UNIT - 5

COMPLEX STRESSES AND PLANE TRUSSES

PART-A

1. **Define principle stresses and principle plane. (AU April/May 2017)**

   Principle stress: The magnitude of normal stress, acting on a principal plane is known as principal stresses.

   Principle plane: The planes which have no shear stress are known as principal planes.

2. **What are the assumptions made in finding out the forces in a frame? (AU April/May 2017)**

   The assumptions made in finding out the forces in a frame are:

   a. The frame is a perfect frame

   b. The frame carries load at the joints

   c. All the members are pin-joined

3. **What is mean by deficient frame? (AU Nov/Dec 2016)**

   If the number of member in a frame are less than (2j -3), then the frame is known as deficient frame.

4. **What is mean by redundant frame? (AU Nov/Dec 2016)**

   If the number of member in a frame are more than (2j -3), then the frame is known as deficient frame.

5. **What is the use of Mohr's circle? (Nov/Dec 2016)**

   To find out the normal, resultant stresses and principle stress and their planes.

6. **What are principle planes? (AU Nov/Dec 2015)**

   The planes, which have no shear stress, are known as principal planes. These planes carry only normal stresses.

7. **What are the advantages of method of sections over method of joints? (AU Nov/Dec 2015)**

This method is very quick

When the forces in few members of the truss are to be determined, then the method of section is mostly used.

8. **What is Mohr's circle? (AU Nov/Dec 2014)**

The transformation equations for plane stress can be represented in graphical form by a plot known as Mohr's Circle.

9. **What are the methods for Analysis the frame? (AU Nov/Dec 2014)**

- Methods of joints
- Methods of sections
- Method of tension coefficient
- Graphical method.

10. **What is the radius of Mohr's circle?**

Radius of Mohr's circle is equal to the maximum shear stress.

11. **List the methods to find the stresses in oblique plane?**

- Analytical method
- Graphical method

12. **A bar of cross sectional area 600 mm^2 is subjected to a tensile load of 50 KN applied at each end. Determine the normal stress on a plane inclined at 30° to the direction of loading.**

A= 600 mm2 Load, P = 50KN

$\theta = 30°$

Stress, σ = Load/Area

= 50*102/600

= 83.33 N/mm2

Normal stress, $\sigma n = \sigma \cos 2\theta$

= 83.33*cos230°

= 62.5 N/mm^2

13. In case of equal like principle stresses, what is the diameter of the Mohr's circle?

Answer: Zero

14. What are the different types of frames?

The different types of frame are:

- Perfect frame
- Imperfect frame.

15. What is mean by perfect frame?

If a frame is composed of such members, which are just sufficient to keep the frame in equilibrium, when the frame is supporting the external load, then the frame is known as perfect frame. For a perfect frame, the number of members and number of joints are not given by, n = 2j – 3.

16. What is mean by Imperfect frame?

A frame in which number of members and number of joints are not given by n = 2j – 3 is known as imperfect frame. This means that number of members in an imperfect frame will be either more or less than (2j – 3).

17. How will you Analysis of a frame?

Analysis of a frame consists of

1) Determinations of the reactions at the supports

2) Determination of the forces in the members of the frame

18. How method of joints applied to Trusses carrying Horizontal loads.

If a truss carries horizontal loads (with or without vertical loads) hinged at one end and supported on roller at the other end, the support reaction at the roller support end will be normal, whereas the support reaction at the hinged end will consist of i)horizontal reaction and (ii) vertical reaction

19. How method of joints applied to Trusses carrying inclined loads.

If a truss carries inclined loads hinged at one end and supported on roller at the other end, the support reaction at the roller support end will be normal. Whereas the support reaction at the hinged end will consist of (i) horizontal reaction and (ii) vertical reaction. The inclined loads are resolved into horizontal and vertical components.

20. What is mean by compressive and tensile force?

The forces in the member will be compressive if the member pushes the joint to which it is connected whereas the force in the member will be tensile if the member pulls the joint to which it is connected.

21. How will you determine the forces in a member by method of joints?

In method of joint after determining the reactions at the supports, the equilibrium of every support is considered. This means the sum all vertical forces as well as the horizontal forces acting on a joint is equated to zero. The joint should be selected in such a way that at any time there are only two members, in which the forces are unknown.

PART-B

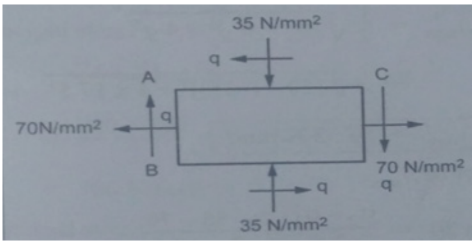
1. Two planes AB and AC which are right angles carry shear stress of intensity 17.5 N/mm² while these planes also carry a tensile stress of 70 N/mm² and a compressive stress of 35 N/mm² respectively. Determine the principal planes and the principal stresses. Also determine the maximum sgear stress and planes on which it acts.

(AU April / may 2017) [Anna Univ.-Civil-Nov 2001]

Given: $\sigma_1 = 70$ N/mm²

$\Sigma_2 = - 35$ N/mm² [Compressive]

Q =17.5 N/mm²



To find: 1. Principal planes and principal stresses

2. Maximum shear stress & plane

Solution:

$$\text{Principal plane, } \tan 2\theta = \frac{2q}{\sigma_1 - \sigma_2} = \frac{2 \times 17.5}{70 - (-35)}$$

$$\tan 2\theta = 0.333$$

$$\Rightarrow \quad 2\theta = 18.43$$

$$\boxed{\theta = 9.217^\circ \text{ or } 99.217^\circ}$$

Maximum principal stress,

$$\sigma_{n1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4g^2}$$

$$= \frac{70 - 35}{2} + \frac{1}{2}\sqrt{(70 - (-35))^2 + 4 \times 17.5^2}$$

$$\boxed{\sigma_{n1} = 72.83 \text{ N / mm}^2}$$

Minor principal stress,

$$\sigma_{n2} = \frac{\sigma_1 + \sigma_2}{2} - \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4q^2}$$

$$= \frac{70-35}{2} - \frac{1}{2}\sqrt{[70-(-35)]^2 + 4\times17.5^2}$$

$$\boxed{\sigma_{n2} = -37.83\,N/mm^2}$$

Maximum shear stress,

$$(\sigma_1)_{max} = \frac{1}{2}\sqrt{(\sigma_1-\sigma_2)^2 + 4q^2}$$

$$= \frac{1}{2}\sqrt{[70-(-35)]^2 + 4\times17.5^2}$$

$$\boxed{(\sigma_1)_{max} = 55.33\,N/mm^2}$$

Direction of $(\sigma_1)_{max°}$

$$\tan 2\theta = \frac{\sigma_2-\sigma_1}{2q} = \frac{-35-70}{2\times17.5}$$

$$2\theta = -71.56$$

$$\boxed{\theta = -35.78°}$$

Result:

Principal plane, θ = 9.217° or 99.217°

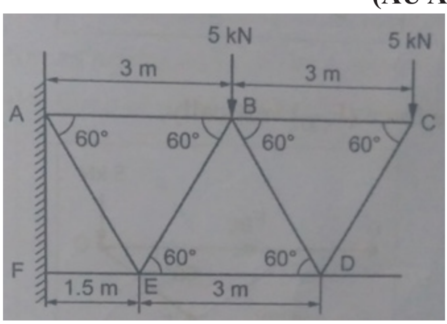
Major principal stress, σ_{n1} = 72.8/3 N/mm^2

Minor principal stress, σ_{n2}= -37.83 N/mm^2

Major Shear stress $(\sigma_t)_{max}$ = 55.33 N/mm^2

Direction of maximum shear stress, θ = -35.78°

2. Determine the forces in all members of a cantilever truss as shown in Fig. (AU April / May 2016)

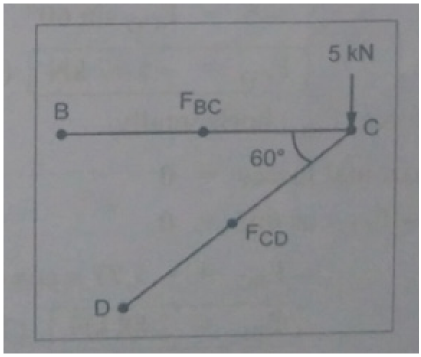


Solution:

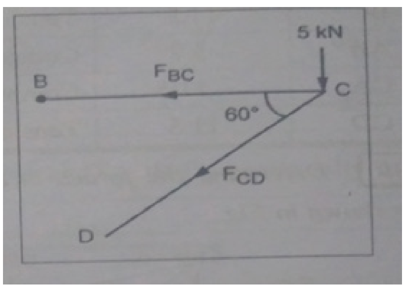
(start the calculation work from free end of cantilever i.e. from C, D, B and E):

Step 1:

Consider the joint C.

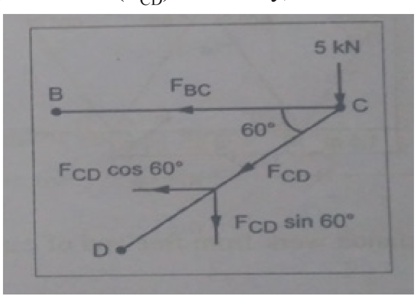


Assume the forces acting on joint C are tensile forces. If we get negative value, the force in that member is compressive.



At joint C:

Resolving the force (F_{CD}) vertically,



Sum of vertical forces = 0

$\Rightarrow \quad -5 - F_{CD} \sin 60^\circ = 0$

$\Rightarrow \quad -5 = F_{CD} \sin 60^\circ$

$\Rightarrow \quad \boxed{F_{CD} = -5.77\,kN}$ (Compressive)

Resolving the force (F_{CD}) horizontally,

Sum of horizontally forces = 0

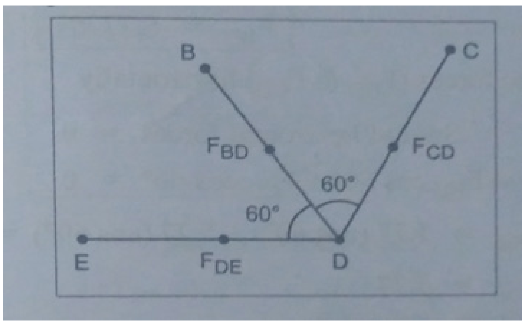
$\Rightarrow \quad -F_{BC} - F_{CD} \cos 60^\circ = 0$

$\Rightarrow \quad -F_{BC} = -5.77 \times \cos 60^\circ$

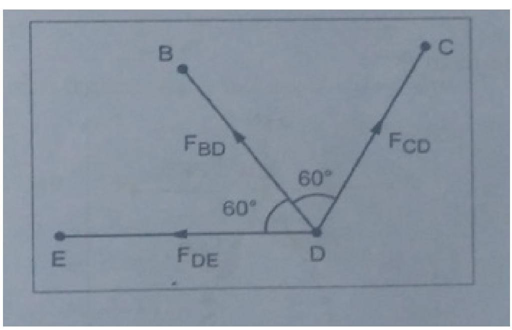
$\Rightarrow \quad \boxed{F_{BC} = 2.88\,kN}$ (Tension)

Step 2:

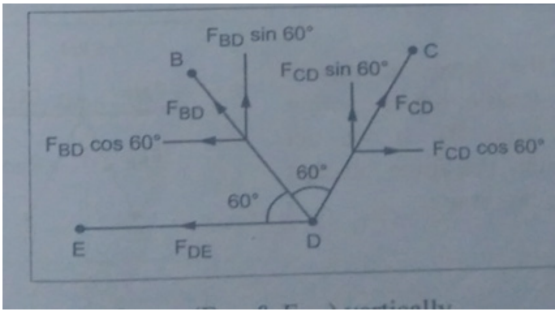
Consider the joint D.



Assume the forces acting on joint D are tensile forces. If we get negative value, the force in that member is compressive.



At joint D:



Resolving the forces (F_{BD} & F_{CD}) vertically,

Sum of vertical forces = 0

$$\Rightarrow \quad F_{BD} \sin 60^\circ + F_{CD} \sin 60^\circ = 0$$

$$\Rightarrow \quad F_{BD} = -F_{CD}$$

$$\Rightarrow \quad \boxed{F_{BD} = 5.77\text{kN}} \quad (\text{Tension})$$

Resolving the forces (F_{BD} & F_{CD}) horizontally

Sum of horizontally forces = 0

$$\Rightarrow \quad -F_{DE} - F_{BD} \cos 60^\circ + F_{CD} \cos 60^\circ = 0$$

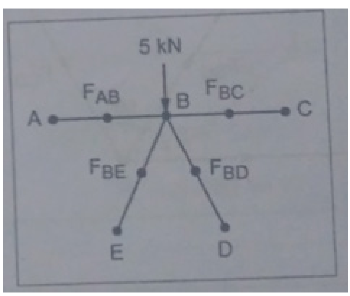
$$\Rightarrow \quad -F_{DE} = 5.77\left(\cos 60^\circ\right) + 5.77\left(\cos 60^\circ\right) = 0$$

$$\Rightarrow \quad -F_{DE} = 5.77$$

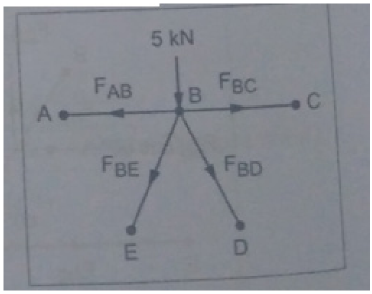
$$\Rightarrow \quad \boxed{F_{DE} = -5.77\,\text{kN}} \; (\text{compression})$$

Step 3:

Consider the joint B.

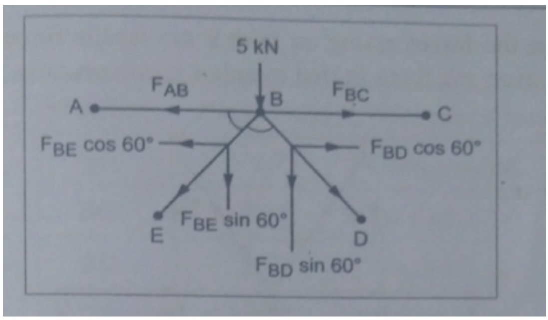


Assume the forces acting on joint B are tensile forces (acting away from joint B). If we get negative value, the force on that member is compressive.



At joint B:

Resolving the forces (F_{BR} & F_{BD}) vertically,



$\Rightarrow \quad -5 - F_{BE}\sin 60^\circ - F_{BD}\sin 60^\circ = 0$

$\Rightarrow \quad -F_{BE}\sin 60^\circ = 5 + F_{BD}\sin 60^\circ$

$\Rightarrow \quad -F_{BE}\sin 60^\circ = 5 + 5.77\sin 60^\circ$

$\Rightarrow \quad \boxed{F_{BE} = -11.5\,\text{kN}}$ (Compression)

Resolving the forces (F_{BE} & F_{BD}) horizontally

Sum of horizontally forces = 0

$\Rightarrow \quad -F_{AB} - F_{BE}\cos 60^\circ + F_{BC} + F_{BD}\cos 60^\circ = 0$

$\Rightarrow \quad -F_{AB} = F_{BE}\cos 60^\circ - F_{BC} - F_{BD}\cos 60^\circ$

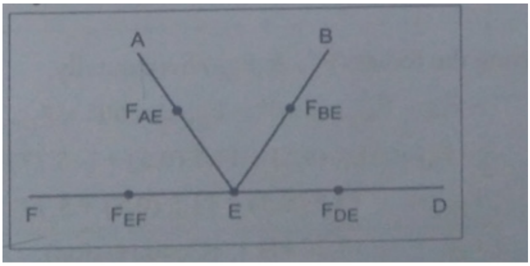
$= -11.5\cos 60^\circ - 2.88 - 5.77\cos 60^\circ = 0$

$\Rightarrow \quad -F_{AB} = -11.5$

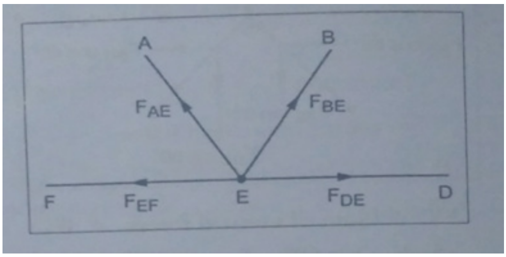
$\Rightarrow \quad \boxed{F_{AB} = 11.5\text{kN}}$ (Tension)

Step 4:

Consider the joint E.



Assume the forces acting on joint E are tensile forces. If we get negative value, the force in that member is compressive.



At joint E:

Resolving the forces (F_{AE} & F_{BE}) vertically,

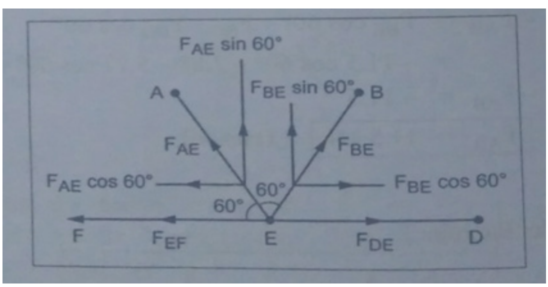
Sum of vertical forces = 0

$$\Rightarrow \quad F_{AE} \sin 60^\circ + F_{BE} \sin 60^\circ = 0$$

$$\Rightarrow \quad F_{AE} \sin 60^\circ = -F_{BE} \sin 60^\circ$$

$$\Rightarrow \quad F_{AE} = -F_{BE}$$

$$\Rightarrow \quad \boxed{F_{AE} = 11.5\,\text{kN}} \quad (\text{Tension})$$



Resolving the forces (F_{AE} & F_{BE}) horizontally,

$$\Rightarrow \quad -F_{BE} - F_{AE}\cos 60^\circ + F_{BE}\cos 60^\circ + F_{DE} = 0$$

$$\Rightarrow \quad -F_{EF} - 11.5(0.5) - 11.5(0.5) + (-5.77) = 0$$

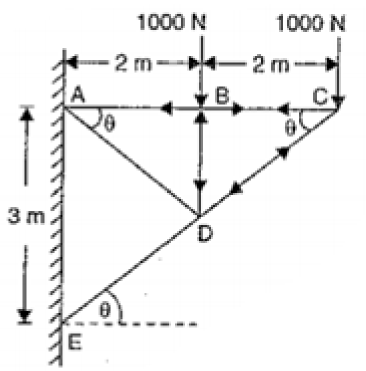
$$\Rightarrow \quad -F_{EF} = +11.5(0.5) + 11.5(0.5) + 5.77$$

$$\Rightarrow \quad \boxed{F_{EF} = -17.27\,\text{kN}} \qquad (\text{Compressive})$$

S.No	Member	Force(KN)	Nature of force
1	CD	-5.77	Compressive
2	BC	2.88	Tension
3	BD	5.77	Tension
4	DE	-5.77	Compressive
5	BE	-11.5	Compressive
6	AB	11.5	Tension
7	AE	11..5	Tension
8	EF	-17.27	Compressive

3. Determine the forces in all the members of the cantilever truss as shown in figure. (Nov/Dec 2015)

Sol, Start the calculations from joint C. From triangle ACE, we have



$$\tan\theta = \frac{AS}{AC} = \frac{3}{4}$$

Also $\quad EC = \sqrt{3^2 + 4^2} = 5$

$$\therefore \quad \cos\theta = \frac{AC}{CE} = \frac{4}{5} = 0.8$$

$$\sin\theta = \frac{AE}{CE} = \frac{3}{5} = 0.6.$$

Joint C

The direction of forces at the joints C are shown in Fig. 11.21

Resolving the forces vertically, we get

$$F_{CD} \sin\theta = 1000$$

$$\therefore \quad F_{CD} = \frac{1000}{\sin\theta} = \frac{1000}{0.6} = 1666.66\text{N}\,(\text{Compressive})$$

Resolving the forces horizontally, we get

$$F_{CD} = F_{CD} d\cos\theta = 1666.66 \times 0.8 = 1333.33\text{N}\,(\text{Tensile})$$

Now consider the equilibrium of joint B.

Joint B

Resolving vertically, we get

$$F_{BD} = 1000\text{N}\,(\text{Compressive})$$

$$F_{BA} = F_{CD} = 1333.33\,(\text{Tensile})$$

Now consider the joint D.

Joint D

The forces in member CD and BD have already been calculated. They are 1666.66 N and 1000 N respectively as shown in Fig. 11.21(a).

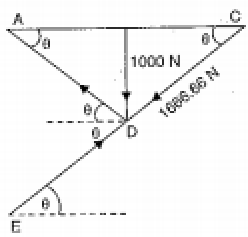


Fig. 11.21 (a)

Let F_{DA} = Force in member DA, and

F_{DE} = Force in member DE

Resolving forces vertically, we get

$$1000 + 1666.66 \sin\theta = F_{AD} \sin\theta + F_{ED} \sin\theta$$

$$\text{or} \quad 1000 + 1666.66 \times 0.6 = F_{AD} \times 0.6 + F_{ED} \times 0.6$$

$$\text{or} \quad F_{AD} + F_{ED} = \frac{1000}{0.6} + 1666.66 = 3333.32 \qquad ....(i)$$

Resolving forces horizontally, we get

$$1666.66 \cos\theta + F_{AD} \cos\theta = F_{gD} \cos\theta$$

Or $1666.66 + F_{AD} = F_{gD}$ or $F_{gD} - F_{AD} = 1666.66$ .....(ii)

Adding equations (i) and (ii), We get

$2F_{ED} = 3333.32 + 1666.66 = 4999.98$

$$\therefore F_{ED} = \frac{4999.98}{2} = 249.99 = 2500\text{N(Compressive)}$$

Substituting this value in equation (i), we get

$$F_{AD} + 2500 = 3333.32$$

$$F_{AD} = 3333.32 - 2500 = 833.32 \text{ N (Tensile)}$$

Now the forces are shown in a tabular from below:

Member	Force in the member	Nature of force
AB	1333.33 N	Tensile
BC	1333.33 N	Tensile
CD	1666.66 N	Compressive
DE	2500 N	Compressive
AD	833.32 N	Tensile
BD	1000 N	Compressive

4. At a point in a strained material the principal stresses are 100 N/mm² (tensile) and 60 N/mm² (compressive). Determine the normal stress, shear stress and resultant stress on a plane inclined at 50° to the axis of major principal stress. Also determine the maximum shear stress at the point. [AMIE, summer 1982]

Sol, Given:

Major principal stress, $\sigma_1 = 100 \text{ N/mm}^2$

Minor principal stress, $\sigma_2 = -60 \text{ N/mm}^2$ (Negative sign due to compressive stress)

Angle of the inclined plane with the axis of major principal stress = 50°

Angle of the inclined plane with the axis of minor principal stress,

$$\Theta = 90 - 50 = 40°.$$

Normal stress (σ_π)

Using equation (3.6),

$$\sigma_\pi = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta$$

$$= \frac{100 + (-60)}{2} + \frac{100 - (-60)}{2}\cos(2 \times 40^\circ)$$

$$= \frac{100 - 60}{2} + \frac{100 + 60}{2}\cos 80^\circ$$

$$= 20 + 80 \times \cos 80^\circ = 20 + 80 \times 1736$$

$$= 20 + 13.89 = 33.89\,\text{N}/\text{mm}^2. \quad \text{Ans.}$$

Shear stress (σ_t)

Using equation (3.7)

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta$$

$$= \frac{100 - (-60)}{2}\sin(2 \times 40^\circ)$$

$$= \frac{100 + 60}{2}\sin 80^\circ = 80 \times 0.9848 = 78.785\,\text{N}/\text{mm}^2.\text{Ans.}$$

Resultant stress (σ_R)

Using equation on (3.8),

$$\sigma_R = \sqrt{\sigma^2{}_\pi + \sigma^2{}_t} = \sqrt{33.89^2 + 78.785^2}$$

$$= \sqrt{114853 + 6207.07} = 85.765\,\text{N}/\text{mm}^2.\text{Ans}$$

Maximum Shear Stress

Using equation (3.9),

$$(\sigma_t)_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{100 - (-60)}{2}$$

$$= \frac{100 + 60}{2} = 80\,\text{N}/\text{mm}^2.\ \text{Ans.}$$

5. The stresses on two mutually perpendicular planes through a point on a body are 30Mpa and 20Mpa both tensile, along with a shear stress of 15Mpa. Find

(i)The position of principal planes and stresses across them.

(ii)The planes of maximum shear stress

(iii)The normal and tangential stress on the plane of maximum shear stress. (AU Nov/Dec 2015)

Given:

$\sigma_1 = 30$ Mpa $\qquad \sigma_2 = 20$ Mpa

$\tau = 15$Mpa

Solution:

(i) Position of principal plans

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2} = 3$$

$$\theta = 35.78^\circ$$

(ii) Plane of Maximum shear stress

$$\tan 2\theta = \frac{\sigma_2 - \sigma_1}{2\tau} = -0.333$$

$$\theta = -9.21^\circ$$

(iii) Normal stress, (σ_n)

$$= \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta$$

$$\sigma_n = 26.58\,\text{N}/\text{mm}^2$$

(iv) Shear stress, (σ_t)

$$= \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta$$

$$\sigma_t = 4.74\,\text{N}/\text{mm}^2$$

6. The principal stresses in the wall of a container are 40 MN/m² and 80 MN/m². Determine the normal, shear, and resultant stresses in magnitude and direction in a plane, the normal of which makes an angle of 30° with the direction of maximum principal stress.

(AU Nov/Dec 2014)

Given:

Principal stress = 40,80 MN/m²

$\sigma_1 = 40$ N/mm²

$\sigma_2 = 80$ N/mm²

$\theta = 30°$

Solution:

$$\text{Normal stress}, \sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta$$

$$= \frac{(40+80)}{2} + \frac{(40-80)}{2}\cos(2\times 30)$$

$$\sigma_n = 60 - 10 = 50\text{N}/\text{mm}^2$$

$$\text{Shear stress}, \sigma_t = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta$$

$$= \left(\frac{40-80}{2}\right)\times \sin(2\times 30)$$

$$\sigma_t = -17.32\text{N}/\text{mm}^2$$

$$\text{Resultant stress}(\sigma_R) = \sqrt{\sigma_n{}^2 + \sigma_t{}^2}$$

$$\sigma_R = 52.91\text{N}/\text{mm}^2$$

7. At a point within a body subjected to two mutually perpendicular directions, the stresses are 80 N/mm² tensile and 40N/mm² tensile. Each of the above the stresses are accompanied by a shear stress of 60 N/mm². Determine the normal stress, shear stress and resultant stress on an oblique plane inclined at an angle 45º with the axis at minor tensile stress

Sol Given:

Major tensile stress, $\sigma_1 = 80$N/mm²

Minor tensile stress, $\sigma_2 = 40$N/mm²

Shear stress, $\tau = 60$N/mm²

Angle of oblique plane, with the axis of minor tensile stress,

$$\theta = 45°.$$

(i) Normal stress (σ_π)

Using equation (3.12),

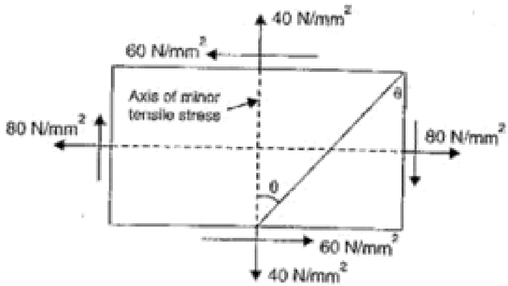
$$\sigma_\pi = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\text{Cos}\,2\theta + \tau \sin 2\theta$$

$$= \frac{80+40}{2} + \frac{80-40}{2}\cos\left(2\times 45°\right) + 60\sin\left(2\times 45°\right)$$

$$= 60 + 20\cos 90° + 60\sin 90°$$

$$= 60 + 20\times 0 + 60\times 1 \qquad \left(\because \cos 90° = 0\right)$$

$$= 60 + 0 + 60 = 120\,\text{N/mm}^2.\text{Ans.}$$



(ii) Shear (or tangential) stress (σ_i)

Using equation (3.13),

$$\sigma_i = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta - \tau\cos 2\theta$$

$$= \frac{80-40}{2}\sin\left(2\times 45°\right) - 60\times\cos\left(2\times 45°\right)$$

$$= 20\times\sin 90° - 60\cos 90°$$

$$= 20\times 1 - 60\times 0$$

$$= 20\text{N/mm}^2.\text{Ans.}$$

(iii) Resultant stress (σ_g)

Using equation,

$$\sigma_g = \sqrt{\sigma_g{}^2 + \sigma_i{}^2}$$

8. At point in a strained material the principal stresses are 100N/mm² (tensile) and 60 N/mm² (compressive). Determine the normal stress, shear stress and resultant stress on a plane included at 50° to the axis major principal stress. Also determine the maximum shear stress at the point.

Sol, Given:

Major principal stress, $\sigma_1 = 100 N/mm^2$

Minor principal stress, $\sigma_2 = -60 N/mm^2$

(Negative sign due to compressive stress)

Angle of the inclined plane with the axis of major principal stress = 50°

Angle of the inclined plane with the axis of minor principal stress,

$$\theta = 90 - 50 = 40°.$$

Normal stress (σ_π)

Using equation (3.6),

$$\sigma_\pi = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta$$

$$= \frac{100 + (-60)}{2} + \frac{100 - (-60)}{2}\cos(2 \times 40^\circ)$$

$$= \frac{100 - 60}{2} + \frac{100 + 60}{2}\cos 80^\circ$$

$$= 20 + 80 \times \cos 80^\circ = 20 + 80 \times 1736$$

$$= 20 + 13.89 = 33.89 \, N / mm^2. \text{Ans}$$

Shear stress (σ_i)

Using equation (3.7),

$$\sigma_1 = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta$$

$$= \frac{100 + 60}{2}\sin 80^\circ = 80 \times 0.9848 = 78.785 N / mm^2. \text{Ans}$$

Resultant Stress (σ_R)

Using equation on (3.8),

$$\sigma_R = \sqrt{\sigma_\pi^{\ 2} + \sigma_i^{\ 2}} = \sqrt{33.89^2 + 78.785^2}$$

$$= \sqrt{1148.53 + 6207.07} = 85.765 \, N / mm^2. \text{Ans.}$$

Maximum Shear stress

Using equation (3.9),

$$(\sigma_t)_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{100-(-60)}{2}$$

$$= \frac{100+60}{2} = 80N/mm^2.\text{Ans.}$$

9. A regulator block of material is subjected to a tensile stress of 110 N/mm² on one plane and a tensile stress of 47/Nmm² on the plane at right angles to the former. Each of the above stresses is accompanied by a shear stress of 63 N/mm² and that associated with the former tensile stress tends to rotate the block anticlockwise. Find:

(i) The direction and magnitude of each of the principal stress and

(ii) Magnitude of the greatest shear stress.

Sol, Given:

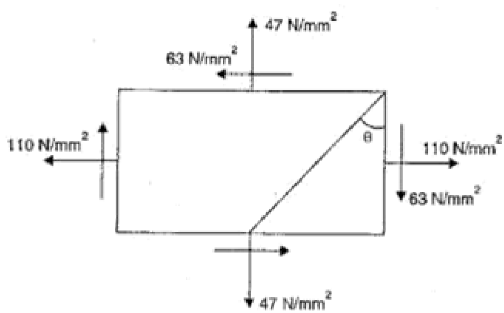
Major tensile stress, $\sigma_1 = 110$ N/mm²

Minor tensile stress, $\sigma_2 = 47$ N/mm²

Shear stress, $\tau = 63$ N/mm²

(i) Major principal stress is given by equation (3.15),

$$\therefore \text{Major principal stress} = \frac{\sigma_1+\sigma_2}{2} + \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}.$$



$$= \frac{110+47}{2} + \sqrt{\left(\frac{110-47}{2}\right)^2 + 63^2}$$

$$= \frac{157}{2} + \sqrt{\left(\frac{63}{2}\right)^2 + (63)^2}$$

$$= 78.5 + \sqrt{31.5^2 + 63^2} = 785 + \sqrt{992.25 + 3969}$$

$$= 78.5 + 70.436 = 148.936 \text{ N / mm}^2. \text{Ans}$$

Minor principal stress is given by equation (3.16),

$$\therefore \text{Minor principal stress, } \quad \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{110+47}{2} - \sqrt{\left(\frac{110-47}{2}\right)^2 + 63^2} = 78.5 - 70.436$$

$$= 8.064 \text{ N / mm}^2. \text{Ans}$$

The directions of principal stresses are given by equation (3.14).

Using equations (3.14),

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2} = \frac{2 \times 63}{110 - 47}$$

$$= \frac{2 \times 63}{63} = 2.0$$

$$\therefore \qquad 2\theta = \tan^{-1} 2.0 = 63^\circ 26^\circ \text{ or } 243^\circ\ 26^\circ$$

$$\therefore \qquad \theta = 31^\circ\ 43 \text{ or } 121^\circ\ 43^\circ, \quad \text{Ans.}$$

(ii) Magnitude of the greatest shear stress

Greatest shear stress is given by equation (3.18).

Using equation (3.18),

$$(\sigma_t)_{max} = \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{(100-47)^2 + 4 \times 63^2}$$

$$= \frac{1}{2}\sqrt{63^2 + 4 \times 63^2} = \frac{1}{2} \times 63 \times \sqrt{5}$$

$$= 70.436 \text{ N / mm}^2. \text{Ans.}$$

10. At a certain point in a material under the intensity of the resultant stress on a vertical plane is 1000 N/cm² inclined at 30° to the normal to that plane and the stress on a horizontal plane has a normal tensile component of intensity 600 N/cm² as shown in Fig. 3.16 (c). Find the magnitude and direction of the resultant stress on the horizontal plane and the principal stresses.

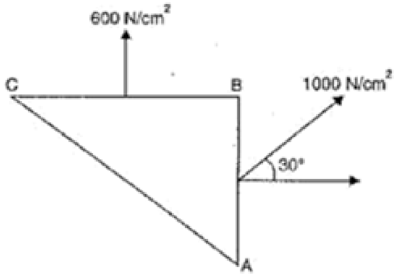


Fig. 3.16 (c)

Sol, Given:

Resultant stress on vertical plane AB = 1000N/cm²

Inclination of the above stress = 30°

Normal stress on horizontal plane BC = 600N/cm²

The resultant stress on plane AB is resolved into normal and tangential component.

The normal component,

$$= 1000 \times \cos 30° = 866 \text{N/cm}^2$$

Tangential component

$$= 1000 \times \sin 30° = 500 \text{N/cm}^2.$$

Hence a shear stress of magnitude 500 N/cm² is acting plane AB. To maintain the wedge in equilibrium, another shear stress of the same magnitude but opposite in direction must act on the plane BC. The free-body diagram of the element ABCD is shown in Fig. 3.16(d), showing normal and shear stress acting on different faces in which:

$$\sigma_1 = 866 \text{ N/cm}^2,$$

$$\sigma_2 = 600 \text{ N/cm}^2,$$

and $\tau = 500 \text{ N/cm}^2$

(i) Magnitude and direction of resultant stress on horizontal plane BC. Normal stress on plane BC, σ_2 = 600 N/cm²

Tangential stress on plane BC, τ = 500 N/cm²

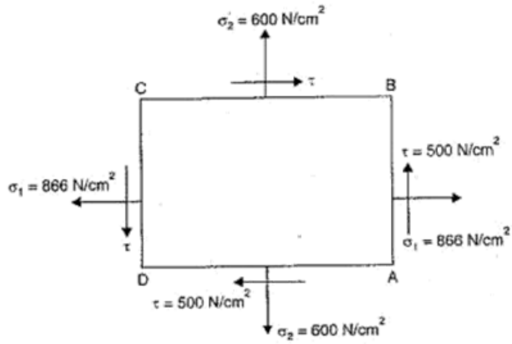


Fig. 3.16 (*d*)

$$\therefore \text{Resultant stress} = \sqrt{{\sigma_2}^2 + \tau^2}$$

$$= \sqrt{600^2 + 500^2} = 781.02 \text{ N/cm}^2. \text{ Ans.}$$

The direction of the resultant stress with the horizontal plane BC is given by,

$$\tan\theta = \frac{\sigma_2}{\tau} = \frac{600}{500} = 1.2$$

$$\theta = \tan^{-1} 1.2 = 50.19°. \qquad \text{Ans.}$$

(iii) Principal stress

The major and minor principal stresses are given by equations (3.15) and (3.16).

$$\therefore \text{Principal Stress} = \frac{\sigma_1 + \sigma_2}{2} = \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{866 + 600}{2} = \sqrt{\left(\frac{800 - 600}{2}\right)^2 + 500^2}$$

$$= 733 \simeq 517.38$$

$$= (733 + 517.38) \text{ and } (733 - 517.38)$$

$$= 1250.38 \text{ and } 215.62 \text{ N/cm}^2.$$

Major principal stress = 1250.38 N/cm² . Ans.

Minor principal stress = 215.62 N/cm². Ans.

11. Find the forces and length of members AB, Ac and BC of the truss as shown in figure by method of joints.

Find the forces in the members AB,AC and BC of the truss shown in fig. 11.5.

Sol, first determine the reactions R_B and R_C. The line of action of load of 20kN acting at A is vertical. This load is at a distance of AB × cos 60° from the point B. Now let us find the distance AB.

The triangle ABC is a right-angled triangle with angle BAC = 90°. Hence AB will be equal to BC × cos60°.

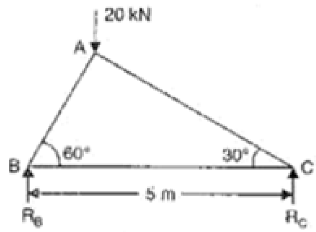


Fig. 11.5

$$\therefore \quad AB = 5 \times \cos 60^\circ = 5 \times \frac{1}{2} = 2.5\text{m}$$

Now the distance of line of action of 20kN from B is

$$AB \times \cos 60^\circ \text{ or } 2.5 \times \frac{1}{2} = 1.25\text{m}.$$

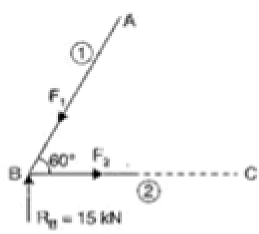


Fig. 11.6

Taking moment about B, we get

$$R_C \times 5 = 20 \times 1.25 = 25$$

$$\therefore \quad R_C = \frac{25}{5} = 5\text{kN}$$

$$\text{and} \quad R_B = \text{Total load} = R_C \times 20 - 5 = 15\text{kN}$$

Now let us consider the equilibrium of the various joints

Joint B

Let F_1 = Force in member AB

F_2 = Force in member BC

Let the force F_1 is acting the joint, B and the force F_2 is acting anyway" from the joint B as shown in Fig. 11.6 (The reaction R_B is acting vertically up. The force F_2 is horizontal. The reaction R_B will be balanced by the vertical component of F_1. The vertical component F_1 must act downwards to balance R_B. Hence F_1 must act downwards the joint b so that its vertical component is downward. Now the horizontal component of F_1 is towards the joint B. Hence force F_2 must act away from the joint to balance the horizontal component of F_1.).

Resolving the force acting on the joint B, vertically

$F_1 \sin 60° = 15$

$$\therefore \quad F_1 = \frac{15}{\sin 60^\circ} = \frac{15}{0.866} = 17.32\,\text{kN}\,(\text{Compressive})$$

As F, is pushing the joint B, hence this force will be compressive, Now resolving the forces horizontally, we get

$$F_2 = F_1 \cos 60^\circ = 17.32 \times \frac{1}{2} = 8.66\,\text{kN}\,(\text{tensile})$$

As F_1 is pulling the joint B, hence this force will be tensile,

Joint C

Let F_1 = Force in member AC

F_2 = Force in the member BC

The force F_2 has already been calculated in magnitude and direction. We have been that force F_2 is tensile and hence it will pull the joint C. Hence it must act away from the joint C as shown in Fig. 11.7.

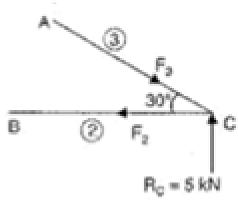


Fig. 11.7

Resolving forces vertically, we get

$$F_3 \sin 30^\circ = 5\text{kN}$$

$$\therefore \qquad F_3 = \frac{5}{\sin 30^\circ} = 10\text{kN}(\text{compressive})$$

As the force F_3 is pushing the joint C, hence it will be compressive.

12. Determine the forces in the truss as shown in figure, which carries a horizontal load of 12kN and a vertical load of 18kN.

Sol, The truss is supported on rollers at b and hence the reaction at B must be normal to the roller base i.e. the reaction at B, in this case, should be vertical.\

At the end A, the truss is hinged and hence the support reactions at the hinged end A will consists of a horizontal reaction H_A and a vertical reaction R_A.

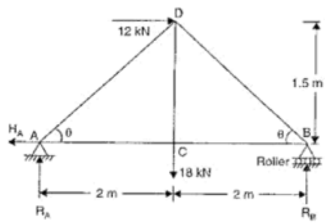


Fig. 11.22

Taking moments of all forces at A, we get

$$R_B \times 4 = 18 \times 2 + 12 \times 1.5 - 36 + 18 = 54$$

$$\therefore \qquad R_B = \frac{54}{4} = 13.5\text{kN}(\uparrow)$$

$$\therefore \qquad R_A = \text{Total vertical load} - R_b = 18 - 13.5 = 4.5\,\text{kN}(\uparrow)$$

And $\quad H_A$ = Sum of all horizontal loads = 12 kN $(\leftarrow)$

Now forces in the members can be calculated.

In triangle BCD,

$$BD = \sqrt{BC^2 + CD^2} = \sqrt{2^2 + 1.5^2} = 2.5\text{m}$$

$$\therefore \qquad \cos\theta = \frac{BC}{BD} = \frac{2}{2.5} = 0.8$$

$$\sin\theta = \frac{CD}{BD} = \frac{1.5}{2.5} = 0.6$$

Let us first consider the equilibrium of joint A.

Joint A

The reaction R_A and H_A are known in magnitude and direction. Let the direction of the forces in the members AC and AB are as shown in Fig. 11.22(a),

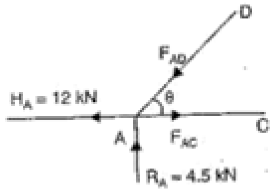


Fig. 11.22 (*a*)

Resolving the forces vertically, we get

$$F_{AD} \sin\theta = R_A$$

or $$F_{AD} = \frac{R_A}{\sin\theta} = \frac{4.5}{0.6} = 7.5\text{kN} \qquad (\text{Compressive})$$

Resolving the forces horizontally, we get

$$F_{AC} = H_A + F_{AD}\cos\theta$$
$$= 12 + 7.5 \times 0.8 = 18\text{kN}(\text{Tensile})$$

Now consider the joint C.

Joint C

At the joint C, force 0 in member CA and vertical load 18kN are known in magnitude and directions. For equilibrium of the joint C,

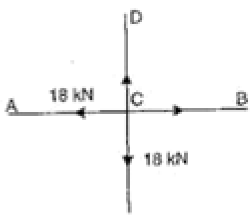


Fig. 11.22 (*b*)

$$F_{BC} = F_{CA} = 18\text{kN}(\text{Tensile})$$
$$F_{CD} = 18\text{kn}(\text{Tensile})$$

Now consider the joint C.

Joint B

At the B,R_B and force F_{BC} are known in magnitude and direction.

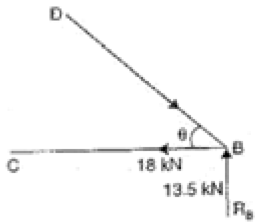


Fig. 11.22 (c)

Let F_{AD} is the force in member BD.

Resolving the forces vertically, we get

$$F_{BD} \times \sin\theta = R_B$$

$$\therefore \quad F_{BD} = \frac{R_B}{\sin\theta} = \frac{13.5}{0.6}$$

$$= 22.5\text{kN}\left(\text{Compressive}\right)$$

Now the forces are shown in a tabular form below:

Member	Force in the member	Nature of force
AC	18kN	Tensile
AD	7.5kN	Compressive
CD	18kN	Tensile
CB	18kN	Tensile
BD	2.5kN	Compressive

13. A truss of span 5m is loaded as shown in figure using method of section.

Sol, Let us first diameter the reaction R_A and R_B

Triangle ABD is a right-angled having angle

ABD = 90°

AD = AB cos 60° = 5 × 0.5 = 2.5m

The distance of line of action the vertical load 10kN

From point A will be AD cos60° or 2.5 × 0.5 = 1.25m

From triangle ACD, we have

$$AC = AD = 2.5m$$

$$BC = 5 - 2.5 = 2.5m$$

In right-angled triangle CEB, we have

$$BE = BC\cos 30^\circ = 2.5 \times \sqrt{\frac{3}{2}}$$

The distance of line of action of vertical load 12kN

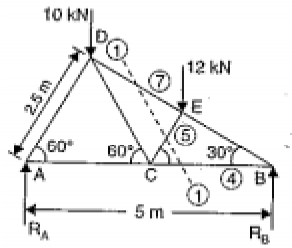


Fig. 11.28

From the point B will be BE cos30° or $BE \times \sqrt{\frac{3}{2}}$

$$= \left(2.5 \times \frac{\sqrt{3}}{2}\right) \times \sqrt{\frac{3}{2}} = 1.875m$$

The distance of the line of action of the load of 12kN from point A will be

$$(5 - 1.875) = 3.125m$$

Now taking the moments about A, we get

$$R_B \times 5 = 10 \times 1.25 + 12 \times 3.125 = 50$$

$$\therefore \quad R_B = \frac{50}{5} = 10kN \text{ and } R_A = (10 + 12) - 10 = 12kN$$

Now draw a section line (1-1), cutting the members 4, 5 and 7 in which forces are to be determined. Consider the equilibrium of the right part of the truss (because it is smaller than the left part).

This part is shown in Fig.11.29.Let F_4, F_5 and F_7 are the forces in members 4,5 and 7. Let their directions are assumed as shown in Fig. 11.29.

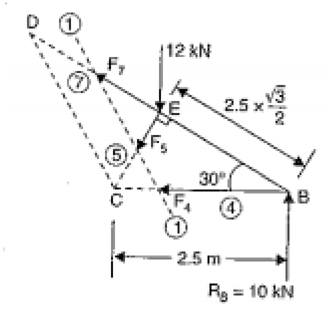


Fig. 11.29

Now taking the moments of all the forces acting on the right part about point E, we get

$$R_B \times BE\cos 30^\circ = F_4 \times \left(BE \times \sin 30^\circ\right)$$

or $$10 \times \left(2.5 \times \frac{\sqrt{3}}{2}\right) \times \frac{\sqrt{3}}{2} = F_4 \times 2.5 \times \frac{\sqrt{3}}{2} \times 0.5$$

Now taking the moment of all the forces about point B acting on the right part, we get

$$12 \times BE\cos 30^\circ + F_5 \times BE = 0$$

or $$12 \times \cos 30^\circ + F_5 = 0$$

$\therefore$ $$F_5 = -12 \times \cos 30^\circ = -10.392\text{kN}$$

-ve sign indicates that F_2 is compressive,

F_5 = 10.392 kN (Compressive). Ans.

Now taking the moment about point C of all the forces acting on the right parts, We get

$$12 \times \left(2.5 - BE\cos 30^\circ\right) = F_7 \times CE + R_B \times BC$$

or $$12 \times \left(25 - 25 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}\right) = F_7 \times 2.5 \times \sin 30^\circ + 10 \times 2.5$$

or $$12 \times (2.5 - 1.875) = F_7 \times 1.25 + 25 \quad \text{or} \quad 7.5 = 1.25F_7 + 25$$

or $$F_7 = 14\text{kN}(\text{compressive}).\ \text{Ans.}$$

14. Determine the forces in all the members of the cantilever truss as shown in figure.

Their assumed directions are shown in Fig. 11.20

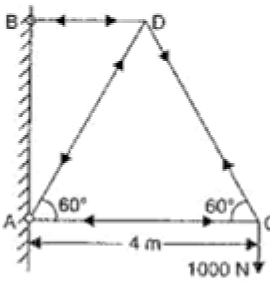


Fig. 11.20

Resolving the force vertically, we get

$$F_{CD} \times \sin 60^\circ = 1000$$

$$\therefore \qquad F_{CD} = \frac{1000}{\sin 60^\circ} = \frac{1000}{0.866} = 1154.7\,\text{N}\,(\text{Tensile})$$

Resolving the forces horizontally, we get

$$\begin{aligned} F_{CA} &= F_{CD} \times \cos 60^\circ \\ &= 1154.7 \times 0.5 \\ &= 577.35\,\text{N}\,(\text{Compressive}) \end{aligned}$$

Now consider the equilibrium of the joint D.

Joint D

[see Fig. 11.20 (a)]

The force F_{CD} = 1154.7 N (tensile) is already calculated.

Let F_{AD} = force in member AD, and

F_{BD} =Force in member BD

Their assumed in directions are shown in Fig. 11.20 (a).

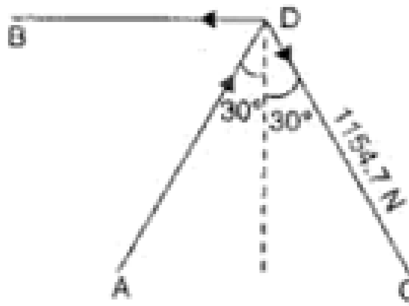


Fig. 11.20 (a)

Resolving in the forces vertically, we get

$$F_{AD} \cos 30^\circ = 1154.7 \cos 30^\circ$$

$$\therefore \quad F_{AD} = \frac{1154.7 \cos 30^\circ}{\cos 30^\circ} = 1154.7\,\text{N}$$

(Compressive)

Resolving then forces horizontally, we get

$$F_{BD} = F_{AD} \sin 30^\circ + F_{DC} \sin 30^\circ$$

$$= 1154.7 \times 0.5 + 1154.7 \times 0.5 = 1154.7\,\text{N}\,(\text{Tensile})$$

Now the forces are shown in a tabular from below:

Member	Force in member	Nature of force
AC	577.35N	Compressive
CD	1154.7N	Tensile
AD	1154.7N	Compressive
BD	1154.7N	Tensile

15. The principal stresses in the wall of a container (Fig. 2.13) are 40 MN/m² and 80 MN/m². Determine the normal, shear and resultant stresses in magnitude and direction in a plane, the normal of which makes an angle of 30° with the direction of maximum principal stress.

Solution:

Given:

$\sigma_x = 80$ MN/m² (tensile);

$\sigma_y = 40$ MN/m² (tensile);

$\theta = 30°$

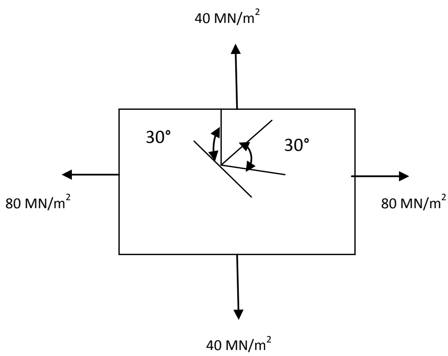


Fig . 2.13

(i) Normal Stress, σ_n:

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta \qquad (\text{Eqn.2.6})$$

$$= \frac{80+40}{2} + \frac{80-40}{2}\cos 60^\circ$$

= 60 + 10 = **70 MN/m²** **(Ans.)**

(ii) Shear stress, τ:

$$\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta = \frac{80-40}{2}\sin 60^\circ \qquad \text{..........}(\text{Eqn.2.7})$$

= 17.32 MN/m² **(Ans.)**

(iii) Resultant stress, σ_r, Ø:

$$\sigma_r = \sqrt{\sigma_n^{\,2} + \tau^2} = \sqrt{70^2 + 17.32^2}$$

i.e, σ_r= 72.11 MN/m² (Ans.)

If Ø si the angle that the resultant makes with the normal to the plane, then

$$\tan\phi = \frac{\tau}{\sigma_n} = \frac{17.32}{70} = 0.2474$$

Ø = 13°54' (Ans).

16. The principal stresses at a point across two perpendicular planes are 75 MN/m²(tensile) and 35 MN/m² (tensile). Find the normal, tangential stresses and the resultant stress and its obliquity on a plane at 20° with the major principal plane.

Solution: Refer to Fig. 2.14,

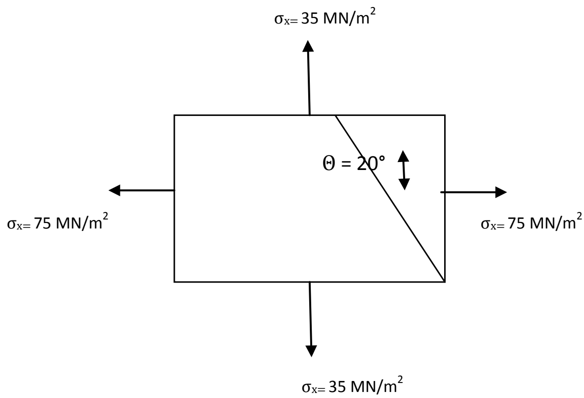


Fig. 2.14

Analytical method:

Given, σ_x = 75 MN/m²(tensile);

σ = 35 MN/m² (tensile), θ = 20°,

Normal stress,

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta = \frac{75+35}{2} + \frac{75-35}{2}\cos\left(2\times 20^\circ\right)$$

$$= 55 + 20\cos 40^\circ = 70.32\,\text{MN}/\text{m}^2\left(\text{tensile}\right)$$

σ_n = 70.32 MN/m² (tensile) (Ans)

Tangential stress,

$$\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta = \frac{75-35}{2}\sin 40^\circ = 20\sin 40^\circ = 12.85\,\text{MN}/\text{m}^2$$

Hence, $\tau = 12.85$ MN/m² **(Ans)**

Resultant stress, $\sigma_r = \sqrt{{\sigma_n}^2 + \tau^2} = \sqrt{(70.32)^2 + (12.85)^2} = 71.48\,\text{MN}/\text{m}^2$

Hence, $\boldsymbol{\sigma_r = 71.48}$**MN/m²** **(Ans.)**

Obliquity ∅: $\tan\phi = \frac{\tau}{\sigma_n} = \frac{12.85}{70.32} = 0.1827$

∴ ∅ = 10°21’ **(Ans.)**