UNIT - 2

SHEAR AND BENDING IN BEAMS

PART - A

1. **Differentiate statically determinate and indeterminate beams.** **(AU April/May 2017)**

   Determinate beams are analysed just by the use of basic equilibrium equations. By this analysis, the unknown reactions are found for the further determination of stresses. Indeterminate beams are not capable of being analysed by mere use of basic equilibrium equations. Along with the basic equilibrium equations, some extra conditions are required to be used like compatibility conditions of deformations etc to get the unknown reactions for drawing bending moment and shear force diagrams.

2. **Define point of contra flexure? In which beam it occurs?** **(AU April/May 2017)**

   It is the point where the B.M is zero after changing its sign from positive to negative or vice versa. It occurs in overhanging beam.

3. **Write the assumptions in the theory of simple bending?** **(AU Nov/Dec 2016)**

   - The material of the beam is homogeneous and isotropic.
   - The beam material is stressed within the elastic limit and thus obey hooke"s law.
   - The transverse section which was plane before bending remains plains after bending also.
   - Each layer of the beam is free to expand or contract independently about the layer, above or below.
   - The value of E is the same in both compression and tension.

4. **Define: Beam**

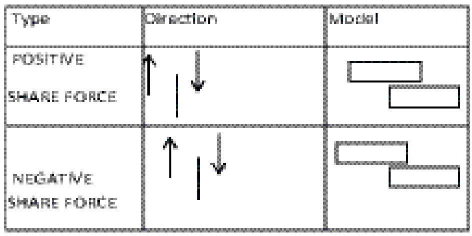
   BEAM is a horizontal structural member subjected to transverse loads.

5. How bending moment, shear force and intensity of loading are related? (AU Nov/Dec 2015 & 16)

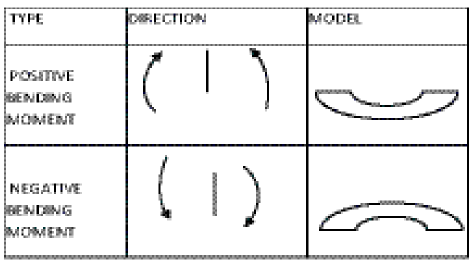
$$\frac{dF}{dx} = -w.$$

$$\frac{dM}{dx} = F$$

Sign Conventions



Bending Moment



6. Define: Moment of resistance (AU Nov/Dec 2015)

Due to pure bending, the layers above the N.A are subjected to compressive stresses, whereas the layers below the N.A are subjected to tensile stresses. Due to these stresses, the forces will be acting on the layers. These forces will have moment about the N.A. The total moment of these forces about the N.A for a section is known as moment of resistance of the section.

7. What is Cantilever beam? (AU Nov/Dec 2014)

A beam whose one end free and the other end is fixed is called cantilever beam.

8. What is simply supported beam?

A beam supported or resting free on the support at its both ends is called simply supported beam.

9. What is mean by over hanging beam?

If one or both of the end portions are extended beyond the support then it is called over hanging beam.

10. What is mean by concentrated loads?

A load which is acting at a point is called point load.

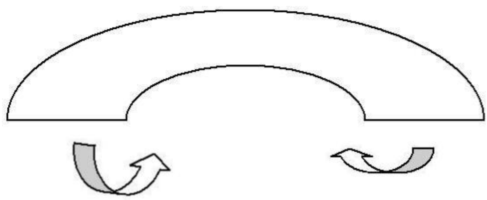
11. What is uniformly distributed load (udl).

If a load which is spread over a beam in such a manner that rate of loading „w" is uniform through out the length then it is called as udl.

12. What is mean by positive or sagging BM?

The BM is said to be positive if moment of the forces on the left side of beam is clockwise and on the right side of the beam is anti-clockwise.

The BM is said to be positive if the BM at that section is such that it tends to bend the beam to a curvature having concavity at the top.



13. Define shear force and bending moment? (AU Nov/Dec 2014)

SF at any cross section is defined as algebraic sum of all the vertical forces acting either side of beam.

BM at any cross section is defined as algebraic sum of all the moments of all the forces which are placed either side from that point.

14. When will bending moment is maximum?

BM will be maximum when shear force change its sign.

15. What is the maximum bending moment in a simply supported beam of span 'L'subjected to UDL of 'w' over entire span?

Max BM = $wL^2/8$

16. In a simply supported beam how will you locate point of maximum bending moment?

The bending moment is max. when SF is zero. Writing SF equation at that point and equating to zero we can find out the distances „x‟ from one end .then find maximum bending moment at that point by taking moment on right or left hand side of beam.

17. What is shear force and bending moment diagram?

It shows the variation of the shear force and bending moment along the length of the beam.

18. What are the types of beams?

- Cantilever beam
- Simply supported beam
- Fixed beam
- Continuous beam
- over hanging beam

19. What are the types of loads?

- Concentrated load or point load
- Uniform distributed load (udl)
- Uniform varying load(uvl)

20. Write the theory of simple bending equation?

The equatiuon of bending is :

$$M/I = \sigma b/y = E/R$$

Where,

M = B.M. or moment of Resistance of the section in Nmm.\

I = MOI of the section about N.A. in mm4

σb = Bending stress at distance y from N.A. in N/mm2

y = distance of fibre from N.A. in mm

E = Young's modulus of elasticity in N/mm2

R = Radius of curvature of N.A. in mm

21. Define: Neutral Axis

The N.A of any transverse section is defined as the line of intersection of the neutral layer with the transverse section.

22. What is mean by transverse loading on beam?

If a load is acting on the beam which perpendicular to the central line of it then it is called transverse loading.

23. Define: Section modulus

Section modulus is defined as the ratio of moment of inertia of a section about the N.A to the distance of the outermost layer from the N.A.

Section modulus (Z)

$Z = I/Y_{max}$

Where, I – M.O.I about N.A

Ymax - Distance of the outermost layer from the N.A

24. What is the formula to find a shear stress at a fiber in a section of a beam?

The shear stress at a fiber in a section of a beam is given by

$$q = \frac{FAy}{Ib}$$

Where , F = shear force acting at a section

A = Area of the section above the fiber

ў = Distance of C G of the Area A from Neutral axis

I = Moment of Inertia of whole section about N A b = Actual width at the fiber

25. What is the shear stress distribution rectangular section?

The shear stress distribution in a rectangular section is parabolic and is given by

$$q = \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right)$$

Where,

d - Depth of the beam

y - Distance of the fiber from NA

26. State the main assumptions while deriving the general formula for shear stresses

- The material is homogeneous, isotropic and elastic .
- The modulus of elasticity in tension and compression are same.
- The shear stressis constant along the beam width
- The presence of shear stress does not affect the distribution of bending stress.

27. Define: Shear stress distribution

The variation of shear stress along the depth of the beam is called shear stress distribution

28. What is the ratio of maximum shear stress to the average shear stress for the rectangular section?

Qmax is 1.5 times the Qavg.

29. What is the ratio of maximum shear stress to the average shear stress in the case of solid circular section?

Qmax is 4/3 times the Qavg.

30. What is the shear stress distribution value of Flange portion of the I-section?

$$q = \frac{F}{2I}\left(\frac{D^2}{4} - y\right)$$

Where, D- depth

y- Distance from neutral axis

31. Where the shear stress is max for Triangular section?

In the case of triangular section, the shear stress is not max at N A. The shear stress is max at a height of h/2

32. What are the different sections in which the shear stress distribution is to be obtained?

Rectangular section

Circular section

I- section

T- section

Miscellaneous section

33. What do you mean by shear stress in beams?

The stress produced in a beam, which is subjected to shear forces is known as shear stress.

34. What is the shear stress distribution Circular section?

$$q = F/3I\ [R^2 - y^2]$$

35. What is the value of maximum of minimum shear stress in a rectangular cross section?

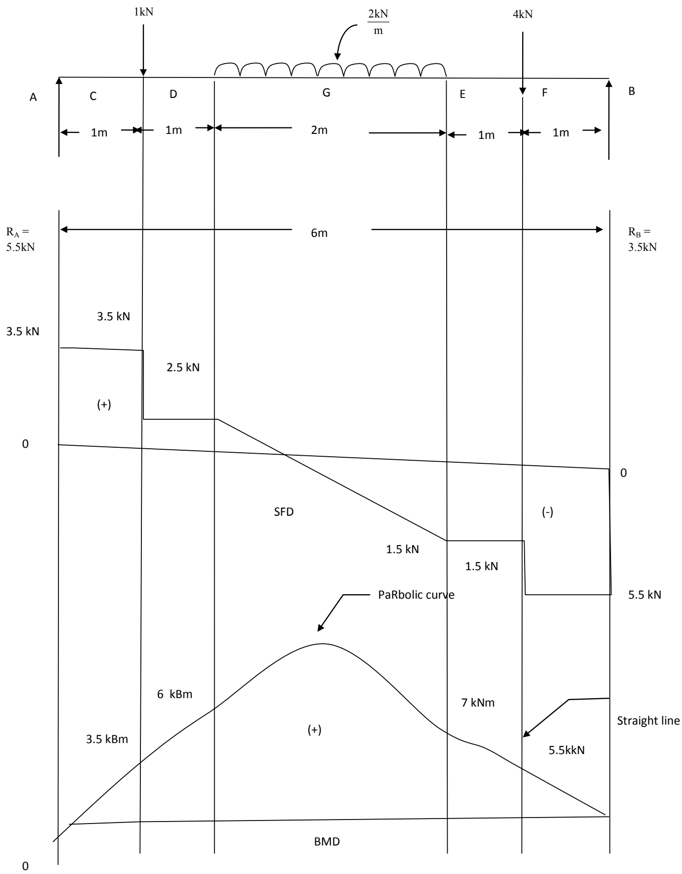
$Q_{max} = 3/2 * F/ (bd)$

36. How will you obtained shear stress distribution for unsymmetrical section?

The shear stress distribution for Unsymmetrical sections is obtained after calculating the position of N A.

PART-B

1. **(AU April/May 2017)**



Reactions (R_A & R_B)

Take moment about – 'A'

$$+(R_B \times 6)-(4\times 5)-(2\times 2)\left(\frac{2}{2}+2\right)-(1\times 1)=0$$

$$+\,6\,R_B - 20 - 12 - 1 = 0$$

$$6\,R_B = 33$$

$$R_B = 55\text{kN}$$

$$R_A + R_B = 1+(2\times 2)+4$$

$$\Sigma\uparrow = \Sigma\downarrow$$

$$R_A = 9-5.5 = 8.5\text{kN}$$

SFD [(+ ↓ (-) ↑]

SF @ 'B'

To 'R' of 'B' = 0

To 'L' of 'B' = - 5.5kN

SF @ 'E'' = -1.5kN

SF @ 'D' = -1.5 + (2 × 2)

SF @ 'C' = + 2.5kN

To 'R' of 'C' = + 2.5kN

To 'L' of 'C' = + 3.5kN

SF @ 'A'

To 'R' of 'A' = +3.5kN

To 'L' of 'A' = 0

SF @ 'F'

To 'R' of 'F' = -5.5kN

To 'L' of 'F' = -5.5 + 4

= -1.5kN

BMD [(-) ↓ (+) ↑]

BM @ 'B' = 0

BM @ 'F' = + (5.5 × 1) = 5.5 kNm

BM @ 'E' = + (5.5 × 2) – (4 × 1)

= + 7kNm

BM @ 'D' = + (5.5 × 4) – (4 × 3) – (2 × 2) ×)2/2)

= 22 – 12 – 4 = +6 kNm.

BM @ 'C' =+(5.5 × 5) – (4 × 4) – (2 × 2) $\left(\frac{2}{2}+1\right)$ = + 3.5kNm

BM @ 'A' = 0.

2. Draw SFD and BMD for a cantilever beam carrying point load (W) At the free end. (AUC NOV / DEC 2016)

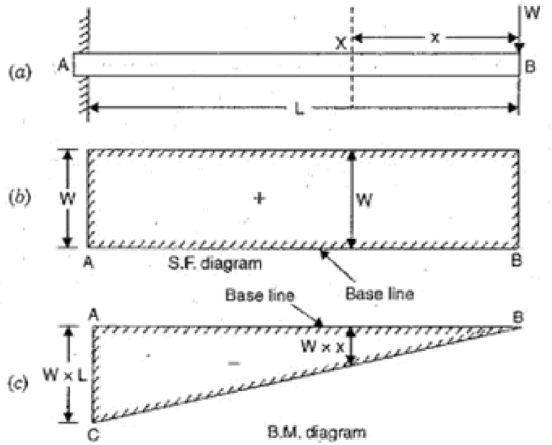


Fig. 6.14

Let F_x = Shear force at X, and

M_x = Bending moment at X.

Take a section X at a distance x from the free end. Consider the right portion of the section.

The shear force at this section is equal to the resultant force acting on the right portion at the section X is W and acting in the downward direction. But a force on the right portion acting downwards is considered positive. Hence shear force at X is positive.

$$F_A = +W$$

The shear force will be constant at all sections of the cantilever between A and Bas there is no other load between A and B. The shear force diagram is shown in Fig. 6.14 (b).

Bending Moment Diagram

The bending moment at the section X is given by

$$M_x = -W \times x \qquad \text{......(i)}$$

(Bending moment will be negative as for right portion of the section, the moment of W at X is clockwise. Also the bending of cantilever will take place in such a manner that convexity will be at the top of the beam).

From equation (i), it is clear that B.M at any section is proportional to the distance of the section from the free end.

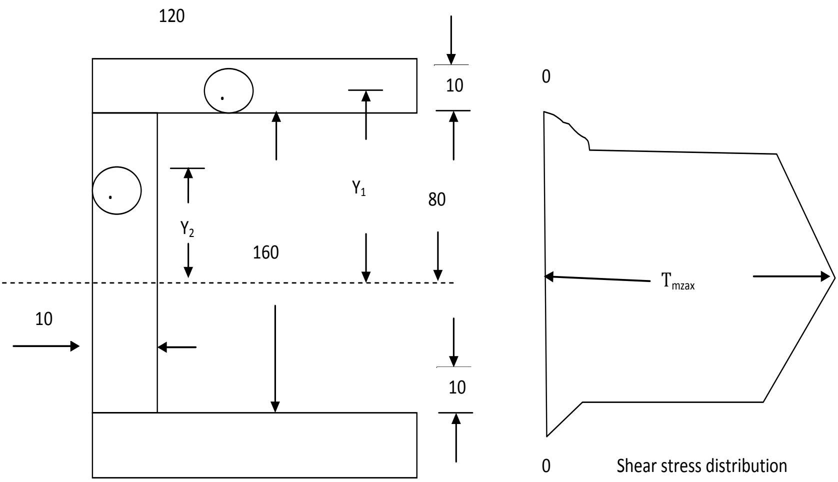
At x = 0 i.e., at B, B.M. = 0

At x = L i.e. at A, B.M, = W × L

Hence B.M follows the straight line law. The B.M diagram is shown in Fig. 6.14 (c). At point A, take AC = W × L in the downward direction. Join point B to C.

3. A channel section made with 120mmx10mm horizontal flanges and 160mmx10mm vertical web is subjected to a vertical shearing force of 120kN. Draw the shear stress distribution diagram across the section (AU Nov/Dec 2015)

ll Drive methods are in mm



$$\tau = \frac{FA\bar{y}}{Ib}$$

Where

F = Shear Force = 120kN = 120×10^3N

$$I = M.I = \frac{BD^3}{12} - \frac{bd^3}{12} \Rightarrow$$

$$B = 120 \;\; D = 180$$

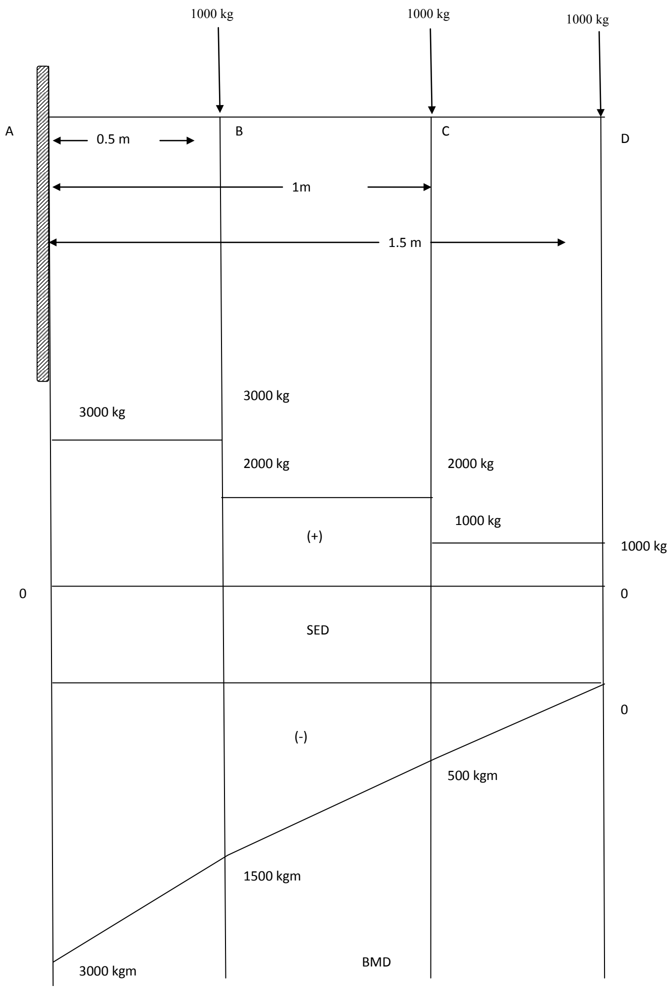
$$b = 110 \;\; d = 160$$

B = Width at N.A = 10mm

$A\bar{y}$ = Area moment about N.A

$$A\bar{y} = A_1y_1 + A_2y_2 = \left[(120\times10\times85)+(10\times80\times40)\right]$$

4. A cantilever beam 1.5m long, fixed at A is carrying point loads of 1000kg at B,C and D each at distances of 0.5m, 1.0m and 1.5m from the fixed end. Calculate the shear force and bending moments at salient points. (AU Nov/Dec 2014)



SFD [Consider From left to Right]

SF @ D¹

TO 'R' of 'D' = 0 [(+) ↓ (-) ↑]

To 'L' of 'D' = + 1000 kg

SF @ 'C'

To 'R' of 'C' = +1000kg

To 'L' of 'C' = + 2000kg

SF @ 'B'

To 'R' of 'B' = +2000 kg

To 'L' of 'B' = + 3000kg

SF @ 'A'

To 'R' of 'A' = +3000 kg

To 'L' of 'A' = 0

BMD [(-)↓ (+)↑]

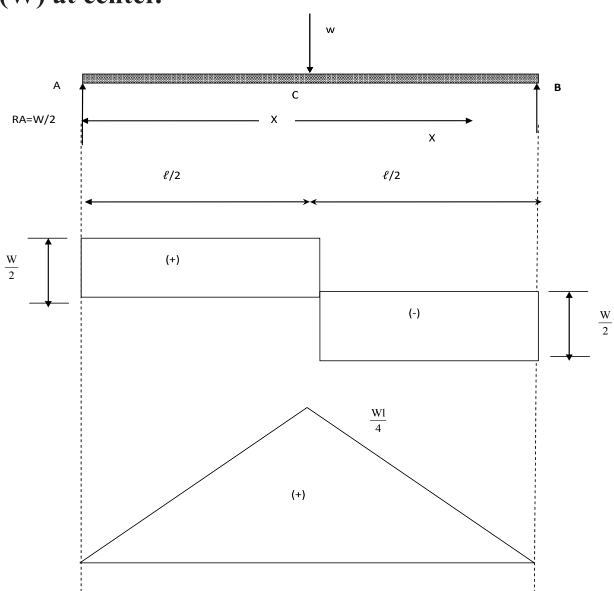
BM @ 'D' = 0

BM @ 'C' = -(1000 × 0.5) kg m

BM @ 'B' = - (1000 × 1) – (1000 × 0.5) kgm

BM @ 'A' = - (1000 × 1.5) – (1000 ×1) – (1000 × 0.5) kgm

5. Draw SFD and BMD for a simply supported beam carrying point load (W) at center.



$$R_A = R_B = \frac{w}{2}(t) \qquad \text{Total load} = \text{Total reaction}$$

At section x; distance x from 'A'

$$F_x = +R_A$$

$$\therefore F_x = +\frac{w}{2}.(\text{cons tan t})$$

$$\therefore F_A = +\frac{w}{2} \qquad \text{and}$$

$$F_c\,(\text{lcH}) = +\frac{w}{2}$$

For portion CB $(x > L/2)$

$$F_x = +RA - W$$

$$= +\frac{w}{2} - w$$

$$\therefore F_x = -\frac{W}{2}\ (\text{cons tan t})$$

B.M.D

For the portion AC,

$$M_x = R_A.x; \qquad = \frac{W}{2}x$$

$$\text{At } x = 0; \qquad M_A = 0$$

$$\text{At } x = \frac{\ell}{2}; \qquad M_c = \frac{W}{2}.\frac{\ell}{2}$$

$$M_c = \frac{W.\ell}{4}$$

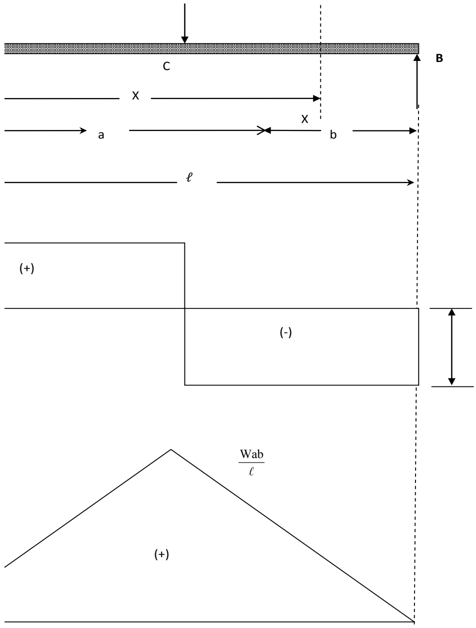
For the portion CB,

$$M_x = R_A.x - W\left(x - \frac{\ell}{2}\right)$$

$$= \frac{W}{2}.x - W\left(x.\frac{1}{2}\right)$$

At $\quad x = \ell.$

6. Draw SFD and BMD for a simply supported beam carrying non central point load (W).



Taking moment about A

$$\left(R_a \times \ell\right) - \left(W \times a\right) = 0$$

$$\therefore R_a = \frac{W_a}{\ell}(\downarrow)$$

We know, Total load = $R_A + R_B$.

$\sum v = 0.$

$$R_A = W - \frac{WR}{\ell}$$

$$\therefore R_A = \frac{W_b}{\ell}(\uparrow)$$

SHARE FORCE DIAGRAM:

S.F. @ A :- $F_A = R_A - w\frac{Wb}{\ell}$

S.F. @ C :- $F_c = R_A - W$

$$F_c = (-)R_A - W$$

$$= (+)\frac{Wb}{\ell} - W$$

$$\therefore F_C = -\frac{Wa}{\ell}$$

S.F @ B :- $F_B = (+) R_A - W - R_B$

$$= (+)\frac{Wb}{\ell} - W - \frac{Wa}{\ell}$$

$$= +\frac{Wb}{\ell} - \left(-\frac{Wa}{\ell} + \frac{Wb}{\ell}\right) - \frac{Wa}{\ell}$$

$$= +\frac{Wb}{\ell} + \frac{Wa}{\ell} - \frac{Wb}{\ell} - \frac{Wa}{\ell}$$

$$F_B = 0$$

BENDING MOMENT DIAGRAM:

B.M. @ B :- $M_B = 0$

B.M. @ C:- $M_C = (R_B \times b) - (W \times 0)$

$$= \frac{-Wa}{\ell} \times b$$

7. Draw SFD and BMD for a simply supported beam carrying udl for the entire span.

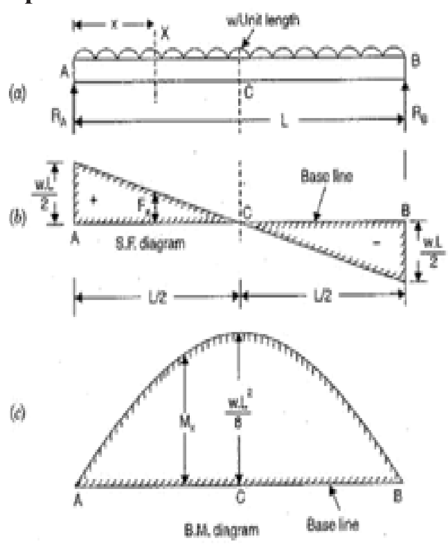


Fig. 6.27

Let R_A = Reaction at A, and

R_B = Reaction at B

$$R_A = R_B = \frac{\omega.L}{2}$$

Consider any section X at a distance x from the left end A. The shear force at the section (i.e. F_x) is given by,

$$F_x = +R_A - \omega.x = +\frac{\omega.L}{2} - \omega.x \qquad ....(i)$$

From equation (i) it is clear that the shear from varies according to straight line law.

The values of shear force at different points are:

At A, x = 0 hence $F_A = +\frac{\omega.L}{2} - \frac{\omega.0}{2} = +\frac{\omega.L}{2}$

At B, x = L hence $F_B = +\frac{\omega.L}{2} - \omega.L = -\frac{\omega.L}{2}$

At C, x = L/2 hence $F_C = +\frac{\omega.L}{2} - \omega.\frac{L}{2} = 0$

The shear force diagram is drawn as shown in Fig. 6.27 (b).

The bending moment at the section X at a distance x from left end A is given by,

$$M_x = +R_A.x - \omega.x.\frac{x}{2}$$

$$= \frac{\omega.L}{2}.x - \frac{\omega.x^2}{2} \qquad \left(\because R_A = \frac{\omega.L}{2}\right)........(ii)$$

The values of B.M at different points are:

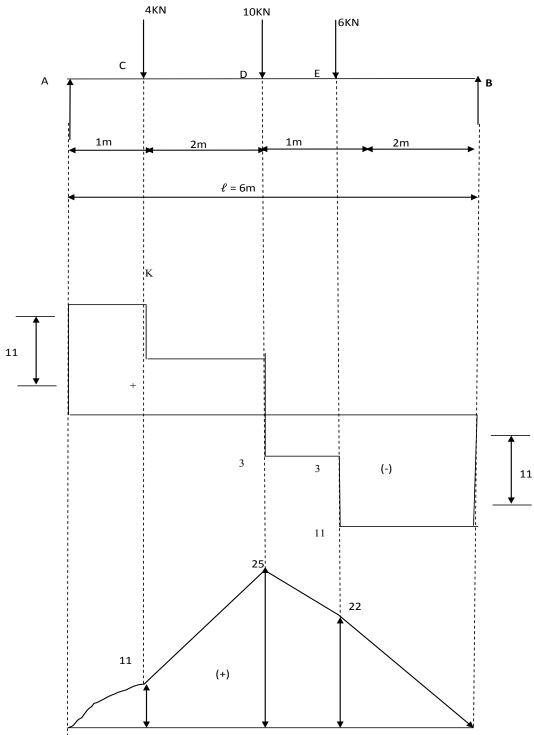
At A, x = 0 hence $M_A = \frac{\omega.L}{2}.0 - \frac{\omega.0}{2} = 0$

At B, x = L hence $M_B = \frac{\omega.L}{2}.L - \frac{\omega}{2}.L^2 = 0$

At C, x =L/2 hence $M_C = \frac{\omega.L}{2}.\frac{L}{2} - \frac{\omega}{2}.\left(\frac{L}{2}\right)^2 = \frac{\omega.L^2}{4} - \frac{\omega.L^2}{8} = +\frac{\omega.L^2}{8}.$

Thus the B.M increases according to parabolic law from zero at A to $+\frac{\omega.L}{8}$ at the middle point of the beam and from this value the B.M decreases to zero at B according to the parabolic law.

8. Draw SFD and BMD for a simply supported beam as shown in figure.



εm = 0

Taking moment about π:

$$6R_B - (8\times4) - (10\times3) - (4\times1) = 0$$

$$6R_B = 66KN. \qquad R_B = 11KN.$$

Ev = 0 $\qquad R_A + R_B$ = Total load

$R_A = 22 - 11$

$R_A = 11KN.$

SHARE FORCE DIAGRAM:

S.F. At A:- $\qquad F_A = R_A = 11KN.$

S.F. at C:- $\qquad F_C = R_A - 4 = 11 - 4$

$F_C = 7KN.$

S.F at D:- $F_D = R_A - 4 - 10 = (11 - 4 - 10)$

$F_D = -3KN.$

S.F. at E:- $F_E = R_A - 4 - 10 = 11 - 4 - 3 - 10) - 8$

$F_E = -11kN.$

S.F at B:- $F_B = R_A - 4 - 10 - 8 + R_B = 11 - 4 - 10 - 8 + 11\backslash$

$F_B = 0$

BENDING MOMENT DIAGRAM:

B.M. at B:- $M_B = 0$

B.M. at E:- $M_E = R_B \times 2 = 11 \times 2 = 22KN.m$

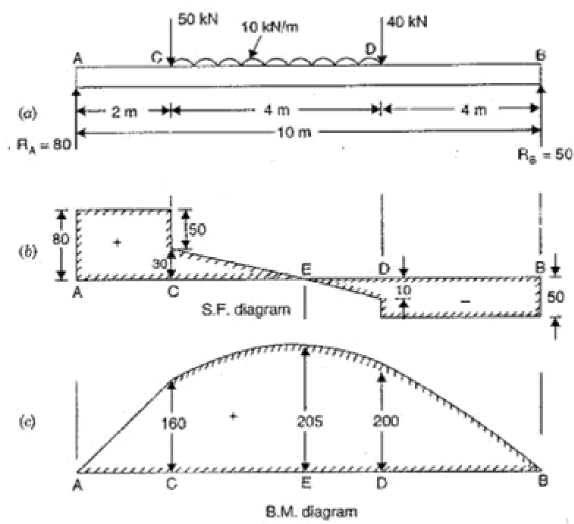
B.M. at D:- $M_D = (R_B \times 3) - (8 \times 1) = 33 - 8$

$M_D = 25KN.m$

B.M at C:- $M_C = (R_B \times \mathbf{5}) - (\mathbf{8} \times \mathbf{3}) - (\mathbf{10} \times \mathbf{2})$

$= 55 - 24 - 20.$

9. Draw SFD and BMD for a simply supported beam carrying loads as shown in figure.



Sol, First calculate the reactions R_A and R_B. Taking moments of all forces about A, we get

$$R_B \times 10 = 50 \times 2 + 10 \times 4 \times \left(2 + \frac{4}{2}\right) + 40(2+4)$$

$$= 100 + 160 + 240 = 500$$

$$\therefore \quad R_B = \frac{500}{10} = 50\text{kN}$$

$$R_A = \text{Total load on beam} - R_B$$

$$= (50 + 10 \times 4 + 40) - 50 = 130 - 50 = 80\text{kN}$$

S.F Diagram

The S.F at A, $F_A = R_A = +80\text{kN}$

The S.F. will remain constant between A and C and equal to + 80kN

The S.F just on R.H.S. of C = R_A -50 = 80 – 50 = 30kN

The S.F. just on L.H.S. of D = R_A -50 -10 × 4 =80 -50 -40 = -10kN

The S.F between C and D varies according to straight line law.

The S.F. just on R.H.S. D = R_A – 50 -10 × 4 – 40 = 80 -50 -40 - 40 = -50kN

The S.F. at B = -50kN

The S.F. remains constant between D and B and equal to – 50kN

The shear force diagram is drawn as shown in Fig. 6.31 (b).

The shear force is zero at point E between C and D.

Let the distance of E from point A is x.

Now shear force at E = R_A- 50 – 10 × **(x – 2)**

= 80 – 50 – 10x + 20 =50 -10x

But shear force at E = 0

$$\therefore \quad 50 - 10x = 0 \text{ or } x = \frac{50}{10} = 5\text{m}$$

B.M Diagram

B.M. at A is zero

B.M. at B is zero

B.M. at C, $M_C = R_A$ **× 2 = 80 ×2 =160kNm**

B.M. at D, $M_D = R_A \times 6 - 50 \times 4 - 10 \times 4 \times 4/2$

$$= 80 \times 6 - 200 - 80 = 480 - 200 - 80 = 200\text{kNm}$$

At E, x = 5m and hence B.M. at E,

$$M_E = F_A \times 5 - 50(5-2) - 10 \times (5-2) \times \left(\frac{5-2}{2}\right)$$

$$= 80 \times 5 - 50 \times 3 - 10 \times 3 \times \frac{3}{2} = 400 - 150 - 45 = 205\text{kNm}$$

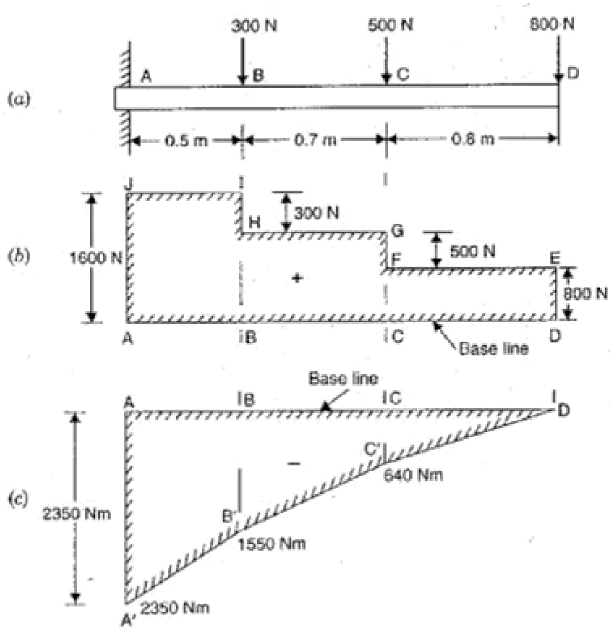
The B.M. between C and D varies according to parabolic law reaching a maximum value at E. The B.M. between A and C also between B and D varies according to linear law. The B.M. diagram is shown in Fig. 6.31(c).

Maximum B.M

The maximum B.M is at E, where S.F becomes zero after changing its sign.

Maximum B.M. = M_E = 205 kNm. Ans

10. Draw SFD and BMD for a cantilever beam carrying loads as shown in figure.



Shear Force Diagram

The shear force at D is + 800 N. This shear force remains constant between D and C. At C, due to point load, the shear force becomes (800 + 500) = 1300 N. Between C and B, the shear force remains 1300 N. At B again, the shear force becomes (1300 + 300) = 1600N. The shear force between B and A remains constant and equal to 1600 N. Hence the force at different points will be as given below:

S.F. at D, $F_D = +800N$

S.F. at C, $F_C = +800 + 500 = +1300N$

S.F. at B, $F_B = +800 + 500 + 300 = 1600N$

S.F. at A, $F_A = +1600N.$

The shear force, diagram is shown in Fig. 6.15 (b) which is drawn as:

Draw a horizontal line AD as base line. On the base line mark the points B and C below the point loads. This the ordinate DE = 800N in the upward direction. Draw a line EF parallel to AD. The point F is vertically above C. Take vertical line FG = 500N. Through G, draw a horizontal line GH in which point H is vertically above B. Draw vertical line HI = 300N. From I draw a horizontal line IJ. The point J is vertically above A. This completes the shear from diagram.

Bending Moment Diagram

The bending moment at D is zero:

i) The bending moment at any section between C and D at a distance x and D is given by,

$M_x = -800 \times x$ Which follows a straight line law.

At C, the value of x = 0.8m.

B.M at C, $M_c = -800 \times 0.8 = -640Nm.$

ii) The B.M at any section between B and C at a distance x from D is given by (At C, x = 0.8 and at B, x = 0.8 + 0.7 = 1.5m. Hence here x varies from 0.8 to 1.5)

$M_x = -800x - 500(x - 0.8)$ ....(i)

Bending moment between B and C also varies by a straight line law.

B.M at B is obtained by substituting x = 1.5m in equation (i),

$$M_B = - 800 \times 1.5 - 500\,(1.5 - 0.8)$$

$$= - 1200 - 350 = - 1550 \text{ Nm.}$$

iii) The B.M at any section between A and B at a distance x from D is given by (At B, x = 1.5 and at A, x = 2.0m. Hence here x varies from 1.5m to 2.0m)

$$M_x = - 800x - 500(x - 0.8) - 300(x - 1.5) \qquad \ldots\ldots\ldots\text{(ii)}$$

Bending moment between A and B varies by a straight line law.

B.M. at A is obtained by substituting x = 2.0m in equation (ii),

$$M_A = - 800 \times 2 - 500(2 - 0.8) - 300(2 - 1.5)$$

$$= - 800 \times 2 - 500 \times 1.2 - 300 \times 0.5$$

$$= - 1600 - 600 - 150 = - 2350 \text{ Nm.}$$

Hence the bending moment at different points will be as given below:

$M_D = 0$

$M_C = - 640$ Nm

$M_B = - 1550$ Nm

And $M_A = - 2350$ Nm.

11. Draw SFD and BMD for a cantilever beam carrying loads as shown in figure.

A cantilever of length 2m carries a uniformly distributed load of 1.5 kN/m run over the whole length and a point load of 2 kN at a distance of 0.5m from the free end. Draw the S.F and B.M diagrams for the cantilever.

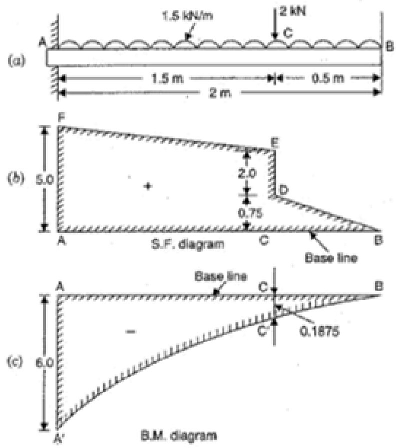
Sol, Given:

Length, $L = 2$m

U.D.L, $\boldsymbol{\omega} = 1.5$ kN/m run

Point Load, $W = 2$kN

Distance of point load from free end = 0.5m\Refer of Fig. 6.19



Shear Force Diagram

i) Consider any section between C and B at a distance x from the end. The shear force at the section is given by,

$F_x = +\,\omega.\,X$ (+ ve sign is due to downward Force on right portion)

$= 1.5 \times x$ ……. (i)

In equation (i), x varies from 0 to 0.5. The equation (i) shows that shear force varies by a straight line law between B and C.

At B, x = 0 hence $F_B = 1.5 \times 0 = 0$

At C, X = 0.5 hence $F_c = 1.5 \times 0.5 = 0.75\text{kN}$

(ii) Now consider any section between A and C at a distance x from free end B. The shear force at the section is given by

$F_x = +\,\omega.x + 2\text{kn}$ (+ve sign is due to downward

On right portion of the section)

$= 1.5x + 2$ ……..(ii)

In equation (ii), x varies from 0.5 to 2.0. The equation (ii) also shows that shear force varies by a straight line law between A and C.

At C, x = 0.5 hence $F_C = 1.5 \times 0.5 + 2 = 2.75\text{kN}$

At A, x = 2.0 hence $F_A = 1.5 \times 2.0 + 2 = 5.0\text{kN}$

Now draw the shear force diagram as shown in Fig. 6.19 (b) in which CD = 0.75kN, DE = 2.0kN or CE = 2.75kN and AF = 5.0kN. The point B is joined to point D by a straight line whereas the point E is also joined to point E is also joined to point F by a straight line.

Bending Moment Diagram

i) The bending moment at any section between C and B at a distance a from the free end B is given by

$$\begin{aligned} M_x &= -(\omega.x).\frac{x}{2} \\ &= -(1.5\times x).\frac{x}{2} \qquad (\therefore \omega = 1.5kN/m) \\ &= -0.7x^2 \qquad .....(iiii) \end{aligned}$$

(The bending moment will be negative as for the right portion of the moment at the section is clockwise).

In equation (iii), x varies from 0 to 0.5 Equation (iii) shows that B.M varies between C and B by a parabolic law.

At, B, x = 0 hence M_B =-0.75 × 0 =0

At C, x = 0.5 hence M_C = -0.75 × 0.5^2 = -0.1875kNm.

ii) The bending moment at any section between A and C at a distance x from the free end B is given by

$$M_x = -(\omega.x).\frac{x}{2} - 2(x-0.5) = -(1.5\times x).\frac{x}{2} - 2(x-0.5)$$

$$(\therefore \qquad \omega = 1.5kN/m)$$

$$= -0.75x^2 - 2(x-0.5) \qquad ....(iv)$$

In equation (iv), x varies from 0.5 to 0.2.Equation (iv) shows that B.M varies by a parabolic law between A and C.

At C, x = 0.5 hence M_C = 0.75 × 0.5^2 – 2(0.5 – 0.5) =-0.1875 knm

At A, x = 2.0 hence M_A = 0.75 × 2^2 -2(2.0 – 0.5) kNm = -3.0 – 3.0 = -6.0kNm^2

Now the bending moment diagram is drawn as shown in Fig.6.19 (c). In this diagram line CC^1 = 0.1875 and AA^1 = 6.0. The points A^1, C^1 and B are on parabolic curves.

12. What are the assumptions made in theory simple bending and derive the bending equation.

ASSUMPTIONS MADE IN THEORY OF SIMPLE BENDING:

(i) The material of the beam is homogeneous and isotropic.

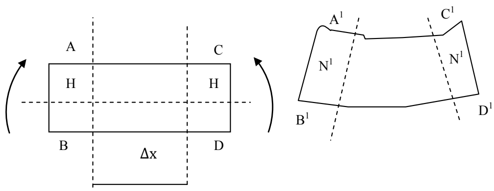
(ii) The value of young's modulus of elasticity is the same in tension and compression.

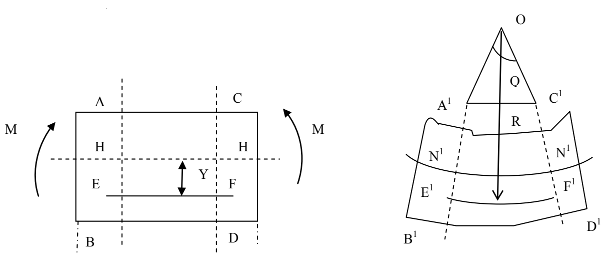
(iii) The transverse sections, which were plane before bending, remain plane after bending also.

(iv) The beam is initially straight and all longitudinal filaments bend into circular arcs with a common centre of curvature.

(v) The radius of curvature is large compared with the dimensions of the cross-section.

(vi) Each larger of the beam is free to expand(or) contract, independently of the larger, above or below it.





Let us consider a beam acted upon by two equal moments (M) at the ends as shown in Fig.

The B.M induced by the moments tends to bend in a concave manner.

So, the top surfaces (AC) are subjected to compressive stresses and contract while the bottom

Surfaces (BD) are subjected to tensile stresses and extend.

However, there is a layer M.n in between the top and bottom, which will remain its original length even after bending (N: N)

`R' is the radius of curvature of the portion of the neutral layer in bend beam.

The following steps are involved in the development of bending theory.

(i) Determination of strain in layer E^1 F^1

(ii) Evaluation of stress in this layer by means of young's modulus.

(iii) Determination of load carried by the strip of cross-section at a distance "y" from neutral plane.

(iv) Calculating the moment produced by this load about neutral plane, and summation of the total moment of all such strip loads.

(i) Determination of strain in layer E^1 F^1:

Strain in layer EF =

$$= \frac{\text{change in length}}{\text{Original length}}$$

$$= \frac{E'F' - EF}{EF}$$

But, we know, $EF = N - N$

$$N - N = N' - N'$$

Expressing the above equation in terms of R and θ,

The arc length E^1 F^1 = $(R + y)\,\boldsymbol{\theta}$

The arc length $N^1 - N^1 = R\boldsymbol{\theta}$

$$\therefore \text{strain in layer } E^1F^1 = \frac{E^1F^1 - N^1N^1}{N^1N^1}$$

$$= \frac{(R+y)\theta - R\theta}{R\theta}$$

$$= \frac{R\theta + y\theta - R\theta}{R\theta}$$

$$= \frac{y\theta}{R\theta}$$

$$\text{Strain in layer } E^1F^1 = \frac{y}{R}$$

(ii) Stress (**σ**) in layer E^1 F^1:

We know young's modulus,

$$E = \frac{\text{stress}(\sigma)}{\text{strain}(e)}$$

$$\text{stress}(\sigma) = E \times e$$

$$\sigma = E \times \frac{y}{R} \qquad \left[\because e = \frac{y}{R}\right]$$

$$\frac{\sigma}{E} = \frac{y}{R}$$

Let a **– Area of c/s of strip at E^1 F^1**

We know that,

$$\text{Stress} = \frac{\text{Load}}{\text{Area}}$$

$$\text{Load} = \text{stress} \times \text{Area}$$

$$= \frac{E(y)}{R} \times A$$

$$= \frac{E}{R} \times Ay$$

$$\therefore \boxed{\text{Load}(y) = \frac{E}{R}.Ay}$$

(iv) Moment of Layer at E^1 F^1:

Moment (M) of the load on this strip about neutral layer,

M =load × distance

$$= \left(\frac{E}{R} \times Ay\right) \times y$$

$$= \frac{E}{y} Ay^2$$

The total moment of the beam section trade up of all such moments

$$= \sum \frac{E}{R} Ay^2$$

$$= \frac{E}{R} \sum Ay^2$$

$$\therefore \boxed{\frac{E}{R} = \frac{M^2}{I}} \quad ......(2)$$

Combining these two equations, we can get bending equation,

$$\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}}$$

Where,

M = Moment of resistance ……… N.mm

I = Moment of inertia ……. Mm^4

σ = Bending stress…….. N/mm^2

Y = Centroidal distance …..mm

E = young's modulus ……… N/mm^2.

13. Draw the S.F. and B.M. diagram for the overhanging beam carrying uniformly distributed load of 2kN/m over the entire length and a point load of 2kN as shown in Fig. 6.36 locate the point of contraflexure.

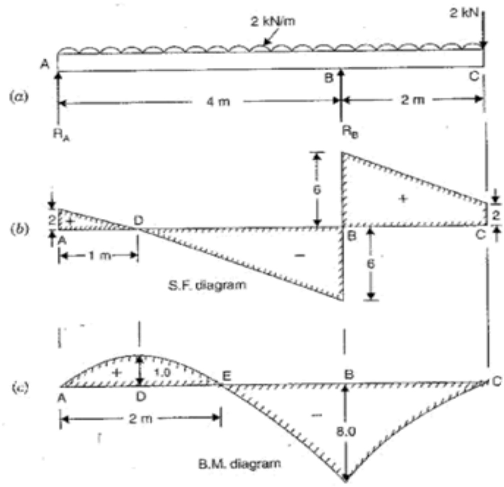
Sol. First calculate the reactions R_A and R_B.

Taking moments of all forces about A, we get

$$R_B \times 4 = 2 \times 6 \times 3 + 2 \times 6 = 36 + 12 = 48$$

$$R_B = \frac{48}{4} = 12kN$$

And $\quad R_A$ = Total load – R_B = $(2 \times 6 + 2) - 12 = 2kN$



S.F. Diagram

S.F. at A = + R_A = + 2kN

i) The S.F. at any section between A and B at a distance x from A is given by,

$F_x = + R_A - 2 \times x$

$= 2 - 2x$ ....(i)

At A, x = 0 hence $F_A = 2 - 2 \times 0 = 2$kN

At B, x = 4 hence $F_A = 2 - 2 \times 4 = -6$kN

The S.F. between A and B varies according to straight line law. At A, S.F. is positive and at B, S.F. is negative. Hence between A and B, S.F. is zero. The point of zero S.F. is obtained by substituting $F_x = 0$ in equation (i).

$$\therefore \quad 0 = 2 - 2x \quad \text{or} \quad x = \frac{2}{2} = 1\text{m}$$

The S.F. is zero at point D. Hence distance of D from A is 1m.

ii) The S.F. at any section between B and C at a distance x from A is given by,

$F_x = + R_A - 2 \times 4 + B_B - 2(x-4)$

$= 2 - 8 + 12 - 2(x-4) = 6 - 2(x-4)$ ....(II)

At B, x = 4 hence $F_B = 6 - 2(4 - 4) = +6kN$

At C, x =6 hence $F_C = 6 - 2(6 - 4) = 6 - 4 = 2Kn$

The S.F. diagram is drawn as shown in Fig. 6.36(b).

B.M. Diagram

B.M. at A is zero

(i) B.M. at any section between A and B at a distance x from A is given by,

$M_A = R_A \times x - 2 \times x \times x/2 = 2x - x^2$ ......(iii)

The above equation shows that the B.M. between A and B varies according to parabolic law.

At A, x = 0 hence $M_A = 0$

At B, x = 4 hence $M_B = 2 \times 4 - 4^2 = -8kNm$

Max. B.M. is at D where S.F. is zero after changing sign

At D, x = 1 hence $M_D = 2 \times 1 - 1^2 = 1kNm$

The B.M. at C is zero. The B.M also varies between B and C according to parabolic law. Now the B.M diagram is drawn as shown in Fig. 6.36 (c).

Point of Contraflexure

The point is at E between A and B, where B.M. is zero after changing its sign. The distance of E from A is obtained by putting $M_x = 0$ in equation (iii).

$$0 = 2x - x^2 = x(2 - x)$$

$$2 - x = 0$$

And **x = 2m. Ans.**

14. A flitched beam is made up of a wooden joist 100mm wide and 200mm deep strengthened by two steel plates 10mm thick and 20cm deep as shown in figure. If the maximum stress in the wooden joist is 7N/mm^2 Find the corresponding maximum stress attained in steel, find also the moment of resistance of the composite section. Take young's modulus for steel = 2x10^5 N/mm^2 and for wood = 1x10^4 N/mm^2.

Let suffix 1 represent steel and suffix 2 represent wooden joist.

Width of wooden joist, b_2 =10cm

Depth of wooden joist, d_2 = 20cm

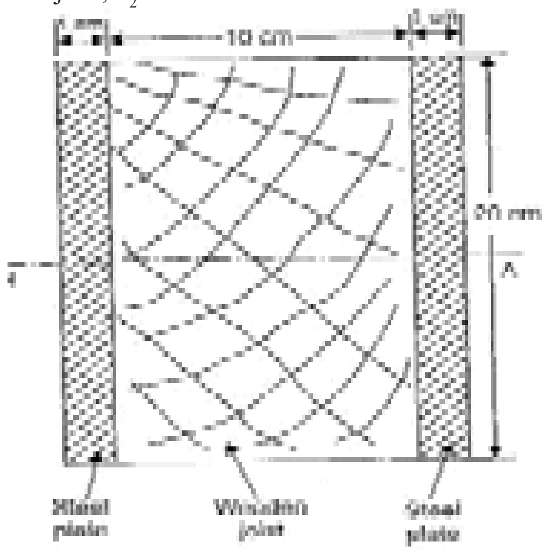


Fig. 7.20

Width of one steel plate, b_1 = 1cm

Depth of one steel plate, d_1 = 20cm

Number of steel plates = 2

Max. stress in wood, σ_2 = 7N/mm^2

E for steel, $E_1 = 2 \times 10^5$ N/mm^2

E for steel, $E_2 = 1 \times 10^4$ N/mm^2

Now M.O.L. of wooden joist about N.A

$$I_2 = \frac{b_2 d_2{}^2}{12} = \frac{10 \times 20^2}{12}$$
$$= 6666.66\,\text{cm}^4$$
$$= 6666.66 \times 10^4\,\text{mm}^4$$

M.O.I of two steel plates about N.A.,

$$I_1 = \frac{2 \times b_1 d_1{}^2}{12} = \frac{2 \times 1 \times 20^2}{12}$$
$$= 1333.33\,\text{cm}^4 = 1333.33 \times 10^4\,\text{mm}^4.$$

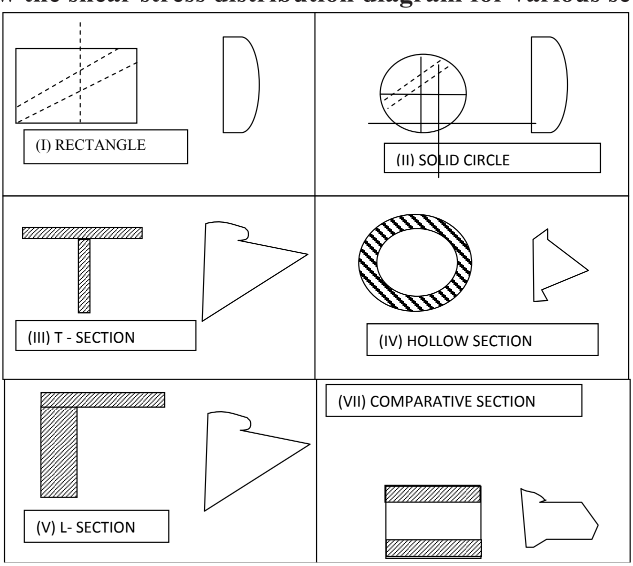
Now modular ratio between steel and wood is given by,

$$m = \frac{E_1}{E_2} = \frac{2 \times 10^5}{1 \times 10^4} = 20$$

The equivalent moment of inertia (I) is given by equation (7.13).

$$\therefore \quad M = \frac{\sigma_2}{y} \times I$$
$$= \frac{7 \times 10^4 \times 33333.2}{10 \times 10} \qquad (\because \quad y = 10\text{cm} = 10 \times 10\text{mm})$$
$$= 233332.4 \times 10^2\,\text{Nmm} = 23333.24\,\text{nm}. \quad \text{Ans.}$$

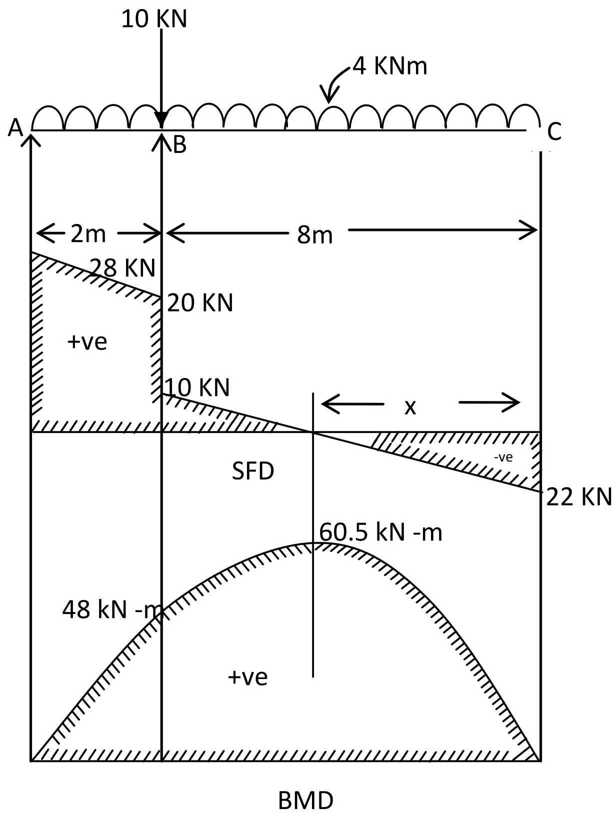
15. Draw the shear stress distribution diagram for various sections



16. A simply supported beam of span 10m carries a concentrated load of 10kN at 2m from the left support and a uniformly distributed load of 4 klN/m over the entire length. Sketch the shear force and bending moment diagram for the beam.

[Madras univ – EEE – Apr -95] (AU Nov/Dec 2016)

Given: As shown in Fig. 2.35 (a)



To draw: SFD and BMD

Solution: Taking moment about A.

$$R_c \times 10 = 4 \times 10 \times \frac{10}{2} + 10 \times 2$$

$$R_C = 22kN$$

$$R_A + R_C = 4 \times 10 + 10 = 50$$

$$R_A = 50 - R_C$$

$$= 50 - 22 = 28kN$$

SF solution

SF at C = $-R_C$ = - 22kN

SF at B (without point load)

= - 22 + 4 × 8 = 10kN

SF at B (with point load)

= 10 + 10 = 20kN

SF at A = R_A = 28kN

Join all values as shown in Fig. 2.35 (b).

BM calculation

$$\text{BM at C} = 0$$

$$\text{BM at B} = R_C \times 8 - 4 \times 8 \times \frac{8}{2} = 48kN - m$$

$$\text{BM at A} = 0$$

The SF changes its sign at a distance of 'x' m from c.

SF equation at that point is

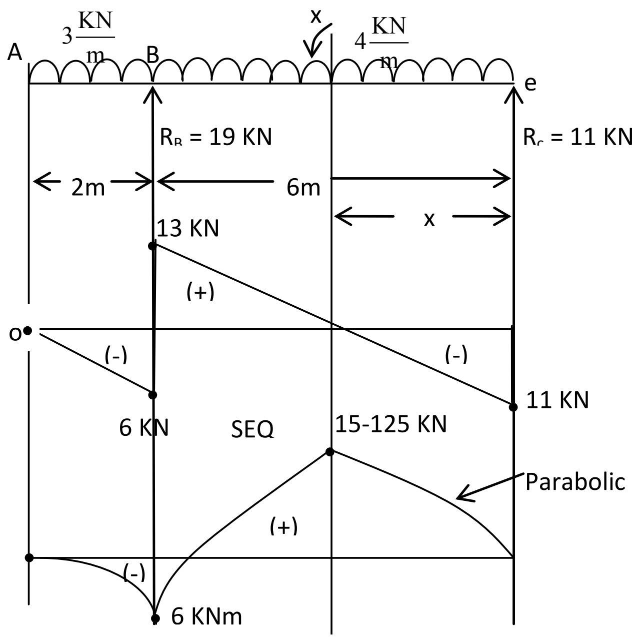
SF_x = - 22 + 4 x =0

X = 5.5m from C

The maximum BM,

$$M_{max} = 22 \times 5.5 - 4 \times 5.5 \times \frac{5.5}{2} = 60.5kN - m$$

17. An overhanging beam ABC of length 8m is simply supported at B & C over a span of 6m and the portion AB overhangs by 2m. Draw SFD and BMD if it is subjected to udl of 3kN/m over the portion AB and 4kN/m over the portion BC.



Reaction (R_B& R_c)

Take moment about – 'c'[(-) ↓ (+) ↑]

$4\ (R_B \times 6) - (3 \times 2)(1 + 6) - (4 \times 6) \times (6/2) = 0$

$6R_B - 4Z - 72 = 0$

$R_B = 19KN$

$\boxed{\delta V = 0}\ \Sigma \uparrow = \Sigma \downarrow$

$R_B + R_c = (3 \times 2) + (4 \times 6)$

$R_c = 11KN$

SFD [(+)↓ (-)↑]

SF @ 'c'

To 'R' of 'c' = 0

To 'L' of 'c' = -11 KN

SF @ ;B'

To 'R' of 'B' = - 11 + (4 × 6) = 13KN

To 'L' of 'B' = + 13 – 19 = - 6KN

SF @ 'A' = 0

BMD [(-) ↓ (+) ↑]

BM @ 'c' = 0

BM @ 'B' = + (11 × 6) – (4 × 6) (3)

= -6KNm

BM @ A = 0

Point of contraflexure AU 2015

Maximum B.M

Consider a section xx at a distance 'x' from the right support 'C'

SF @ xx = - 11 + (4 × x) = 0

4x = 11

x = 2.75 m

B.M @ xx = + (11 × 2.75) – (4 × 2.75)$\left(\frac{2.75}{2}\right)$ = 15.125 KNm