1.4T_{mean} \Rightarrow T_{max}$ = 50133.7Nm = 50133700Nmm

$$T_{max} = \frac{\pi}{16} x \tau x \left[\frac{D^4 - d^4}{D}\right]$$

$$50133700 = \frac{\pi}{16} x 60 x \left[\frac{D^4 - (0.6D)^4}{D}\right]$$

D = 169.2mm $\approx$ 170mm

d = 0.6 x 170 = 102mm

4) A solid cylindrical shaft is to transmit 300kW power at 100rpm. (a) If the shear stress is not to exceed , find its diameter. (b) What percent saving in weight would be obtained if this shaft is replaced by a hollow one whose internal diameter equals to 0.6 of the external diameter, the length, the material and maximum shear stress being the same. (Apr/May 2018)

$a)$

$$P=\frac{2\pi NT}{60}$$

$$300000=\frac{2\pi x 100 x T}{60} \Rightarrow T=28647.8Nm = 28647800Nmm$$

$$T = \frac{\pi}{16} \text{ x } \tau \text{ x } D^3$$

$$28647800= \frac{\pi}{16} \text{ x } 80 \text{ x } D^3 \Rightarrow D = 121.8 \approx 122mm$$

b)

$D_i = 0.6 D_o$

Torque transmitted by solid shaft is equal to torque transmitted by hollow shaft

$$T = \frac{\pi}{16} \text{ x } \tau \text{ x } \frac{D_o^4 - D_i^4}{D_o}$$

$$T = \frac{\pi}{16} \text{ x } \tau \text{ x } \frac{D_o^4 - (0.6D_o)^4}{D_o}$$

$$28647800 = \frac{\pi}{16} \text{ x } 80 \text{ x } \frac{D_o^4 - (0.6D_o)^4}{D_o}$$

$D_o = 127.6mm \approx 128mm$

$D_i = 0.6 D_o = 0.6 \text{ x } 128 = 76.8mm$

W_s = Weight of solid shaft = Weight density x Area of solid shaft x Length

W_h = Weight of hollow shaft = Weight density x Area of hollow shaft x Length

$$\text{Percentage saving in weight} = \frac{W_s - W_h}{W_s} \text{ x } 100$$

(Both of same material and same length weight density and length get cancelled)

$$\text{Percentage saving in weight} = \frac{D^2 - (D_o^2 - D_i^2)}{D^2} \text{ x } 100$$

$$= \frac{122^2 - (128^2 - 76.8^2)}{122^2} \text{ x } 100 = 29.55\%$$

5) A closely coiled helical spring made of 10mm diameter steel wire has 15 coils of 100mm mean diameter. The spring is subjected to an axial load of 100N. Calculate (i) The maximum shear stress induced (ii)The deflection and (iii) Stiffness of the spring Take C=8.16 x 10^4 N/mm^2 (Nov/Dec 2017)

d = 10mm

n = 15

D = 100mm

$R = \frac{D}{2} = 50mm$

W = 100N

$C = 8.16 \text{ x } 10^4 \text{ N/mm}^2$

i) Maximum shear stress induced

$$\tau = \frac{16WR}{\pi d^3} = \frac{16x100x50}{\pi x10^3} = 24.46N/mm^2$$

ii) Deflection (δ)

$$\delta = \frac{64WR^3n}{Cd^4} = \frac{64x100x50^3x15}{8.16 \text{ x } 10^4 x \text{ } 10^4} = 14.7mm$$

iii) Stiffness of the spring (k):

$$k = \frac{W}{\delta} = \frac{100}{14.7} = 6.802 \text{ N/mm}$$

6) A hollow shaft, having an inside diameter 60% of its outer diameter, is to replace a solid shaft transmitting in the same power at the same speed. Calculate percentage saving in material, if the material to be is also the same. (May 2017)

Given:

Let D_0 = outer diameter of the hollow shaft

D_i = Inside diameter of the hollow shaft

$= 60\% \text{ of } D_0 = \frac{60}{100} \times D_0 = 0.6D_0$

D = Diameter of the solid shaft

P = power transmitted hollow (or) solid shaft

N = speed of each shaft

τ = maximum shear stress induced in each shaft since material of both is same

$$p = \frac{2\pi NT}{60}$$

$$T = \frac{p \times 60}{2\pi N} = \text{constant}$$

Torque transmitted by solid shaft is the same as the torque transmitted by hollow shaft

$$T = \frac{\pi}{16}\tau D^3 \quad \text{(Solid shaft)} \quad \rightarrow(1)$$

Torque transmitted by hollow shaft

$$T = \frac{\pi}{16}\tau\left[\frac{D_0{}^4 - D_i{}^4}{D_0}\right] = \frac{\pi}{16}\tau\left[\frac{D_0{}^4 - (0.6D_0)^4}{D_0}\right]$$

$$= \frac{\pi}{16}\tau\left[\frac{D_0{}^4 - 0.1296D_0{}^4}{D_0}\right] = \frac{\pi}{16}\tau \times 0.8704D_0{}^3 \rightarrow (2)$$

Torque transmitted is same, hence equating equations (1) and (2)

$$\frac{\pi}{16}\tau D^3 = \frac{\pi}{16}\tau \times 0.8704D_0{}^3$$

$D = (0.8704)^{1/3}\ D_0 = 0.9548D_0$

Area of solid shaft = $\frac{\pi}{4}D^2 = \frac{\pi}{4}(0.9548D_0)^2 = 0.716D_0{}^2$

Area of hollow shaft = $\frac{\pi}{4}\left[D_0^2 - D_i^2\right]$

$$= \frac{\pi}{4}\left[D_0{}^2 - (0.6D_0)^2\right]$$

$$= \frac{\pi}{4}\left[D_0{}^2 - 0.36D_0{}^2\right]$$

$$= \frac{\pi}{4} \times 0.64D_0{}^2 = 0.502D_0{}^2$$

For the shaft of same material, the weight of the shaft is proportional to the areas.

$\therefore$ saving in material = saving in area $= \frac{\text{Area of solid shaft} - \text{Area of hollow shaft}}{\text{Area of solid shaft}}$

$$= \frac{0.716D_0{}^2 - 0.502D_0{}^2}{0.716D_0{}^2}$$

$$= 0.2988$$

$\therefore$ percentage saving in material = 0.2988 x 100 = 29.88

7) Derive a relation for deflection of a closely coiled helical spring subjected to an axial compressive load 'w'. (May 2017)

Expression for deflection of spring

Length of one coil = πD (or) $2\pi R$

$\therefore$ Total length of the wire = length of one coil x No of coils

$\ell = 2\pi R \times n$

Strain energy stored by the spring

$$U = \frac{\tau^2}{4C}.\text{volume} = \frac{\tau^2}{4C}\text{volume}$$

$$= \left[\frac{16W.R}{\pi d^3}\right]^2 \times \frac{1}{4C} \times \left[\frac{\pi}{4} d^2 \times 2\pi R.n\right]$$

$$\left[\tau = \frac{16WR}{\pi d^3} \text{ and volume} = \frac{\pi}{4} d^2 \times \text{Total length of wire}\right]$$

$$= \frac{32W^2R^2}{Cd^4}.R.n = \frac{32W^2R^3n}{cd^4}$$

Work done on the spring = Average load x Deflection

$$= \frac{1}{2} W \times \delta$$

Equating the work done on spring to energy stored

$$\frac{1}{2} W.\delta = \frac{32W^2R^3.n}{cd^4}$$

$$\delta = \frac{64WR^3.n}{cd^4}$$

8) A solid shaft has to transmit the power 105kw at 2000 r.p.m. The maximum torque transmitted in each revolution exceeds the mean by 36% . Find the suitable diameter of the shaft, if the shear stress is not to exceed 75N/mm^2 and maximum angle of twist is 1.5 in a length of 3.30m and G = 0.80 x 10^5 N/mm^2.

(AU Nov 2016 – 8Marks)

Given data

Power = p =105 kw

Speed = N = 2000 rpm

T_{max} = 1.36 T_{mean}

Shear stress (τ) = 75 N/mm^2

Angle of twist (Q) = 1.5° $= 1.5 \times \frac{\pi}{180} = 0.026$ radians

Length = L = 3.30m = 3300mm

$G = 0.80 \times 10^5$ N/mm^2

To find

(i) diameter of the shaft

Solution:

We know that

$$\text{Power (p)} = \frac{2\pi NT}{60}$$

$$\text{Torque (T)} = \frac{P \times 60}{2 \times 3.14 \times 2000}$$

T = 0.5015 kN-m

$$\boxed{T = 501.59\,\text{Nm}}$$

T_{max} = 1.36 x 501.59

T_{max} = 682.16 Nm

Considering shear stress (z)

$$\text{Torque, T} = \frac{\pi}{16} \times \tau \times D^3$$

$$D^3 = \frac{T \times 16}{\pi \times \tau} = \frac{682.16 \times 10^3 \times 16}{3.14 \times 75}$$

$$D = 35.92\,\text{mm}$$

Considering angle of twist (θ)

$$\frac{T_{max}}{J} = \frac{c\theta}{\ell} \quad \text{where } J = \frac{\pi}{32} \times D^4$$

$$\frac{682.16 \times 10^3}{\frac{\pi}{32} \times D^4} = \frac{0.8 \times 10^5 \times 0.026}{3300}$$

$$\frac{682.16 \times 10^3 \times 3300 \times 32}{3.14 \times 0.8 \times 10^5 \times 0.026} = D^4$$

D = 57.62

D = 58mm

From above two cases we find that suitable diameter for the shaft is 58mm(ie, greater of the two values)

9) A laminated spring carries a central load of 5200 N and it is made of 'n' number of plates, 80mm wide, 7 mm thickness and length 500 mm . Find the number of plates is the maximum deflection is 10mm. Let $E = 2.0 \times 10^5$ N/mm^2 (AU Nov/Dec -2016 -8 marks)

Given data

Load = w = 5200 N

Width of plate = b =80 mm

Thickness of plate = t = 7 mm

Length of plate = ℓ = 500 mm

Maximum deflection = δ = 10 mm

`young modulus = E = 2.0×10^5 N/mm^2

To find:

(i) number of plates = (n)

We know that deflection equation for semi-elliptical spring is

$$\delta = \frac{3}{8}\frac{w\ell^3}{nEbt^3}$$

$$n = \frac{3}{8}\frac{w\ell^3}{Ebt^3\delta}$$

$$n = \frac{3}{8}\frac{5000\times 500^3}{2.0\times 10^5\times 80\times 7^3\times 10}$$

n = 4.27

n = 5 number of plates = 5

10) A closed coiled helical spring is to be made out of 5mm diameter wire, 2m long, so that it deflects by 20mm under an axial load of 50 N. Determine mean diameter of the coil Take $c = 8.1 \times 10^4$ N/mm^2 (Nov/Dec 2016) 16 Marks

Given

Diameter of wire = d = 5mm

Length (ℓ) = 2m = 2000 mm

Deflect (δ) = 20 mm

Axial load (w) = 50 N

C = 8.1×10^4 N/mm^2

To find:

(i) mean diameter of the coil (D)

Solution:

Deflection (δ) = $\frac{64wR^3n}{cd^4}(or)\frac{8wD^3n}{cd^4}$

Length of spring (ℓ) πD_n

$$n = \frac{\ell}{\pi D} = \frac{2000}{3.14\times D}$$

$$\delta = \frac{8wD^3n}{cd^4}$$

$$\frac{8cd^4}{8w_n} = D^3$$

$$\frac{20\times 8.1\times 10^4\times(5)^4\times 3.14\,D}{8\times 50\times 2000} = D^3$$

$$\boxed{D = 63.04mm}$$

11) A solid circular shaft 200 mm in diameter is to be replaced by a hollow shaft the ratio of external diameter to internal diameter being 5:3 . Determine the size of the hollow shaft if max shear stress is to be the same as that of a solid shaft. Also find the percentage saving in mass (March/June 2016) 16 Marks

Solid shaft dia (D) = 200 mm

Hollow shaft internal dia(d) = ?

Hollow shaft external dia(D_1) = ?

$$D_1/d = 5/3$$

$$d = 3/5\,D_1 = 0.6D_1$$

Torque transmitted by solid shaft (T) = $\pi/16\times\tau\times D^3$

$$\boxed{T = 1570796.32\,\tau}$$

Torque transmitted by solid hollow shaft (T) = $\pi/16\times\tau\times\left(\frac{D_1^n - d^n}{D_1}\right)$

D = $0.6D_1$

$$T = \frac{\pi}{16} \times \tau \times \left[\frac{D_1^4 - (0.6D_1)^4}{D_1} \right]$$

$$= \frac{\pi}{16} \times \tau \times \frac{D_1^4}{D_1} \left[1 - (0.6)^4 \right]$$

$$= \frac{\pi}{16} \times \tau \times D_1^3 \times 0.8704$$

$$\boxed{T = 0.1709\,\tau D_1^3} \qquad \rightarrow (2)$$

Equate (1) & (2) , we get

$1570796.32 = 0.1709\ D_1^3$

`$D_1^3 = 9191176.43$

$$\boxed{D_1 = 209.47\,\text{mm} \approx 210\text{mm}}$$

% Saving in weight = $\frac{\text{weight of solid shaft} - \text{weight of hollow shaft}}{\text{weight of solid shaft}}$

Weight of solid shaft = A x ρ x ℓ

$$= \frac{\pi}{4}\left(D_1^2 - d^2\right)\rho^{\ell}$$

$$= \frac{\pi}{4}\left(210^2 - 126^2\right)\rho^{\ell}$$

$$= 31415.9\rho^{\ell}$$

% savings in weight = $\frac{31415.93\rho^{\ell} - 22167.08\rho^{\ell}}{31415.93\rho^{\ell}} \times 100$

$$\boxed{\%\,\text{Savings in weight} = 29.44\%}$$

12) A closely coiled helical spring made from round steel rod is required to carry a load of 1000 N for a stress of 400MN/m^2 , the spring stiffness being 20N/mm^2 the dia of the helix is 100mm and G for the material is 80GN/m^2 . Calculate (1) the diameter of the wire and and (2) the number of turns required for the spring (8) (May/June 2016)

W = 1000 N

K = 20N/mm

D = 100 mm

τ = 400 MN/m^2

= 400 N/mm^2

d = ? C = 80 x 10^9 x 10^{-6} N/mm^2

n =? = 80 x 10^3 N/mm^2

$$\tau = \frac{16wR}{\pi d^3}$$

$$d^3 = \frac{16wR}{\pi\tau} = \frac{16\times1000\times\left(100/2\right)}{\pi\times400}$$

$$d^3 = 636.62$$

$$\boxed{d = 8.60\,\text{mm}}$$

$$k = \frac{cd^4}{64R^3n}$$

$$n = \frac{cd^4}{64R^3k} = \frac{80\times10^3\times8.6^4}{64\times\left(\frac{100^3}{2}\right)\times20}$$

$$\boxed{n = 2.73 \approx 3}$$

13) A spiral spring is made of 10mm diameters wire has to close coils, each 100mm mean diameter. Find the axial load the spring will carry if he stress is not exceed 200N/mm^2 Also determine the extension of the spring. Take G = 0.8 x 10^5 N/mm^2 (May/June-2016) 8 Marks

d = 10 mm c (or) G = 0.8 x 10^5 N/mm^2

n = 20

D = 100 mm ⟹ R = D/2 = 50 mm

τ = 200 N/mm^2

W = ?

δ = ?

$$\tau = \frac{16WR}{\pi d^3}$$

$$W = \frac{\tau\times\pi\times d^3}{16R} = \frac{200\times\pi\times10^3}{16\times50} = 785.40N$$

$$\delta = \frac{64WR^3n}{cd^4} = \frac{64\times785.4\times50^3\times20}{0.8\times10^5\times10^4}$$

$$\boxed{\delta = 157.08\,\text{mm}}$$

14) A hollow shaft of external dia 120 mm transmit 300 kw power at 200 rpm. Determine the maximum internal dia. If the max stress in the shaft is not to exceed 60 N/mm^2 (Nov/Dec -2015) 16 Marks

D = 120 mm P = 300 kw N = 200 rpm d = ?

τ = 60 N/mm^2

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{P \times 60}{2\pi N} = \frac{300 \times 10^3 \times 60}{2\pi \times 200} = 14323.914\,\text{Nm}$$

$$T = \frac{\pi}{16} = \left(\frac{D^4 - d^4}{D}\right)$$

$$14323.94 \times 10^3 = \frac{\pi}{16} \times 60 \times \left[\frac{120^4 - d^4}{120}\right]$$

$$\frac{14323.94 \times 10^3 \times 16 \times 120}{\pi \times 60} = 120^4 - d^4$$

$$145902454.8 = 120^4 - d^4$$

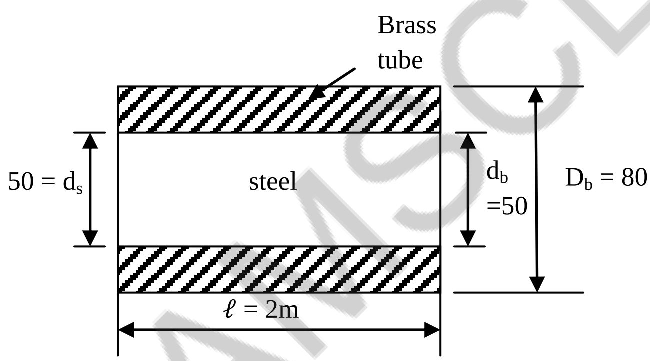
$$d^4 = 61457545.24$$

$$\boxed{d = 88.54\text{mm} \simeq 89\text{mm}}$$

15) A brass tube of external dia. 80mm and internal dia 50mm is closely fixed to a steel rod of 50mm dia to form a composite shaft. If a torque of 10 KNm is to be resisted by this shaft, find the max stresses developed in each material and the angle of twist in 2m length. Take modulus of rigidity of brass and steel as 40 x 10^3 N/mm^2 respectively. (Apr/May 2015) 16 Marks

D_b = 80 mm d_b = 50 mm d_s = 50mm T = 10kNm θ =

C_b = 40 x 10^3 N/mm^2 c_s = 80 x 10^3 N/mm^2 ℓ = 2 m



$$\frac{T_S}{J_S} = \frac{\tau_S}{r_s}$$

$$\tau_S = \frac{T_S}{J_S} \times r_S$$

$$= \frac{10\times10^6}{\frac{\pi}{32}\times50^4} \times \left(\frac{52}{2}\right)$$

$$\tau_S = \frac{10\times10^6\times25}{613592.31} = 407.44\,\text{N/mm}^2$$

$$\boxed{\tau_S = 407.44\,\text{N/mm}^2}$$

$$\frac{T_b}{J_b} = \frac{\tau_b}{R_b}$$

$$\tau_b = \frac{T_b}{J_b} \times R_b$$

$$= \frac{10\times10^6}{\frac{\pi}{32}\times(80^4-50^4)} \times \left(\frac{80}{2}\right)$$

$$\tau_S = \frac{10\times10^6\times40}{3407646.28} = 117.38\,\text{N/mm}^2$$

$$\boxed{\tau_S = 117.38\,\text{N/mm}^2}$$

$$\theta = \frac{T_S \ell_S}{C_S J_S}$$

$$= \frac{10\times10^6\times2000}{80\times10^3\times\pi/32\times50^4} = \frac{10\times10^6\times2000}{4.9087\times10^{10}}$$

$$= 0.407\,\text{rod}$$

$$\theta = 0.407\times\left(\frac{180}{\pi}\right)$$

$$\boxed{\theta = 23.34}$$

$$\boxed{\theta = \theta_s = \theta_b = 23.34^\circ}$$

16) A close-coiled helical spring is to have a stiffness of 900 N/m in compression, with a max. Load of 45N and a max. Shearing stress of 120N/mm^2. The solid length of the spring (i.e coils touching) is 45 mm. Find

(i) the wire dia
(ii) the mean coil radius
(iii) the number of coils. Take c = 0.4 x 10^5 N/mm^2 (Apr/May 2015) 16 Marks
k = 900 N/m w = 45 N τ = 120N/mm^2

(i) d

$$\delta = \frac{64WR^3n}{cd^4}$$

$$k = W/s = \frac{cd^4}{64R^3n}$$

$$0.9 = \frac{0.4\times10^5\times d^4}{64R^3n}$$

$$d^4 = \left(\frac{0.9\times64}{0.4\times10^5}\right)R^3n \qquad \rightarrow (1)$$

$$\tau = \frac{16WR}{\pi d^3}$$

$$120 = \frac{16\times45\times R}{\pi d^3}$$

$$R = \frac{120\pi d^3}{16\times45}$$

$$\boxed{R = 0.52d^3} \qquad \rightarrow (2)$$

solid length of spring when coils are touching } $nd = 45$

$$\boxed{n = 45/d} \qquad \rightarrow (3)$$

Substituting equation (2) & (3) values in equation (1)

$$d^4 = \left(\frac{0.9\times64}{0.4\times10^5}\right)\left(0.52d^3\right)^3\times\frac{45}{d}$$

$$= \left(\frac{0.9\times64}{0.4\times10^5}\right)(0.52)^3\times45d^8$$

$$d^4 = \frac{0.4\times10^5}{0.9\times64\times(0.52)^3\times45} = 109.75$$

$$d = (109.75)^{1/4} = 3.24mm$$

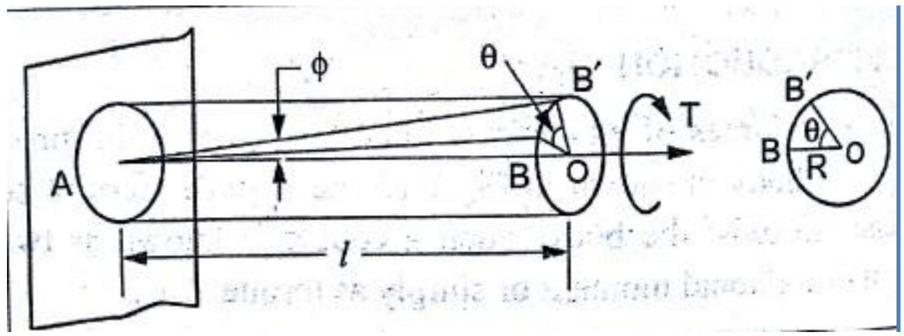
(ii) $R = 0.52d^3$

$R = 0.52 \times (3.24)^3 = 17.68$ mm

(iii) $n = 45/d$

$n = 45/3.24 = 13.88 \simeq 14$

17) Derive torsion equation **(AU Nov/Dec 2014 – 16 Marks)**



Consider a shaft of length L, radius R, fixed at one end and subjected to a torque 'T' at the other end is shown in figure.

Let '0' be the centre of circular section 'B' a point on surface and AB be the line on the shaft parallel to the axis of the shaft.

When the shaft is subjected to torque (T), B is moved to B' if 'ϕ' is shear strain and 'θ' is the angle of twist in length 'ℓ'.

Then $R\theta = BB' = \ell\phi \quad \rightarrow (1)$

If 'τ' is the shear stress and 'c' is the modulus of rigidity then

$$\phi = \frac{\tau}{c}$$

Substitute ϕ value in equation (1)

$$R\theta = \ell\phi$$

$$R\theta = \ell \times \frac{\tau}{c}$$

$$\boxed{\frac{c\theta}{\ell} = \frac{\tau}{R}} \quad \rightarrow (2)$$

Polar moment of inertia (J)

From equation (2) we know that

$$\frac{c\theta}{\ell} = \frac{\tau}{R}$$

Where, c – modulus of rigidity – N/mm^2

θ – angle of twist - radian

ℓ - length - mm

τ - shear stress - N/mm^2

R –Radius - mm

We know that

Torque, $T = \frac{\pi}{16} \times \tau \times D^3$

$$\tau = \frac{16 \times T}{\pi D^3}$$

Substitute τ value in equation (2)

$$\frac{c\theta}{\ell} = \frac{\frac{16 \times T}{\pi D^3}}{R} = \frac{\frac{16 \times T}{\pi D^3}}{\frac{D}{2}}$$

$$= \frac{32\,T}{\pi D^3 \times D} = \frac{32\,T}{\pi D^4}$$

$$\frac{c\theta}{\ell} = \frac{T}{\frac{\pi}{32} \times D^4} = \frac{T}{J}$$

Where J (polar moment of inertia) $= \frac{\pi}{32} D^4$

$$\frac{T}{J} = \frac{c\theta}{\ell}$$

Torsional equation = $\boxed{\frac{T}{J} = \frac{c\theta}{\ell} = \frac{\tau}{R}}$

For hollow shaft,

Polar moment of inertia , $J = \frac{\pi}{32}\left[D^4 - d^4\right]$

Where d – inner diameter

D – outer diameter

18) **The stiffness of a close coiled helical spring is 1.5 N/mm of compression under a maximum load of 60 N. The maximum shearing stress produced in the wire is 125 N/mm^2 . The solid length of the spring (when the coils are touching) is given as 50 mm. Find:**

(i) The diameter of wire
(ii) The mean diameter of the coils
(iii) Number of coils required.
Take C = 4.5 x 10^4 N/mm^2 (Nov/Dec 2018)(Apr/May 2018)(Nov/Dec – 2014)

Given data:

Stiffness, k = 1.5 N/mm

Load, w = 60 N

Stress, τ = 125 N/mm^2

Solid length = n x d = 50 mm

Modulus of rigidity = c =4.5 x 10^4 N/mm^2

To find

(i) diameter of the wire, d

` (ii) diameter of the coil, D

(iii) number of coils , n

Solution:

We know that,

$$\text{Stiffness, } k = \frac{cd^4}{64R^3n}$$

$$1.5 = \frac{4.5\times10^4\times d^4}{64R^3n}$$

$$\boxed{2.133\times10^{-3} = \frac{d^4}{R^3n}} \qquad \rightarrow (1)$$

$$\text{Shear stress, } \tau = \frac{8WD}{\pi d^3} = \frac{16WR}{\pi d^3}$$

$$125 = \frac{16\times60\times R}{\pi d^3}$$

$$\boxed{0.4090 = \frac{R}{d^3}} \qquad \rightarrow (2)$$

nd = 50

$$\boxed{d = \frac{50}{n}}$$

Substitute ‘d’ value in equation (1) & (2)

$$\text{eqn}(1) \Rightarrow \frac{\left(\frac{50}{n}\right)^4}{R^3 n} = 2.133 \times 10^{-3}$$

$$\frac{(50)^4}{R^3 n^5} = 2.133 \times 10^{-3}$$

$$\boxed{R^3 n^5 = 2930 \times 10^9} \qquad \rightarrow (3)$$

$$\text{Eqn}(2) \quad 0.4090 = \frac{R}{\frac{(50)^3}{n^3}}$$

$$51.125 \times 10^3 = Rn^3$$

$$\boxed{R = \frac{51.125 \times 10^3}{n^3}} \qquad \rightarrow (4)$$

Substitute R value in equation (3)

$$\left[\frac{51.125 \times 10^3}{n^3}\right]^3 n^5 = 2.930 \times 10^9$$

$$\frac{1.336 \times 10^4}{n^4} = 2.930 \times 10^9$$

$$n = 14.62 = 15$$

Number of turns, $\boxed{n = 15}$

Substitute n value in equation (4)

$$R = \frac{51.125 \times 10^3}{(14.62)^3}$$

R = 16.36 mm

Diameter of coil $\boxed{D = 32.72\text{mm}}$

We know that nd = 50

14.62 x d = 50

D = 3.42 mm

Diameter of wire, d = 3.42 mm

Results: 1) d = 3.42 mm

2) D = 32.72 mm

3) n = 15

19) Determine the bending stress, shear stress and total work done on an open coiled helical spring subjected to axial force having mean radius of each coil as 'r' and 'n' number of turns. (May/June 2014) 16 Marks

Let,

W = axial load

P = pitch of the spring

d = wire diameter

R = mean radius of spring

Axial load

Torque, T = WR cos d

Bending moment, M = WR sin d

Shear stress, $\tau = \frac{16T}{\pi d^3} = \frac{16WR\cos\alpha}{\pi d^3}$

Bending stress, $\sigma_b = \frac{32M}{\pi d^3} = \frac{32WR\sin\alpha}{\pi d^3}$

Work done

Deflection, $\delta = \frac{64WR^3n}{cd^4}$

The average external work done on the spring under load,

$$w = \frac{1}{2}W\delta$$
$$= \frac{1}{2}W \times \frac{64WR^3n}{cd^4}$$

Work done = $w = \frac{32WR^3n}{cd^4}$

UNIT – IV

BEAM DEFLECTION

PART – A

1) Write the equation giving maximum deflection in case of a simply supported beam subjected to a point load at mid span (Apr/May 2018)

12.4. DEFLECTION OF A SIMPLY SUPPORTED BEAM CARRYING A POINT LOAD AT THE CENTRE

A simply supported beam *AB* of length *L* and carrying a point load *W* at the centre is shown in Fig. 12.3.

As the load is symmetrically applied the reactions R_A and R_B will be equal. Also the maximum deflection will be at the centre.

[redacted]

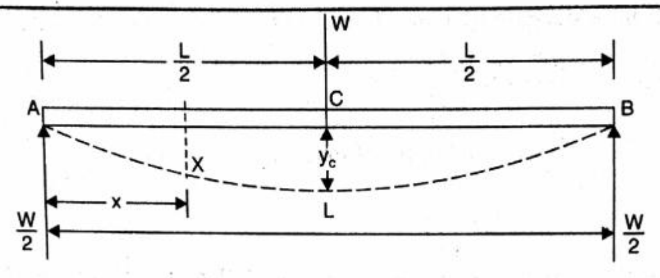


Fig. 12.3

Now $$R_A = R_B = \frac{W}{2}$$

Consider a section X at a distance x from A. The bending moment at this section is given by,

$$M_x = R_A \times x$$
$$= \frac{W}{2} \times x$$ (Plus sign is as B.M. for left portion at X is clockwise)

But B.M. at any section is also given by equation (12.3) as

$$M = EI\frac{d^2y}{dx^2}$$

Equating the two values of B.M., we get

$$EI\frac{d^2y}{dx^2} = \frac{W}{2} \times x \qquad ...(i)$$

On integration, we get

$$EI\frac{dy}{dx} = \frac{W}{2} \times \frac{x^2}{2} + C_1 \qquad ...(ii)$$

where C_1 is the constant of integration. And its value is obtained from boundary conditions. The boundary condition is that at $x = \frac{L}{2}$, slope $\left(\frac{dy}{dx}\right) = 0$ (As the maximum deflection is at the centre, hence slope at the centre will be zero). Substituting this boundary condition in equation (*ii*), we get

$$0 = \frac{W}{4} \times \left(\frac{L}{2}\right)^2 + C_1$$

or $$C_1 = -\frac{WL^2}{16}$$

Substituting the value of C_1 in equation (*ii*), we get

$$EI\frac{dy}{dx} = \frac{Wx^2}{4} - \frac{WL^2}{16} \qquad ...(iii)$$

The above equation is known the *slope equation*. We can find the slope at any point on the beam by substituting the values of x. Slope is maximum at A. At A, $x = 0$ and hence slope at A will be obtained by substituting $x = 0$ in equation (*iii*).

$$\therefore \quad EI\left(\frac{dy}{dx}\right)_{\text{at } A} = \frac{W}{4} \times 0 - \frac{WL^2}{16}$$

$$\left[\left(\frac{dy}{dx}\right)_{\text{at } A} \text{ is the slope at } A \text{ and is represented by } \theta_A\right]$$

or
$$EI \times \theta_A = -\frac{WL^2}{16}$$

$$\therefore \quad \theta_A = -\frac{WL^2}{16EI}$$

The slope at point B will be equal to θ_A, since the load is symmetrically applied.

$$\therefore \quad \theta_B = \theta_A = -\frac{WL^2}{16EI} \qquad \text{...(12.6)}$$

Equation (12.6) gives the slope in radians.

Deflection at any point

Deflection at any point is obtained by integrating the slope equation (*iii*). Hence integrating equation (*iii*), we get

$$EI \times y = \frac{W}{4} \cdot \frac{x^3}{3} - \frac{WL^2}{16} x + C_2 \qquad \text{...}(iv)$$

where C_2 is another constant of integration. At A, $x = 0$ and the deflection (y) is zero.

Hence substituting these values in equation (*iv*), we get

$$EI \times 0 = 0 - 0 + C_2$$

or
$$C_2 = 0$$

Substituting the value of C_2 in equation (*iv*), we get

$$EI \times y = \frac{Wx^3}{12} - \frac{WL^2 \cdot x}{16} \qquad \text{...}(v)$$

The above equation is known as *the deflection equation.* We can find the deflection at any point on the beam by substituting the values of x. The deflection is maximum at centre point C, where $x = \frac{L}{2}$. Let y_c represents the deflection at C. Then substituting $x = \frac{L}{2}$ and $y = y_c$ in equation (v), we get

$$EI \times y_c = \frac{W}{12}\left(\frac{L}{2}\right)^3 - \frac{WL^2}{16} \times \left(\frac{L}{2}\right)$$

$$= \frac{WL^3}{96} - \frac{WL^3}{32} = \frac{WL^3 - 3WL^3}{96}$$

$$= -\frac{2WL^3}{96} = -\frac{WL^3}{48}$$

$$\therefore \quad y_c = -\frac{WL^3}{48EI}$$

(Negative sign shows that deflection is downwards)

$$\therefore \quad \text{Downward deflection, } y_c = \frac{WL^3}{48EI} \qquad \text{...(12.7)}$$

2) State the two theorems of conjugate beam method (Apr/May 2018)

Conjugate Beam Theorem I :

"The slope at any section of a loaded beam relative to the original axis of the beam, is equal to the shear in the conjugate beam at the corresponding section."

We know that, load $= w = \dfrac{M}{EI}$

$\therefore$ Shear $= S_x = \int_0^x w \cdot dx = \int_0^x \dfrac{M}{EI}\, dx$

But, $\int_0^x \dfrac{M}{EI}\, dx = \int_0^x \dfrac{d^2 y}{dx^2} = \dfrac{dy}{dx} = slope$

Conjugate Beam Theorem II :

"The deflection at any given section of a loaded beam, relative to the original position, is equal to the bending moment at the corresponding section of the conjugate beam."

We know that, shear $S_x = \int_0^x \dfrac{M}{EI}\, dx$

$\therefore$ Bending moment, $M_x = \int_0^x S_x \cdot dx = \int_0^x\int_0^x \dfrac{M}{EI}\, dx$

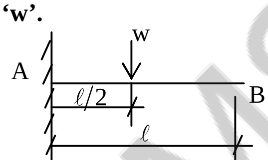
But, $\int_0^x\int_0^x \dfrac{M}{EI}\, dx = \int_0^x\int_0^x \dfrac{d^2 y}{dx^2} = \int_0^x \dfrac{dy}{dx} = y = deflection$ ...*Proved*

The following points are worth noting for the **conjugate beam method:**

(*i*) This method can be *directly used only for simply supported beams*.

(*ii*) In this method for cantilevers and fixed beams, artificial constraints need to be applied to the conjugate beam so that it is supported in a manner consistent with the constraints of the real beam.

3) Write down the equation for the maximum deflection of a cantilever beam carrying a central point load 'w'. (May / June 2017)



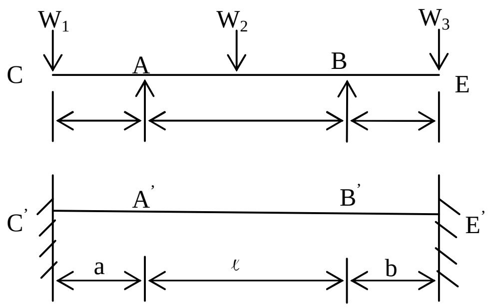
$$y_b = \frac{w_a{}^3}{3EI} + \frac{w_a{}^2}{2EI}(\ell - a)$$

$$= \frac{w}{3EI}(\ell/2)^3 + \frac{w}{2EI}(\ell/2)^2 \times (\ell/2)$$

$$= \frac{w\ell^3}{24EI} + \frac{w\ell^3}{16EI} = \frac{2w\ell^3 + 3w\ell^3}{48EI}$$

$$= \frac{5w\ell^3}{48EI}$$

4) Draw conjugate beam for a double side over hanging beam (May / June 2017)



5) List out the method's available to find the deflection of the beam. (Nov / Dec 2015, 2016)

The available methods to find the deflection of beam are

i) Double integration method
ii) Macaulay's method
iii) Moment Area method
iv) Conjugate beam method

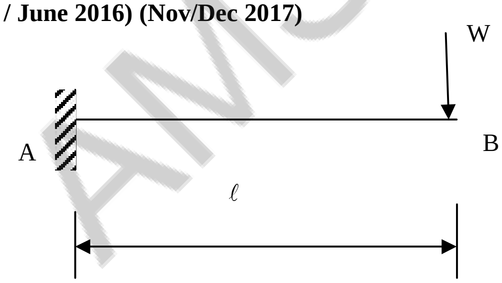
6) State Maxwell's reciprocal theorem (Nov / Dec 2016) (May / June 2016) (Nov / Dec 2017) (Nov/Dec 2018) (Apr/May 2019)

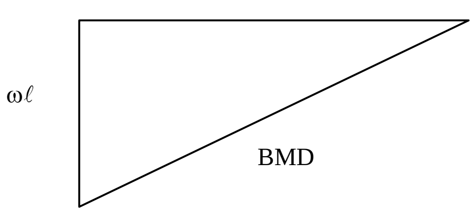
The Maxwell reciprocal theorem states that, "the work done by the first system of load due to displacement caused by a second system of load equal the work done by the second system of load due to displacement caused by the first system of load".

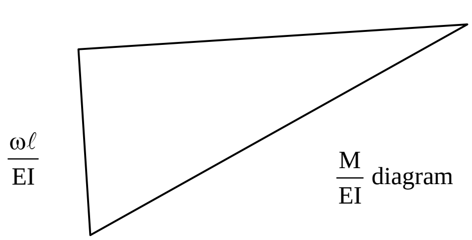
$$\sum_{i=1}^{n}(P_i)_A(\delta_i)_{\mathbf{B}} = \sum_{j=1}^{m}(P_j)_A(\delta_j)_{\mathbf{B}}$$

7) How the deflection & slope is calculated for the Cantilever beam by conjugate beam method?

(May / June 2016) (Nov/Dec 2017) (Nov/Dec 2018)







Total load on conjugate beam = Area of load diagram

$$P = A = \frac{1}{2} \times \ell \times \frac{\omega\ell}{EI} = \frac{-\omega\ell^2}{2EI}$$

We know that,

Slope at B = shear force at B for the conjugate beam

$$\boxed{\theta_B = -P = \frac{\omega\ell^2}{2EI}}$$

Deflection at B = B.M at B for the conjugate beam

$$= -p \times \frac{2}{3} \times \ell$$

$$= \left[\frac{\omega\ell^2}{2EI} \times \frac{2}{3} \times \ell\right]$$

$$y_B = \frac{\omega\ell^3}{3EI}$$

8) What is the equation used in the case of double integration method? (Nov / Dec 2015)

The B.M at any point is given by the differential equation

$$M = EI\frac{d^2y}{dx^2}$$

Integration the above equation, we get,

$$\int M = \int EI\frac{d^2y}{dx^2} = EI\frac{dy}{dx}$$

Integration above equation twice, we get , → slope equation

$$\iint M = \iint EI\frac{d^2y}{dx^2} = EIy$$

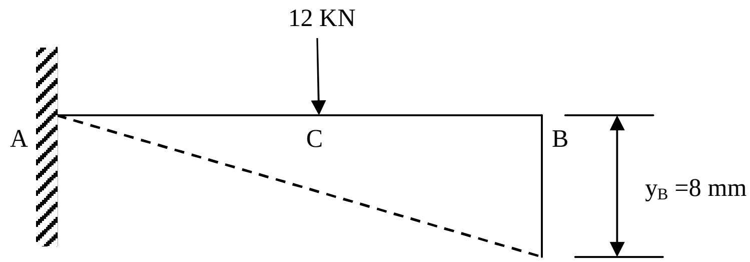
$$\rightarrow \text{Deflection equation}$$

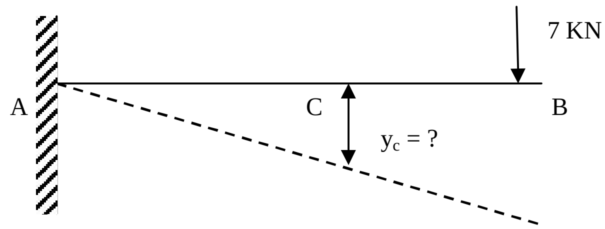
9) What are the advantages of Macaulay's over other method for the calculation of slope & deflection? (Apr / May 2015)

The procedure of finding slope and deflection for a SSB with an eccentric point load is very Laborious. There is a convenient method, that method was devised by Mr. M.H.Macaulay and is known as Macaulay's method.

In this method, B.M at any section is expressed and the integration is carried out.

10) In a cantilever beam, the measured deflection at, free end was 8 mm when a concentrated load of 12 KN was applied at it's mid span. What will be the deflection at mid – span when the same beam carries a concentrated load of 7KN at the free end? (Apr / May 2015)





Maxwell Reciprocal theorem,

$$\sum \frac{1}{2} P_i \delta_i = \sum \frac{1}{2} P_j \delta_j$$

$$12 \times 8 = 7 \times y_c$$

$$\frac{12 \times 8}{7} = y_c$$

$$y_c = 13.71 \text{ mm}$$

11) What is the limitation of double integration method? (Nov / Dec 2014)

* This method is used only for single load

* This method for finding slope & deflection is very laborious

12) Define strain energy? (Nov / Dec 2014)

When an elastic material is deformed due to application of external force, internal resistance is developed in the material of the body, Due to deformation, some work is done by the internal resistance developed in the body, which is stored in the form of energy. This energy is known as strain energy. It is expressed in Nm.

13) What is the relation between slope, deflection and radius of curvature of a beam?

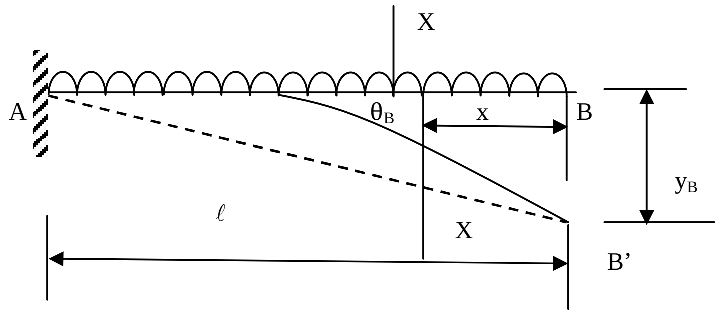
$$\frac{1}{R} = \frac{d^2y}{dx^2}$$

Where, R = radius of curvature

$\theta = dy/dx =$ slope

y = Deflection

14) State the expression for slope and deflection at the free and of a Cantilever beam of length 'l' subjected to a uniformly distributed load of 'w' per unit length.



Consider a section X at a distance x from the free end B,

$$\text{B.M at section XX} = M_{XX} = \frac{-\omega x^2}{2}$$

$$M = EI\frac{d^2y}{dx^2} = \frac{-\omega x^2}{2}$$

Integrate the above equation

$$EI\frac{dy}{dx} = -\frac{\omega x^3}{6} + C_1 \qquad \text{...1}$$

Integration again,

$$EIy = \frac{-\omega x^4}{24} + C_1 x + C_2 \qquad \text{...2}$$

C_1 & C_2 values are obtined from boundary condition.

i) when $x = \ell$; slope $\frac{dy}{dx} = 0$

ii) when $x = \ell$; deflection $y = 0$

Applying BC (i) to equation 1

$$0 = \frac{-\omega \ell^3}{6} + C_1$$

$$\boxed{C_1 = \frac{\omega \ell}{6}}$$

Substitute the C_1 values in equation 1

$$EI\frac{dy}{dx} = \frac{-\omega x^3}{6} + \frac{\omega \ell^3}{6} \qquad \rightarrow 3(\text{slope equ})$$

Max slope →substituting x=0 in equation 3

$$EI = \frac{dy}{dx} = \frac{\omega \ell^3}{6}$$

$$\text{max slope}, \theta_B = \frac{dy}{dx} = \frac{\omega \ell^3}{6EI}$$

Applying B.C(iii) to equation 2

$$0 = \frac{-\omega \ell^4}{6} - \frac{\omega \ell^4}{24} = \frac{-3\omega \ell^4}{24} = \frac{-\omega \ell^4}{8}$$

Substitute C_1 & C_2 values in equation 2

$$EIy = \frac{-\omega\ell^4}{24} + \frac{\omega\ell^4}{6}x - \frac{-\omega\ell^4}{8} \qquad \rightarrow 4$$

Max deflection occurs at the end, → substituting x=0 in equation 4

$$EIy_B = 0 - 0 - \frac{\omega\ell^4}{8}$$

$$y_B = \frac{-\omega\ell^4}{8EI}$$

Max deflection, $\boxed{y_B = \frac{-\omega\ell^4}{8EI}}$

15) In a support beam of 3m span carrying uniformly distribution load throughout the length the slope at the support is 1°. What is the max deflection in the beam? (Apr/May 2019)

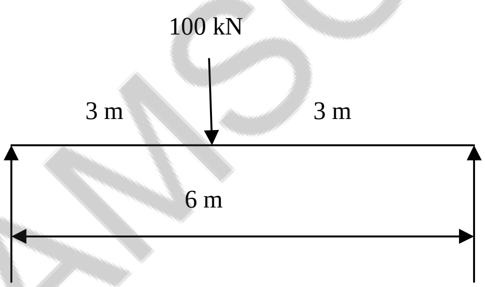
$$\theta_A = \frac{\omega\ell^3}{24EI} = 1° = \frac{\pi}{180°}$$

Max deflection (y_{max}) = $\frac{5}{384}\frac{\omega\ell^4}{EI}$

$$= \frac{\omega\ell^3}{24EI} \times \frac{5\ell}{16} = \frac{\pi}{180°} \times \frac{5 \times 3}{16}$$

$$\boxed{y_{max} = 0.0164}$$

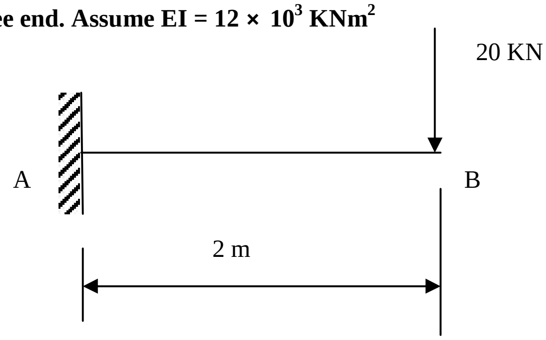
16) Calculate the maximum deflection of a simply support beam carrying a point load of 100 KN at mid span. Span = 6m; EI = 20,000 KN/m²



$$y_{max} = \frac{\omega\ell^3}{48EI} = \frac{100 \times 6^3}{48 \times 20000} = 0.0225\,m$$

$$\boxed{y_{max} = 22.5 \text{ mm}}$$

17) A cantilever beam of span 2m is carrying a point load of 20 KN in the free end. Calculate the slope at the free end. Assume EI = 12 × 10³ KNm²



$$\theta_B = \frac{\omega \ell^2}{2EI}$$

$$= \frac{20 \times 2^2}{2 \times 12 \times 10^3}$$

$$\boxed{\theta_B = 0.0033 \text{ rad}}$$

18) State the two theorems in the moment area method.

Mohr's theorem 1:

The change of slope between any two point is equal to the net area of the BM diagram between these points divided by EI.

Mohr's theorem 2:

The total deflection between any two point is equal to the moment of the area of the BM diagram between these two point about the last point divided by EI.

19) Define Resilience and proof resilience?

Resilience is ability of a material to absorb energy under elastic deformation and to recover this energy upon removal of load. Resilience is indicated by the area under the stress strain curving to the point of elastic limit. In a technical sense, resilience is the property of a material that allow it return to its original shape after being de formed.

Proof resilience is defined as the maximum energy that can be absorbed within the elastic limit without creating a permanent distortion.

20) Define the term modulus of resilience.

It is the ratio of the proof resilience to the volume of the body.

21) Why moment area method is more useful when compared with double integration?

Moment area method is more useful, as compared to double integration method because many problem which do not have a simple mathematical solution can be simplified by the ending moment area method.

22) Explain the theorem for conjugate beam method?

Theorem I: The slope at any section of a loaded beam, relative to the original axis of the beam is equal to the shear in the conjugated beam at the corresponding section.

Theorem II: the deflection at any given section of a loaded beam, relative to the original position is equal to the bending moment at the corresponding section of the conjugated beam.

23) Define method of singularity function?

In Macaulay's method a single equation is formed for all loading on a beam, the equation is constructed in such a way that the constant of integration apply to all portion of the beam. This method is also called of singularity function.

24) What are the point to be worth for conjugate beam method.

1) This method can be directly used for simply support beam

2) In this method for cantilever and fixed beam, artificial constraints need to be supplied to the conjugate beam so that it is support in a manner consistent with the constraints of the real beam.

PART – B

1) A beam of length 5m and of uniform rectangular section is simply supported at its end. It carries a uniformly distributed load of 9kN/m run over the entire length. Calculate the width and depth of the beam if permissible bending stress is 7 N/mm^2 and central deflection is not to exceed 1cm. Take E=1x10^4 N/mm^2 (Apr/May 2019) (Nov/Dec 2018)

$$L = 5m = 5000mm$$

$$w = 9kN/m$$

$$W = wL = 9x5 = 45kN = 45000N$$

$$\pm_b = 7N/mm^2$$

$$y_C = 1cm = 10mm$$

$$E = 1x10^4 N/mm^2$$

$$I = \frac{bd^3}{12}$$

$$y_C = \frac{5}{384}\frac{WL^3}{EI}$$

$$10 = \frac{5}{384}\frac{45000x5000^3x12}{1x10^4 xbd^3}$$

$$bd^3 = 878.906x10^7 mm^4 \rightarrow 1$$

$$M = \frac{wl^2}{8} = \frac{Wl}{8} = \frac{45000x5000}{8} = 28125000Nmm$$

Bending equation

$$\frac{M}{I} = \frac{\pm_b}{y}$$

$$\frac{28125000}{\frac{bd^3}{12}} = \frac{7}{\frac{d}{2}} \Rightarrow bd^2 = 24107142.85mm^3 \rightarrow 2$$

divide equation 1 by equation 2, we get,

d=364.58mm subs. in equation 2, we get

b=181.36mm

2) A simply supported beam of length 5m carries a point load of 5kN at a distance of 3m from the left end. If E=2x10^5 N/mm^2 and I=10^8 mm^4 determine the slope at the left support and deflection under the point load using conjugate beam method. (Apr/May 2019) (Nov/Dec 2017)

Sol. Given :

Length, $L = 5$ m

Point load, $W = 5$ kN

Distance AC, $a = 3$ m

Distance BC, $b = 5 - 3 = 2$ m

Value of $E = 2 \times 10^5$ N/mm^2 $= 2 \times 10^5 \times 10^6$ N/m^2

$= 2 \times 10^5 \times 10^3$ kN/m^2 $= 2 \times 10^8$ kN/m^2

Value of $I = 1 \times 10^8$ mm^4 $= 10^{-4}$ m^4

Let R_A = Reaction at A

and R_B = Reaction at B.

Taking moments about A, we get

$$R_B \times 5 = 5 \times 3$$

$$\therefore \quad R_B = \frac{5 \times 3}{5} = 3 \text{ kN}$$

and R_A = Total load $- R_B = 5 - 3 = 2$ kN

The B.M. at $A = 0$

B.M. at $B = 0$

B.M. at $C = R_A \times 3 = 2 \times 3 = 6$ kNm.

Now B.M. diagram is drawn as shown in Fig. 14.3 (b).

Now construct the conjugate beam as shown in Fig. 14.3 (c). The vertical load at C^* on conjugate beam

$$= \frac{\text{B.M. at } C}{EI} = \frac{6 \text{ kNm}}{EI}$$

Now calculate the reaction at A^* and B^* for conjugate beam

Let R_A^* = Reaction at A^* for conjugate beam

R_B^* = Reaction at B^* for conjugate beam.

Taking moments about A^*, we get

$R_B^* \times 5$ = Load on $A^*C^*D^*$ × distance of C.G. of $A^*C^*D^*$ from A^*

+ Load on $B^*C^*D^*$ × Distance of C.G. of $B^*C^*D^*$ from A^*

$$= \left(\frac{1}{2} \times 3 \times \frac{6}{EI}\right) \times \left(\frac{2}{3} \times 3\right) + \left(\frac{1}{2} \times 2 \times \frac{6}{EI}\right) \times \left(3 + \frac{1}{3} \times 2\right)$$

$$= \frac{18}{EI} + \frac{6}{EI} \times \frac{11}{3} = \frac{8}{EI} + \frac{22}{EI} = \frac{40}{EI}$$

$$\therefore \quad R_B^* = \frac{40}{EI} \times \frac{1}{5} = \frac{8}{EI}$$

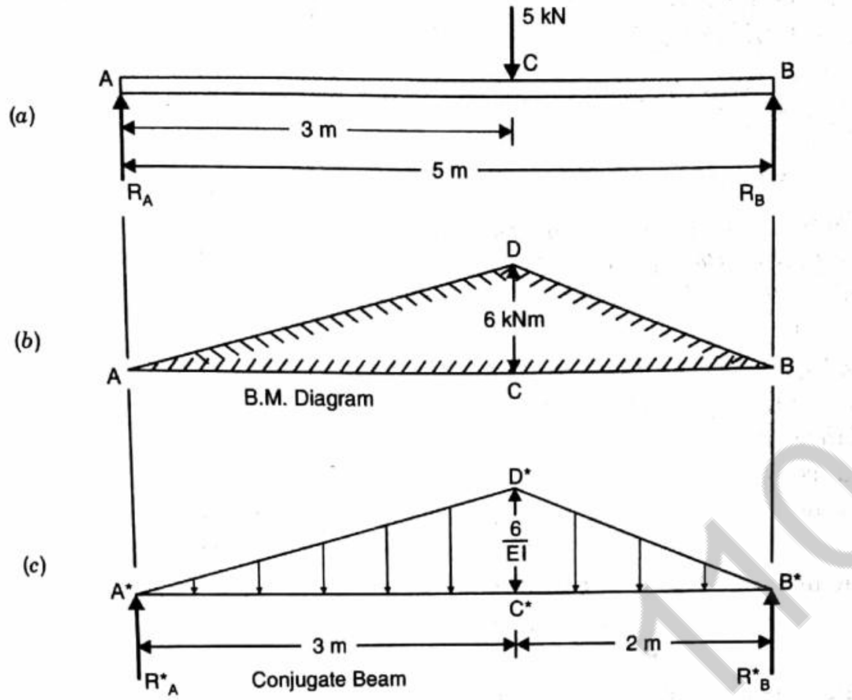


Fig. 14.3

$\therefore$ $R_A^* = \text{Total load } (i.e., \text{ load } A^*B^*D^*) - R_B^*$

$$= \left(\frac{1}{2}\times 5\times \frac{6}{EI}\right) - \frac{8}{EI}$$

$$= \frac{15}{EI} - \frac{8}{EI} = \frac{7}{EI}$$

Let θ_A = Slope at A for the given beam *i.e.*, $\left(\frac{dy}{dx}\right)$ at A

y_C = Deflection at C for the given beam

Then according to conjugate beam method,

θ_A = Shear force at A^* for conjugate beam = R_A^*

$$= \frac{7}{EI} = \frac{7}{2\times 10^8 \times 10^{-4}} \quad (\because E = 2\times 10^8 \text{ kN/m}^2 \text{ and } I = 10^{-4} \text{ m}^4)$$

= **0.00035 radians. Ans.**

y_C = B.M. at C^* for conjugate beam

$= R_A^* \times 3 - \text{Load } A^*C^*D^* \times \text{Distance of C.G. of } A^*C^*D^* \text{ from } C^*$

$$= \frac{7}{EI}\times 3 - \left(\frac{1}{2}\times 3\times \frac{6}{EI}\right)\times\left(\frac{1}{3}\times 3\right)$$

$$= \frac{21}{EI} - \frac{9}{EI} - \frac{12}{EI}$$

$$= \frac{12}{2\times 10^8\times 10^{-4}} = \frac{6}{10^4} \text{ m} = \frac{6\times 1000}{10000} \text{ mm} = \mathbf{0.6 \text{ mm.}} \quad \textbf{Ans.}$$

3) Derive the equation for slope and deflection of a simply supported beam of length 'L' carrying point load 'W' at the centre by Mohr's theorem. (Nov/Dec 2018)

Fig. 12.20 (*a*) shows a simply supported beam *AB* of length *L* and carrying a uniformly distributed load of *w*/unit length over the entire span. The B.M. diagram is shown in Fig. 12.20 (*b*). This is a case of symmetrical loading, hence slope is zero at the centre *i.e.*, at point *C*.

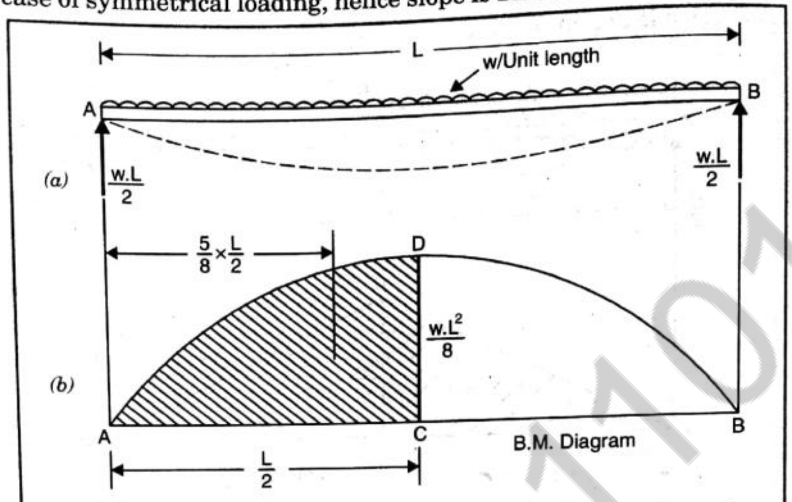


Fig. 12.20

(*i*) Now using Mohr's theorem for slope, we get

Slope at $A = \dfrac{\text{Area of B.M. diagram between } A \text{ and } C}{EI}$

But area of B.M. diagram between *A* and *C*

$$= \text{Area of parabola } ACD$$

$$= \frac{2}{3} \times AC \times CD$$

$$= \frac{2}{3} \times \frac{L}{2} \times \frac{wL^2}{8} = \frac{w \cdot L^3}{24}$$

$\therefore$ Slope at $A = \dfrac{w \cdot L^3}{24EI}$

(*ii*) Now using Mohr's theorem for deflection, we get from equation (12.17) as

$$y = \frac{A\bar{x}}{EI}$$

where A = Area of B.M. diagram between *A* and *C*

$$= \frac{w \cdot L^3}{24}$$

and $\bar{x}$ = Distance of C.G. of area *A* from *A*

$$= \frac{5}{8} \times AC = \frac{5}{8} \times \frac{L}{2} = \frac{5L}{16}$$

$$\therefore \quad y = \frac{\frac{w.L^3}{24} \times \frac{5L}{16}}{EI} = \frac{5}{384} \frac{w.L^4}{EI}.$$

4. A cantilever of length 2m carries a uniformly distributed load of 2.5kN/m run for a length of 1.25m from the fixed end and a point load of 1kN at the free end. Find the deflection at the free end, if the section is rectangular 12cm wide and 24cm deep and E=1x10^4 N/mm^2 (Apr/May 2018)

Length, $L = 2 \text{ m} = 2000 \text{ mm}$

U.d.l., $w = 2.5 \text{ kN/m} = 2.5 \times 1000 \text{ N/m}$

$= \dfrac{2.5 \times 1000}{1000} \text{ N/mm} = 2.5 \text{ N/mm}$

Point load at free end, $W = 1 \text{ kN} = 1000 \text{ N}$

Distance AC, $a = 1.25 \text{ m} = 1250 \text{ mm}$

Width, $b = 12 \text{ cm}$

Depth, $d = 24 \text{ cm}$

Value of $I = \dfrac{bd^3}{12} = \dfrac{12 \times 24^3}{12}$

$= 13824 \text{ cm}^4 = 13824 \times 10^4 \text{ mm}^4 = 1.3824 \times 10^8 \text{ mm}^4$

Value of $E = 1 \times 10^4 \text{ N/mm}^2$

Let y_1 = Deflection at the free end due to point load 1 kN alone

y_2 = Deflection at the free end due to u.d.l. on length AC.

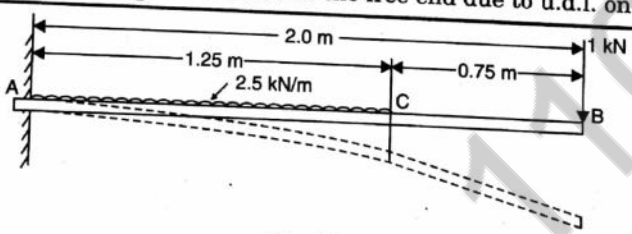


Fig. 13.7

(*i*) Now the downward deflection at the free end due to point load of 1 kN (or 1000 N) at the free end is given by equation (13.2 A) as

$$y_1 = \frac{WL^3}{3EI} = \frac{1000 \times 2000^3}{3 \times 10^4 \times 1.3824 \times 10^8} = 1.929 \text{ mm.}$$

(*ii*) The downward deflection at the free end due to uniformly distributed load of 2.5 N/mm on a length of 1.25 m (or 1250 mm) is given by equation (13.8) as

[redacted]

$$y_2 = \frac{wa^4}{8EI} + \frac{w \cdot a^3}{6EI}(L - a)$$

$$= \frac{2.5 \times 1250^4}{8 \times 10^4 \times 1.3824 \times 10^8} + \frac{2.5 \times 1250^3}{6 \times 10^4 \times 1.3824 \times 10^8}(2000 - 1250)$$

$$= 0.5519 + 0.4415 = 0.9934$$

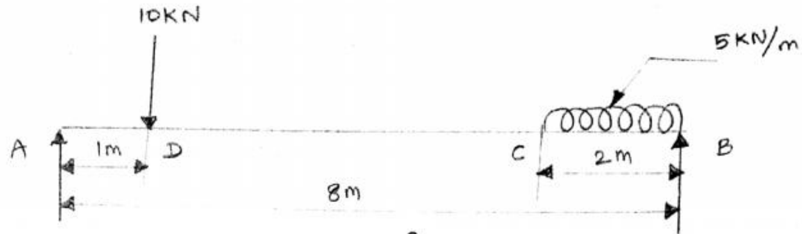
$\therefore$ Total deflection at the free end due to point load and u.d.l.

$= y_1 + y_2 = 1.929 + 0.9934 =$ **2.9224 mm. Ans.**

5) A beam AB of 8m span is simply supported at the ends. It carries a point load of 10kN at a distance of 1m from the end A and a uniformly distributed load of 5kN/m for a length of 2m from the end B. If I= 10 x 10^{-6}m^4, determine : i) Deflection at the mid span ii) Maximum deflection iii) slope at the end A (Apr/May 2018)

14)
b)

Apr - 2018



$I = 10\times10^{-6}\ m^4 \qquad E = 200\times10^{9}\ N/m^2$

To find, R_A & R_B, $\qquad R_A + R_B = 10 + 5\times2 = 20KN \rightarrow (1)$

$\Sigma M_A = 0 \Rightarrow \quad -10\times1 - (5\times2)\times\left(6+2/2\right) + 8R_B = 0$

$-10 - 70 + 8R_B = 0$

$8R_B = 80$

$\boxed{R_B = 10KN}$ subs. in (1),

we get $\boxed{R_A = 10KN}$

$$M_x = EI\frac{d^2y}{dx^2} = 10x \Big| -10(x-1) \Big| -5(x-6)\times\frac{(x-6)}{2} \rightarrow (2)$$

Integrating, we get,

$$EI\frac{dy}{dx} = \frac{10x^2}{2} + C_1 \Big| -\frac{10(x-1)^2}{2} \Big| -\frac{5}{2}\frac{(x-6)^3}{3}$$

$$= 5x^2 + C_1 \Big| -5(x-1)^2 \Big| -\frac{5}{6}(x-6)^3 \rightarrow (3)$$

Integrating again, we get,

$$EIy = \frac{5x^3}{3} + C_1x + C_2 \Big| -\frac{5(x-1)^3}{3} \Big| -\frac{5}{6}\frac{(x-6)^4}{4}$$

$$= \frac{5x^3}{3} + C_1x + C_2 \Big| -\frac{5}{3}(x-1)^3 \Big| -\frac{5}{24}(x-6)^4 \rightarrow (4)$$

when $x = 0$; $y = 0$ subs. in (4) $\Rightarrow \boxed{C_2 = 0}$

$x = 8$; $y = 0$ subs. in (4) $\quad 0 = \frac{5(8)^3}{3} + 8C_1 - \frac{5}{3}(7)^3 - \frac{5}{24}(2)^4$

$0 = 853.33 + 8C_1 - 571.67 - 3.33$

$-278.33 = 8C_1$

$\boxed{C_1 = -34.79}$

Hence slope & Deflection eqns. are,

$$EI\frac{dy}{dx} = 5x^2 - 34.79 - 5(x-1)^2 - \frac{5}{6}(x-6)^3$$

→ slope eqn.

$$EI\,y = \frac{5x^3}{3} - 34.79x - 5/3\,(x-1)^3 - 5/24\,(x-6)^4$$

→ Deflection eqn.

(i) Deflection at mid span:

$$EI\,y_{mid} = 5/3\,(4)^3 - 34.79(4) - 5/3\,(3)^3 = -77.49$$

$(x=4m)$

$$y_{mid} = \frac{-77.49}{EI} = \frac{-77.49}{200\times10^9\times10\times10^{-6}} = 3.87\times10^{-5}\ m$$

$$\boxed{y_{mid} = 0.0387\ mm}$$

(ii) Max. Deflection:

For max. defle., equate slope at the section to zero,

we get, $EI\frac{dy}{dx} = 5x^2 - 34.79 - 5(x-1)^2 = 0$

$$5x^2 - 34.79 - 5(x^2 - 2x + 1) = 0$$

$$5x^2 - 34.79 - 5x^2 + 10x - 5 = 0$$

$$10x = +39.79$$

$$\boxed{x = +3.979\ m}$$

$$EI\,y_{max} = 5/3\,(3.979)^3 - 34.79(3.979) - 5/3\,(2.979)^3$$

$$EI\,y_{max} = -77.49$$

$$\boxed{y_{max} = 0.0387\ mm}$$

(iii) slope at end A,

putting $x=0$ in slope eqn., we get

$$EI\frac{dy}{dx} = -34.79$$

$$\theta_A = \frac{-34.79}{EI} = ~~1.74\times10^{-5}~~ -1.74\times10^{-5}$$

$$\boxed{\theta_A = -0.00099^\circ}$$

6) A cantilever of length 3m is carrying a point load of 50kN at a distance of 2m from the fixed end. If E=2x10^{5} N/mm^{2} and I=10^{8} mm^{4} find i) slope at the free end and ii) deflection at the free end. (Nov/Dec 2017)

$$L = 3m = 3000mm$$

$$W = 50kN = 50000N$$

$$a = 2m = 2000mm$$

$$I = 10^8 mm^4$$

$$E = 2x10^5 N/mm^2$$

$i)slope$

$$-_B = \frac{Wa^2}{2EI} = \frac{50000x2000^2}{2x2x10^5x10^8} = 0.005rad$$

$ii)Deflection$

$$y_B = \frac{Wa^3}{3EI} + \frac{Wa^2}{2EI}(\text{L} - \text{a})$$

$$y_B = \frac{50000x2000^3}{3x2x10^5x10^8} + \frac{50000x2000^2}{2x2x10^5x10^8}(3000 - 2000)$$

$$y_B = 11.67mm$$

7) Determine the slope at the two supports and deflection under the loads. Use conjugate beam method E = 200 GN/m^{2}, I for right half is 2 x 10^{8}mm^{4}, I for left half is 1 x 10^{8} mm^{4} the beam is given in fig. Q.14 (b). (May / June 2017)

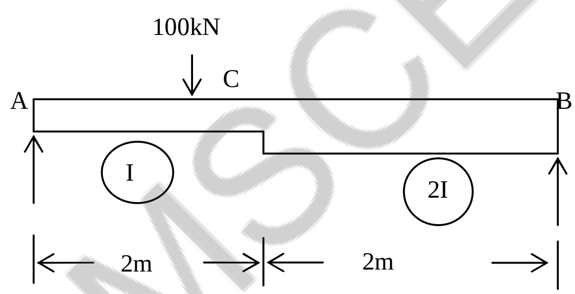


Fig. Q .14 (b)

Solution.

Given:

Length, L = 4m

Length AC = Length BC = 2m

Point load, W = 100kN

Moment of inertia for AC

$$\text{I} = 1 \text{ x } 10^8 \text{ mm}^4 = \frac{10^8}{10^{12}}\text{m}^4 = 10^{-4}\text{m}^4$$

Moment of inertia for BC

$= 2 \times 10^8 mm^4$

$= 2 \times 10^{-4} m^4 = 2I$

Value of $E = 200\ GN/m^2 = 200 \times 10^9\ N/m^2$

$= 200 \times 10^6\ kN/m^2$.

The reactions at A and B will be equal, as point load is acting at the centre,

$$\therefore \quad R_A = R_B = \frac{100}{2} = 50kN$$

Now B.M. at A and B are zero.

B.M. at C = R_A x 2 = 50 x 2 = 100 kNm

Now B.M. can be drawn as shown in Fig.14 (b)

Now we can construct the conjugate beam by dividing B.M. at any section by the product of E and M.O.I.

The conjugate beam is shown in Fig.14 (c). The loading are shown on the conjugate beam. The loading on the length A^*C^* will be $A^*C^*D^*$ whereas the loading on length B^*C^* will be $B^*C^*E^*$.

$$\text{The ordinate } C^*D^* = \frac{\text{B.M. at C}}{E \times \text{M.O.I for AC}} = \frac{100}{EI}$$

$$\text{The ordinate } C^*E^* = \frac{\text{B.M. at C}}{\text{product of E and M.O.I for BC}} = \frac{100}{E \times 2I} = \frac{50}{EI}$$

Let R_A^* = Reaction at A^* for conjugate beam

R_B^* = Reaction at B^* for conjugate beam

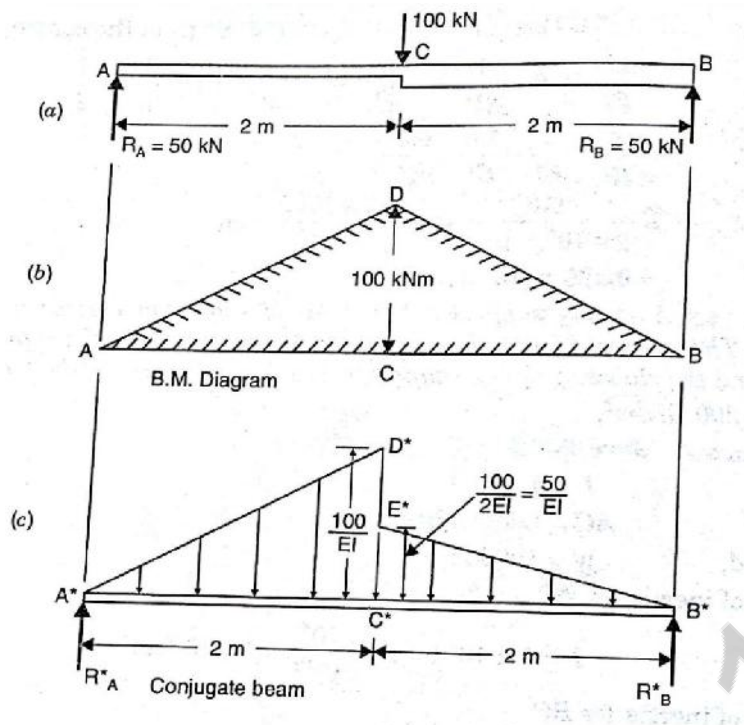


Fig.14

First calculate R_A^* and R_B^*

Taking moments of all forces about A^*, we get

R_B^* x 4 = Load $A^*C^*D^*$ x Distance of C.G. of $A^*C^*D^*$ from A +

Load $B^*C^*E^*$ x Distance of C.G. of $B^*C^*E^*$ from A^*

$$= \left(\frac{1}{2}\times 2\times\frac{100}{EI}\right)\times\left(\frac{2}{3}\times 2\right)+\left(\frac{1}{2}\times 2\times\frac{50}{EI}\right)\times\left(2\times\frac{1}{3}\times 2\right)$$

$$= \frac{400}{3EI}+\frac{400}{3EI}=\frac{800}{3EI}$$

$$R_B^* = \frac{200}{3EI}$$

R_A^* = Total load on conjugate beam - R_B^*

$$= \left(\frac{1}{2}\times 2\times\frac{100}{EI}+\frac{1}{2}\times 2\times\frac{50}{EI}\right)-\frac{200}{3EI}$$

$$= \frac{150}{EI}-\frac{200}{3EI}=\frac{250}{3EI}$$

i) Slopes at the supports

Let θ_A = Slope at A i.e., $\left(\frac{dy}{dx}\right)$ at A for the given beam

$$\Theta_B = \text{Slope at B i.e., } \left(\frac{dy}{dx}\right) \text{ at B for the given beam}$$

Then according to the conjugate beam method,

$$\Theta_A = \text{shear force at } A^* \text{ for conjugate beam} = R_A^*$$

$$= \frac{250}{3EI}$$

$$= \frac{250}{3 \times 200 \times 10^6 \times 10^4} = 0.004166\text{rad.} \quad \text{Ans.}$$

$$\Theta_B = \text{shear force at } B^* \text{ for conjugate beam} = R_B^*$$

$$= \frac{200}{3EI}$$

$$= \frac{200}{3 \times 200 \times 10^6 \times 10^{-4}} = 0.003333\text{rad.} \quad \text{Ans.}$$

(iii) Deflection under the load

Let y_c = Deflection at C for the given beam.

Then according to the conjugate beam method,

Y_c =B.M. at point C^* of the conjugate beam

$= R_A^*$ x 2 –(Load $A^*C^*D^*$) x Distance of C.G. of $A^*C^*D^*$ from C^*

$$\frac{250}{3EI} \times 2 - \left(\frac{1}{2} \times 2 \times \frac{100}{EI}\right) \times \left(\frac{1}{3} \times 2\right)$$

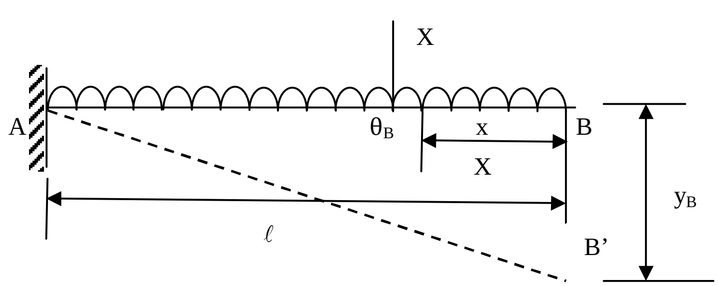
$$= \ = \frac{500}{3EI} - \frac{200}{3EI} = \frac{100}{EI}$$

$$= \frac{100}{200 \times 10^6 \times 10^{-4}} \text{m}$$

$$= \frac{1}{200} \text{m} = \frac{1}{200} \times 1000 = 5\text{mm.} \quad \text{Ans..}$$

8) Cantilever of length (l) carrying uniformly distributed load w KN per unit run over whole length. Derive the formula to find the slope and deflection at the free end by double integration method. Calculate the deflection if w = 20 KN/m, l = 2.30 m and EI = 12000 KNm^2 (13)

(Nov / Dec 2016)



Cantilever AB of length (l) fixed at and free at end B carrying a UDL of w per unit length over the whole span,

Consider section XX at a distance x from the free end B

B.M at section XX = $\omega x \, x/z = \frac{-\omega x^2}{2}$

$$M = EI\frac{d^2y}{dx^2} = \frac{-\omega x^2}{2}$$

Integration the above equation, $EI\frac{dy}{dx} = \frac{-\omega x^3}{6} + C_1$ ...1

Integration again, $EIy = \frac{-\omega x^4}{24} + C_1 x + C_2$ ...2

C_1 & C_2 → values are obtained from the boundary condition,

i) When $x = \ell$, slope $\frac{dy}{dx} = 0$

ii) When $x = \ell$, slope $y = 0$

Applying Boundary condition i) in equation 1 we get,

$$0 = \frac{-\omega \ell^3}{6} + C_1$$

$C_1 = \frac{\omega \ell^3}{6}$ sub in equa 1 we get

slop equation $EI\frac{dy}{dx} = \frac{-\omega x^3}{6} + \frac{\omega \ell^3}{6}$ ...3

Max slop can be determine by substituting x = 0 in equ 3

$$EI\left(\frac{dy}{dx}\right)_{B \text{ at } (x=0)} = \frac{\omega \ell^3}{6}$$

$$EI\,\theta_{B \text{ at } (x=0)} = \frac{\omega \ell^3}{6}$$

$$\boxed{\theta_B = \frac{\omega \ell^3}{6EI}}$$

Apply ii) Boundary condition to equation 2,

$$0 = \frac{-\omega \ell^4}{24} + \frac{\omega \ell^3}{6}\ell + C_2$$

$$\boxed{C_2 = \frac{\omega \ell^4}{6} - \frac{\omega \ell^4}{24} = \frac{-3\omega \ell^4}{24} = \frac{-\omega \ell^4}{8}}$$ ...4

Sub, C_1 & C_2 value in equation 2 we get, Deflection equation $\boxed{EI\,y = \frac{-\omega z^4}{24} + \frac{\omega \ell^3}{6}x - \frac{\omega \ell^4}{8}}$ ...5

Max deflection occur at z = 0 in equation 5

$$EI\,y_{B\,(x=0)} = \frac{-\omega \ell^4}{8}$$

$$\boxed{y_B = \frac{-\omega \ell^4}{8EI}} \qquad [\text{sign indicate downward deflection}]$$

$\omega = 20\ KN/m \quad \ell = 2.30\,m \quad EI = 12000\,KNm^2$

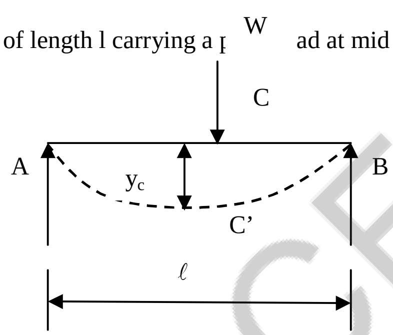
$$y_B = \frac{20\times10^3 \times 2.3^4}{8\times12000\times10^3} = 5.83\times10^{-3}\ m$$

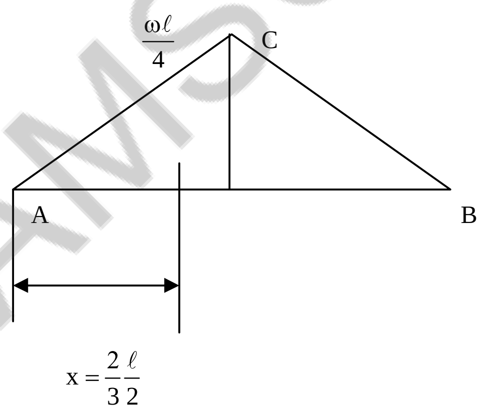
$$\boxed{y_B = 5.38\ mm}$$

9) Derive the formula to find the deflection of a simply supported beam with point load w at the centre by moment area method (8 mark)

(Nov / Dec 2016)

A SSB of length l carrying a p ad at mid – span,





Loading is symmetric the maximum deflection occurs at mid span C. The slope at C is zero. Slope at A & B is maximum,

$$\text{Slope at } A = \theta_a = \frac{\text{Area of BMD between A \& C}}{EI} = \frac{A}{EI}$$

$$A = \frac{1}{2} \times \frac{\ell}{2} \times \frac{\omega \ell}{4} = \frac{\omega \ell^2}{16}$$

$$\boxed{\theta_a = \frac{\omega \ell^2}{16EI}}$$

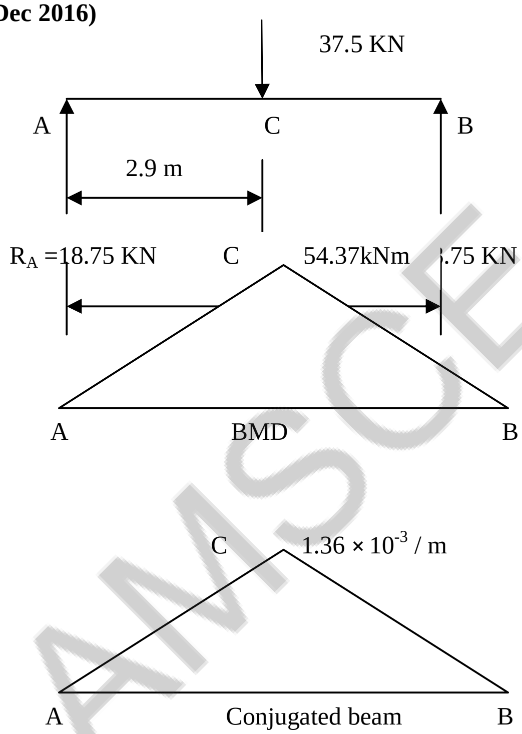
$$\bar{x} = \frac{2}{3}\frac{\ell}{2} = \frac{\ell}{3}$$

$$y_c = \frac{A\bar{x}}{EI} = \frac{\frac{\omega \ell^2}{16} \times \frac{\ell}{3}}{EI}$$

$$\boxed{y_c = \frac{\omega \ell^3}{48EI}}$$

10) A simply supplied beam of span 5.80 m carries a central point load of 37.5 KN, Find the max. slope and deflection, Let EI = 40000 KNm^2. Use conjugate beam method, (5)

(Nov /Dec 2016)



BMD:

R_A & R_B $\qquad R_A + R_B = 37.5$ KN $\qquad$ ...1

$\sum M_A = 0 \qquad 5.8R_B = 37.5 \times 2.9$

$\boxed{R_B = 18.75 \text{ KN}}$ substitute in 1, we get

$\boxed{R_A = 18.75 \text{ KN}}$

$$M_A = M_B = 0$$

$$M_c = 18.75 \times 2.9$$

$$= 54.375 \text{ KNm}$$

$$\frac{M}{EI} = \frac{54.375}{40000} = 1.36 \times 10^{-3} / \text{m}$$

Total load on conjugated beam =Area of M/EI diagram

$$P = \frac{1}{2} \times 5.8 \times 1.36 \times 10^{-3}$$

$$P = 3.94 \times 10^{-3}$$

Reaction at each support for conjugate beam,

$$R_A = R_B = \frac{1}{2}P = 1.972 \times 10^{-3} \text{ radians}$$

Deflection at c = B.M at C for the conjugate beam,

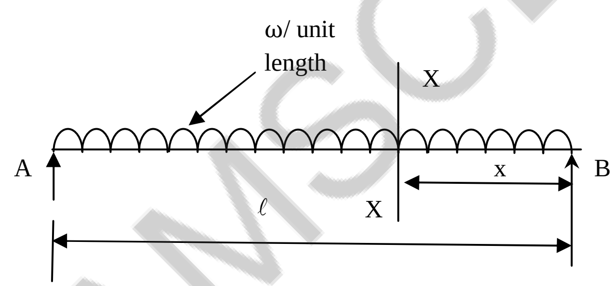
$$= 1.972 \times 10^{-3} \times 2.9 - \frac{1}{2} \times 2.9 \times 1.36 \times 10^{-3} \times \frac{1}{3} \times 2.9$$

$$= 5.7188 \times 10^{-3} - 1.9062 \times 10^{-3}$$

$$y_c = 3.8125 \times 10^{-3} \text{m}$$

$$\boxed{y_c = 3.8125 \text{ mm}}$$

11) A SSB subjected to UDL of w KN/m for the entire span. Calculate the maximum deflection by double integration method (16 mark) (Apr / May 2016)



SSI $R_A = \frac{\omega\ell}{2}$ d carrying a UDL of ω per m length $R_B = \frac{\omega\ell}{2}$ e span

The reaction at A & B are, $R_A = R_B = \frac{\omega\ell}{2}$

Consider a section XX at a distance x from B

B.M at XX = $\frac{\omega\ell}{2}x - \omega x\frac{x}{2}$

$$M_x = \frac{\omega\ell}{2}x - \frac{\omega x^2}{2}$$

$$M_x = EI\frac{d^2y}{dx^2} = \frac{\omega\ell}{2}x - \frac{\omega x^2}{2} \qquad \text{...1}$$

Integrating the above equation

$$EI\frac{dy}{dx} = \frac{\omega\ell}{4}x^2 - \frac{\omega x^3}{6} + C_1 \qquad ...2$$

Integration again,

$$EI\,y = \frac{\omega\ell x^3}{12} - \frac{\omega x^4}{24} + C_1x + C_2 \qquad ...3$$

Varies of C_1 & $C_2 \rightarrow$ obtained by applying Boundary condtion,

i) when $x = \frac{\ell}{2} \Rightarrow \text{slop}\,\frac{dy}{dx} = 0$

ii) when $x = 0 \Rightarrow \text{deflection}\ y = 0$

Apply B.C i) to equation 2

$$0 = \frac{\omega\ell}{4}\left(\frac{\ell}{2}\right)^2 - \frac{\omega}{6}\left(\frac{\ell}{2}\right)^3 + C_1$$

$$0 = \frac{\omega\ell^3}{16} - \frac{\omega\ell^3}{48} + C_1$$

$$\boxed{C_1 = \frac{\omega\ell^3}{48} - \frac{\omega\ell^3}{16} = -\frac{\omega\ell^3}{24}} \quad \text{sub in Equ 2}$$

Slop equation $\boxed{EI\frac{dy}{dx} = \frac{\omega\ell}{4}x^2 - \frac{\omega x^3}{6} - \frac{\omega\ell^3}{24}}$ ...4

Max slop occur between A & B

Max slop substitute x=0 ; in equa 4

$$EI\frac{dy}{dx} = EI\theta_B = \frac{-\omega\ell^3}{24}$$

$$\boxed{\theta_B = \frac{-\omega\ell^3}{24\,EI}} \quad -\text{ve sign slop in neg direction}$$

$$\boxed{\theta_A = \theta_B = \frac{\omega\ell^3}{24\,EI}}$$

Applying boundary condition ii) ,in equation 3

$C_2 = 0$

Substitute C_1 & C_2 values ion equation 3, we get

$$EIy = \frac{\omega\ell x^3}{12} - \frac{\omega x^4}{24} - \frac{\omega\ell^3 x}{24} \qquad ...5$$

the deflection is minimum at mid point C.

To find max deflection x= $\frac{\ell}{2}$ sub in equa 5

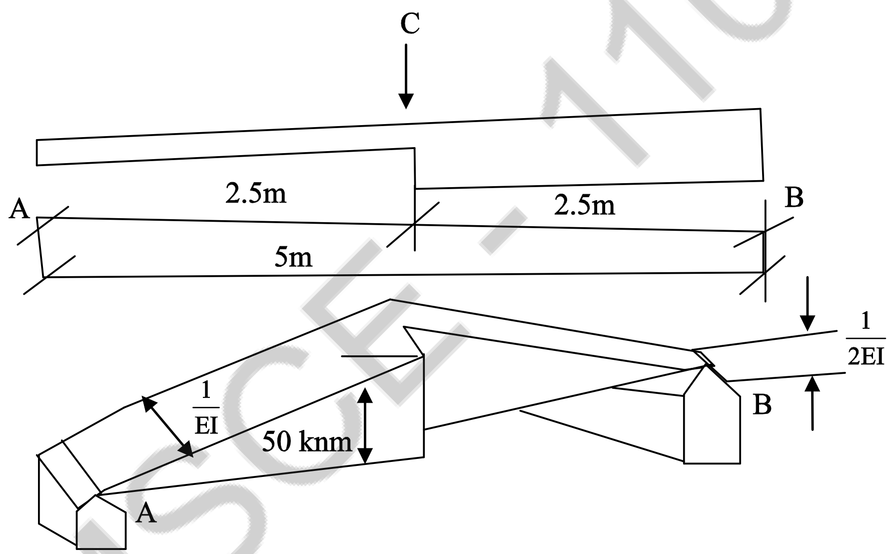
$$EIy_c = \frac{w\ell\left(\frac{\ell}{2}\right)^3}{12} - \frac{\omega}{24}\left(\frac{\ell}{2}\right)^4 - \frac{\omega\ell^3}{24}\left(\frac{\ell}{2}\right)$$

$$= \frac{\omega\ell^4}{96} - \frac{\omega\ell^4}{384} - \frac{\omega\ell^4}{384} = \frac{5\omega\ell^4}{384}$$

$$y_c = \frac{5w\ell^4}{384\ EI}$$

12) A SSB AB of span 5m carries a point of 40 KN at its centre. The values of moments of inertia for the left half is 2 × 10 ^{8} mm^4 and for the right half of portion is 4 × 10^8 mm^4. Find the slope at the two support and deflection under the load. Take E = 200 GN/m^2 (16 mark)

(Apr / May 2016)



Slope at two supports, $(BM)_{max} = \frac{\omega\ell}{4} = \frac{40\times5}{4} = 50\,KNm$

Draw conjugate beam, Take M_A

$$R_B\times5 = \frac{1}{EI}\left[\frac{1}{2}\times50\times2.5\times\frac{5}{3}\right] + \frac{1}{2EI}\left[\frac{1}{2}\times50\times2.5\right]\left[2.5+\frac{2.5}{3}\right]$$

$$= \frac{1}{3EI}[312.5] + \frac{1}{2EI}\left[\frac{1}{2}\times50\times2.5\times\frac{10}{3}\right]$$

$$5R_B = \frac{312.5}{3EI} + \frac{312.5}{3EI} = \frac{625}{3EI}$$

$$\boxed{R_B = \frac{125}{3EI}\ KN}$$

$$R_A + R_B = \frac{1}{EI}\left[\frac{1}{2}\times 50\times 2.5\right]+\frac{1}{2EI}\left[\frac{1}{2}\times 50\times 2.5\right]$$

$$= \frac{62.5}{EI}+\frac{62.5}{2EI}=\frac{187.5}{2EI}$$

$$R_A = \frac{187.5}{2EI}-\frac{125}{3EI}=\frac{562.5-250}{6EI}=\frac{312.5}{6EI}\ KN$$

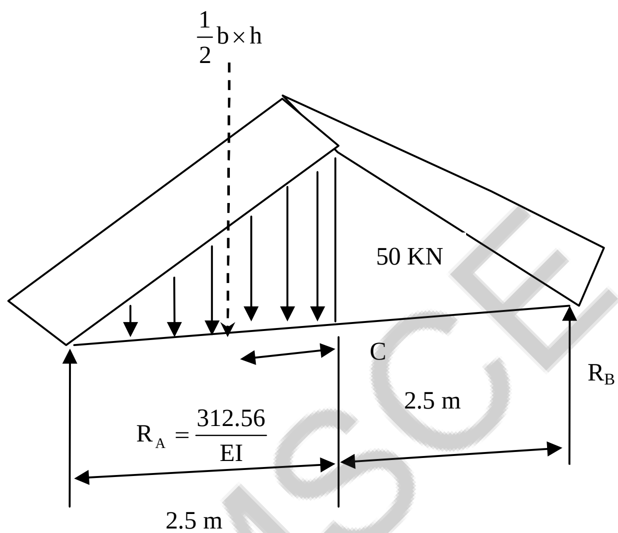
$$\text{shear force at A}, \theta_A = F_A = \frac{312.5}{6EI}=\frac{312.5\times 10^3}{6\times 200\times 10^9\times 2\times 10^{-4}}$$

$$\theta_A = 0.0013 \text{ rad}$$

$$\text{shear force at A}, \theta_B = F_B = \frac{125}{3EI}=\frac{125\times 10^3}{3\times 200\times 10^9\times 2\times 10^{-4}}$$

$$\theta_B = 0.00104 \text{ rad}$$

Deflection under load(y_c)



$$M_c = R_A\times 2.5 - \frac{1}{EI}\left[\frac{1}{2}\times 50\times 2.5\times\frac{2.5}{3}\right]$$

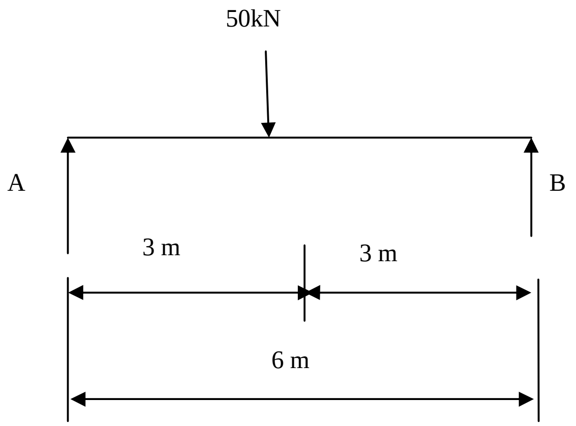
$$M_c = \frac{312.5}{6EI}\times 2.5 - \frac{312.5}{6EI}=\frac{468.75}{6EI}$$

$$y_c = M_c = \frac{468.75}{6EI}=1.95\times 10^{-3}\,m$$

$$\boxed{y_c = 1.95 \text{ mm}}$$

13) A beam 6m long, simply supported at its end, is carrying a point load of 50 KN at its centre. The moments of inertia of the beam is given as equal to 78 × 10^6mm^4. If E for the material of the beam = 2.1 × 10 ^{5} N/mm^2, calculate i) deflection at the centre of the beam & ii) slope at the supports (16 mark)

(Nov / Dec 2015)



$\ell = 6\text{m} \quad \omega = 50\,\text{KN} \qquad I = 78\times10^{6}\,\text{mm}^{4} \quad E = 2.1\times10^{5}\ \text{N/mm}^{2}$

Double Integration method

ii) $\theta_A = \theta_B = \dfrac{\omega\ell^2}{16\,EI} = \dfrac{50\times10^3\times6000^2}{16\times2.1\times10^5\times78\times10^6} = 6.87\times10^{-3}$ radians

i) $y_c = \dfrac{\omega\ell^3}{48\,EI} = \dfrac{50\times10^3\times6000^3}{48\times2.1\times10^5\times78\times10^6}$

$y_c = 13.74$ mm

14) A beam of length 6m is simply supported at ends and carries two point loads of 48 kN and 40 kN at distance of 1m and 3m respectively from the left support as shown in fig.

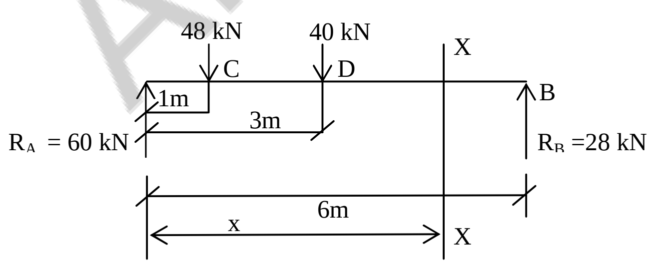
Using Macauley's method find

(i) deflection under each load

(ii) maximum deflection &

(iii) the point at which maximum deflection occurs,

Given, $E = 2 \times 10^5$ N/mm^2 & $I = 85 \times 10^6$ mm^4 (Nov/Dec 2015) (16)



R_A & R_B

$R_A + R_B = 88$ kN $\rightarrow(1)$

$\Sigma M_A = 0 \Longrightarrow -48\text{x}1 - 40\text{x}3 + 6R_B = 0$

$6R_B = 168$

$$\boxed{R_B = 28\,kN}$$

Substitute in eqn (1)

$$\boxed{R_A = 60\,kN}$$

Consider the selection X in the last part of the beam at a distance x from the left support A. The BM at this selection is given by,

$$EI\frac{d^2y}{dx^2} = R_A x \left| -48(x-1) \right| - 40(x-3)$$
$$= 60x \left| -48(x-1) \right| - 40(x-3)$$

Integrating the above equation, we get,

$$EI\frac{dy}{dx} = 60\frac{x^2}{2} + c_1 \left| -48\frac{(x-1)^2}{2} \right| - 40\frac{(x-3)^2}{2}$$
$$= 30x^2 + c_1 \left| -24(x-1)^2 \right| - 20(x-3)^2 \rightarrow (1)$$

Integrate the above equation, again,

$$EIy = 30\frac{x^3}{3} + c_1 x + c_2 \left| \frac{-24(x-1)^3}{3} \right| - \frac{20(x-3)^3}{3}$$
$$= 10x^3 + c_1 x + c_2 \left| -8(x-1)^3 \right| \frac{-20}{3}(x-3)^3 \rightarrow (2)$$

To find values of c_1 & c_2 use, two boundary condition,

(i) At x = 0; y = 0

(ii) At x = 6m ; y = 0

Substitute boundary condition (i) in equation (2) we get

x = 0 ; y = 0 $\Rightarrow \boxed{c_2 = 0}$

$\Downarrow$

lies first part of the beam so consider equation, upto first line

substitute boundary condition (ii) in equation (2) , we get,

x = 6m ; y = 0

$0 = 10 \times 6^3 + c_1 \times 6 + 0 - 8(6-1)^3 - 20/3\ (6-3)^3$

$0 = 2160 + 6c_1 - 8 \times 5^3 - 20/3 \times 3^3$

$0 = 2160 + 6c_1 - 1000 - 180 = 980 + 6c_1$

$$\boxed{c_1 = \frac{-980}{6} = -163.33}$$

Substitute c_1 & c_2 value in equation (2),

$$EIy = 10x^3 - 163.33x \left| -8(x-1)^3 \right| - \frac{20}{3}(x-3)^3 \qquad \rightarrow (3)$$

(1) Deflection under each load:

At point c,

Substitute x =1 in equation (3) upto first part of vertical line,

$EIy_c = 10 \times 1^3 - 163.33 \times 1$

$= -153.33\ kNm^3$

$EIy_c = -153.33 \times 10^{12}\ Nmm^3$

$$y_c = \frac{-153.33\times10^{12}}{EI} = \frac{-153.33\times10^{12}}{2\times10^5\times85\times10^6}$$

$$\boxed{y_c = -9.016\,mm}$$

At point D,

Substitute x = 3 in eqn (3) upto second part of vertical line,

$EIy_D = 10 \times 3^3 - 163.33 \times 3 - 8(3-1)^3$

$= 270 - 489.99 - 64 = -283.99\ kNm^3$

$= -283.99 \times 10^{12}\ Nmm^3$

$$y_D = \frac{-283.99\times10^{12}}{EI} = \frac{-283.33\times10^{12}}{2\times10^5\times85\times10^6}$$

$$\boxed{y_D = -16.7\,mm}$$

(2) Maximum Deflection;

Deflection is max between section C & D

For maximum deflection, dy/dx = 0 substituting in eqn (1)

Consider the eqn (1) upto second vertical line,

$30x^2 + c_1 - 24(x-1)^2 = 0$

$6x^2 + 48x - 187.33 = 0$

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-48\pm\sqrt{48^2-4\times6\times187.33}}{2\times6} = 2.87m$$

Substitute , x = 2.87m in eqn(3) ,upto second vertical line, we get,

$EIy_{max} = 10 \times 2.87^3 - 163.33 \times 2.87 - 8(2.87-1)^3$

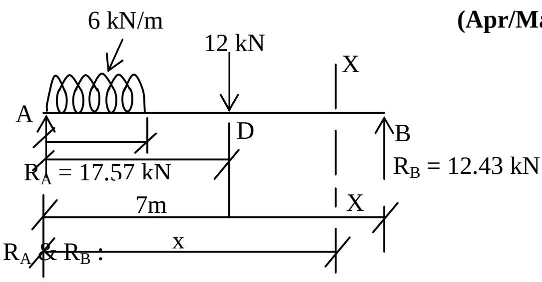
$= 284.67\ kNm^3 = -284.67 \times 10^{12}\ Nmm^3$

$$y_{max} = \frac{-284.67\times10^{12}}{2\times10^{5}\times85\times10^{6}} = -16.745\text{mm}$$

$$\boxed{y_{max} = 16.745\text{mm}}$$

15) A horizontal beam of uniform section and 7m long is simply supported at it ends. The beam is subjected to a UDL of 6 KN/m over a length of 3m from the left end and a concentrated load of 12 KN at 5m from the left end. Find the maximum deflection in the beam using Macauley's method.

(Apr/May 2015) 16 Marks



To find R_A & R_B :

$R_A + R_B = 6 \times 3 + 12 = 30$ KN $\rightarrow (1)$

$\Sigma M_A = 0 \Rightarrow 7R_B = 12 \times 5 + 3 \times 3 \times 3/2 = 87$

$$\boxed{R_B = 12.43\,\text{KN}}$$

$$\boxed{R_A = 17.57\,\text{KN}}$$

$M_{XX} = R_A x - 6X\,3X\,(x-1.5) - 12X\,(x-5)$

$M_{XX} = EI\,d^2y/dx^2 = 17.57x\,|-18(x-1.5)| - 12(x-5)$

$$EI\frac{d^2y}{dx^2} = 17.57x\,|-18(x-1.5)\,| -12(x-5)$$

Integrate

$$EI\frac{dy}{dx} = 17.57\,x^2/2 + c_1\left|\frac{-18(x-1.5)^2}{2}\right| - \frac{12(x-5)^2}{2}$$

$$EI\frac{dy}{dx} = 8.785x^2 + c_1\left|-9(x-1.5)^2\right| - 6(x-5)^2 \qquad \rightarrow (1)$$

Integrate

$$EIy = 8.785\,x^3/3 + c_1x + c_2\left|\frac{-9(x-1.5)^3}{3}\right| - \frac{6(x-5)^3}{3}$$

$$= 2.93x^3 + c_1x + c_2\left|-3(x-1.5)^3\right| - 2(x-5)^3 \qquad \rightarrow (2)$$

To find values of c_1 & c_2 use boundary

(i) At x = 0 $\Longrightarrow$ y = 0 $\longrightarrow$ (i)

(ii) At x = 7m $\Longrightarrow$ y = 0 $\longrightarrow$ (ii)

Substitute B.C in equation (2), we get, consider the term upto first vertical line

$$\boxed{0 = c_2}$$

Substitute B.C (ii) in equation (2) , we get

X = 7m ; y = 0

$$0 = 2.93(7)^3 + 7c_1 – 3(7-1.5)^3 -2(7-5)^3$$

$7c_1 = -2.93(7)^3 + 3(7-1.5)^3 +2(7-5)^3$

$7c_1 = -489.865$

$$\boxed{c_1 = -69.98}$$

Substitute c_1 & c_2 values in equation (2) , we get,

$$EIy = 2.93x^3 -69.98\ x\ |-3(x-1.5)^3\ |\ -2(x-5)^3 \rightarrow (3)$$

Assume deflection maximum between c & D_1 we get,

For maximum deflection dy/dx = 0

Substitute in equation (1),

Consider upto second vertical line

$0 = 8.785x^2 -69.98 -9(x-1.5)^2$

$= 8.785x^2 - 69.98 - 9(x^2-3x+2.25)$

$= 8.785x^2 - 69.98 – 9x^2 + 27x – 20.25$

$0 = -0.215x^2 + 27x – 90.23$

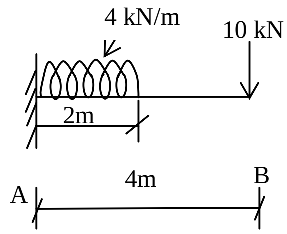
$0 = 0.215x^2 -27x + 90.23$

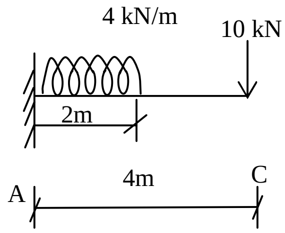
$\boxed{x = 3.435m}$ substituting in equation (3) upto second vertical line,

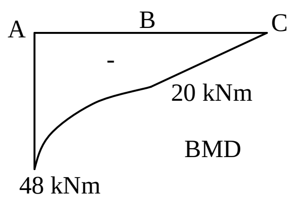
$$EIy_{max} = 2.93(3.435)^3 – 69.98(3.435) -3(3.435-1.5)^3$$

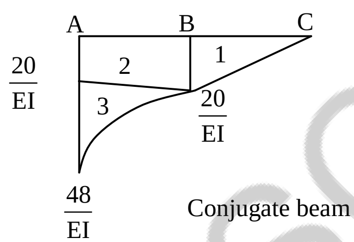
$$\boxed{y_{max} = \frac{-143.36}{EI}}$$

16) A cantilever of span 4m carries a UDL of 4 KN/m over a length of 2m from the fixed end and a concentrated load of 10 KN at the free end. Determine the slope and deflection of the cantilever at the free and using conjugate beam method. Assume EI uniform throughout.









B.M at C = 0

B.M at B = -10 x 2 = -20 KNm

B.M at A = $-10 \times 4 - 4 \times 2 \times \left(\frac{\not{2}^{1}}{\not{2}}\right)$

$= -10 \times 4 = 4 \times 2 \times 1 = -48$ KNm

Total load on beam = Area of $\frac{M}{EI}$ diagram

$$P = -\frac{1}{2} \times 2 \times \frac{20}{EI} - 2 \times \frac{20}{EI} - \frac{1}{3} \times 2 \times \left(\frac{48}{EI} - \frac{20}{EI}\right)$$

$$= \frac{-20}{EI} - \frac{40}{EI} - \frac{1}{3} \times 2 \times \frac{28}{EI}$$

$$P = \frac{-60 - 120 - 56}{3EI} = \frac{-236}{3EI}$$

Slope at C,

$$\theta_c = SF \text{ at } C = -P = \frac{236}{3EI}$$

For finding BM at C for conjugate beam the total load can be considered as UVL and which is divided into one triangle & one rectangle and one parabolic curve on conjugate beam

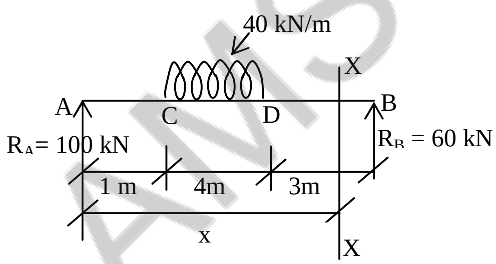
$$\text{B.M at } c = \left[\frac{1}{2} \times 2 \times \frac{20}{EI} \times \frac{2}{3} \times 2\right] + \left[2 \times \frac{20}{EI} \times \left(2 + \frac{2}{2}\right)\right] + \left[\frac{1}{2} \times 2 \times \left[\frac{48}{EI} - \frac{20}{EI}\right] \times \left(\frac{3}{4} \times 2 + 2\right)\right]$$

$$= \frac{80}{3EI} + \frac{120}{EI} + \left[\frac{1}{3} \times 2 \times \frac{28}{EI} \times \frac{7}{2}\right]$$

$$\boxed{\left.\begin{matrix}\text{Deflection} \\ \text{at c}\end{matrix}\right\} = \left.\begin{matrix}\text{BM at} \\ \text{c}\end{matrix}\right\} = \frac{80}{3EI} + \frac{360}{3EI} + \frac{196}{3EI} = \frac{636}{3EI}}$$

17) Determine the deflection of the beam at its midspan and also the position of maximum deflection & max. Deflection Take E = 2 x 10^5 N/mm^2 and I = 4.3 x 10^8 mm^4 . Use Macaulay's method. The beam is given in fig

(Nov/Dec 2014) (May / June 2017) (16)



R_A & R_B :

$R_A + R_B = 40 \text{ X } 4 = 160 \quad \rightarrow (1)$

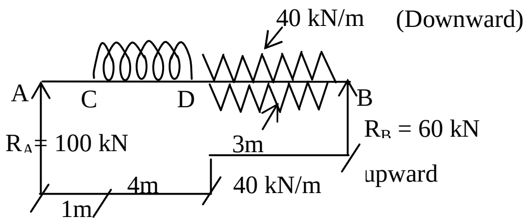
$\Sigma M_A = 0 \Rightarrow 8R_B = 40 \text{ X } 4 \text{ X } 3$

$8R_B = 480$

$\boxed{R_B = 60\,KN}$ substitute in (1)

$\boxed{R_A = 100\,KN}$

To obtain general expressions for the B.M at a distance x from the left end A, which will apply for all values of x, it is necessary to extend the UDL upto the support B, compensating with an equal upward load of 40 KN/m over the span DB as shown in figure, now Macauley's method can be applied.



B.M at any section at a distance x from end A is given by,

$$EI\frac{d^2y}{dx^2} = R_A x \left| -40(x-1)\frac{(x-1)}{2} \right| + 40(x-5)\frac{(x-5)}{2}$$

$$EI\frac{d^2y}{dx^2} = 100x \left| -20(x-1)^2 \right| + 20(x-5)^2 \qquad \rightarrow (1)$$

Integrate the above equation, we get,

$$EI\frac{dy}{dx} = \frac{100x^2}{2} + c_1 \left| -20\frac{(x-1)^3}{3} \right| + 20\frac{(x-5)^4}{3} \qquad \rightarrow (2)$$

Integrate again, we get,

$$EIy = 50\, x^3/3 + c_1 x + c_2 \left| \frac{-20}{3}\frac{(x-1)^4}{4} \right| + \frac{20}{3}\frac{(x-5)^4}{4}$$

$$= 50\, x^3/3 + c_1 x + c_2 \left| \frac{-5}{3}(x-1)^4 \right| + \frac{5}{3}\frac{(x-5)^4}{4} \qquad \rightarrow (3)$$

The value of c_1 & c_2 are obtained from boundary condition (i) x = 0 ; y = 0 9ii) x = 8m y = 0

Substituting x = 0; y = 0 in equation (3) upto first dotted line, we get $\boxed{c_2 = 0}$

Substituting (ii) B.C x = 8; y = 0 in equation(3),

$$0 = \frac{50}{3}\times 8^3 + c_1 \times 8 + 0 - 5/3\,(8-1)^4 + 5/3\,(8-5)^4$$

$$0 = 8533.33 + 8c_1 - 4001.66 + 135$$

$$8c_1 = -4666.67$$

$$\boxed{c_1 = \frac{-4666.67}{8} = -583.33}$$

Substituting the values of c_1 & c_2 in equation (3) we get,

$$EIy = \frac{50}{3}x^3 - 583.33x \Big| - \frac{5}{3}(x-1)^4 \Big| + \frac{5}{3}(x-5)^4 \qquad \rightarrow (4)$$

a) Deflection at centre

substitute x = 4 in equation (4) , upto second vertical line,

$$EIy_{(x=4)} = \frac{50}{3}4^3 - 583.33 \times 4 - \frac{5}{3}(4-1)^4$$
$$= -1401.66 \times 10^3 \times 10^9 \text{ Nmm}^3$$
$$= -1401.66 \times 10^{12} \text{ Nmm}^3$$

$$y = \frac{-1401.66 \times 10^{12}}{2 \times 10^5 \times 4.5 \times 10^8} = -16.29\,\text{mm}$$

(– sign indicates downward)

b) Position of maximum deflection

For maximum deflection dy/dx = 0; equating the slope given by eqn (2) upto second vertical line;

$$0 = 50x^2 + c_1 - \frac{20}{3}(x-1)^3$$
$$\boxed{0 = 50x^2 - 583.33 - 6.667(x-1)^3} \qquad \rightarrow (5)$$

The above equation is solved by trial & error method

Let,

x =1 ; R.H.S of equation of eqn (5),

$= 50(1)^2 - 583.33 – 6.667(1-1)^3$

= -533.33

x =2 ; then R.H.S

$= 50 \times 4 - 583.33 – 6.667 (1)^3$

= -390.00

x = 3; then R.H.S

$= 50 \times 9 – 583.33 – 6.667(2)^3$

=-136.69

x = 4; then R.H.S

$= 50 \times 16 - 583.33 – 6.667(3)^3$

= + 36.58

x value lies between x = 3 & x = 4

Let x = 3.82 then R.H.S

$= 50 \times 3.82 - 583.33 - 6.667(3.82-1)^3$

$= -3.22$

X = 3.83 then R.H.S

$= 50 \times 3.83 - 583.33 - 6.667(3.83-1)^3$

$= -0.99$

Maximum deflection will be at a distance of 3.83 m from support A.

c) Maximum deflection

substitute x = 3.83 m in eqn (4) upto second vertical line, we get maximum deflection,

$$EIy_{max} = \frac{50}{3}(3.83)^3 - 583.33 \times 3.83 - \frac{5}{3}(3.83-1)^4$$
$$= -1404.69\,KNm^3 = -1404.69 \times 10^{12}\,Nmm^3$$

$$\boxed{y_{max} = \frac{-1404.69 \times 10^{12}}{2 \times 10^5 \times 4.3 \times 10^8} = -16.33mm}$$

UNIT-5

PART-A (2 MARKS)

THIN CYLINDERS, SPHERES AND THICK CYLINDERS

1) How does a thin cylinder fail due to internal fluid pressure? (May / June 2017)

Thin cylinder failure due to internal fluid pressure by the formation of circumferential stress and longitudinal stress.

2) Name the stress develops in the cylinder. [NOV/DEC 2016]

The stresses developed in the cylinders are:

1. Hoop or circumferential stresses.
2. Longitudinal stresses
3. Radial stresses

3) Define radial pressure in thin cylinder. [NOV/DEC 2016]

The internal pressure which is acting radially inside the thin cylinder is known as radial pressure in thin cylinder.

4)Differentiate between thin and thick cylinders [MAY/JUNE 2016] [APR/MAY 2015](Nov/Dec 2018) (Apr/May 2019)

S.No	Thin	Thick
1	Ratio of wall thickness to the diagram of cylinder is less than 1/20.	Ratio of wall thickness to the diagram of cylinder is more than 1/20
2	Hoop stress is assumed to be constant throughout the wall thickness.	Hoop stress varies from inner to outer wall thickness.

5) Describe the lame's theorem: [MAY/JUNE 2016][NOV/DEC 2014] [MAY/JUNE 2017] (Apr/May 2018)

(Apr/May 2019)

Ratio stress, $\sigma_{r} = b/r^2 - a$

Hoop stress, $\sigma_c = b/r^2 + a$

6) State the expression for max shear stress in a cylinder shell [NOV/DEC 2015]

In a cylindrical shell, at any point on it circumference there is a set of two mutually perpendicular stresses $\sigma_c \sigma_\gamma$ which are principal stresses and as such the planes in which these act are the principal planes.

$$\tau_{max} = \frac{\sigma_c - \sigma_\gamma}{2} = \frac{\frac{pd}{2t} - \frac{pd}{4t}}{2} = \frac{pd}{8t}$$

$$\boxed{\tau_{max} = \frac{pd}{8t}}$$

7) Define-hoop stress & longitudinal stress **[NOV/DEC 2015](Apr/May 2018)**

(i) Hoop stress: (σ_c)

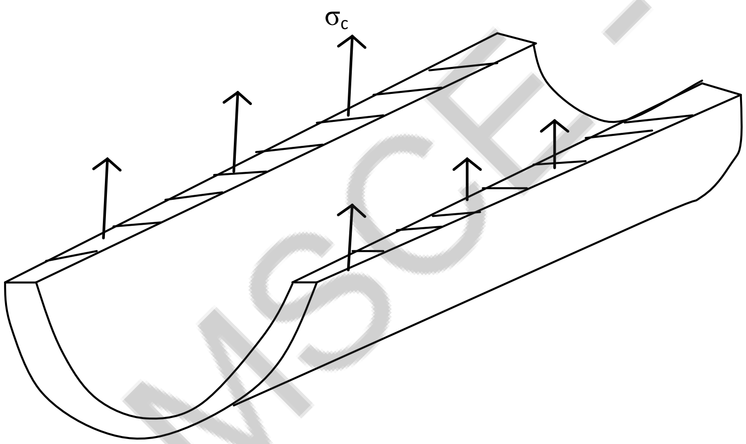
These act in a tangential dirn, to the circumference of the shell.

$$\sigma_c = \frac{pd}{2t}$$

(ii) Longitudinal stress: (σ_ℓ)

The stress in the longitudinal direct due to tendency of busting the cylinder along the transverse place is called longitudinal stress

$$\sigma_l = \frac{pd}{4t}$$



8) State the assumption made in lame's theorem for thick cylinder analysis. [APR/MAY 2015] [NOV/DEC 2017] [NOV/DEC 2018]

1. The material is homogeneous and Isotropic.
2. The material is stressed within elastic limit.
3. All the fibers of the material are to expand (or) contract independently without being constrained by the adjacent fibers.
4. Plane section perpendicular to the longitudinal axis of the cylinder remain plane after the application of internal pressure.

9) What is meant by circumferential stress? **[NOV/DEC 2014]**

The stress in the circumferential direction in due to tendency of bursting the cylinder along the longitudinal axis is called circumferential stress (or) hoop stress.

$$\sigma_c = \frac{pd}{2t}$$

10) A storage tank of internal diameter 280 mm is subjected to an internal pressure of 2.56 MPa. Find the thickness of the tank. If the hoop & longtudital stress are 75 MPa and 45 MPa respectively

$\sigma_c = 75$ MPa, $\sigma_l = 45$ MPa, d = 280 mm, p = 2.5MPa

$\sigma_c > \sigma_l \Rightarrow$ use σ_c

$$\sigma_c = \frac{pd}{2t}$$

$$t = \frac{pd}{2\sigma_c} = \frac{2.5 \times 280}{2 \times 75}$$

$$\boxed{t = 4.66 \text{ mm}}$$

11) A spherical shell of 1m internal diameter undergoes a diameter strain of 10^{-4} due to internal pressure. What is the corresponding change in volume?

$$\delta V = e_v \times V$$

$$= 3 \times e \times V = 3 \times 10^{-4} \times \frac{\pi}{6} \times (1000)^3$$

$$\delta V = 157.079 \text{ mm}^3$$

12) A thin cylindrical closed at both ends is subjected to an internal pressure of 2 MPa. Internal diameter is 1m and the wall thickness is 10mm. What is the maximum shear stress in the cylinder material?

$p = 2\,\text{mPa} = \frac{2N}{mm^2}$ $\quad$ $d = 1m = 100mm \quad t = 10mm$

$$\sigma_c = \frac{pd}{2t} = \frac{2 \times 1000}{2 \times 10} = 100\,N/mm^2$$

$$\sigma_l = \frac{pd}{4t} = \frac{2 \times 1000}{4 \times 10} = 50\,N/mm^2$$

$$\tau_{max} = \frac{\sigma_c - \sigma_l}{2} = \frac{100 - 50}{2} = \frac{50}{2}$$

$$\tau_{max} = 25\ N/mm^2$$

13) Find the thickness of the pipe due to an internal pressure of 10N/mm^2 if the permissible stress is 120 N/mm^2 and the diameter of the pipe is 750 mm

$p = 10\ N/mm^2$, $\sigma_c = 120\,N/mm^2$, $d = 750\,mm$

$$\sigma_c = \frac{pd}{2t}$$

$$t = \frac{pd}{2\sigma_c} = \frac{10 \times 750}{2 \times 120} = 31.25 \text{ mm}$$

14) A spherical shell of 1m diameter is subjected to an internal pressure 0.5 N/mm^2. Find the thickness if the allowable stress in the material of the shell is 75N/mm^2.

$d = 1m = 1000$ mm, $\quad p = 0.5\,N/mm^2 \quad \sigma_c = 75N/mm^2$

$$\sigma_c = \frac{pd}{4t}$$

$$t = \frac{pd}{4\sigma_c}$$

$$= \frac{0.5 \times 1000}{4 \times 75} = \boxed{1.67\,mm}$$

15) Define thick cylinder

When the ratio of thickness (t) to internal diameter of cylinder is more than 1/20 then the cylinder is known as thick cylinder

16) In a thick cylinder will the radial stress is vary over the thickness of wall?

Yes, in thick cylinder radial stress is maximum at inner and minimum at the outer radius.

17) Define thin cylinder. (Nov/Dec 2017)

If the thickness of wall of the cylinder vessel is less than 1/15 to 1/20 of its internal diameter, the cylinder vessels is known as thin cylinder.

18) In a thin cylinder will the radial stress over the thickness of wall?

No, In the cylinder radial stress developed in its wall is assumed to be constant since the wall thickness is very small as compared to the diameter of cylinder

19) What is the ratio of circumference stress to longitudinal stress of a thin cylinder?

The ratio of circumferential stress to longitudinal stress of a thin cylinder is two.

20) Distinguish between cylinder shell and spherical shell.

S.No.	Cylindrical shell	Spherical shell
1.	Circumferencial stress is twice the longitudinal stress	Only hoop stress presents
2.	It withstands low pressure than spherical shell for the same diameter	It withstand more pressure than cylinder shell for the same diameter

21) What is the effect of riveting a thin cylinder shell?

Riveting reduce the area offering the resistance. Due to this, the circumferential and longitudinal stresses are more. It reduces the pressure carrying capacity of the shell.

PART-B

1) A cylindrical thin drum 80cm in diameter and 3m long has a shell thickness of 1cm. If the drum is subjected to an internal pressure of 2.5 N/mm^2, determine (i) change in diameter (ii) change in length and (iii) change in volume $E=2\times10^5 N/mm^2$ and poisons ratio=0.25 (Apr/May 2019)

d = 80cm

L = 3m = 300cm

t = 1cm

$p = 250 N/cm^2$

$E = 2 \times 10^7 \ N/cm^2$

$\mu = 0.25$

Change in diameter (δd)

$$\delta d = \frac{pd^2}{2tE}\left[1-\frac{\mu}{2}\right]$$
$$= \frac{250\times80^2}{2\times1\times2\times10^7}\left[1-\frac{0.25}{2}\right]$$
$$\boxed{\delta d = 0.35cm}$$

Change in length (δℓ)

$$\delta\ell = \frac{pdL}{2tE}\left[\frac{1}{2}-\mu\right]$$
$$= \frac{250\times80\times300}{2\times1\times2\times10^7}\left[0.5-0.25\right]$$
$$\boxed{\delta\ell = 0.0375cm}$$

Change in volume (δv)

$$\frac{\delta V}{V} = 2\frac{\delta d}{d} + \frac{\delta l}{l}$$
$$\frac{\delta V}{V} = 2\frac{0.035}{80} + \frac{0.0375}{300} = 0.001$$
$$\text{original volume, } V = \frac{\pi}{4}d^2\times\ell = \frac{\pi}{4}\times80^2\times300$$
$$V = 1507964.473 cm^3$$

$\delta V = 0.001 \times V = 0.001 \times 1507964.473 = 1507.96 \ cm^3$

2) A spherical shell of internal diameter 0.9m and of thickness 10mm is subjected to an internal pressure of $1.4N/mm^2$. Determine the increase in diameter and increase in volume. $E=2\times10^5 N/mm^2$ and poissons ratio=1/3 (Apr/May 2019)

$d = 0.9\text{m} = 900\text{mm}$

$t = 10\text{mm}$

$p = 1.4\text{N/mm}^2$

$E = 2\text{x}10^5\text{N/mm}^2$

$\mu = \frac{1}{3}$

Change in diameter: (δd)

$$\delta d = \frac{pd^2}{4tE}\left[1-\frac{1}{m}\right]$$
$$= \frac{1.4\times 900^2}{4\times 10\times 2\times 10^5}\left[1-\frac{1}{3}\right]$$
$$\boxed{\delta d = 0.0945\text{mm}}$$

Change in volume (δv)

$$e_V = 3\text{x}\frac{\delta d}{d} = 3\text{x}\frac{0.0945}{900} = 315\text{x}10^{-6}$$
$$\frac{\delta V}{V} = 315\text{x}10^{-6}$$
$$V = \left(\frac{\pi}{6}\right)\text{x}d^3 = \left(\frac{\pi}{6}\right)\text{x}900^3$$
$$\delta V = 12028.5\text{mm}^3$$

3) A boiler shell is to be made of 15mm thick plate having tensile stress of 120 N/mm^2 If the efficiencies of the longitudinal and circumferential joints are 70% and 30%. Determine the maximum permissible diameter of the shell for an internal pressure of 2 N/mm^2 (Nov/Dec 2018)

Maximum diameter of circumference stress

$$\sigma_c = \frac{pd}{2t\eta_l}$$
$$120 = \frac{2\times d}{2\times 15\times 0.7}$$
$$d = \frac{120\times 2\times 15\times 0.7}{2}$$
$$d = 1260\text{mm}$$

Maximum diameter for longitudinal stress

$$\sigma_l = \frac{pd}{4t\times \eta_c}$$
$$120 = \frac{2\times d}{4\times 15\times 0.3}$$
$$d = \frac{120\times 4\times 15\times 0.3}{2}$$
$$d = 1080\,\text{mm}$$

4) A thin cylindrical shell with following dimensions is filled with a liquid at atmospheric pressure. Length=1.2m, external diameter=20cm, thickness of metal=8mm, Find the value of the pressure exerted by the liquid on the walls of the cylinder and the hoop stress induced if an additional volume of 25cm^3 of liquid is pumped into the cylinder. Take E=2.1×10^{5}N/mm^2 and poisons ratio=0.33 (Nov/Dec 2018)

$$L = 1.2m = 1200mm$$
$$D = 20cm = 200mm$$
$$t = 8mm$$
$$d = D - 2t = 184mm$$
$$\delta V = 25cm^3 = 25000mm^3$$
$$E = 2.1x10^5 N/mm^2$$
$$\mu = 0.33$$

$$\text{Volume, } V = \frac{\pi}{4} \times d^2 \times \ell$$
$$= \frac{3.14}{4} \times 184^2 \times 1200$$
$$= 31908528 mm^3$$
$$\delta V = V \times \frac{pd}{2tE}\left(\frac{5}{2} - \frac{2}{m}\right)$$
$$25000 = 31908528 \times \frac{p \times 184}{2 \times 8 \times 2.1 \times 10^5}\left[\frac{5}{2} - 2(0.33)\right]$$
$$p = 7.7 N/mm^2$$
$$\sigma_c = \frac{pd}{2t} = \frac{7.7x184}{2x8} = 89.42 N/mm^2$$

5) A cylindrical shell 3m long which is closed at the ends has an internal diameter of 1.5m and a wall thickness of 20mm. Calculate the circumferential and longitudinal stresses induced and also change in the dimensions of the steel. If it is subjected to an internal pressure of 1.5 N/mm^2 Take E=2×10^{5}N/mm^2 and poisons ratio=0.3 (Apr/May 2018)

$$l = 3m = 3000mm$$
$$t = 20mm$$
$$d = 1.5m = 1500mm$$
$$p = 1.5N/mm^2$$
$$E = 2x10^5 N/mm^2$$
$$\mu = 0.3$$

Hoop stress, $\sigma_c = \frac{pd}{2t} = \frac{1.5 \times 1500}{2 \times 20} = 56.25$

$$\sigma_c = 56.25 N/mm^2$$

Longitudinal stress, $\sigma_\ell = \frac{pd}{4t} = \frac{1.5\times1500}{4\times20} = 28.125$

$\sigma_\ell = 28.125 N/mm^2$

Change in diameter (δd)

$$\delta d = \frac{pd^2}{2tE}\left[1-\frac{\mu}{2}\right]$$

$$= \frac{1.5\times1500^2}{2\times20\times200\times10^3}\left[1-\frac{0.3}{2}\right]$$

$$\boxed{\delta d = 0.7225mm}$$

Change in length (δℓ)

$$\delta\ell = \frac{pdL}{2tE}\left[\frac{1}{2}-\mu\right]$$

$$= \frac{1.5\times1500\times3000}{2\times20\times200\times10^3}\quad[0.5-0.3]$$

$$\boxed{\delta\ell = 0.16875mm}$$

Change in volume (δv)

$$\frac{\delta V}{V} = \frac{pd}{2tE}\left[\frac{5}{2}-\frac{2}{m}\right]$$

original volume, $V = \frac{\pi}{4}d^2\times\ell = \frac{\pi}{4}\times1500^2\times3000$

$V = 5301437603mm^3$

$$\delta V = \frac{1.5\times1500x5301437603}{2\times20\times200\times10^3}\left[\frac{5}{2}-2\times0.3\right]$$

$$\boxed{\delta V = 2832955.72mm^3}$$

6) A compound cylinder formed by shrinking one tube to another is subjected to an internal pressure of 90MN/m^2. Before the fluid is admitted, the internal and external diameter of the compound cylinders are 180mm and 300mm respectively and the diameter at the junction is 240mm. If after shrinking on, the radial pressure at the common surface is 12MN/m^2. Determine the final stresses developed in the compound cylinder (Apr/May 2018)

Solution. Internal pressure in the cylinder,

$$p_1 = 90 \text{ MN/m}^2$$

Internal radius of the cylinder, $r_1 = \dfrac{180}{2} = 90 \text{ mm} = 0{\cdot}09 \text{ m}$

External radius of the cylinder, $r_3 = \dfrac{300}{2} = 150 \text{ mm} = 0{\cdot}15 \text{ m}$

Radius at the junction, $r_2 = \dfrac{240}{2} = 120 \text{ mm} = 0{\cdot}12 \text{ m}$

Radial pressure at the common surface after shrinking on,

$$p = 12 \text{ MN/m}^2$$

Final stresses developed:

Let the *Lame's equations* be:

For inner tube: $\sigma_r = \dfrac{b}{r^2} - a$

and, $\sigma_c = \dfrac{b}{r^2} + a$

For outer tube: $\sigma_r = \dfrac{b'}{r^2} - a'$

and, $\sigma_c = \dfrac{b'}{r^2} + a'$

(a) ***Before the fluid is admitted:***

Inner tube:

At, $r = r_1 = 0{\cdot}09$ m, $\sigma_r = 0$

$$\frac{b}{0{\cdot}0081} - a = 0$$

∴ $123{\cdot}456\, b - a = 0$ ...(i)

or,

At, $r = r_2 = 0{\cdot}12$ m,

$\sigma_r = 12$ MN/m²

$$\frac{b}{0{\cdot}0144} - a = 12$$

∴ $69{\cdot}44\, b - a = 12$ ...(ii)

or,

From eqns. (i) and (ii), we get

$$b = -0{\cdot}222 \text{ and } a = -27{\cdot}41$$

Hence circumferential stress at any point in the *inner tube* will be given by

$$\sigma_c = -\frac{0{\cdot}222}{r^2} - 27{\cdot}41$$

The *minus sign indicates that the stress will be wholly compressive.*

At, $r = r_1 = 0{\cdot}09$ m,

$$\sigma_{c(0{\cdot}09)} = -\frac{0{\cdot}222}{0{\cdot}09^2} - 27{\cdot}41 = 54{\cdot}82 \text{ MN/m}^2 \text{ (comp.)}$$

At, $r = 0{\cdot}12$ m,

$$\sigma_{c(0{\cdot}12)} = -\frac{0{\cdot}222}{0{\cdot}12^2} - 27{\cdot}41 = 42{\cdot}82 \text{ MN/m}^2 \text{ (comp.)}$$

Outer tube:

At, $r = 0{\cdot}15$ m, $\sigma_r = 0$

$$\therefore \quad \frac{b'}{0{\cdot}15^2} - a' = 0$$

or, $44{\cdot}44\, b' - a' = 0$ ...(iii)

At, $r = 0{\cdot}12$ m, $\sigma_r = 12$ MN/m²

$$\therefore \quad \frac{b'}{0{\cdot}12^2} - a' = 0$$

or, $69{\cdot}44\, b' - a' = 12$ ...(iv)

From eqns. (iii) and (iv), we get

$$b' = +0{\cdot}48, \text{ and } a' = +21{\cdot}33$$

Hence the circumferential stress at any point in the *outer tube* will be given by

$$\sigma_c = \frac{0{\cdot}48}{r^2} + 21{\cdot}33$$

At, $r = 0{\cdot}12$ m,

$$\sigma_{c(0{\cdot}12)} = \frac{0{\cdot}48}{0{\cdot}12^2} + 21{\cdot}33 = 54{\cdot}66 \text{ MN/m}^2 \text{ (tensile)}$$

At, $r = 0{\cdot}15$ m,

$$\sigma_{c(0{\cdot}15)} = \frac{0{\cdot}48}{0{\cdot}15^2} + 21{\cdot}33 = 42{\cdot}66 \text{ MN/m}^2 \text{ (tensile)}$$

(b) *After the fluid is admitted:*

Let the *Lame's equation* be:

$$\sigma_r = \frac{b}{r^2} - a$$

At, $r = 0{\cdot}09$ m, $\sigma_r = 90$ MN/m²

$$\therefore \quad 90 = \frac{b}{0{\cdot}09^2} - a$$

or, $90 = 123{\cdot}45\, b - a$ ...(v)

At, $r = 0{\cdot}15$ m, $\sigma_r = 0$

$$\therefore \quad 0 = \frac{b}{0{\cdot}15^2} - a$$

or $0 = 44{\cdot}44\, b - a$ ...(vi)

From eqns. (v) and (vi), we get

$$b = 1{\cdot}139 \text{ and } a = 50{\cdot}61$$

Hence, the circumferential stress at any point in the compound tube is given by,

$$\sigma_c = \frac{b}{r^2} + a$$

At, $r = 0{\cdot}09$ m, $\sigma_{c(0\cdot09)} = \frac{1{\cdot}139}{0{\cdot}09^2} + 50{\cdot}61 = 191{\cdot}23$ MN/m² (tensile)

$r = 0{\cdot}12$ m, $\sigma_{c(0\cdot12)} = \frac{1{\cdot}139}{0{\cdot}12^2} + 50{\cdot}61 = 129{\cdot}71$ MN/m² (tensile)

$r = 0{\cdot}15$ m, $\sigma_{c(0\cdot15)} = \frac{1{\cdot}139}{0{\cdot}15^2} + 50{\cdot}61 = 101{\cdot}23$ MN/m² (tensile)

The *final circumferential stresses* at different points are tabulated below:

Tensile stress........ +

Compressive stress..... –

Circumferential (or hoop) stress (MN/m²)	*Inner tube*		*Outer tube*	
	$r = 0{\cdot}09$ m	$r = 0{\cdot}12$ m	$r = 0{\cdot}12$ m	$r = 0{\cdot}15$ m
(i) Initially	– 54·82	– 42·82	+ 54·66	+ 42·66
(ii) Due to fluid pressure	+ 191·23	+ 129·71	+ 129·71	+ 101·23
Final	+ 136·41	+ 86·89	+ 184·31	+ 143·89

Hence the *final circumferential stresses are:*

Inner tube: $\sigma_{c\,(0\cdot09)}$ **= 136·41 MN/m²** *(tensile)*

$\sigma_{c\,(0\cdot12)}$ **= 86·89 MN/m²** *(tensile)* **(Ans.)**

Outer tube: $\sigma_{c\,(0.12)}$ **= 184·31 MN/m²** *(tensile)*

$\sigma_{c\,(0\cdot15)}$ **= 143·89 MN/m²** *(tensile)* **(Ans.)**

7) Determine the maximum and minimum hoop stress across the section of a pipe of 400 mm internal diameter and 100 mm thick, when the pipe contains a fluid at a pressure of 8 N/mm². Also sketch the radial pressure distribution and hoop stress distribution across the section. (May 2017) (Nov/Dec 2017)

Solution,

Given:

Internal dia = 400 mm

∴ Internal radius, $r_1 = \frac{400}{2} = 200\,\text{mm}$

Thickness = 100 mm

∴ External radius $r_2 = \frac{600}{2} = 300\,\text{mm}$

Fluid pressure, $p_0 = 8\,\text{N/mm}^2$

or at $x = r_1$, $p_x = p_0 = 8\,\text{N/mm}^2$

The radial pressure (p_x) is given by equation (18.1) as

$$p_x = \frac{b}{x^2} - a$$

Now apply the boundary conditions to the above equation. The boundary conditions are:

1. At $x = r_1 = 200$ mm, $p_x = 8\,\text{N/mm}^2$

2. At $x = r_2 = 300$ mm, $p_x = 0$

Substituting these boundary conditions in equation(i), we get

$$8 = \frac{b}{200^2} - a = \frac{b}{40000} - a \qquad \ldots(ii)$$

and

$$0 = \frac{b}{300^2} - a = \frac{b}{90000} - a \qquad \ldots(iii)$$

subtracting equation (iii) from equation (ii), we get

$$8 = \frac{b}{40000} - \frac{b}{90000} = \frac{9b - 4b}{360000} = \frac{5b}{360000}$$

$$b = \frac{360000 \times 8}{5} = 5760000$$

Substituting this value in equation (iii), we get

$$0 = \frac{5760000}{90000} - a \quad \text{or} \quad a = \frac{5760000}{90000} = 6.4$$

The values of 'a' and 'b' are substituted in the hoop stress.

Now hoop stress at any radius x is given by equation (18.2) as

$$\sigma_x = \frac{b}{x^2} + a = \frac{576000}{x^2} + 6.4$$

At x = 200 mm, $\sigma_{200} = \frac{576000}{200^2} + 6.4 = 14.4 + 6.4 = 20.8 \mathrm{N/mm^2}$. Ans.

At x = 300 mm, $\sigma_{300} = \frac{576000}{300^2} + 6.4 = 6.4 + 6.4 = 12.8 \mathrm{N/mm^2}$. Ans.

Fig.15 Shows the radial pressure distribution and hoop stress distribution across the section. AB is taken a horizontal line. AC = 8N/mm^2. The variation between B and C is parabolic. The curve BC shows the variation of radial pressure across AB.

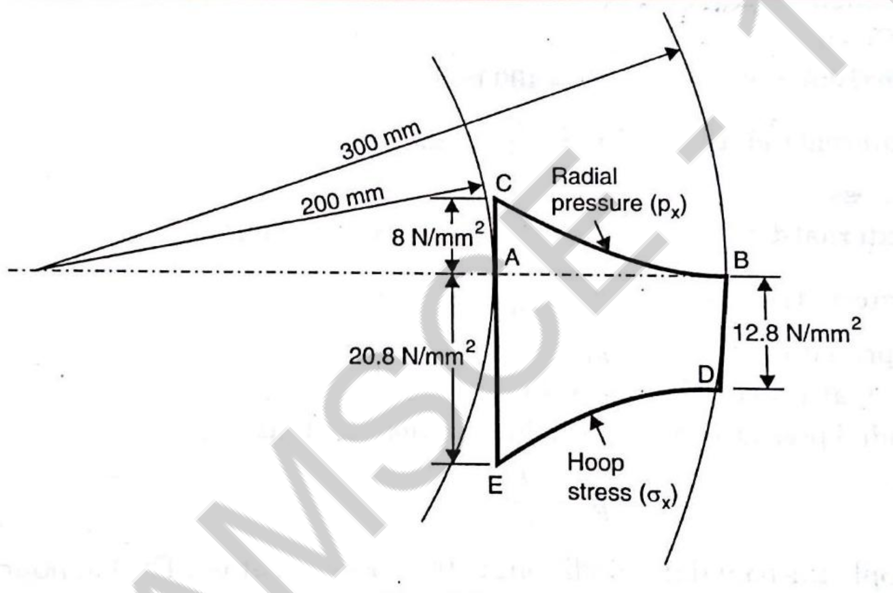


Fig. 15

The curve DE which is also parabolic, shows the variation of hoop stress across AB. Value BD = 12.8 N/mm^2 and AE = 20.8 N/mm^2 . The radial pressure is compressive whereas the hoop stress is tensile.

8) A cylindrical vessel is 2m diameter and 5m long is closed at ends by rigid plates. It is subjected to an internal pressure of 4N/mm^2 of the maximum principal stress is not to exceed 210N/mm^2. Find the thickness of the shell. Assume E=2×10^{5}N/mm^2 and poisons ratio=0.3, find the change in diameter, length and volume of the shell. [MAY/JUNE 2016-8 marks]

Given data:

Diameter, d=2m=2000mm

Length, l=5m=5000mm

Initial pressure, p=4N/mm^2

Maximum principal stress means the circumferential stress=σ_c=210N/mm^2

Young modulus =E=2×10^{5}N/mm^2

Poisons ratio=u=0.3

To find:

1.) Thickness of the shell (t)

2.) Change in diameter (∫d)

3.) Change in length and (∫ℓ)

4.) Change in volume (∫v)

Solution:

$$\sigma_c = \frac{pd}{zt}$$

$$t = \frac{pd}{2\times\sigma_c} = \frac{4\times 2000}{2\times 210} = 19.047\text{mm}$$

Change in diameter (∫d)

$$\int d = \frac{pd^2}{2tE}\left[1-\frac{1}{2}\times\mu\right]$$

$$= \frac{4\times 2000^2}{2\times 19.047\times 2\times 10^5}[1-0.5\times 0.3]$$

$$\int d = 1.785\text{mm}$$

Change in length (∫ℓ)

$$\int \ell = \frac{pd\ell}{2tE}\left[\frac{1}{2}-\mu\right]$$

$$= \frac{4\times 2000\times 5000}{2\times 19.047\times 2\times 10^5}\left[\frac{1}{2}-0.3\right]$$

$$\int \ell = 1.050\text{mm}$$

Change in volume (∫v)

$$\frac{\int v}{v} = \frac{pd}{2tE}\left[\frac{5}{2}-2\times\mu\right] = \frac{4\times 2000}{2\times 19.047\times 2\times 10^5}\left[\frac{5}{2}-2\times 0.3\right]$$

$$\boxed{\int v / v = 1.995\times 10^{-3}\text{mm}^3} \qquad \left[V = \frac{\pi}{4}\times d^2\times L\right]$$

$$\int v = 1.995\times 10^{-3}\times\frac{\pi}{4}\times 2000^2\times 5000$$

$$\boxed{\int v = 313121500\text{mm}^3}$$

9) A spherical sheet of 1.50m internal diameter and 12mm shell thickness is subjected to pressure of $2N/mm^2$. Determine the stress induced in the material of the shell [APR/-MAY/JUNE 2016-8marks]

Given data:

Internal diameter, d=1.5m=1500mm

Shell thickness, t=12mm

Pressure, $P=2N/mm^2$

To find:

(1) Stress induced in the material of shell

$$\sigma_1 = \frac{p}{4t}$$
$$= \frac{2\times1500}{4\times12}$$
$$= 62.5N/mm^2$$

10) A spherical shell of internal diameter 1.2m and of thickness 12mm is subjected to an internal pressure of $4N/mm^2$. Determine the increase in diameter and increase in volume. Take $E=2\times10^5 N/mm^2$ and μ=0.33. [APR.MAY/JUNE 2016] 8marks

Given data:

Internal diameter of spherical shell, d=1.2m=1200mm

Thickness of spherical shell, t=12mm

Internal pressure, $P=4N/mm^2$

Young's modulus, $E=2\times10^5 N/mm^2$

Poisons ratio $=\mu=\frac{1}{m}=0.33$

To find:

(i) Increase in diameter, δd

(ii) Increase in volume, δv.

Change in diameter: (δd)

$$\delta d = \frac{pd^2}{4tE}\left[1-\frac{1}{m}\right]$$
$$= \frac{4\times1200^2}{4\times12\times2\times10^5}\left[1-0.33\right]$$
$$\boxed{\delta d = 0.402mm}$$

Change in volume (δv)

$$\delta v = v \times ev$$
$$= v \times \frac{3pd}{4tE}\left[1-\frac{1}{m}\right]$$
$$= \frac{\pi d^2}{6} \times \frac{3pd}{4tE}\left[1-\frac{1}{m}\right]$$
$$= \frac{\pi pd^4}{8tE}[1-0.33]$$
$$= \frac{3.14\times4\times1200^4}{8\times12\times2\times10^5}[1-0.33]$$
$$\boxed{\delta = 908,841.6mm^3}$$

Result:

1) Change in diameter =δd=0.402mm

2.) Change in volume =δv =908841.6mm

11) A steel cylinder of 300mm external diameter is to be shrunk to another steal cylinder of 150mm internal diameter. After shrinking the diameter at the function is 250mm and radial pressure at the common function is 28N/mm^2. Find the original difference in radial function. Take E=2×10^{5}N/mm^2 [Apr/May 2016-8 marks]

Given:

External diameter of outer cylinder =300mm

Radius of outer cylinder = r_2=150mm

Internal diameter of inner cylinder =150mm

Radius of inner cylinder =r_1=75mm

Diameter at the function =250mm

□radius at the function =r^*=125mm

Radial pressure at the function, P^*=N/mm^2

Young modulus =E=2×10^{5}N/mm^2

Original difference of radius at the function = $\frac{2r^*}{E}(a_1 - a_2) - - - (1)$

Find the values of a_1 and a_2 using the lame's equation.

For outer cylinder

$$P_x = \frac{b_1}{x_1^{\,2}} - a_1$$

(i) At function x=r^* =125mm and P^*=28N/mm^2

(ii) At x=150mm, P_x=0

Substitute in above equation, we get

$$28 = \frac{b_1}{125^2} - a_1 = \frac{b_1}{15625} - a_1 ----(2)$$

$$0 = \frac{b_1}{150} - a_1 = \frac{b_1}{22500} - a_1 -----(3)$$

solving equation (2)×(3) we get

$$b_1 = 1432000 \qquad a_1 = 63.6$$

For inner cylinder

$$P_x = \frac{b_2}{x^2} - a_2$$

(i) At function x=r* =125m $\quad P_x = P^* = 28N/mm^2$

(ii) At x=75mm, $P_x = 0$

Substitute these two condition ion above equation

$$28 = \frac{62}{75^2} - a_2 = \frac{b_2}{15625} - a_2 -----(4)$$

$$0 = \frac{b_2}{75^2} - a_2 = \frac{b_2}{15625} - a_2 ------(5)$$

solving equation (4) & (3) we get

$$b_2 = -246100$$

$$a_2 = -43.75$$

substitute the valuies of a_2 & a_1 in equation

$$= \frac{2r^*}{E}(a_1 - a_2)$$

$$= \frac{2\times125}{2\times105} \qquad [63.6-(-43.75)]$$

$$= \frac{125}{105}\times107.35$$

$$= 0.13mm$$

12) Calculate (i) the change in diameter (ii) Change in length and (iii) Change in volume of a thin cylindrical shell 100cm diameter, 1cm thick and 5m long, when subjected to internal pressure of $3N/mm^2$. Take the value of $E=2\times10^5 N/mm^2$ and poison's ratio, μ=0.3 (Nov/Dec 2017)[Nov/Dec 2016][13 marks] [Nov/Dec 2015]

Given data:

Diameter of cylindrical shell, (d) =100cm =1000mm

Thickness of shell (t) =1cm=10mm

Length of the shell (ℓ) =5m=5000mm

Internal pressure $=P=3N/mm^2$

Young modular$=E=2\times10^5 N/mm^2$

Poison's ratio =μ=0.3

Solution:

Longitudinal stress, $\sigma_1 = \frac{pd}{4t} = \frac{3\times1000}{4\times10} = 75$

$\boxed{\sigma_1 = 75 N/mm^2}$

Hoop stress, $\sigma_c = \frac{pd}{2t} = \frac{3\times1000}{2\times10} = 150$

$\boxed{\sigma_c = 150 N/mm^2}$

(i) Change in diameter

$$\delta d = \frac{pd^2}{2tE}(1 - \frac{1}{2m})$$
$$= \frac{3\times1000^2}{2\times10\times2\times10^5}\left[1 - \frac{1}{2}\times0.3\right]$$
$$\boxed{\delta d = 0.637mm}$$

(ii)Change in length (**δ**ℓ)

$$\delta\ell = \frac{pdL}{2tE}\left[\frac{1}{2} - \mu\right]$$
$$= \frac{3\times1000\times5000}{2\times10\times200\times10^3} \quad [0.5 - 0.3]$$
$$\boxed{\delta\ell = 0.75mm}$$

(iii) Change in volume ,

$$\delta v = v\times\frac{pd}{2tE}\left(\frac{5}{2}\frac{-2}{m}\right)$$
$$\text{Volume}, v = \frac{\pi}{4}\times d^2\times\ell$$
$$= \frac{3.14}{4}\times1000^2\times5000$$
$$= 39.25\times10^8 mm^3$$
$$\delta v = 39.25\times10^8\times\frac{3\times1000}{2\times10\times2\times\times10^5}\left[\frac{5}{2} - 2(0.3)\right]$$
$$\delta v = 5593125mm^3$$

Result:

(i) Change in diameter (δd) =0.637mm

(ii) Change in length (δℓ) =0.75mm

(iii) Change in length (δv) =5593125mm^3

13) Calculate the thickness of metal necessary for a cylindrical shell of internal diameter 16mm ton with slant of internal pressure of 25mN/m$_2$. If maximum permissible shell stress is 125MN/m$_2$. [NOV/DEC-2016]

Given data:

Internal diameter, d=160mm.

Internal pressure, P=25MN/m^2 =25N/Mm^2

Maximum permissible shell stress =125MN/m^2=125N/mm^2

To find:

Thickness (t)

Solution:

$$\sigma_{max} = \frac{pd}{8t}$$
$$125 = \frac{25\times160}{8\times t}$$
$$t = \frac{25\times160}{125\times8}$$
$$t = 4mm$$
$$\text{Thickness of cylinderrical shell is 4mm}$$

14) A boiler is subjected to an internal steam pressure of 2N/mm^2. The thickness of boiler plate is 2.6cm and permissible tensile stress is 120N/mm^2. Find the maximum diameter, when efficiency of longitudinal joint is 90% and that of circumference joint is 40%. [NOV/DEC 2015 , 16marks]

Given data:

Internal steam pressure, P=2N/mm^2

Thickness boiler plate, t=2.6cm & 26mm

Permissible tensile stress (σ) =120N/mm^2

Efficiency of longitudinal joint, η_l =90% =0.90

Efficiency of circumferences joint, η_c =40% =0.40

In case of joint the permissible stress may be longitudinal (or) circumferential stress.

To find:

Maximum diameter (d)

Solution:

Maximum diameter of circumference stress

$$\sigma_c = \frac{pd}{2t\eta_l}$$
$$120 = \frac{2\times d}{2\times0.90\times2.6}$$
$$d = \frac{120\times2\times0.90\times26}{2}$$
$$d = 2808mm$$

Maximum diameter for longitudinal stress

$$\sigma_2 = \frac{pd}{4t \times \eta_c}$$
$$120 = \frac{2 \times d}{4 \times 26 \times 0.40}$$
$$d = \frac{120 \times 4 \times 0.40 \times 26}{2}$$
$$d = 2496\,\text{mm}$$

The longitudinal (or) circumferential stresses induced in the material directly proportional to diameter (d). Hence the stress induced will be less if the value of 'd' is less. Hence take the minimum value of diameter.

Hence, diameter (d) =249.6cm

15) A thin cylindrical shell 2.5 long has 700 mm internal diameters and 8mm thickness, if the shell is subjects to an internal pressure of 1Mpa, find

(i) The hoop and longitudinal stresses developed

(ii) Maximum shell stress induced and

(iii) The change in diameter, length and volume. Take modulus of elasticity of the wall material as 200Gpa and poison's ratio as 0.3 [AP/MAY 2015- 16 marks]

Given data:

Length of cylindrical shell, ℓ=2.5m=2500mm

Internal diameter ∓ d, =700mm

Thickness of shell, t=8mm

Internal pressure, P=1mpa=1N/mm^2

Modulus of elasticity = E=200Gpa=200×10^{3}N/mm^2

Poison's ratio = μ=0.3

To find:

1.) Hoop stress and longitudinal stress

2.) Maximum shell stress induced.

3.) Change in diameter, (δd)

4.) Change in volume,(δv)

5.) Change in length (δ ℓ)

Solution:

Hoop stress, $\sigma_c = \frac{pd}{2t} = \frac{1 \times 700}{2 \times 8} = 43.75$

$$\sigma_c = 43.75\,\text{N/mm}^2$$

Longitudinal stress, $\sigma_\ell = \frac{pd}{ut} = \frac{1\times 700}{4\times 8} = 21.87$

$\sigma_\ell = 21.875 N/mm^2$

Change in diameter (δd)

$$\delta d = \frac{pd^2}{2tE}\left[1-\frac{\mu}{2}\right]$$

$$= \frac{1\times 700^2}{2\times 8\times 200\times 0^3}\left[1-\frac{0.3}{2}\right]$$

$$\boxed{\delta d = 0.130mm}$$

Change in length (δℓ)

$$\delta\ell = \frac{pdL}{2tE}\left[\frac{1}{2}-\mu\right]$$

$$= \frac{1\times 700\times 2500}{2\times 8\times 200\times 10^3} \quad [0.5-0.3]$$

$$\boxed{\delta\ell = 0.109mm}$$

Change in volume (δv)

$$\delta v = \frac{pdv}{2tE}\left[\frac{5}{2}-\frac{2}{m}\right]$$

$$\text{original volume}, V = \frac{\pi}{4}d^2\times\ell = \frac{\pi}{4}\times 700^2\times 2500$$

$$V = 961625000mm^3 = 96.16\times 10^7 mm^3$$

$$\delta v = \frac{1\times 700\times 96.16\times 10^7}{2\times 8\times 200\times 10^3}\left[\frac{5}{2}-2\times 0.3\right]$$

$$\boxed{\delta v = 399665mm^3}$$

Maximum shell stress induced (σmax)

$$\sigma_{max} = \frac{pd}{t} = \frac{1\times 700}{8\times 8} = 10.937 N/mm^2$$

$$\sigma_{max} = 10.937 N/mm^2$$

Result:

1.) Hoop stress σ_c =43.75N/mm^2

2.) Longitudinal stress, σ_ℓ=21.875N/mm^2

3.) Maximum shell stress, σ_{max} =10.937N/mm^2

4.) Change in diameter, δd=0.130mm

5.) Change in length, δℓ=0.109mm

6.) Change in length, δv=399665mm^3

16) A thick cylinder with external diameter 320mm and internal diameter 160mm is subjected to an internal pressure of 8N/mm^2. Draw the variation of radial and hoop stresses in the cylinder wall. Also determine the maximum shell stress in the cylinder wall. [APR/MAY- 2015 -16marks]

Given data:

Internal diameter, d_1 =160mm

External diameter, d_2 =320mm

Internal radius, r_1 =80mm

External radius, r_2 =160mm

Internal pressure, P_1 =[8N/mm^2

To find:

1.) To draw variation of radial and hoop stress.

2.) The maximum shell stress in the cylinder.

Solution: we know that by lame's equation

$$\sigma_r = \frac{b}{r^2} - a - - - - - (1)$$
$$\sigma_c = \frac{b}{r^2} + a - - - - - (2)$$

At, r=r_1 =80, and σ_r =P_1 =8N/mm^2

R=r_2 =160mm and σ_r =P_2 =0

Substitute in equation (1)

$$8 = \frac{b}{(80)^2} - a \Rightarrow 8 = 1.562 \times 10^{-4} b - a - - - - (3)$$
$$0 = \frac{b}{(160)^2} - a \Rightarrow 0 = 3.9 \times 10^{-5} b - a - - - - - (4)$$

Equation (3) and (4) becomes

$$a - 1.562 \times 10^{-4} b = -8 - - - - (5)$$
$$a - 3.9 \times 10^{-4} b = 0 - - - - - (6)$$

Solving equation (5) and (6)

A=13.34

B=34217.27

Substitute values of a and b in equation (2)

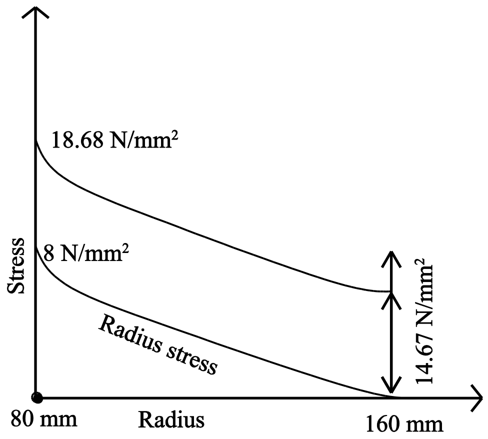
$$\sigma_c = \frac{b}{(80)^2} + a \Rightarrow = \frac{34217.27}{80^2} + 13.34$$

$$\sigma_c = 18.686 N/mm^2$$

At $r = r_2 = 160mm$

$$\sigma_c = \frac{b}{(160)^2} + a \Rightarrow \frac{34217.27}{(160)^2} + 13.34$$

$$\sigma_c = 14.67 N/mm^2$$



17) Desire relations for change in dimensions and change in volume of a thin cylinder subjected to internal pressure P. (May / June 2017) [NOV/DEC 2014]-16marks

Due to Internal pressure, the cylindrical shells are subjected to lateral and linear strain. Thus the change in dimensions such as length, diameter may increases.

We know that

$$e_c = \frac{\delta d}{d} = \frac{\sigma_c}{E} - \frac{\sigma_a}{mE}$$

Where , δd – change in diameter

$\frac{1}{m}$ = poison's ratio

E – young's Modulus

Circumferential stress,

$$e_c = \frac{pd}{2tE} - \frac{pd}{\mu t\, mE}$$

$$\boxed{e_c = \frac{pd}{2tE}\left[1 - \frac{1}{2m}\right]}$$

Change in diameter,

$$\delta d = e_c \times d$$

$$\delta d = \frac{pd^2}{2tE}\left[1 - \frac{1}{2m}\right]$$

$$e_a = \frac{\delta\ell}{\ell} = \frac{\sigma_a}{E} - \frac{\sigma_c}{mE}$$

Longitudinal strain, $= \frac{pd}{4tE} - \frac{pd}{2tmE}$

$$\boxed{e_a = \frac{pd}{2tE}\left[\frac{1}{2} - \frac{1}{m}\right]}$$

Change in length,

$$\delta\ell = e_a \times \ell$$

$$\delta\ell = \frac{pd\ell}{2tE}\left[\frac{1}{2} - \frac{1}{m}\right]$$

Volume strain,

$$e_v = \frac{\text{final volume} - \text{initial volume}}{\text{initial volume}}$$

$$= \frac{\frac{\pi}{4}(d + \delta d^2)(\ell + \delta\ell) - \frac{\pi}{4}d^2\ell}{\frac{\pi}{4}d^2\ell}$$

By neglecting higher order terms of $\delta\ell$ and δd

$$e_v = \frac{2\delta d}{d} + \frac{\delta\ell}{\ell}$$

$$= 2e_c + e_a$$

$$= \frac{2pd}{2tE}\left(1 - \frac{1}{2m}\right) + \frac{pd}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$$

$$= \frac{pd}{2tE}\left[2 - \frac{2}{2m} + \frac{1}{2} - \frac{1}{m}\right]$$

$$= \frac{pd}{2tE}\left[2 + \frac{1}{2} - \frac{2}{m}\right]$$

$$e_v = \frac{pd}{2tE}\left[\frac{5}{2} - \frac{2}{m}\right]$$

Change in volume,

$$\delta v = e_v \times v$$

$$= \frac{pdv}{2tE}\left[\frac{5}{2} - \frac{2}{m}\right]$$

$$\delta v = v \times \frac{\sigma_c}{E}\left(\frac{5}{2} - \frac{2}{m}\right)$$

18) Find the thickness of metal necessary for a thick cylindrical shell of internal diameter 160mm to withstand an internal pressure on 8N/mm^2. The maximum hoop stress in section is not to exceed 35N/mm^2. [NOV/DEC- 2014 -] [16 marks]

Given data:

Internal diameter, d_1 =160mm

Internal radius = $r_1 = \frac{d_1}{2} = \frac{160}{2} = 80mm$

Internal pressure,=P_1 =8N/mm^2

Maximum hoop stress =σ_c =35N/mm^2

To find:

Thickness of metal (t)

Solution:

The lame's equation's are

$$\sigma_r = \frac{b}{r^2} - a -----(1)$$

$$\sigma_c = \frac{b}{r^2} + a -----(2)$$

At $r = r_i = 80mm$ and $\sigma_r = P_1 = 8N/mm^2$

$$(\sigma_c)_{max} = 35N/mm^2$$

substituting in equation (1) and (2), we get

$$8 = \frac{b}{(80)^2} - a \Rightarrow 8 = 1.56\times10^{-4} b - a ----(3)$$

$$35 = \frac{b}{(80)^2} + a \Rightarrow 35 = 1.56\times10^{-4} b + a ----(4)$$

Equation (3) and (4) becomes

$$a - 1.56\times10^{-4} b = -8 -----(5)$$

$$-a - 1.56\times10^{-4} b = -35 ----(6)$$

Solving equation (5) and (6), we get

$$(5)\times1 \quad -a + 1.56\times10^{-4} b = -8$$

$$(6)\times1 \quad -a - 1.56\times10^{-4} b = -35$$

$$-2a = -27$$

$$\boxed{a = 13.5}$$

Substitute (a) value in equation (5)

$$13.5 - 1.56\times10^{-4}\,b = -8$$
$$-1.56\times10^{-4}\,b = -8 - 13.5$$
$$-1.56\times10^{-4}\,b = -21.5$$
$$b = \frac{21.5}{1.56\times10^{-4}}$$
$$\boxed{b = 137.82}$$

19) A cylindrical shell in diameter and 3m length is subjected to an internal pressure of 2MPa. Calculate the maximum thickness if the stress should not exceed 50MPa. Find the change in diameter and volume of shell. Assume poisson's ratio of 0.3 and young's modulus of 200kN/mm^2. [MAY/JUNE -2014-16marks]

Given data:

Diameter of cylindrical shell, d=1m=1000mm

Length of cylindrical shell, ℓ=3, m=3000mm

Internal pressure, P=2Mpa = 2N/mm^2

Maximum stress,. σ_c =50Mpa =50N/mm^2

Young's modulus =E =200KN/mm^2 =2×10^5N/mm^2

Poison's ratio, $\frac{1}{m} = 0.3$

To find:

(i) Change in diameter, δd

(ii) Change in volume, δv.

Solution:

$$\sigma_c = \frac{pd}{2t} = \frac{2\times1000}{2\times t}$$

Hoop stress, $50 = \frac{2\times1000}{2\times t}$

$$t = 20\text{mm}$$

Change in diameter, δd

$$\delta d = \frac{Pd^2}{2tE}\left[1 - \frac{1}{2m}\right]$$
$$= \frac{2\times(1000)^2}{2\times20\times2\times10^5}\left[1 - \frac{1}{2}\times0.3\right]$$
$$\delta d = 0.2125\text{mm}$$

Change in volume,

$$\delta v = \frac{pdv}{2tE}\left[\frac{5}{2} - \frac{2}{m}\right]$$

Volume of cylinder, $V = \frac{\pi}{4} d^2 \times \ell$

$$= \frac{\pi}{4}(1000)^2 \times 3000$$

$$= 2.355 \times 10^9 \, mm^3$$

$$\delta v = \frac{Pdv}{2tE}\left[\frac{5}{2} - \frac{2}{m}\right]$$

$$= \frac{2 \times 1000 \times 2.35 \times 10^9}{2 \times 20 \times 2 \times 10^5} \quad [2.5 - 0.6]$$

$$\boxed{\delta v = 118625 mm^3}$$

Result:

(i) Thickness of cylinder. t=20mm

(ii) Change in diameter. δd=0.2125mm

(iii) Change in volume, δv=1118625mm^3.